HANDBOOK OF APPLIED HYDRAULICS

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HANDBOOK

of

APPLIED HYDRAULICS

CALVIN VICTOR DAVIS, Editor-in-Chief

Vice President, Harza Engineering Company, Chicago; formerly Chief Engineer, Ambursen Engineering Corporation, New York

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HANDBOOK OF APPLIED HYDRAULICS

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CONTRIBUTORS

James S. Bowman, B.E.; A.S.C.E., A.S.M.E.; Chief Water Control Planning Engineer, Tennessee Valley Authority, Knoxville, Tennessee.

Joseph R. Bowman, B.S. in C.E.; A.S.C.E.; Civil and Hydraulic Engineer, Harza Engineering Company, Chicago.

Edgar H. Burroughs, M.E., C.E.; A.S.C.E.; President, Ambursen Engineering Corporation, New York.

Thomas R. Camp, S.M.; A.S.C.E., Boston S.C.E., A.W.W.A., N.E.W.W.A., N.E.-S.W.A., Fellow A.P.H.A.; Consulting Engineer, Camp, Dresser and McKee, Boston.

Calvin V. Davis, B.S.; A.S.C.E.; Vice President, Harza Engineering Company, Chicago; formerly Chief Engineer, Ambursen Engineering Corporation, New York; formerly Project Engineer, Fontana Dam, Tennessee Valley Authority.

Samuel A. Greeley, A.B., B.S.; Honorary Member, 1951, A.S.C.E., Honorary A.P.-H.A., A.P.W.A., Western Society of Engineers; Consulting Engineer, Greeley and Hansen, Chicago.

George H. Hickox, B.E., M.S., Ph.D.; A.S.C.E., A.S.M.E., A.S.E.E., American Geophysical Union, International Association for Hydraulic Research; Associate Director, Engineering Experiment Station, University of Tennessee, Knoxville, Tenn.

Julian Hinds, C.E.; A.S.C.E., A.W.W.A., A.C.I.; General Manager and Chief Engineer, United Water Conservation District, Santa Paula, Calif.

Ivan E. Houk, B.E.; A.S.C.E., A.C.I., A.A.A.S., Colo. Soc. Engrs., American Geophysical Union, American Meterological Society; Consulting Engineer, Denver, Colo.

Phillip A. Kinzie, C.H.K.; F.R.C., A.S.M.E.; Consulting Engineer, Rancho Santa Fe, Calif.; formerly Senior Engineer, Gates, Valves and Heavy Machinery, Bureau of Reclamation, Denver Office, Colorado.

Phillip Z. Kirpich, B.S. in C.E.; A.S.C.E., New York State Society of Professional Engineers, American Geophysical Union, International Society for Hydraulic Research; Lecturer on Hydraulic Structures, Columbia University; Hydraulic Engineer, Knappen-Tippetts-Abbett-McCarthy, New York.

Theodore T. Knappan, Late Consulting Engineer.

Emory W. Lane, C.E.; A.S.C.E.; Consulting Hydraulic Engineer, U.S. Bureau of Reclamation, Denver, Colo.; formerly Professor of Hydraulic Engineering, State University of Iowa, Iowa City.

John Lowe, III, B.S., M.S.; A.S.C.E.; Secretary, National Council Soil Mechanics and Foundation Engineering; Chief Soils, Foundations and Paving Engineer, Knappen-Tippetts-Abbett-McCarthy, Engineers, New York.

Lewis F. Moody, M.S.; Hon. Mem. A.S.M.E., Fellow A.A.A.S., Mem. Franklin Inst., A.S.E. Educ.; Professor of Hydraulic Engineering, Emeritus, Princeton Univ.; formerly Consulting Engineer, I. P. Morris Div., Baldwin Locomotive Works, and Worthington Pump & Machinery Corp.

George R. Rich, B.S. in C.E., D. Eng.; A.S.C.E., A.S.M.E., Boston S.C.E., American Mathematical Society; Consulting Engineer, Chas. T. Main, Inc., Boston; formerly Chief Design Engineer, Tennessee Valley Authority, and Lecturer to Graduate Students in Hydraulic Engineering, Columbia University.

Kenneth E. Sorensen, M.S. in C.E.; A.S.C.E.; Structural Engineer, Harza Engineering Company, Chicago.

William E. Stanley, B.S. in C.E., C.E.; A.S.C.E., A.W.W.A., B.S.C.E., N.E.S. & W.A., N.E.W.W.A.; Professor of Sanitary Engineering, Massachusetts Institute of Technology.

I. C. Steele, B.S. in C.E.; A.S.C.E.; Consulting Engineer, Piedmont, Calif.; formerly Vice President and Chief Engineer, Pacific Gas and Electric Company, San Francisco.

J. C. Stevens, B.S. in C.E., C.E., Dr. Eng.; A.S.C.E., A.I.E.E., A.I.C.E.; Consulting Engineer, Stevens and Koon, Portland, Ore.

James H. Stratton, B.S., C.E.; Brig. Gen., U.S. Army (Ret.); A.I.C.E., S.A.M.E., A.S.C.E.; Partner, Knappen-Tippetts-Abbett-McCarthy, New York; formerly New England Division Engineer, U.S. Engineers; Supervising Engineer, Panama Canal; Chief of Engineering, Office of the Chief Engineer, War Department.

William L. Voorduin, C.E.; A.S.C.E.; Chief Hydraulic Engineer, Gibbs & Hill, Inc., New York.

PREFACE TO THE SECOND EDITION

The objectives of the second edition are identical with those of the first. Ten years of use in connection with the practical design of hydraulic works has resulted in many requests for enlargement of the book. In response to these requests new sections have been added on water hammer, surge tanks, speed regulation, and navigation locks. The basic policy of drawing illustrative examples from the design files of actual projects has been followed throughout.

Other sections have been either rewritten or brought up to date. The treatment of gravity dams has been enlarged to include recently built structures with special reference to structural behavior. The section on spillways and streambed protection works includes much new material on discharge coefficients and recent advances in design. The section on hydroelectric plants has been enlarged to include important new installations with especial reference to underground stations.

A major change has been made in the treatment of hydrology. A new section written strictly from the design viewpoint replaces the more general treatment of the subject presented in the first edition.

A separate section on drainage has not been included in the second edition. The elements concerned with hydraulic design will be found in the various sections. Adequate treatment of the subject would have required considerable expansion and correlation with the soil sciences. Space requirements did not permit.

Reference material on hydraulics has been presented as an appendix in the second edition. A new section, Appendix B, covers the graphical analyses of many of the problems presented in the various sections. Practical experience in design offices has established the merits of these methods.

The editor wishes to express sincere appreciation to L. F. Harza, President, E. Montford Fucik, Vice President, and Joseph R. Bowman, hydraulic engineer, all of the Harza Engineering Company, for helpful assistance with the preparation of the various sections. Grateful acknowledgment is made to Miss Ellen Crigley and Miss Ruth Crigley for general aid in the preparation of manuscripts.

CALVIN VICTOR DAVIS

Chicago, Ill.
June, 1952

PREFACE TO THE FIRST EDITION

This book has been written in response to the need for a general reference volume on hydraulic engineering. The authors have aimed, first, to present clearly and concisely the fundamental principles which are basic to each subdivision of hydraulic engineering and, second, to demonstrate the practical applications of these principles by examples which have been drawn largely from the records of recently constructed projects.

The work is intended to serve both engineers and engineering students. To the engineer it furnishes practical reference data on the planning and design of modern hydraulic works; to the student it furnishes, in one volume, complete texts on hydrology water supply, sewerage, water power, hydraulic structures, and other related subjects which are usually presented in engineering courses.

A knowledge of elementary hydraulics is presupposed. General formulas, tables, and diagrams that are more or less common to each section of the book are summarized in convenient reference form in [Appendix A]; special applications of hydraulics, on the other hand, are set forth in the sections to which they are related.

Much progress in the hydraulic field has resulted from the river-development programs which have been completed or are now under way in this country. The completion of some of the most important dams, power plants, aqueducts, and other hydraulic structures affords a convenient point at which to summarize these advances.

Research on the structural behavior of high dams is another important outgrowth of these river-development programs. Observations now in progress are throwing much light on the validity of current designing practices. To record and interpret the results of this research, as the findings become conclusive, will be a continuing function of this book.

The rapid progress in hydraulic engineering during the past decade has necessarily resulted from the work of many engineers. In order to make this volume completely authoritative, its contributors have been selected from the members of the profession who have been most prominently identified with recent advances. The work represents the joint efforts of contributors with approximately equal representation by engineers engaged either in private practice or employed by public utilities, engineers employed by government agencies, and engineering educators.

For helpful assistance with the preparation of various parts of the text the editor wishes to express his appreciation to S. W. Stewart, President, Ambursen Dam Company, New York; E. H. Sargent, Chief Engineer, Hudson River Regulating District, Albany, N.Y.; Franklin Hudson, Consulting Engineer, Elizabeth, N.J.; Raymond A. Hill, Consulting Engineer, Los Angeles, Calif.; Lieutenant Alfred B. Bourgo, U.S A.; E. S. Brosell, with TVA; and Franklyn C. Rogers, Rutgers University. Acknowledgment is made to Miss C. Louise Brooks for general aid in preparing the manuscripts and the index.

CALVIN VICTOR DAVIS

Knoxville, Tenn.
June, 1942

CONTENTS

SECTION 1

RIVER REGULATION BY RESERVOIRS

BY THEODORE T. KNAPPEN, JAMES H. STRATTON, AND CALVIN V. DAVIS

GENERAL DISCUSSION

Storage reservoirs are used to control floods, to conserve water, and to regulate stream flow. Reservoirs may be of two types: single-purpose or multipurpose. Aside from location and structural problems, the planning for a single-purpose reservoir leads to simple relationships among the available water supply, the water demand, and the volume of reservoir storage to be provided. These relationships are much more complex for a multipurpose reservoir since they involve the seasonal distribution of stream flow and the reconciliation thereto, and seasonal and other varying demands for the several purposes for which the reservoir is intended. If a specified volume of reservoir storage is dedicated to more than one use on the basis of seasonal distribution of flow, as, for example, to provide additional power storage in the nonflood season, the problem of planning becomes even more complex. The fluctuation of reserve levels for sanitary purposes and for malarial control introduces additional problems.

Reservation of storage in a single-purpose or in a multipurpose reservoir for flood control requires that the storage space be available at any season when the natural flow may cause damage stages. The extent of reservoir flood storage to be provided must be related to downstream tributary inflow. Similarly, low-water-regulation storage in which water is held over in a reservoir from periods of excess flow for release in times of deficient flow must be related to downstream tributary inflow to ensure a dependable supply to cities, industries, navigation, hydroelectric generating plants, irrigation diversions, and recreation developments to the extent that these are involved.

CLASSIFICATION OF STORAGE

In dealing with reservoir storage many qualifying terms will be used. These are defined in the following paragraphs:

Conservation storage is impounded for later release, as required for some useful purpose, such as municipal supply, power, or irrigation.

Flood-control storage is reserved solely to reduce downstream flood flows; water stored during floods is usually released as rapidly as channel capacities permit.

Valley storage is the volume occupied by a stream in flood after it has overflowed its banks. In some cases, such as in an alluvial valley, this may be great. Valley storage in the Mississippi Valley probably has exceeded 50 million acre-ft.

Effective storage is the volume available for the designed purpose. Storage below outlet levels is not effective. In power reservoirs, only storage above the minimum drawdown level is effective. In flood-control reservoirs, the effective storage is the difference between actual capacity above outlet elevation and valley storage, or the storage that the flood waters would have utilized had the reservoir not been constructed.

Units. *Cubic Foot per Second* (*cfs*). This is the usual unit of flow in the English system.

Acre-foot (*acre-ft*). Although reservoir storage is sometimes measured in gallons or in cubic feet, the commonest unit is the acre-foot, which is an acre 1 ft deep, or 43,560 cu ft.

Day-second-foot (*dsf*). This is a convenient unit of storage for flood computations. One dsf equals 86,400 cu ft, or almost 2 acre-ft. This relation is convenient in reservoir computations.

Inches of Runoff. Volume of runoff is frequently given in terms of inches of depth over the drainage area. This is a convenient unit for developing the rainfall-runoff relation. One inch of runoff from 1 sq mile equals 53.33 acre-ft.

RESERVOIR DESIGN STUDIES

Area-volume Curves. The first essentials in reservoir planning are curves, typified by Fig. 1, showing total volume in acre-feet and the total area in acres plotted against reservoir elevation. The best basis for such curves is a topographic survey of the reservoir area. In the event a suitable topographic map is not available, the storage which may be developed may be determined with sufficient accuracy for use in preliminary studies by a stream profile and valley cross sections at suitable intervals. Recent improvements in aerial mapping now make it possible to map large areas for only a fraction of the cost of obtaining similar results by the older mapping methods. Aerial photographs are also of material aid in evaluating the principal reservoir features involved in acquisition.

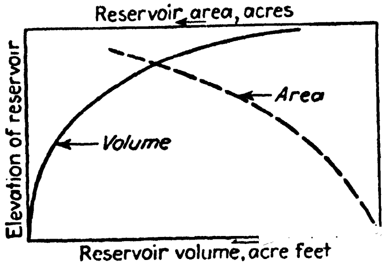

FIG. 1.—Typical reservoir area and volume curves.

Hydrographs. The hydrograph, a plot of flow against time, is the most common expression of inflow characteristics. The time period may be an hour, day, week, or month. Figure 2 is a hydrograph of this type.

In designing a reservoir for conservation purposes, the critical period is the minimum flow period during the driest year of record. It may be necessary to cover a period of several successive dry years in determining storage requirements and when giving consideration to holdover storage to meet demands in periods of critical low flow.

In applying hydrographs, it is frequently desirable to superimpose on the hydrograph of flow the hydrograph of demand or use. The demand hydrograph may be a straight line in the case of a base-load hydroelectric project or a navigation project. In other cases, as in a water-supply or irrigation project, there may be a material seasonal variation. In the case of a peak-load hydroelectric development, there may be wide variations in the demand at various seasons of the year. Figure 3 is a flow hydrograph with the demand hydrograph superimposed. The flow deficiency that must be made up from storage is the area between the hydrographs where the demand exceeds the unregulated discharge.

Duration Curves. Duration curves are used to represent the relation between flow and time. Figure 4, curve 1, shows for a given period the percentage of the time that each flow is equaled or exceeded. Where a duration curve represents a long period of time, it is useful for estimating the amount of energy that can be produced under average conditions by converting the discharge scale to power units corresponding to the available head. The effect of reservoir control is represented by a modified

flow-duration curve such as curve 2, Fig. 4. Duration curves are useful in studying navigation problems as well as in power, water-supply, irrigation, and sanitation studies.

Mass Curves. Mass curves show cumulative flow and can be used to indicate cumulative utilization and storage requirements.

Adjustments for evaporation and other losses must be made in determining the net volume available from accumulated stream flow. This may be done by sub-

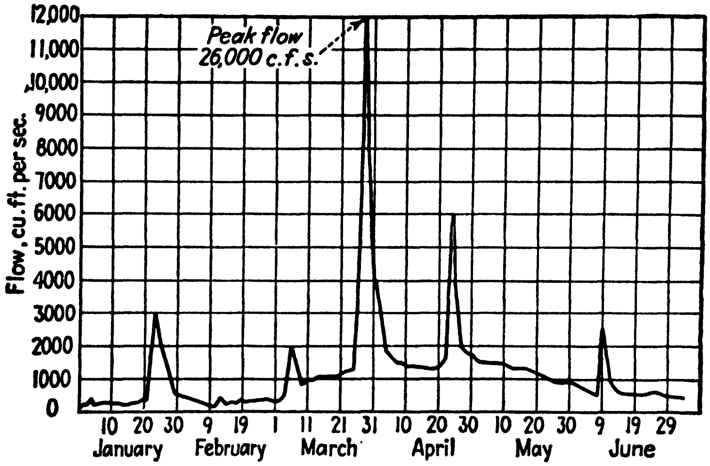

FIG. 2.—Typical flow hydrograph.

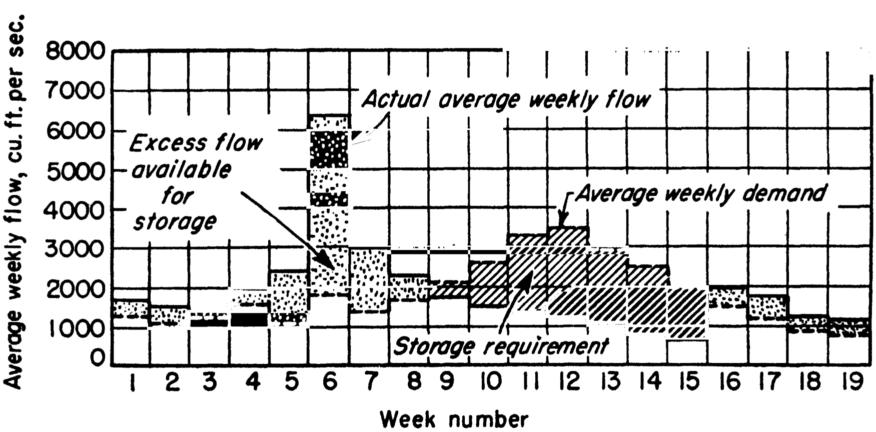

FIG. 3.—Flow and demand hydrograph.

tracting from the natural flow the losses anticipated in order to ensure the required net yield. In Fig. 5 the curve OA, the accumulated flow, has been adjusted by subtracting the expected losses before the plot was made. To illustrate, Fig. 5 shows an accumulation of stream flow OA and an accumulation of demand OB. If a line parallel to OB is drawn tangential to point a, the beginning of the longest dry period of record, the ordinate dc will represent the volume of storage required to maintain a rate of flow not less than that represented by the slope of the line OB.

The mass curve of water utilization need not be a straight line. Figure 6 shows a curve of irregular demand plotted with a curve of supply for a typical dry period. In this figure the lower curve shows the total natural flow in acre-feet from Oct. 1 to May 7 to be 110,000. On Dec. 15 the total was 15,000 acre-ft; on Apr. 1 it was 40,000 acre-ft; likewise the total flow from Oct. 1 to other dates may be read from the curve.

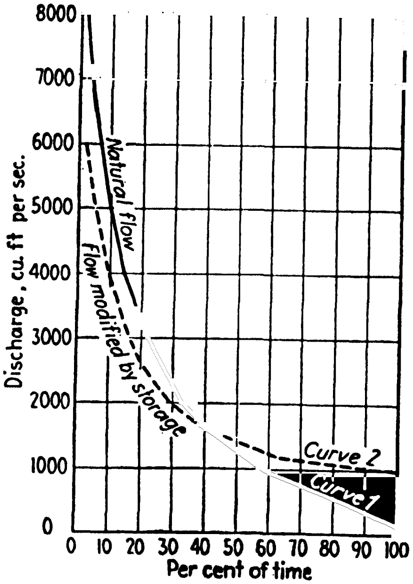

FIG. 4. Flow-duration curve.

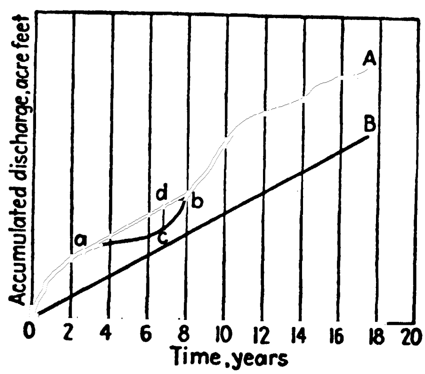

FIG. 5.—Typical mass curve.

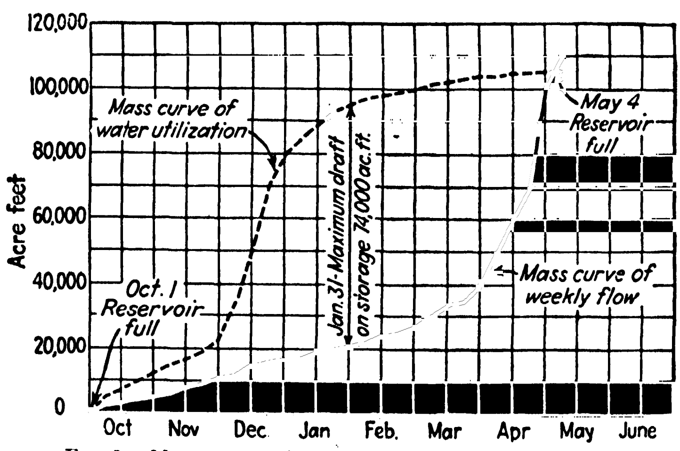

FIG. 6.—Mass curves of water utilization and weekly flow.

The maximum vertical distance between the curves is the storage required to meet the needs of the project. If the worst period of record is selected for the study, the storage requirements are obtained from the mass curve. In the illustration, it was assumed that the reservoir was full on Oct. 1, the beginning of the period. The greatest amount of storage was used on Jan. 31, 74,000 acre-ft. The reservoir was full again on May 4, when the two curves intersected. The mass curve, of course, is

plotted from the hydrograph of reservoir inflow. Graphical analyses of this type are particularly adapted to preliminary studies. For final studies a tabular computation is usually required.

Effect of Valley Storage. For conditions of normal flow where the reservoir is of large capacity, no serious error is introduced if it is assumed that the hydrograph of inflow is the same as the hydrograph at the dam site. However, the effect of valley

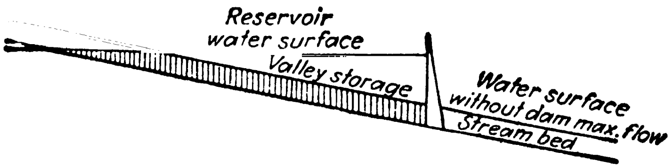

FIG. 7.—Valley-storage diagram.

storage must be taken into account in dealing with flood-control reservoirs where the valley storage at time of flood is of significant proportions in relationship to the reservoir storage volume. The volume of water contained by a stream within its banks may be small in relation to the volume at time of flood when the stream is out of banks, depending on the valley cross section. To illustrate, if the average cross section of a stream is 1,000 sq ft for 1 mile, the volume of water is 5,280,000 cu ft, or about 120 acre-ft. Now the same stream in flood may use 2,000 acre-ft per mile in valley storage. With a reservoir 10 miles long, this might be 20,000 acre-ft at time of maximum discharge at the dam site. In using the hydrograph at the dam site, one should not lose sight of the fact that this 20,000 acre-ft has become available for modification of the hydrograph as is clear from an examination of Fig. 7.

Moreover, the hydrograph of inflow had an earlier and greater maximum discharge as is illustrated in Fig. 8 which shows the effect of valley storage on the hydrograph of reservoir inflow. Curve 1 is the hydrograph of inflow and curve 2 the hydrograph at the dam site. In this instance, it will be noted that the peak discharge has been decreased about 20 per cent and the time of peak retarded. The total flow, of course, is not changed, and area *a* in the diagram is equal to area *b*.

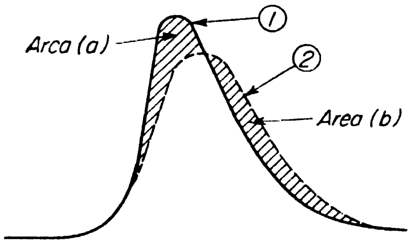

FIG. 8.—Effect of valley storage on the hydrograph of inflow during flood.

It should be noted that the effect of change in timing after the construction of a reservoir is an important consideration and may effect a coincidence of peak reservoir outflow with peak downstream tributary inflows. This critical condition may require the provision of additional storage to avoid such a possibility.

To apply a correction for valley storage, it is first necessary to compute its volume under various conditions. This requires that the reservoir be divided into sections and the volume in each section be computed for various reservoir water surfaces for several discharges may then be calculated and plotted as a family of curves as in Fig. 9 with discharge as a parameter.

The effect of valley storage is important in flood-control reservoirs. Usually it is not economical to provide sufficient storage for the entire volume of a flood, so that it is necessary to discharge some flow during the filling period. This flow is fixed usually by channel conditions below. The rate of discharge having been fixed, the inflow hydrograph, modified by discharge, determines the reservoir size. The effect of valley

storage may be to require a reservoir as much as 30 per cent larger than would be required if the effects of valley storage are ignored or lost sight of. This results from the fact that under normal conditions, the effect of valley storage is to reduce downstream flood peaks. Accordingly, credit thereto must be given, particularly if such effect is large, in computing the size of reservoir required to produce a desired degree of flood reduction downstream of the dam.

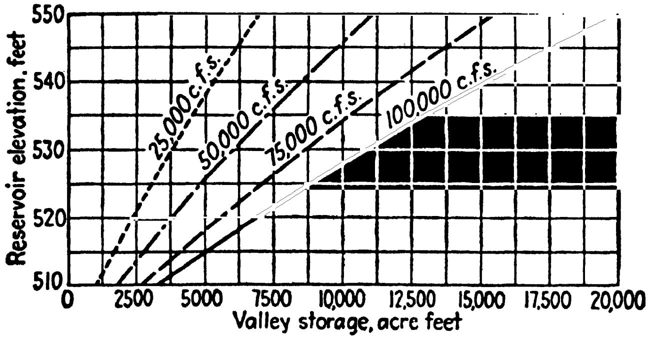

FIG. 9.—Valley-storage curves.

To facilitate reservoir routing studies, when making the correction for valley storage, modified or net capacity, curves for use with the hydrograph at the dam site are often constructed. A set of these curves is illustrated in Fig. 10 with various values of discharge at the dam site as parameters. For each of these discharges, the net effective storage is plotted against elevation. Values for intermediate flow are determined by interpolation.

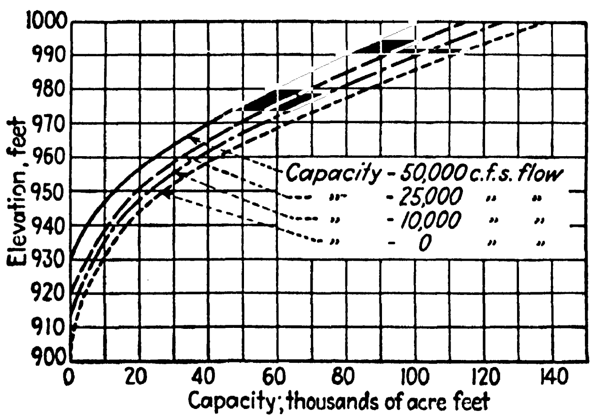

FIG. 10.—Reservoir-capacity curves corrected for valley storage.

Likewise, where spillway design is based upon a particular type of flood with the hydrograph at the dam site, a material increase in required capacity may result when correction is made for valley storage.

It is now the general practice in the Department of the Army to develop inflow hydrographs for all principal drainage areas contributing to the reservoir. The inflow to the water surface of the reservoir is the volume of rainfall falling upon it. Present development in hydrologic practice, utilizing correlations between unit-hydrographs as well as fairly reliable synthetic unit-hydrographs, permits construction of runoff

hydrographs from ungaged areas. In important projects, ungaged areas are relatively small, thus errors in their computed hydrographs are relatively unimportant. In this manner, inflow hydrographs can be developed and routed through gross storage generally with more satisfactory results than with the net storage method. It should be noted that valley storage profiles and volumes for floods much greater than those of record (such as spillway design floods) are quite approximate.

Reservoir Flood Routing. There are many ways that may be used in routing floods through reservoirs. A general solution is given herein which permits a rapid adjustment for variations in discharge and correction for valley storage.

The basic equation, for any interval of time, is

$$\text{Inflow} = \text{storage} + \text{outflow}$$

Inflow is obtained from the reservoir inflow hydrograph, if available. The storage measured in acre-feet is plotted against elevation as in curve *a*, Fig. 11.

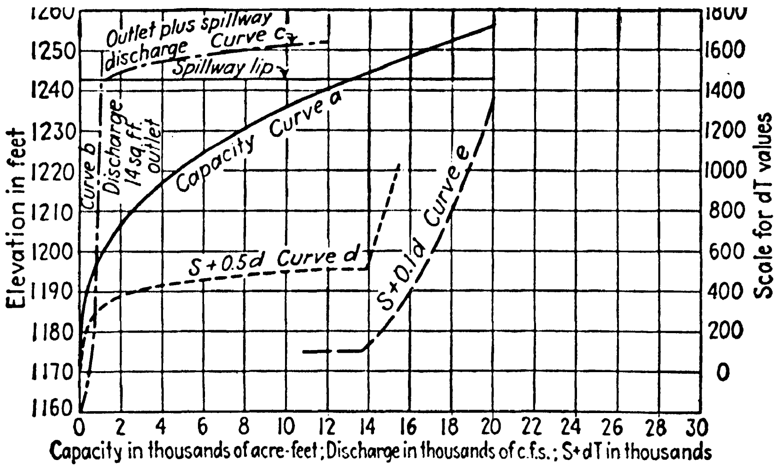

FIG. 11.—Typical reservoir flood-routing curves.

Where the hydrograph is the one at the dam site, capacity curves corrected for valley storage should be used, in which case there will be a family of storage curves as in Fig. 11. Outflow may be through the outlets or over the spillway, or both. For the outlets, the momentary discharge in cubic feet per second may be plotted against elevation as in curve *b* (Fig. 11). Likewise, spillway discharge may be plotted against elevation. In Fig. 11, outlet plus spillway discharge is plotted in curve *c*.

With reference to the basic relation given above and by using the following nomenclature, Eqs. (1) and (2) below are derived:

I = inflow, acre-ft during any time interval, T.
T = interval of time, days.
d_1 = momentary discharge, cfs at beginning of time interval
d_2 = same at end of time interval T.
S_1 = storage, acre-ft in reservoir at beginning of time interval T.
S_2 = same at end of time interval T.

$$I = (d_1 + d_2)T + S_2 - S_1 \tag{1}$$
$$I + S_1 - d_1T = S_2 + d_2T \tag{2}$$

NOTE: The average rate $(d_1 + d_2)/2$ for time interval T is not used because $(d_1 + d_2)T$ is in acre-feet and 1 dsf = 2 acre-ft.

In Eq. (2), the value of S_1 is obtained from curve a (Fig. 11), the reservoir water surface being known; d_1 is assumed in the case of an empty reservoir or is determined from curve b. The algebraic sign of $I + S_1 - d_1T$ may then be determined, which equals $S_2 + d_2T$. Since the value of S_2 and d_2T for one time interval equals S_1 and d_1T for the next time interval, it is necessary to determine S_2 and d_2T. Curves d and e give values of dT in terms of $S + dT$, and S is obtained by subtracting DT from $S + dT$.

Curves d and e are plotted for definite values of T, which are taken in this case as 0.5 or one-half day for curve d and 0.1 or one-tenth day for curve e. These two values are selected to permit a rapid computation by using curve d in the reservoir filling stage and greater accuracy by using curve e in the critical range above spillway crest. Of course, any suitable values of T may be utilized.

To illustrate the construction of these curves, consider some elevation, say 1,245. The storage in the reservoir at this elevation read from curve a is 14,400 acre-ft. The combined outlet and spillway discharge at the same elevation is 2,054 cfs. Then for curve d with $T = 0.5$ day, the ordinate against which the curve is plotted is dT, or $0.5 \times 2.054 = 1{,}027$. The abscissa is $S + dT$, or $14{,}400 + 1{,}027 = 15{,}427$. Likewise for curve e, with $T = 0.1$, $dT = 0.1 \times 2{,}054 = 205.4$, and $S + dT = 14{,}605.4$.

TABLE 1.—TYPICAL FLOOD-ROUTING COMPUTATION USING FIG. 11

(1) Day	(2) Inflow, acre-ft I	(3) Storage, acre-ft S	(4) Discharge dT	(5) $I + S_1 - d_1T = S_2 + d_2T$	(6) Elevation of water in reservoir	(7) Surcharge over spillway, ft
0.0	......	0	50	250		
	300					
0.5	......	30	220	2,390		
	2,580					
1.0	......	2,005	385	11,480		
	9,860					
1.5	......	10,990	490	14,552		
	3,660	......	98[1]			
1.6	......	14,352	200	17,312		
	3,160					
1.7	......	16,632	680	18,492	1,249.5	
	2,540					
1.8	......	17,552	940	18,812	1,251	
	2,220	......	.....	......	(1,251.5)	8.5
1.9	......	17,512	1,300	18,152	1,251	
Total	24,300	17,512	4,363			

[1] Change of time interval from $T = 0.5$ to $T = 0.1$

$$490 \div \frac{0.5}{0.1} = 98$$

Check of computations

$(4.363 \times 2) - (50 + 490 + 98 + 1{,}300) = 6{,}788$ acre-ft discharge
17,512 acre-ft storage
24,300 acre-ft inflow

Table 1 shows the routing computations for the reservoir of Fig. 11 with inflow as given in column 2 of the table. Column 1 is the time in days corresponding to the spillway design flood hydrograph. Column 2 is the inflow in acre-feet for each time interval as determined from the hydrograph. Columns 3, 4, and 5 are determined as the routing proceeds. On the assumption of a base discharge of 50 cfs, it is placed in

column 4 opposite 0.0 for the initial value of dT. Column 5 is the algebraic sum of columns 2, 3, and 4, *i.e.*, $I + S_1 - d_1T$, thus $300 + 0 - 50 = 250$, which is entered in column 5. Since this is also $S_2 + d_2T$, entering curve d, the value of d_2T is found to be 220, which is entered in column 4. This result being subtracted from column 5, the value of the storage S is found to be 30, which is entered in column 3, as the storage in acre-feet at the end of the first half day. Likewise, for the next time interval, 2,580 is the inflow, 30 is the value of S, and 220 is d_1T so $I + S_1 - d_1T = 2{,}390$, which in turn is the new $S_2 + d_2T$ and is entered in column 5. The steps are continued until the storage (column 3) begins to decrease. Care must be exercised at the time the water surface approaches the spillway crest, where a change in time interval is necessary. At this point of change, the dT for the longer interval is divided by the ratio of the longer interval to the shorter interval to obtain the dT for the shorter interval. Storage column 3 may be converted into water-surface elevation (column 6) by the use of the curve a.

Where the hydrograph is for the dam site, additional capacity a curves should be drawn corresponding to net storage, as previously described. For each additional a curve, new d and e curves will be required. These having been established, the set of curves most nearly corresponding to the inflow should be used in each step.

Reservoir Losses. There is always some loss in a reservoir from evaporation and seepage. In any study of reservoir operations, consideration should be given to the effect that such losses might have on reservoir efficiency.

Seepage losses are not important in most cases, but may have a vital effect in others. Where a reservoir is underlain by porous strata which have ample outlets beneath the surrounding hills, or under the dam, seepage may amount to several hundred cubic feet per second. In the usual case, a maximum loss of 10 cfs. would be large. The basin of a reservoir should be carefully studied, and if porous conditions are present, experts in geology and soil mechanics should be called in to analyze the problem, where seepage losses are an economic factor.

Evaporation is a direct function of reservoir area and is usually expressed in inches of depth. The rate of evaporation varies directly with temperature and wind velocity, and inversely with humidity. Reservoirs at high elevations generally show lower rates of evaporation, because of lower temperatures. Approximately 75 per cent of the annual evaporation occurs in the six months from April to September, and about 20 per cent occurs in the maximum month.

In the United States, east of the Mississippi, evaporation varies from 20 in. per year, along the Great Lakes and Canadian boundary, to 43 in., at Birmingham, Ala. From there to the Gulf, there is a slight decrease owing to increasing humidity. The rate increases to 40 in. in the upper Missouri basin and to 70 in. in southwestern Texas. In the Rocky Mountain states of the west, the range is wide, depending on elevation. In the intermountain region, the range is from 38 in. in northern Nevada to 80 in. in southern Arizona. In the Pacific states, the range is from 20 in. on the north coast to over 90 in. in the Imperial Valley in Southern California. For comprehensive records on evaporation, the reader is referred to a paper by Robert Follansbee.[1]

It should be noted that evaporation, which is a factor in almost all conservation dams, may be disregarded in the design of flood-control reservoirs. Seepage, likewise, is of no economic importance in flood-control reservoirs.

PRINCIPLES OF RESERVOIR PLANNING

Single-purpose Conservation Reservoirs. If the water supply available for any given purpose, such as the generation of hydroelectric energy, the regulation of low

[1] *Trans. A. S. C. E.*, **99**, 74, 1934.

water flows, or the domestic and industrial requirements of a city, is in excess of the demand at one season and deficient during another, a conservation storage reservoir is required. The basic principles of planning are governed by the use that the reservoir is to serve. In planning conservation reservoirs, consideration should be given to the possible future increases in demand based on estimates for population or load growth.

Single-purpose reservoirs, located on important streams, should be designed to fit in with an ultimate system that will develop most efficiently the major water resources of the basin. It cannot be emphasized too strongly that the general system plan should be completed before work is started on a single element of the scheme of development. Anything short of complete planning is almost certain to result in the waste of valuable water resources.

Single-purpose Flood-control Reservoirs. *Retarding Reservoirs.* The simplest type of flood-control reservoir is the retarding basin with uncontrolled outlets. The outlet capacity is such that the unregulated discharge with the reservoir full does not exceed the permissible carrying capacity of the channel through the protected areas. An outstanding illustration is the Miami Conservancy District system in western Ohio. All five of the Miami reservoirs are designed with uncontrolled outlets of sufficient capacity to handle a storm one-third greater than that of March, 1913, without overtopping the spillways. The leveed channels through the towns in turn are of sufficient capacity to handle the outlet discharge, plus the flow from unregulated areas.

Retarding basins have one major advantage in that the human element is eliminated from reservoir operation, for no gates are provided. Another advantage is that the expensive gate and gate-control installation cost is avoided. The further apparent advantage of smaller conduit capacity is usually offset by the necessity for greater capacity during the construction period.

A serious disadvantage in connection with retarding basins is that automatic regulation may cause the coincidence of flood crests farther downstream. The travel time of the tributary and main river flood waves should be given consideration in planning reservoirs of this type.

An ideal installation for a retarding basin is the case of a single reservoir located immediately above the city for which the protection is designed. A similar case would be that of reservoirs located above the confluence of two or more streams where the protected area is immediately below their confluence. This case is exemplified by the Miami installation.

Detention Basins. A flood-control reservoir provided with outlet control is termed a detention basin. Outlet control is intended to achieve greater usefulness through flexibility of operations of the reservoir. The disadvantages of the detention type of reservoir result from the possibility of human error in operation and the fact that gate installations add to the construction cost and to the cost of operation. The advantages of this type are several and should be weighed in each case to determine whether gate installations are advisable. Where the area under control is large in size and the area to be protected extends for some distance downstream of the reservoir, the advantages of the detention basin type of control outweigh its disadvantages.

Since a large by-pass capacity is generally necessary for stream diversion during construction, it will be found at most dams, particularly those of earth construction, that adaptation of the diversion facilities to serve as gated outlets can be accomplished at a relatively small additional cost. Gated outlets permit releases at any desired rate up to the maximum, thus making possible the more effective use of the storage capacity of the reservoir by maximum utilization of the capacity of the downstream channel in the early stages of flooding. Moreover, as channel conditions permit, the

reservoir may be emptied more rapidly following a flood to prepare it for the acceptance of subsequent flood flows.

In a system of reservoirs, the detention type of control gives greater flexibility to meet the variety of conditions which are a consequence of variations in rainfall and runoff over the watershed in any one storm as well as permitting operation of the system to meet wide variations in storm and flood patterns. When conditions require, the outlets of the reservoirs which control tributaries experiencing low flows can be closed and the main channel reserved for discharges from uncontrolled tributaries and from reservoirs experiencing high concentrations of runoff. Detention reservoirs make possible the avoidance of coincidental arrival of tributary peak flows in the main channel which otherwise might not be possible.

Preliminary Investigations. In planning a flood-control system in which reservoirs are utilized, the important considerations are reservoir location, area controlled by reservoir, reservoir capacity, and outlet discharge. Usually a reservoir system to be effective must get a large portion of the area under control. In the Muskingum system, 50 per cent of the drainage area was above reservoirs, and 65 per cent of the area above the principal flood-damage center at Zanesville was controlled. An empirical equation for flood discharges from a drainage area is represented by the expression

$$Q = C\sqrt{A}$$

in which Q = discharge, cfs.

C = a constant which may vary from 2,000 to 6,000.

A = drainage area, sq miles.

From this, it will be seen that, if 50 per cent of the area is controlled, a 30 per cent reduction in discharge may be expected. In practice, with favorable reservoir locations, a better result may be obtained, and with unfavorable location, the result may be less satisfactory. This rough method of determining reservoir effect is valuable in preliminary studies to determine what reservoirs have economic possibilities. In the final analysis of a system of flood-control reservoirs, the costs for various systems should be compared with the benefits.

For each reservoir in a proposed system, there is a definite relation between reservoir and outlet capacity. Ordinarily, the system is designed by selecting a project storm and applying a runoff factor. In planning the Muskingum project, the design basis was a 10-in. 5-day rainfall, having a 90 per cent runoff factor. This storm was distributed 1¼ in. first day, 2 in. second day, 4 in. third day, 1¾ in. fourth day, and 1 in. fifth day. By the use of unit hydrographs and distribution graphs, the flood hydrograph at the dam site was produced. In designing a retarding basin, the permissible outlet size is assumed and the flood routed through the reservoir to determine the required capacity to spillway elevation. With detention basins, the outlet discharge hydrograph must be assumed and the same process carried out.

Where reservoir capacity is limited by some other circumstance, the outlets for retarding must be designed by trial until the proper size is obtained. In designing outlets for detention reservoirs, the range of operation desired must be considered. Usually the minimum capacity will be that required to discharge the maximum safe capacity of the channel below with the reservoir storage capacity about 25 per cent full. In gate design, it is advisable to have sufficient gate area to develop full outlet capacity with one gate closed.

Capacities of flood-control reservoirs in the United States usually vary between 4 and 9 in. of runoff from the drainage area. Generally 6 to 7 in. is a desirable capacity for reservoirs in the Eastern states. In the Gulf region larger volumes are desirable, and in the Western section the capacities may be smaller, except on small drainage areas. Outlet capacities vary with reservoir capacity and discharge but usually fall

between the limits of 20 and 40 csm for detention reservoirs at maximum water surface.

The relationship of reservoir capacity to downstream channel conditions is an important one and involves consideration not only of the effects of sudden changes in releases which may cause bank caving and other deleterious effects, but also of the consequences of long-sustained flows. In the case of the latter the sustained flows, even though well within banks, may result in holding the ground water adjacent to the stream at such levels as to result in severe agricultural losses. The fact that flood-plain utilization is generally increased after flood-control reservoirs are built often precludes the full use of the theoretical channel capacities as a basis for reservoir outflow determinations. Hence larger storage capacities than otherwise required may be necessary where this is a consideration.

Multipurpose Reservoirs. Multipurpose reservoirs are designed for two or more uses. For example, a reservoir located on the tributary of a major river might be

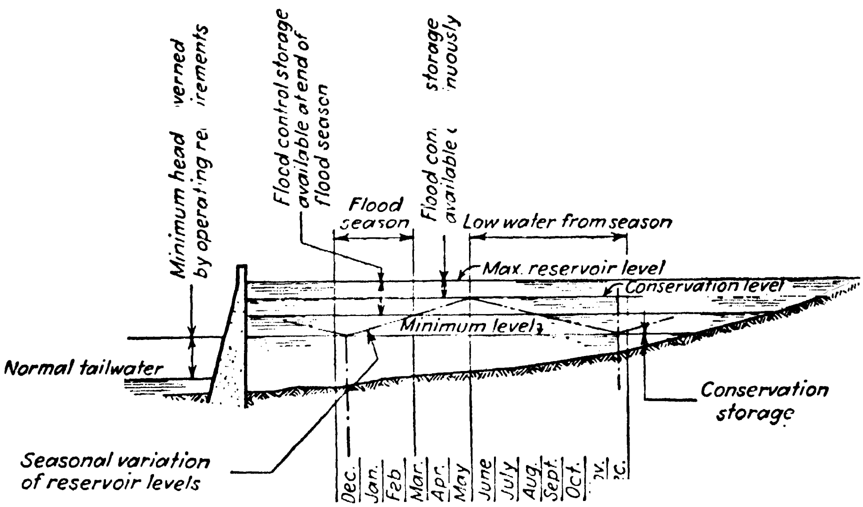

FIG. 12.—Diagram illustrating multiple-purpose reservoir operation.

designed to protect the downstream river towns and cities against disastrous floods, increase the dependable water supply, and generate hydroelectric energy. The principles of multipurpose reservoir planning may best be understood by referring to Fig. 12, which illustrates a typical schedule of operations. Assume that the reservoir shown by Fig. 12 is located on the tributary of a river that is subject to severe floods between December and March. Assume further that this reservoir will be designed to lower the flood stages on the main river and to generate hydroelectric energy. Owing to the seasonal pattern of maximum flood distribution, it would be necessary to keep empty, during the flood season, the storage volume between the maximum and the Mar. 15 reservoir levels.

It would be permissible to fill, during the season of high flow, the space between the minimum and the Mar. 15 levels. This storage would be held for later release, during the low-water season, to increase the primary energy output of the hydroelectric plant at the dam and possibly the energy output of other plants located downstream. It would also be permissible to conserve water above the Mar. 15 level, provided sufficient storage space were reserved above the maximum level to control spring and summer floods.

The minimum level would be governed by operating requirements and by the economic balance between the value of the developed head at the site and at the downstream plants. The relation between drawdown and minimum heads should be such that a reasonable turbine efficiency will be maintained throughout the range of operating reservoir levels. Usually only a small part of the reservoir capacity is below the minimum reservoir level.

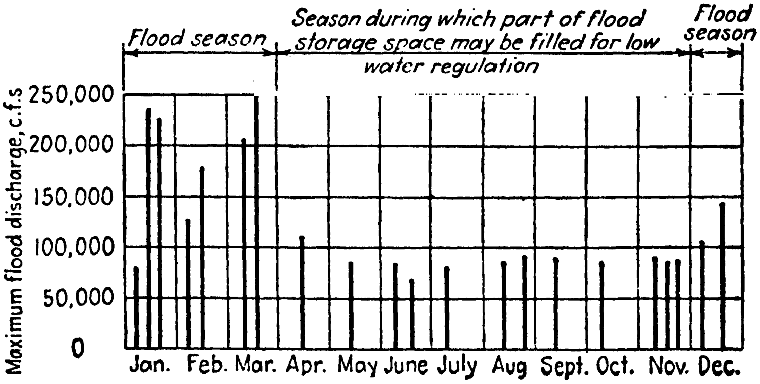

FIG. 13.—Distribution of floods over 50,000 cfs, occurring during 50-year period, at point on main stem of large river system.

The dual-purpose use of the flood storage space above the Mar. 15 level should be permitted only if the pattern of flood distribution is distinctly seasonal. Stream-flow records taken over a long period of time are required to demonstrate the feasibility of this type of operation. If maximum floods have been known to occur during all the seasons or if the stream-flow records are of short duration, safety demands that sufficient storage space to meet flood-control requirements be held available continuously. In the case shown by Fig. 14, the reservoir would not be filled above the Mar. 15 level.

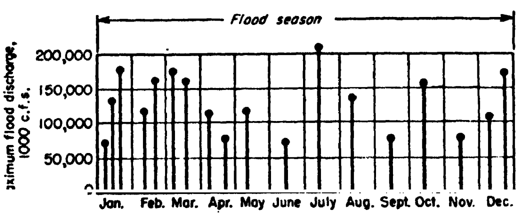

FIG. 14.—Distribution of floods over 50,000 cfs occurring during 50-year period at point on upper tributary of large river system.

Two distinct patterns of seasonal flood distribution that would require different types of operation are shown by Figs. 13 and 14. Figure 13 shows the record of the floods that have occurred on the main stem of a large river, Fig. 15 shows similar data for an upper tributary of this same river. These graphs illustrate that the maximum flood occurrences on the main river are distinctly seasonal and that floods on the upper tributary have occurred in the summer as well as the winter and therefore have no seasonal pattern. If a reasonable reserve of flood-control storage space were held throughout the year for contingencies, it would be possible to operate a multipurpose reservoir designed to control the main river in such a way that flood control and power usage would overlap. The same principles of planning would govern the design of

storage space required for each of the several purposes as would be applied to the design of single-purpose reservoirs.

RESERVOIR ECONOMICS

The principal measures of reservoir economy are (1) the cost of attaining a given objective and (2) the return on the investment. Construction costs offer a basis of comparison for alternate schemes for a single reservoir or for a reservoir system designed to accomplish objectives from which the economic benefits may be constant and to some extent independent of the system design. Equivalent navigation benefits, for example, might result from either a high-dam or a low-dam system, provided that proper adjustments were made for the value of the lost time in making the additional lockages required by the shorter reservoirs. Likewise, if a system were designed for the single purpose of providing complete protection against floods to a city, the economic benefits would be constant. In this case, the most economic system would be the lowest cost combination of reservoirs and local protection works that would adequately serve the purpose. Economic justification would depend upon the ratio of benefits to cost.

If, however, either a flood-control or a power-reservoir system served an extensive area and, furthermore, if increases in flood storage gave corresponding increased benefits, there would be some economic limit to the extent of reservoir development. In this case, the most economic scheme would be that which would yield the maximum return in benefits per dollar of investment. In some cases, it might be desirable to extend the system beyond the most economic stage of development to a point where the return on the additional investment would be satisfactory, even though somewhat less than the maximum obtainable.

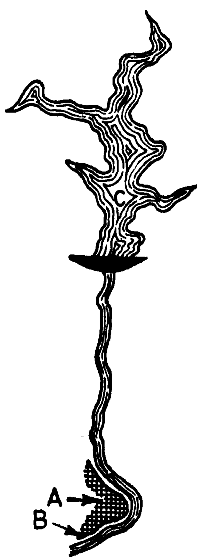

FIG. 15.—Typical example of flood protection by reservoir and levees for a city.

Although the principles of reservoir economics are relatively simple, their application to actual problems is complicated by the obstacles experienced in evaluating the various benefits. It is extremely difficult, for example, to obtain a satisfactory estimate of flood-control benefits. Power benefits, on the other hand, are more direct and easier to evaluate. These principles will be demonstrated by two typical cases.

CASE I. FLOOD PROTECTION FOR A CITY BY LEVEES AND SINGLE-PURPOSE FLOOD-CONTROL RESERVOIR

Let it be assumed that city *A*, shown by Fig. 15, will be protected from floods by local levees *B* and an upstream reservoir *C*. It is desired to obtain the most economic combination of reservoir and levees that will protect the city.

In designing this flood-control system, it would be convenient to express both reservoir and levee costs in terms of either channel capacities or their corresponding flood stages at the city. If, for example, the maximum flood expected at the city (Fig. 16) were reduced from a peak flow *a* to a regulated flow *b*, it would be necessary to provide levees higher than the river stage corresponding to discharge *b* and in addition to provide sufficient storage capacity to impound at least the shaded area of the hydrograph of uncontrolled discharge. Increasing the regulated discharge *b* would increase the cost of levee protection and decrease the cost of the reservoir, as shown by Fig. 17. Likewise, decreasing the regulated flow would decrease levee costs and increase the cost of the reservoir. At some stage corresponding to discharge, say *b*,

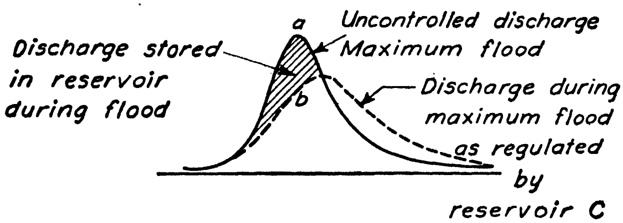

FIG. 16.—Flood hydrographs for use in typical example.

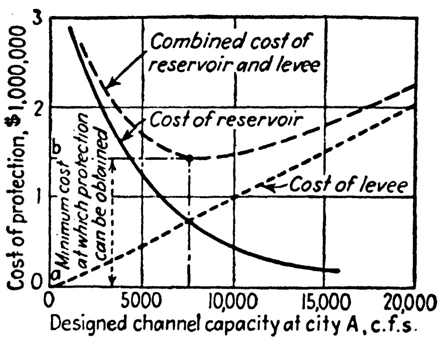

FIG. 17.—Economic analysis for typical example.

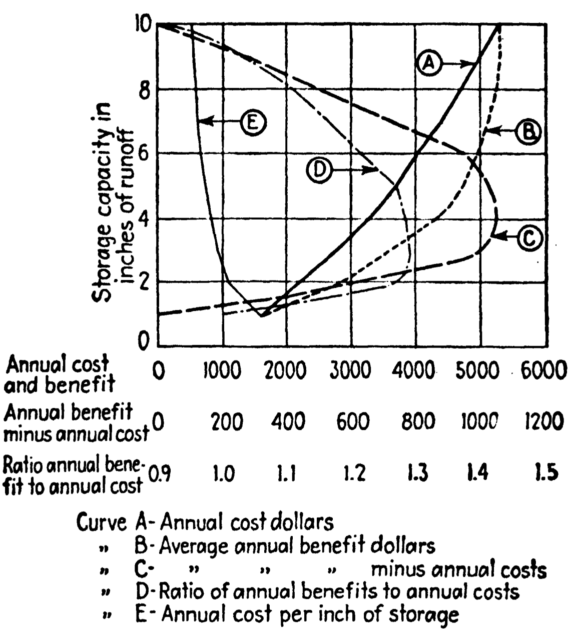

FIG. 18.—Economic analysis of reservoir costs.

the combined costs of the levees and the reservoir would be a minimum. The most economic reservoir in this case would be that which would result in a minimum combined levee and reservoir cost *ab* (Fig. 17). If regulated flow is plotted against height of reservoir, it is also possible to establish, by this procedure, the most economic reservoir level.

Case II. Flood-control Reservoir Serving Large Area

If reservoir *C* (Fig. 15) would afford general flood-control benefits to a large region, it is possible that increases in storage capacity would result in corresponding increases in benefit values. In this case, the most economic reservoir would be that which would result in the maximum net yield in annual benefits value per dollar of annual costs. Reference to Fig. 18 will make this principle clear. Curve *D* indicates that the most economic height of dam would be that for which the ratio of annual charges to annual benefits would be a maximum with a reservoir capacity corresponding to 3 in. of runoff from the drainage area. From curve *C* it is seen that a reservoir having a 5-in. capacity would still yield a satisfactory rate of return on the additional investment over that required for the 3-in. reservoir, and it might be desirable to construct the dam to the height that would give this capacity.

These principles apply also to multipurpose reservoirs designed for both flood control and the generation of electric energy. In this case, the combined benefits and charges would replace the single-purpose benefits described in the preceding paragraph.

RESERVOIR OPERATION

Conservation Reservoirs. A reservoir operation plan is devised to achieve the greatest value or benefit from the storage capacity. The plan must be based (1) on a knowledge of the flow characteristics of the stream, *i.e.*, a history of its part performance. (2) The purpose or purposes of the reservoir must be analyzed to determine how the hydrographs of flow shall be altered to produce the greatest benefits. (3) Special considerations, such as the effect of sudden releases on the stream banks and the effect of long-sustained flows from the reservoir (even though well within banks) on agricultural developments in the valley below the reservoir.

If the reservoir furnishes water to a city or regulates low water flows for navigation, it would be imperative to maintain a predetermined minimum flow, the aim being to utilize the available storage to increase the dependable flow. This might be termed the insurance method of operation. In carrying out the method, studies would be made to determine what flow could be maintained by the reservoir in question during the driest year of record. Only enough water would be released from the reservoir during the dry period to maintain the predetermined minimum flow at the point of regulation; all other conservation storage would be held in reserve for the next dry period that might be expected. From the water-power point of view, there are several objections to this method. In all years except the driest, only partial benefits would be obtained from the reservoir, and in some years little or no water would be released from storage.

In contrast to the insurance method of operation, the annual use method aims to use practically all the available storage each year without special regard for maintaining a minimum regulated flow. This method provides for the release of water from the reservoir just as soon as the natural flow of the stream falls below the amount necessary to operate the plants at full capacity. The principal objection to this method is that the reservoir might be emptied in extremely dry years before the end of the drought.

Modern practice in hydroelectric planning combines the best features of the insurance and annual use methods to provide a dependable minimum flow and at the same time use to the best advantage the total volume of stored water each year. In

carrying out this method, rule curves of operation are devised which indicate how the best results could have been obtained from the reservoir on the basis of past experience. These rule curves are then applied to future operations with the knowledge that the most efficient use will be made of the stored water in all except the extremely wet years, which are of infrequent occurrence.

If the stored water is to benefit a number of power plants located downstream, it is important to operate the reservoir so that the maximum benefits to the power developments in the stream as a whole will result. It is obvious that equal regulation cannot be obtained at all downstream plants; consequently, regulation must be planned for that point on the stream which will yield the maximum benefit to the whole system. Under ordinary conditions, the greatest benefit will result from locating the point of regulation as near as possible to the center of gravity of power for the system. The center of gravity of power may be based on either wheel capacities or developed head and drainage areas.

Flood-control Reservoirs. Where no gate control is provided for the outlets, as in retarding basins, operation is automatic. Sometimes, as in the case of the Pleasant Hill Reservoir in the Muskingum system, gated outlets are provided in addition to the automatic sluices. These are installed where the capacity is large, 7 in. of runoff in this case, and the uncontrolled outlets, therefore, are so small that the emptying period is unduly long. The operation of these gates is quite simple; they are regulated to discharge as much water as can be carried without damage in the channels below the reservoir.

For the detention type of flood-control reservoirs, *i.e.*, reservoirs with controlled outlets, operation is primarily governed by the requirement that no water shall be released that will cause material flood damage below. Where there are only one or two reservoirs in the system, the problem of operation is quite simple, but where there is a large number as in the Muskingum system, 14, or in the Pittsburgh and lower Ohio systems, the problem is more complicated. It is easy enough to work out a plan for a uniform flood or a historic flood, but where the area is large, no future flood can surely be expected to have the characteristics of any past occurrence. The solution to the problem lies in analysis, information, and communications.

The operating personnel must study many possible flood combinations to determine an effective plan for each. An adequate system of rainfall and runoff measurement must be installed together with a system of communications that will get the information to the central office rapidly. Likewise, there must be a good system of communication with the operating personnel. In addition, a basic plan of operation must be installed at each reservoir. If the reservoir is near the flood-damage centers, it can be operated so as to start storing at such a time as to be effective when damaging stages become imminent. Ordinarily, this will mean that the gates are open in the early part of the flood. In the case of remote reservoirs, all early runoff must be stored, and no releases are made until falling stages are imminent, or the reservoir capacity is exhausted or is approaching exhaustion.

Multipurpose Flood-control Reservoirs. Most storage reservoirs are constructed with the idea of impounding flood waters for the purpose of utilizing them during periods of low flow. In such reservoirs, there is no guarantee that they will have sufficient available capacity at the incidence of a flood to provide effective flood control. Flood-control reservoirs are operated on the principle that they are never to be used except for the temporary storage of flood waters, which are subsequently released as rapidly as channel conditions will permit without damage. Sometimes these two types are combined in reserve storage reservoirs, where the lower levels are used for conservation storage and the higher levels reserved for flood-control purposes. Several of the reservoirs of the Muskingum system are of this type. On that project the top

storage, equivalent to 7 in. of runoff, is reserved for flood control and the remaining capacity dedicated to conservation. The operation of reservoirs of this type is divided into the operation of the component parts and has their characteristics. The only deviation is that during flood periods releases are avoided when water is below conservation level.

Where storage is expensive and the ability of the benefited interests to meet costs is limited, there is frequently economic pressure to utilize storage capacity for both flood-control and conservation purposes.

Justification for this method of operation is based upon the relative needs and values at various seasons when the frequency and severity of flood occurrences within the period of record are established as greatest during certain seasons of the year. Let us consider a reservoir in New England. During the winter months, precipitation occurs in the form of snow with only moderate runoff. Toward the end of March the breakup occurs; when this is combined with heavy precipitation, serious floods result. Again in the fall months, serious floods have developed from tropical disturbances. Although serious summer rains have occurred, they have never caused anything but local floods. But it must be recognized that there is a chance of serious flood at any time of the year; past experience has shown that the greatest frequency is in the spring and the fall. Now under these conditions, a reservoir may be designed to provide flood protection during those seasons by storing the runoff and releasing it during the summer and winter months for conservation purposes.

To be effective for combined purposes, a reservoir must be of large capacity. For any degree of positive operation, it must be tested against a long period of record. From this analysis, a rule curve of operation is developed. Similar curves are used in the operation of irrigation and power reservoirs without flood-control features.

SEDIMENTATION OF RESERVOIRS

General. Sedimentation of reservoirs refers to the deposition of all material transported by flowing waters, whether suspended matter or bed load. Sedimentation is the result of erosion, a natural process over which man has little control. The control or handling of sediment deposits in reservoirs is a vital part of any reservoir-design study. In general, the problem is more serious in arid regions where the ground does not support a good vegetable cover. As a result, heavy rains cause excessive sheet erosion and a high percentage of sediment in the streams. In humid climates, the effect of vegetable cover prevents serious erosion. However, where reservoir capacity is small compared with annual stream flow, sedimentation may be an important consideration.

Percentage of Sediment. The percentage of sediment carried in various streams has a wide variation. Table 2 lists representative and approximate figures for the average sediment content of various streams in percentage by weight.[1] These figures in general are average figures; the maximum during great floods may be 10 or more times as great, and during low flow periods, the minimum is much less. According to Stevens, the bed load is usually 10 to 20 per cent of the total, but may be as much as 50 per cent.

Reservoir Sedimentation. The importance of this problem may be illustrated by the degree of sedimentation which has occurred in Lake Mead (Hoover Dam) and in the Elephant Butte Reservoir. In the latter reservoir in the period January, 1915, when storage began, to Sept. 30, 1940, there was deposited 443,000 acre-ft of sediment,

[1] The data are taken from J. C. Stevens, The Silt Problem, *Trans. A. S. C. E.*, **101**, 207, 1936; from J. C. Stevens, Future of Lake Mead and Elephant Butte Reservoir, *Trans. A. S. C. E.*, **111**, 1946; and from Lt. Col. D. O. Elliott, Corps of Engineers, U.S. Army, "The Improvement of the Lower Mississippi."

TABLE 2

River	Approx drainage area, sq miles	Sediment content, %
Colorado	242,000	1.0
Salt	5,760	0.2
Mississippi above Missouri	170,000	0.01
Mississippi below Missouri	700,000	0.2
Mississippi below Ohio	900,000	0.05
Illinois	13,000	0.004
Missouri	500,000	0.4
Ohio	200,000	0.02
Upper Rio Grande	30,000	1.4
Lower Rio Grande	180,000	0.4
Sacramento	9,300	0.3
Feather	3,640	0.01
Yangtze (China)	......	0.04
Yellow (China)	....	4.0

or approximately 16 per cent of its capacity. The average rate of deposit is estimated at 1,600 tons/sq mile per year. The sediment is computed by Stevens to have a weight of 65 lb/cu ft. On the basis of past experience, the Elephant Butte reservoir will have a total life of 158 years; however, the useful life for the purposes intended, will be much less. By the end of this century 50 per cent of its capacity will be lost and its usefulness greatly impaired. The fact that the rate of sedimentation is decreasing in recent years and the possibility of upstream improvements that will result in a decrease in transported sediment are factors which have a bearing on the continued usefulness of the Elephant Butte Reservoir.

In Lake Mead the estimated average annual rate of sedimentation is 198,000 acre-ft, which if continued will result in the filling of the reservoir to spillway crest (capacity 28,632,000 acre-ft) in the period ending 144 years after storage began. Stevens' studies show that the life of Lake Mead could be extended to 233 years with the construction of five upstream reservoirs.

Many reservoirs in the arid west have already had their usefulness destroyed by sedimentation. Even in humid regions, sedimentation may be a factor to be con-

TABLE 3

	Date of survey			
	Spring, 1913	June, 1928	June, 1938	June, 1946
Sediment deposits during period, acre-ft	....	109,250	33,300	24,784
Avg annual sediment deposits during period, acre ft	....	7,283	3,330	3,098
Total sediment deposits to date, acre-ft	....	109,250	142,550	167,334
Avg annual sediment deposits to date, acre-ft	....	7,283	5,702	5,071
Reservoir capacity, acre-ft	479,550	370,300	337,000	312,216
% reduction original capacity	0	22.8	29.7	34.9

Drainage area above the dam is 119,000 sq miles.
Mean annual runoff for period of record is 44,300,000 acre-ft.

sidered. The history of the sedimentation of the Keokuk Reservoir, Mississippi River, is given in Table 3.

In the Hales Bar Reservoir, Tennessee, deposits of sediment during the first 17 years consumed 19 per cent of the capacity. The rate of sedimentation over the past 10 years has been reduced to a fraction of 1 per cent per year.

Control of Sedimentation. Only in rare instances is the removal of sediment from a reservoir by mechanical means justified economically. The removal of sediment, once deposited, by reservoir sluices has never been particularly effective. However, where large gates can be installed and reservoir operation permits the passing flood water, a large portion of the sediment can be passed through the reservoir. This is the practice followed in navigation dams to a large extent.

Usually though, watershed control is the only practical solution. In humid climates, reforestation and planting to grass are effective except in the most severe storms and reduce the sediment load. Sometimes certain small areas in a watershed are major sediment contributors. In such cases, there may be economic justification for special check dams and debris barriers. Another solution, applicable in certain localities, is the construction of a sediment barrier above the head of the reservoir. Owing to the tendency of debris to be deposited on a slope, there may develop large effective capacities. Subsequent willow growth will help further by causing sediment to deposit above the reservoir.

Special Problems Arising from Sedimentation of Reservoirs. Control methods for reducing the sedimentation of reservoirs are costly, particularly for the upstream storage of sediments. The cost of the sediment works and controls is properly assessable against the project whose life and usefulness are to be extended. Inquiry into the possibility of such costs being incurred at any time during the life of the project should be studied when new projects are under investigation, as a part of the study to determine the effect of sediment encroachment on the useful life of the project.

The deposition of sediments at the head of a reservoir may so alter the stream regimen as seriously to affect the capacity of the stream to carry flows within its banks. Levees or flood walls to prevent such flooding which otherwise might not occur may be found necessary. Water supply intakes and sewage outfalls may also be affected and, if channel depositions are excessive, may require pumping or other special arrangements. The provision of works to counteract such adverse effects should be studied as a part of the reservoir planning.

RESERVOIR BACKWATER CURVES

Backwater curves for reservoirs are computed by dividing the channel above the dam into reaches such that the velocity and hydraulic radius will be substantially constant throughout each reach, and, beginning with the dam, to compute for each reach the mean values of velocity, hydraulic radius, and friction head. Accumulated friction heads, modified by changes in velocity head, for successive reaches above the dam furnish the basis for plotting the backwater curve. For further treatment on the subject of backwater curves reference should be made to Appendix B.

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SECTION 2

GRAVITY DAMS

BY CALVIN V. DAVIS

Any dam that does not depend on arch action to resist the forces imposed on it might be termed a *gravity dam.* The term, however, is customarily restricted to solid masonry or concrete dams of roughly triangular section which are straight or only slightly curved in plan. Dams of this type depend for their stability almost entirely upon their own weight and, in spite of their impressive bulk, have a small factor of safety. This fact must be kept constantly in mind at all stages in the design and construction of a gravity dam.

NOMENCLATURE

The following symbols are used in this section:

H = total height of dam from base to crest
h = height of section considered to water surface, depth of water
b = base or thickness of dam from the face to the back measured horizontally
e = eccentricity, distance from point of application of resultant to center of base
y_1 = distance from center of base to downstream face
y_2 = distance from center of base to upstream face
w = density of water, 62.5 lb/cu ft
R = resultant, foundation reaction, or equilibrant
F = total force, see Fig. 1 for subscripts
U = total uplift force
ΣH = algebraic summation of all active horizontal forces
ΣV = algebraic summation of all active vertical forces
ΣM = algebraic summation of all moments
σ_x = vertical normal stress
σ_1 = first principal stress
σ_2 = second principal stress
τ_{xy} = horizontal and vertical shearing stress
θ = angle made by downstream face with vertical
α = angle made by upstream face with vertical
a = ratio of acceleration due to earthquake forces to the acceleration of gravity
f = sliding factor
f' = coefficient of maximum static friction between two surfaces
q = unit shear resistance of foundation material
Q = shear-friction factor of safety

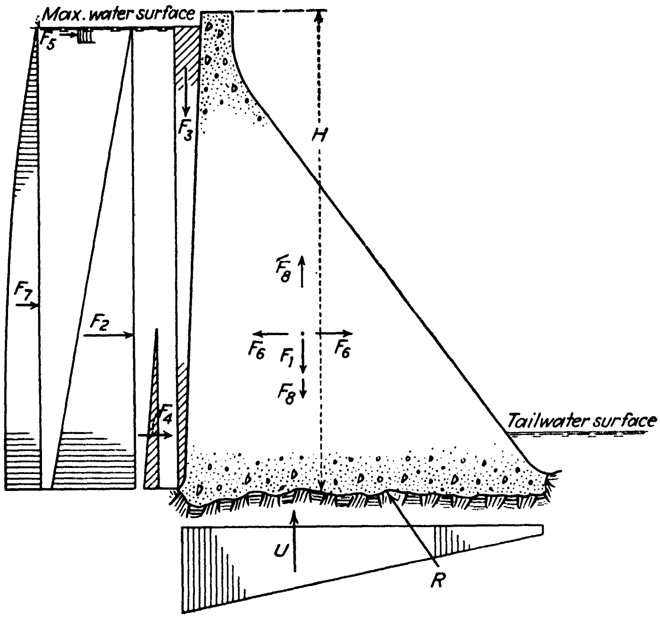
FORCES TO BE RESISTED

The primary function of any dam is to raise the level of the water in the stream valley above it. Obviously, the principal external force to be resisted is the pressure of the water so impounded. Of the other forces which must be dealt with in designing a gravity dam, the greatest is the weight of the dam itself. The various forces that act on a gravity dam are shown diagrammatically in Fig. 1.

Vertical Weight. The force of gravity acting on the mass of the dam (F_1) is the largest force of all. This is both fortunate and necessary, because the vertical weight of a gravity dam, except as it is increased by vertical water load, is the only active force that does not tend to produce failure. Hence, for any given conditions, the safety of a gravity dam is roughly proportional to the density of the concrete or masonry of which it is built. By use of modern construction methods, a concrete

weight of 155 lb/cu ft is frequently attained. In preliminary designs, it is conservative to assume that the weight will be 150 lb/cu ft.

Foundation pressures with an empty reservoir are also proportional to the density of the material. Vertical weight likewise influences the magnitude and direction of stress when the reservoir is full. Consequently, tests should be made to determine in advance of design the weight and strength of the concrete or masonry that could economically be made available for construction at the site of the particular dam under consideration.



F_1 Gravity acting upon the concrete.
F_2 Hydrostatic pressure of water on the upstream face.
F_3 Gravity acting upon the water on the upstream face.
F_4 Excess fluid pressure on the upstream face due to silt.
F_5 Pressure of ice on the upstream face.
F_6 Inertia force of the concrete due to horizontal earthquake acceleration.
F_7 Inertia force of water against upstream face due to earthquake acceleration to left.
F_8 Inertia force of the concrete due to vertical earthquake acceleration.
U Uplift acting on base of dam due to hydrostatic pressure.
R Equilibrant of foundation reaction upon the base of the dam.

FIG. 1.—External forces acting on gravity dam.

Water Pressure. The unit pressure of water increases in proportion to its depth. The horizontal force due to water pressure can thus be represented by a triangular load whose resultant is at two-thirds of the distance from the water surface to the base of the section under consideration. The formula for this water pressure is

$$F_2 = \tfrac{1}{2}wh^2$$

For construction and other reasons, the upstream face of a gravity dam is usually inclined slightly. The vertical water load (F_3) is of course proportional to the horizontal pressure if the slope of the upstream face is constant. When the slope of the upstream face is not a constant, the total vertical load on any section is represented

by the area of water vertically above that section, and the resultant of the vertical load is through the centroid of that area.

Silt Pressure. In most reservoirs, finely divided silt or clay is deposited in substantially horizontal layers against the upstream face of the dam. These deposits are produced by flows of muddy water along the bottom of the reservoir and are to be distinguished from the deltas of silt and sand that form in the upper end of reservoirs. The total pressure F_4 that may develop from this cause and the location of the resultant are quite indeterminate.

When such fine clayey material is first deposited, the percentage of solid matter by weight is relatively small. Under the load of subsequent deposits, the excess water is progressively displaced. At the point of complete saturation, the weight of the clayey material in any unit volume is substantially equal to the weight of the water. If it is assumed that at this point of saturation the mixture is completely fluid, the resulting pressure is equivalent to that of a liquid having a unit weight of 90 lb/cu ft. In order for the weight of a unit volume of such deposits to be greater than about 90 lb/cu ft, water must be expelled by reduction of the void ratio under the weight of overlying deposits. It will then act as a true fluid only under abnormal conditions. At considerable distances below the surface of a silt deposit, the unit pressure against the dam may become less than the pressure of water at the same depth.

In general, it may be assumed that, if a gravity dam is sufficiently heavy to resist all other forces, any slow accumulation of silt against it will not materially decrease its stability.

Ice Pressure. In the colder sections of the world, ice pressure F_5 must likewise be considered. The amount of this pressure naturally varies greatly, depending upon the thickness of the ice that will form on the reservoir and upon other factors, including the slope of the banks of the reservoir and the shape of the upstream face of the dam itself. The magnitude of ice pressure has been variously estimated from 5,000 to 30,000 psf of contact with the vertical face of a dam. It is believed that an allowance of 10,000 psf would be ample under any ordinary circumstance.

Ice of substantial thickness forms slowly even during extreme low temperatures. Consequently, the greatest ice pressure will develop at the water level that is maintained during the winter. In the case of dams built to develop head on power plants, ice may form at the full reservoir level, but there can be no material ice pressure above that level.

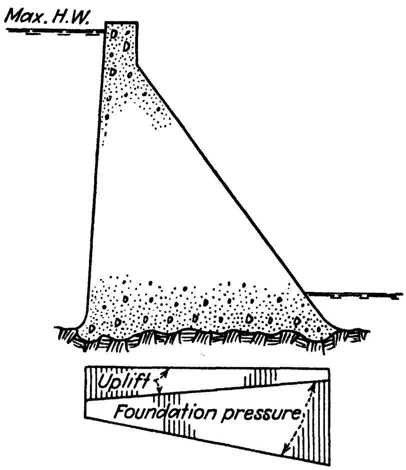


FIG. 2.—Effect of uplift on foundation reaction.

Uplift Pressure. Regardless of measures taken to prevent percolation, some water under pressure will find its way between the dam and the foundation, along pour joints in the dam, or in horizontally bedded joints under the foundation. Wherever this occurs, part of the weight of the dam is supported by water and the direct foundation reaction is correspondingly reduced. This effect is termed *uplift*. The total uplift force U to be allowed for in any design is largely a matter of judgment based upon the character of the foundation, the steps taken to eliminate percolation, the probable efficiency of foundation drains, and the methods of construction to be used.

In designing the Hiwassee Dam, it was assumed that the uplift pressure would vary from 100 per cent of the hydrostatic head at the upstream face to 50 per cent at the line of drains and to 0 per cent or tail water at the downstream face, all acting over 66⅔ per cent of the areas.[1] This assumed uplift pressure distribution as compared with measured uplift pressures at the base of the Hiwassee Dam is compared in Fig. 3A.[2] The pattern of uplift at Hiwassee is typical of that under all T.V.A. tributary dams. Earlier measurements of uplift pressures have been described by Houk.[3]

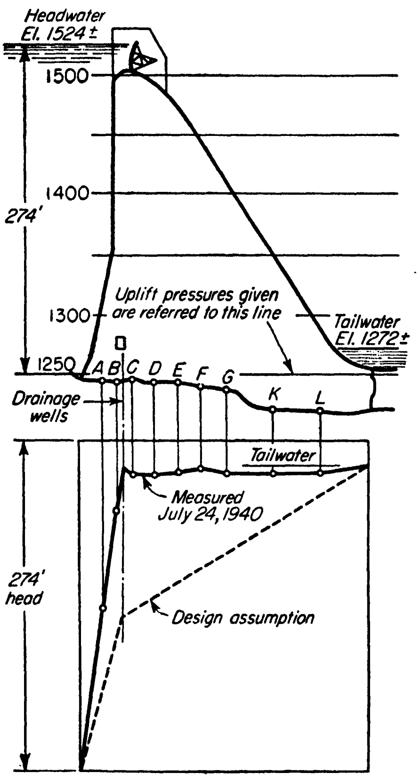


FIG. 3A.—Uplift pressures at the base of Hiwassee Dam.[1]

In designing the Shasta Dam, it was assumed that uplift pressure would vary as a straight line from full reservoir pressure at the upstream face to zero at the downstream face and would act over two-thirds of the horizontal area of the base.[4] The general pattern of measured uplift at Shasta was similar to that at Hiwassee.[5]

These assumptions have not been verified by experiment. Many designers are now turning to the more conservative assumption that uplift pressures actually are exerted on 100 per cent of the area of the base. By mathematical deduction, L. F. Harza has offered rigorous proof that uplift pressures actually are exerted on 100 per cent of the area of the base.[6] The Harza analysis offers the only present rational basis for design.

As indicated by Fig. 3A, uplift pressures are influenced materially by the presence of drains in the foundation. The pattern of flow in the vicinity of a drain is similar to that around a well. There has been much controversy among designers concerning the long-time effectiveness of the drains. Some claim that drains will eventually clog and should be ignored; other designers, like those of T.V.A., take advantage of uplift reductions by drains in proportioning their structures.

The effect of uplift on the foundation reaction, as usually assumed, is shown by Fig. 2.

It is of importance to attain vertical foundation pressures near the heel of the dam considerably in excess of the uplift pressures which may develop at that point. If the uplift pressure exceeds the foundation pressure, the entire vertical load may be progressively shifted downstream until failure takes place.

In stability and stress computations, it is customary to neglect uplift when com-

[1] PEARCE, C. E., Design of Hiwassee Dam—Basic Considerations, *Civil Eng.*, June, 1940, p. **340**.

[2] RIEGEL, Structural Features of Hydraulic Structures, *Trans. A.S.C.E.*, **111**, 1164.

[3] HOUK, I. E., Uplift Pressure in Masonry Dams, *Civil Eng.*, September, 1932, p. 576.

[4] Bigger than Boulder, *Eng. News-Record*, **120**, 649–650, 1938.

[5] KEENER, KENNETH B., Uplift Pressures in Concrete Dams, *Proc. A.S.C.E.*, **76** (Separate No. 25), June, 1950.

[6] HARZA, L. F., The Significance of Pore Pressures in Hydraulic Structures, *Trans. A.S.C.E.*, **114**, 193.

puting stresses and to include it when computing safety against sliding, shear resistance, and overturning.

Measured stress distribution in high-gravity dams is not in close agreement with calculated stress distribution. Studies of the structural behavior of Hiwassee Dam[1] indicate that the dam behaves more like a pile of concrete than an elastic structure.

Seismic Forces. No part of the world is entirely free from earthquakes; hence, adequate allowance should be made for seismic forces in the design of most gravity dams. The magnitude of such forces depends on both the amplitude and the frequency of the induced waves. It has been usual to consider only the horizontal forces produced by the inertia of the dam (F_6) and the momentarily increased pressure of the water (F_7) as the foundation shifts laterally. A close approximation of the force F_7

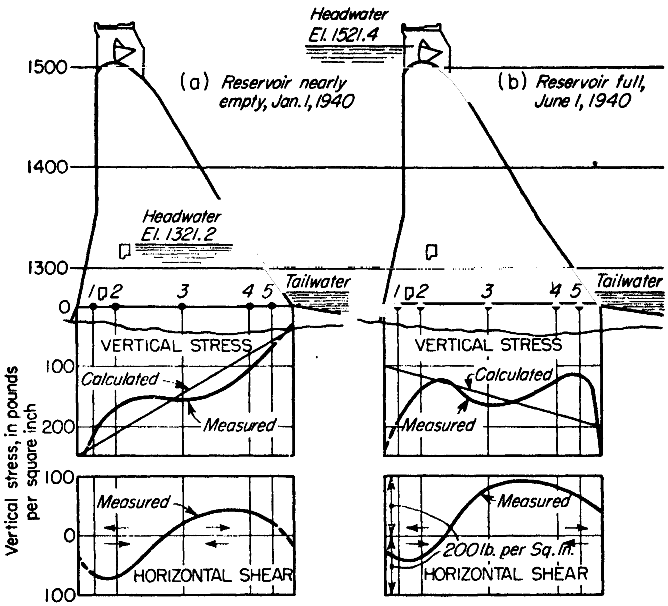


FIG. 3B.—Comparison of measured and calculated stress distribution, Hiwassee Dam.[1]

is given by the following equation developed by von Karman:[2] $F_7 = 0.555awh^2$ acting at a point $(4/3\pi)h$ above the base.

Seismic forces, however, may and do act in all directions. Vertical accelerations of 3 to 6 ft/sec² are to be expected during a severe earthquake; far greater acceleration has been reported. Any upward acceleration (F_8) opposes the acceleration of gravity, the effective weight of the dam upon which its stability depends being thereby momentarily reduced.

PRELIMINARY INVESTIGATIONS

For maximum safety, every condition must be avoided in the selection of the site for a gravity dam which would reduce the effectiveness of the vertical weight of the dam. It is not sufficient to assume that the narrowest section of a canyon is the best site for construction of a dam whose general location has been fixed by economic and other functional considerations. The topographic and geologic limitations of all possible sites in that locality must be weighed.

[1] RIEGEL, *loc. cit.*

[2] Pressures on Dams during Earthquakes, discussion by von Karman, *Trans. A.S.C.E.*, **98**, 434, 1933.

Examination of Site. The first step should be the preparation of an accurate topographic map of each possible site. The scale of the maps should be large enough for layout purposes, and the accuracy and detail should be sufficient to permit close visual identification on the ground of topographic features shown on the maps. The second step should be a field examination by a structural geologist. This examination should be directed primarily toward a determination of conditions that might render unusable one or more of the possible sites, such as the presence of faults or the absence of rock of adequate strength. At the conclusion of the preliminary geologic examination, tentative paper locations should be made and the outlines of the dam should then be flagged out on the ground.

At this point in the selection of the site, it becomes necessary to ascertain the actual conditions that will be encountered in the construction of the dam.

Foundation Exploration. Rarely will conditions be so favorable as to warrant the design and construction of a gravity dam without exploring the foundation. Where the structural geology is simple and clearly indicated by surface exposure, diamond drilling is generally sufficient. Where conditions are less definite, the drilling of large-diameter holes or the sinking of shafts and the driving of tunnels becomes advisable.

It is significant that the actual amount of excavation for the foundations of gravity dams has almost always exceeded the estimate of what would be required, owing primarily to insufficient data on existing conditions at the site. Experience thus indicates the advisability of much more exploratory work than is customary.

In any case, a sufficient number of holes should be drilled to determine the probable depth of excavation to sound rock. Some of the holes should be drilled deep enough to disclose any weakness in the underlying rock structure, especially in sedimentary formations and near intrusions or flows. Slanting holes should be drilled to intersect seams or faults that would not be disclosed by vertical holes. Shafts should be sunk near the upstream face of the dam on the axis of all faults. In locations where there is considerable overburden, trenches should be excavated along the entire length of the proposed dam to expose the surface of the rock.

Geology of Site. Much of this exploratory work would naturally be carried on under the direction of an experienced geologist, who should furnish the engineer with a detailed geologic report covering all factors that would have material bearing upon the final location and design of the dam to be constructed. The location of all faults, contacts, and other structural features should be accurately indicated on the topographic map of the site. The characteristics of the foundation materials and their limitations should be clearly and accurately defined. The probable required depth of excavation at all points should be set forth.

In brief, by exploration and geologic study, there should be disclosed to the designer the foundation upon which the gravity dam must stand and, by its own weight, resist all forces upon it.

Topographic Position. The next step preliminary to actual design of a gravity dam should be the preparation of a topographic map of the sound rock surface, *i.e.*, the surface of the foundation on which the dam is to be built. Paper locations should then be made on this topographic map as a base, some typical cross section being used for comparison of quantities.

In making a paper location, it is generally advisable to select the best position for each block of the dam, without regard to the desirable position of other blocks, and keep in mind at all times that the toe of the dam should be as high or higher than the heel. In order to bring these individual blocks into a consistent pattern, some must be shifted upstream and others downstream.

Usually, the best location for a gravity dam is just far enough upstream from the narrowest section of the valley or gorge so that the trace of the downstream toe is on

the axis of the ridges that form the narrowest section. The upstream face of the dam may be straight in plan or curved, depending upon the topographic form of the sound rock surface. A dam straight in plan is to be preferred from the standpoint of design, unless the curvature is great enough to ensure effective arch action. A dam slightly curved in plan is sometimes desirable for construction reasons, as this tends to bring the centers of gravity of all the blocks into a straight line.

DESIGN

Conditions affecting the proportions of gravity dams vary so widely for different sites that it is not possible to set up a design procedure which is generally applicable. Consideration must be first be given to the basic assumptions that apply specifically to the site under consideration and then to the effects of these assumptions on the proportions of the structure.

Basic Assumptions. Regardless of the data available, the design of a gravity dam must be predicated on certain basic assumptions which may or may not be in strict accordance with existing conditions. Any deviation from these assumptions in order to adjust for possible variations in foundation or masonry properties may lead to extreme complications.

Houk and Keener[1] list 25 basic assumptions involved in the design of important masonry dams. These are as follows:

1. The rock that constitutes the foundation and abutments at the site is strong enough to carry the forces imposed by the dam with stresses well below the elastic limit at all places along the contact planes.

2. The bearing power of the geologic structure along the foundation and abutments is great enough to carry the total loads imposed by the dam without rock movements of detrimental magnitude.

3. The rock formations are homogeneous and uniformly elastic in all directions, so that their deformations may be predicted satisfactorily by calculations based on the theory of elasticity, by laboratory measurements on models constructed of elastic materials, or by combinations of both methods.

4. The flow of the foundation rock under the sustained loads that result from the construction of the dam and the filling of the reservoir may be adequately allowed for by using a somewhat lower modulus of elasticity than would otherwise be adopted for use in the technical analyses.

5. The base of the dam is thoroughly keyed into the rock formations along the foundation and abutments.

6. Construction operations are conducted so as to secure satisfactory bond between the concrete and rock materials at all areas of contact along the foundation and abutments.

7. The concrete in the dam is homogeneous in all parts of the structure.

8. The concrete is uniformly elastic in all parts of the structure, so that deformations due to applied loads may be calculated by formulas derived on the basis of the theory of elasticity or may be estimated from laboratory measurements on models constructed of elastic materials.

9. Effects of flow of concrete may be adequately allowed for by using a somewhat lower modulus of elasticity under sustained loads than would otherwise be adopted for use in the technical analyses.

10. Construction joints are properly grouted under adequate pressures, or open slots are properly filled with concrete, so that the dam may be considered to act as a monolith.

[1] HOUK and KEENER, Masonry Dams, a Symposium; Basic Design Assumptions, *Proc. A. S. C. E.*, May, 1940, p. 813.

11. Sufficient drains are installed in the dam to reduce such uplift pressures as may develop along areas of contact between the concrete and rock materials.

12. Effects of increases in horizontal pressures caused by silt contents of flood waters usually may be ignored in designing high-storage dams, but may require consideration in designing relatively low diversion structures.

13. Uplift forces adequate for analyzing conditions at the base of the dam are adequate for analyzing conditions at horizontal concrete cross sections above the base.

14. Internal stresses caused by natural shrinkage and by artificial cooling operations may be adequately controlled by proper spacing of contraction joints.

15. Internal stresses caused by increases in concrete temperature after grouting are beneficial.

16. Maximum pressures used in contraction-joint grouting operations should be limited to such values as may be shown to be safe by appropriate stress analyses.

17. No section of the United States may be assumed to be entirely free from the occurrence of earthquake shocks.

18. Assumptions of maximum earthquake accelerations equal to one-tenth of gravity are adequate for the design of important masonry dams without including additional allowances for resonance effects.

19. Vertical as well as horizontal accelerations should be considered, especially in designing gravity dams.

20. During the occurrence of temporary abnormal loads, such as those produced by earthquake shocks, some increases in stress magnitudes and some encroachments on usual factors of safety are permissible.

21. Effects of foundation and abutment deformations should be included in the technical analyses.

22. In monolithic straight gravity dams, some proportions of the loads may be carried by twist action and beam action at locations along the sloping abutments, as well as by the more usually considered gravity action.

23. Detrimental effects of twist and beam action in straight gravity dams, such as cracking caused by the development of tension stresses, may be prevented by suitable construction procedure.

24. In monolithic curved gravity and arch dams, some proportions of the loads may be carried by tangential shear and twist effects, as well as by the more usually considered arch and cantilever actions.

25. The distribution of loads in masonry dams may be determined by bringing the calculated deflections of the different systems of load transference into agreement at all conjugate points in the structure.

Not all of these assumptions, of course, will be applied to every case.

Factors of Safety. Before the proper cross section of a gravity dam can be determined, certain minimum factors of safety consistent with the foregoing basic assumptions must be adopted. In the selection of these arbitrary values, economic considerations must be weighed against those of safety. The decision demands experience and judgment, because the margin of safety is not large in any case.

Overturning. The factor of safety against overturning is usually defined as the ratio of the righting moments to the overturning moments about the toe of the dam. When so defined, all forces, excepting the direct foundation reaction, are deemed active. Ordinarily the factor of safety against overturning is between 2 and 3.

Sliding. Clear distinction must be made between sliding factor and the factor of safety against sliding. The former is more accurately defined as the coefficient of friction required to prevent sliding of the dam upon its base under particular loading conditions. A third term, coefficient of static friction, is a limiting factor and is equal

to the maximum horizontal force that can be applied to a body of unit weight without causing sliding on a horizontal plane. If the plane upon which the unit weight rests is inclined gradually, the tangent of the angle between the horizontal and the maximum slope at which sliding does not occur is equal to this coefficient of static friction. The coefficient of static friction is variously reported from 0.65 to 0.75 for the materials ordinarily encountered in the construction of a gravity dam.

The sliding factor of a gravity dam, when the base is horizontal, is equal to the tangent of the angle between the perpendicular to the base and the direct foundation reaction under a given loading condition. When the sliding factor is greater than the coefficient of static friction, the dam is unsafe. The ultimate resistance of the dam to sliding varies, depending upon the loading condition, and is equal to the product of the net normal direct foundation reaction multiplied by the coefficient of static friction.

The factor of safety against sliding is properly defined as the ratio of the coefficient of static friction f' to the tangent of the angle between a perpendicular to the base and the direct foundation reaction.

The small factor of safety against sliding possessed by gravity dams is not generally appreciated. A gravity dam of conservative cross section has a factor of safety against sliding of only about 1.5, even when uplift pressure and seismic forces are disregarded. Inclusion of uplift on a horizontal plane reduces the factor of safety to about 1.2, and, when accelerations equal to one-tenth of gravity are taken into consideration, to allow for seismic forces, the factor of safety against sliding may become less than unity. These values are for sliding on a horizontal plane; if the foundation slopes downstream, the factors of safety against sliding on the plane of the base are correspondingly reduced.

It is obvious, therefore, that the resistance of a gravity dam to sliding is primarily dependent upon the development of sufficient shearing strength. The factor of safety due to combined shearing and sliding resistance may be expressed by the formula

$$Q = \frac{[(\Sigma V - U) \times f'] + (b \times q)}{\Sigma H}.$$

In practice, this resistance is attained in part by stepping the foundation and by measures taken to ensure bond between concrete and rock and successive pours of concrete. In a true masonry dam, adequate shearing strength results from the interlocking of the blocks of masonry; in a concrete dam, however, the resistance to shear on any approximately horizontal construction joint above the foundation depends entirely upon the bond developed between successive pours. The specifications for construction of a gravity dam thus are as much a matter of design as the analysis of stresses.

Shear. Regardless of the fact that the ability of a gravity dam to withstand the forces acting upon it depends largely upon the strength of the material in shear, little is known about the actual strength of concrete in shear.

The strength of concrete in compression is generally measured by the loads required to break test cylinders in a compression machine. Ordinary concrete suitable for placing in a gravity dam should test at least 2,000 psi at the end of 28 days; much stronger concrete is customary. It is generally assumed that the unit shearing strength of concrete is about one-fifth of the breaking stress of standard cylinders. This would indicate shearing strength of 400 to 800 psi for concrete of the character used in the construction of gravity dams. A factor of safety of 4 would make the unit working stress in shear 100 to 200 psi.

The distribution of shear along a horizontal plane of a gravity dam is from about zero at the heel to a maximum near the toe, so that the intensity of shear at the toe is roughly twice the average. Furthermore, bond may not be developed uniformly between successive concrete pours. The average shear on any construction joint should thus be limited to about one-twentieth of the compressive strength of the concrete as determined by standard crushing tests. Unless there is assurance that all surfaces will be thoroughly cleaned before placing of concrete, little reliance should be placed upon shearing strength to resist the tendency of a gravity dam to fail by sliding.

Structural Analysis. In final designs for high gravity dams, consideration should be given to combined beam and cantilever action, the effect of rock movements and the effects of twist and beam action along sloping abutments in addition to the conventional stability and stress analyses. Only the more conventional analysis for a vertical section having a width of 1 ft is considered in this section. Reference may be made to the various publications set forth in the appended bibliography for descriptions of the more advanced methods of design.

Ordinarily the following steps will suffice:

1. Compute the righting and overturning moments on selected horizontal planes, taking into consideration all of the forces which may act on the section.

2. Compute the vertical normal stresses σ_x on each selected plane of analysis by the formulas

$$\sigma_x \text{ max (at downstream face)} = \frac{\Sigma V}{144b}\left(1 + \frac{6e}{b}\right)$$

$$\sigma_x \text{ min (at upstream face)} = \frac{\Sigma V}{144b}\left(1 - \frac{6e}{b}\right)$$

Vertical normal stresses may be assumed to have straight-line variation between the toe and the heel as shown by Fig. 2. Usually these stresses are computed on the assumption that no uplift pressure is acting on the base.

3. Compute the principal and shearing stresses at the downstream face. The first principal stress $\sigma_1 = \sigma_x/\cos^2\theta$ in which θ = the angle between the downstream face and the vertical. The horizontal shearing stress at the toe is $\tau_x = \sigma_x \tan\theta$.

4. Estimate the probable distribution of uplift pressure on the foundation, and determine its effects upon the stability of the section.

5. Considering the effects of uplift on the sliding factor, compute the shear-friction factor of safety and the factor of safety against overturning.

In important structures, it may be desirable to determine the intensities and directions of the principal and maximum shearing stresses at various points throughout the section. The usual procedure in analyzing these stresses is to divide the section into elementary prisms and consider each prism to be held in equilibrium by the stresses acting upon it. By starting with vertical normal stresses acting on the upper and lower surfaces of each elementary prism, it is possible to determine by successive integration the vertical and horizontal shearing stress intensities, the horizontal normal stresses, and the first and second principal stresses. Reference is made to Sec. 4, Buttress Dams, for a complete description of this method.

Case 1 and Case 2 illustrate the principles of stability computation applied to the section through Norris Dam shown by Fig. 4. These computations apply only to a horizontal plane at El. 800. Forces due to earthquakes are not included. The results of the analysis for other planes are also shown by Fig. 4. For a complete structural analysis of the Norris Project see *Technical Report* 1, The Norris Project, Tennessee Valley Authority, United States Government Printing Office, 1940.

CASE 1. RESERVOIR EMPTY

Dimensions	*Volume of concrete, cu ft*	*Moment arm, ft*	*Righting moment*
$\frac{1}{2} \times 75 \times 15 =$	563.	193.09	109,000
$260 \times 20 =$	5,200.	178.09	926,000
$240.13 \times 168.09 \times \frac{1}{2} =$	20,182.	112.06	2,262,000
	25,945	127.05	3,297,000
	150 lb/cu ft		150 lb/cu ft
Weight of concrete =	3,892,000 lb	127.05	494,437,000 ft-lb

Eccentricity $e = 127.05' - \frac{203.09'}{2} = 25.50'$

Average vertical normal stress $= \frac{3{,}892.000 \text{ lb}}{203.09' \times 144} = 133$ psi

Maximum vertical normal stress at upstream face $\sigma_{x\,max} = 133 \text{ psi} \left(1 + \frac{6 \times 25.50'}{203.09'}\right)$

$= 233$ psi, upstream face

Minimum vertical normal stress at downstream face $\sigma_{x\,min} = 133 \text{ psi} \left(1 - \frac{6 \times 25.50'}{203.09'}\right)$

$= 33$ psi downstream face

CASE 2. RESERVOIR FULL

Water surface El. 1052, tail water El. 847, no uplift on base

	Force, lb	*Moment arm, ft*	*Righting moment, ft-lb*
Weight of concrete =	3,892,000	127.05	494,437,000
Vertical water load:			
Upstream face			
$177 \times 15 \times 62.5 =$	166,000	195.59	32,456,000
$75 \times 15 \times \frac{1}{2} \times 62.5 =$	35,000	198.09	6,964,000
Downstream face			
$47 \times 32.9 \times \frac{1}{2} \times 62.5 =$	48,000	10.97	530,000
$\Sigma V =$	4,141,000	129.05	534,387,000
Horizontal water pressure:			
Upstream face			
$\frac{1}{2} \times \overline{252}^2 \times 62.5 =$	1,985,000	84.00	166,698,000
Downstream face			
$\frac{1}{2} \times \overline{47}^2 \times 62.5 =$	69,000	15.67	1,082,000
$\Sigma H =$	1,916,000	86.44	165,616,000
Sliding factor $f =$	$\frac{1{,}916{,}000}{4{,}141{,}000} = 0.463$		368,771,000

Point of application of resultant, distance from downstream face $= \frac{368{,}771{,}000 \text{ ft-lb}}{4{,}141{,}000 \text{ lb}} = 89.05$ ft

$$e = \frac{203.09'}{2} - 89.05' = 12.50'$$

$$\sigma_{x\,av} = \frac{4{,}141{,}000 \text{ lb}}{203.09 \times 144} = 142 \text{ psi}$$

$$\sigma_{x\,max} = 142\left(1 + \frac{6 \times 12.50'}{203.09'}\right) = 194 \text{ psi at downstream face}$$

$$\sigma_{x\,min} = 142\left(1 - \frac{6 \times 12.50'}{203.09'}\right) = 89 \text{ psi at upstream face}$$

First principal stress, σ_1 parallel to downstream face

$$= \sigma_x(1 + \tan^2 \theta) = \sigma_x/\cos^2 \theta$$
$$= 194 \text{ psi } (1 + 0.49) = 289 \text{ psi}$$

Maximum horizontal shearing stress $= \tau_x = (\sigma_x - \text{uplift pressure}) \tan \theta$

$= (194 \text{ psi} - 20 \text{ psi} = 174 \text{ psi}) \times .7 = 122$ psi

Effect of uplift pressure on sliding factor

(Assume uplift varies from full static head at upstream face to tail water at downstream face applied to two-thirds of area of face)*

$$\text{Uplift } U = \frac{2}{3} \times 62.5 \text{ lb/cu ft} \times 203.09 \text{ sq ft/ft} \times \frac{(252' + 47)}{2} = 1{,}265{,}000 \text{ lb}$$

* The author recommends that this factor be increased to 1 in future designs.

$\Sigma H = 1{,}916{,}000$ lb

ΣV, without uplift $= 4{,}141{,}000$ lb

U (uplift) $= 1{,}265{,}000$ lb

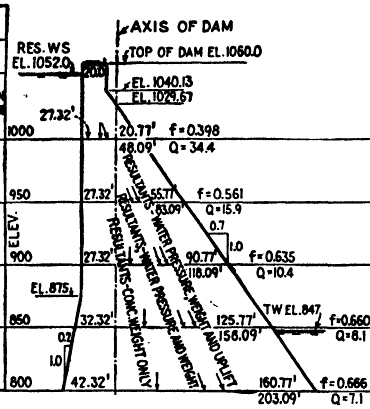
ΣV with uplift $= 2{,}876{,}000$ lb

$$\text{Sliding factor } f = \frac{1{,}916{,}000 \text{ lb}}{2{,}876{,}000 \text{ lb}} = 0.666$$

Shear friction factor of safety, considering effect of uplift

$$-Q = \frac{(2{,}876{,}045 \times 0.65^{*}) + (203.09 \text{ sq ft/ft} \times 400^{\dagger} \text{ psi} \times 144 \text{ psi})}{1{,}916{,}000 \text{ lb}} = 7.1$$

Stresses at upstream face in pounds per square inch: Reservoir empty, Vertical found. pressure	Reservoir empty, Conc. stress parallel to face	Reservoir W S Elev. 1052, Water pressure	Reservoir W S Elev. 1052, Vertical found. pressure	Reservoir W S Elev. 1052, Conc. stress parallel to face	Reservoir W S Elev. 1052, Shear stress on horiz. plane	Elev.	Stresses at downstream face in pounds per square inch: Reservoir W S Elev. 1052, Vertical found. pressure	Reservoir W S Elev. 1052, Conc. stress parallel to face	Reservoir W S Elev. 1052, Shear stress on horiz. plane	Reservoir empty, Vertical found. pressure	Reservoir empty, Conc. stress parallel to face
76	76	23	49	49	0	1000	27	40	19	0.8	1.1
127	127	44	61	61	0	950	66	98	46	-0.9	-1.4
179	179	66	69	69	0	900	108	161	76	-1.0	-1.6
211	219	88	78	81	2	850	151	224	105	12	18
233	242	109	89	93	4	800	194	289	122	33	49



MAXIMUM ABUTMENT SECTION

FIG. 4.—Structural analysis, bulkhead section, Norris Dam.

Typical sections through existing gravity bulkhead sections are shown by Figs. 5 and 7 to 10. Figure 6 shows a view of the recently completed Hiwassee Dam.

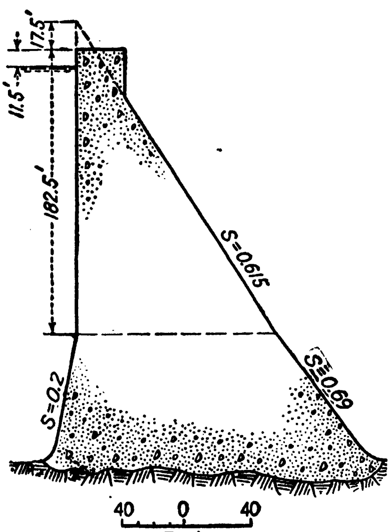


FIG. 5.—Hiwassee Dam, Hiwassee River, North Carolina.

* Coefficient of internal friction.

† Unit shear resistance of foundation rock.

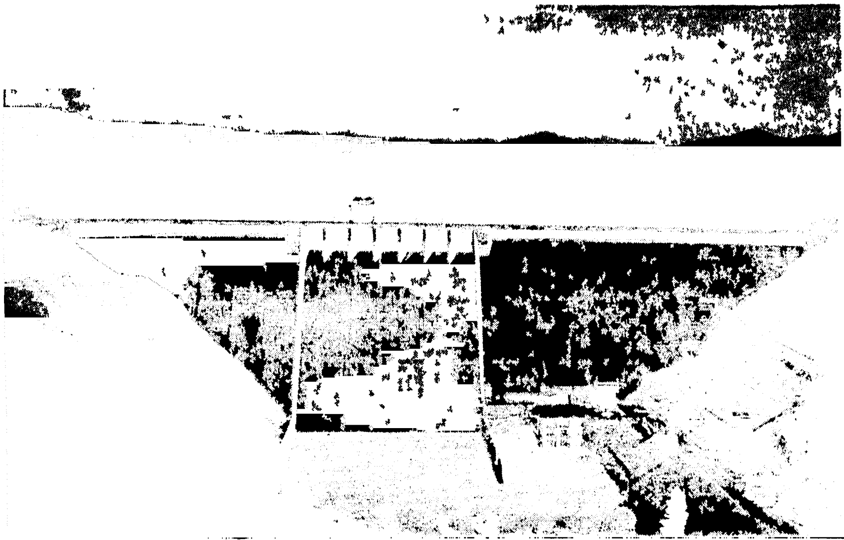


FIG 6.—Hiwassee Dam, Hiwassee River, North Carolina. (*Tennessee Valley Authority*)

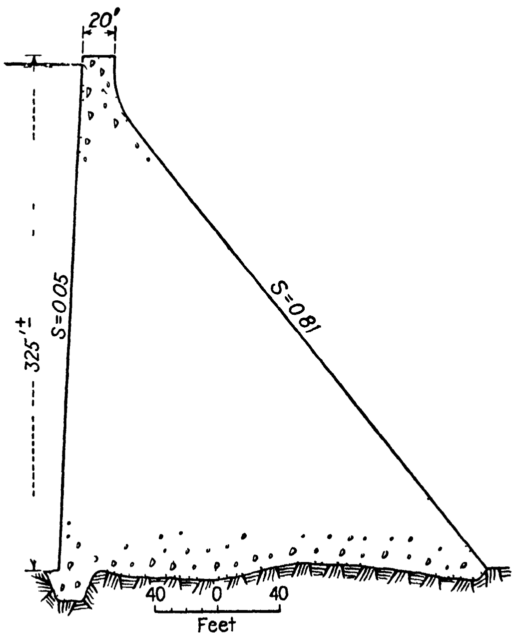


FIG 7.—Morris Dam San Gabriel River California.

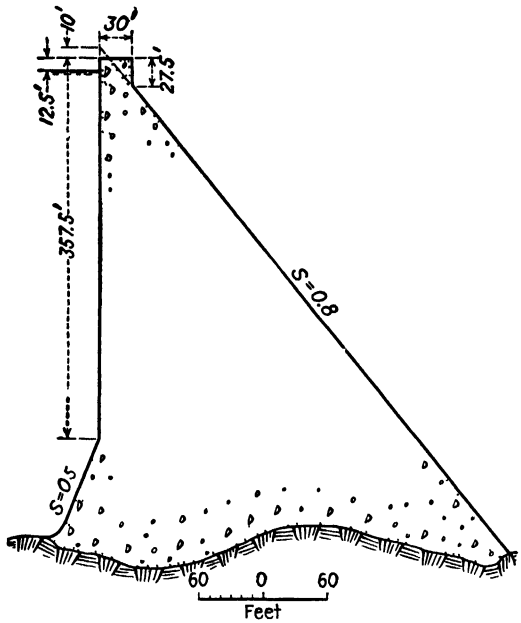


FIG 8 —Shasta Dam, Sacramento River, California

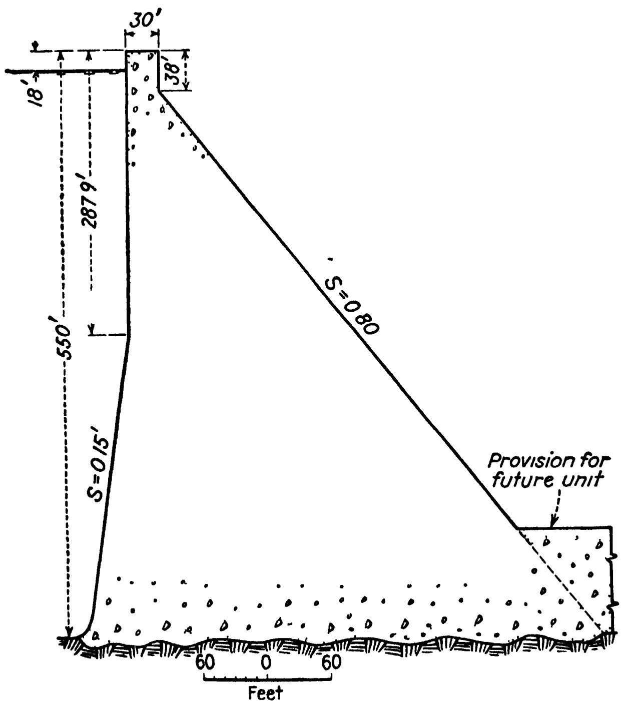
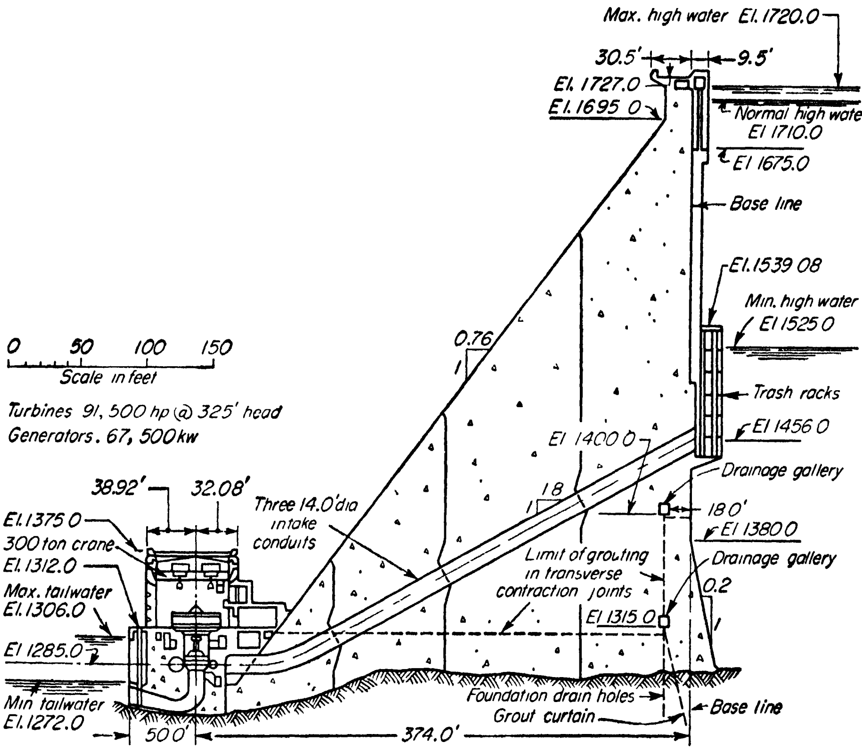


FIG 9 —Grand Coulee Dam, Columbia River, Washington.

The Morris, Shasta, and Grand Coulee Dams, Figs. 7, 8, and 9, respectively, have in common a downstream face batter equal to or flatter than 0.8. The Fontana profile, Fig. 10, also approaches these proportions. Recent studies of the structural behavior of Fontana indicate a possible zone of tension near the downstream face during periods of high air temperature. Such zones have been observed also for other high-gravity dams having relatively flat downstream slopes. Such studies of structural behavior point to the desirability of modifying the design of high-gravity dams by using liberal batters on the upstream face, as indicated by Fig. 8, and per-



Section A-A

FIG. 10.—Fontana Dam and Powerhouse. (*Civil Engineering, July*, 1943, p. 306.)

haps by steepening somewhat the slope of the downstream face in the vicinity of the foundation.

Important features of some designs are the use of fillets at the upstream and downstream toes as shown by Fig 8. Photoelastic experiments show that high stress concentrations exist at the foundation rock at the toe of the dam when the reservoir is full and at the heel of the dam when the reservoir is empty; they further show that rounded fillets lessen these stresses quite appreciably.

Crack Control. Properly spaced contraction joints and temperature regulation of the concrete are the two most important methods of controlling cracks. Actually contraction joints are designed cracks which replace the ragged, structurally objectionable cracks caused by shrinkage within large monoliths.

The ideal spacing for contraction joints would vary from block to block depending upon the position in the structure and the topographic features of the dam site. This ideal spacing is seldom attained, however, and economic and structural considerations

usually require the use of standard form panels and uniform joint spacings throughout substantial portions of the structure.

A few years ago, it was common practice to space vertical joints 75 to 100 ft apart. Recently, however, the trend has been toward the use of closer spacings. At present, 50 ft seems to be the most popular spacing for joints normal to the axis of the dam although in designing the 500-ft-high Hungry Horse Dam in Montana the Bureau of Reclamation has adopted a spacing of 80 ft. The spacings of both longitudinal and transverse joints in typical modern gravity dams are given in Table 1.[1]

TABLE 1.—COMPARATIVE DATA ON CONTRACTION JOINTS IN GRAVITY DAMS

Name	River	Location	Year finished	Volume, thousands of cu yd	Dimensions, ft			Joint spacing, ft		Height of lifts, ft
					Height above		Length	Transverse	Longitudinal	
					Foundations	Low water				
Black Canyon	Payette	Idaho	1924	79	184	.....	1,134	75, 50	.....	3.5
Bonneville	Columbia	Ore.-Wash.	1938	500	170	90±	1,250	60	.....	5
Hoover	Colorado	Nev.-Ariz.	1935	3,250	726	.....	1,282	25–66	30–50	5
Bull Run	Bull Run	Oregon	1929	222	200	.....	1,000	40	.....	4.5
Calderwood	Little Tennessee	Tennessee	1930	280	232	200	916	50±	.....	10
Cheoah	Little Tennessee	N. Carolina	1919	200	225	200	750	50	.....	5
Chute-a-Caron	Saguenay	Quebec	1931	460	200	170	3,040	50–55	.....	10
Conchas	South Canadian	New Mexico	1939	750	235	190	1,250	40–50	30–50	5
Don Pedro	....	California	1923	282	288	278	1,040	32.5, 65, 130	.....	4
Elephant Butte	Rio Grande	New Mexico	1916	605	306	.....	1,155	100, 50	.....	4
Exchequer	Merced	California	1926	369	330	309	955	25, 50, 75	.....	5
Fifteen Mile Falls	Connecticut	Vermont	1930	.....	175	.....	.....	30–50		
Friant	San Joaquin	California	1943	1,900	300	255	3,430	50	.....	5
Grand Coulee	Columbia	Washington	1940	10,000	540	368	4,140	50	50	5
Hiwassee	Hiwassee	N. Carolina	1940	770	307.5	.....	1,265	38–50	.....	5
Kensico	....	New York	1916	.....	307	168	1,843	73.5–79		
Madden	Chagres	Panama	1934	525	223	.....	974	56	.....	5
Marshall Ford	Colorado	Texas	1939	969	265	.....	2,325	52	42	5
Martin	Tallapoosa	Alabama	1926	395	168	157	1,300	30–72	.....	10
Morris	San Gabriel	California	1934	450	328	245	756	50	.....	5
Norris	Clinch	Tennessee	1935	1,000	265	.....	1,570	56	.....	5
O'Shaughnessy	Toulumne	California	1923	398	344.5	226.5	605	97	.....	5
O'Shaughnessy (Raised)	Toulumne	California	1938	675	430	312	900	48.5	.....	5
Owyhee	Owyhee	Oregon	1932	504	405	.....	850	50	.....	4
Pardee	Mokolumne	California	1929	640	358	350	1,320	37.5, 75, 150	.....	5
Shasta	Sacramento	California	1943	5,400	560	528	2,860	50	50	5
Tygart	Tygart	W. Virginia	1937	1,200	240	.....	1,850	52, 60	.....	5, 8, 10
Wilson	Tennessee	Alabama	1926	1,240	137	.....	4,860	38, 48, 54	.....	4–6

Properly designed contraction joints constitute only one provision for the control of cracks; other steps are as follows:

1. The construction of shallow vertical lifts preferably not more than 5 ft high in large dams
2. The use of a low cement content
3. Refrigeration of mixing water
4. Allowance of sufficient time between the construction of vertical lifts to permit a large loss of heat from the surface
5. Cooling the concrete by circulating water through pipes embedded in the concrete

[1] Masonry Dams, A Symposium, Construction Joints, by Byram W. Steele, *Proc. A. S. C. E.*, May, 1940, pp. 908–941.

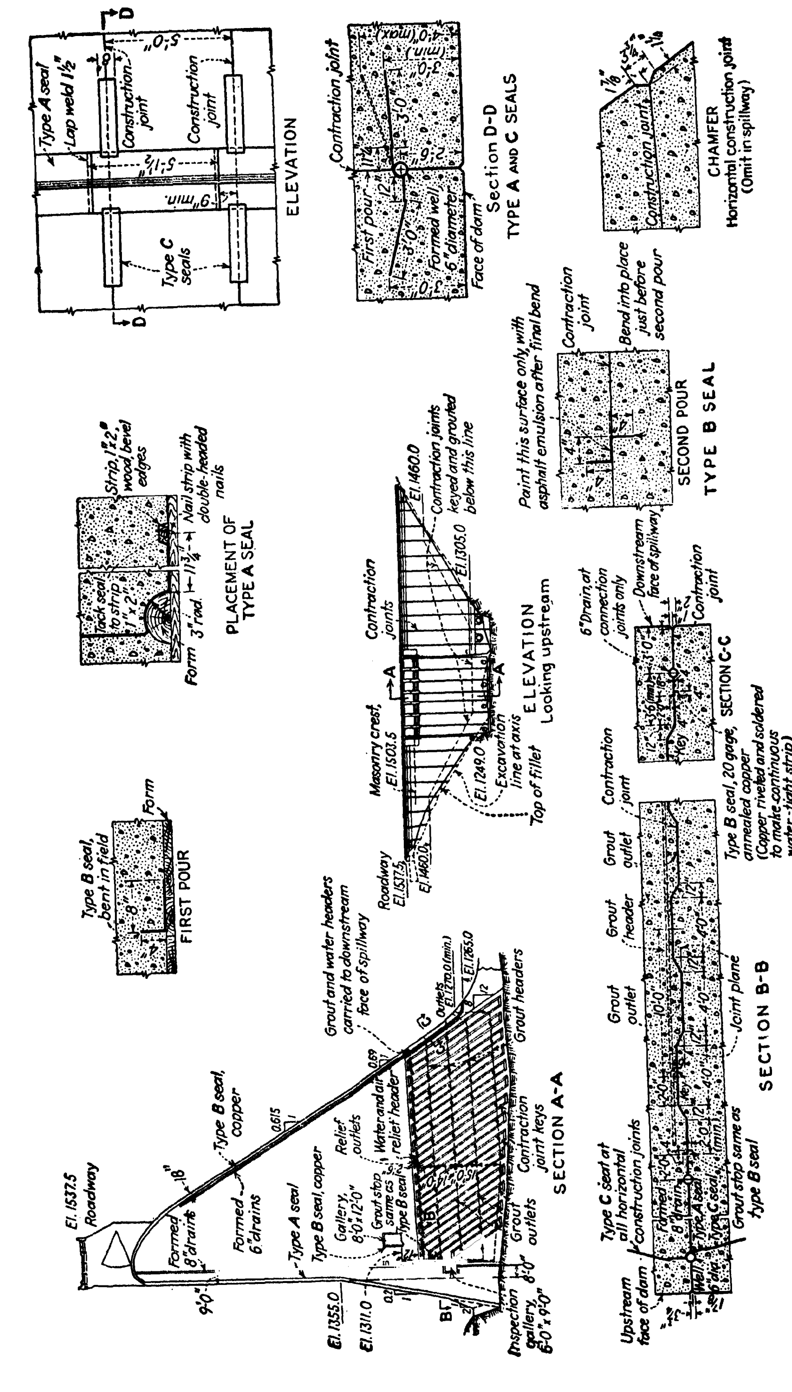


FIG. 11.—Construction details, Hiwassee Dam. (*From Cecil E. Pearce, Design of Hiwassee Dam, Civil Engineering, July, 1940, p. 433.*)

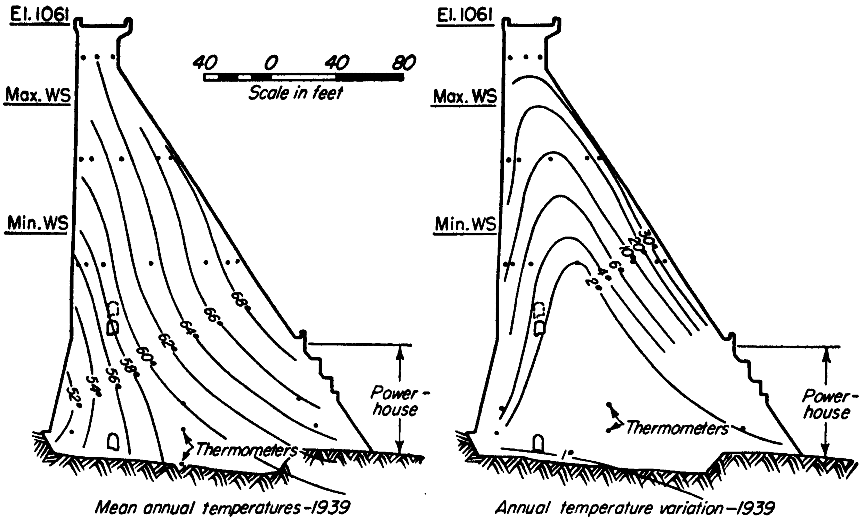


FIG. 12.—Final temperatures, Norris Dam, concrete, blocks 30 and 36. (*Measurements of the Structural Behavior of Norris Dam, Technical Monograph No. 53, TVA.*)

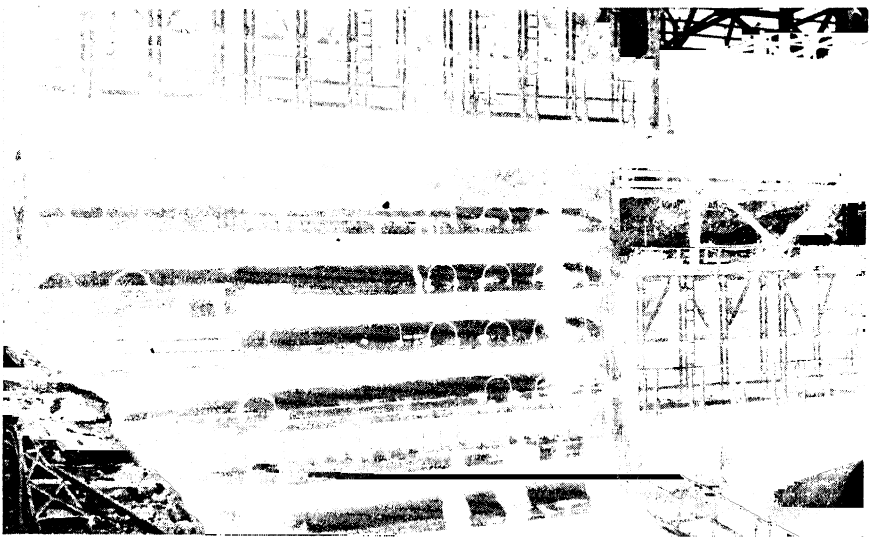


FIG. 13.—Fontana Dam, vertical longitudinal joints, showing metal returns of cooling system. (*Tennessee Valley Authority.*)

Recently the construction of shallow lifts has been open to question. In the construction of the 200-ft-high Stewartville Dam, near Ontario, the highest lift that was practical to form—in the neighborhood of 50 ft was poured. Rate of placing was the governing factor rather than height of lift.[1]

The cooling systems used in the construction of Grand Coulee, Shasta, and other high dams recently built by the Bureau of Reclamation are made up of embedded coils,

[1] Ontario Hydro Pushes Work on Power Dam at Stewartville, *Eng. News-Record*, Sept. 2, 1948, p. 65.

placed in parallel and served by supply and return headers. The coils consist of 1 in. outside diameter, thin-walled metal tubing about 800 ft long, coupled with expansion-type couplings.

The vertical spacing of the pipes is usually 5 ft, the tubing being placed on the top of each lift after the concrete has hardened. Horizontally the spacing varies between 2 and 6 ft, depending upon the extent of cooling required.

The velocity of flow through the embedded coils is not less than 2 fps, or about 4 gpm in the 1-in. tubing. Water may be either pumped through the coils or circulated by a gravity system. When river water is used, the warmed water, after passing through the coils, is wasted. When refrigerated water is used, the warmed water is returned to the refrigerating plant and used repeatedly. The cooling pipes in Fontana Dam are shown by Fig. 13.

The temperature of the concrete is determined by resistance thermometers either embedded in the concrete or inserted in pipes embedded in the concrete.

Basic to the design of any crack control system is the formula for stress in any point in a dam due to temperature change.[1]

$$f = CE_cR(t_p + t_r - t_f)$$

where C = coefficient of thermal expansion of concrete, usually about 0.000005 in.
E_c = modulus of elasticity of concrete, usually between 4 and 5 million psi for 28-day concrete.
R = restraint factor.
t_p = placing temperature of the mix.
t_r = temperature rise of concrete after placing due to heat of hydration of cement.
t_f = final stable temperature of concrete.

In the time required to produce temperature stress, plastic flow and other factors will produce an effective value to E_c varying between 2 and 3 million psi.

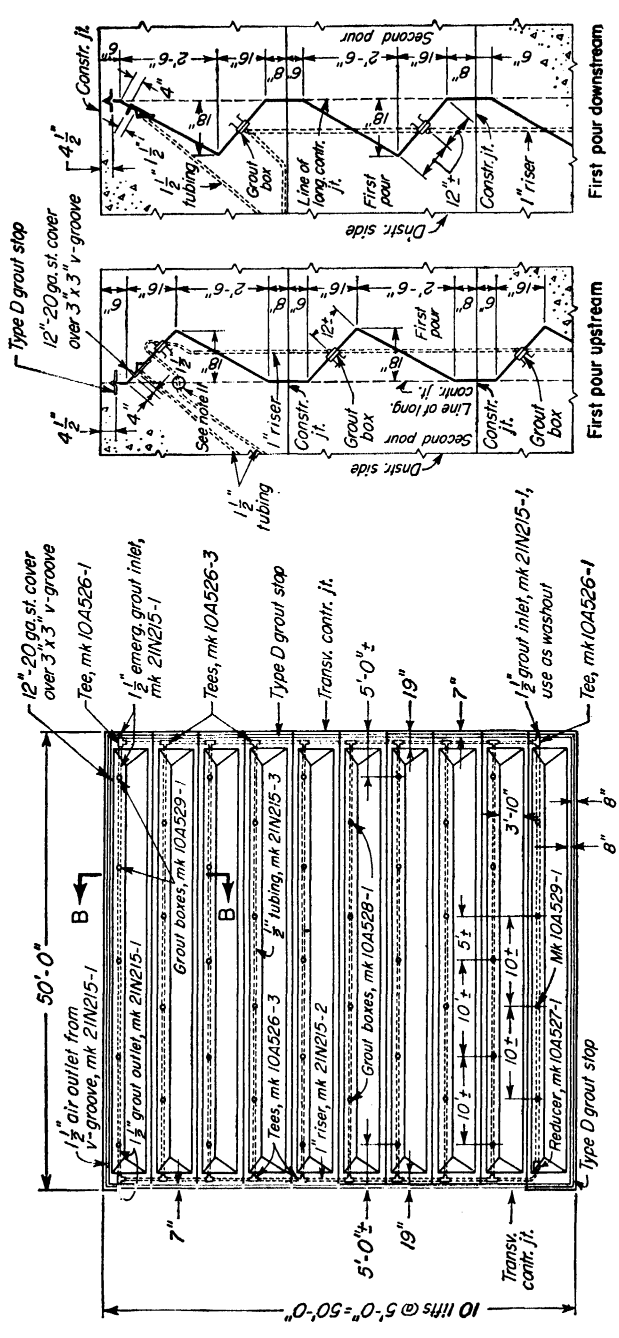
The restraint factor R is 100 per cent at the foundation. This decreases very rapidly above the foundation. In designing the Fontana Dam, the following relations were used: for a length of block L, R reduces to 50 per cent for a height of $0.15L$ and to zero at a height of $0.5L$ above the foundation. R will increase appreciably above any level where concreting stops for more than 10 days or 2 weeks. In order to minimize restraint and to limit tensile stresses due to shrinkage and temperature change, the Fontana section was divided by three longitudinal joints spaced 83 to 100 ft as shown by Fig. 10.

t_p is dependent mostly on the temperatures of the water, cement, and aggregates entering the mixing plant. With no special control except winter heating, it will approximate air temperatures with variations from 40F in winter to 80F in summer.

t_r depends upon the heat-generating characteristics of the cement. It will be about 36F for a mix using 0.8 bbl of Type B cement per cubic yard and placed in 5-ft lifts at 3-day intervals. Lower values may be obtained by other combinations of lifts and time intervals.

t_f is affected by climatical conditions, such as annual average air temperature, reservoir water temperature, foundation temperature, and exposure of the dam to the sun. At Fontana, t_f ranges from 45F near the upstream face to 65F near the downstream face. These temperatures are also approximated in Norris as shown (Fig. 12).

[1] Design of Fontana Dam for Stresses Caused by Temperature Change, *Report* 17-19, T.V.A.



SECTION B-B

TYPICAL KEYS-AREAS OF HIGH SHEAR

Showing first pour upstream

Details similar in long. jt. C-D, except height of grout area

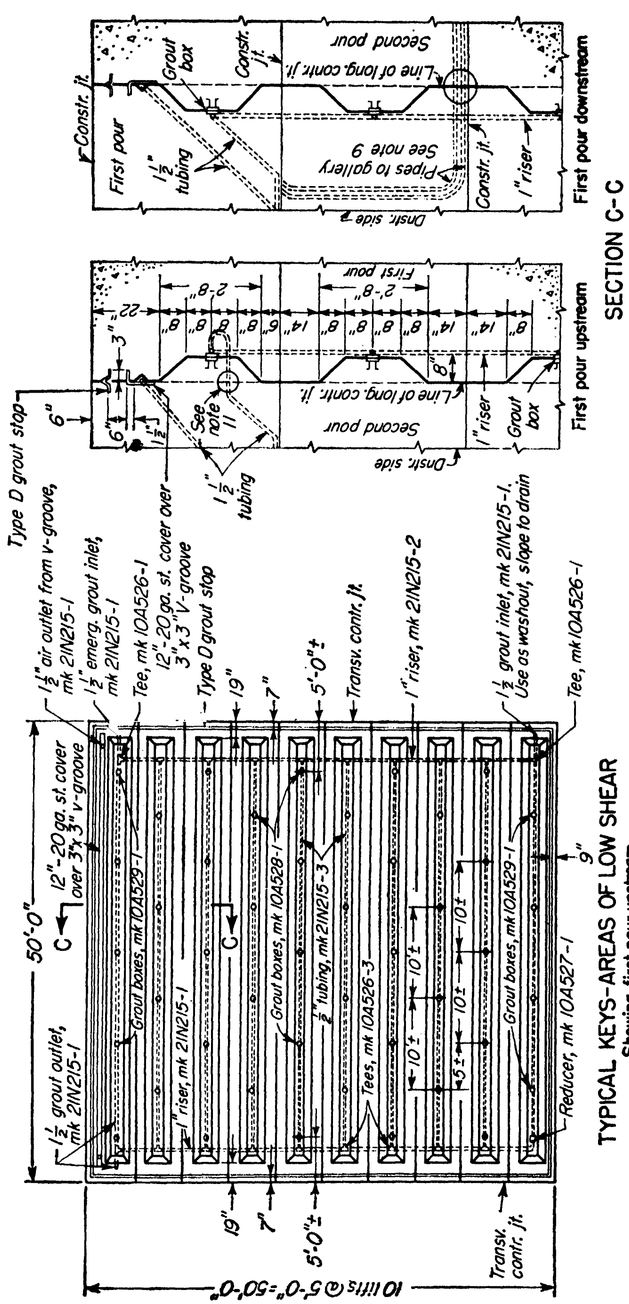


FIG. 14.—Fontana Dam—typical detail of longitudinal joints.

The problem of control can be summarized as follows

1. Lower the placing temperature t_p, by cooling the mix and reduce the temperature rise, t_r, through a proper arrangement of the pouring schedule and by circulating cooling water through the concrete immediately after a lift is poured

2 Reduce the volume of concrete where high restraint occurs combined with a careful arrangement of pouring schedule and forced cooling This method was adopted at Fontana

In the design of high-gravity dams, it is desirable to keep f below 200 psi.

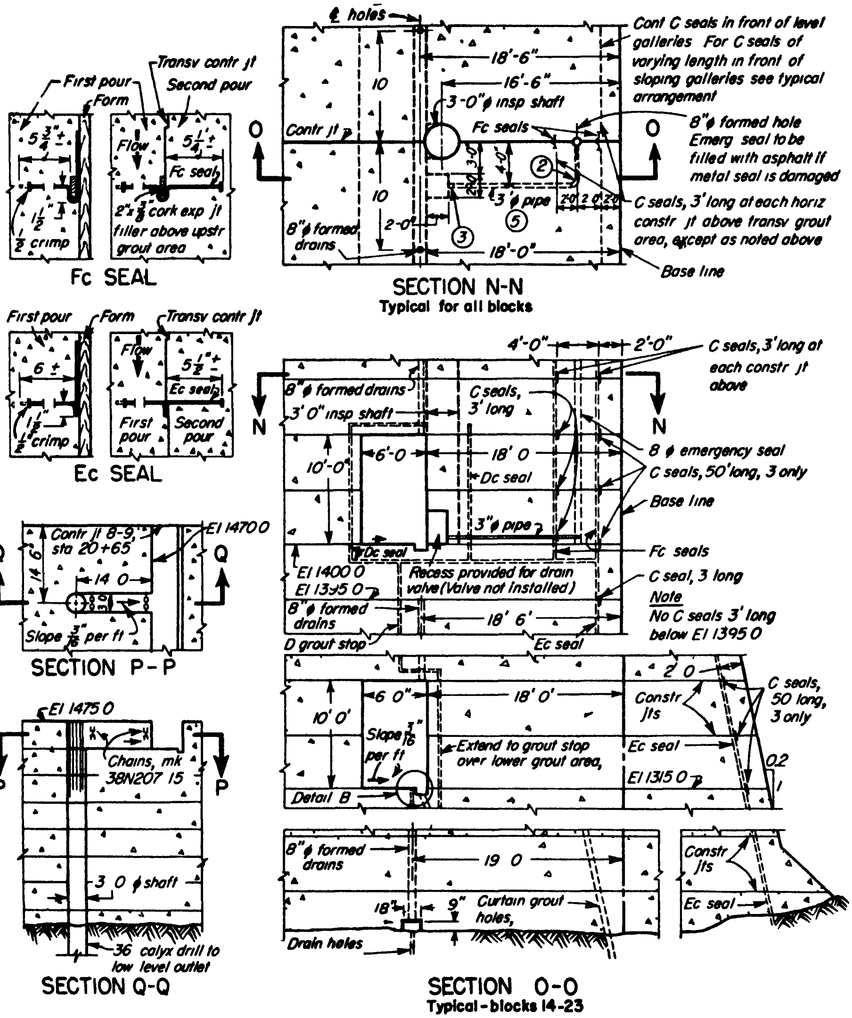


FIG 15 —Fontana Dam—typical arrangement of drains, seals, and water stop.

Water Stops. Metal water stops are used frequently as adjuncts of construction joints in order to deter the passage of water through these joints In the United States, it has become nearly standard practice to use 20-oz soft annealed copper for them Monel metal is considered more durable, however, and for this reason it was selected for the water stops in Hoover Dam Stainless-steel strips, 20 gage, were used for the Hiwassee Dam

The ultimate effectiveness of any metal water stop is open to question. It is possible that in time the copper strips may become brittle and crack. The various types of water stops used in Hiwassee Dam are shown by Fig. 11. The arrangement of seals and water stops for Fontana is shown by Fig. 15.

A yielding type of rubber or plastic seal has been used for a number of dams. Rubber should be used only in locations which are always wet and dark. Such seals are usually about 3/8 in. thick by 9 in. long with a 1/2-in.-radius bulb at each end and a cored 3/4-in.-radius bulb at the insersection of the joint.

Keyways. There are two schools of thought concerning the use of keys in transverse contraction joints. One urges the complete elimination of all keys on the ground that they have no structural value; the other favors keys on the principle that the structure as a whole should be as nearly monolithic as possible and that keys assist materially in this direction. Examples of designs of both types are plentiful.

In the Hiwassee Dam, keys were placed in the lower third of the area of vertical joints, as shown by Fig. 11. These keys were inclined parallel to the downstream face of the dam or approximately parallel to trajectories of principal stresses. If the reentrant corners of these keys have any influence on the formation of cracks, such cracks will be parallel to the trajectories of principal stress. The parts of the joints that were keyed were grouted according to usual practice. In most of the high dams constructed by the Bureau of Reclamation, transverse contraction joints were grouted throughout the entire section.

Longitudinal contraction joints must necessarily be keyed in order to transmit vertical shearing stresses across the section. In several recently constructed high dams, these keys extend horizontally and are roughly saw-toothed in form. In order to minimize shearing stresses along the keys when the reservoir is full, alternate surfaces are placed approximately on trajectories of principal stress. After the concrete has received its shrinkage, longitudinal joints must be grouted throughout. Figures 13 and 14 show typical details of a longitudinal joint for the Fontana Dam.

There appears to be no standard practice governing the size and spacing of keys in contraction joints. Extreme variation is revealed by an examination of existing designs indicating that the judgment of the designer is the principal factor in determining the size and spacing of keys.

Drainage. In order to relieve uplift, it is customary to provide both vertical and horizontal drainage galleries. In the Hiwassee Dam, 8-in. round drains, placed on the contraction joints, connect with a longitudinal gallery as shown by Fig. 11. A row of 5 in. diameter drain holes parallel to the axis and 8 to 12 ft downstream from it discharge into the lowest drainage gallery. The arrangement of the drains in the Fontana Dam is shown by Fig. 15.

Foundation Treatment. In order to obtain a watertight foundation and assist in eliminating uplift, it is usually necessary to grout the foundations. Two general methods are generally used: curtain grouting on a line of grout holes near the upstream face of the dam and blanket grouting or general grouting over the entire remaining area of the foundation.

In curtain grouting for the Hiwassee Dam, holes were spaced on 5-ft centers on a single line upstream from the drain holes. Blanket grouting followed no particular pattern, but the holes, varying from 5 to 40 ft in depth, were spotted where needed to reach all seams, joints, faults, and cracks.

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SECTION 3

ARCH DAMS

BY IVAN E. HOUK

An arch dam is a curved dam that carries a major part of its water load horizontally to the abutments by arch action, the part so carried being primarily dependent on the amount of curvature. Massive masonry dams, slightly curved, are usually considered as gravity dams, although some parts of the loads may be carried by arch action. Many early arch dams were built of rubble, ashlar, or cyclopean masonry. However, practically all arch dams constructed during recent years have been built of concrete.

Arch principles have been used in bridges and buildings since about 2000 B.C. Apparently, Pontalto Dam, built in Austria in 1611 A.D., was the first arch dam recorded in engineering history.[1] The 64-ft Bear Valley Dam, built in the San Bernardino Mountains of southern California in 1883, was the first arch dam constructed in America. It was followed by the 95-ft Sweetwater Dam, in 1888, and the 88-ft Upper Otay Dam, in 1900, both built near San Diego, Calif. Lake Cheesman Dam, a 236-ft curved gravity dam constructed near Denver, Colo., in 1904, was the first high dam for which a careful attempt was made to analyze arch action. Since 1904, many arch dams have been built in the United States and certain foreign countries.

ARCH-DAM TYPES

Arch dams are usually classified on the basis of thickness, symmetry with respect to the crown section, or characteristics of extrados and intrados curves. For instance, they may be referred to as thin-arch, thick-arch, constant-thickness, variable-thickness, symmetrical-arch, nonsymmetrical-arch, single-arch, compound-arch, constant-radius, or variable-radius types, or by other more or less self-explanatory designations. During the last 20 years there has been a growing tendency to adopt the last two terms as the general basis for classification. Typical examples of actual designs are given later.

Constant-radius Dams. A constant-radius arch dam generally has a vertical upstream face. However, an unusually high structure, such as Hoover Dam, may have extrados curves of gradually increasing radii in the lower part of the canyon, to provide a vertical batter near the base of the higher cross sections. Intrados curves may be concentric or nonconcentric with reference to extrados curves. They usually have decreasing radii as the depth below the crest increases, to provide the increased thickness needed for the higher reservoir pressures. Constant-radius arch types are particularly adapted to U-shaped canyons, where relatively large proportions of the water load at the lower elevations are carried by cantilever action.

Variable-radius Dams. A variable-radius arch dam, also known as a constant-angle arch dam, usually has extrados and intrados curves of gradually decreasing radii as the depth below the crest increases.[2] This is to keep the central angle as large

[1] NOETZLI, FRED A., Pontalto and Madruzza Arch Dams, *West. Constr. News*, Apr. 10, 1932, pp. 451–452.

[2] JORGENSEN, LARS R., The Constant-angle Arch Dam, *Trans. A. S. C. E.*, **78**, 685–733, 1915.

and as nearly constant as possible, so as to secure maximum arch efficiency at all elevations. Variable-radius arch dams often have vertical, or even overhanging, faces at the upstream side near the abutments and at the downstream side near the crown. Face slopes may be relatively flat at the upstream side near the crown and usually are relatively flat at the downstream side near the abutments. The variable-radius type of arch dam is frequently adapted to narrow V-shaped canyons.

GENERAL THEORY OF ARCH DAMS

The general theory of arch dams now used in design constitutes a comparatively recent development in engineering science. The mathematical principles, laws of mechanics, and theories of elasticity involved in an arch-dam analysis have been known for many years. Nevertheless, the proper methods of applying such knowledge in arch-dam design have been largely developed during the last two decades. Summarized arch-dam formulas are given later. This section is confined to the general theory of arch-dam action.

Arch Action Only. Many arch dams have been designed on the theory that all horizontal water loads are carried horizontally to the abutments by arch action and that only the dead load weights, plus the vertical water loads in the case of a sloping upstream face, are carried vertically to the foundations by cantilever action. In some of the earlier designs, arch thicknesses were determined by the unreliable thin-cylinder formula $t = RP/S$, where t is the thickness of the arch, R the radius of the upstream face, P the water load, and S the allowable concrete stress. In other cases, thicknesses were determined by analyses of elastic arches, formulas being used such as those developed by the late William Cain.[1]

Designs that ignore centilever action can seldom be considered wholly satisfactory. The vertical cantilevers that make up the dam are restrained at the foundation. They must bend until their deflected positions coincide with the deflected positions of the arch elements. Since the cantilever bending can be produced only by the transfer of water load through the cantilever elements to the foundation, the theory that the entire water load is carried horizontally to the abutments by arch action is obviously incorrect.

Cantilever and Arch Action. The most satisfactory and now commonly accepted theory of arch dams is that the horizontal water load is divided between the arches and cantilevers; so that the calculated arch and cantilever deflections are equal at conjugate points in all parts of the structure. The load distribution required to fulfill this criterion is determined by trial. Consequently, the procedure is called the trial-load method. When the deflections are in satisfactory agreement, the arch and cantilever stresses are calculated and considered to be the true stresses in the dam.

The first analyses are usually made on the theory that any element can move in a radial direction without being restrained by adjacent elements and without being subjected to tangential or twisting deformations. Since this theory is inaccurate, the discrepancies must be corrected by subsequent trial-load adjustments which make adequate allowances for tangential shear and twist effects.

The cantilever elements are assumed to be fixed at the foundation; and the arch elements, fixed at the abutments. However, the rock formations may be moved by loads transferred through the dam and by direct reservoir pressures. Although foundation and abutment materials are probably never uniformly elastic, owing to the presence of cracks, fissures, faults, and bedding planes, their movements may be roughly calculated by elastic formulas and included in the analyses of arch and cantilever deflections. Since the dam is curved, the cantilever elements are vertical

[1] CAIN, WILLIAM, The Circular Arch under Normal Loads, *Trans. A. S. C. E.*, **85**, 233–283, 1922.

slices, bounded by vertical radial planes. Arch elements are horizontal slices, with constant vertical thicknesses from abutment to abutment.

Basic Assumptions.—The basic assumptions usually made in designing arch dams may be briefly listed as follows:[1]

1. The foundation and abutment rock is homogeneous, isotropic, and uniformly elastic.
2. The concrete is homogeneous, isotropic, and uniformly elastic.
3. The stresses are well within the elastic limit, and Hooke's law applies.
4. Stresses vary as a straight line between the upstream and downstream faces of the dam in both arch and cantilever elements.
5. Plane surfaces in the unloaded structure remain plane after the load is applied.
6. Temperature changes in the arches vary with the horizontal thickness, but are constant throughout each element.
7. Temperature strains and stresses are proportional to temperature changes.
8. Effects of flow of concrete and rock materials may be neglected.
9. Tension stresses are relieved by cracking; so that all loads are carried by compressive and shearing stresses in the uncracked portions of the dam.
10. Radial construction joints are grouted or open slots filled; so that the dam acts as a monolith.
11. Vertical shrinkage is completed before the joints are grouted or the slots filled; so that no loads are transferred laterally by vertical arching.

LOADS ON ARCH DAMS

Loads on arch dams are essentially the same as loads on gravity dams, except that temperature changes, which usually are not important considerations in straight dams, cause important deflections and stresses in curved dams. The principal dead load is the concrete weight. The principal live load is the reservoir water pressure. Additional loads may be imposed by tail-water pressure, uplift pressure, upward water pressure under overhanging sections, deposition of silt on sloping faces, presence of silt in flood flows, and formation of ice surfaces. Earthquake accelerations cause momentary changes in water pressure and an additional live load due to the inertia of the concrete.

The general subject of forces on dams is treated in Secs. 2 and 4. Discussions presented herein are confined to additional considerations required in designing arch dams.

Uplift Pressure. Uplift pressure seldom has an important bearing on the safety of an arch dam. If no cracking occurs, it can be neglected. If cracking occurs, uplift pressure in the cracks causes increases in downstream deflections, changes in load distribution, and increases in maximum compressive stresses in both arch and cantilever elements. Uplift in horizontal cantilever cracks usually has a greater effect on stress conditions than uplift pressure in vertical arch cracks.[2]

Ice Pressure. Ice pressure causes a continuous concentrated load along the arch element at the elevation of the ice. This load is carried partly by arch action and partly by cantilever action. The actual distribution can be determined by a trial-load analysis. The transference of ice loads to the foundation and abutments can be facilitated by placing vertical reinforcing at the faces of the dam. Concentration of reinforcing at the downstream face, along the elevation of the ice, increases the proportion of ice load carried by arch action.

Temperature Loads. Temperature changes cause internal forces that move the dam upstream during the summer and downstream during the winter, the former condition working against the reservoir load and the latter with it. Consequently the winter condition is usually the more important in the stress analysis.

Since zero temperature stresses occur at the time of closing the arches, the closures should be made after the setting heat has been developed and dissipated and after a winter season has reduced concrete temperatures to their minimum values. However,

[1] HOUK, IVAN E. and KENNETH B. KEENER, Masonry Dams—Basic Design Assumptions, *Trans. A. S. C. E.*, **106**, 1115–1130, 1941.
[2] HOUK, IVAN E., Uplift Pressure in Gravity Dams, *Western Constr. News*, July 25, 1930, pp. 344–349.

practical considerations usually prevent closure at the most desirable time. Unless the concrete is artificially cooled, as at Hoover Dam,[1] it may be necessary to include some of the setting-heat effects in analyzing temperature stresses.[2]

If closure can be deferred until the setting heat has been fully developed and completely dissipated, the designer may assume that the temperature changes to be considered in the arch analyses will be the reductions from mean annual to minimum concrete temperatures expected during full reservoir load. Figure 1 shows the maxi-

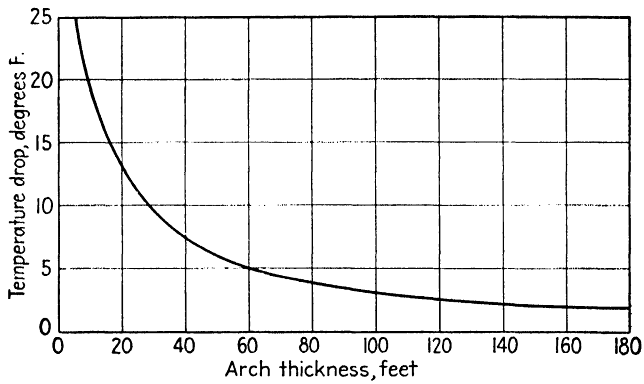


FIG. 1.—Maximum drop in average concrete temperature, below mean annual.

mum drop in average concrete temperature, below mean annual, which may occur in arches of different thickness.[3]

STRESS DISTRIBUTION IN ARCH DAMS

The distribution of stress in an arch dam varies with the horizontal curvature, shape of vertical cross sections, general dimensions of the structure, and uniformity of canyon profile. Pronounced humps in the rock surface cause stress concentrations in adjoining concrete, sometimes resulting in the formation of diagonal cracks. Maximum cantilever stresses often occur at such humps, even though the elevations are appreciably higher than the base of the maximum cross section.

Cantilever Stresses. Maximum cantilever stresses in arch dams, built at sites free from pronounced irregularities, usually occur at the base of the highest cantilever. During full reservoir load, maximum compressive stresses usually occur at the downstream edge of the base, but may occur at the upstream edge in comparatively high and thick dams provided with an upstream batter. Tension often occurs at the upstream edge of the base in relatively thin arch dams. During the empty condition of the reservoir, maximum compressive stresses in the crown cantilever usually occur at the upstream edge of the base in constant-radius dams and at the downstream edge of the base in variable-radius dams.

Arch Stresses. Arch stresses in the central and upper portions of arch dams are commonly higher than in the lower portions. Maximum arch stresses usually occur at the crown and abutment sections. At the crown section, relatively high compressive stresses usually occur at the upstream face of the dam and relatively low compressive or tension stresses at the downstream face. At the abutment sections,

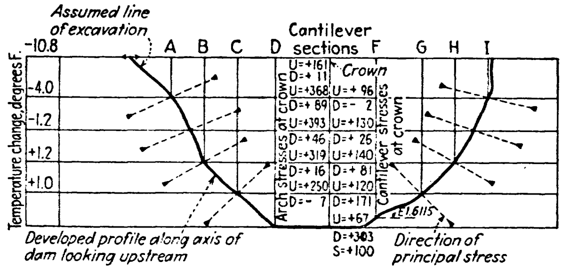
[1] STEELE, BYRAM W., Cooling Boulder Dam Concrete, *Eng. News-Record*, **133**, 451–455, 1934.

[2] HOUK, IVAN E., Setting Heat and Concrete Temperature, *Western Constr. News*, Aug. 10, 1931, pp. 411–415.

[3] HOUK, IVAN E., Temperature Variations in Concrete Dams, *Western Constr. News*, Dec. 10, 1930, pp. 601–608.

Cantilever stress at A	Cantilever stress at B	Cantilever stress at C	Cantilever stress at D	Arch stress at abut	Principal stress at abutment P_2	Principal stress at abutment P_1	Location of direct stress	Maximum shearing stress, rock plane	Maximum shearing stress, arch abut	Elev
				- 193	0	-193	U.S.	+37	+45	6361
				- 22	0	- 22	D.S.			
+63	+88	+96	+99	- 73	+92	-102	U.S.	+63	+64	6300
+31	+ 3	- 4	- 8	+367	-50	+451	D.S.			
	+100	+120	+136	- 82	+95	- 96	U.S.	+87	+81	6250
	+56	+35	+18	+410	+ 8	+485	D.S.			
	+85	+114	+145	- 46	+101	- 62	U.S.	+124	+103	6200
	+139	+105	+72	+324	+ 49	+431	D.S.			
		+90	+131	- 12	+110	- 31	U.S.	+134	+94	6150
		+202	+155	+187	+ 17	+395	D.S.			
			+94				U.S.			6100
			+272				D.S.			

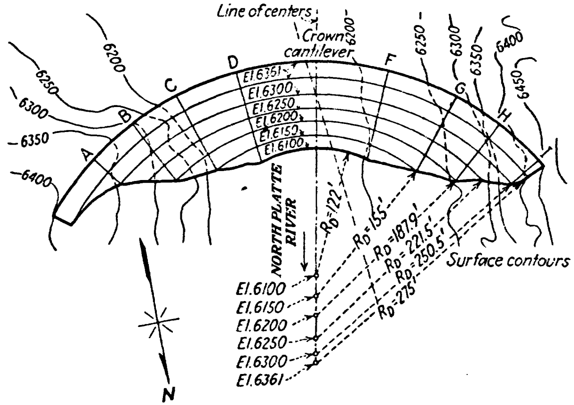
STRESSES AT LEFT OF CROWN



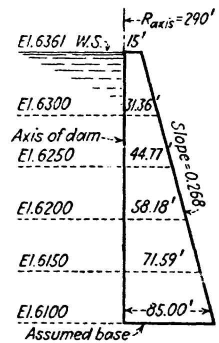
STRESSES AT CROWN AND DIRECTION OF MAJOR PRINCIPAL STRESS AT DOWNSTREAM FACE AT ABUTMENTS

Elev	Maximum shearing stress, arch abut.	Maximum shearing stress, rock plane	Location of direct stress	Principal stress at abutment P_1	Principal stress at abutment P_2	Arch stress at abut	Cantilever stress at F	Cantilever stress at G	Cantilever stress at H	Cantilever stress at I
6361	+44	+36	U.S.	-101	0	-101				
			D.S.	+225	0	+225				
6300	+50	+46	U.S.	-143	+121	- 99	+98	+97	+84	+77
			D.S.	+457	- 12	+429	- 7	- 5	+ 8	+15
6250	+75	+76	U.S.	-106	+110	- 84	+132	+123	+99	
			D.S.	+457	+ 28	+411	+23	+31	+57	
6200	+100	+119	U.S.	- 55	+113	- 34	+136	+117	+92	
			D.S.	+415	+ 43	+310	+84	+102	+131	
6150	+93	+134	U.S.	- 26	+111	- 4	+106	+89		
			D.S.	+396	+ 8	+178	+189	+203		
6100			U.S.				+64			
			D.S.				+290			

STRESSES AT RIGHT OF CROWN



PLAN



MAXIMUM SECTION

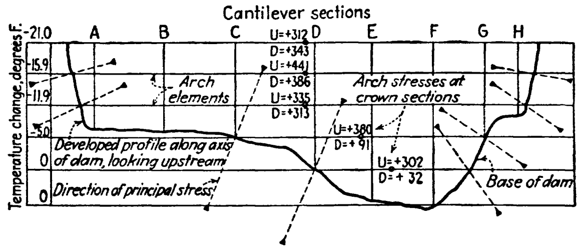
NOTES

Upstream and downstream cantilever stresses act parallel to edges of cantilevers
Upstream and downstream arch stresses act parallel to edges of arches
U = Upstream stress D = Downstream stress
+ = Compression − = Tension
S = Maximum shearing stress
+S means downstream shear
All stresses are pounds per square inch
Assumed earthquake shock has acceleration = 0.10 gravity, period of vibration = 1.0 second, and acts horizontally along line of centers
Arches closed at concrete temperatures 5°F. below mean annual
Effects of twist, tangential shear and abutment deformations included
U.S. = Upstream face D.S. = Downstream face

FIG. 2.—Stress distribution in Seminoe Dam.

CANTILEVER STRESS AT A	B	C	D	ARCH STRESS AT ABUT.	PRINCIPAL STRESS AT ABUTMENT P_2	P_1	LOCATION OF DIRECT STRESS	MAXIMUM SHEARING STRESS ROCK PLANE	ARCH ABUT.	ELEV.
				C +611			U.S. D.S.		+35	1530
+55 + 7	+71 C	+92 C	+140 C	C +488	- 41	+540	U.S. D.S.	+ 68	+66	1490
+37 +76	C +125	C +203	+44 +110	C +490	-58	+626	U.S. D.S.	+ 95	+88	1450
C +187*	C +591*	C +859	C +338	C +304	+202	+965	U.S. D.S.	+331	+115	1410
			C +862	- 22 +237	+145	+990	U.S. D.S.	+305	+122	1370
							U.S. D.S.			1325

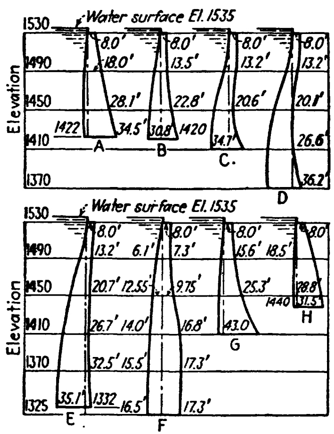
STRESSES IN LEFT PART OF DAM



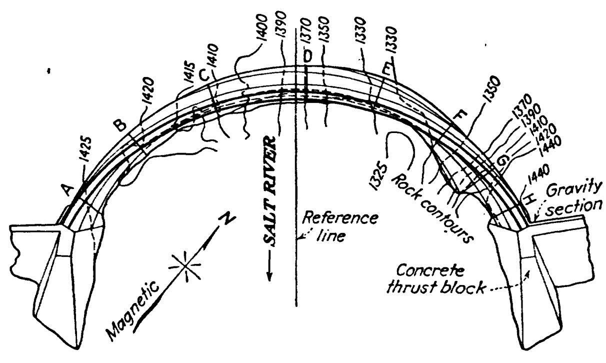
ARCH STRESSES AT CROWN SECTIONS AND DIRECTIONS OF MAJOR PRINCIPAL STRESSES AT DOWNSTREAM FACE OF ABUTMENTS

ELEV.	MAXIMUM SHEARING STRESS ARCH ABUT.	ROCK PLANE	LOCATION OF DIRECT STRESS	PRINCIPAL STRESS AT ABUTMENT P_1	P_2	ARCH STRESS AT ABUT.	CANTILEVER STRESS AT E	F	G	H
1530	+41		U.S. D.S.			C +625				
1490	+48	+49	U.S. D.S.	+493	- 13	C +475	+116 C	+78 C	+17 +50	+39 +23
1450	+92	+102	U.S. D.S.	+520	-65	- 57 +270	+57 +88	+36 +90	C +152	C +144
1410	+182	+197	U.S. D.S.	+605	- 9	- 49 +340	C +228	C +154	C +181	C* +245
1370	+41	+73	U.S. D.S.	+654	+ 18	- 9 +229	C +429	C +293		
1325			U.S. D.S.				C +735	C +453		

STRESSES IN RIGHT PART OF DAM



CROSS SECTIONS AT CANTILEVERS



PLAN

NOTES

Upstream and downstream cantilever stresses act parallel to edges of cantilevers

Upstream and downstream arch stresses act parallel to edges of arches

U=Upstream stress D=Downstream stress

+=Compression −=Tension

C Cracked to point of zero stress

* Stress at base of cantilever

\+ Shearing stress means downstream shear

All stresses are pounds per square inch

∘ Location of arch crown

Reservoir water surface at top of parapet

Tailwater surface at elevation 1413

Uplift pressures vary from reservoir head at upstream face to zero or tailwater head at downstream face

Uplift pressures are applied over 50 per cent of the area

Effect of twist, tangential shear, and abutment deformations included

U.S.=Upstream face D.S.=Downstream face

FIG. 3.—Stress distribution in Stewart Mountain Dam.

stress conditions are usually reversed in accordance with the change in moment sign which generally takes place near the quarter points. Stress conditions at the abutments may be somewhat different in the top arches of long thin dams, owing to the upstream deflections that sometimes occur near such locations. Shearing stresses at the crown section are zero in symmetrical arches symmetrically loaded.

Principal Stresses. Major principal stresses along the contact between concrete and rock usually act in planes approximately horizontal at the top of the dam, practically vertical at the base of the maximum cross section, and at gradually varying inclinations along the intervening parts of the profile.

Stress Examples. Figure 2 shows the stress distribution in Seminoe Dam, central Wyoming, under full reservoir load combined with horizontal earthquake accelerations, construction joints grouted at concrete temperatures 5 deg below mean annual being assumed. Seminoe Dam is a 261-ft constant-radius arch dam, located on the Kendrick Irrigation Project. General dimensions of the structure are given in Table 8.

Figure 3 shows the stress distribution in Stewart Mountain Dam, central Arizona, determined by a recent trial-load analysis for maximum flood conditions, the arch elements being assumed closed at mean annual concrete temperatures. Stewart Mountain Dam is a 212-ft variable-radius arch dam, located on the Salt River Valley Irrigation Project. General dimensions are given in Table 9. This dam was designed for a maximum stress of 650 psi, the full water load being assumed to be carried by the arch elements.

DESIGN OF ARCH DAMS

The design of an arch dam is a cut-and-try problem. Preliminary plans must be prepared, stresses analyzed, and costs compiled. The best design will have the stresses as uniformly distributed as possible, tension stresses as low as possible, maximum compressive and shear stresses kept within allowable limits, and the total cost of the structure held to a minimum. The following sections briefly discuss technical problems involved in determining the best design.[1] Details of structural features and construction methods are not considered.

Allowable Stresses. Stresses in arch dams analyzed by trial-load methods, on the assumption of 3,000-lb concrete, a straight line distribution of stress, and no loads carried by tension, should not exceed 600 psi compression, or 300 psi shear, during maximum reservoir loads. Increases of 10 to 15 per cent may be permissible, momentarily, during intense earthquake shocks. However, decreases of 25 to 35 per cent should be made if the dam is analyzed by approximate methods, such as placing the full water load on the arch elements or bringing the arch and cantilever deflections into agreement at the crown section only.

Ordinarily, vertical tension at the upstream face may be as high as 100 psi without analyzing secondary cantilevers, when the corresponding compression at the downstream face does not exceed 500 psi. Horizontal tension at the upstream face may be as much as one-third the corresponding compression at the downstream face without analyzing secondary arches, when the sum of the tension and compression does not exceed 600 psi. In an unusually high massive structure, provided with an upstream batter, compressive stresses somewhat higher than 600 psi may be permitted along the upstream face near the base of the crown cantilever, owing to the triaxial conditions of compression occurring at such locations.

Maximum Stresses. Table 1 gives maximum arch, cantilever, and principal stresses in some arch dams recently designed or analyzed by trial-load methods. Effects of tangential shear and twist action were included in all cases except Gibson

[1] HOUK, IVAN E., Technical Design of High Masonry Dams, *Engineer*, Aug. 4, 1933, pp. 105–106; Aug. 11, 1933, pp. 128–130.

TABLE 1.—MAXIMUM STRESSES IN ARCH DAMS DETERMINED BY TRIAL-LOAD ANALYSES
(Psi)

Name of dam	Type[3]	Max height, feet[4]	Cantilever stresses			Arch stresses			Principal stresses[6]			Loading condition
			Comp.	Tens.	Shear	Comp.	Tens.	Shear	Comp.	Tens.	Shear	
Hoover	C.R.	731	565	None	154	231	31	120	565	16	160	Full reservoir, 5 deg subcooling
Owyhee	C.R.	421	358	6	86	294	242	175	413	344	143	W.S. at top of dam[7]
Arrowrock[1]	C.R.	356	466	39	164	305	314	128	496	314	207	Full reservoir with earthquake
Parker	C.R.	335	289	C[5]	65	542	1	76	451	C	95	Full reservoir with earthquake 7 deg subcooling
Ariel	V.R.	313	808	C	...	560	107	...	.....	...	...	W.S. at top of parapet
Horse Mesa	V.R.	305	956	C	250	1061	C	273	1,074	C	297	W.S. at top of parapet
Seminoe	C.R.	261	303	8	100	429	193	103	485	193	134	Full reservoir with earthquake, 5 deg subcooling
Mormon flat	V.R.	229	633	C	74	893	C	181	1,095	C	322	W.S. at top of parapet
Stewart Mountain	V.R.	212	862	C	56	625	C	182	990	C	331	W.S. at top of parapet
Gibson[2]	C.R.	199	605	C	...	364	66	...	.....	...	...	W.S. at top of parapet
Deadwood	C.R.	168	472	C	...	360	184	...	.....	...	...	W.S. at top of parapet
Cat Creek	V.R.	118	413	C	...	277	114	...	.....	...	...	W.S. 1.5′ above top of dam

1 Analysis for 5-ft increase in height.
2 Tangential shear and twist not included.
3 C.R., constant radius; V.R., variable radius.
4 Above lowest foundation, see Tables 8 and 9 for additional dimensions.
5 C, cracked, no load carried by tension.
6 Along abutment planes.
7 W.S., water surface.

Dam. Effects of rock deformations were considered in all cases. General dimensions of the dams are given in Tables 8 and 9.

Constants Needed in Analyses. Table 2 gives general values of constants needed in analyzing arch dams. These values may be used in preliminary studies where more accurate information is not available. They should be replaced by data based on field and laboratory measurements before adopting final designs. Tabulated values of modulus of elasticity are for sustained load conditions. Great accuracy in determining elastic properties of canyon rock is not necessary since effects of foundation and abutment movements are of a secondary nature. The modulus of elasticity for direct stress may be assumed to be the same for tension and compression, for both rock and concrete materials. The modulus for shear can be computed by the formula $E_s = E/2(1 + \mu)$, where μ is Poisson's ratio, E the modulus for direct stress, and E_s the modulus for shear.

TABLE 2.—CONSTANTS NEEDED IN ANALYZING ARCH DAMS

Constant	Material	Values	Units
Weight, saturated	Concrete	150	lb per cu ft
Weight, saturated	Silt	110–120	lb per cu ft
Weight, saturated	Sand	110–120	lb per cu ft
Temperature coefficient	Concrete	0.0000040–60	ft per ft per deg F
Poisson's ratio	Concrete	0.15–0.22	
Poisson's ratio	Rock	0.10–0.30	
Modulus of elasticity	Concrete	2–2.5 million	psi
Modulus of elasticity	Limestone	1–2 million	psi
Modulus of elasticity	Granite	2–4 million	psi
Modulus of elasticity	Sandstone	1–1.5 million	psi

Preliminary Plans. In preparing preliminary plans for an arch dam, the engineer should study designs adopted for similar sites, where dimensions and curvature were accurately determined by trial-load analyses. Published descriptions of constructed dams and data included in Tables 8, 9, and 10 will be helpful in preliminary investigations. In order to avoid high-tension stresses at the reservoir face and to secure maximum arch efficiency, central angles should be as large as possible. Theoretical considerations, based on the thin-cylinder formula, show that a central angle of 133°34′ is most advantageous from the viewpoint of economy.[1] However, practical considerations, together with topographical conditions, usually prevent the adoption of such angles for the lower arch elements.

The extrados and intrados curves should be located so that the ends of the arches converge in a downstream direction. Otherwise radial buttresses at the abutments may be needed to carry the loads transferred horizontally by radial shear. Such buttresses are often necessary where arch elements abut against gravity tangents. Radial arch ends are most desirable; but smaller amounts of convergence usually suffice where radial construction requires excessive excavation, as in large thick dams.

Top widths of arch dams are usually made constant from abutment to abutment. Arch thicknesses at lower elevations may be constant or may increase toward the abutments, depending on stress conditions. Thickening toward the abutments may begin at the crown sections, at the quarter points, or may be secured by providing long radius fillets at the ends of the intrados curves. Abutment thickening should be warped between adjacent arch elements, so as to avoid undesirable appearances at the downstream face.

[1] JORGENSEN, *op. cit.*, p. 689.

The ratio of length to thickness at the top of the dam should not exceed about 60. Usually the ratio will be smaller, owing to the desirability of stiffening the upper part of the structure or the necessity for providing a roadway along the top. Considerations of slenderness ratio are not important at the lower arches. The additional thicknesses needed from the stress viewpoint, together with the reduced widths of the canyon, will reduce the ratio to satisfactory values. Furthermore, the restraining effect of the cantilevers on the bending of the arch elements increases as the depth below the top increases.

Foundations and Abutments. Depths of required excavation must be estimated in determining dimensions for preliminary analyses. Sometimes humps in rock profiles, which may cause stress concentrations, can be removed in preparing rock surfaces. Sometimes deep holes, or relatively narrow gorges, can be plugged with concrete and treated as parts of the foundation instead of parts of the dam. Excavated surfaces should be gradually warped between adjacent elevations, pronounced stepping along abutment planes being avoided. Adequate grouting and draining should always be specified. Geological conditions at the dam site should be approved by competent foundation experts before proceeding with detailed designs.

ANALYSES OF PRELIMINARY PLANS

Analyses of preliminary plans for arch dams are usually made for full reservoir load plus maximum temperature drop. Analyses for other loads, which seldom require major changes in dimensions, can be made after general designs are tentatively adopted. Analyses of preliminary plans may be made by the following methods:

1. Assigning full horizontal loads to arch elements.
2. Dividing horizontal loads between arch and cantilever elements on the basis of a radial adjustment of deflections at the crown section.
3. Dividing horizontal loads between arch and cantilever elements on the basis of radial adjustments at several vertical sections.

The method to be used in a particular case depends on the shape of the canyon and the type, height, and importance of the structure. If the rock profile contains pronounced irregularities, or the shape of the canyon is not symmetrical, the analyses should be made by the trial-load method, listed as 3, regardless of the size of the dam. If the canyon is V-shaped, with comparatively uniform sides, and the dam of nominal size and importance, the second method may suffice. If the canyon is relatively regular and narrow, and the dam of low height, so that a symmetrical thin arch structure with large central angles can be adopted, the first method may be sufficient.

Full Load on Arches. Formulas for analyzing circular arches of constant thickness, under uniform radial loads, have been developed by various engineers. The studies made by William Cain were especially noteworthy.[1] Slightly modified forms of Cain's equations for thrust and moment at the crown and abutment sections, due to uniform water loads, are as follows:

Thrust at crown, $$H_0 = pr - \frac{pr}{D} 2\varphi \sin \varphi \frac{t^2}{12r^2} \tag{1}$$

Moment at crown, $$M_0 = -(pr - H_0)r\left(1 - \frac{\sin \varphi}{\varphi}\right) \tag{2}$$

Thrust at abutments, $$H_a = pr - (pr - H_0) \cos \varphi \tag{3}$$

Moment at abutments, $$M_a = r(pr - H_0)\left(\frac{\sin \varphi}{\varphi} - \cos \varphi\right) \tag{4}$$

In the preceding formulas, r is the radius to the center line of the arch, p the normal radial pressure at the center line, t the horizontal arch thickness, and φ the angle

[1] CAIN, *loc. cit.*

between the crown and abutment radii. The center-line pressure p is the extrados pressure times the ratio of the upstream radius to the center-line radius (see Fig. 4).

If shear is neglected, values of D are given by the equation

$$D = \left(1 + \frac{t^2}{12r^2}\right) \varphi \left(\varphi + \frac{\sin 2\varphi}{2}\right) - 2 \sin^2 \varphi \tag{5}$$

In order to simplify the formulas for crown thrust, D has been used in Eqs. 1 and 8, in lieu of the lengthy right-hand part of Eq. 5, which appears in the original formulas.

When shear is included, D is replaced by D_s the value of which is given by

$$D_s = \left(1 + \frac{t^2}{12r^2}\right) \varphi \left(\varphi + \frac{\sin 2\varphi}{2}\right) - 2 \sin^2 \varphi + 3.00 \frac{t^2}{12r^2} \varphi \left(\varphi - \frac{\sin 2\varphi}{2}\right) \tag{6}$$

Thrusts and moments having been calculated, intrados and extrados stresses may be found by the usual formula

$$S = \frac{H}{t} \pm \frac{6M}{t^2} \tag{7}$$

More complicated formulas, referred to the neutral axis, with water pressures referred to the extrados, were later developed by Cain.[1] Frederick Hall Fowler,

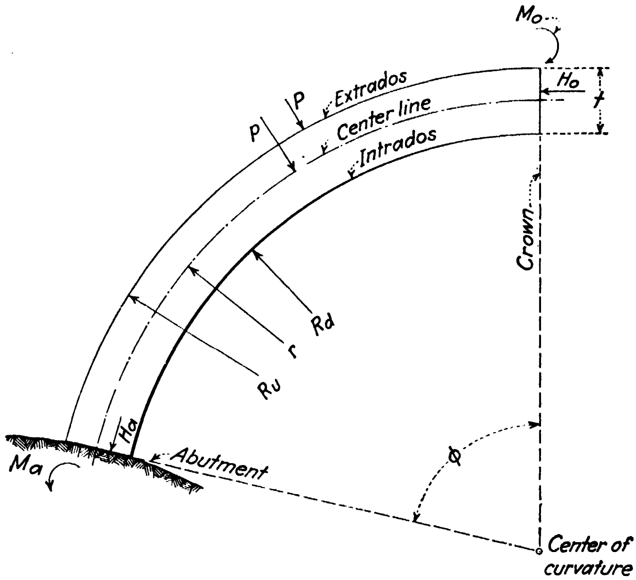


FIG. 4.—Constant-thickness circular arch, fixed at abutments.

using Cain's formulas as a basis, worked out diagrams from which intrados and extrados stresses at the crown and abutment sections may be easily obtained for different values of central angle and ratio of thickness to radius.[2] Philip Cravitz later prepared similar diagrams which included effects of abutment deformations.[3]

[1] CAIN, WILLIAM, discussion of Stresses in Thick Arches of Dams by B. F. Jakobsen, *Trans. A. S. C. E.*, **90**, 522–547, 1927.

[2] FOWLER, F. H., A Graphic Method for Determining the Stresses in Circular Arches under Normal Loads by the Cain Formulas, *Trans. A. S. C. E.*, **92**, 1512–1560, 1928.

[3] CRAVITZ, PHILIP, Analyses of Thick Arch Dams, including Abutment Yield, *Trans. A. S. C. E.*, **101**, 501–523, 1936.

For temperature loads, Cain's equations, shear being neglected, are as follows:

$$H_0 = \frac{2\varphi \sin \varphi}{D} \times \frac{Et^3cT}{12r^2} \tag{8}$$

$$M_0 = H_0 r\left(1 - \frac{\sin \varphi}{\varphi}\right) \tag{9}$$

$$H_a = H_0 \cos \varphi \tag{10}$$

$$M_a = H_0 r \operatorname{vers} \varphi - M_0 \tag{11}$$

In the preceding formulas, E is the modulus of elasticity, c the coefficient of thermal expansion, and T the change in concrete temperature. Other quantities are the same as before. In the preceding equations, the moment of inertia I has been replaced by the quantity $t^3/12$ which applies to rectangular sections. Formulas for temperature thrusts and moments, including shear, are given in the subsequent section on arch analyses.

Radial Adjustment at Crown. In analyzing an arch dam by dividing horizontal loads between arch and cantilever elements on the basis of a radial deflection adjustment at the crown section, formulas for cantilever and arch deflections are needed. Since such methods assume the partial water loads on the arch elements to be constant from abutment to abutment, Cain's arch equations may be used. His crown deflection equations for constant thickness, circular arches, slightly modified, are as follows:

Water-load deflection, $$\Delta = \frac{PR_u rC}{Et} \tag{12}$$

Temperature deflection, $$\Delta_t = crTC \tag{13}$$

In these equations, P is the normal radial pressure at the extrados, R_u the radius of the extrados, and C a coefficient depending on r, t, and φ, previously defined. If shear is neglected, C is given by the formula

$$C = \frac{(\varphi - \sin \varphi)(1 - \cos \varphi)}{\left(\varphi + \frac{\sin 2\varphi}{2}\right) - \left[\frac{1 - \cos 2\varphi}{\varphi\left(1 + \frac{t^2}{12r^2}\right)}\right]} \tag{14}$$

When shear is included, C is replaced by C_s, the value of which is given by the formula

$$C_s = \frac{(1 - \cos \varphi)\left[\left(1 + \frac{t^2}{12r^2}\right)(\varphi - \sin \varphi) + \frac{t^2}{4r^2}(\varphi + \sin \varphi)\right]}{\left(1 + \frac{t^2}{12r^2}\right)\left(\varphi + \frac{\sin 2\varphi}{2}\right) - \left(\frac{1 - \cos 2\varphi}{\varphi}\right) + \frac{t^2}{4r^2}\left(\varphi - \frac{\sin 2\varphi}{2}\right)} \tag{15}$$

Figure 5 shows values of C_s for different values of φ and the ratio t/r.[1]

Cantilever forces, moments, deflections, and stresses may be calculated by methods described in the subsequent section on cantilever analyses. If the dam is relatively thin, cantilevers may be considered as vertical slices with parallel sides 1 ft apart. However, they generally should be considered as vertical slices with radial sides 1 ft apart at the upstream face or at a circular vertical plane passing through the upstream edge of the top, herein referred to as the axis of the dam. Analyses of cracked cantilevers seldom are necessary in preliminary studies.

In determining the water-load distribution, temperature deflections must be added to water-load deflections in the case of the arch elements, but not in the case of the cantilever elements. The load distribution having been determined, arch stresses

[1] HOUK, IVAN E., Arch Deflections and Temperature Stresses in Curved Dams, No. II, *Engineer*, Apr. 9, 1937, pp. 414, 415.

may be obtained from the Fowler or Cravitz diagrams. Arch stresses due to temperature changes may be calculated by formula 7, after thrusts and moments have been computed by Eqs. (8) to (11).

Radial Adjustment at Several Sections. In analyzing an arch dam by adjusting radial deflections at several vertical sections, trial proportions of water load are assigned to different horizontal and vertical elements until the calculated arch and cantilever deflections coincide in all parts of the structure. Arch and cantilever

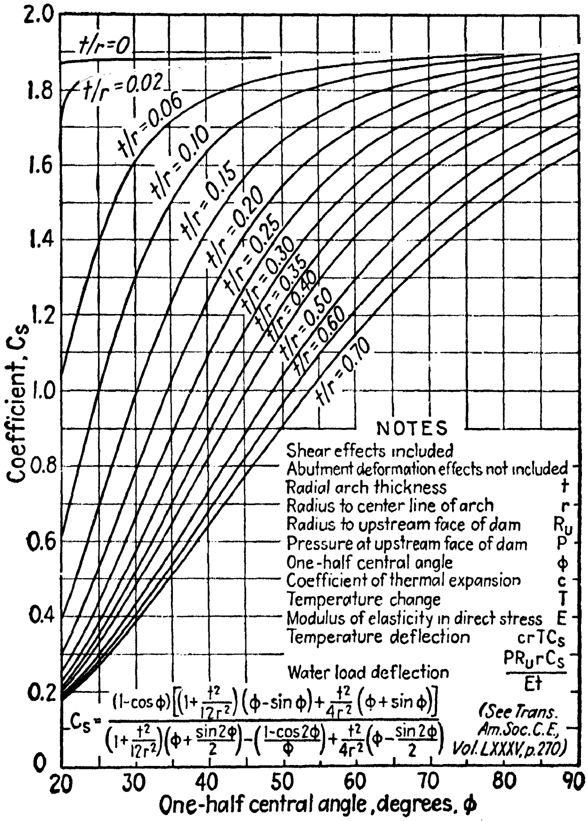


FIG. 5.—Values of C_s, Eq. (15).

stresses are then computed for the final load distribution. The analysis of five or six arch elements and an equal number of cantilever elements usually is sufficient in preliminary studies.

Cantilever elements may be analyzed by methods given later. Arch elements must be analyzed by more complicated methods than those given above, for the loads are not constant along the extrados curves. In preliminary trial-load calculations, arch elements may be analyzed by the voussoir summation process[1] or by theoretical formulas given in the subsequent section on arch analyses. In relatively thick dams, arch analyses should include radial shear effects. Effects of tangential shear, twist, rock deformations, and other secondary influences usually may be omitted in preliminary trial-load computations.

[1] HOWELL, C. H., and A. C. JAQUITH, Analysis of Arch Dams by the Trial Load Method, *Trans. A. S. C. E.*, **93**, 1191–1316. 1929.

ANALYSES OF ADOPTED PLANS

General plans of arch dams, adopted on the basis of preliminary investigations, should be reanalyzed by detailed trial-load methods, including all important secondary effects. Necessary alterations in dimensions or curvature can then be made before beginning construction. Final analyses should consider all possible load conditions, including earthquake shocks, ice forces, silt pressures, maximum flood stages, and maximum temperature increases, as well as normal full reservoir loads plus maximum temperature reductions. However, special load conditions generally may be analyzed on the basis of radial adjustments of deflections. One complete analysis, including effects of tangential shear and twist action, usually is adequate. Repeated trial-load studies have shown that such effects for the same dam under different conditions of loading are of the same sign and very similar magnitude, unless the change in applied loads is sufficient to change the direction of the deflections as may sometimes be true in the case of using maximum temperature increases instead of maximum temperature reductions. The consideration of eight or ten arch elements and an equal number of cantilever elements usually is sufficient in the final analyses.

Radial shear buttresses at the ends of the arched section, if needed, are analyzed by methods used for gravity dams. Radial shear forces are added to direct water pressures and are assumed to decrease uniformly from maximum values at the edge of the buttress, adjoining the arched section, to zero at the opposite edge.

THE TRIAL-LOAD METHOD

The development of the trial-load method was begun by the Bureau of Reclamation in 1923,[1] about the time a similar method was being investigated in Europe.[2] The Bureau's first use of the method included effects of thrust, moment, and temperature in the arch analyses and thrust, moment, and horizontal radial shear in the cantilever elements. The first analyses brought the deflections into adjustment in the radial direction only. The next step in the development was the inclusion of radial shear effects in the arch calculations. Since that time the method has been gradually amplified; so that now effects of rock deformations, tangential shear, twist action, and other secondary considerations may be included whenever necessary. The introduction of tangential shear and twist effects requires adjustments of deflections in circumferential and angular directions as well as in radial directions.[3]

Adjustments. In the present use of the method, adjustments of deflections are first made in radial directions, including effects of radial shear and rock deformations in both arch and cantilever elements. If considerations of tangential shear and twist effects are necessary, adjustments are next made in circumferential directions, then in angular directions. In circumferential adjustments, equal and opposite tangential shear loads are introduced, by trial, to compensate for differences in tangential movements caused by radial loads, one set of loads being applied to the arch elements and the balancing set, to the cantilever elements. In angular adjustments, equal and opposite twist loads are applied to arch and cantilever elements, to compensate for discrepancies in rotation caused by radial loads. Radial movements, caused by tangential shear and twist loads, are then considered in a radial readjustment and their effects considered in circumferential and angular readjustments, until resultant deflections are in agreement in all three directions.[4]

[1] *Ibid.*

[2] STUCKY, ALFRED. Study of Arch Dams. *Bull. tech. Suisse Romande*, Lausanne, 1922.

[3] HOUK, IVAN E., Trial Load Analyses of Curved Concrete Dams, *Engineer*, July 5, 1935, pp. 2–5.

[4] WESTERGAARD, H. M., Arch Dam Analysis by Trial Loads Simplified, *Eng. News-Record*, **106**, 141–143, 1931.

Rock Movements. Considerations of rock movements and their effects on the action of arch dams may be based on approximate formulas.[1] If the ends of the arch elements are vertical, and the bases of the cantilever elements, horizontal, rock rotations and deflections of elements with parallel sides 1 ft apart may be calculated by the following equations:

Rotation due to moment, $$\alpha' = \frac{MK_1}{E_r t^2} \tag{16}$$

Deflection due to thrust, $$\beta' = \frac{HK_2}{E_r} \tag{17}$$

Deflection due to shear, $$\gamma' = \frac{VK_3}{E_r} \tag{18}$$

Rotation due to twist, $$\delta' = \frac{M_t K_4}{E_r t^2} \tag{19}$$

Rotation due to shear $$\alpha'' = \frac{VK_5}{E_r t} \tag{20}$$

Deflection due to moment, $$\gamma'' = \frac{MK_5}{E_r t} \tag{21}$$

In the preceding equations, M and V are the arch and cantilever moments and shears, H the arch thrust, M_t the cantilever twisting moment, E_r the elastic modulus of the rock, t the radial thickness of the element, and K_1, K_2, K_3, K_4, and K_5 constants depending on Poisson's ratio and the ratio of the average length of the dam b to the average width a. Table 3 gives values of K constants for a Poisson's ratio of 0.20 and different values of b/a.

Equations (16), (18), (20), and (21) give movements at the ends of the arch and cantilever elements. Equation (17) gives horizontal movements caused by arch thrusts. Vertical movements at cantilever bases and twist movements at arch abutments are not needed. Equation (19) gives twist movements at cantilever bases. Rotations and deflections given by Eqs. (20) and (21) are of a secondary nature and relatively unimportant.

TABLE 3.—VALUES OF K CONSTANTS IN EQS. (16) TO (21), FOR POISSON'S RATIO = 0.20

Values of b/a	Values of K				
	K_1	K_2	K_3	K_4	K_5
1.0	4.32	0.62	1.02	4.65	0.345
1.5	4.65	0.78	1.23	4.86	0.413
2.0	4.84	0.91	1.39	5.18	0.458
3.0	5.04	1.10	1.60	5.64	0.515
4.0	5.15	1.25	1.77	5.90	0.550
5.0	5.22	1.36	1.89	6.08	0.574
6.0	5.27	1.47	2.00	6.20	0.592
8.0	5.32	1.63	2.17	6.37	0.614
10.0	5.36	1.75	2.31	6.46	0.630
15.0	5.41	1.98	2.55	6.59	0.653
20.0	5.43	2.16	2.72	6.66	0.668

[1] VOGT, FREDRIK, Ueber die Berechnung der Fundamentdeformation, *Det Norske Videnskaps-Akademi*, 1925.

If pounds, feet, and radians are used as dimensional units, calculated deflections and rotations are feet and radians, respectively. Further discussions of rock movements in trial-load analyses are given in subsequent sections on cantilever and arch analyses.

CANTILEVER ANALYSES

In cantilever analyses, the vertical elements are divided into sections by horizontal planes at small increments of height, as shown in Fig. 6. Total loads, shears, and moments, acting on the horizontal planes, are then summated from the top downward to the foundation; and slopes of neutral axis, moment deflections, and shear deflections are summated from the foundation upward to the top, rock deformations being inserted as initial movements in beginning the upward summations. Radial deflections at assumed horizontal planes are then found by adding moment and shear deflections.

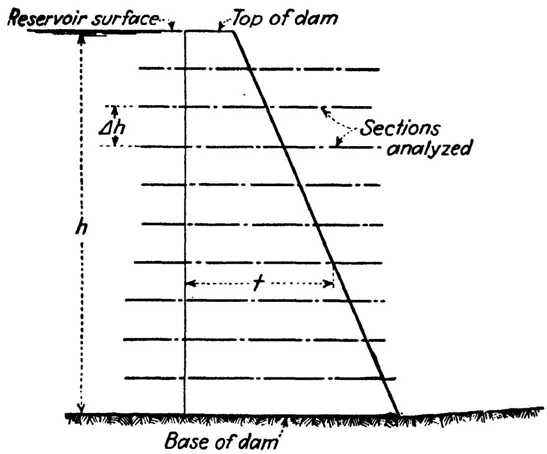


FIG. 6.—Vertical element of arch dam.

In the following discussions, cantilevers with parallel sides, radial sides, and upstream cracking are treated from the viewpoint of radial loads. Effects of tangential shear and twist loads on uncracked elements with radial sides are then considered separately.

Cantilever with Parallel Sides. For an uncracked cantilever with parallel vertical sides, 1 ft apart, increments of concrete weight, vertical water loads on the upstream face, where sloped, horizontal water pressures, centers of gravity, shears, moments, and moments of inertia are easily calculated by usual methods. Slopes of the neutral axis, moment deflections, and shear deflections are then obtained by the following summation formulas:

Slope of neutral axis, $$\frac{dy}{dh} = \alpha' + \alpha'' + \sum \frac{12M}{Et^3}\,\Delta h \tag{22}$$

Moment deflection, $$\Delta_m = \sum \left(\alpha' + \alpha'' + \sum \frac{12M}{Et^3}\,\Delta h\right)\Delta h \tag{23}$$

Shear deflection, $$\Delta_s = \left(\gamma' + \gamma'' + \sum \frac{KV}{tE_s}\,\Delta h\right) \tag{24}$$

In these equations, M is the resultant bending moment, V the total horizontal shear in the radial direction, Δh the increment of height, dy a differential movement in the horizontal radial direction, E_s the shearing modulus of elasticity, and K a con-

stant allowing for nonuniform distribution of shear, usually taken as 1.25. Other quantities are the same as in preceding sections. The moment of inertia I has been replaced by its equivalent $t^3/12$.

Vertical stresses at the faces of the dam may be computed by Eq. (7), the vertical force W being used instead of the horizontal thrust H. Inclined stresses at the edges of the cantilever, acting parallel to the slopes, and unit shearing stresses on horizontal and vertical planes at the edges of the cantilever may be calculated by the formulas

Inclined stress, $S_i = S \sec^2 \alpha - p \tan^2 \alpha$ (25)

Shearing stress, $N = \mp (S - p) \tan \alpha$, (-at upstream face) (26)

where S = vertical stress.
p = water pressure.
α = angle between the face of the dam and the vertical direction.

If the face of the dam is vertical, the shearing stress at the edge of the cantilever is zero.

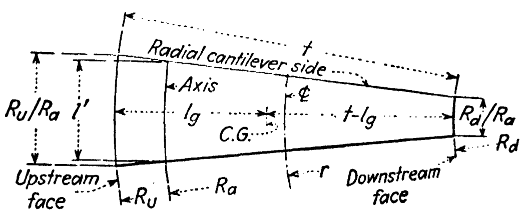


FIG. 7.—Horizontal section of cantilever with radial sides.

Cantilever with Radial Sides. Properties of an uncracked cantilever with radial sides 1 ft apart at the axis of the dam, as shown in Fig. 7, may be calculated by the following formulas:

Area, $$A = \left(\frac{R_u + R_d}{2R_a}\right) t \qquad (27)$$

Distance to center of gravity, $$l_g = \frac{t}{3}\left(\frac{R_u + 2R_d}{R_u + R_d}\right) \qquad (28)$$

Moment of inertia, $$I = \frac{t^3}{36}\left(\frac{R_u^2 + 4R_uR_d + R_d^2}{R_a(R_u + R_d)}\right) \qquad (29)$$

Vertical stress at upstream face, $$S_u = \frac{W}{A} - \frac{Ml_g}{I} \qquad (30)$$

Vertical stress at downstream face, $$S_d = \frac{W}{A} + \frac{M(t - l_g)}{I} \qquad (31)$$

In these equations R_u is the upstream radius, R_d the downstream radius, R_a the radius to the axis, and W the total vertical load. Other quantities are the same as before.

By using A, l_g, and I in their proper places and increasing upstream widths to R_u/R_a before calculating water pressures, loads, moments, shears, slopes of neutral axis, and deflections may be determined by summation methods, as in the preceding section. Rock movements at the cantilever base should be multiplied by R_a/r before being included in the summations.

Cracked Cantilevers. In analyzing cracked cantilevers as shown in Fig. 8, the depth of cracking may be calculated by the formula

$$\frac{R_u - l_g - e}{R_d} = \frac{1}{2}\left[\frac{1 + \frac{2R_d}{R_0} + 3\left(\frac{R_d}{R_0}\right)^2}{\frac{R_d}{R_0} + 2\left(\frac{R_d}{R_0}\right)^2}\right] \qquad (32)$$

In this equation, R_0 is the radius to the upstream limit of the uncracked area, and e the distance from the center of gravity of the entire area to the location of the resultant. The equation is found by taking moments of the stress solid about the center of gravity. Values R_u and R_d are known; l_g is given by formula 28; and e is calculated by usual methods. These terms determine the fraction at the left of the equation.

Probably the best way to use Eq. (32) is to plot a diagram showing values of the function at the right for different values of R_d/R_0. Calculated values of the fraction at the left may then be used to obtain R_d/R_0 from the diagram. If R_d/R_0 is less than R_d/R_u, no cracking occurs. If R_d/R_0 is greater than R_d/R_u, the section is cracked.

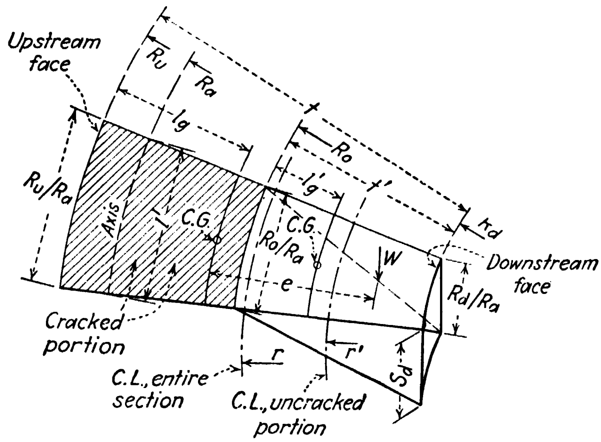


FIG. 8.—Section of cracked cantilever.

Having determined R_d/R_0, the uncracked thickness t' may be obtained from the formula

$$t' = \frac{R_d}{\dfrac{R_d}{R_0}} - R_d \tag{33}$$

Equations for area A', distance from the point of zero stress to the center of gravity l_g', and moment of inertia I', for the uncracked part of the cross section may be obtained by substituting t' for t and R_0 for R_u in Eqs. (27), (28), and (29). The moment of the redistributed forces about the new center of gravity and the vertical stress at the downstream face may be computed by the following formulas:

Moment,
$$M' = \frac{Wt'}{6}\left[\frac{1 + \dfrac{4R_d}{R_0} + \left(\dfrac{R_d}{R_0}\right)^2}{1 + \dfrac{3R_d}{R_0} + 2\left(\dfrac{R_d}{R_0}\right)^2}\right] \tag{34}$$

Vertical stress,
$$S_d = \frac{W}{A'} + \frac{M'(t' - l_g')}{I'} \tag{35}$$

Deflections may be computed by formulas given in preceding sections, by using t', M', I' and other quantities pertaining to the uncracked portions of the cross sections. Rock movements should be multiplied by R_a/r', before being included in the summations, r' being the radius to the center line of the uncracked area.

Tangential Shear Loads. Tangential shear loads, equal and opposite to corresponding loads on the arch elements, are multiplied by r/R_a, to allow for the reduced

width at the center line of a cantilever with radial sides, and are then applied along the center line. Shear loads are summated from the top downward to the foundation, and shear deflections, from the foundation upward to the top, as in the case of radial loads.

Deflections are computed by the summation formula

$$\Delta_s = \left(\frac{R_a}{r}\gamma' + \sum \frac{V}{AE_s}\Delta h\right) \tag{36}$$

No correction factor is needed in the second term, for tangential shear is assumed to be uniformly distributed.

Twist Loads. Twist loads in foot-pounds per square foot, equal and opposite to corresponding loads on the arch elements, are multiplied by r/R_a and applied at the center line. The loads act in horizontal planes, as in the case of radial and tangential shear loads. Loads are summated from the top down, and resulting angular deflections, from the foundation up, as before.

Angular deflections are computed by the summation formula

$$\Delta\theta = \left(\frac{R_a}{r}\delta' + \sum \frac{M_t}{2E_sI}\Delta h\right) \tag{37}$$

All quantities in the formula are the same as previously explained. The moment of inertia I is taken about the same axis as in the case of radial loads. Consequently, its value is determined by Eq. (29). The foundation rotation δ' caused by the twisting moment M_t is computed by Eq. (19).

ARCH ANALYSES

Moments, forces, and movements of arch elements, caused by radial, tangential, twist, and temperature loads, may be analyzed by flexure formulas for curved cantilever beams, amplified to allow for rib-shortening and transverse shear effects. The method consists of cutting the loaded arch at the crown, introducing initial moments, thrusts, and shears to compensate for crown displacements, developing equations for crown movements for both parts of the arch, equating the two sets of formulas, and solving for crown forces. Equations for moments, thrusts, and shears may then be written in terms of crown forces, and moments, thrusts, and shears due to external loads. The moments, thrusts, and shears having been determined, stresses may be calculated by usual formulas.

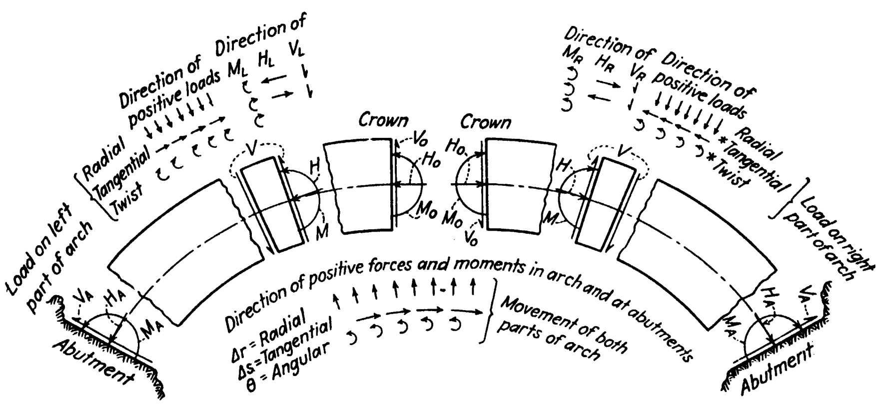
Abutment movements, determined by Eqs. (16) to (21), may be inserted in the general deflection formulas. However, for the sake of simplicity, such movements are neglected in the following treatment. Temperature effects are discussed separately for the same reason.

General formulas are given for a circular arch subjected to symmetrical or nonsymmetrical loads. Special formulas for constant-thickness circular arches are then given for the terms that are functions of the arch properties, called *arch constants;* for the moments, thrusts, and shears due to external loads, called *load formulas;* and for the terms that are functions of both arch and load properties, called *load constants.* The formulas for load constants and for moments, thrusts, and shears due to external loads include equations needed in analyzing effects of uniform and triangular loads. The loads considered include tangential and twist loads as well as radial loads. Deflections for symmetrical or nonsymmetrical nonuniform loads may be obtained by adding deflections for different combinations of uniform and triangular loads.

Notation. The following notation is used, all quantities being measured in horizontal planes:

R_u = radius to upstream face.
R_d = radius to downstream face.
r = radius to center line.
t = radial arch thickness.
A = area of radial cross section.
I = moment of inertia of radial cross section about axis along arch center line.
s = length along center line.
φ = angle from arch point under consideration to any point on arch.
x = $r \sin \varphi$.
y = r vers φ.
φ_a = angle from arch point where deflections are desired to abutment.
φ_0 = angle from arch point where deflections are desired to beginning of external load.
φ_1 = angle from beginning of external load to abutment.
M = moment.
H = thrust.
V = shear.
P = intensity of external load.
E = modulus of elasticity of concrete in tension and compression.
E_s = modulus of elasticity of concrete in shear.
μ = Poisson's ratio.
K = constant to allow for nonuniform distribution of shear.
c = coefficient of thermal expansion of concrete.
T = temperature change, positive when rising.
θ = angular movement of arch center line.
Δr = radial deflection of arch center line.
Δs = tangential deflection along center line.

The subscript 0 means at the crown, a at the abutment, L at the left of the crown, and R at the right of the crown. In the case of M, H, and V, subscripts L or R mean that the moment, thrust, and shear are due to external loads on the left or right portions of the arch, respectively.



* *Uniform tangential and twist loads are continuous along the arch and their directions are those for load on left part of arch*

FIG. 9.—Direction of positive loads, forces, moments, and movements.

If μ equals 0.20, and K 1.25, the ratio K/E_s in some of the subsequent equations may be replaced by $3/E$.

Signs. The convention of signs, shown on Fig. 9, is as follows:

Positive moments cause compression at the extrados.

Positive thrusts cause compression.

Positive shears produce positive moments on the section of the arch at the left in the case of the left part of the arch and positive moments at the right in the case of the right part, except V_0 which acts as shown in Fig. 9.

Radial loads are positive when acting toward the arch center. Uniform tangential loads are positive when acting from left to right, in both parts of the arch. Triangular

tangential loads are positive when acting from the abutments toward the crown. Uniform twist loads are positive when acting clockwise in both parts of the arch. Triangular twist loads are positive when acting clockwise in the left part and counterclockwise in the right part.

Positive moments, thrusts, and shears (M_L, H_L, V_L, or M_R, H_R, V_R) due to external loads are in the same direction as the moments, thrusts, and shears of positive radial loads. Following this convention, moments, thrusts, and shears of all positive triangular loads are positive except thrusts of tangential loads, which are negative. Since the portion of the uniform tangential or twist load on the right part of the arch is applied in the same direction as the load on the left part, the M_R, H_R, and V_R of these loads will change sign.

Positive radial deflections are upstream.

Positive tangential deflections are toward the right.

Positive angular movements are counterclockwise.

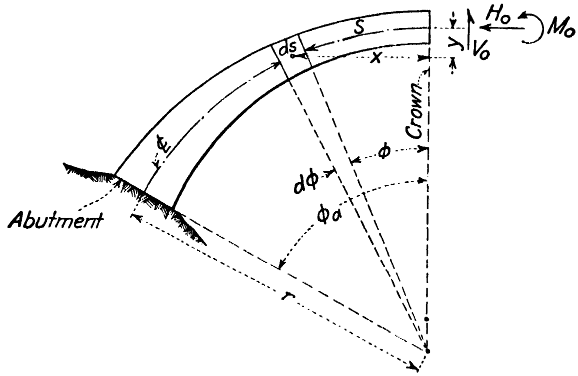


FIG. 10.—Left part of arch cut at crown.

General Formulas. Consider a differential element of length ds in the left part of an arch cut at the crown, as shown in Fig. 10. From mechanics, the equations for the arch movements at the crown, due to a moment, thrust, and shear acting on the element, are

$$d\theta_0 = \frac{M\,ds}{EI} \tag{38}$$

$$d(\Delta r)_0 = \frac{Mx\,ds}{EI} - \frac{H \sin \alpha\,ds}{EA} + \frac{KV \cos \varphi\,ds}{E_sA} \tag{39}$$

$$d(\Delta s)_0 = -\frac{My\,ds}{EI} - \frac{H \cos \varphi\,ds}{EA} - \frac{KV \sin \varphi\,ds}{E_sA} \tag{40}$$

The first term in Eqs. (39) and (40) gives the movement caused by bending; the second term, the movement caused by rib-shortening; and the third term, the movement caused by shear.

By integrating the preceding equations and using s to designate the total length along the center line from crown to abutment, the following equations are obtained for the left part of the arch:

$$\theta_0 = \int_0^s \frac{M\,ds}{EI} \tag{41}$$

$$\Delta r_0 = \int_0^s \frac{Mx\,ds}{EI} - \int_0^s \frac{H \sin \varphi\,ds}{EA} + \int_0^s \frac{KV \cos \varphi\,ds}{E_sA} \tag{42}$$

$$\Delta s_0 = -\int_0^s \frac{My\,ds}{EI} - \int_0^s \frac{H \cos \varphi\,ds}{EA} - \int_0^s \frac{KV \sin \varphi\,ds}{E_sA} \tag{43}$$

Quantities M, H, and V may be replaced by their equivalents in terms of moment, thrust, and shear at the crown (M_0, H_0, and V_0) and moment, thrust, and shear due to external loads between the differential element and the crown (M_L, H_L and V_L).

$$M = M_0 + H_0 y + V_0 x - M_L \tag{44}$$
$$H = H_0 \cos\varphi - V_0 \sin\varphi + H_L \tag{45}$$
$$V = H_0 \sin\varphi + V_0 \cos\varphi - V_L \tag{46}$$

If these substitutions are made and the ratio K/E_s replaced by $3/E$, the following formulas are obtained:

$$\theta_0 = M_0 \int_0^s \frac{ds}{EI} + H_0 \int_0^s \frac{y\,ds}{EI} + V_0 \int_0^s \frac{x\,ds}{EI} - \int_0^s \frac{M_L\,ds}{EI} \tag{47}$$

$$\begin{aligned} \Delta r_0 = {} & M_0 \int_0^s \frac{x\,ds}{EI} + H_0 \left(\int_0^s \frac{xy\,ds}{EI} - \int_0^s \frac{\sin\varphi\cos\varphi\,ds}{EA} + 3\int_0^s \frac{\sin\varphi\cos\varphi\,ds}{EA} \right) \\ & + V_0 \left(\int_0^s \frac{x^2\,ds}{EI} + \int_0^s \frac{\sin^2\varphi\,ds}{EA} + 3\int_0^s \frac{\cos^2\varphi\,ds}{EA} \right) \\ & - \left(\int_0^s \frac{M_L x\,ds}{EI} + \int_0^s \frac{H_L \sin\varphi\,ds}{EA} + 3\int_0^s \frac{V_L \cos\varphi\,ds}{EA} \right) \end{aligned} \tag{48}$$

$$\begin{aligned} \Delta s_0 = {} & -M_0 \int_0^s \frac{y\,ds}{EI} - H_0 \left(\int_0^s \frac{y^2\,ds}{EI} + \int_0^s \frac{\cos^2\varphi\,ds}{EA} + 3\int_0^s \frac{\sin^2\varphi\,ds}{EA} \right) \\ & - V_0 \left(\int_0^s \frac{xy\,ds}{EI} - \int_0^s \frac{\sin\varphi\cos\varphi\,ds}{EA} + 3\int_0^s \frac{\sin\varphi\cos\varphi\,ds}{EA} \right) \\ & + \left(\int_0^s \frac{M_L y\,ds}{EI} - \int_0^s \frac{H_L \cos\varphi\,ds}{EA} + 3\int_0^s \frac{V_L \sin\varphi\,ds}{EA} \right) \end{aligned} \tag{49}$$

If symbols are substituted for the multipliers of M_0, H_0, and V_0 and for the terms depending on load, the preceding equations may be written

$$\theta_0 = A_1 M_0 + B_1 H_0 + C_1 V_0 - D_1 \tag{50}$$
$$\Delta r_0 = C_1 M_0 + B_2 H_0 + C_2 V_0 - D_2 \tag{51}$$
$$\Delta s_0 = -B_1 M_0 - B_3 H_0 - B_2 V_0 + D_3 \tag{52}$$

Similar equations for the right part of the arch may be developed in the same manner. In this case, the values of M, H, and V are

$$M = M_0 + H_0 y - V_0 x - M_R \tag{53}$$
$$H = H_0 \cos\varphi + V_0 \sin\varphi + H_R \tag{54}$$
$$V = H_0 \sin\varphi - V_0 \cos\varphi - V_R \tag{55}$$

The resulting equations for the right part of the arch are

$$\theta_0 = -A_1' M_0 - B_1' H_0 + C_1' V_0 + D_1' \tag{56}$$
$$\Delta r_0 = C_1' M_0 + B_2' H_0 - C_2' V_0 - D_2' \tag{57}$$
$$\Delta s_0 = B_1' M_0 + B_3' H_0 - B_2' V_0 - D_3' \tag{58}$$

Crown Forces. The moment, thrust, and shear at the crown M_0, H_0, and V_0 may be obtained by equating the values of θ_0, Δr_0, and Δs_0 for the two parts of the arch as given by formulas 50, 51, 52, 56, 57, and 58. The equations so derived are

$$(A_1 + A_1')M_0 + (B_1 + B_1')H_0 + (C_1 - C_1')V_0 = (D_1 + D_1') \tag{59}$$
$$(C_1 - C_1')M_0 + (B_2 - B_2')H_0 + (C_2 + C_2')V_0 = (D_2 - D_2') \tag{60}$$
$$(B_1 + B_1')M_0 + (B_3 + B_3')H_0 + (B_2 - B_2')V_0 = (D_3 + D_3') \tag{61}$$

If the quantities in parentheses are replaced by a, b, c, and d, the equations may be written

$$a_1M_0 + b_1H_0 + c_1V_0 = d_1 \tag{62}$$
$$c_1M_0 + b_2H_0 + c_2V_0 = d_2 \tag{63}$$
$$b_1M_0 + b_3H_0 + b_2V_0 = d_3 \tag{64}$$

By solving Eqs. (62), (63), and (64) simultaneously and introducing an additional symbol K', the following equations for M_0, H_0, and V_0 are obtained:

$$M_0 = \frac{1}{K'}\left[d_1(b_3c_2 - b_2^2) - d_3(b_1c_2 - c_1b_2) - d_2(b_3c_1 - b_1b_2)\right] \tag{65}$$

$$H_0 = \frac{1}{K'}\left[-d_1(b_1c_2 - b_2c_1) + d_3(a_1c_2 - c_1^2) + d_2(b_1c_1 - a_1b_2)\right] \tag{66}$$

$$V_0 = \frac{1}{K'}\left[-d_1(b_3c_1 - b_1b_2) + d_3(b_1c_1 - a_1b_2) + d_2(a_1b_3 - b_1^2)\right] \tag{67}$$

The value of K' is given by the equation

$$K' = a_1(b_3c_2 - b_2^2) - b_1(b_1c_2 - c_1b_2) - c_1(b_3c_1 - b_1b_2) \tag{68}$$

In the case of a symmetrical arch, the preceding equations reduce to

$$M_0 = \frac{1}{K'}(d_1b_3 - d_3b_1) \tag{69}$$

$$H_0 = \frac{1}{K'}(-d_1b_1 + d_3a_1) \tag{70}$$

$$V_0 = \frac{d_2}{c_2} \tag{71}$$

$$K' = a_1b_3 - b_1^2 \tag{72}$$

TABLE 4.—FUNCTIONS NEEDED IN DETERMINING CROWN FORCES

Term	Functions	
	Left part	Right part
a_1	$\int_0^s \frac{ds}{EI}$	$\int_0^s \frac{ds}{EI}$
b_1	$\int_0^s \frac{y\,ds}{EI}$	$\int_0^s \frac{y\,ds}{EI}$
c_1	$\int_0^s \frac{x\,ds}{EI}$	$-\int_0^s \frac{x\,ds}{EI}$
b_2	$\int_0^s \frac{xy\,ds}{EI}$ $-\int_0^s \frac{\sin\phi\cos\phi\,ds}{EA}$ $3\int_0^s \frac{\sin\phi\cos\phi\,ds}{EA}$	$-\int_0^s \frac{xy\,ds}{EI}$ $\int_0^s \frac{\sin\phi\cos\phi\,ds}{EA}$ $-3\int_0^s \frac{\sin\phi\cos\phi\,ds}{EA}$
c_2	$\int_0^s \frac{x^2\,ds}{EI}$ $\int_0^s \frac{\sin^2\phi\,ds}{EA}$ $3\int_0^s \frac{\cos^2\phi\,ds}{EA}$	$\int_0^s \frac{x^2\,ds}{EI}$ $\int_0^s \frac{\sin^2\phi\,ds}{EA}$ $3\int_0^s \frac{\cos^2\phi\,ds}{EA}$
b_3	$\int_0^s \frac{y^2\,ds}{EI}$ $\int_0^s \frac{\cos^2\phi\,ds}{EA}$ $3\int_0^s \frac{\sin^2\phi\,ds}{EA}$	$\int_0^s \frac{y^2\,ds}{EI}$ $\int_0^s \frac{\cos^2\phi\,ds}{EA}$ $3\int_0^s \frac{\sin^2\phi\,ds}{EA}$
d_1	$\int_0^s \frac{M_L\,ds}{EI}$	$\int_0^s \frac{M_R\,ds}{EI}$
d_2	$\int_0^s \frac{M_Lx\,ds}{EI}$ $\int_0^s \frac{H_L\sin\phi\,ds}{EA}$ $3\int_0^s \frac{V_L\cos\phi\,ds}{EA}$	$-\int_0^s \frac{M_Rx\,ds}{EI}$ $-\int_0^s \frac{H_R\sin\phi\,ds}{EA}$ $-3\int_0^s \frac{V_R\cos\phi\,ds}{EA}$
d_3	$\int_0^s \frac{M_Ly\,ds}{EI}$ $-\int_0^s \frac{H_L\cos\phi\,ds}{EA}$ $3\int_0^s \frac{V_L\sin\phi\,ds}{EA}$	$\int_0^s \frac{M_Ry\,ds}{EI}$ $-\int_0^s \frac{H_R\cos\phi\,ds}{EA}$ $3\int_0^s \frac{V_R\sin\phi\,ds}{EA}$

The functions included in the a, b, c, and d terms of Eqs. (62), (63), and (64) are given in Table 4. These are the quantities needed in determining the moment, thrust, and shear at the crown. In the case of the b_2, c_1, and d_2 terms, the signs of the quantities for the right part of the arch are negative, in accordance with the signs in Eqs. (59), (60), and (61). Consequently, the algebraic sums of the a, b, c, and d terms in Table 4 may be substituted directly in Eqs. (62) to (72). Evaluations of the integrals in Table 4 are given in subsequent sections.

Deflections. The deflections at any point on an arch may be obtained by considering the portion of the arch between the given point and the abutment as a curved cantilever beam. The desired movements are the sum of the movements due to the moment, thrust, and shear at the point and the movements due to the external load between the point and the abutment. Consequently, the movements may be calculated by the general formulas given in the preceding section.

In calculating deflections at a point in the left part of the arch, the moment, thrust, and shear at the crown are first determined as previously explained. The moment, thrust, and shear at the point are then obtained from Eqs. (44), (45), and (46), by using formulas for M_L, H_L, and V_L given in the subsequent section on load formulas. The deflections at the point are then obtained from Eqs. (47), (48), and (49), the radial section through the point being considered as a new crown section and the moment, thrust, and shear at the point being used as new values of M_0, H_0, and V_0. Deflections at points in the right part of the arch may be determined by similar methods.

Arch Constants. The quantities A_1, B_1, B_2, B_3, C_1, and C_2 in Eqs. (50), (51) and (52) and the similar quantities in Eqs. (56), (57), and (58) consist of integrals or groups of integrals which are functions of the arch properties. Consequently, they are designated *arch constants*. These constants are really deflections at a point due to a unit force or moment at the point. Their meanings may be briefly stated as follows:

A_1 = angular movement due to a unit moment.
B_1 = angular movement due to a unit thrust, or the tangential deflection due to a unit moment.
C_1 = angular movement due to a unit shear, or the radial deflection due to a unit moment.
B_2 = radial deflection due to a unit thrust, or the tangential deflection due to a unit shear.
C_2 = radial deflection due to a unit shear.
B_3 = tangential deflection due to a unit thrust.

If the arch has a constant thickness and the center line is used instead of the neutral axis, quantities I, s, ds, and A in Eqs. (47), (48), and (49) may be replaced by $t_3/12$, $r\varphi$, $r\,d\varphi$, and t. Since E is a constant, the integrals of the arch constants, for either side of the arch, may then be evaluated and the constants determined by the following equations in which φ_a is the angle from the point where the deflections are desired to the abutment.

$$A_1 = \frac{12r}{Et^3}[\varphi_a] \tag{73}$$

$$B_1 = \frac{12r^2}{Et^3}[\varphi_a - \sin\varphi_a] \tag{74}$$

$$C_1 = \frac{12r^2}{Et^3}[\text{vers}\,\varphi_a] \tag{75}$$

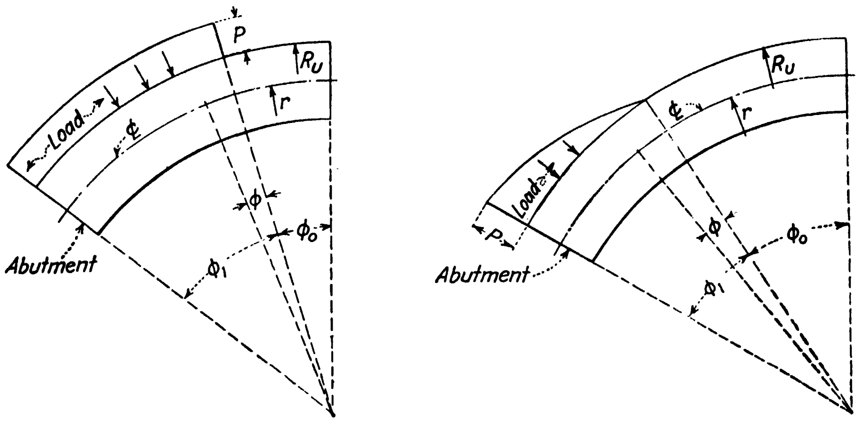
$$B_2 = \frac{12r^3}{Et^3}\left[\text{vers}\,\varphi_a - \frac{\sin^2\varphi_a}{2}\right] + \frac{r}{Et}[\sin^2\varphi_a] \tag{76}$$

$$C_2 = \frac{12r^3}{Et}\left[\frac{\varphi_a - \sin\varphi_a\cos\varphi_a}{2}\right] + \frac{r}{Et}\left[\frac{(\varphi_a - \sin\varphi_a\cos\varphi_a)}{2} + \frac{3(\varphi_a + \sin\varphi_a\cos\varphi_a)}{2}\right] \tag{77}$$

$$B_3 = \frac{12r^3}{Et^3}\left[\varphi_a - 2\sin\varphi_a + \frac{(\varphi_a + \sin\varphi_a\cos\varphi_a)}{2}\right] + \frac{r}{Et}\left[\frac{(\varphi_a + \sin\varphi_a\cos\varphi_a)}{2} + \frac{3(\varphi_a - \sin\varphi_a\cos\varphi_a)}{2}\right] \tag{78}$$

Since the quantities contained in the brackets of Eqs. (73) to (78) depend only on the arch angle, suitable tables may be prepared for use in calculating arch constants.

Load Formulas. Formulas for moment, thrust, and shear due to external loads M_L, H_L, and V_L or M_R, H_R, and V_R must be obtained before the D terms in the preceding equations can be evaluated. Such formulas may be written in terms of the external load P, the upstream radius R_u, the center-line radius r, the total central angle sub-



UNIFORM LOAD TRIANGULAR LOAD

FIG. 11.—Uniform and triangular radial loads.

tended by the load φ_1, and different functions of the central angle φ from the beginning of the load to any point on the loaded section of the arch. Equations for the uniform and triangular, radial, tangential, and twist loads shown on Figs. 11, 12, and 13 are given in Table 5. Equations for M_R, H_R, and V_R due to similar loads on the right side of the arch are the same as the equations for M_L, H_L, and V_L, except as noted in the section on signs.

P is usually expressed in pounds per square foot in the case of radial and tangential loads, and in foot pounds per square foot in the case of twist loads. Since the twist

TABLE 5.—FORMULAS FOR MOMENT, THRUST, AND SHEAR DUE TO EXTERNAL RADIAL, TANGENTIAL, AND TWIST LOADS

Loads	Force	Formulas for loads shown in Figs. 11, 12, and 13	
		Uniform loads	Triangular loads
Radial loads (see Fig. 11)	M_L	$PR_ur \operatorname{vers} \phi$	$\frac{PR_ur}{\phi_1}(\phi - \sin\phi)$
	H_L	$PR_u \operatorname{vers} \phi$	$\frac{PR_u}{\phi_1}(\phi - \sin\phi)$
	V_L	$PR_u \sin\phi$	$\frac{PR_u}{\phi_1} \operatorname{vers} \phi$
Tangential loads (see Fig. 12)	M_L	$Pr^2(\phi - \sin\phi)$	$\frac{Pr^2}{\phi_1}\left(\frac{\phi^2}{2} - \operatorname{vers}\phi\right)$
	H_L	$-Pr \sin\phi$	$-\frac{Pr}{\phi_1} \operatorname{vers}\phi$
	V_L	$Pr \operatorname{vers}\phi$	$\frac{Pr}{\phi_1}(\phi - \sin\phi)$
Twist loads (see Fig. 13)	M_L	$Pr\phi$	$\frac{Pr}{\phi_1}\frac{\phi^2}{2}$

loads are couples applied along the arch center line, they do not produce thrusts or shears. Consequently no formulas for H_L or V_L due to twist appear in the table. The load formulas may be used to calculate moment, thrust, and shear due to external loads at the right of any point between the beginning of the load and the abutment.

Load Constants. The quantities D_1, D_2, and D_3 in Eqs. (50), (51), and (52) and the similar quantities in Eqs. (56), (57), and (58) consist of integrals or groups of integrals that depend on both arch properties and external loads. These quantities

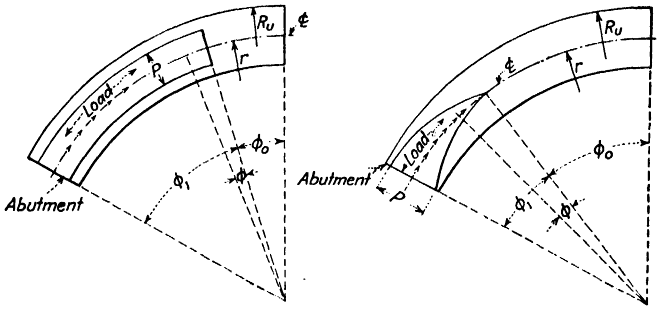


FIG. 12.—Uniform and triangular tangential loads.

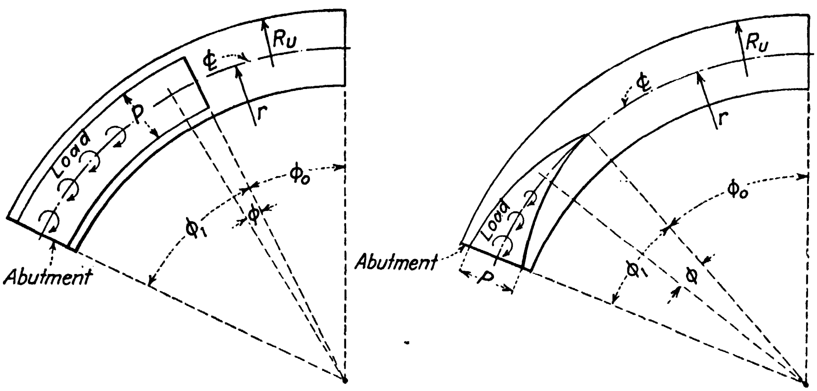


FIG. 13.—Uniform and triangular twist loads.

are designated *load constants*. They are really deflections at a point due to the loads applied between the point and the abutment. D_1 is the angular movement, D_2 the radial movement, and D_3 the tangential movement. Their values for the left part of a constant-thickness arch, loaded from the abutment to an angular distance φ_1, as shown on Figs. 11, 12, and 13, again using the center line instead of the neutral axis, are given by the following formulas:

$$D_1 = \frac{12}{Et^3}\int_0^{\varphi_1} M_L r\, d\varphi \tag{79}$$

$$D_2 = \frac{12}{Et^3}\int_0^{\varphi_1} M_L r^2 \sin\varphi\, d\varphi + \frac{1}{Et}\int_0^{\varphi_1} H_L r \sin\varphi\, d\varphi + \frac{3}{Et}\int_0^{\varphi_1} V_L r \cos\varphi\, d\varphi \tag{80}$$

$$D_3 = \frac{12}{Et^3}\int_0^{\varphi_1} M_L r^2 \operatorname{vers}\varphi\, d\varphi - \frac{1}{Et}\int_0^{\varphi_1} H_L r \cos\varphi\, d\varphi + \frac{3}{Et}\int_0^{\varphi_1} V_L r \sin\varphi\, d\varphi \tag{81}$$

TABLE 6.—FORMULAS FOR LOAD CONSTANTS FOR UNIFORM AND TRIANGULAR RADIAL AND TANGENTIAL LOADS

Trigonometric part of formula*	Radial loads				Tangential loads			
	Uniform		Triangular		Uniform		Triangular	
	Designation	Multiplier	Designation	Multiplier	Designation	Multiplier	Designation	Multiplier
$\sin\phi_0\left(\frac{\sin^2\phi_1}{2}\right)+\cos\phi_0\left(\frac{\phi_1-\sin\phi_1\cos\phi_1}{2}\right)$	D_3 2d term 2d part	$\frac{PR_ur}{ET}$			D_2 2d term 1st part	$-\frac{Pr^2}{ET}$		
$\cos\phi_0\left(\frac{\sin^2\phi_1}{2}\right)-\sin\phi_0\left(\frac{\phi_1-\sin\phi_1\cos\phi_1}{2}\right)$	D_2 2d term 2d part	$\frac{PR_ur}{ET}$			D_3 2d term 1st part	$\frac{Pr^2}{ET}$		
$\phi_1-\sin\phi_1$	D_1 D_3 1st term 1st part	$\frac{PR_ur^2}{EI}$ $\frac{PR_ur^3}{EI}$						
$\sin\phi_0\left(\sin\phi_1-\frac{\phi_1+\sin\phi_1\cos\phi_1}{2}\right)+\cos\phi_0\left(\text{vers }\phi_1-\frac{\sin^2\phi_1}{2}\right)$	D_2 1st term D_2 2d term 1st part	$\frac{PR_ur^3}{EI}$ $\frac{PR_ur}{ET}$	D_3 2d term 2d part	$\frac{PR_ur}{\phi_1ET}$	D_3 2d term 2d part	$\frac{Pr^2}{ET}$	D_2 2d term 1st part	$-\frac{Pr^2}{\phi_1ET}$
$\cos\phi_0\left(\sin\phi_1-\frac{\phi_1+\sin\phi_1\cos\phi_1}{2}\right)-\sin\phi_0\left(\text{vers }\phi_1-\frac{\sin^2\phi_1}{2}\right)$	D_3 1st term 2d part D_3 2d term 1st part	$-\frac{PR_ur^3}{EI}$ $-\frac{PR_ur}{ET}$	D_2 2d term 2d part	$\frac{PR_ur}{\phi_1ET}$	D_2 2d term 2d part	$\frac{Pr^2}{ET}$	D_3 2d term 1st part	$\frac{Pr^2}{\phi_1ET}$
$\frac{\phi_1^2}{2}-\text{vers }\phi_1$			D_1 D_3 1st term 1st part	$\frac{PR_ur^2}{\phi_1EI}$ $\frac{PR_ur^3}{\phi_1EI}$	D_1 D_3 1st term 1st part	$\frac{Pr^3}{EI}$ $\frac{Pr^4}{EI}$		
$\sin\phi_0\left(\phi_1\sin\phi_1-\text{vers }\phi_1-\frac{\sin^2\phi_1}{2}\right)+\cos\phi_0\left(\sin\phi_1-\frac{\phi_1-\sin\phi_1\cos\phi_1}{2}-\phi_1\cos\phi_1\right)$			D_2 1st term D_2 2d term 1st part	$\frac{PR_ur^3}{\phi_1EI}$ $\frac{PR_ur}{\phi_1ET}$	D_2 1st term	$\frac{Pr^4}{EI}$	D_3 2d term 2d part	$\frac{Pr^2}{\phi_1ET}$
$\cos\phi_0\left(\phi_1\sin\phi_1-\text{vers }\phi_1-\frac{\sin^2\phi_1}{2}\right)-\sin\phi_0\left(\sin\phi_1-\frac{\phi_1-\sin\phi_1\cos\phi_1}{2}-\phi_1\cos\phi_1\right)$			D_3 1st term 2d part D_3 2d term 1st part	$-\frac{PR_ur^3}{\phi_1EI}$ $-\frac{PR_ur}{\phi_1ET}$	D_3 1st term 2d part	$-\frac{Pr^4}{EI}$	D_2 2d term 2d part	$\frac{Pr^2}{\phi_1ET}$
$\sin\phi_1-\phi_1+\frac{\phi_1^3}{6}$							D_1 D_3 1st term 1st part	$\frac{Pr^3}{\phi_1EI}$ $\frac{Pr^4}{\phi_1EI}$
$\sin\phi_0\left(\frac{\phi_1^2}{2}\sin\phi_1+\phi_1\cos\phi_1-2\sin\phi_1+\frac{\phi_1+\sin\phi_1\cos\phi_1}{2}\right)+\cos\phi_0\left(\phi_1\sin\phi_1-\frac{\phi_1^2}{2}\cos\phi_1-2\text{ vers }\phi_1+\frac{\sin^2\phi_1}{2}\right)$							D_2 1st term	$\frac{Pr^4}{\phi_1EI}$
$\cos\phi_0\left(\frac{\phi_1^2}{2}\sin\phi_1+\phi_1\cos\phi_1-2\sin\phi_1+\frac{\phi_1+\sin\phi_1\cos\phi_1}{2}\right)-\sin\phi_0\left(\phi_1\sin\phi_1-\frac{\phi_1^2}{2}\cos\phi_1-2\text{ vers }\phi_1+\frac{\sin^2\phi_1}{2}\right)$							D_3 1st term 2d part	$-\frac{Pr^4}{\phi_1EI}$

* Trigonometric functions, times multipliers, give formulas for terms in designation columns.

Formulas for the right side of the arch are the same as above except that M_L, H_L, and V_L are replaced by M_R, H_R, and V_R.

By substituting M_L, H_L, and V_L from Table 5 in Eqs. (79), (80), and (81) and integrating between the limits of 0 and φ_1, equations for D_1, D_2, and D_3 at the point where the load begins may be obtained. By introducing proper functions of φ_0, the angular distance beyond the loaded section, the equations may be amplified to give D terms at points on the unloaded portion of the arch.

Formulas for load constants, for the uniform and triangular loads shown on Figs. 11, 12, and 13, may be compiled from data given in Tables 6 and 7. Table 6 gives data needed for radial and tangential load constants; Table 7 gives data needed for twist-load constants. Since some of the lengthy trigonometric functions appear in several of the equations, the trigonometric parts of the formulas are given in the first columns of the tables, and the places where they appear, together with their multipliers, are indicated in subsequent columns.

TABLE 7.—FORMULAS FOR LOAD CONSTANTS FOR UNIFORM AND TRIANGULAR TWIST LOADS

Trigonometric part of formula*	Twist loads			
	Uniform		Triangular	
	Designation	Multiplier	Designation	Multiplier
$\frac{\phi_1^2}{2}$	D_1 D_3 1st part	$\frac{Pr^2}{EI}$ $\frac{Pr^3}{EI}$		
$\frac{\phi_1^3}{6}$			D_1 D_3 1st part	$\frac{Pr^2}{\phi_1 EI}$ $\frac{Pr^3}{\phi_1 EI}$
$\sin \phi_0(\phi_1 \sin \phi_1 - \text{vers } \phi_1) + \cos \phi_0(\sin \phi_1 - \phi_1 \cos \phi_1)$	D_2	$\frac{Pr^3}{EI}$		
$\cos \phi_0(\phi_1 \sin \phi_1 - \text{vers } \phi_1) - \sin \phi_0(\sin \phi_1 - \phi_1 \cos \phi_1)$	D_3 2d part	$-\frac{Pr^3}{EI}$		
$\sin \phi_0 \left(\frac{\phi_1^2}{2} \sin \phi_1 + \phi_1 \cos \phi_1 - \sin \phi_1\right) + \cos \phi_0 \left(\phi_1 \sin \phi_1 - \frac{\phi_1^2}{2} \cos \phi_1 - \text{vers } \phi_1\right)$			D_2	$\frac{Pr^3}{\phi_1 EI}$
$\cos \phi_0 \left(\frac{\phi_1^2}{2} \sin \phi_1 + \phi_1 \cos \phi_1 - \sin \phi_1\right) - \sin \phi_0 \left(\phi_1 \sin \phi_1 - \frac{\phi_1^2}{2} \cos \phi_1 - \text{vers } \phi_1\right)$			D_3 2d part	$-\frac{Pr^3}{\phi_1 EI}$

* Trigonometric functions, times multipliers, give formulas for terms in designation columns.

In order to simplify the tabulations, the D_2 and D_3 constants are divided into two terms. The first term gives the effect of bending, and the second term, the effects of rib shortening and shear. In order to still further simplify the tabulations, the first and second terms are sometimes subdivided into first and second parts. The subdivisions of the first terms have no special significance. In the case of the second-term subdivisions, the first part gives the effects of rib shortening, and the second part the effects of shear.

Since the D terms are deflections due to loads between the point considered and the abutment, D terms for intermediate points along a triangularly loaded arch cannot be obtained by integrating Eqs. (79), (80), and (81) up to values of φ less than φ_1. Values of D terms for such intermediate points must be calculated by integrating revised forms of the equations in which the M_L, H_L, and V_L parts apply to the uniform and triangular loads comprising the total external load from the abutment to the point considered.

Temperature Loads. Formulas for temperature thrusts, moments, and shears at the crown and abutment sections of a constant-thickness arch, including effects of shear but not effects of abutment deformations, are as follows:

$$H_0 = \frac{cTr \sin \varphi}{\frac{12r^3}{Et^3}\left[(\varphi - 2\sin\varphi) + \frac{(\varphi + \sin\varphi\cos\varphi)}{2}\right] + \frac{r}{Et}\left[\frac{(\varphi + \sin\varphi\cos\varphi)}{2} + \frac{3(\varphi - \sin\varphi\cos\varphi)}{2}\right] - \frac{12r^3}{\varphi E t^3}[\varphi - \sin\varphi]^2} \tag{82}$$

$$M_0 = -\frac{r(\varphi - \sin\varphi)H_0}{\varphi} \tag{83}$$

$$V_a = 0 \tag{84}$$

$$H_a = H_0 \cos\varphi \tag{85}$$

$$M_a = -\frac{r(\varphi - \sin\varphi)H_0}{\varphi} + rH_0 \operatorname{vers}\varphi \tag{86}$$

$$V_a = H_0 \sin\varphi \tag{87}$$

Diagrams showing stresses caused by a 1-deg drop in temperature, including effects of abutment deformations, have recently been published for arch elements having different values of φ and t/r.[1]

Temperature deflections of the arch elements must be included with other arch-load deflections before adjusting with the cantilever deflections. Effects of temperature loads may be included in the general formulas for arch movements by adding the following terms to Eqs. (39) and (40), respectively:

$$d(\Delta r)_0 = \int_0^s cT \sin\varphi \, ds$$

$$d(\Delta s)_0 = \int_0^s cT \cos\varphi \, ds$$

T is the temperature change, positive when rising, and c the coefficient of thermal expansion.

The preceding additions to Eqs. (39) and (40) result in the following values of D_2 and D_3 for temperature loads:

$$D_2 = -cTy_a = -cTr \operatorname{vers}\varphi_1 \tag{88}$$

$$D_3 = cTx_a = cTr \sin\varphi_1 \tag{89}$$

Arch Stresses. When the arch deflections have been brought into satisfactory agreement with the cantilever deflections in all parts of the dam, arch thrusts, moments, and shears, caused by the combined radial, tangential, twist, and temperature loads, are calculated for the locations where stresses are desired, usually the crown and abutment sections. Direct stresses at the extrados and intrados curves are then computed by Eq. (7). Stresses at different depths in the concrete may be computed on the assumption of a straight-line variation between the faces of the dam.

[1] HOUK, IVAN E., Arch Deflections and Temperature Stresses in Curved Dams, No. I, *Engineer*, Apr. 2, 1937, pp. 382–384.

Average shearing stresses are the shearing forces divided by the areas on which they act. Shearing forces at crown sections of symmetrical arches, symmetrically loaded, are zero. Horizontal shearing stresses at the upstream and downstream edges of arch elements, acting in vertical tangential planes, may be computed by the formula

$$N = \mp (S - p) \tan \beta \qquad (-\text{ at extrados}) \qquad (90)$$

where S = direct stress acting normal to the plane.
p = water pressure at the face of the dam.
β = angle between the normal to the plane and a line tangent to the edge of the arch element.

In circular constant-thickness arches, shearing stresses at the extrados and intrados are zero; and maximum shearing stresses in the interior may be estimated at 3⁄2 the average shearing stress, a parabolic distribution being assumed. If tangential shear and twist effects are analyzed, Eq. (90) should include the correction $\pm N_t \tan \alpha$, where N_t is the tangential shear stress at the edge of the arch and α the angle between the edge of the cantilever and the vertical direction, as in Eq. (26), the plus sign being used for the extrados.

INVESTIGATIONS AT ARCH DAM

Research measurements of technical factors involved in design have been made at several arch dams during the last 15 years. The most noteworthy were those conducted at Stevenson Creek Test Dam near Fresno, Calif.,[1] and at Ariel Dam near Portland, Ore.[2] Additional arch dams at which investigations have been made, or are now in progress, include Clear Creek, Emigrant Creek, Gerber, Gibson, Calderwood, Owyhee, and Hoover dams. Experimental measurements have also been made at some European arch dams.[3]

Measurements Made. The investigations included measurements of rock deformations, radial concrete deflections, circumferential concrete movements, concrete strains, crack formations, opening and closing of construction joints, seasonal concrete temperature variations, temperature changes caused by generation and dissipation of setting heat, water temperatures at the upstream face of the dam, uplift pressures at the base of the dam, uplift pressures at horizontal construction joints, efficiency of drainage installations, and other miscellaneous factors that affect the structural action and safety of arch dams. Only the more important investigations and resulting conclusions can be discussed herein.

Stevenson Creek Test Dam. The Stevenson Creek Test Dam, a constant-radius structure 60 ft. high, was built solely for research measurements. It was purposely designed with a relatively thin section, so that strains and deflections would be of measurable magnitudes. It was constructed under rigid technical control and was arranged so that the full water load could be applied or removed in a few hours. Testing operations were conducted by a staff of research specialists, supervised by the Engineering Foundation Arch Dam Committee. The measurements obtained were so accurate and comprehensive that the research engineers were able to satisfactorily analyze the action of the structure. For instance, they were able to determine the proportions of water load carried by arch and cantilever action in different parts of the dam; the moments, thrusts, and shears in the arch and cantilever elements; the vertical, horizontal, and principal strains at the downstream face; the spreading of

[1] "Report on Arch Dam Investigation," vols. I and III, The Engineering Foundation, 1927 and 1933.

[2] LARNED, A. T., and W. S. MERRILL, Actual Deflections and Temperatures in a Trial-load Arch Dam, *Trans. A. S. C. E.*, **99**, 897–961, 1934.

[3] LANG, W., "Deformationsmessungen an Staumauern," Verlag der Obteilung für Landestopographie, Berne, 1929.

the canyon walls; the radial movements of the dam; and the relations between the actual deformations of the structure and those calculated by deflection formulas.

Results of Investigations. The results of the investigations at Stevenson Creek Test Dam, together with careful studies of experimental measurements at other arch dams, lead to the following conclusions:

1. Concrete temperature changes are especially important, not only as affecting stress magnitude and distribution, but also as affecting structural movements and crack formations.
2. Chemical heat generated during the curing period should be thoroughly dissipated before closing the arches by grouting radial contraction joints or filling radial slots.
3. Appreciable proportions of horizontal water loads are carried by cantilever action, even in dams of the thin-arch type.
4. Horizontal cracks may develop along the upstream face of thin-arch dams, near the foundation, owing to tension stresses in the cantilever elements.
5. Movements of arch dams may be satisfactorily estimated from calculations by elastic formulas if proper allowances are made for rock deformations, twist action, and tangential shear effects.
6. Proportions of horizontal water loads carried by arch elements may be uniform from crown to abutment sections at certain elevations in symmetrical dams, but are usually too variable to warrant the preparation of important final designs on such a basis.
7. The trial-load method of analyzing arch and cantilever action in curved concrete dams furnishes a satisfactory basis for the design of arch dams of any type and size.

MODEL INVESTIGATIONS

Several model investigations of the action of arch dams have been made since 1926. Some were conducted on models of the entire prototype. Others were conducted on slab models representing arch or cantilever elements. In both cases, the models included substantial portions of the foundations and abutments. The more comprehensive tests were as follows:

1. Celluloid model of Stevenson Creek Test Dam.[1]
2. Concrete model of Stevenson Creek Test Dam.[2]
3. Concrete model of Gibson Dam.[2]
4. Plaster-celite model of Hoover Dam.[3]
5. Litharge-rubber model of Hoover Dam.[4]
6. Plaster-celite model of crown cantilever of Hoover Dam.[4]
7. Plaster-celite model of an arch element of Hoover Dam.[4]
8. Plaster-celite model of a cantilever element of a thin-arch dam.
9. Litharge-rubber model of Calderwood Dam.[5]

Mercury was used as a loading medium in 1, 2, 3, and 4; water, in 5 and 9; and mechanical systems of weights and levers, in 6, 7, and 8. In 6 and 8, vertical loads were applied, so that the relation between model weights and external loads would be the same as the prototype. Anyone referring to the article cited for model tests 9 should carefully study the discussions by A. W. Simonds, Eldred D. Smith, and Elmer O. Bergman.

Results of Investigations. The results of the model investigations agreed satisfactorily with data secured at existing dams, particularly in the Stevenson Creek tests. Trial-load analyses of the Stevenson Creek, Gibson, and plaster-celite Hoover models showed close agreements between calculated and measured deflections. Although tests of the litharge-rubber model of Hoover Dam showed a fairly close agreement of deflections, they were not wholly satisfactory, owing to the high Poisson's ratio of the model material, about 0.50, and the large variations in the modulus of elasticity in different directions.

[1] "Report on Arch Dam Investigation," The Engineering Foundation, vol. I, 1927, pp. 219–230; vol. III, 1933, pp. 150–174.

[2] *Ibid.*, vol. II, 1931, 542 pages.

[3] SAVAGE, J. L. and IVAN E. HOUK, Checking Arch Dam Designs with Models, *Civil Eng.*, May, 1931, pp. 695–699; also SAVAGE, J. L., and IVAN E. HOUK, Model Tests Confirm Design of Hoover Dam, *Eng.-News Record*, **108**, 494–499, 1932.

[4] *Bur. Reclamation, U.S. Dept. Interior, Bull.* 2, Slab Analogy Experiments, *Bull.* 3, Model Tests of Boulder Dam, and *Bull.* 6, Model Tests of Arch and Cantilever Elements; Boulder Canyon Project Final Reports, Part V, Technical Investigations, 1938, 1939, and 1940.

[5] KARPOV, A. V., and R. L. TEMPLIN, Model of Calderwood Arch Dam, *Trans. A. S. C. E.*, **100**, 185–262. 1935.

Conclusions. The general conclusions which may be drawn from the model investigations are as follows:

1. Satisfactory predictions of deflections and strains in thin-arch dams may be made from tests of celluloid models, loaded with mercury, particularly if tension stresses are relieved at locations where cracks would be expected in the prototypes.
2. Satisfactory predictions of deflections and strains in thin-arch dams may be made from tests of concrete models, loaded with mercury.
3. Plaster celite is the best material thus far developed for use in building models of large, thick-arch dams or models of horizontal or vertical elements of such structures.
4. Trial-load analyses of model strains and deflections check the experimental measurements satisfactorily.
5. Model tests of horizontal and vertical elements of arch dams furnish valuable data regarding the stress distribution in such structures.
6. In the case of unusually large, thick-arch dams, where dead-load stresses are important and where prototype construction is necessarily complicated by contraction joints, contraction-joint grouting, concrete temperature control, variable concrete curing, partial water loads during construction, irregular rock profiles, differences in geological formations, and other field factors that cannot be definitely determined until the work is completed, carefully controlled model tests are more valuable in checking fundamental methods of design than in making direct predictions of deformations and stresses in the prototypes.

EXAMPLES OF ARCH DAMS

The following examples illustrate practical designs of arch dams that have been built and are operating satisfactorily. Hoover Dam represents an unusually high and massive, arched-gravity type of constant-radius dam, Deadwood Dam a more usual height of constant-radius structure, and Pacoima Dam a high type of variable-radius dam.

Hoover Dam. Hoover Dam, built by the Bureau of Reclamation, was completed on Colorado River near Las Vegas, Nev., in 1936. Figure 14 shows a general plan of the structure, a vertical cross section along the line of centers, and a tabulation of arch data at 100-ft intervals of elevation. Principal dimensions are given in Table 8. The dam was designed on the basis of trial-load analyses. It is curved on a radius of 500 ft to the upstream edge of the crest, has extrados curves of gradually increasing radii as the depth below the crest increases, and is provided with long radius fillets at the abutment ends of the intrados curves in the regions of pronounced arch stress. Short radius fillets connect the dam with the abutment and foundation rock along the entire profile at the upstream face. Several articles describing its design and construction have been published.[1]

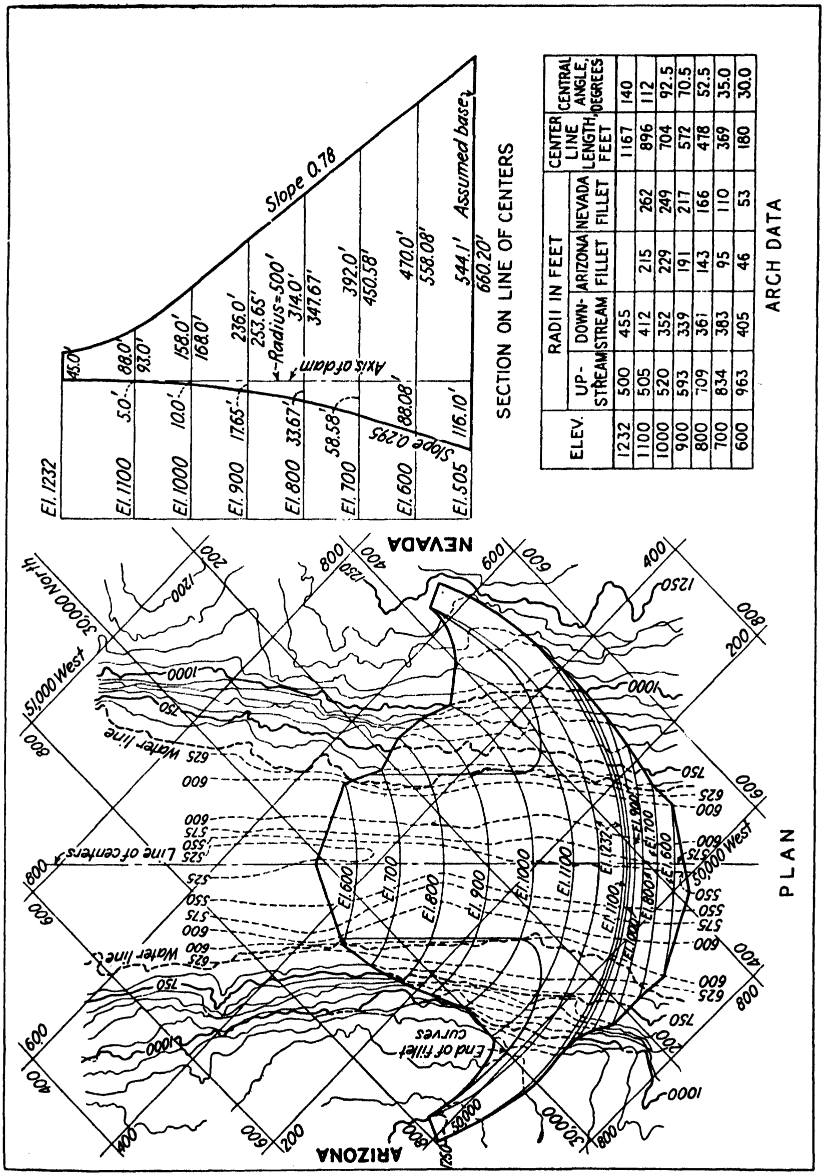
Deadwood Dam. Deadwood Dam, a 168-ft structure built by the Bureau of Reclamation, was completed on Deadwood River near Boise, Idaho, in 1930. Figure 15 shows a general plan of the dam, a developed elevation of the upstream face, vertical cross sections at the spillway and valve-house locations, typical cross sections of the thrust block and gravity tangent at the right end of the structure, and a tabulation of arch and cantilever stresses. Principal dimensions are given in Table 8. The arched section was designed by the trial-load method. It is curved on a radius of 290 ft to the upstream edge of the crest and has a vertical upstream face from abutment to abutment. Long-radius fillets were provided near the abutment ends of the intrados curves at the lower elevations. The thrust block was designed to carry radial shear loads, transferred through the arch elements, as well as direct water pressures at the upstream face. A spillway section, 100 ft long, with its crest 6 ft below the top of the nonoverflow sections, was provided at the river channel.[2]

Pacoima Dam. Pacoima Dam, a 372-ft structure, built by the Los Angeles County Flood Control District, was completed on Pacoima Creek near Los Angeles, Calif.,

[1] HOUK, IVAN E., Technical Design Studies for Hoover Dam, *Western Constr. News*, Apr. 10, 1932, pp. 187–193. Also see *Eng. News-Record*, **104**, Feb. 6, 1930; **109**, Dec. 15, 1932; **111**, Dec. 21, 1933; and **115**, Dec. 26, 1935.

[2] NEWELL, R. J., Deadwood Dam, Boise Project, Idaho, *New Recl. Era*, August, 1931. pp. 175–177.

in 1928. Figure 16 shows the general plan, profile, and maximum cross section of the dam, also a tabulation of arch data, furnished through the courtesy of C. H. Howell, chief engineer. Principal dimensions are given in Table 9. The dam was originally

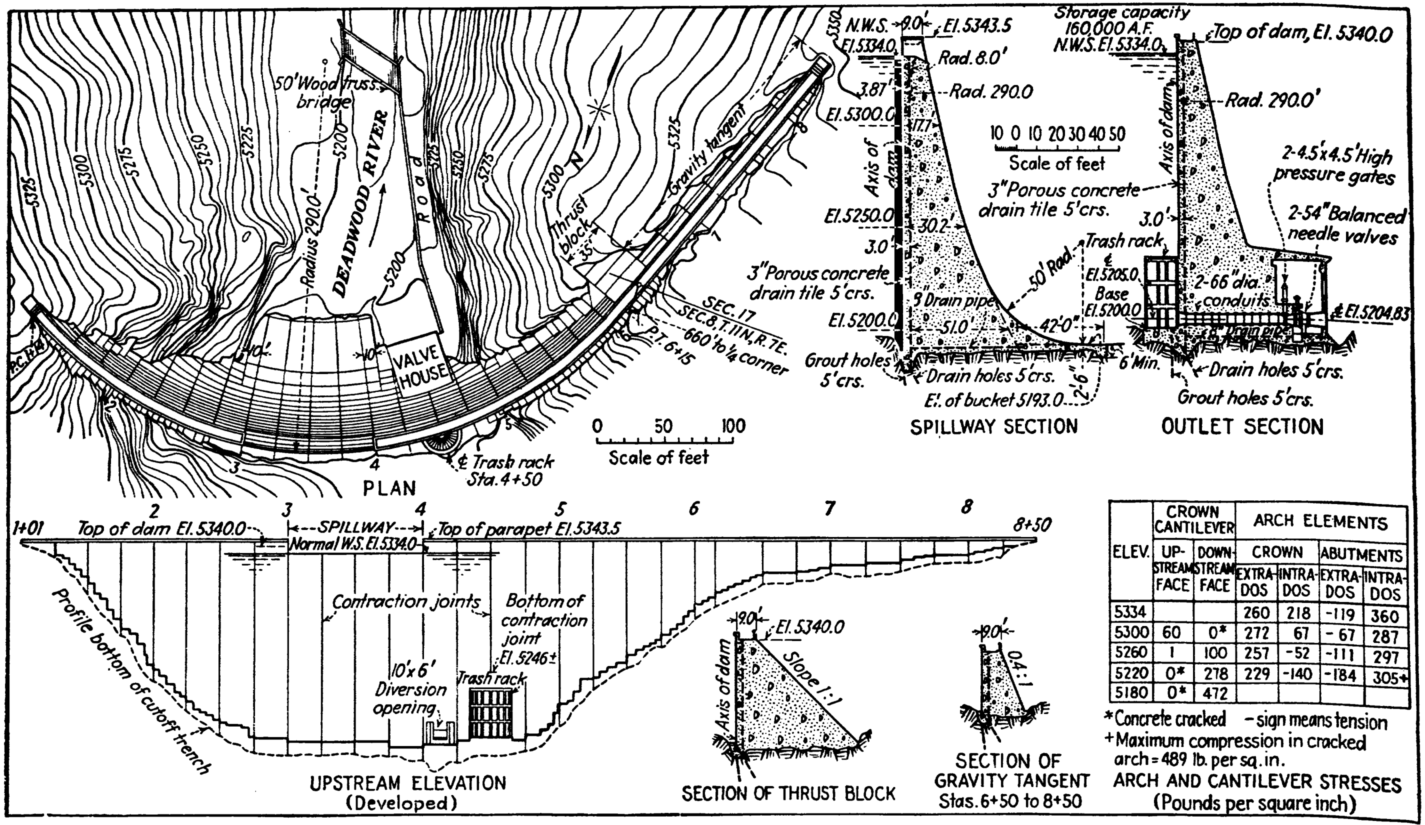


ELEV.	RADII IN FEET				CENTER LINE LENGTH, FEET	CENTRAL ANGLE, DEGREES
	UP-STREAM	DOWN-STREAM	ARIZONA FILLET	NEVADA FILLET		
1232	500	455			1167	140
1100	505	412	215	262	896	112
1000	520	352	229	249	704	92.5
900	593	339	191	217	572	70.5
800	709	361	143	166	478	52.5
700	834	383	95	110	369	35.0
600	963	405	46	53	180	30.0

FIG. 14.—Plan and maximum section of Hoover Dam.

designed by the thin-cylinder formula, but was later analyzed by a trial-load method which brought the arch and cantilever deflections into adjustment at the crown section.[1] Final analyses showed a maximum compressive arch stress of 500 psi at

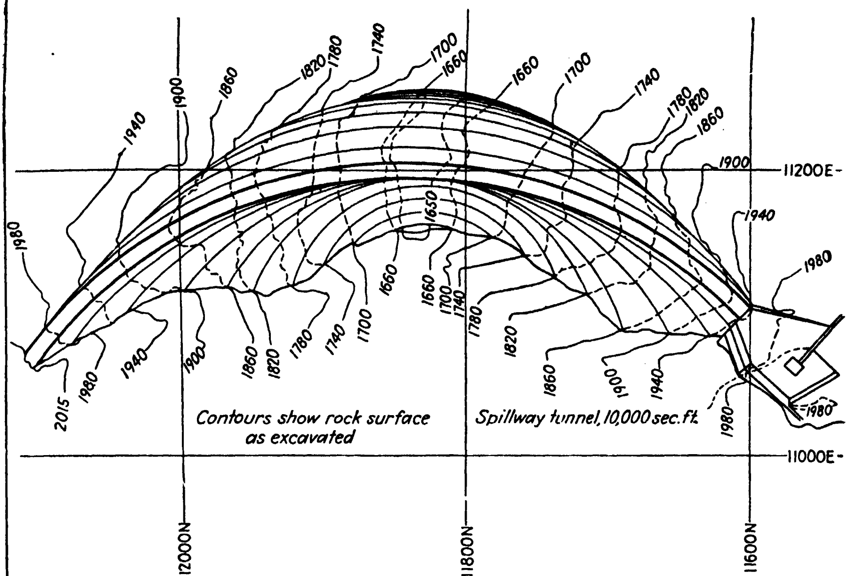
[1] NOETZLI, FRED A., The Pacoima Arch Dam, California, *Engineering*, Jan. 18, 1929, pp. 70–71.



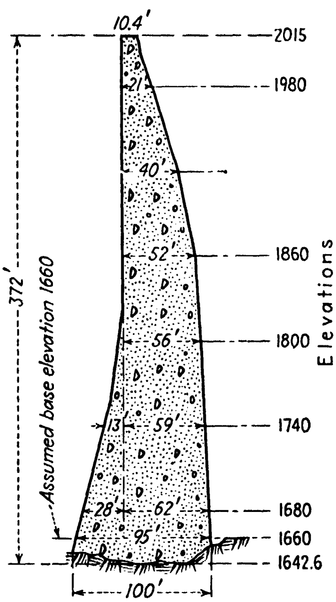
ELEV.	CROWN CANTILEVER UP-STREAM FACE	DOWN-STREAM FACE	ARCH ELEMENTS CROWN EXTRA-DOS	INTRA-DOS	ABUTMENTS EXTRA-DOS	INTRA-DOS
5334			260	218	-119	360
5300	60	0*	272	67	- 67	287
5260	1	100	257	-52	-111	297
5220	0*	218	229	-140	-184	305+
5180	0*	472				

*Concrete cracked - sign means tension
+Maximum compression in cracked arch = 489 lb. per sq. in.
ARCH AND CANTILEVER STRESSES
(Pounds per square inch)

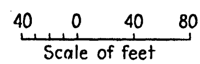
FIG. 15.—Plan, elevation, and sections of Deadwood Dam.



PLAN



MAXIMUM CROSS SECTION



ELEVATION	RADIUS-FEET		THICKNESS-FEET	
	EXTRADOS	INTRADOS	CROWN	ABUT.
2015	335.80	325.40	10.4	10.4
1980	318.29	297.29	21.0	21.0
1920	281.55	232.85	40.0	43.4
1860	251.73	152.00	52.0	72.0
1800	234.47	111.30	59.7	78.0
1740	212.00	79.00	72.0	82.0
1680	171.34	64.00	90.0	
1660			95.0	
1642			100.0	

ARCH DIMENSIONS

FIG. 16.—Plan, sections, and dimensions of Pacoima Dam.

Table 8.—Constant-radius Dams in the United States More than 100 Ft High

All concrete except Cheesman and Pathfinder, which are rubble masonry with ashlar faces

Name of dam	Location	Stream	Year completed	Purpose[2]	Max height, ft[4]	Upstream radius, ft	Thickness, ft		Top length, ft		Volume in dam, 1,000 cu yds	Remarks
							Top	Base	Arch	Total[5]		
Hoover	Arizona-Nevada	Colorado	1936	[3]	731	500	45	659	1,220	1,220	3,241	Inclined-shaft spillways in abutments
Owyhee	Oregon	Owyhee	1932	I.	421	500	30	255	528	820	521	Gravity sections at ends; shaft spillway at right[6]
Arrowrock	Idaho	Boise	1915	I.	351	672	15.5	223	1,100	1,100	585	Side-channel spillway at right end
Parker	Arizona-California	Colorado	U.C.[1]	D.	335	315	39	105	820	820	260	Central overflow; 50 × 50 ft stoney gates
Shoshone	Wyoming	Shoshone	1910	I.	328	155	10	108	200	200	79	Tunnel spillway around left end
Shannon	Washington	Baker	1926	P.	263	250	20	135	493	493	132	Overflow section with gates
Seminoe	Wyoming	North Platte	U.C.	I.	261	290	15	85	563	563	161	Inclined shaft spillway at right end
Cushman 2	Washington	No. Fork of Skokomish	1931	P.	240	140	8	40	330	450	38	Gravity section at left end; thrust block and spillway at right
Cheesman	Colorado	So. Fork of South Platte	1904	W.S.	236	400	18	176	710	710	103	Saddle spillway beyond left end
Salmon River	Idaho	Salmon	1912	I.	225	225	12	119	480	480		
Pathfinder	Wyoming	North Platte	1909	I.	218	156	11	94	225	432	60	Gravity sections at ends; saddle spillway at left
Melones	California	Stanislaus	1926	I.	214	238	20	110	545	591	93	Overflow section with gates; gravity sections at ends
Gibson	Montana	No. Fork of Sun	1929	I.	199	405	15	87	960	960	161	Shaft spillway at left abutment
Gibralter	California	Santa Inez	1920	W.S.	185	243	8	65	984	1,100	60	Thrust block at left end; spillway beyond
Deadwood	Idaho	Deadwood	1930	I.	168	290	9	62	501	749	55	Center spillway; thrust block and gravity section at right end
Saw Pit	California	Saw Pit	1927	F.C.	162	265	8	58	.....	539	56	Gravity sections at ends
Eleven Mile Canyon	Colorado	So. Fork of South Platte	1932	W.S.	150	270	15	53	445	445	50	Spillway around left end
Big Creek 6	California	San Joaquin	1923	P.	135	180	8	39	.....	495	20	Overflow section; gravity sections at ends
Malibu	California	Malibu	1925	I.	125	...	3	12	.....	150	4.2	
Warm Springs	Oregon	Malheur	1919	I.	109	190	8	27	469	549	19	Central overflow section

[1] U.C., under construction.
[2] I., irrigation; D., diversion; W.S., municipal water supply; P., power development; F.C., flood control.
[3] I., W.S., F.C., P., and silt control.
[4] Above lowest foundation.
[5] Including thrust blocks, gravity sections, and other parts of main dam; but not spillways separated from the dam.
[6] Looking downstream.

TABLE 9.—VARIABLE-RADIUS DAMS IN THE UNITED STATES MORE THAN 100 FT HIGH
(All built of concrete)

Name of dam	Location	Stream	Year completed	Purpose[2]	Maximum height, ft[3]	Upstream radius, ft		Thickness, ft		Top length, ft		Volume in dam 1,000 cu yds	Remarks
						Top	Base	Top	Base	Arch	Total[5]		
Diablo	Washington	Skagit	1930	P.	426	390	149	16	140	588	1,180	350	Gravity overflow sections at ends
Pacoima	California	Pacoima	1928	F.C.	372	336	170	10.4	100	640	640	225	Spillway tunnel at left end[6]
Ariel	Washington	Lewis	1931	P.	313	397	247	19.5	93	728	1,250	307	Thrust block, overflow section and nonoverflow section at right end
Horse Mesa	Arizona	Salt	1927	P.	305	251	82	8	43	520	784	147	Overflow sections at ends
Spaulding	California	So. Fork of Yuba	1919[1]	P.	290	428	250	11	186[4]	521	800	192	Gravity sections at ends
Cushman 1	Washington	No. Fork of Skokomish	1926	P.	280	210	118	8	52	447	857	90	Gravity sections at ends; saddle spillway beyond right end
Big Tujunga 1	California	Big Tujunga	1931	F.C.	253	190	120	8	73	400	800	79	Gravity section at left end; spillway and earth section at right
Calderwood	Tennessee	Little Tennessee	1930	P.	230	338	281	25	48	736	814	400	Overflow section with gates
Mormon Flat	Arizona	Salt	1925	P.	229	187	100	8	20	360	623	43	Spillway at left end
Big Santa Anita	California	Big Santa Anita	1927	F.C.	225	305	129	7	48	605	605	76	Spillway at right end
Stewart Mountain	Arizona	Salt	1930	P.	212	273	170	8	33	580	1,260	122	Gravity abutments and wing walls at ends; spillway at left end
Glines Canyon	Washington	Elwah	1927	P.	200	128	62	4	28	152	435	16	Thrust block and gravity section at E. end; spillway at W. end
Waterville	North Carolina	Big Pigeon	1929	P.	200	323	...	16	40	...	.....	120	Overflow section with gates
Bullards Bar	California	No. Fork of Yuba	1924	P., Deb.	198	240	94	6	43	520	520	49	Overflow section, designed for 90 lb per cu ft pressure
East Canyon Creek	Utah	East Canyon	1916	I.	193	99	70	5	26	128	273	16	Reinforced; deck section and spillway at right end; gravity section at left

TABLE 9.—VARIABLE-RADIUS DAMS IN THE UNITED STATES MORE THAN 100 FT HIGH.—(*Continued*)

Name of dam	Location	Stream	Year completed	Purpose[2]	Maximum height, ft[3]	Upstream radius, ft.		Thickness, ft		Top length, ft		Volume in dam 1,000 cu yds	Remarks
						Top	Base	Top	Base	Arch	Total[5]		
Salmon Creek	Alaska	Salmon	1914	P.	170	331	148	6	48	545	640	52	Spillway at right end
Juncal	California	Santa Inez	1930	W.S.	160	237	84	6	64	...	440	34	Overflow section
Caneadea	New York	Caneadea	1928	P.	143	262	...	5	44	440	620	66	Brick faced, gravity abutments, spillway around left end
Hogan	California	Calaveras	1930	F.C.	137	349	205	7	48	616	1,366	122	Overflow sections, gravity abutments; gravity and earth sections at right end
Hubbart	Montana	Little Bitter Root	1923	I.	131	220	188	5	24	503	503	17	Top 25 ft reinforced; central spillway
Sun River	Montana	No. Fork of Sun	1914	I. D.	131	80	50	7.5	37.5	163	254	6	Overflow section; gravity section at left end
Emigrant Creek	Oregon	Emigrant	1924	I.	127	165	132	5	23	297	430	15	Reinforced; gravity tangent and auxiliary arch at left end; siphon at right
Lost Creek	California	Lost	1924	I.	120	200	90	4	24	486	486	11	Overflow section
Cat Creek	Nevada	Cat	1932	W.S.	118	110	58	6	23	237	237		
Ashland	Oregon	Ashland	1928	W.S.	116	230	158	7	25	465	465	...	Overflow section at center; horizontal joint 10 ft above base
Kerkhoff	California	San Joaquin	1919	P.	114	205	136	10	36	440	570	23	Overflow section with gates; gravity sections at ends
Safford	Arizona	Frye	1929	W.S.	110	80	14	2	5	180	180	1	Reinforced and dowelled to abutment rock

[1] Built 225 ft high in 1913; raised 35 ft in 1916; raised 15 ft more in 1919.
[2] I., irrigation; F.C., flood control; P., power development; D., diversion; Deb., debris control; W.S., municipal water supply.
[3] Above lowest foundation.
[4] Base built for 305-ft dam; reduced to 94 ft at elevation 64 ft above base.
[5] Including gravity abutments, overflow, or other connected sections; but not spillways separated from the dam.
[6] Looking downstream.

TABLE 10.—ARCH DAMS IN THE UNITED STATES LESS THAN 100 FT HIGH

Name of dam	Location	Stream	Type[1]	Year completed	Purpose[2]	Max height, ft[3]	Upstream radius ft		Thickness, ft		Top length, ft		Volume in dam 1,000 cu yds	Remarks
							Top	Base	Top	Base	Arch	Total[6]		
Van Giesen	California	Bear	V.R.	1928	I., P.	99	205	190	5	22	440	800	22	Overflow section with non-overflow gravity abutments
Grizzly Forebay	California	Grizzly	V.R.	1928	P.	98	215	78	6	23	420	...	17	Central overflow; gravity sections at ends
Toltec	New Mexico	Bluewater	C.R.	1927	I.	97	175	175	...	....	340	500	14	Gravity section with 2 siphon spillways at right end[7]
Humphreys	Colorado	Goose	C.R.	1924	R.	96	85	85	3.5	17	186	186	2.9	Upper 50 ft. reinforced; saddle spillway beyond left end
Halligan	Colorado	No. Branch, Cache La Poudre	C.R.	1910	I.	94	324	324	2	55[4]	350	350	18	Reinforced; central spillway
Bucks Creek	California	Bucks	V.R.	1927	D.	93	213	158	6	26	410	...	19	Gravity section at right end; gravity spillway at left; central emergency spillway
Big Creek 4	California	Big	C.R.	1913	P.	89	150	150	7.5	29	190	286	6.9	Gravity sections at ends
Upper Otay	California	Otay	C.R.	1901	W.S.	88	359	359	4	14	350	350	3.3	Reinforced with steel plates and cables
Gerber	Oregon	Miller	V.R.	1925	I.	85	197	155	5	18	478	478	12	Upper part reinforced; central spillway
Clear Creek	Washington	Clear	V.R.	1918	I.	84	128	106	3	9.8	160	404	4.1	Gravity sections at ends
Big Creek 5	California	Big	C.R.	1921	P.	80	150	150	7.5	22	260	260	3.8	
Crowley Creek	Oregon	Crowley	C.R.	1914	I.	70	70	70	3	5.2	158	...	1.1	Gravity sections at ends; faulty arch construction
Bear Valley	California	....	C.R.	1884	I.	64	335	335	3	20[5]	300	...	3.4	Multiple arch dam built below in 1911
Stevenson	California	Stevenson	C.R.	1926	T.	60	100	100	2	7.5	140	140	0.5	No spillway; dam not keyed into foundation
Morris	California	James	C.R.	1927	I., P.	56	90	90	2.8	8.7	...	140	0.8	Reinforced; overflow section
Malibu Lake	California	Triunfo	C.R.	1923	R.	52	135	135	3	9	...	210	0.9	
Las Vegas	New Mexico	Gallinas	C.R.	1911	W.S.	50	250	250	4	15.5	210	210	2.7	Dry gulch; reservoir filled by pipe line; no spillway
Butte Creek	California	Butte	V.R.	1916	P.	45	...	...	4	8	...	110	0.7	Channel overflow
East Park Diversion	California	Stony	C.R.	1914	I., D.	44	100	100	3	6	155	155	1.8	Reinforced; gravity abutment at one end
Six Mile Creek	New York	Six Mile	D.	1903	W.S.	35	60	...	2.5	7.8	67	...	...	Designed for 90-ft height
Yadkin	North Carolina	Yadkin	C.R.	1919	P.	28	76	76	...	5	...	160	...	Gravity wing at one end
Buffalo Creek	North Carolina	Buffalo	C.R.	1916	P.	27	100	100	..	5.3	170	170		
Campbells	New York	Cascadilla	C.R.	1904	....	25	70	70	2.5	4	...	88	...	Reinforced; gravity wings at ends

[1] V.R., variable radius; C.R., constant radius; D., dome.
[2] I., irrigation; P., power development; R., recreation; W.S., municipal water supply., T., testing.
[3] Above lowest foundation.
[4] Base for gravity dam, offset to 27 ft at height of 22 ft.
[5] Offset to 8.5 ft, at height of 16 ft.
[6] Including thrust blocks, gravity sections, and other parts of main dam; but not spillways at other locations.
[7] Looking downstream.

the abutment intrados near the base of the dam and slightly less than 100 psi tension at the extrados. The dam is curved on a radius of 336 ft at the top extrados and has gradually decreasing radii of curvature at both faces as the depth below the top increases. Radii of curvature at an elevation 37 ft above the base are 171 ft for the extrados and 64 ft for the intrados.

DATA ON ARCH DAMS

Tables 8, 9, and 10 give principal dimensions of some arch dams in the United States. Table 8 includes constant-radius dams more than 100 ft high, Table 9 variable-radius dams more than 100 ft high, and Table 10 some miscellaneous arch dams less than 100 ft high. All dams are listed in order of maximum height. In most cases, the maximum height is the distance from the lowest foundation level to the top of the parapet. The tables were compiled from construction drawings, personal records, and published material. Conflicting data were carefully reviewed and an effort made to give the most accurate information. Some of the dams, such as Bear Valley, Upper Otay, and Stevenson Creek Test Dam, were included because of their historic interest. Although these structures served the purposes for which they were built, their designs are too light to be recommended for ordinary service. The upper part of the Upper Otay Dam was reinforced with steel plates and railway cables.

FAILURES OF ARCH DAMS

No important arch dam has failed thus far. A small arch dam near Manitou, Colo., about 50 ft high and 300 ft long, suffered a partial failure in 1924 owing to disintegration of poor concrete used in its construction.[1] A thin-arch dam on Moyie River near Bonner's Ferry, Idaho, 53 ft high and 154 ft long, failed in 1926 owing to undermining of the timber-lined spillway channel which had been built in soft stratified rock near the left abutment. A low-arch dam on Vaughn Creek at Lake Lanier, N. C., 62 ft high and 236 ft long, also failed in 1926 owing to washing out of a cyclopean masonry abutment built on soft decomposed material at one end of the arch section. The Moyie River and Vaughn Creek dams both stood intact, under nearly full reservoir pressures, while the abutments were being washed away; so that they were still in place after the water escaped.[2]

A small-arch dam on Purisima Creek, San Mateo County, California, failed completely during the first filling of the reservoir. It was 40 ft high, 100 ft long, 2 ft thick at the top, 4 ft thick at the base, and was curved on a radius of 120 ft. It was built in a loosely bedded, shale gully and apparently failed by shearing of the foundation and abutment rock.[3] No other records of arch-dam failures have been found in engineering literature.

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[2] Two Arch Dams Fail through Undermining of Abutments, *Eng. News-Record*, **97**, 616–618, 1926.

[3] GRUNSKY, C. E., Comments on a Few Dams and Reservoirs, *Mil. Eng.*, January–February, 1931, pp. 53–54.

SECTION 4

BUTTRESS DAMS

BY EDGAR H. BURROUGHS

I. THE BUTTRESS PRINCIPLE

The principal structural elements of a buttress dam are the water-supporting upstream face, or deck, and the buttresses. These water-bearing upstream members are supported upon the buttresses and span between them; the buttresses are equally spaced, triangular walls proportioned to transmit to the foundation the water load and the weight of the structure. The basic principles of buttress construction will be made clear by reference to Figs. 1, 2, and 3, which show construction views of the 145-ft high Stony Gorge Dam, completed by the U.S. Reclamation Bureau in 1928 near

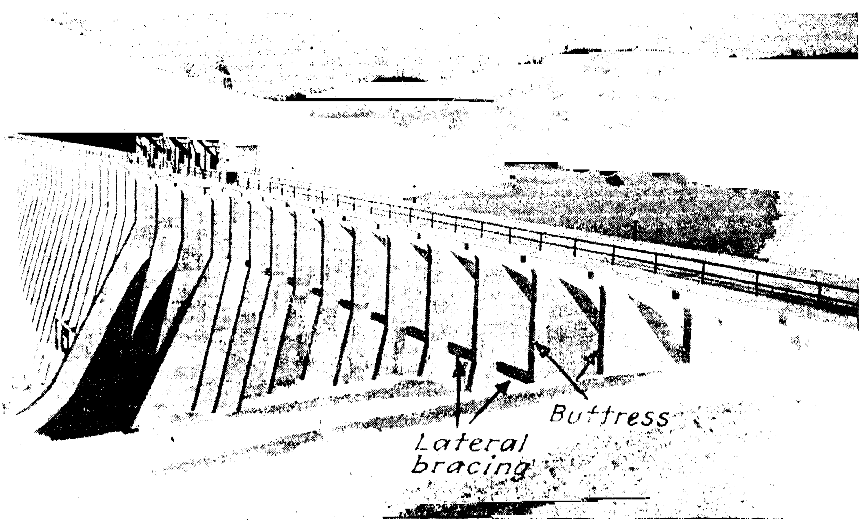


FIG. 1.—Stony Gorge Dam, California. General view. Ambursen type.

Orland, Calif. Figure 1 shows a general view of the completed dam, Fig. 2 an upstream view of the buttresses during the placement of the deck slabs, and Fig. 3 a side view of typical buttresses in different stages of construction. The decks (Fig. 2), in this instance of the Ambursen or flat-slab type, are supported on the buttresses by haunches or corbels. Additional structural elements of secondary importance are lateral bracing and buttress contraction joints. These features will be described in detail subsequently in this chapter.

The fundamental concepts of buttress design may be illustrated by comparing the loading action of the conventional gravity dam with that of the buttress dam. This can be done most conveniently by assuming each dam to consist of a series of elementary curved, inclined columns, each of which is structurally independent of adjacent columns. Such a comparison for typical columns of this type is shown in Fig. 4.

The column $ABCDE$ for the gravity dam and the column $A_1B_1C_1D_1E_1$ for the buttress dam are each designed to transmit water loads Hw and Hw_1 from equal heads on the upstream face through the structure to the foundation. It is a simple matter to

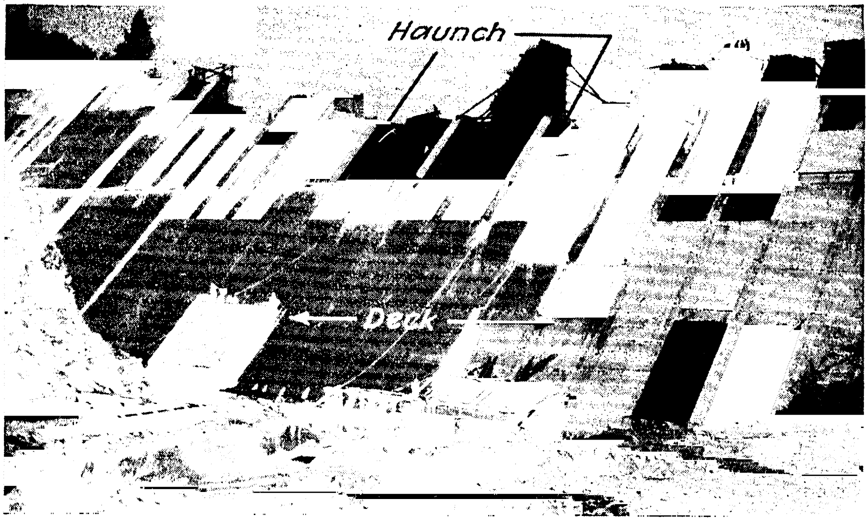


FIG. 2.—Stony Gorge Dam, California. Upstream view of buttresses and deck. Ambursen type.

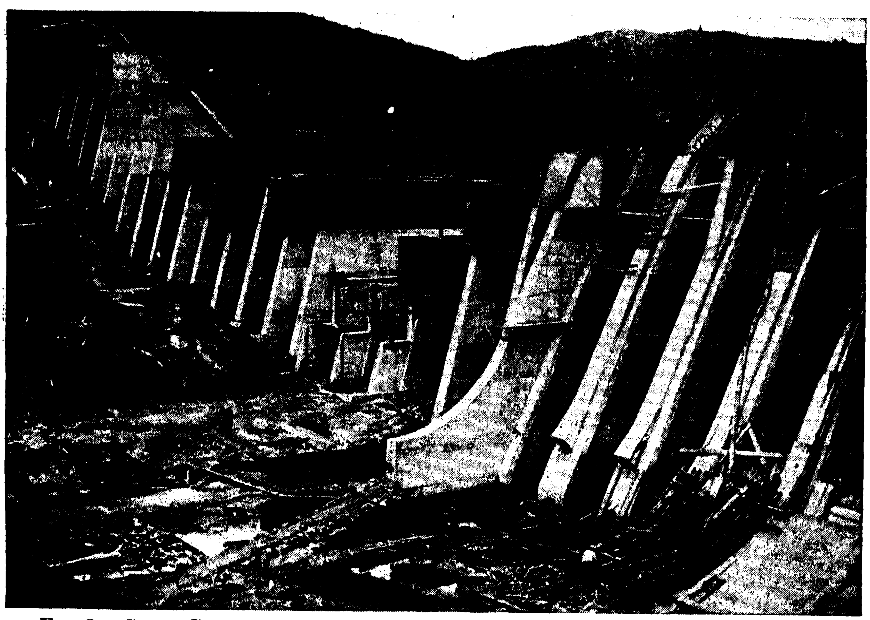


FIG. 3.—Stony Gorge Dam, California. Side view of buttresses. Ambursen type.

determine by means of force polygons the weights of masonry—W_{G1}, W_{G2}, W_{G3}, and W_{G4}, for the gravity dam, and W_{B1}, W_{B2}, W_{B3}, and W_{B4}, for the buttress dam—that are required to keep each polygon in equilibrium.

A comparison of the force polygons shows that the total weight of masonry W_1, required to keep the downstream column of the gravity dam in equilibrium, is much greater than the total weight of masonry W_2, required to keep the downstream column of the buttress dam in equilibrium. The saving in masonry effected through the adoption of the buttress principle is represented approximately by the difference

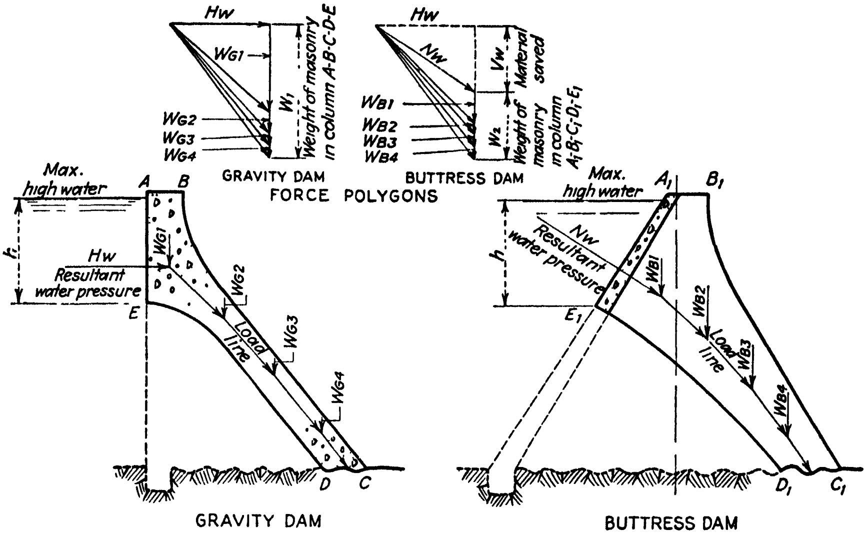


FIG. 4. Comparison of loading action in gravity and buttress dams.

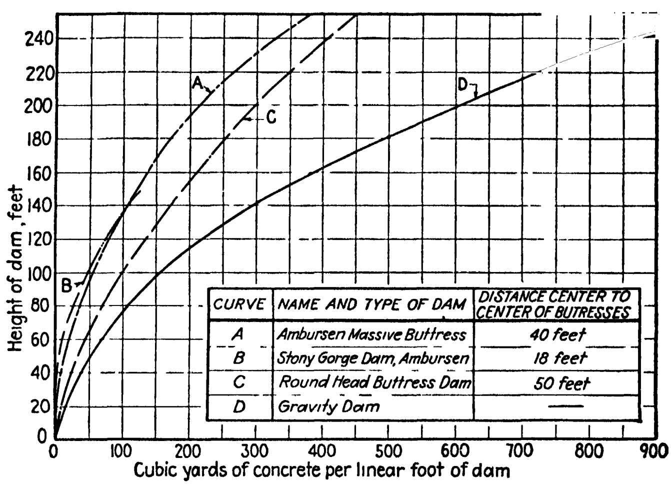


FIG. 5.—Comparison of quantity of materials in dams of gravity and buttress types.

between the total vertical weight of each column, or by the expression $(W_1 - W_2)$. This total saving, made possible by the application of the buttress principle, may be illustrated by comparing the quantity curves of several buttress dams with the curve of the conventional gravity dam as shown by Fig. 5.

It may be noted that not only has great economy been effected in the buttress dam for which quantity curves are plotted in Fig. 5, but also that the safety factors

of these structures have been increased as well. The vertical component of the inclined water load on the buttress column tends to stabilize this structural element

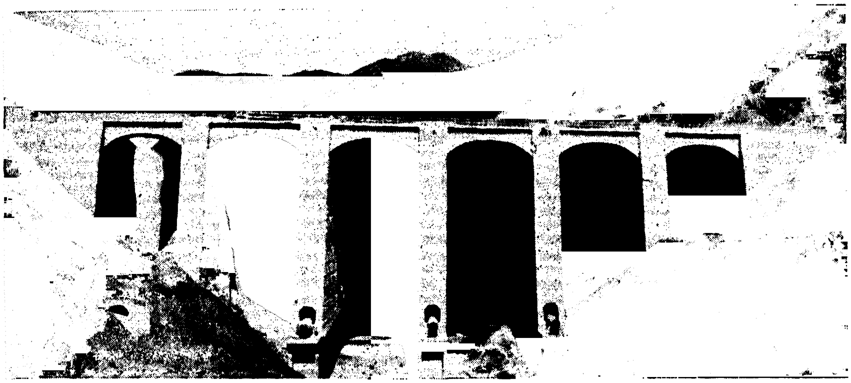


FIG. 6.—Big Dalton Dam, Los Angeles Flood Control District, California. General view. Multiple-arch type with hollow buttresses. Elevation looking upstream.

against both overturning and sliding. The stabilizing effect of this vertical load on the buttress structure as a whole usually results in a factor of safety substantially greater than that obtained in the gravity dam.

FIG. 7.—Big Dalton Dam, California. View of arches. Upstream face. Multiple-arch type.

II. CLASSIFICATION, HISTORY, AND DESCRIPTION

The first reinforced-concrete buttress dam to be constructed making use of the principle of utilizing the weight of the impounded water in combination with the weight of the structure to assist in stabilizing the dam against overturning, uplift, and sliding was designed and built by Nils F. Ambursen at Theresa, N. Y., in 1903.[1] Since that time, this basic principle has been proposed for a large number of different types of dams, with a great many modifications of different individual types, only a few of which have ever reached the construction stage. Other types of dams that have been built making use of this principle are the multiple-arch dam, the round-head buttress dam, the diamond-head buttress dam, the Ransom flat-slab dam, the columnar-buttress and the truss-buttress flat-slab dams, the cantilever-deck dam, and the multiple-dome dam.

Buttress dams may be classified as either rigid or articulated. Rigid-type dams have their upstream water-supporting members constructed monolithically and

[1] *Eng. News*, **50**, No. 19, 1903, p. 403.

unyieldingly with their buttresses, with no provisions for unequal foundation settlement or for expansion and contraction between adjacent bays. Subdivisions of this classification are the multiple-arch and multiple-dome dams, shown by Figs. 6 to 9.

Between the rigid type of buttress dam, such as the multiple-arch and the multiple-dome dams, and the articulated Ambursen type of dam is found the round-head buttress dam and the diamond-head buttress dam, the designs for which eliminate much of the undesirable rigidity of the multiple-arch and multiple-dome designs, although not securing the high degree of flexibility or articulation incidental to the Ambursen flat-slab design.

1. Multiple-arch-type Dams. The Meer Allum Dam in India, built about 1800, was the earliest recorded example of a dam consisting of a series of arches transmitting the water load to buttresses. These buttresses, however, were built with vertical upstream faces. Probably the Australian engineer J. D. Derry was the first to advocate, in 1891, a masonry design somewhat similar to the present-day multiple-arch dam.

Typical multiple-arch details, as shown by Figs. 7 and 8, consist of an arch barrel supported either directly on the sloping upstream edge of the buttresses or on buttress haunches or corbels. The construction joints between the arches and the buttresses are provided with steel dowels which preferably are continuous with the temperature steel in the deck.

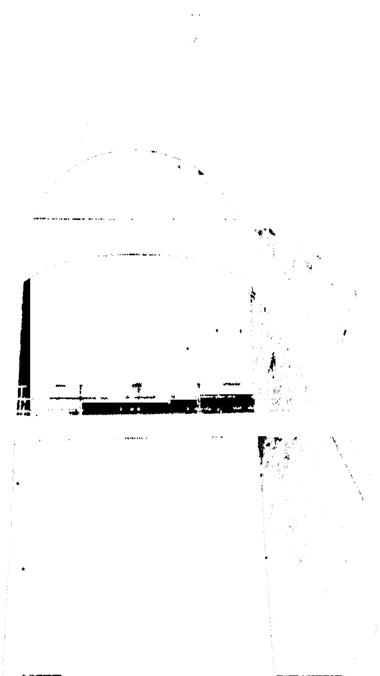


FIG. 8.—Big Dalton Dam, California. View of arches, looking downstream. Multiple-arch type.

The best available records show 71 examples of the multiple-arch type of dam actually built or under construction, varying in height from 20 to 260 ft, distributed geographically as follows:

United States	44	Australasia	2
Europe	18	Canada	2
Africa	3	Latin America	2

Nearly all these examples have been built upon sound ledge-rock foundations, the material best suited to this type of construction. Both the multiple-arch and the multiple-dome types are unsuited to soft or yielding foundations unless special foundation structures are provided, the cost of which is generally prohibitive because of the wide buttress spacing. The Sherman Island Dam on the Hudson River, New York,[1] is the only multiple-arch dam of importance constructed on soft foundation. The main portion of this dam is founded on coarse sand. Extremely flat deck slopes were required to keep the sliding factor within the low limit allowable for sand. Protection against unequal settlement and low foundation pressures was obtained by supporting the buttresses on a continuous reinforced concrete foundation slab.

[1] *Trans., A. S. C. E.*, **88**, 1257, 1925.

The Cave Creek multiple-arch dam in Arizona,[1] with a maximum height of 120 ft, was built in 1922 upon cemented gravel, most of the concrete in the structure, however, being below the ground level. Although the design of the structure was daring, there is no record of trouble having been experienced with it. In 1927, the construction of the 200 ft high Sutherland multiple-arch dam, upon decomposed granite foundations, was begun by the city of San Diego, Calif. After the dam was about 45 per cent complete, doubt of the ability of the structure to resist unequal foundation settlement caused the abandonment of the project. On the other hand, the Beni Bahdel multiple-arch dam in Algeria,[2] 160 ft high, was successfully constructed upon spread footings on a marly shale having a very low bearing value.

Fig. 9.—Coolidge Dam, Arizona. General view. Multiple-dome type.

2. Ambursen-type Dams. The basic flat-slab and buttress type of dam has borne the name of its inventor since 1903. These articulated buttress dams are provided with expansion joints between the decks and the buttress, as shown by Fig. 10, a typical section through the deck of an Ambursen massive-buttress-type dam. The deck consists of reinforced concrete deck slabs, separated by the buttress tongues, and supported by reinforced haunches which are constructed monolithically with the buttresses.

Of the Ambursen type of dam there are recorded 391 examples completed or under construction, varying in height from a minimum of about 6 ft to a maximum of 250 ft, distributed geographically as follows:

United States	335	Australasia	4
Canada	25	Japan	3
Europe	16	Africa	2
Latin America	6		

[1] *Trans. A. S. C. E.*, **87**, 402, 1924.
[2] Essais geotechniques des terrains de fondation, Second Congress on Large Dams, 1936.

The foundations of these structures represent a wide range of materials, varying from fine sand through coarse sand, gravel, boulders, clay, and hardpan to ledge rock, and in varying combinations of these materials.[1]

3. Miscellaneous Types of Buttress Dams. *Ransom Dam.* This flat-slab type of dam was introduced by W. M. Ransom about 1908. In this design, the buttresses were constructed at an angle of 30 deg with the axis of the dam instead of at 90 deg as in the case of all other buttress dams. Alternate buttresses were turned 30 deg to the right, with the adjoining buttresses turning 30 deg to the left, this construction resulting in a honeycomb or cellular interior. Although the deck was of conventional Ambursen type with a reasonable degree of articulation, the intersecting buttress construction was monolithic throughout the length of the dam, a very undesirable feature.

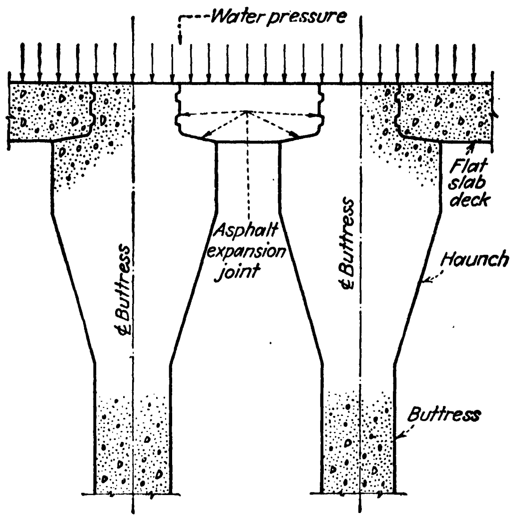


FIG. 10.—Ambursen massive-buttress type. Section through deck and buttress.

No economy was found in this type of structure, and the only two recorded examples are found at Columbia, N. J., and near Cleveland, Ohio.

Austin Dam. In 1915 a somewhat similar plan was used for the second Austin dam in Texas,[2] based upon designs suggested by W. S. Edge about 1911. In this case, the usual buttresses constructed at right angles to the axis were supplemented by longitudinal walls, continuous through six 20-ft bays and inclined upstream, which carried the deck slabs, the result being an anterior divided into relatively small rectangular cells. In the case of this structure, the customary buttresses were used in part as braces between the longitudinal buttresses and in part to carry the downstream spillway apron. The cost of construction was found to be so great that there is no other recorded example of this type. Shortly after the dam was finished, dirft clogged the narrow crest gate openings to such an extent that part of the superstructure was washed away. In 1940, this dam was entirely rebuilt, much of the old structure being salvaged.[3]

[1] *Trans. A. S. C. E.*, **100**, 1303–1307, 1935.
[2] *Eng. News-Record*, **76**, No. 14, 1916, p. 636.
[3] *Eng. News-Record*, **52**, 1941, p. 1.

Columnar-buttress Dam. The columnar-buttress dam, a modification of the typical Ambursen dam, substituted for each buttress a series of inclined columns terminating in spread footings on the foundation and carrying on their tops or upstream ends a heavy inclined girder which supported the deck slabs. This type of design appears to have been originated about 1910 by W. S. Morton, and only one example is recorded as having been constructed, 300 ft long and 45 ft high, built about 1927 in Missouri. It was suitable only for the best of ledge-rock foundations, and little or no economy was obtained through the substitution of heavily reinforced-concrete columns for the plain concrete buttress of the conventional Ambursen dam, particularly as costly struts and diagonal braces were required.

Truss-buttress Dam. This type of dam, of which the only recorded example was constructed in French Indo-China about 1912, was similar to the columnar-buttress

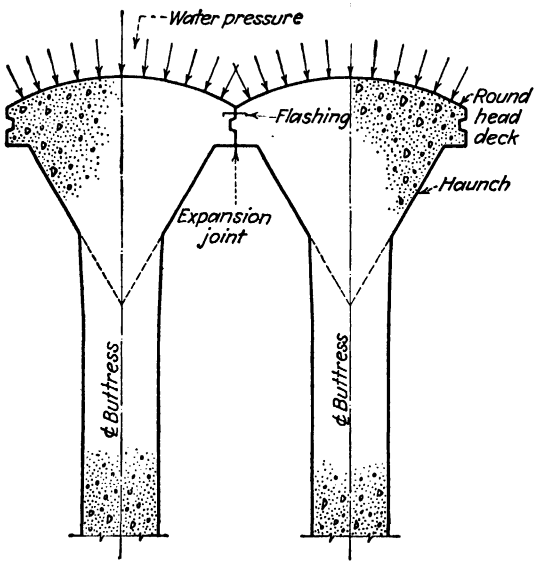


FIG. 11.—Round-head buttress. Section through head and buttress.

dam described above except that heavy vertical trusses of reinforced concrete, instead of columns, took the place of the customary solid buttresses.

Continuous Deck Dam. Buttress dams having continuous reinforced-concrete deck slabs, supported directly by the buttresses and without the aid of haunches or contraction joints, have not been used frequently. One successful structure of this type is the Lake Walk Dam on Devils River in Texas, built in 1928. A section through the spillway is shown by Fig. 37.

Round-head Buttress Dam. The upstream water-supporting member of the round-head buttress dam, as originated by F. A. Noetzli in 1926, is provided with a radial face (Fig. 11) which transmits the water pressure in direct compression through the flared water-bearing member to the buttress below. This type of deck has several distinct advantages: (1) the entire deck is in compression under full water load; (2) little steel reinforcement is required as bending and diagonal tension stresses are theoretically eliminated; and (3) savings in construction cost may in some instances

be effected through the use of mass concrete-construction methods. Of the round-head buttress type of dam the first example built was the Don Martin Dam for the National Irrigation Commission of Mexico.[1] In 1936, a second example, 85 ft high and 279 ft long, was built in Switzerland. A third structure of this type was started in Mexico, but because of unexpected foundation defects the project was abandoned before the dam was complete. A fourth, the Cruz del Eje Dam, is being built in Argentina. The construction of another round-head buttress dam on the Rio Negro in Uruguay was begun in 1938, but at this time (1950) no record is found of any other use of this type of dam. It requires firm ledge-rock foundations.

Diamond-head Buttress Dam. This structure is a modification of the round-head buttress design. As its name implies, it substitutes for the curved upstream face of the buttress head a series of three planes. The only structure of this type recorded as having been built is at Haweswater in England.

Multiple-dome Dam. The 250 ft high Coolidge Dam, constructed in Arizona in 1928,[2] is the only existing structure of the multiple-dome type. The principal features, illustrated by Fig. 9, are three domes supported by massive buttresses spaced 180 ft center to center. In the case of the Coolidge site, a multiple-dome dam was found to be slightly more economical than the conventional multiple-arch type. In this instance, it emphasized the important principle that thick buttresses and wide buttress spacings, used in connection with high dams, may offer greater economy than obtainable with close buttress spacings and relatively thin masonry construction. This principle will be treated more fully subsequently in the chapter. Unsightly cracks in this structure have resulted because of uncontrollable contraction and expansion and because of the pulling away of the arches from the rock foundation at the abutments. This type of dam was originated and patented by C. R. Olberg.

4. Steel Dams. To complete the history and description of dams utilizing the buttress principle wholly or in part, it is necessary to discuss dams built of steel, of which three examples have been built. F. H. Bainbridge appears to have been the first to advocate, in 1895, the construction of an all-steel dam consisting of an inclined upstream face supported on steel columns. In 1898, a dam of this type was constructed near Ash Fork, Ariz., 180 ft in length and 46 ft high.[3] The Redridge Dam of somewhat similar design was constructed in Michigan in 1901, 464 ft long and 74 ft high.[4] Both of these dams were built on good rock foundation and are still in service. The third and last dam of this type to be built, the Hauser Lake Dam in Montana,[5] with a length of 630 ft and maximum height of 81 ft, was constructed upon a rock foundation at each end but about 300 ft in the middle of the river was placed upon gravel into which a steel-pile cutoff wall 35 ft deep was driven, an upstream blanket of fine material 20 ft deep and extending 300 ft above the dam being installed to prevent underflow. The dam was completed in 1907 and failed the following year, the failure, however, being due to inadequate foundation provisions and not to the design of the dam. If the question of high cost is disregarded, the principal objection advanced to steel dams has been the possibility of excessive corrosion of the water-supporting steel face. The Ash Fork Dam has been unwatered and repainted at considerable intervals of time, approximately every seven years, whereas the Redridge Dam is said to have been unwatered and repainted even less frequently.

[1] *Trans. A. S. C. E.*, **96**, 835, 1932.
[2] *Eng. News-Record*, p. 438, 1928.
[3] *Eng. News*, May 12, 1898, p. 279.
[4] *Eng. News*, Aug. 15, 1901, p. 107.
[5] *Eng. News*, **58**, No. 20, 501, 1908; **59**, No. 18, 491 1908.

III. FORCES ON BUTTRESS DAMS

In designing a buttress dam, consideration must be given to the following forces:

1. Headwater pressure
2. Weight of structure
3. Tail-water pressure
4. Silt
5. Ice thrust
6. Temperature expansion and contraction
7. Earthquake accelerations

1. Headwater Pressure. A buttress dam utilizes the water load upon its inclined deck as a stabilizing force. Reference to Fig. 12, showing the principal forces acting

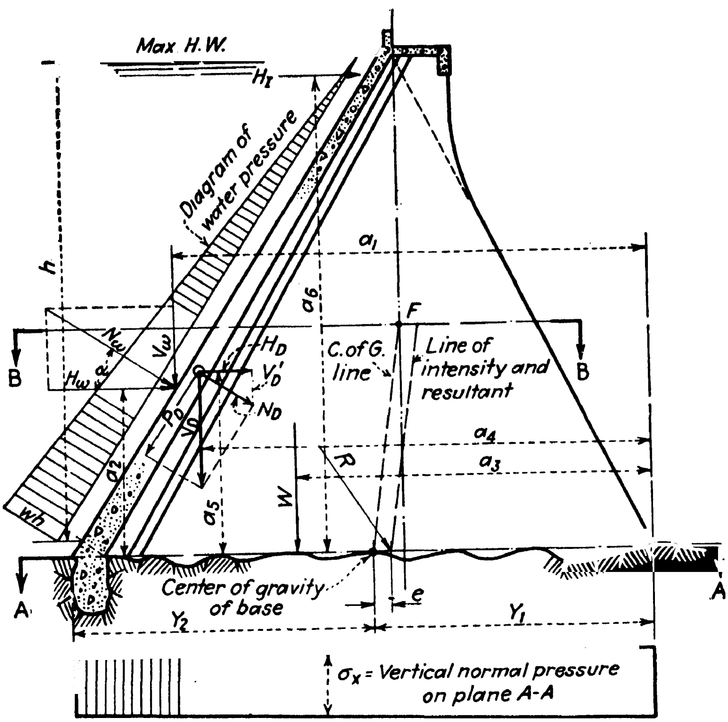


FIG. 12.—Forces acting on buttress dams.

on a buttress dam, will make this property clear. The resultant water load N_w, acting normal to the deck, may be resolved into a vertical component V_w, exerting a righting moment V_wa_1, and a horizontal component H_w, exerting an overturning moment H_wa_2. In addition to the moment effect of the various loads, there is a tendency to slide and shear across the foundation material. Resistance to horizontal movement is commonly expressed by the shear-friction factor of safety (see page 31).

The headwater uplift pressure on a buttress dam is usually of insignificant proportions, varying in an arbitrary manner from the maximum pressure at the upstream face of the water-bearing member to either zero or tail-water pressure at the downstream face. The fact that this pressure is applied near the upstream face of the dam tends to increase the overturning moment, but in most cases its net effect is

slight and can be ignored. The total intensity of such headwater uplift is usually extremely small because the water-supporting member is comparatively thin and also for the reason that in special cases requiring spread footings of considerable width or very thick buttresses at the foundation, adequate cross drains are installed under the buttress.

For relatively small buttress dams, the effect of pore-water pressure can likewise be ignored. However, for high dams pore-water pressure in the thick water-supporting members may have an appreciable effect on the internal buttress stresses and a recognition of this factor is required in the complete analysis of a high buttress. It is considered sufficient to figure the pore-water uplift pressure, at any point in the water supporting member or on any construction joint in that member, equivalent to full headwater at the upstream face to either zero or tail-water pressure at the downstream face, over two-thirds of the normal cross-sectional area. The total area for such uplift pressure also should include a strip of the upstream edge of the buttress equal in depth to the thickness of the deck.

2. Weight of Structure. It is most convenient to treat the weight of the structure as divided into two parts: (1) the buttress load and (2) the deck load. The buttress load is transmitted down through masonry directly to the foundation material and is handled in computations in a manner similar to usual masonry loads. The manner in which the deck load is transmitted to the foundation depends entirely upon the way in which the slab is placed on the upstream end of the buttress. If the deck is constructed monolithic with, or properly keyed into, the buttress, the masonry weight is transmitted in its entirety down through the buttress to the foundation, in a manner similar to the buttress load. With reference to Fig. 12, it will be seen that the righting moment of the buttress is equal to Wa_3 and that of the deck is V_Da_4.

When the deck is completely articulated and merely rests against the upstream face of the buttress or upon the haunches, the weight of the deck is broken down into two components, one normal to the upstream edge of the buttress and the other parallel to the face. The normal component is transmitted down through the buttress masonry to the foundation, and the parallel component travels down through the deck slab directly to the foundation. Again with reference to Fig. 12, it will be noted that the righting moment of the buttress is equal to Wa_3. However, in this case the righting moment of the deck is equal to V'_Da_4. The overturning moment exerted by a completely articulated deck is equal to H_Da_5.

3. Tail-water Pressure. The force on the deck and the buttress due to tail water reduces the effective headwater pressure and, consequently, is beneficial in this respect. However, the loss of effective weight due to partial submergence of the structure frequently offsets whatever advantage is gained by the presence of tail water. It is usually advisable to analyze the structure for both maximum and minimum tail water conditions. Where the structure is not submerged, the uplift due to tail water will of course be either negligible or zero. Where the buttress is submerged to considerable depth, the uplift pressure will tend to increase the sliding factor near the water level but reduce the sliding factor near the foundation of the dam. In this respect, its effect is definitely beneficial. The effect of tail-water uplift on the stability of the dam depends upon the physical dimensions of the buttress. In many cases, a beneficial righting moment will result.

4. Silt. An additional load due to silting of the reservoir will be placed on the deck near the bottom of the dam. The height to which the silt load may extend up the deck of the dam will depend on the quantity of suspended matter carried by the stream, the velocity of the water through the reservoir, and the size and shape of the reservoir. For streams bearing heavy silt loads, it is usual to assume that the depth of silt will build up from one-quarter to one-half the full height of the dam. Conven-

tional methods of computing earth pressures with proper allowances for the effect of saturation should be applied in making the analysis.

5. Ice Thrust. The effects of ice thrust are illustrated in Fig. 13. The maximum ice-thrust pressure that can develop on the upstream face of a dam near the water line is governed by the minimum temperatures of the particular location, the degree of exposure of the reservoir area, and the general physical outline of the reservoir area. In the northern section of the United States and in Canada, where the minimum winter temperatures are sufficient to maintain an ice sheet several feet thick throughout several months, it is not unusual to allow pressures of 5 to 10 tons/lin ft for a dam with vertical upstream face. However, this force cannot introduce a harmful overturning moment on the sloping upstream face of a buttress dam because there is insufficient frictional resistance or adhesion between the deck and the ice to overcome the component of ice thrust parallel to the deck. The result is a breaking up of the ice sheet adjacent to the deck.

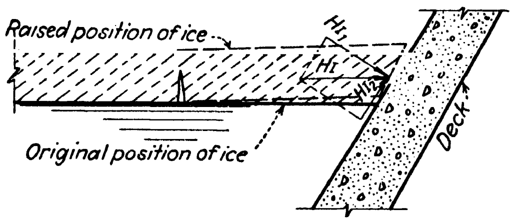


FIG. 13.—Action of ice on deck slab.

6. Temperature Expansion and Contraction. Temperature changes both during and after construction are of major importance in buttress design. To eliminate the effects of serious temperature stresses during construction, the buttress and deck concrete is poured and cured in relatively small sections or blocks. Temperature stresses developing after construction are either carried by horizontal steel or relieved by contraction and expansion joints. The design requirements for temperature provisions are set forth under Contraction Joints.

7. Earthquake Accelerations. The articulated flat-slab buttress-type dam offers an inherent resistance to earthquake forces. Two examples of the selection of this type of structure specifically for construction in earthquake zones are the Stony Gorge Dam (Figs. 1, 2, and 3) built by the U.S. Reclamation Bureau and the Rodriguez Dam (Figs. 53 and 54) built by the National Irrigation Commission of Mexico. Each structure was built across a geological fault, the Rodriguez Dam being designed with alternate pairs of buttresses tied together by reinforced concrete diaphragms so as to form a tower construction of great stability, heavy unanchored beams or struts being installed in the intermediate bays.[1]

The more rigid multiple-arch dam, when properly designed for earthquake conditions, is satisfactory for areas of restricted earthquake action but offers less inherent resistance. The Bartlett Dam, built by the Bureau of Reclamation in 1939, was tested for an earthquake acceleration of 3.2 ft/sec^2 (0.1 g), and a large amount of steel was added to the buttresses to carry the additional stresses.

When it is necessary to make special provisions for earthquakes, the stresses due to acceleration are determined by applying at the center of gravity of the structure a force equal to one-tenth (or whatever other factor is being used) of the vertical weight of the dam. If the dam has an inclined upstream face, one-tenth of the water load on that face should be included. If the proposed dam is high, the hydrodynamic

[1] *Jour. A.W.W.A.*, **25**, No. 3, 355, 1933.

effect of the inertia of the water against the upstream face should be computed.[1] Consideration should also be given to the shearing, torsional, and bending stresses in the dam induced by the elastic deformation of the foundation material.

A well-designed buttress dam has such a large factor of safety against overturning that the additional overturning force of one-tenth of the vertical load will be negligible. This does not apply, however, to the lateral stability of the buttresses. If the earthquake hazard is severe, special lateral bracing in the form of either beams or diaphragms should be provided. This subject is treated in more detail under Lateral Bracing. See page 123. Articulated dams, with special provisions for lateral bracing, are more earthquake resistant than the rigid types.

IV. THEORY OF BUTTRESS DESIGN

The fundamental principles of buttress design and analysis are applicable to all types of buttress dams (Ambursen, multiple-arch, round-head, diamond-head, con-

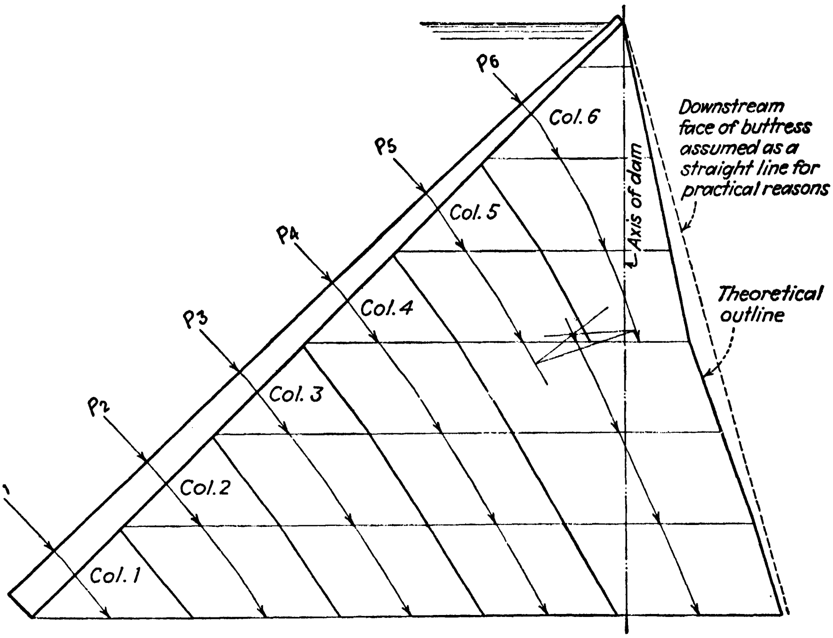


FIG. 14.—Column design of buttress.

tinuous-deck, multiple-dome). When judiciously applied, the principles can be extended to include, in addition to the usual solid-wall buttress, the hollow-buttress and the diaphragm-buttress types.

The design of a buttress involves two steps: (1) the selection of a trial design, (2) the analysis of this design to determine whether it satisfies stress and stability requirements. In selecting the proportions of the trial design, the buttress outline and thicknesses may be computed directly by using simplifying assumptions or determined from the results of previous experience with a similar structure.

1. Trial Design Using Simplifying Assumptions. Preliminary investigations are based on the simplifying assumption that a buttress consists of a system of columns (see Fig. 14), each carrying the incident water load by column action to the founda-

[1] *Trans. A. S. C. E.*, **98**, 418, 1933.

tion. These columns are proportioned to develop a uniform compressive stress and curved to avoid any serious eccentricity on any normal or horizontal plane when the water and concrete loads are resolved. It is possible to determine, graphically, the approximate outline and thicknesses of a buttress by progressively designing a series of column sections, starting at the foundation and continuing to the crest, such that the load carried by the individual columns produces no shear between adjacent columns. This method of design is based on the assumptions (1) that the buttress columns are free to act independently of each other, and (2) that by placing

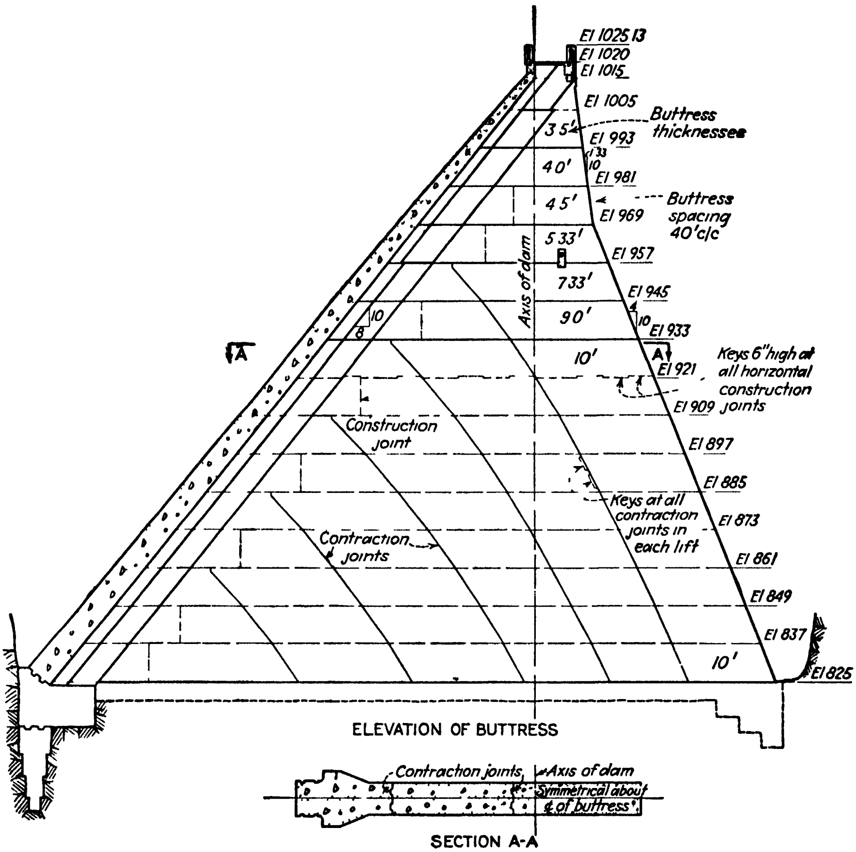


FIG. 15.—Possum Kingdom Dam (renamed Morris Sheppard Dam). Bulkhead buttress and sections; keys in contraction joints (inclined joints).

all columns concentric with their respective loadings, the second principal stress will be zero for maximum load conditions. If the buttress is constructed as a monolith, the columns become merely hypothetical and cannot act independently. It is obvious that in this case deformations and, consequently, stress will be transmitted between adjacent (hypothetical) columns. However, for small dams the monolithic buttress offers the additional inherent stability of a larger and continuous member. Analysis indicates that in a monolithic buttress proportioned in strict accordance with the direct-design method the first principal stress will not remain uniform, nor will the second principal stress be zero throughout the buttress. If the buttress is

jointed so as to preserve the columnar structure (as shown in Fig. 15), the action and the stresses will more nearly approximate the theoretical. The decision as to whether the buttress should be jointed or monolithic rests with the designer. The jointed buttress has the advantages of (1) minimizing stresses due to setting and temperature, (2) eliminating the necessity of placing a substantial amount of horizontal or inclined steel in the main body of the buttress, (3) facilitating construction, and (4) approaching the theoretical action.

It is important to note that any variation from the buttress shape and thicknesses determined by the above-described trial-design method, such as the addition of haunches or corbels at the upstream edge of the buttress or the addition of pilasters at the downstream edge, will, in proportion to the relative size of the addition, alter the principal stress values from those used as a design requirement. In view of its many uncertainties, the direct-design results should be carefully checked by a thorough stress analysis except, perhaps, in the case of a low, simple buttress.

With reference to Fig. 14, which shows a simple buttress of approximately triangular shape, the incident water load is divided into six sections of approximately equal length along the water-bearing face. By starting at the foundation, the upper boundary of column 1 is determined so that the trajectory, described by resolving the water load P_1 and the masonry weight, passes through the center of gravity of either horizontal or normal sections through column 1. This procedure is repeated for columns 2, 3, 4, 5, and 6 in succession. The lower boundary for each column is the upper edge of the preceding column. A practical guide to the most desirable horizontal column width is the limitation of the horizontal length of pour that can safely be made without inviting shrinkage cracks. The limiting length is about 35 or 40 ft, proper curing of the green concrete being assumed. Thicknesses are determined so as to give uniform major compressive stress throughout the buttress. The maximum allowable stress is governed by the slenderness ratio as in the design of common columns. It is, usually, more economical to use struts or diaphragms to keep the unsupported length at the greatest allowable value $\left(\frac{l}{d} = 15\right)$ with which the maximum compressive stress may be used.

2. Stability and Stress Investigations. After the trial design has been selected, subsequent detailed investigations will involve computations for the following:

1. Stability against overturning and sliding (movement downstream)
2. Normal foundation pressures and normal pressures on selected horizontal planes through the buttress at various elevations
3. Shearing, horizontal normal and principal stresses

In most cases, steps 1 and 2 will indicate desirable changes in the trial design. It will save considerable time, therefore, if the initial computations for these steps are made with a slide rule. After indicated adjustments have been made, the entire analysis should be recomputed on a calculating machine.

Stability against overturning and sliding are determined in the conventional manner by summing all moments, vertical forces, and horizontal forces at the base of the dam. The buttress-type dam has inherently great stability against overturning. This consideration is seldom a governing factor in design because the resultant of all forces usually passes through the approximate center of gravity of the base of the section. Stability against sliding can be increased at will by flattening the slope of the upstream face of the dam. This relation is discussed later in this section under Buttress Spacing and Proportions. See page 113.

For ordinary computations, it is sufficiently accurate to assume that normal stresses on horizontal planes have straight-line distribution, as shown for the plane AA (Fig. 12). This assumption is more nearly accurate when the median line of the

buttress is normal, or nearly so, to the plane of analysis. When the angle formed by the median line and the plane of analysis approaches 45 deg, the distribution of the vertical normal stress varies noticeably from a straight line. Normal foundation

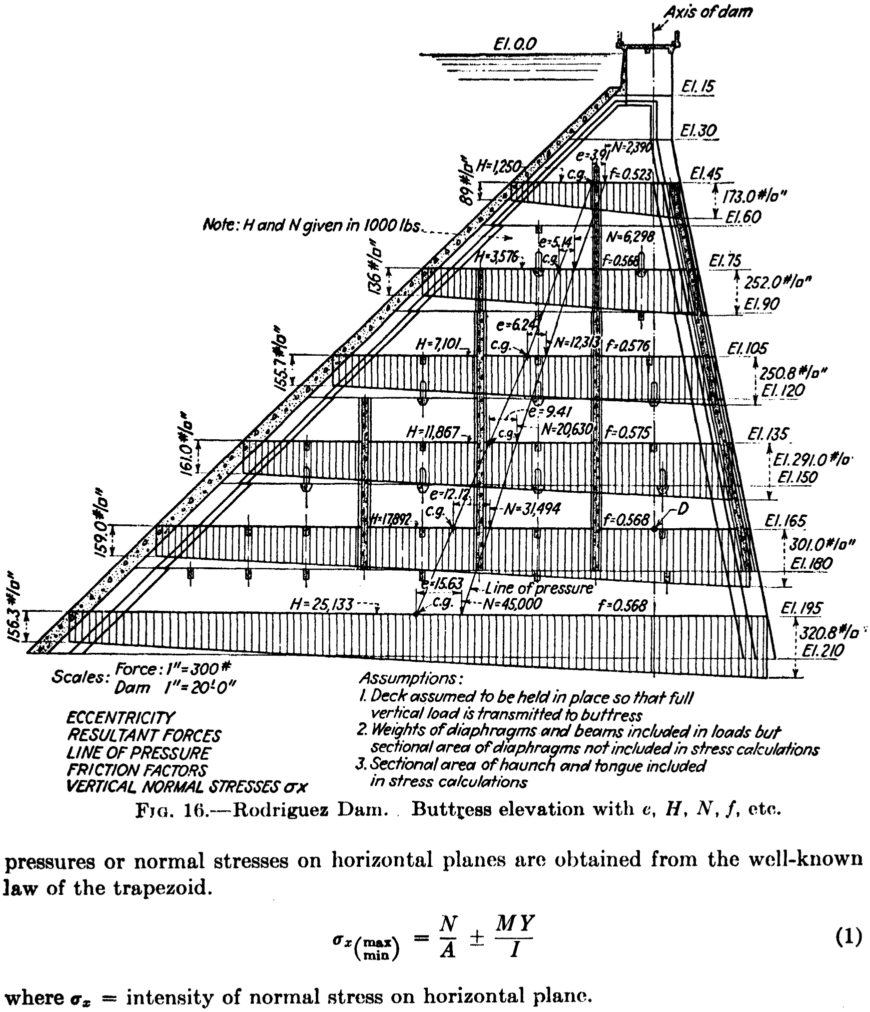


FIG. 16.—Rodriguez Dam. Buttress elevation with e, H, N, f, etc.

pressures or normal stresses on horizontal planes are obtained from the well-known law of the trapezoid.

$$\sigma_{x\binom{\max}{\min}} = \frac{N}{A} \pm \frac{MY}{I} \tag{1}$$

where σ_x = intensity of normal stress on horizontal plane.

N = total vertical load on section (masonry + water).

A = sectional area of base.

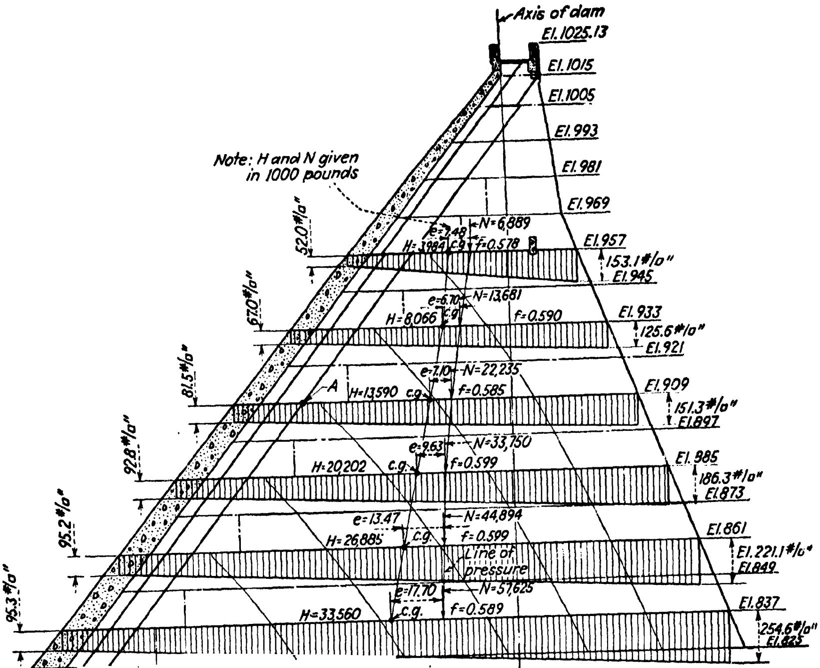
M = moment = Ne.

e = eccentricity (distance from point of application to center of gravity of section).

Y = distance from center of gravity to most remote fiber.

I = moment of inertia of horizontal section.

Location of resultants, sliding factors, and vertical normal pressures for Rodriguez Dam and for Possum Kingdom Dam are shown in Figs. 16 and 17, respectively.



Scales: Force 1" = 300#
Dam 1" = 20'-0"

Assumptions:
1. Deck assumed to be held in place so that full vertical load is transmitted to buttress
2. Sectional area haunch and tongue included in stress calculations

ECCENTRICITY
RESULTANT FORCES
LINE OF PRESSURE
FRICTION FACTORS
VERTICAL NORMAL STRESSES σx

FIG. 17.—Possum Kingdom Dam. Buttress elevation with e, H, N, f, etc.

SAMPLE COMPUTATIONS—HORIZONTAL SHEAR STRESS (See p. 102)

$$pd = 217.87^k \qquad \gamma_t = 1.500$$
$$p'd = 2.312 \qquad m = 0.4$$
$$k = 0.7464 \qquad l_0 = 134.67'$$
$$k' = 0.00182 \qquad z = 9.6'$$
$$S_z = a + b_z - cz^2$$
$$a = mpd = 0.4 \times 217.87 \qquad = 87.1$$
$$b = p'd - mk - \gamma_t$$
$$= 2.312 - 0.4 \times 0.7464 - 1.500 \qquad = 0.513$$
$$c = \frac{k'}{2} = \frac{1}{2} \times 0.00182 \qquad = 0.00091$$
$$S = 87.1 \times 0.513 \times 9.6 - 0.00091 \times \overline{9.6}^2 = 91.9^k$$
$$\tau = \frac{S}{t} = 9.19^k/\text{sq ft or } 63.8 \text{ psi}$$

FIG. 18.—Possum Kingdom Dam. Sample computation. Horizontal shear stress.

3. Shearing, Horizontal, Normal, and Principal Stresses. Briefly stated, the method of computing the principal stresses in a buttress consists of dividing the buttress by horizontal and vertical planes into a number of elementary prisms which are then considered to be in vertical and horizontal equilibrium. Equations are written expressing the unknown stress functions in terms of the stresses which may be determined directly from Eq. (1).

If vertical normal stresses, horizontal normal stresses, and vertical shearing stresses are thus evaluated, both the principal stresses and their directions may be determined either analytically, by the principal stress equations, or graphically, by

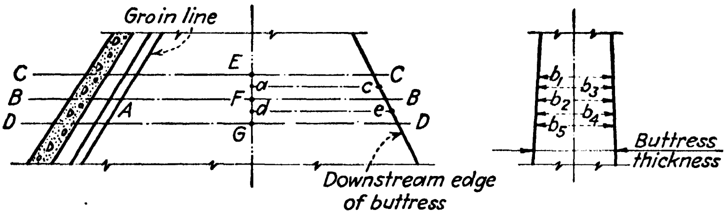


FIG. 19.—Partial side elevation of buttress. Planes for principal stress analysis.

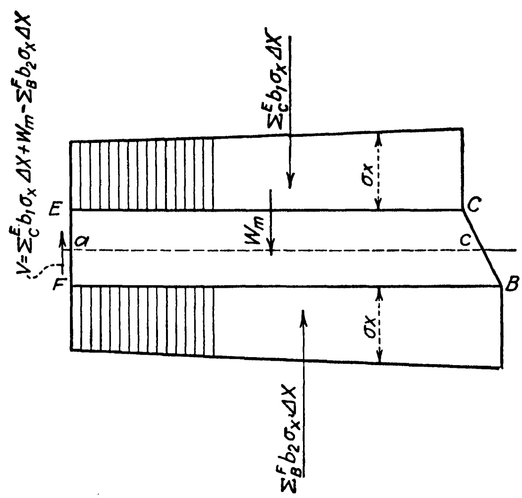


FIG. 20.—Unit block removed from Fig. 19.

Mohr's circle. In the following, this procedure is developed more fully. Assume for example that it is desired to determine the intensities and directions of the principal stresses at point F, plane BB (Fig. 12). For convenience, an enlarged section of Fig. 12 is shown by Fig. 19. Horizontal planes CC and DD (Fig. 18) are plotted equal distances above and below plane BB, and the values of σ_x (max) and σ_x (min) are computed for each plane by Eq. (1). The distance between these planes must be relatively small if sufficient accuracy is to be maintained.

Next construct a vertical plane intersecting the three horizontal planes at E, F, and G. The elementary prism $ECBF$ may then be visualized as separated from the buttress as shown in Fig. 20 and to be held in equilibrium by the forces acting on it. These forces are the total vertical normal pressure on the plane EC, $\sum_C^E b_1\sigma_x\Delta x$; the

weight of the masonry in the prism W_m; and the total vertical normal pressure on the plane FB, $\sum_B^F b_2\sigma_x\Delta X$.

The total shear V across the plane EF may now be determined by writing the equation for vertical equilibrium for the prism $ECBF$:

$$V_{EF} = \sum_C^E b_1\sigma_x\Delta X + W_m - \sum_B^F b_2\sigma_x\Delta X \tag{2}$$

The average intensity of shear v_1 on the section EF is equal to V_{EF} divided by the sectional area between E and F. By a similar procedure, the average shearing stress v_2 between F and G may be determined. The shearing stress at the point F on the plane BB is equal to:

$$\tau_{xy} = \frac{v_1 + v_2}{2} \tag{3}$$

The next step is to separate the prism $acde$ from the buttress and consider it in horizontal equilibrium under the shearing forces acting upon it. As horizontal and vertical shearing stress intensities are equal, the total shear acting on the planes ac and de may be determined by the procedure outlined above. The differences between these total shearing forces will be a horizontal force N_y over the area ad and normal to the plane EF. These forces are expressed in the following equation:

$$N_y = \sum_c^a b_3\tau_{xy}\Delta X - \sum_e^d b_4\tau_{xy}\Delta X \tag{4}$$

The average intensity of horizontal normal pressure σ_y equals N_y divided by the sectional area between a and d.

It is now possible to determine the principal stresses and the state of stress at the point F by using from the following principal stress equations:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \tag{5}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \tag{6}$$

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \tag{7}$$

Both the first and second principal stresses and their directions may be easily determined graphically by means of Mohr's circle as illustrated by Fig. 21. The construction of Mohr's circle is self-explanatory, and the derivation may be readily determined by inspection. Figure 21a,b,c, shows three cases: (1) the analysis of a point at the groin line with the second principal stress in compression, (2) the analysis of a point at the groin line with the second principal stress in tension, and (3) the analysis of a point at the downstream face of the dam where the second principal stress is zero. The fact that incorrect proportioning of the buttress may readily cause σ_2 to be a dangerously high tensional stress emphasizes the importance of computing the normal stresses on all planes through the point of analysis.

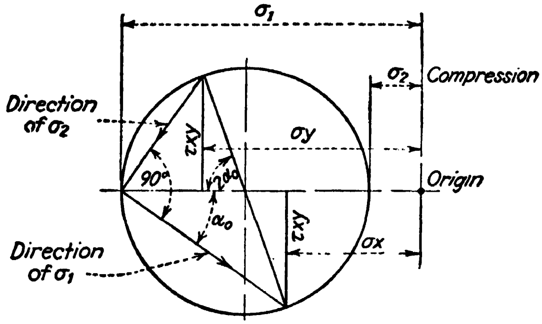
In Fig. 22 are shown three general cases of normal stress variation around a point: (a) for both principal stresses in compression, (b) for the first principal stress in compression and the second principal stress equal to zero, and (c) for the first principal stress in compression and the second principal stress in tension. After the intensities

and directions of both the first and second principal stresses are computed, this variation may be determined at any point by the following equation:

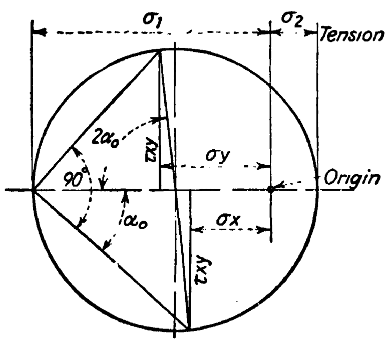
$$\sigma = \sigma_1 \cos^2 \phi + \sigma_2 \sin^2 \phi \qquad (8)$$

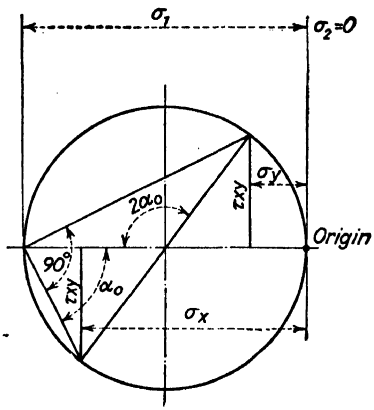
in which σ is the normal stress on any plane acting at an angle of ϕ with the direction of σ_1.

If in a particular design, for purposes of economy, to secure a low sliding factor or to satisfy other design considerations, it is decided to allow tension in the second



(a) Second principal stress in compression.





(b) Second principal stress in tension. (c) Second principal stress zero.

FIG. 21.—Analysis of principal stresses by Mohr's circle.

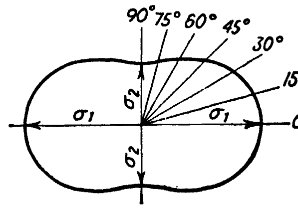
principal stress, inclined steel in an amount sufficient to take the entire tensional stress in the buttress should be provided.

Although the basic theory of buttress analysis is readily understandable, its application to a specific case offers several problems, the major of which is to determine the degree of accuracy required in the stress computations. As in almost all stress computations, the amount of work involved varies directly with the degree of accuracy, and it is only by experience and a thorough knowledge of all the factors concerned that the relation between accuracy and method can be appreciated.

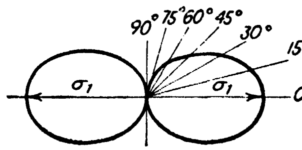
Two factors affecting the accuracy of stress-analysis computations are the shape of the buttress and the method of applying the theory of analysis. Stress computations are relatively simple for a triangular-shaped buttress of constant thickness and become increasingly difficult as the shape is complicated by changes in thickness, the

addition of haunches or corbels, and the addition of flanges or pilasters at the downstream end. Further difficulty is experienced in the analysis of buttresses carrying spillway gate piers or having other special treatment. Unfortunately, it is usually the more complicated shapes that require the most accurate and exhaustive stress analysis.

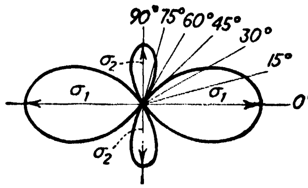
The methods of applying the theory of principal stress analysis vary for the most part in the selection of the size of block or the distance between the planes in question (dimension EF, Fig. 19). In the general derivation of the theory, the planes are considered as being relatively close and it is on this close spacing that the claim to accuracy of the method is based. The use of a spacing of 1 ft between planes (total height of two blocks equals 2 ft) gives accurate results and is entirely satisfactory for use on a simple buttress shape. However, for extremely complicated shapes the computations may have to be carried with as many as 20 significant figures to ensure accuracy of the small differences used in the final substitutions. The method using small blocks is fully analytical, summing, in every case, the full distance from the downstream edge of the buttress to the point in question. It is necessary only to substitute constants derived from the vertical normal stress computations on each of the three planes in the equations to evaluate the vertical and horizontal shears (τ_{xy}) and the horizontal normal stress (σ_y). This method has the advantage of being particularly applicable to analyzing isolated points and sections of the buttress. This advantage becomes a disadvantage when a complete buttress analysis is required because every portion of the buttress must be treated as an isolated area.



(a) Both principal stresses in compression.



(b) First principal stress in compression and second principal stress zero.



(c) First principal stress in compression and second principal stress in tension.

FIG. 22.—Distribution of stresses around a point.

In order to avoid greatly extended computations, block heights greater than 2 ft are commonly used, the selection being influenced by the height of the dam. When using the greater height, it is frequently more convenient to subdivide the entire buttress into blocks of approximately square elevation and carry the necessary summations as numerical rather than analytical operations. The horizontal summations are extended as with the smaller blocks by successive steps from the downstream end toward the upstream end of the buttress. The use of larger blocks in combination with numerical summation relies on linear interpolation for accuracy, and in many cases the variation of stresses from the assumed linear distribution is sufficient to introduce appreciable error in the final principal stress determination. Because of the size of the blocks, the summation cannot be carried up under the haunch (point A, Fig. 19) where the stress conditions tend to be most critical.

A typical example will illustrate the application of the principal stress analysis using larger blocks to a practical design. The problem, illustrated by Figs. 16 and 23, is to determine the principal stresses at point D in the buttress of a 210-ft-high Ambursen dam having buttresses spaced 22 ft center to center. As indicated by Fig. 16, the buttress is constructed in 15-ft vertical lifts, or pours. The vertical normal pressures, as determined from stability computations, are also shown by

Fig. 16. These stresses may be computed readily by an application of Eq. (1) and involve no unusual difficulties.

To determine the principal stresses, the first step is to separate from the buttress the prisms $A\ B\ C\ D$ and $C\ D\ E\ F$ as shown in Fig. 23. Each prism is then assumed to be held in equilibrium by the normal and shearing stresses and by the weight of the masonry within the boundaries of the prism. Figure 23 illustrates the successive steps in computing the principal stresses.

If the directions of principal stresses are determined at a large number of points, it will be observed that the trajectories of principal stresses follow definite mathematical relations as illustrated by Fig. 24. These trajectories indicate that the water

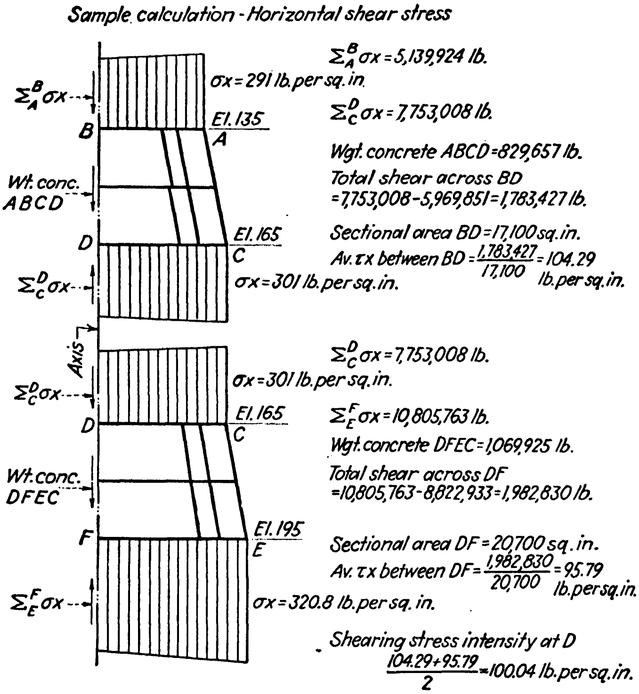


FIG. 23.—Rodriguez Dam. Unit block with sample calculation. Horizontal shear stress.

load normal to the deck is transmitted through the buttress to the foundation by means of a series of elementary curved, inclined columns substantiating, in general, the theory presented under Trial Design Using Simplifying Assumptions. Each of these elementary columns is in equilibrium under the influence of the water load, the second principal stresses on each side, and the weight of masonry in the column.

These trajectories of the principal stresses have an important bearing on buttress-dam design as they represent planes on which construction or contraction joints may safely be placed so that under maximum loading the monolithic action of the buttress as a whole will not be destroyed. The reason for this monolithic action is that the shearing stresses along the planes of principal stress are zero, and there is no tendency for relative motion between the columns along the joints when the reservoir is full. However, under partial reservoir loading, the trajectories will vary from the pattern for full reservoir loading and the continuance of monolithic action depends on the transmittal of light shearing stresses across the joints. For this reason, keys (as

shown in Fig. 15) are placed in the joints. An additional benefit derived from the use of keys along the joints is the stiffening effect that the shorter columns have on the adjacent longer columns.

The inclined-column concept is illustrated further by Figs. 25*a* and 25*b*, which show graphically the relationship between the forces acting on a column visualized as separated from the buttress (illustrated by Fig. 14) along a trajectory of principal stress. It will be noted that the column is held in equilibrium by the weight of the masonry, the water load on the upstream face, and the second principal stresses. A

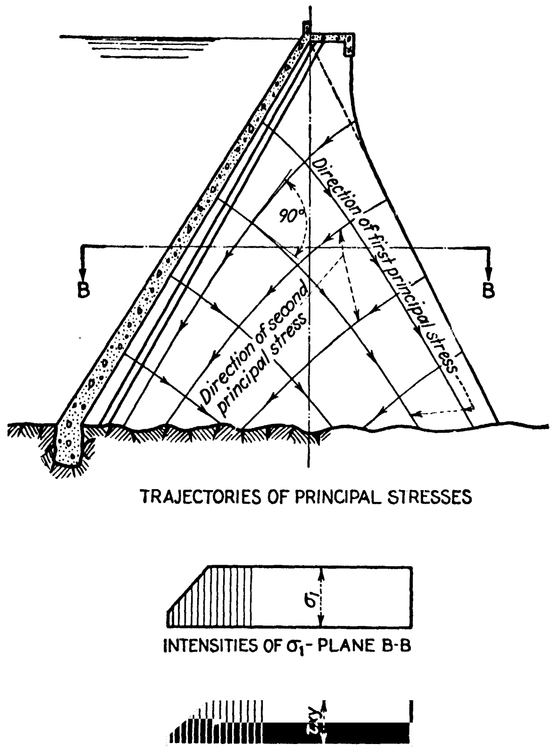


FIG. 24.—Trajectories of principal stresses.

graphical analysis of this type offers a convenient method of checking the stability of sections of buttress separated by joints.

The disadvantage of using small blocks lies in the handling of the extremely large values of the vertical normal pressures necessary to evaluate correctly the small stress differentials across a 1-ft height. The advantage with small blocks is the ease with which any point, including the area directly under the haunch, may be analyzed. The use of larger blocks facilitates the basic computations of the vertical normal stresses but then introduces undesirable errors by extending the use of linear interpolation to include not only the vertical normal stresses but also the horizontal normal and the vertical and horizontal shearing stresses. This method becomes inaccurate to the point of being unreliable in the area just under the upstream haunch. A combination of the desirable features of both methods by using blocks of medium height (6- to 12-ft

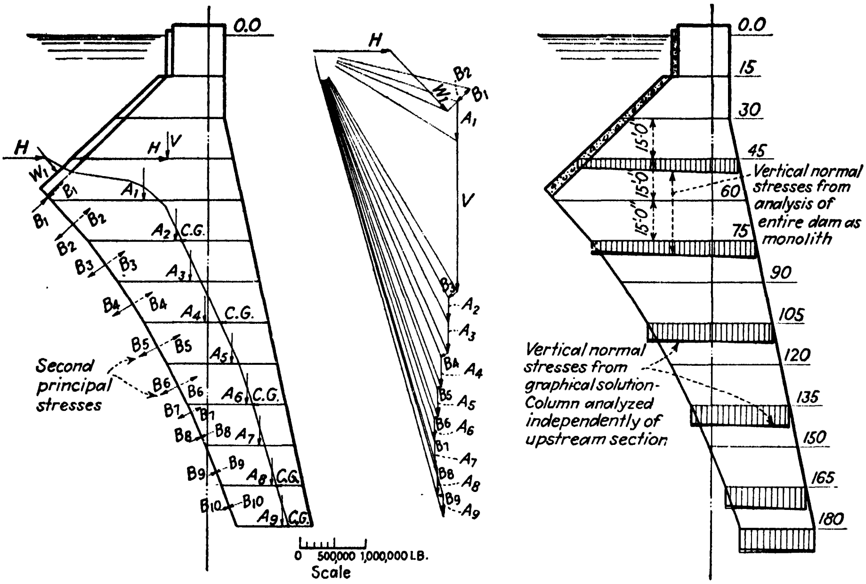


FIG. 25a. Rodriguez Dam. Graphic analysis of inclined column including effect of second principal stresses.

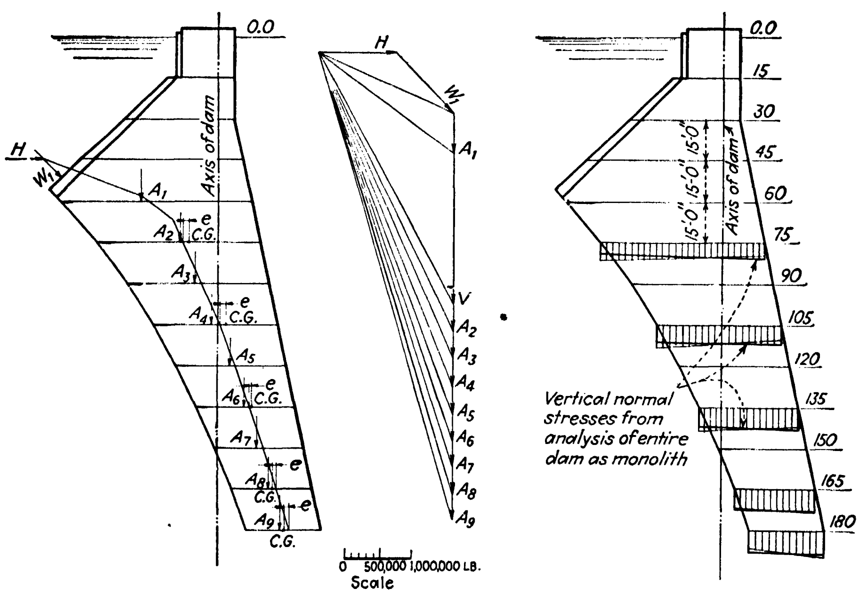


FIG. 25b.—Rodriguez Dam. Graphic analysis of inclined column assuming no second principal stresses.

blocks have been found satisfactory) and then evaluating the vertical normal stress by interpolation offers a more rapid and accurate analysis particularly applicable to triangular-shaped buttresses. The general equations of internal equilibrium [Eqs. (2) and (4)] are written in the differential form.

$$\frac{d\sigma_y}{dx} + \frac{d\tau_x}{dy} = 0 \tag{9}$$

$$\frac{d\sigma_x}{dy} - \frac{d\tau_y}{dx} + w_m = 0 \tag{10}$$

$$\tau_x + \tau_y = 0 \tag{11}$$

In order to include the influence of the variable buttress thickness t, the values of the normal and shearing stresses are expressed as a value per unit length (equal to the stress in an imaginary buttress of uniform thickness, $t = 1$).

$$p_x = t\sigma_x \tag{12}$$

$$p_y = t\sigma_y \tag{13}$$

$$s_x = t\tau_x; \qquad s_y = t\tau_y \tag{14}$$

$$w_{mt} = tw_m \tag{15}$$

Then Eqs. (9), (10), and (11) become

$$\frac{dp_y}{dx} + \frac{ds_x}{dy} = 0 \tag{16}$$

$$\frac{dp_x}{dy} - \frac{ds_y}{dx} + w_{mt} = 0 \tag{17}$$

$$s_x + s_y = 0 \tag{18}$$

After evaluating p_x, p_y, s_x, and s_y, and thereby σ_x, σ_y, and τ_{xy}, the principal unit stresses σ_1 and σ_2 are easily determined by Mohr's circle or by Eqs. (5), (6), and (7).

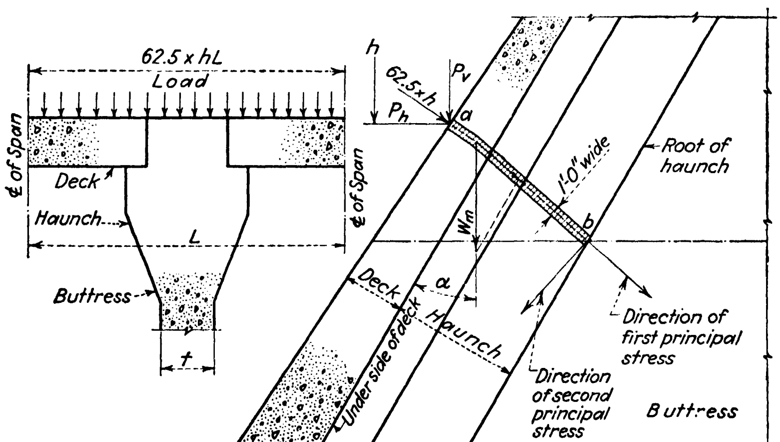


FIG. 26.—Quick-stress analysis at groin line.

As a further refinement, Eqs. (16), (17), and (18), by expanding and grouping terms, are reduced to expressions of constants and variables. The variables are evaluated for the entire buttress elevation and the results plotted in curve form. It is then possible to determine rapidly the principal stresses at any point by selecting the proper values from the curves and substituting in the basic equations.

Figure 17 shows an Ambursen bulkhead buttress 178 ft high used for the Possum Kingdom Dam on the Brazos River, Texas. The vertical normal stresses [as computed by Eq. (1)], eccentricities, and sliding factors are given in Fig. 17 for six planes or lift lines. Values of p_d, the intensity of the vertical normal (trapezoidal) stress at the downstream end of any horizontal section; of k, the rate of change of the vertical

FIG. 27a.

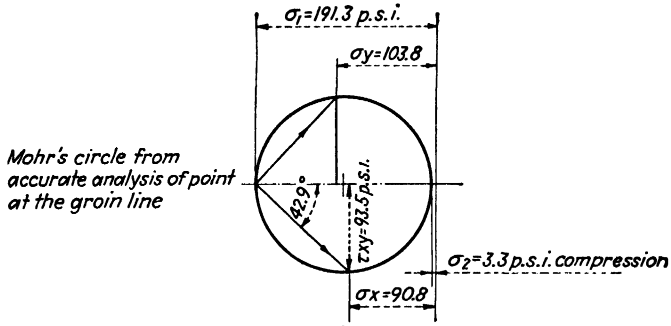


FIG. 27b.

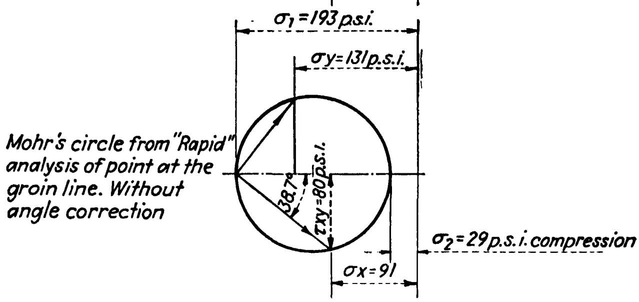


FIG. 27c.

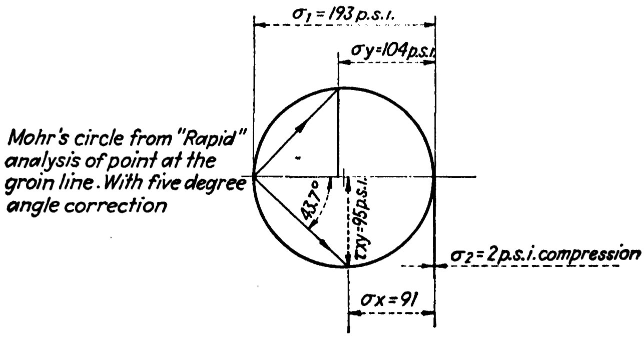


FIG. 27a.—Possum Kingdom Dam. Mohr's circle at groin, exact method.

FIG. 27b.—Possum Kingdom Dam. Mohr's circle at groin, quick method with no angle correction.

FIG. 27c.—Possum Kingdom Dam. Mohr's circle at groin, quick method with angle correction.

normal stress; and of the first and second differentials of both p_d and k are determined from the stress diagrams shown in Fig. 17. The shear s and the horizontal normal stress p_y are evaluated by Eqs. (19) and (20) in which m expresses the slope of the downstream edge of the buttress, w_{mt} the masonry weight as previously explained, and z the distance from the downstream edge of the buttress to the point being analyzed.

$$s = m(p_d) + (p_d' - mk - w_{mt})z - \frac{1}{2}k'z^2 \tag{19}$$

$$p_y = m^2(p_d) + m(2p_d' - mk - w_{mt})z + \frac{1}{2}(p_d'' - 2mk' - w_{mt}')z^2 - \frac{1}{6}k''z^3 \tag{20}$$

The normal and shearing stress values for point A (see Fig. 17) at El. 909 and the Mohr's circle for this point are shown in Fig. 18.

A sample computation to determine the horizontal shear stress at any point in the buttress is shown by Fig. 18. In the example the point investigated is located 9.6 ft from the downstream edge of the buttress at El. 909.

4. Rapid Stress Analysis of Area under Haunch. In the preliminary stages of design, it is usually sufficient to examine the area under the haunch where the most severe stress conditions tend to occur. A study of Eqs. (5), (6), and (7) or of the Mohr's circles in Fig. 21 demonstrates that, of the six variables σ_x, σ_y, σ_1, σ_2, τ_{xy}, and α, only three need be known to determine the remaining three. The analyses presented under Trial Design using Simplifying Assumptions require the evaluation of σ_x, σ_y, and τ_{xy} for the final determination of the two principal stresses and their directions. For a rapid stress analysis, the vertical normal stress σ_x is determined in the usual manner; the first principal stress σ_1 is computed by resolving the water load on the deck and the weight of a curved strip of masonry extending from the water-supporting face back to the point being analyzed. The angle α is evaluated by increasing the angle of the incident water load slightly (3 to 10 deg, depending on the size and type of structure and the location of the point with respect to the water level) to correct for the curvature of the principal stress trajectory from the water-supporting face to the point of analysis. It is then possible to determine τ_{xy}, σ_y, and σ_2.

Figure 26 demonstrates the application of this approximate method to an Ambursen dam. The first principal stress σ_1 is evaluated by the following equation:

$$\sigma_1 = \frac{P_b}{144t} = \frac{\sqrt{P_h^2 + (P_v + W_m)^2}}{144t} \tag{21}$$

in which P_h and P_v are, respectively, the horizontal and vertical components of the total water load on a horizontal strip of deck 1 ft wide and one bay in length, W_m the weight of a curved strip of masonry extending from point a to point b, t the buttress thickness at point b, and P_b the total resolved force on the 1-ft section at point b.

If the value of σ_x and σ_1 are known, the various stress functions may be determined from Mohr's circle or analytically. The following equations express the equations analytically:

$$\tau_{xy} = (\sigma_1 - \sigma_x)\tan\alpha \tag{22}$$

$$2\tau_{max} = (\sigma_1 - \sigma_x)(1 + \tan^2\alpha) \tag{23}$$

$$\sigma_2 = \sigma_1 - 2\tau_{max} \tag{24}$$

$$\sigma_y = \sigma_1 - \tau_{xy}\tan\alpha \tag{25}$$

Comparative principal stresses computed by the approximate and exact methods for point A (Fig. 17) are shown by Fig. 27.

V. BUTTRESS SPACING AND PROPORTIONS

1. Buttress Spacing. The selection of an economical and satisfactory buttress spacing for any particular site is influenced largely by (1) the mean height of the dam, (2) the presence of spread footings or a continuous floor slab, (3) the presence of a spillway over the dam, (4) the slope of the upstream water-supporting member, and (5) the unusual foundation or sidehill conditions. The first four considerations affect the quantities of the more important construction items: concrete, reinforcing steel, form area, and, occasionally, excavation. The last factor (5) sometimes demands an

arbitrary revision of buttress spacing to eliminate difficult and expensive foundation or sidehill treatment.

Any change in the buttress spacing produces a fluctuation of the quantities required for the structure. For wider spacings, the concrete quantity and the necessary reinforcing steel for the water-supporting decks are materially increased, whereas the large form area of the buttresses is distributed over a greater length of dam. Smaller spacings have the advantage of reducing the amount of concrete and reinforcement in the water-supporting member but increasing the form area per unit length of dam,

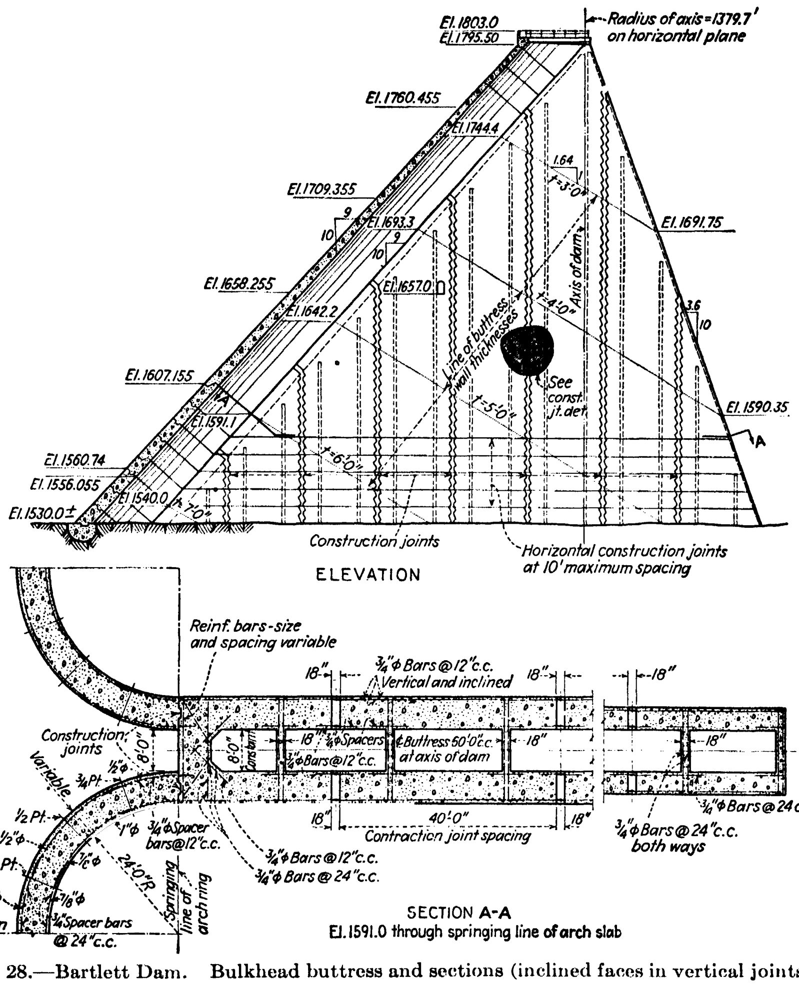


FIG. 28.—Bartlett Dam. Bulkhead buttress and sections (inclined faces in vertical joints).

consequently, the total form costs. The spacing that will give the most economical balance of concrete, reinforcing steel, and form area, satisfying also design requirements, is determined by the relative quantities and, hence, total cost of the above-listed items.

The height of the dam is of the greatest importance in selecting the buttress spacing. For high dams, greater buttress spacings are economical and satisfactory, whereas for low dams the buttress spacing will be proportionally less. Buttress

spacing should be selected to suit the average or mean buttress height rather than the maximum height. If the foundation is sound rock, the most economical buttress spacings for Ambursen dams are about as follows:

Mean Height, ft	Economic Buttress Spacing Distance between Center Lines, ft
50–100	15–20
100–150	30–40
Above 150	40–50

Spread footings, continuous floor slabs, and the crest and apron slabs of spillways over buttress dams respond to changes in spacing in the same manner as the water-supporting member: the quantities vary directly with the buttress spacing. It is therefore more economical to use smaller spacings for these cases.

The slope of the upstream deck is a minor factor in selecting the buttress spacing although it materially affects the second principal stress. As the slope becomes

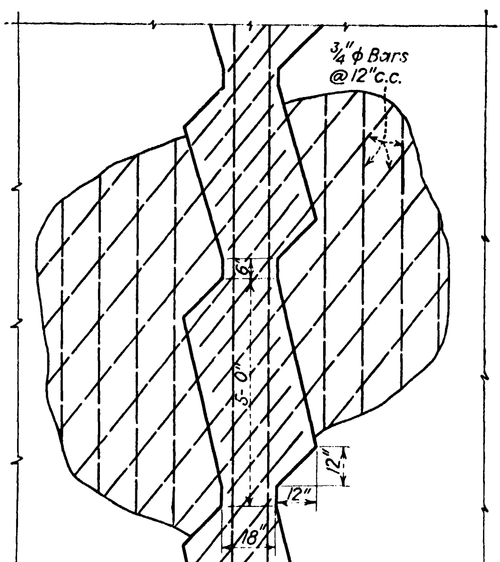


FIG. 28a.—Bartlett Dam. Detail of vertical contraction joints.

flatter, the inclined length of the deck tends to increase and the deck quantities assume greater importance. In general, it may be said that slightly greater economy will be realized if smaller spacings are used for dams with extremely flat upstream decks.

Foundation conditions may influence the spacing by the presence of fault zones or steep, unstable sidehills which frequently require an arbitrary selection of buttress spacing to suit the particular condition. Fault zones, when relatively narrow, can be straddled by using a wide buttress spacing. Wider fault zones can be bridged with a special arch or girder structure which transmits the buttress loads to either side of the fault. In this case, a lesser spacing may be desirable so as to spread the buttress loads more uniformly. Steep or unstable sidehill conditions can be treated likewise, either increasing the spacing to straddle the difficult areas or decreasing the spacing to distribute the loading. The proper solution of foundation problems of this type cannot be reached analytically; complete familiarity with the subsurface conditions and the application of mature judgment are the only safe guides.

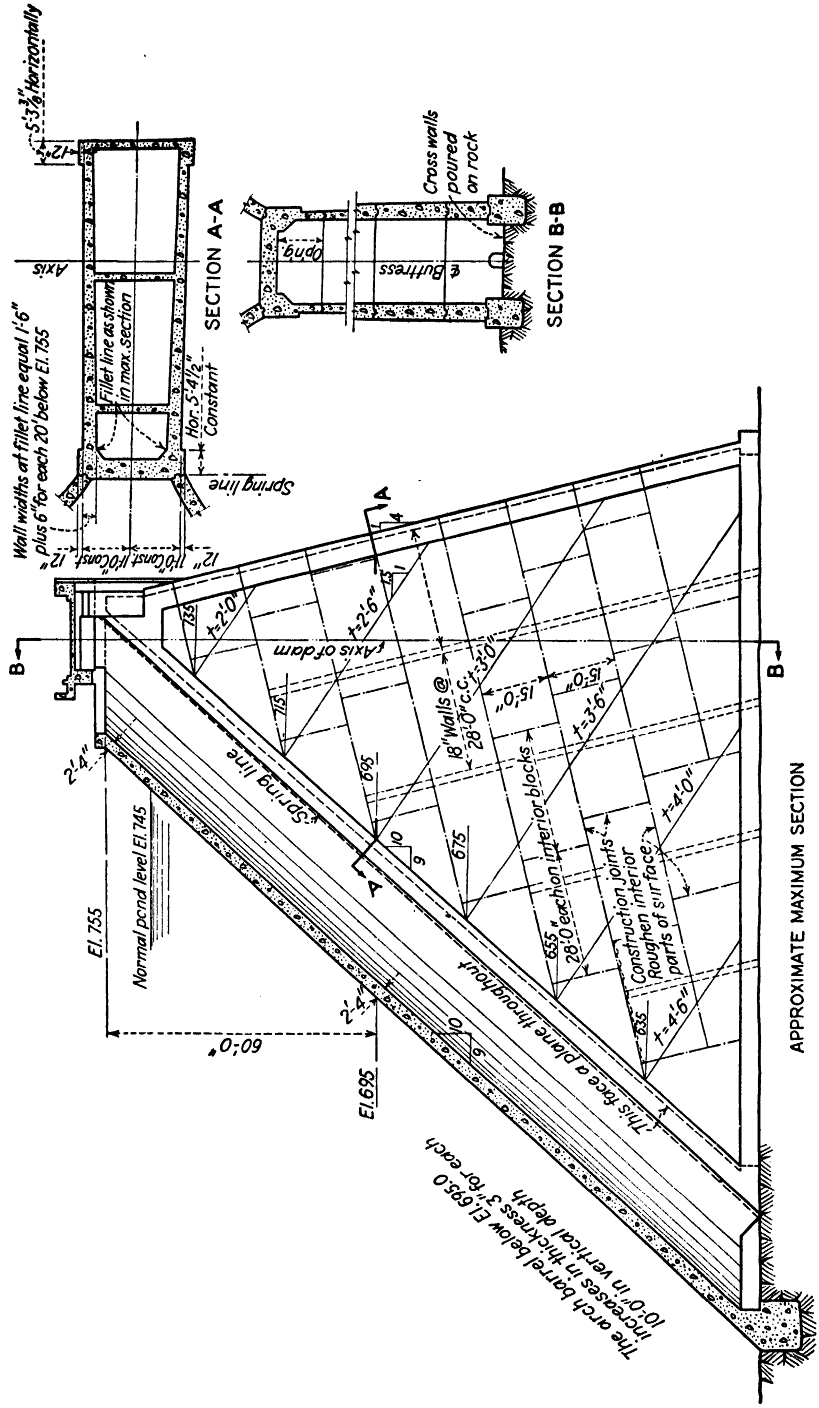


FIG. 29.—Pensacola Dam. Buttress elevation and sections (inclined joints).

Buttress spacings varying from 30 ft to 80 ft have been used in connection with modern multiple-arch dams. A buttress spacing of 60 ft was used for the Bartlett Dam[1] (Fig. 28) and 84 ft. for the Pensacola Dam (Fig. 29) both of the hollow-buttress type.[2] For Buchanan Dam (Fig. 30), the buttress spacing is 70 ft.

In designing both Ambursen and multiple-arch dams on rock foundation, a wide choice of buttress spacings is possible without materially affecting the cost. The Stony Gorge Dam (Fig. 1) would not have been materially increased in cost if the buttress spacing had been changed from 18 ft, as constructed, to 40 ft. Comparative estimates for the Coolidge Dam (Fig. 9) indicated that a standard multiple-arch dam, having buttresses spaced 60 ft center to center, would have cost only 2 per cent more than the multiple-dome dam, with buttresses spaced 180 ft center to center.

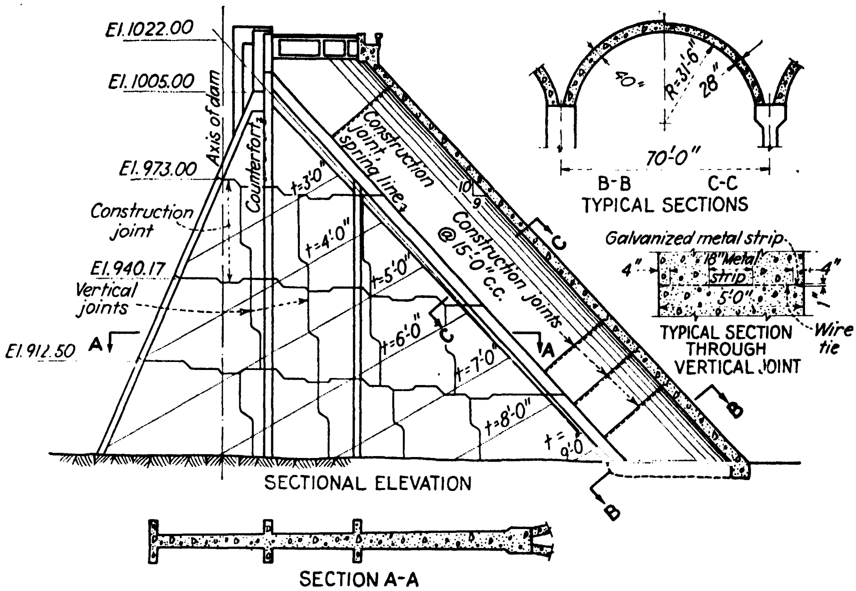


FIG. 30.—Buchanan Dam. Buttress elevation and sections.

Other factors remaining constant, buttress thicknesses will vary directly with the buttress spacing. The methods of determining and the effects of variations in the buttress thickness are discussed under Theory of Buttress Design. See page 99.

2. Upstream and Downstream Buttress Slopes and Sliding Factor. The slope of the upstream face of a buttress is governed by sliding-factor requirements which, in turn, are affected by foundation conditions. Within the range of usual sliding-factor values, the cost of the structure decreases as the sliding factor increases, and, consequently, for any particular foundation condition it is desirable to use the highest safe sliding-factor value. Sliding factors varying between 0.3 and 0.5 are generally selected for buttress dams constructed on soft or pervious foundations, whereas a sliding factor as high as 0.9 may be permissible for a buttress thoroughly keyed into sound rock. In general, the stress distribution and intensities are more satisfactory in buttresses with steeper upstream decks, and, in the case of flatter slopes, additional buttress thickness may be required to avoid an excessive amount of tension in the second principal stress or to reduce the maximum shear value. However, no definite rule can be given; actual experience with structures of this type is the safest guide.

[1] Constructed by the Reclamation Bureau.

[2] The double buttress dam was originated and patented by F. A. Noltzli.

The downstream slope of the buttress is selected so that the horizontal length at all elevations gives adequate base for stability and sufficient section to hold the stress values at or below the allowable. The base length varies from 1.2 to 1.5 times the depth below maximum headwater.

3. Buttress Thicknesses and Stiffeners. The upstream and downstream buttress slopes and the buttress thicknesses should be selected with a view to approaching uniformity of stress distribution. Maximum economy will be attained if the values of σ_1, σ_x, and τ_{xy} are substantially uniform on each horizontal plane.

A uniform horizontal shearing-stress distribution increases the safety of the structure against sliding. The sliding factor, as usually considered, does not apply uniformly to all points in the foundation, particularly if vertical normal and shearing stresses vary at different rates. Unless buttress proportions are carefully selected, it may be entirely possible that dangerous shearing-stress concentrations will occur in planes where the sliding-factor value is entirely satisfactory.

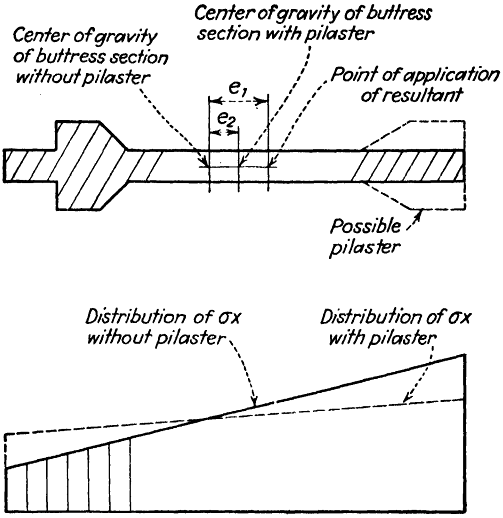


FIG. 31.—Effect of pilaster on stress distribution.

Buttress thicknesses are governed by practical considerations and limiting stress intensities. If the buttresses are closely spaced, say 15 to 20 ft center to center, the minimum thickness of the top lift should not be less than 9 or 12 in.; for wider spacings, the minimum thickness will vary between 18 and 48 in. The point to which these minimum thicknesses can extend down the buttress and the rate of thickness increase below that point are determined by limiting stress intensities.

Stress analyses indicate a subdivision of the buttress, below the portion governed by minimum thickness requirements, into zones in which any single stress function may govern the thickness. The boundaries of the zones depend entirely upon the limiting stresses selected for a given design. In a recent buttress design (Bartlett), a first principal stress value σ_1 of 600 psi and maximum shear values of over 300 psi were permitted. The buttress, in this case, was sufficiently reinforced with inclined steel to take all second principal stress tension. In more conservative designs, the allowable values do not exceed 500 psi for the first principal stress and, when the buttress is properly reinforced with inclined steel, 300 psi for the maximum shearing stress. The

allowable shearing stress should be reduced to 250 psi for buttresses without inclined steel.

Vertical normal and horizontal normal stress intensities will not govern if the principal and shearing stress intensities are held within allowable limits. Exceptions to this may occur in buttresses requiring spread footings to attain low foundation pressures. In such cases, the proportion of the footings, dependent in a large measure upon the total vertical pressure per linear foot of buttress, may control the economic design of the dam.

Stress intensity distribution and buttress dimensions may, to a limited degree, be controlled by modifying the shape of the buttress section. For example, if it were found that the horizontal section, shown in Fig. 31 by the solid lines, gave the objectionable stress distribution shown by the solid line of the stress diagram below, the addition of a pilaster, as shown by the broken lines, would reduce the eccentricity from e_1 to e_2, the moment due to the eccentricity being decreased and a more uniform distribution of the vertical normal pressure being made.

It is advisable to keep the stiffness ratio (l/r or l/d) within the limits allowed for ordinary columns. Since the ratio is expressed in terms of the factors of unsupported length and effective thickness or depth of section, it is possible to satisfy the limitations by adjusting either factor. For the Ambursen-type solid single-wall buttress of moderate spacing, it is usually more economical to control the unsupported length by the use of struts or diaphragms extending continuously from sidehill to sidehill. The diaphragms, when used, and the struts, when diaphragms are not required, are securely tied in alternate bays to adjacent buttresses. In the intermediate bays, to ensure articulation, the struts are fitted into pockets in the buttress face but not tied to the buttress.

For extremely large buttress spacings, struts or diaphragms become decidedly uneconomical and their actual structural worth is questionable. In these cases, it is necessary to adjust the effective thickness, and the hollow or double buttress is sometimes used, as shown in Figs. 28 and 29. Another solution is shown in Fig. 30, Buchanan Dam, in which the effective thickness was increased by the addition of vertical fins on a solid single-wall buttress.

VI. BUTTRESS DETAILS

The more important considerations of the buttress details are the following: height of lift, construction joints, contraction joints, buttress reinforcement, lateral bracing, and openings.

1. Height of Lift. Lift heights, the vertical distances between horizontal construction joints, have been used as follows: Bartlett Dam, 10 ft; Stony Gorge and Possum Kingdom dams, 12 ft; Rodriguez Dam, 15 ft; Pensacola Dam, approximately 15½ ft; Buchanan Dam, approximately 30 ft. A high lift reduces form handling costs but requires stronger and more expensive form panels. For general construction work, lift heights of about 12 ft are commonly recommended, and the elevation of lift lines is maintained the same for all buttresses, the less than standard height lifts being placed at the foundation rather than at the top of the buttress.

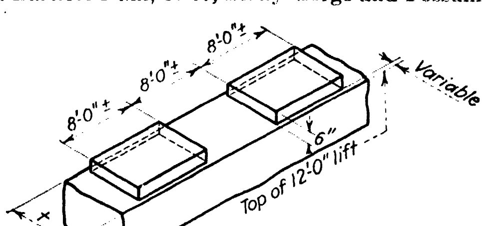


FIG. 32.—Possum Kingdom Dam. Isometric of buttress keys at top of 12-ft. lift.

2. Construction Joints. A satisfactory type of horizontal construction joint is shown by Fig. 32, an isometric view of the top of a typical buttress lift. These joints consist of raised keys, usually 6 in. high, 4 to 8 ft long, having a clear spacing about equal to the key length. It is usually more convenient to make the width of these keys equal to the thickness of the buttress lift above. The resulting shoulder assists materially in placing and holding the forms for the next stage of buttress construction. Various types of horizontal (or nearly horizontal) construction joints are shown in Figs. 15, 28, 29, 30, 32, and 34. The slightly inclined construction joints, as shown by Fig. 29, have been used in relatively few structures.

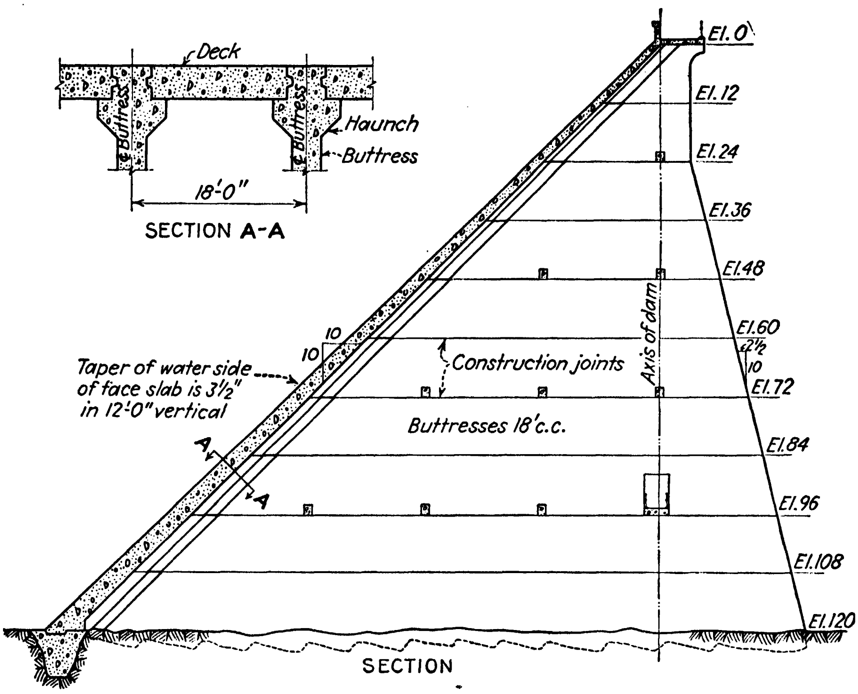


FIG. 33.—Stony Gorge Dam. Bulkhead buttress and sections.

3. Contraction Joints. Volumetric changes in concrete may be due to any of the following causes:

1. Shrinkage due to setting
2. Expansion or contraction due to temperature variations
3. Expansion and contraction due to alternate wetting and drying

Except in extreme cases, the last two can usually be controlled by a nominal amount of reinforcing as in ordinary reinforced-concrete structures.

Shrinkage due to setting is of sufficient importance to justify many special precautions such as the use of low-heat cement to reduce initial expansion and hence the final shrinkage due to setting, careful moist curing to ensure complete hydration and strength to resist cracking, contraction joints to localize shrinkage at a predetermined plane, and horizontal steel to distribute the shrinkage stresses uniformly between contraction joints.

From 1930 on, low-heat cement has been increasingly used until, at present, its use is customarily required in the construction of all large dams. The use of this type of

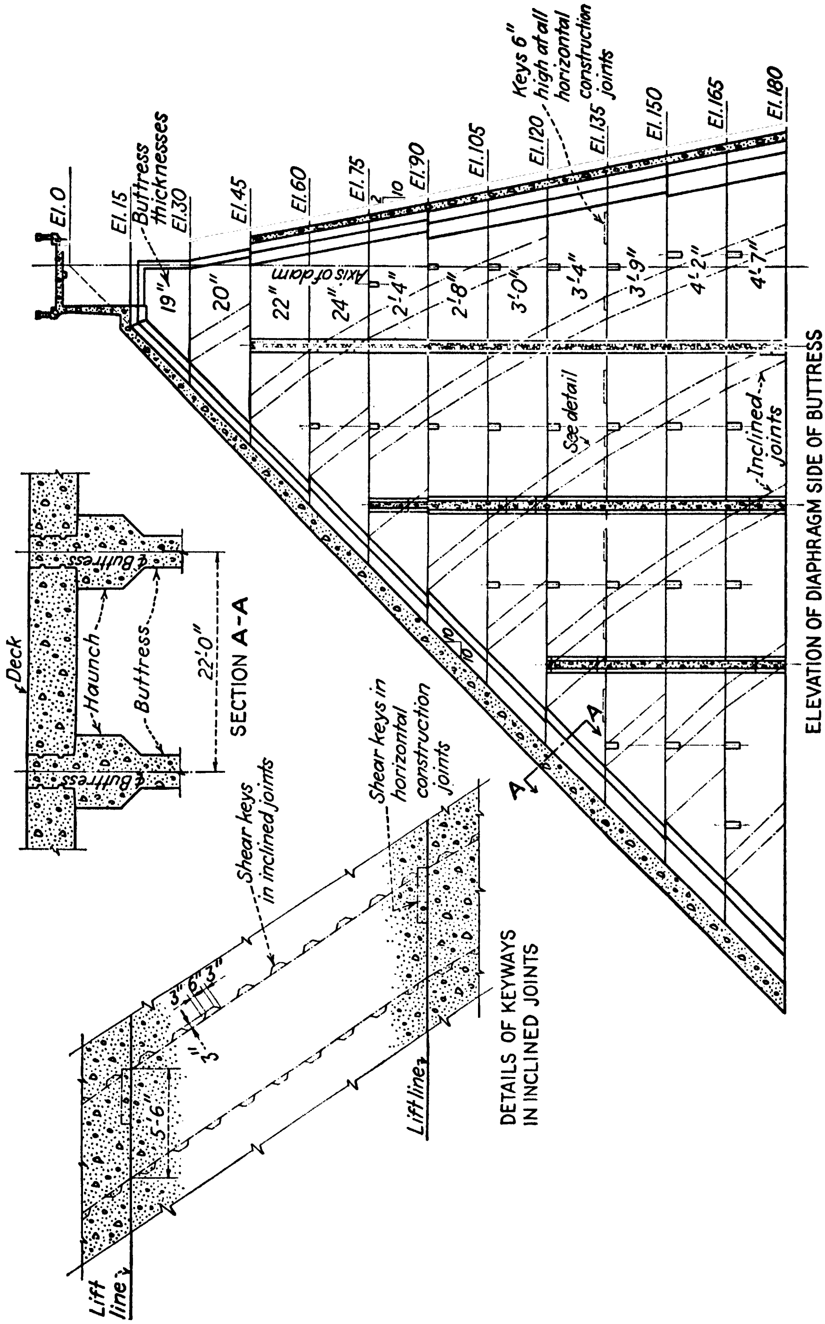


FIG. 34.—Rodriguez Dam. Buttress elevation and sections (inclined joints).

cement plus effective moist curing have contributed to the elimination of serious cracking which occurred in some early buttress dams. However, the possibility of cracking due to setting shrinkage cannot be completely eliminated by the use of low-heat cement and subsequent moist curing; there must be also a judicious placing of contraction joints, or, when the lifts are not sufficiently long to require contraction joints, carefully located reinforcement should be provided.

Buttress contraction joints may be divided roughly into two classes: the single joint which is placed so that a small opening of the joint will not adversely affect the stability of the structure, and the double, or "plug," type of joint in which case compensation for the shrinkage is made by filling the joint after the buttress concrete has attained initial shrinkage. The single joint is shown in Figs. 15, 29, and 30, and the plug-type joint in Figs. 28, 28*a*, and 34.

When the single joint is used, it is customary to pour the alternate blocks of any one lift, allow this group to attain complete shrinkage, and then pour intermediate blocks. A sufficient number of single joints located in a vertical position (or nearly vertical) can be used, as shown in Figs. 29 and 30. However, it is preferable to locate these joints on the trajectories of the first principal stress as shown in Fig. 15. The disadvantage of the former arrangement is the necessity of transmitting shear across the joints by adequate keys, diagonal steel, or a combination of both. Contraction joints placed on the trajectories of the first principal stress are required to transmit no shear under maximum loading conditions and relatively low shearing stresses under partial loading conditions. The latter are transmitted across the joint by a series of keys, as shown in Fig. 15.

The plug-type joint has been used only where special earthquake or other unusual conditions make the elimination of shrinkage openings and a complete tying together of the buttress advisable. This joint is, for obvious reasons, more difficult to build, and, hence, more expensive. In Fig. 34, the faces of the joints are placed on the trajectories of the first principal stress, and in Fig. 28 the faces are placed alternately on planes of first and second principal stresses. In both cases, the buttress reinforcing is made continuous through the opening or plug.

In conventional buttress construction, the horizontal length of pours should be limited to a maximum of 40 ft. If inclined joints are used, they should terminate some distance below the upstream buttress face, preferably at a band of reinforcing parallel to the water-supporting face.

4. Buttress Reinforcement. Horizontal reinforcement concentrated in bands is not so effective in minimizing cracking as that which is evenly distributed on the face. Frequently there are several controlling factors that govern the amount of steel required, but generally speaking a sectional area of steel equivalent to 0.2 per cent of the vertical sectional area is satisfactory for buttresses less than 5 ft thick. Smaller amounts may be used with buttresses thicker than 5 ft, provided that all buttress stresses are in compression and open contraction joints are used. With the massive-type buttress where thicknesses exceed 4 to 5 ft, it is not necessary to reinforce the section with sufficient horizontal steel to prevent shrinkage and temperature cracks. For this type of buttress construction, practically all reinforcement can be eliminated if proper contraction joints are provided. However, in some special cases it may be advisable to reduce the usual spacing of the contraction and construction joints.

Diagonal reinforcement parallel to the water-supporting face of the dam is usually provided either in a band near the upstream edge of the buttress or spread uniformly over the buttress faces. When the second principal stresses are a light compression or approximately zero throughout the buttress, the joints may be left open as shown in Fig. 15 and, with the exception of a band of reinforcing near the upstream face, diagonal buttress reinforcement is unnecessary. If the second principal stresses are

tension, however, it is necessary to carry both horizontal and inclined steel across the joints, as shown by Figs. 28 and 34, or, if the tension values are low and the inclined buttress columns are independently stable, open joints may be used provided that diagonal steel, sufficient in quantity to carry any stress concentration, is placed parallel to the deck in the buttress area just under the haunch.

5. Lateral Bracing. The governing considerations for placing and arrangement of struts and diaphragms for lateral bracing appear under Buttress Spacing and Proportions as part of the discussion of buttress stiffness. See page 113.

The Rodriguez Dam (Fig. 34) was built with vertical diaphragms, spaced 40 ft center to center in alternate bays. The diaphragms were constructed monolithically with the buttresses to form in effect rigid reinforced concrete bents. Spaced between the diaphragms were vertical rows of struts also constructed monolithically with the buttresses in the diaphragm bays. In the nondiaphragm bays, struts supported in asphalted pockets were placed opposite both beams and diaphragms in the adjacent bays. This arrangement provides the ability to transmit lateral loads through the dam without damaging the buttresses and at the same time preserving sufficient articulation to accommodate vertical foundation movements.

The buttresses of the Stony Gorge Dam (Fig. 33) were braced with reinforced-concrete struts tied to the buttresses in alternate bays and supported in asphalted pockets in the intermediate bays. No diaphragms were used.

The method of constructing alternate pairs of buttresses monolithically with connecting diaphragms has also been applied in a modified form to the Pensacola Dam (Fig. 29) and the Bartlett Dam (Fig. 28). The lateral bracing in these dams is not continuous, as it is in the form of diaphragms constructed only within the wide hollow buttresses, serving to stiffen the relatively thin walls and at the same time to increase the base width of the buttress member. No beams are used, and there are no bracing members to connect each hollow buttress with the adjacent one. The diaphragms were placed vertically in the case of the Bartlett Dam and inclined in the case of the Pensacola Dam.

6. Openings. Large openings through buttresses should be avoided wherever possible. If interior walkways are necessary, the tops of the doorways should be circular in form and the buttress area surrounding the doorway opening heavily reinforced.

VII. WATER-SUPPORTING MEMBERS

1. Ambursen Dam Decks and Supports. The decks of Ambursen dams are designed as freely supported slabs. Loading and span assumptions are shown in Fig. 35. The slab thicknesses are controlled by bending moments in the upper lifts of the dam and by shear in the lower lifts. The slab must be tested for both bending and shear as the design progresses from the upper to the lower lifts. Typical deck details for the Possum Kingdom Dam are shown in Fig. 36. It is to be noted that bent bars are provided in the lower lifts, to take diagonal tension.

It is ordinarily desirable to provide for expansion and contraction of the deck slab in two directions longitudinally, *i.e.*, parallel to the axis of the dam; and diagonally, *i.e.*, in an inclined direction parallel to the upstream edge of the buttress. Longitudinal adjustment is made possible by a coating of bituminous mastic placed on the portion of the buttress tongue that is in contact with the deck slab. The mastic serves the double purpose of providing a small clearance (usually about 1/8 in. at each end of slab) for expansion and of allowing the deck slab to withdraw slightly from the tongue face, when contracting, without breaking the water seal. This movement is facilitated by a lubricating bituminous coating on the deck-supporting face of the haunch. The

pull exerted on the haunch by contraction of the deck slab must be included as a factor in design of both the haunch and the deck.

An interesting exception to the usual practice is furnished by the Lake Walk Dam on Devils River, Texas, shown by Fig. 37. The upstream deck slab and the downstream apron are both constructed continuously. This dam has been in successful

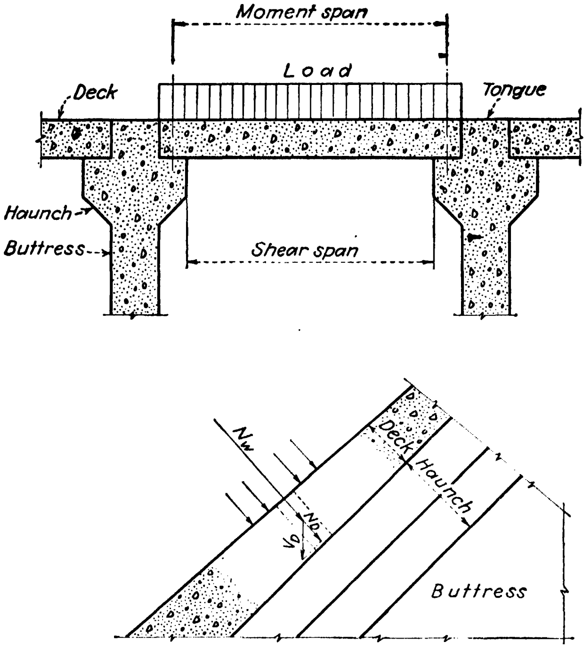


FIG. 35.—Loading assumptions on deck.

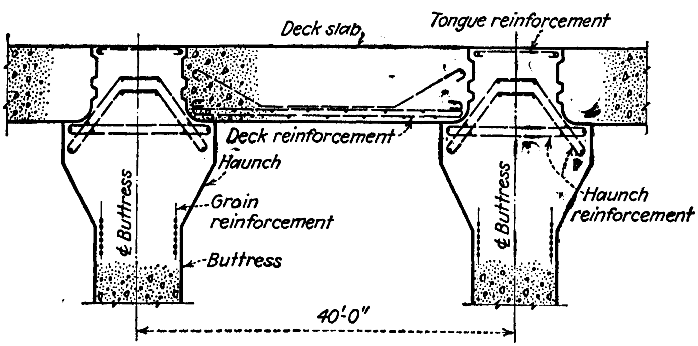


FIG. 36.—Possum Kingdom Dam. Section through haunch and deck.

service for 22 years and has withstood several floods of record magnitude. Most engineers have been reluctant to use continuous-slab decks because the negative reinforcement is only a few inches from the water. The experience with the Lake Walk Dam and with other reinforced-concrete structures appears to justify the view that the reinforcing steel will not deteriorate.

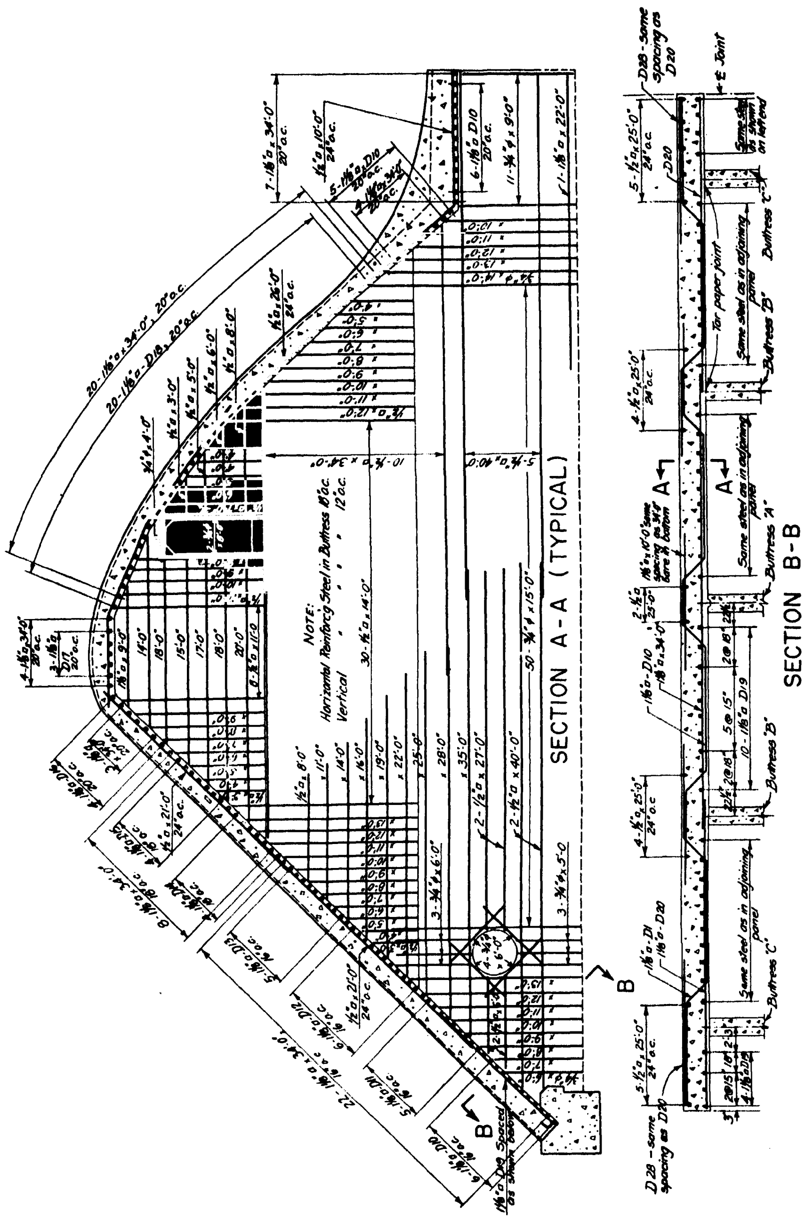


FIG. 37.—Lake Walk Dam, Devils River, Tex.

The movement of the deck slab upward and downward along the inclined haunch may, unless adequately provided for, seriously affect the stability of the structure by causing an overturning moment. The separation of the deck from the buttresses in the Ambursen-type dam, in addition to providing complete articulation, eliminates the development of overturning moments due to temperature changes. It is, however, desirable to get the full benefit of the deck weight so as to improve stability. This is done by forming, in the tongue face, keys designed to transmit the full deck weight to the buttress (see Fig. 38). The lifting effect of deck expansion is made ineffective by strips of compressible material placed in the upper portion of the recessed keys. In the case of high dams, the deck slab is divided by horizontal expansion joints (Fig. 39) so that the accumulated expansion does not, at any point, exceed the compressibility of the material in the keys.

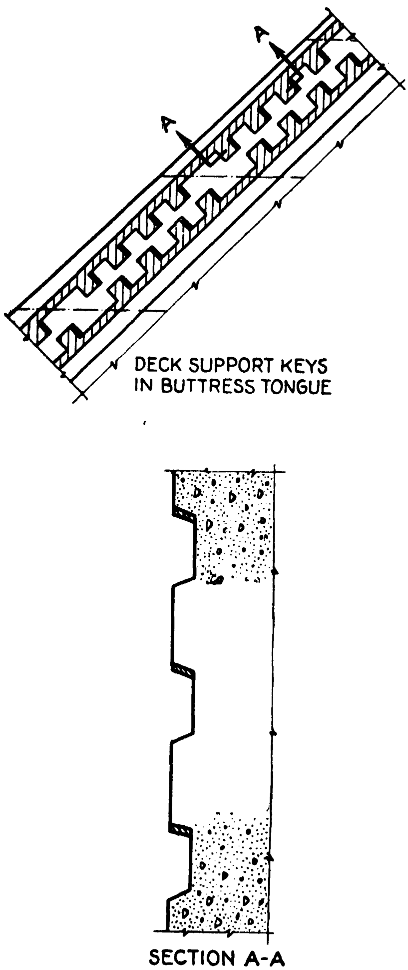


FIG. 38.—Possum Kingdom Dam. Deck support keys in buttress tongue.

Haunch dimensions are governed by the width of bearing face required for the deck reaction and by the depth of section required for shear or, in some cases, bending moment. It is assumed that under the maximum loading the deck will deflect sufficiently to change the load distribution on the haunch bearing face from rectangular to triangular loading with the maximum at the outer edge. For a haunch of uniform average shear strength, this triangular loading gives a section that is parabolic in shape, as shown by the broken line in Fig. 40. The dimension of the flat face parallel to the center line of the buttress *cd* (Fig. 40) is arbitrarily made equal to or somewhat larger than the width of the bearing face. The face *de* is sloped so that the depth requirements for shear are satisfied at all points. Where the slope is 45 deg or flatter, the critical shear section occurs not at the buttress face but at some point out on the haunch. This point can be located by differentiating for the maximum unit shear value or by trial computations.

The standard design formulas for flexure and diagonal tension in beams are not strictly applicable to corbels or haunches with extremely flat sloping faces *de* and of depth materially greater than the projection. Photoelastic investigations by the Bureau of Reclamation and in the author's office indicate that when the inclination between the sloping haunch face *de* and the buttress face is about 60 deg or greater the maximum compressive stress will occur at the outer edge (point *c*) on the haunch bearing face *bc* and the principal stress trajectories will "flow" back to the buttress without

any "pinching" at the reentrant angle. The tensile stresses will attain a maximum near the inside corner (point b) on the haunch bearing face bc, with an average direction of about 60 deg from the center line of the buttress. The total amount of tensile

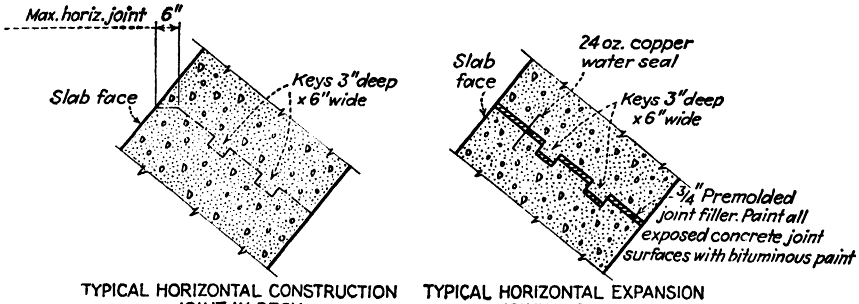


FIG. 39.—Possum Kingdom Dam. Typical horizontal construction and expansion joints.

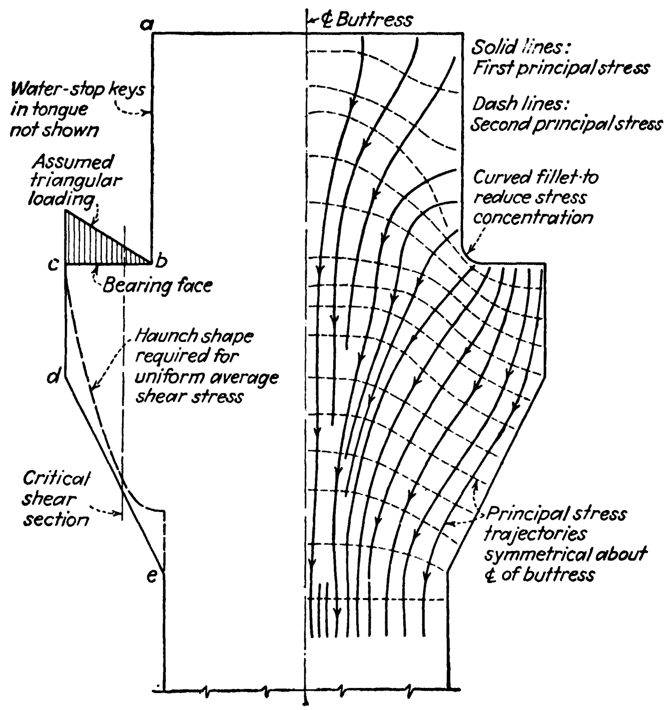


FIG. 40.—Haunch section. Broken line, theoretical outline; solid line, actual. Principal stress trajectories.

stress can be evaluated by resolving the deck reaction into tensile and compressive forces within the haunch. The direction of the two resisting forces can be determined with reasonable accuracy from a brief photoelastic analysis. Reinforcing steel is placed so as to carry all the diagonal tensile stress, and also the tension directly under and parallel to the bearing face. An additional amount of steel should be provided

directly under the haunch bearing face for the tension that will be transmitted from the deck to the bearing surface of the haunch when the deck contracts due to a drop in temperature. The amount of this tension will depend upon the deck reaction on the haunch and the coefficient of friction between the deck and the asphalted surface of the haunch. In early designs, a coefficient of friction as high as 1.0 was used, largely because of the uncertainty of the lubricating efficiency of the bituminous coatings available at that time. In more recent designs, the value of 0.5 has been used. This value has been proven by tests to be well on the conservative side.

It is particularly desirable and economical to have a low friction coefficient for the deeper sections of high dams where the reaction of the deck on the haunch is great. To ensure a dependable and enduring well-lubricated surface, the usual coating of

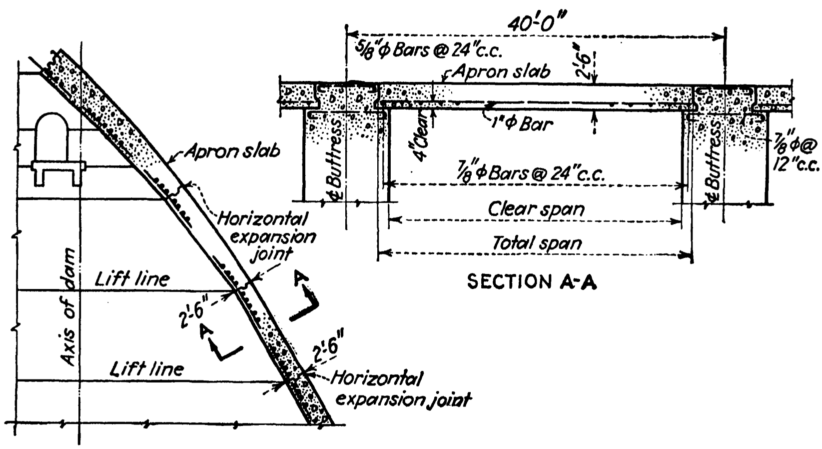


FIG. 41.—Possum Kingdom Dam. Typical apron slab.

mastic is preceded by a priming coat of bitumen and the mastic coating increased in thickness. Where the deck span is large, the bituminous treatment is sometimes augmented by a layer of fabric placed on the haunch bearing face.

2. Spillway Face Slabs or Aprons. It is especially important to proportion the downstream face of the spillway so that the under surface of the discharging water sheet will at no time leave the face of the slab. However, this tendency of the jet to spring clear is prevented by the use of a correctly designed spillway ogee. Experiments have indicated that separation does not occur with discharge heads up to 1.5 to 2 times the ogee design head.[1]

With a correctly designed spillway ogee discharging under the head for which the curve was designed, the pressures at all points along the ogee are zero. At some lesser head, the positive pressures normal to the masonry will reach a maximum and, as the head increases above the design head, the pressures will drop below atmospheric and tend to lift the slab. Usually the component weight of the spillway apron slab is more than sufficient to offset the slight vacuum effect at the face of the slab, and the resultant of the two forces holds the slab firmly on the downstream edges of the buttresses. For high discharges, an added factor of safety is introduced by keying the ends of the slab into pockets provided in the buttress, as shown in Fig. 41. For smaller dams carrying medium depths of spillway discharge, it may be unnecessary to provide keys, the weight of the face slab alone being sufficient to prevent its displacement under all conditions. In this case, the spillway face slab can be constructed as shown in Fig. 42.

[1] *Civil Eng.*, January, 1935, p. 10.

Spillway apron slabs, when designed in the conventional manner for maximum loading conditions, are too thin to be practical, and, consequently, thicknesses are chosen arbitrarily varying from 12 in. for spans of about 15 ft to 36 in. for spans of about 50 ft. Moment reinforcing is placed horizontally in the bottom of the slab, and it is desirable to provide longitudinal steel, as shown by Figs. 41 and 42, which will be continuous throughout the apron and crest, except when horizontal expansion

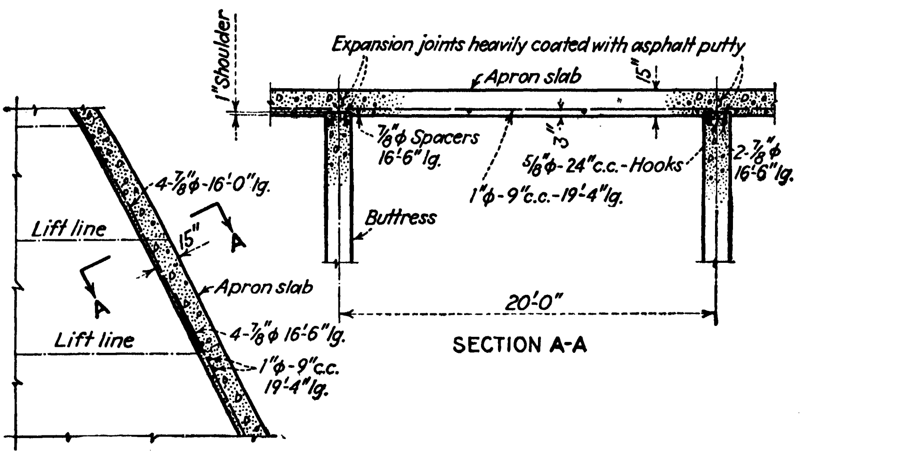


FIG. 42.—Prescott Dam. Typical apron slab.

joints are placed in the slab. The horizontal joints (Fig. 43) provide for upward expansion of the spillway apron slab similar to the special horizontal joints in the deck slab (Fig. 39).

The spillway face slab terminates, at or near the bottom of the dam, in a bucket or curved deflector which serves to turn the discharged sheet of water from a downward path to a horizontal or slightly upward path. Spillway sections with the bucket on rock and raised are shown in Fig. 44. In these cases, the bucket must be designed to carry the large centrifugal force of the discharge sheet.

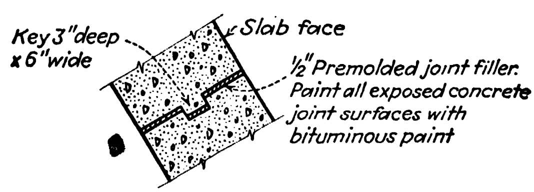


FIG. 43.—Possum Kingdom Dam. Apron expansion joint.

3. Multiple-arch Decks. In analyzing the stresses in the arches of a multiple-arch dam, the arch barrel is assumed to be divided into elementary sections 1 ft wide. Arch thicknesses are determined approximately by the standard cylinder formula and then checked by the elastic method, the stresses due to the water pressure, the weight of the arch, the shrinkage of the concrete, and the changes in temperature being taken into consideration. The water load is generally taken as the average on the arch ring instead of considering the variation that actually occurs throughout any single arch element.

The central angle of the arches usually varies between 120 and 180 deg. Circular arches of uniform thickness (Figs. 45 and 46) are most economical for spans less than 40 ft, and, although slightly less economical for longer spans, offer definite construction advantages. For spans in excess of 40 ft, in many cases greater economy may be

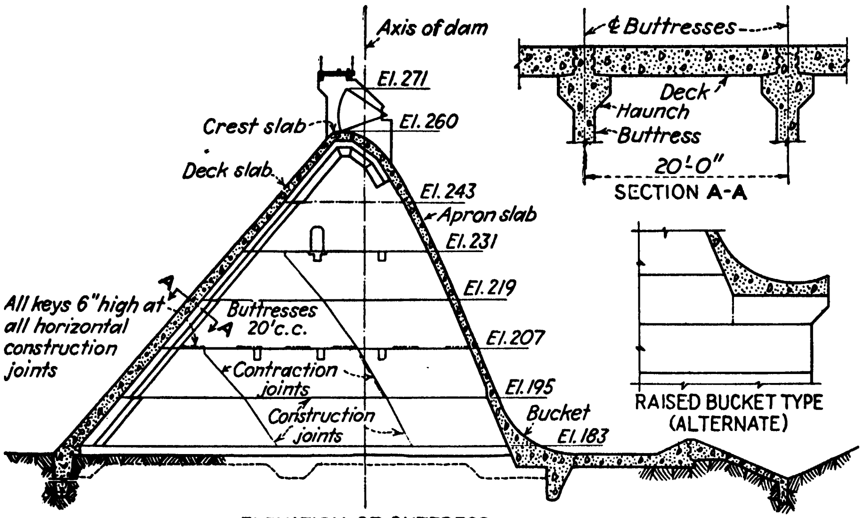


FIG. 44.—Minas Basin Dam, on shale. Spillway buttress elevation and deck section.

obtained by using a compound circular curve with thicknesses varying from a maximum at the springing line to a minimum at the crown (Fig. 47).

Expansion and contraction of the arches due to both temperature changes and loading variations are provided for as described in the following. Longitudinal move-

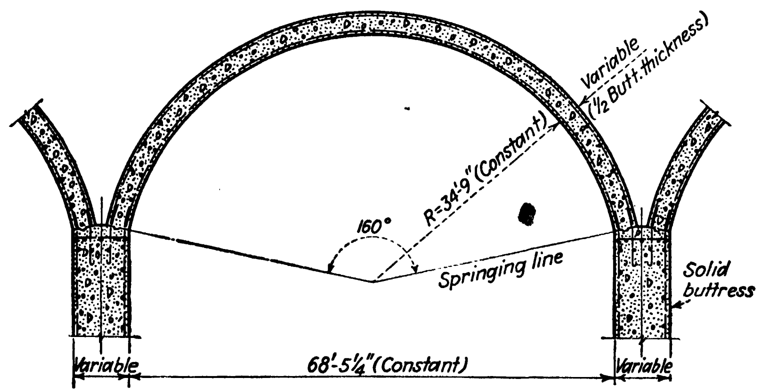


FIG. 45.—Victoria Dam. Section through arch.

ment, parallel to the axis of the dam, is restricted by the nature of the structure, and hence additional reinforcement must be included in the arch section to carry the induced moments. The upward and downward movement of the arch barrel is entirely restricted at the upstream edge of the buttress by the continuous reinforcement extending into the arch. In some cases, contraction joints are provided in the

arch barrel as shown by Fig. 48. The designing procedure for multiple arches is substantially the same as that for arch dams outlined in Sec. 3.[1]

4. Round-head, Diamond-head, and Cantilever Deck. The round-head buttress, as shown in Fig. 11, is an adaptation of the articulated type of structure utilizing a

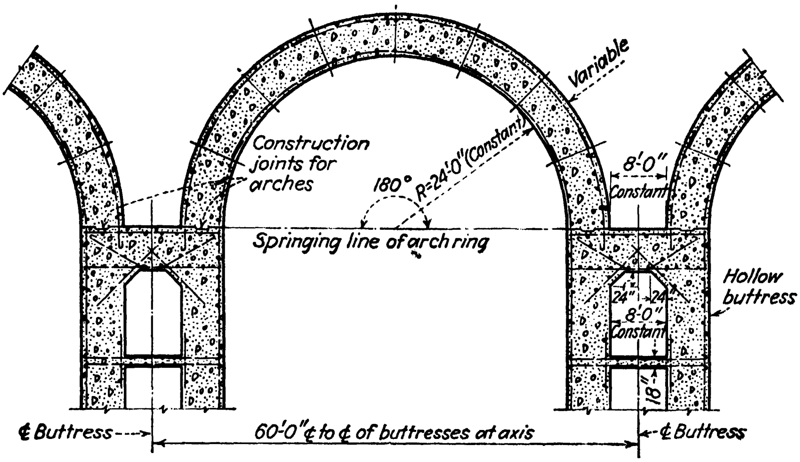


FIG. 46.—Bartlett Dam. Sections through arch.

heavy wedge-shaped mass of concrete as a water-supporting member and achieving a slight degree of articulation. The water pressures are, by virtue of the rounded head, directed toward a point on the buttress center line, thereby eliminating unbalanced moments. The buttress sides are flared so that all the water-pressure vectors fall

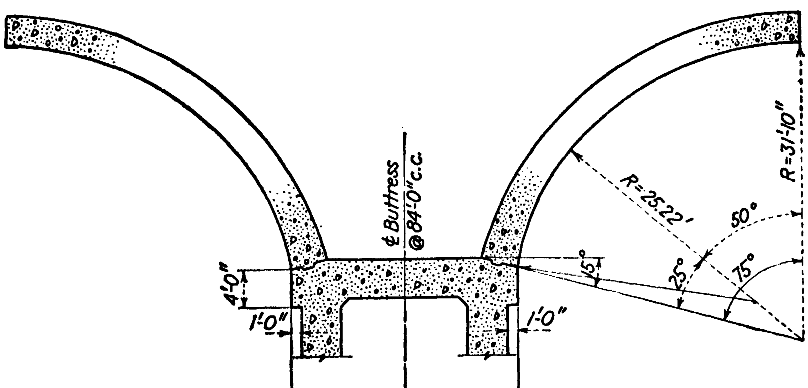


FIG. 47.—Pensacola Dam. Section through arch.

within the outline of the round head. The cantilever moment and the shear on a plane at the buttress face must be examined to detect any undesirable stress concentrations.

Owing to the great size of the buttress head, it is necessary to provide keyed and flashed construction joints in planes that will not alter the structural action. Provi-

[1] See *Trans. A. S. C. E.*, **98**, 1200

sions for longitudinal expansion are made at the midpoint between the buttresses with a keyed and flashed expansion joint.

The diamond-head buttress is a variation of the round-head type in which a series of three or more planes replace the curved upstream face. The design and details are similar to the round head.

The expansion joints are placed as for the round head.

The cantilever deck is of the same general type as the round and diamond head except that the water pressure is not directed toward the center of the buttress. It is

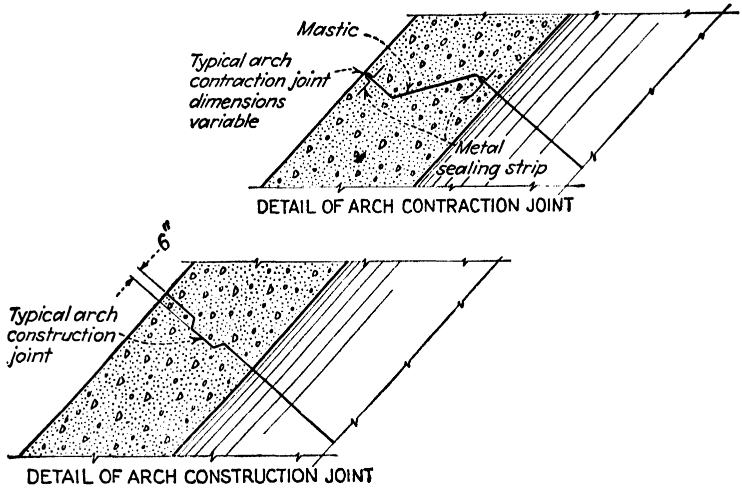


FIG. 48.—Bartlett Dam. Detail of arch construction and contraction joints.

therefore necessary to place heavy reinforcing on the upstream face to carry the moment due to the cantilever action.

In designing the round-head and diamond-head buttress dams, drainage means should be provided at the foundation line under the upstream ends of the buttresses to relieve uplift.

VIII. STRUCTURES FOR DIFFICULT OR SOFT FOUNDATIONS

The appropriation of many of the better dam sites and the increasing demands for water conservation are among the factors that have made necessary the development of many sites having other than sound rock foundation materials. The effect of this requirement is twofold: (1) the technical difficulties of design have been increased tremendously, (2) basically improved structures are being evolved in response to the new needs.

The design of dams on difficult foundations should be developed only by engineers who have had specialized experience in this field. Theoretical analyses may serve as a guide only after practical requirements have been met. In the hands of an inexperienced designer, theories concerning dams on soft foundations may lead to positively dangerous results.

For purposes of description, dams on difficult foundations may be considered as falling into two general classes: (1) dams constructed on porous or soft foundation material reasonably uniform in texture, such as clay, sand, or gravel; (2) dams constructed on foundation materials that are less porous or erodible than the foregoing, such as the soft breccias, badly laminated or cavernous limestones, shales of low bearing value and low resistance to shear or sliding, and various shattered or faulted rock

formations, and also materials having widely varying characteristics. The Rodriguez Dam presents an outstanding example of a high dam falling into the second class. In the river-bed section where this structure attains its maximum height of 250 ft, the foundation materials range from a faulted section containing soft clay gouge in part of this area and broken rock in another part, to sound rock on each side of this faulted section.

1. Dams on Soft or Porous Foundations. The prevention of piping is the principal problem in designing dams on earth foundations. The foundation material beneath the dam may be viewed as a conduit connecting the reservoir upstream with the tail water downstream. The objective is to make this conduit long enough and to create within it sufficient friction to reduce velocities below values capable of moving foundation material.

If the velocity of flow, where water emerges from the foundation material below the dam, is sufficient to move particles of the stream-bed material, the length of the conduit is progressively lessened. As the length of water travel is decreased, both velocity and head increase until ultimately a channel or pipe is formed beneath the dam. Complete failure may follow such a break.

Darcy's law (see also Sec. 1) furnishes a theoretical basis for providing adequate length of water travel beneath a dam. This relation may be expressed as

$$Q = C_1 \frac{HA}{L} \tag{26}$$

where Q = discharge, cfs.
H = head, ft.
L = length, ft.
C_1 = a coefficient that depends on the character of the material.

Substituting for Q, the value Av [Eq. (26)] becomes

$$L = C_1 \frac{H}{V} \tag{27}$$

For each class of foundation material, homogeneity being assumed, there is a definite maximum velocity Vn, at which the water can emerge below the dam without carrying away foundation material and causing the failure of the structure. This value of Vn may be combined with C_1 to form a new coefficient $C_2 = C_1/Vn$.

Substituting C_2 in Eq. (27) for C_1/V, there results

$$Ln = C_2H \tag{28}$$

where Ln = minimum safe length of travel path.
C_2 = a coefficient depending upon the foundation material.

The foregoing theory, which has now been generally accepted, involves two difficulties. The first relates to measuring the length of the path of travel and the second to the selection of a coefficient that will apply to the foundation material under consideration.

The path that water takes beneath a foundation might be determined with reasonable accuracy by analytical methods if foundation materials were homogeneous. This is rarely the case, however, and for purposes of design it is usually necessary to make certain assumptions.

One procedure, commonly called the *line-of-creep method* involves the assumption that the water flows along the line of contact of the dam foundation and cutoff wall and that the total length of this contact may be regarded as distance through which the water travels. This method was suggested by Bligh in the second edition of his "Practical Design of Irrigation Works."

Another approach that has been advanced to some extent is the short-path method which is based on the assumption that the course taken by the percolating water is the shortest path through the foundation material between the headwater and the tail water.

The line-of-creep method has had the widest acceptance for purposes of practical design, although neither method presents a true picture of what actually takes place beneath the foundations. In determining the length of travel by this method, it is not advisable to include the length of long hearths or paving below spillways unless these have sufficient weight to resist uplift pressures.

Seepage and percolation of water through foundation materials are objectionable from the standpoint of engineering only when they exceed safe limits or create undesirable conditions. These limits may be defined and summarized as follows for dam foundations under reservoir heads:

1. The seepage pressure under the foundation that would subject the structure to an undesirable overturning moment or so reduce the vertical forces of the dam as to cause sliding on the foundation.
2. For granular soils, a secondary effect upon their internal stresses due to seepage could reduce the effective stresses to such a point that the shearing stress in the material would be critical and sliding on the foundation become imminent.
3. Percolation velocities in the foundation material that would be great enough to move soil particles and cause piping or scouring of the foundation.
4. If the velocity pressures become great enough to increase sufficiently the neutral or fluid pressure in the soil, the internal shear will be reduced to a point at which flow of the material would occur, and consequent failure by sliding on the foundation.

It will be seen from the foregoing that there are three conditions to be investigated:

1. Seepage uplift
2. Piping and undermining from percolation
3. Reduction of shearing resistance due to either condition (1) or (2), or both.

These factors are affected by the type and location of the water seal or seepage obstruction to be adopted. It is usual practice to place a cutoff wall or steel sheet piling at the upstream face of the dam, as shown in Fig. 49*a*, in order to reduce the uplift pressures under the foundations. In Fig. 49*b*, the cutoff is placed for purpose of illustration at the downstream extremity of the dam. Flow nets have been sketched for these arrangements, a homogeneous foundation soil being assumed. Flow nets can be constructed by the use of a model in a glass tank, sand and a dye solution being used to trace the path of the flow lines. Probably more accurate results are obtained by employing an electrical analogy or electric-potential method in which cuts in a metal sheet simulate the water seals or foundation configuration. The few flow lines and equipotential lines sketched here will be sufficient for the purposes of discussion. The equipotential lines represent points of equal hydrostatic pressure due to the reservoir head H. Also, the rate of flow between flow lines is equal. Comparative merits of the two schemes can be seen by studying the plottings of the flow nets.

From the number of equipotential lines between the upstream and downstream faces of the dam (Fig. 49*a*), a considerable portion of the reservoir head has been dissipated to approximately $\frac{8}{12}H$ at a point *a* below the cutoff wall, and underneath the footing a short distance farther downstream from the wall at point *b*, the head has been further reduced to $\frac{5}{12}H$. The lines of equal flow represented on the diagram serve to indicate that percolation is concentrated (as revealed by the close spacing of the lines) at the bottom of the cutoff wall and at the downstream extremity of the foundation slab. Under the wall, the flow is confined under a weight of overlying material and can do no harm. However, at the downstream edge of the foundation slab, point *c*, where there is also a concentrated flow producing seepage, piping may occur and scouring take place, undermining the base of the dam, as shown in the enlarged detail forming part of Fig. 49*a*. This erosion causes an increased hydraulic

gradient and further undermining and, unless checked, can cause further erosion and eventual failure.

The condition has been corrected in Fig. 49*b* by placing the cutoff downstream, effectively reducing the flow at point *c* and causing the critical pressure to occur at the bottom of the cutoff wall in a depth of material sufficient to reduce its effect. However, in this case the uplift pressure is free to act on the base of the dam and the structure must be designed for this uplift.

A better arrangement, combining both of these advantages, and indicated by presentation of the analysis below, is shown in Fig. 49*c*. From the flow net diagram and Darcy's law of flow through soils, the approximate uplift pressures and percolation velocities can be computed.

If the number of equipotential divisions N_1 in Fig. 49*c* is 18, and the number of flow channels N_2 bounded by flow lines is 5, then

$$\text{Hydraulic gradient per unit head } i = \frac{N_1}{N_2} \tag{29}$$

For a soil of permeability k, void ratio e, and specific gravity s, the flow under a head H is

$$Q = kH\frac{N_1}{N_2} \tag{30}$$

Examination of the foundation for allowable internal pressure which would limit shear in the soil can be determined:

At a point a (Fig. 49*c*) at a depth D, below the surface, a saturated foundation and flow path L being assumed, the total pressure is calculated as follows:

Let p = total stress per unit area.
p_e = effective stress per unit area.
p_n = neutral stress per unit area.
γ_w = specific gravity of water.

Then
$$p = D\left(\frac{s+e}{1+e}\right)\gamma_w$$
$$p_n = D\gamma_w + \frac{D}{L}H\gamma_w$$

and the effective stress in the soil

$$p_e = p - p_n = D\left(\frac{s+e}{1+e}\right)\gamma_w - D\gamma_w - D\frac{H}{L}\gamma_w$$

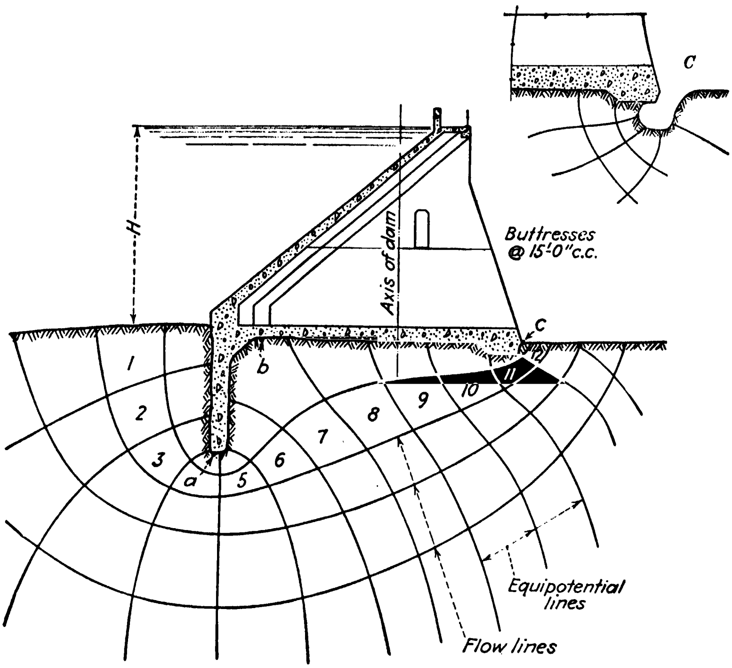
but $1/L \propto i$ and $i = N_1/N_2$, therefore

$$p_e = D\gamma_w\left(\frac{s-1}{1+e} - \frac{N_1}{N_2}H\right) \tag{31}$$

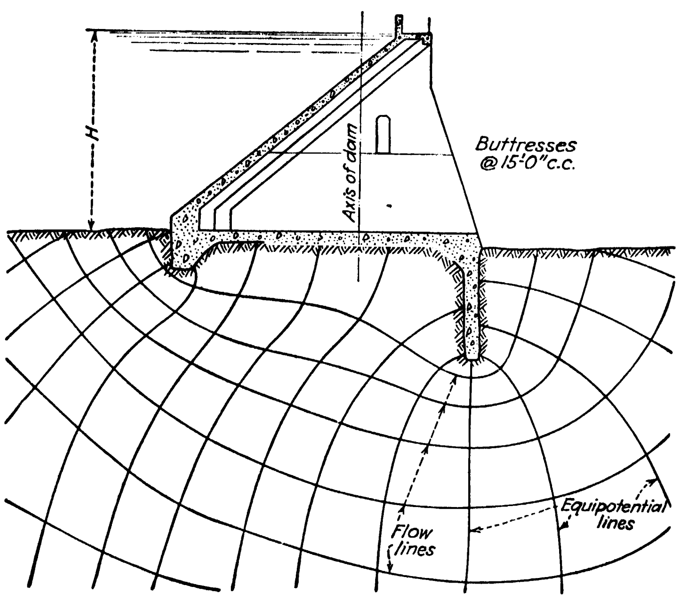
Were the soil structure of such a nature and the reservoir head H great enough to create a large amount of percolation or high seepage pressure, the flow Q would exceed the limits for a safe foundation and the effective pressure p_e would be reduced enough to permit soil distortion and sliding of the foundation material upon itself.

In Fig. 49*d* a graduated trickling filter has been substituted for the downstream cutoff wall. The fine material has been placed first, followed by graded coarser rock of a size to resist the high velocity and increased hydraulic gradient at the downstream extremity of the base.

The foundation can now be investigated for all the limiting conditions described above. As has been previously stated, the weighted creep ratios or Bligh's coefficients may be used as a general criterion for governing the choice of layout or length of foundation creep, but final investigation must determine the limiting factors, for it is

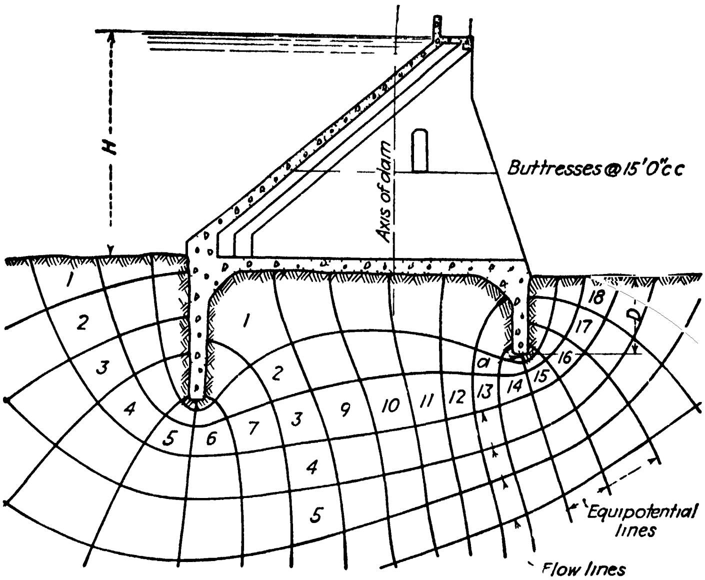


(a)

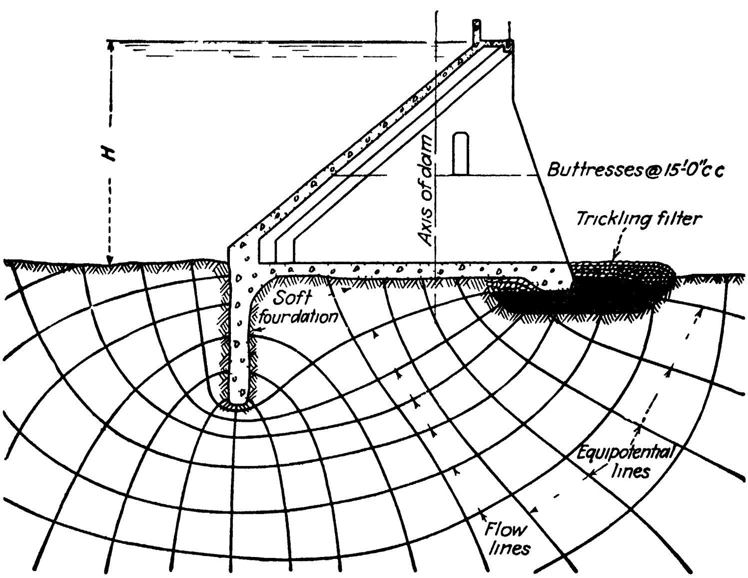


(b)

FIGS. 49 *a*, *b*.—Flow net diagrams.



(c)



(d)

FIGS. 49 c, d. Flow net diagrams

apparent that neither of these methods can reveal the relative merits of location or *arrangement of cutoff wall or other seepage obstructions.*

Hence, it is evident that the values of creep coefficients do not strictly govern unless they are applied with a thorough understanding of the critical engineering limit of their application.

Regardless of the method selected for measuring the length of the line of creep, the practical result must be an adequate cutoff depth and sufficient base length. In applying Darcy's law, as expressed by Eq. (28), the required length of creep may be determined by the direct method of applying coefficients which experience has proven to be amply safe, or by assuming a velocity of flow through the foundation material that would appear to give a margin of safety against piping failure, and then determining the depth of cutoff required to hold to this velocity. The latter method is commonly based on Slichter's formula. Assuming the short-path method, Darcy's law, as modified to include the Slichter formula, gives the following formula, for the depth of the cutoff wall:

$$d = \frac{KH}{2PV} - \frac{b^2PV}{2KH} \tag{32}$$

in which d = depth of cutoff.
K = a transmission constant.
H = head, ft.
P = porosity of the material expressed as a decimal.
V = permissible velocity, fpm.
b = effective base width of the dam.

Comparative applications of these two methods to specific designs invariably indicate that the first, based on observations on the performance of actual structures, results in deeper cutoff walls than those required by Eq. (32). The first method is recommended as being the most conservative.

An important study, undertaken by E. W. Lane,[1] summarizes observations made on 336 dams constructed on soft foundations. In order to determine from the results of practical experience a safe creep ratio for each class of foundation material, Lane plotted length of travel against head for each of about 300 dams. The envelope curve for safe creep ratios was determined from the records for dams that had failed.

Several important facts were revealed by this analysis, Lane concluding "first, several dams have failed from piping with percolation distances which, judged by ordinary standards, should be safe; second, many dams have stood successfully with percolation distances much less than those previously recommended; third, the dams that failed had very little of their creep paths along vertical or steeply sloping surfaces while those that stood, with much smaller distances, had a considerable proportion of such creep."

From an examination of all available data, Lane found that equal values could not be assigned to both horizontal and vertical creep distances. In fact, in many cases it appeared that horizontal creep distance offered no resistance to water travel owing to the separation of the foundation material from the foundation structure due to unequal consolidation or settlement of the foundation material itself, or to settlement between supporting piles. Studies of records on foundation pressures and other related data led Lane to the conclusion that 1 ft of vertical creep is worth at least 3 ft of horizontal creep. Quite arbitrarily, the term vertical creep may be applied to surfaces having slopes steeper than 45 deg and horizontal creep to surfaces having slopes flatter than 45 deg. The resulting equivalent creep distance divided by the head was termed by Lane the *weighted-creep ratio.*

In Table 1 are given the weighted-creep ratios for several classes of foundation

[1] *Trans. A. S. C. E.*, **100**, 1235, 1303–1307, 1935.

materials which Lane's analysis of all available data on existing dams indicated as necessary for safety against failure by piping. These weighted-creep ratios are also compared in the same table with Bligh's values for identical materials.

TABLE 1.—COMPARISON OF WEIGHTED-CREEP RATIOS
(Weight of horizontal creep, one-third)

Material	Safe weighted-creep ratio	Bligh's value
Very fine sand or silt	8.5	18
Fine sand	7.0	15
Medium sand	6.0	
Coarse sand	5.0	12
Fine gravel	4.0	
Medium gravel	3.5	
Gravel and sand	...	9
Coarse gravel, including cobbles	3.0	
Boulders with some cobbles and gravel	2.5	
Boulders, gravel, and sand	...	4–6
Soft clay	3.0	
Medium clay	2.0	
Hard clay	1.8	
Very hard clay, or hardpan	1.6	

The weighted-creep method transfers reliance for resistance to flow from horizontal creep under the foundation to vertical creep created by the cutoff wall. This method is consistent with the practices of the author's company, specializing in the design and construction of dams on difficult foundations. In designing a very large number of dams of this type, the Ambursen Company has placed principal reliance for safety against piping upon deep vertical cutoff walls. As the foundation slabs under most of these structures are drained, little or no dependence was placed upon horizontal creep.

An analysis[1] of the creep ratios actually existing in the case of 186 dams of the Ambursen type built upon soft or porous foundations, many of them having been in successful use for more than 30 years, indicates that as the height of the structure increases, the creep ratio may be decreased. This is shown by Table 2, in which the results of the application of the weighted-creep theory are also given.

In practice, however, each site will usually present special problems which cannot be solved by arbitrary assumptions but which must be made the subject of careful studies that take into consideration the homogeneity and texture of each of the various materials comprising the foundation materials.

In order to attain the ultimate resistance to water travel, sound construction methods are also required. Lack of care or skill in actual construction of soft foundation dams may be even more dangerous to the stability of a dam than errors in designing assumptions.

A concrete cutoff properly built in contact with the sides of the trench is preferable to either steel or timber sheet piling. The latter should be used in no case except where the depth to which it is to be driven is relatively small and the material is susceptible to easy penetration. Wandering, brooming, and splitting is the rule rather than the exception for Wakefield, splined, or tongue-and-groove sheeting.

Although steel sheet piling may be used to advantage at many sites, its uncertainties in hard driving should be recognized as well. The use of jets for sinking diffi-

[1] *Trans. A. S. C. E.*, **100**, 1303–1307, 1935.

cult cutoff piling may well impair its creep-resisting value in many materials through displacement of well-bedded particles and the disturbance of the immediately adjacent material.

Concrete cutoff walls may be just as defective as those of wood or steel sheet piling if proper care in construction is not exercised. Forming and backfilling are not permissible without special precaution to prevent subsequent leakage. If sheeting and bracing are necessary, the withdrawal of the sheeting invites displacement of the foundation material and faulty contact between the concrete and the sides of the trench.

TABLE 2.—ACTUAL AND THEORETICAL CREEP RATIOS FOR 186 DAMS

Foundation material	Number of examples	Height of dam	Mean actual creep ratio	Mean weighted-creep ratio	Lane's creep ratio	Bligh's creep ratio
On very fine sand or silt	8	15′ or less	7.93	5.18		
	8	20′–42′	6.61	4.22		
	Mean for 16 dams		7.01	4.51	8.50	18.00
On medium sand	6	15′ or less	9.24	5.05		
	7	16′–50′	5.38	3.47		
	Mean for 13 dams		6.40	3.89	6.00	
On coarse sand	7	15′ or less	6.85	3.94		
	5	17′–58′	4.34	3.22		
	Mean for 12 dams		5.17	3.45	5.00	12.00
On medium gravel	10	15′ or less	5.95	3.73		
	10	17′–50′	4.73	2.97		
	Mean for 20 dams		5.07	3.18	3.50	
On gravel and sand	13	15′ or less	5.92	4.11		
	8	16′–28′	4.07	2.78		
	Mean for 21 dams		4.89	3.37	....	9.00
On boulders with cobbles and gravel	8	15′ or less	5.95	4.35		
	6	17′–58′	3.69	2.01		
	Mean for 14 dams		4.32	2.67	2.50	
On boulders, gravel and sand	11	15′ or less	6.74	4.68		
	6	20′–50′	5.71	3.89		
	Mean for 17 dams		6.26	4.30	....	4–6
On medium clay	9	15′ or less	8.15	5.43		
	41	16′–250′	4.58	2.83		
	Mean for 50 dams		4.80	2.98	2.00	
On very hard clay and hardpan	6	15′ or less	4.10	2.44		
	17	18′–135′	3.53	2.02		
	Mean for 23 dams		3.60	2.08	1.60	

The assumption regarding creep affects materially the design of the foundation structure and the proportions of the dam. To illustrate, Fig. 50 shows the outlines of a dam constructed on hard clay which has been in successful service for about 15 years. On the assumption that the effective water travel is between points A and B and the conditions of full reservoir and no tail water, a comparison will be made of the weighted-creep ratio, Bligh's ratio, and the accompanying uplift pressures that would result from each.

If it is assumed that the sloping surfaces are equivalent to vertical creep, the total creep distance would be 44 ft and the total horizontal creep distance 42 ft. If 1 ft of

vertical creep is equivalent in resistance to 3 ft of horizontal creep, as stated by Lane, the weighted-creep ratio would be

$$\frac{44 + {}^{42}\!/_{3}}{31} \quad 1.87$$

This ratio is very close to the weighted-creep ratio of 1.8 recommended by Lane for dams constructed on hard clay.

By applying Bligh's method, however, the ratio would be ${}^{86}\!/_{31} = 2.77$, a value substantially lower than that recommended by Bligh as being safe for hard clay. The results obtained in this isolated example agrees with Lane's finding that the principal reliance for resistance to water travel should be placed upon vertical cutoff walls.

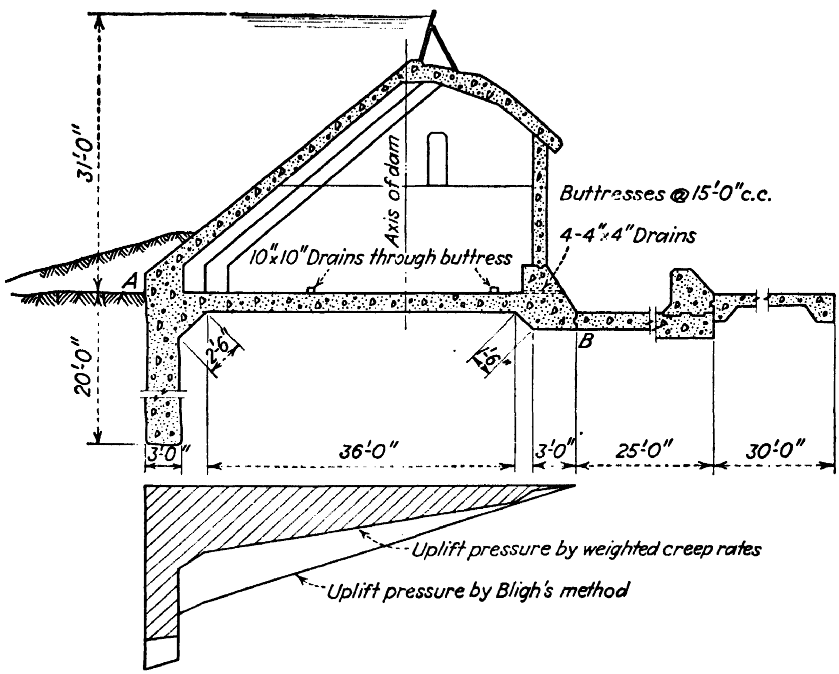


FIG. 50.—Sebasticook River Dam. Section through spillway with line of creep—Lane and Bligh.

The effect of the creep-ratio assumption upon uplift pressure is also shown by Fig. 50. The uplift pressure under the dam would be substantially greater in this case if Bligh's ratio were applied rather than the weighted-creep ratio. A conservative design procedure would be to determine the depth of cutoff wall by using the weighted-creep method and to compute the stability of the dam using Bligh's methods.

Foundation slabs under dams of the type shown by Fig. 50 should be continuous, with extra reinforcement over that required for positive and negative moments in both the top and bottom to provide for the possibility of unequal settlement. A typical foundation detail is shown by Fig. 51. The principal reinforcement A is provided to take the bending and shearing stresses. Where inclined tension steel is required in the buttress, it must be carried down into the foundation slab for proper anchorage. Also, it is preferable to extend the longitudinal steel in this slab and the similar steel of

the deck member into the cutoff wall. However, no credit for increased stability against sliding thus effected should be made in the design.

If the foundation is drained in order to reduce uplift and to lower the sliding factor, continuous gravel-filled trenches should be used. Tile and gravel drains may be used to advantage under larger structures. Isolated gravel pockets are not effective in draining the foundation and in relieving uplift. Naturally, drained slabs cannot be included in figuring horizontal creep.

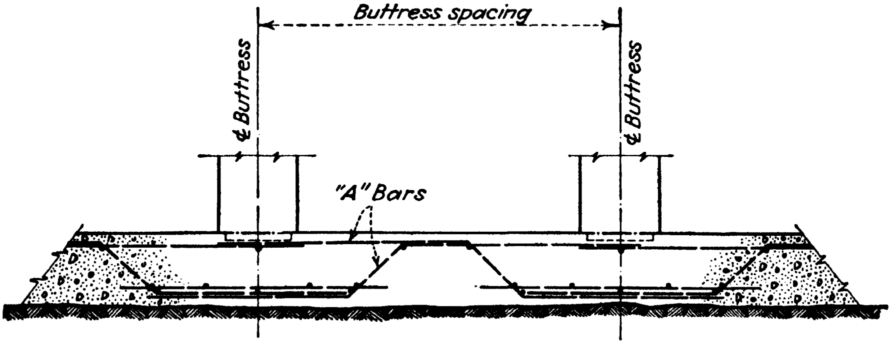


FIG. 51.—Section through typical continuous foundation slab.

Where the controlling conditions necessarily vary so widely between different sites and structures as in the case of dams to be built upon soft or porous foundations, it is impracticable to lay down a rule of practice that will apply even in a general way to all such dams. The following requirements are among those which should be followed:

1. Explore as fully as may be economically feasible the subsurface area beneath and adjacent to the dam.
2. Use a continuous well-reinforced concrete floor beneath any buttress dam for soft foundations, regardless of differences of opinion as to the exact creep value of such construction.
3. Avoid the use of pile foundations or spread footings for dams of any but the lowest heads.
4. Avoid reliance on horizontal creep under long downstream pavements or stilling basins, particularly where articulated.
5. Take advantage of the economical upstream impervious fill to increase seepage distance.
6. Rely principally on a properly constructed upstream masonry cutoff wall for control of underseepage and piping.
7. Install a good downstream cutoff as insurance against removal of foundation material by scour.
8. Where rock or other impervious material may be reached without prohibitive cost, carry a substantial concrete cutoff wall down to it, regardless of theoretical creep limitations.

These principles are among the fundamentals for soft-foundation dam design and construction and are believed worthy of emphasis. There is no structure justifying a higher factor of safety than a dam on soil foundations nor one less susceptible to determination based upon laboratory or theoretical considerations.

2. Dams on Poor Rock or Materials Having Widely Varying Characteristics. No general procedure can be established for the design of dams having such foundations, as each site will in most cases present special problems which must be solved in the light of past experience. The Ambursen type of dam has been shown to be particularly suitable to weak and shattered rock formations because of its fundamentally low and evenly distributed foundation loading. The articulated superstructure readily adapts itself to minor foundation settlements or even to foundation movements of considerable magnitude, without structural failure. The amount of foundation loading is easily controlled in the designs, and the buttress type of dam is especially well adapted to the distribution of pressures by means of spread footings or a continuous floor. In the case of the 190-ft-high Possum Kingdom Dam built on the Brazos River

in Texas,[1] constructed on a pure clay shale without intergranular cementation, foundation loadings were reduced to 5½ tons/sq ft by means of spread footings (Fig. 52). The sliding factor for the spillway section is 0.45, and the spillway is designed to discharge 515,000 cfs.

Many foundations of soft, shattered, cavernous, or badly laminated rock that would not carry a gravity section dam are suitable for the lower loads transmitted to them by the buttress type of dam, or can be improved by conventional methods, including grouting, so as to make them entirely satisfactory for that type of structure. Examples are the 90-ft-high Estacada dam on the Clackamas River in Oregon,[2] constructed upon a very porous and fissured lava or volcanic breccia; the 100-ft-high

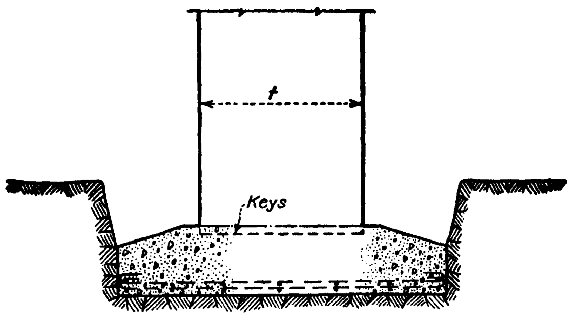


FIG. 52.—Possum Kingdom Dam. Section through footing (shale).

Minas Basin Dam on St. Croix River in Nova Scotia, constructed upon a very soft and badly broken shale; the 65-ft-high Blue Earth Dam in Minnesota, on indurated sand and decayed sandstone; the dams on very cavernous limestone formation at Luray and Stanley on the Shenandoah River in Virginia; the St. Regis Dam on the Black River in New York, built on horizontally laminated and broken limestone; and the Oklahoma City Dam on the North Canadian River, constructed upon horizontally bedded soft shale with clay seams.

The situation encountered at the Rodriguez Dam site[3] presented an unusual combination of difficulties requiring special foundation treatment. In the stream-bed section where the dam attains a maximum height of 250 ft, foundation exploration revealed a faulted area about 100 ft wide. The walls on either side of this fault are of sound rock capable of supporting the foundation loads. Between these walls however, the foundation material varied from soft clay to broken rock and was entirely unsuited for the support of the five high buttresses that would bear entirely or in part on this area.

The final solution for this problem is shown by Figs. 53 and 54. An arch barrel (Fig. 53) capable of carrying the entire masonry and water loads from the five buttresses above to the sound rock at the sides was constructed across the fault. This arch was built over a subintradosal block of mass concrete which was separated from the arch by an oiled joint. No dependence was placed on the subintradosal concrete to support loads from the dam above. It merely served as a filler to provide rigidity and creep travel and as a form for the arch concrete.

The transition between the arch and the cutoff wall consists of a large monolithic block which extends across the faulted section. This block was heavily reinforced in both directions to take both beam and arch actions. In order to increase the rigidity of the entire foundation structure and to provide lateral support for the steep rock

[1] *Eng. News-Record*, p. 71, 1939.
[2] *Eng. News-Record*, p. 1017, 1938. *Trans. A. S. C. E.*, **78**, 447–546, 1915.
[3] *Eng. News-Record*, **105**, 600, 1930.

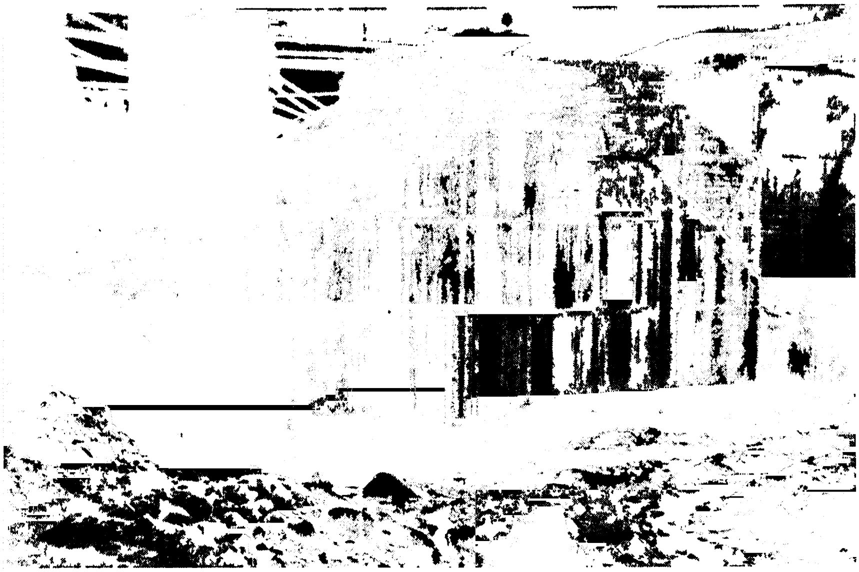


Fig 53 —Rodriguez Dam, Mexico Downstream view of foundation arch and subintradosal concrete fill Ambursen type

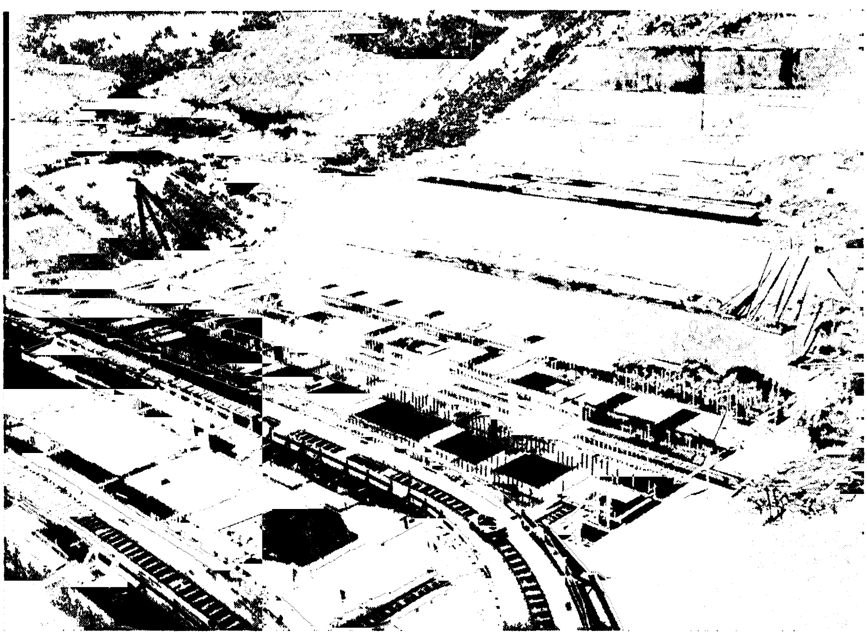


Fig. 54 —Rodriguez Dam, Mexico General view of foundation structure on river-bed arch.

abutments on each side of the fault, 5-ft-thick ribs spaced 20 ft center to center were constructed monolithically with the arch barrel as shown by Fig. 54. These ribs also served to support the floor for the outlet works.

The upstream cutoff wall was constructed to a depth of 300 ft below the stream bed (550 ft below the crest of the dam) by the cellular-wedge method, developed and patented by the Ambursen Company. The procedure consisted of first excavating a trench with sloping sides to a depth that did not require excessive sheeting or bracing. Shafts of ample size to permit several men to work therein were formed in this trench and the spaces between the shafts and trench sides concreted. Before this trench was concreted, a layer of loose materials about 8 in. deep was spread on the bottom for the purposes of facilitating renewed excavation procedure below. Heavy vertical reinforcement in the wall, terminating in hooks that projected into this loose fill, provided a means for anchoring the subsequently constructed lower section of the wall to the initial construction.

After the first section of the wall had been completed, excavation was continued through the shafts and a second cellular wedge-shaped lift was concreted. By this method, the cutoff was constructed downwardly in successive steps from the river bed to a depth of 300 ft below against the full head of the river water. Where deposits of semifluid sand and clay were encountered, sheeting was resorted to. The height of the pours varied from about 4 to 15 ft (Fig. 55) depending entirely upon the stability of the widely varying materials encountered in the fault. The sloping sides of each stage helped, by wedge action, to support the entire wall and to tighten the contact between the masonry and the sides of the trench. Provision was made for adopting compressed-air methods in case serious slides or the inflow of water through the faulted material became too great for convenient handling. However, the work was carried to successful completion over a period of about 15 months, without incident. Each section of the wall was provided with water-stop grooves. Pipes connecting with the construction joints and with the surfaces of the trench served as drains during construction and were later used for grouting the wall, final tightening being done by grouting through to the outside of the concrete wall. Drainage through these pipes and inflow through the foundation material during construction was led to a sump and pumped to the surface.

When the cutoff had reached the designed depth, the shafts were filled with concrete except that small circular shafts were left at two places with provision for attaching in the future steel cylinders at the level of the top of the cutoff wall, so that excavation could be resumed without emptying the reservoir, and the cutoff wall deepened by compressed-air or other methods if after a period of use there appeared to be an undue amount of water escaping beneath the cutoff wall. However, observations carried on over a period of about five years since the reservoir was filled have shown that the leakage under the cutoff wall is considerably less than expected and in fact is so small as to be almost unmeasurable.

The same type of cellular-wedge cutoff wall construction was used at the Ayers Island Dam and powerhouse, Bristol, N. H.[1] The dam was originally constructed in 1923 to develop a head of 50 ft, with provision for subsequent increase to 80 ft. In 1930 when the dam was raised 30 ft, it was necessary to add to the dam and to make the other necessary structural changes without taking the power plant out of service or reducing the pond level. A new cutoff wall had to be constructed in the left bank adjacent to the powerhouse section. This work was done by the cellular-wedge method, the shafts being sunk to a maximum depth of 110 ft through sand, gravel, and clay (Fig. 56) and through a heavy deposit of loose boulders and gravel overlying the bedrock, without the use of compressed air and at very low cost. As the Ayers

[1] *Eng. News-Record*, **111**, No. 7, 185, 1933.

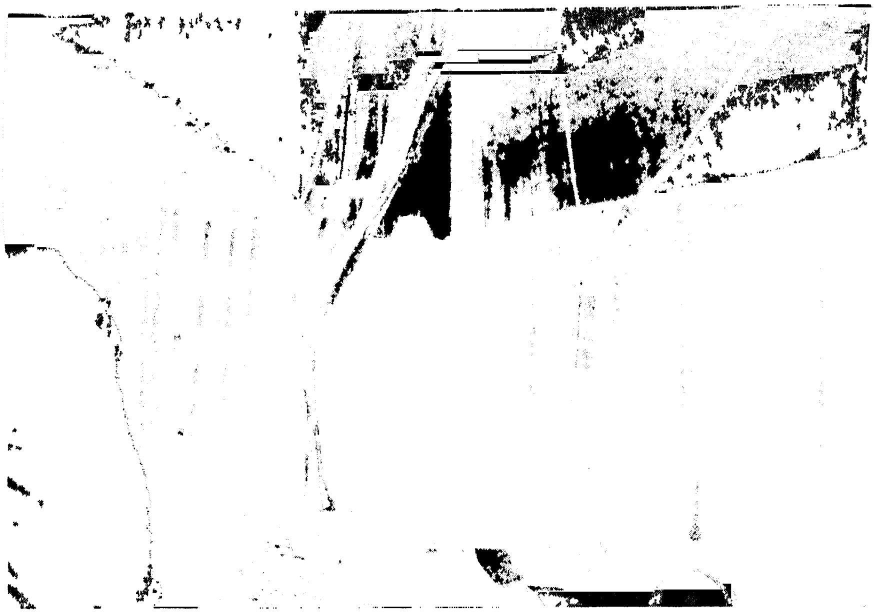


FIG. 55.—Rodriguez Dam, Mexico A section of cellular-wedge core wall at a depth of 265 ft below river bed, ready for forms and concrete.

FIG. 56.—Ayers Island Dam, New Hampshire. A section of cellular-wedge core wall in clay at a depth of 90 ft below pond level.

Island cutoff wall was bottomed out on good rock, the shafts were merely backfilled with earth instead of being filled with concrete.

For dams built upon defective rock formations, just as in the case of dams constructed upon soft foundations, every precaution must be taken to limit underseepage, with principal reliance upon a properly designed and constructed upstream cutoff wall. Complete detailed foundation investigations must be made by the usual methods, and a conservative sliding factor must be adopted. The general design must be adapted to the special conditions at the site, with particular attention to the requirement for suitable flexibility. Adequate stream-bed protection must be provided, and usually some provision should be made to avoid further deterioration of the foundation materials adjacent to the dam.

3. Structural Requirements. The proper design for a dam on soft or porous foundations, even though adequate to prevent failure by piping or excessive leakage through the foundation material, as treated in the preceding pages, must also (1) provide suitably restricted foundation pressures; (2) provide adequate sliding resistance; and (3) still maintain a certain amount of flexibility without loss of structural strength.

A low foundation pressure is easily obtained by the construction of the buttresses upon spread footings, or in special cases upon reinforced concrete slabs or mats which distribute the loading over a substantial base area. The extent of these base slabs can be adjusted to obtain almost any desired foundation pressure. However, experience shows that in most cases the length of the buttress base that will give sufficient sliding resistance will also be enough to produce sufficiently low foundation pressures. Moreover, it is obvious that typical buttress and deck-slab construction necessarily creates a low dead load which aids materially in maintaining a low foundation pressure. Pressures as low as 3 tons/sq ft can be readily obtained, and pressures even as low as 1 ton/sq ft can be economically procured for the occasional dam site where material of very low bearing value is encountered.

An ideal stress distribution condition is found in the buttress dam because the predetermined upstream and downstream slope proportions will produce approximately uniform foundation pressures and uniform distribution of the first principal or maximum inclined stress in the buttress. A uniform foundation pressure distribution is of particular importance, for it maintains the safety of the structure by minimizing unequal settlement on poor foundations. It is of further importance that this selected proportioning for the upstream and downstream slopes produces a high sliding-resistance factor which is uniform for the full length of the foundation base.

With the floor type of Ambursen design for soft foundations, adequate sliding resistance is secured by making the slope of the upstream face of the structure somewhat flatter than is ordinarily used for a dam on rock foundations. This procedure increases the vertical water load on the dam which, together with the dead-load weight of the floor itself, produces a sliding resistance suitable for materials having quite low bearing properties. For hard sand, gravel, and boulders this factor generally varies from 0.4 to 0.5; for extremely poor foundation conditions, it can be readily made as low as 0.35 to 0.3, and even lower, by varying the slope of the upstream and downstream faces. Therefore the value of this factor is easily controlled by the designer.

It is important that for a concrete dam to be constructed on yielding foundations an appreciable amount of flexibility exists, without sacrifice of structural strength. Provision must be made against excessive internal stresses due to foundation settlement. Inherently, because of its form of construction, and because of the customary contraction and expansion joints both in the buttresses and in the water-supporting members, the articulated buttress-type dam adapts itself admirably to these requirements. Lateral braces or ties in the form of beams or diaphragms, or both, can be

arranged to provide a series of two-buttress bents or towers which provide much reserve strength and yet permit a reasonable amount of settlement without injury to the structure.

In the majority of cases where Ambursen dams have been constructed upon soft or porous foundations, they are used for spillway purposes. For the nonoverflow or bulkhead sections of dams on such foundations, the earth-fill, rock-fill, or combination types of dam are often, although by no means always, lower in first cost, but they cannot safely be used for flood discharge. Ample spillway capacity must always be provided, and carefully designed construction must be installed below the dam in the form of a hearth, paving, stilling pool, sill, or other energy-dissipating and scour-preventing arrangement. The velocity of the water before its return to the river bed below the dam must be reduced to very conservative limits to avoid undermining the foundations, and in all cases a suitable downstream cutoff wall should be provided.

Well-constructed cutoff walls or core walls must be provided at each extremity of the dam, extending deep enough and far enough into the river banks to ensure against dangerous underflow around the abutments of the dam or erosion of the banks. These precautions are just as necessary as the provision of an adequate upstream cutoff beneath the structure.

SECTION 5

EARTH DAMS[1]

BY THEODORE T. KNAPPEN AND JOHN LOWE, 3D

Earth dams, dikes, and levees are the commonest structures used to impound water, and innumerable instances of their use exist in all parts of the world. The entire range in soils, from clays to boulders or quarried stone, has been used in their construction.

The construction of a reservoir usually requires one or more dams or dikes, and a spillway and outlet works. The choice between earth structures and masonry or other types of impounding structures depends upon several factors: foundation conditions, availability of borrow material, aggregate, etc. Earth dams usually provide the most economical and most satisfactory solution on soil foundations and frequently on rock foundations. Similarly, except where there are space limitations, earth dikes and levees are usually the most economical and satisfactory for local flood protection works.

Natural soils vary to such a great degree from site to site that field and laboratory investigations are necessary for their identification and for determination of their engineering properties. Because of the great variability of soils, experience, ingenuity, and thoroughness are prime qualifications for an earth-dam designer. By using past experience and applying the latest developments in soil mechanics, a sound, well-balanced design of an earth dam can be made for almost every site.

This section on earth dams has for its purpose the presentation of (1) the basic considerations that enter into earth-dam design, (2) data for formulation of at least a preliminary design, and (3) a list of references for detailed discussion of design procedures and features. Although the term dam is used throughout the text, the discussions generally apply to levees as well.

I. GENERAL

1. Given Conditions for Dam. The general location of a dam, its height, the water conditions under which it must be stable, and the maximum leakage loss permitted are determined primarily by river planning and over-all economic considerations. These considerations are discussed in Sec. 1. Two others taken into account in deciding upon the location of a dam are the topography of the valley and the general geologic conditions of the area. Several different types of dam may be suitable, or an earth dam may be the only feasible one. In any case, an economic comparison of different types of dam and of different types of earth dam is made before final selection is made.

The total height of a dam is determined as the height of the spillway crest plus the surcharge height of the maximum spillway design discharge plus a freeboard height depending on wave action. The water conditions under which a dam must

[1] The authors wish to acknowledge the assistance of S. S. Cooke-Yarborough, E. P. Sorensen, E. J. Zegarra, and other engineers of the Knappen Tippetts Abbett Engineering Co. in connection with the preparation of this section. Data obtained from other sources are acknowledged in the text.

be stable are determined by operating conditions and include maximum design water level and rates of filling and drawdown of reservoir. The amount of leakage to be permitted through the dam and its foundation depends upon the purpose of the reservoir formed by the dam, *i.e.*, whether the reservoir is for flood control or for conservation storage.

The problem of the design of an earth dam may be expressed as follows:

Given: (*a*) Certain reservoir operating conditions and permissible leakage.
(*b*) Certain foundation conditions.
(*c*) Certain available borrow materials.

Problem: Design a dam section and foundation treatment to produce a stable and sufficiently watertight structure.

Usually the solution to the problem is made by trial and error. One or more types of earth dam and foundation treatment are chosen and investigated for stability and watertightness. Several trials are needed to find the slopes and dimensions for which the analysis will give the required safety factors. Cost comparisons can then be made for an economic study of the types investigated.

2. Spillway and Outlet Considerations. Since overtopping of an earth dam usually results in failure and may be extremely disastrous, adequate spillway capacity is essential where an earth dam is involved. The most desirable type of spillway for such a dam is one located some distance from the structure, as through a separate saddle in the reservoir rim. The practice of putting the spillway channel over the embankment has frequently resulted in failure and should be avoided where possible. The reader is referred to Sec. 7 for spillway designs, many of which should be studied before deciding on the proper one for a particular site.

There have been numerous failures of earth dams traceable to improperly designed outlets through the embankment. As a consequence, in conservative design the outlets are, where possible, tunneled through the abutments as far from the structure as practicable or located in separate concrete structures founded on rock. The next best solution usually is to locate the outlet conduits in open cut on rock in the abutments or foundations. Conduits located on earthen foundations under the embankments are subject to high stresses resulting from embankment load and to differential settlement resulting from foundation consolidation. Although such structures can be adequately designed, thorough foundation investigations and tests are required, and the cost of a properly designed structure may be comparatively great. Generally this solution should be avoided.

Wherever a conduit passes through or under an earth dam, gates should be provided at the intake to give a positive closure on the upstream end, collars or seep rings around the conduit to interrupt the path of flow between the masonry and the earth, and, where the conduit emerges from the embankment on the downstream side, inverse filters consisting of successive layers of graded sand, gravel, and rock, parallel to the downstream slope.

It is not good practice to carry a pipe or conduit subject to reservoir pressure through an embankment because of the danger of failure of the dam by piping in case of a break in the line. Gates at the outlet end should, therefore, be avoided unless the pipes carrying pressure are placed inside other conduits.

Neither outlets nor spillways should discharge at high velocity near the toes of earth dams without adequate stilling basins and other protective works because of the danger of destruction of the embankment from scour. On important structures, economy in design of such outlets and spillways usually will result from the use of model studies in well-equipped hydraulic laboratories. Figure 1 is a view of the

stilling basin at the outlet of the tunnels of the Mohawk Dam, Ohio, showing the hydraulic jump at a discharge of 25,000 cfs. The design of this structure was checked in a hydraulic laboratory.

3. Site and Laboratory Investigations. The program of field and laboratory investigations is normally carried out in three stages and in conjunction with three cycles of design. The first stage consists in performing the investigations necessary for determining the specific location for the dam, spillway, and outlet structures from among the several alternatives which usually are present within the general area prescribed for the dam. A topographic survey to a scale of 100 to 200 ft to the

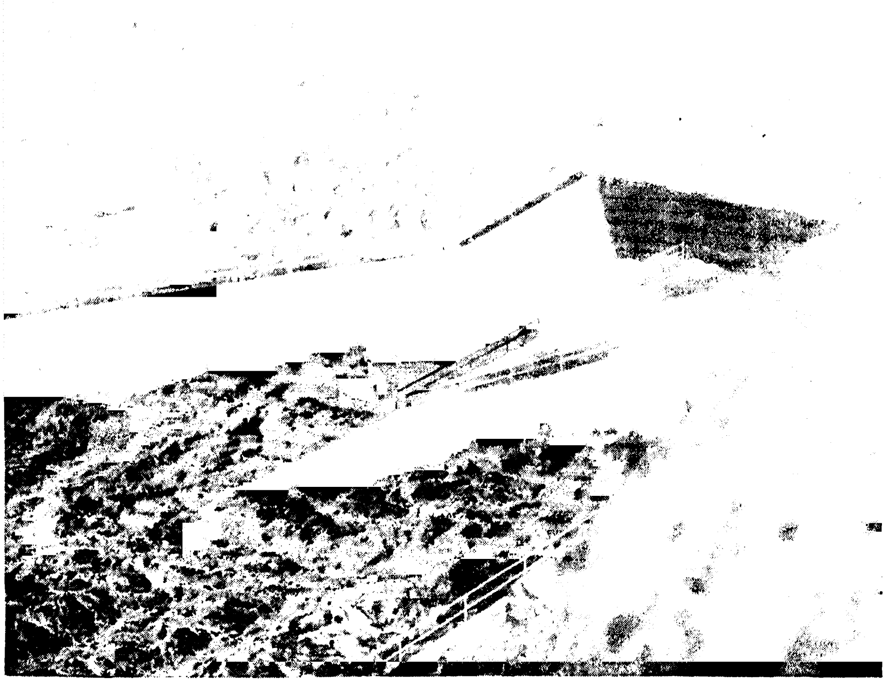


Fig. 1.—Stilling basin, Mohawk Dam.

inch of the particular area or areas involved is convenient in this stage as a basis for geological and subsurface investigations and for studies and preliminary plans. To ascertain the foundation conditions and the availability of embankment materials at the alternate sites, a geological reconnaissance, a limited program of borings and/or other types of subsurface explorations is required. Samples of foundation and borrow soils are classified visually and by simple laboratory tests. Engineering properties of the soils such as permeability, shear strength, and compressibility usually can be estimated from these classifications with sufficient accuracy for use in preparing a rough design. Choice of site is made from quantity or cost estimates based on the rough design. At times, certain foundation or construction conditions will materially affect the cost estimates and thereby eliminate some of the otherwise possible alternate sites from consideration.

In the second stage, additional field and laboratory investigations are made at the selected site to enable determination of the essential features and dimensions of the

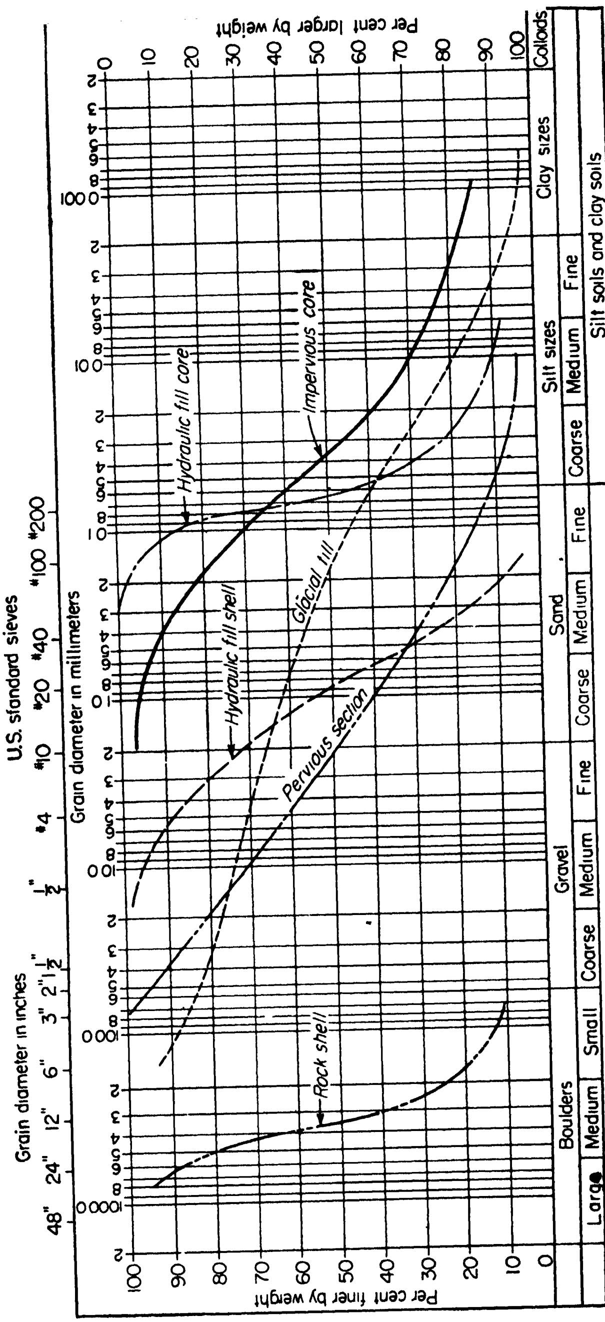


FIG 2 —Grain-size distribution curves of typical embankment materials.

dam and related structures. The above-mentioned topographic survey is adequate for this stage. Additional geologic studies, borings, test pits, and other subsurface investigations are made to disclose further the character of the foundations and to enable drawing of geologic profiles and sections close to all structure. Representative foundation and borrow samples are tested to determine engineering properties of the soil. On the basis of these investigations, a close design can be made.

In the third stage, final design is made. A topographic map to a larger scale and ground profiles and sections may be necessary. Additional borings and test pits are put down at the exact location of structures to fill the gaps in information disclosed by the second-stage design. Additional field tests and special investigations, such as the construction of a test embankment, may be necessary. Supplementary laboratory tests may be performed to expand and confirm the data collected in the second stage.

The three stages need not be carried out separately. Stages 1 and 2 usually are merged, and the amount of investigation done in the third stage sometimes may be very small. However, in order that the field and laboratory investigations supply all the essential data at minimum cost, it is important to follow through the three stages of investigations and cycles of design. Each of the first two stages discloses what information is essential and which areas are important for investigation in the following stage.

A description of methods of subsurface investigation is given in Art. VIII, and a description of laboratory investigations in Art. IX.

4. Choice of Section. In planning an earth dam, the yield of materials excavated from spillway and outlet structures should be utilized where possible in the embankment or other works if economy is to be obtained. Usually there will be little if any difference in cost between such excavations and borrow-pit excavations; thus, it follows that, within the limits of embankment requirements, the designer may plan structures requiring large excavations without materially affecting the cost of the work. Also by varying the width and depth of such excavations some control over the amounts of different types of material can be obtained. The effect of the relative amounts of different types of material on the choice of dam section is discussed in Art. III.

For general information, typical grain-size analyses of materials used in various parts of earth dams are illustrated in Fig. 2.

In evolving a dam section, consideration of the relative permeability of the various available soils is an important item. The chart illustrated in Fig. 8 gives the relationship between soil classification and permeability and is useful in this connection. A permeability of 1 ft per day is commonly used as a dividing line between pervious and impervious soils.

The analysis of existing earth dams in the vicinity of the project, or of earth dams constructed of similar embankment materials on similar foundations, is of assistance in choosing preliminary sections. Several typical designs of earth dams are presented in Art. XI. Additional descriptions of existing dams may be found in Refs. 15 and 16.[1] A general discussion of various types of embankments and foundation treatments is presented in Arts. II and III.

For the case of an embankment composed of cohesionless material on a foundation which is rock or which is much stronger than the embankment material, the accompanying table of slopes may be used as a rough guide in assuming an embankment section.

[1] See end of this section.

Embankment material	Approximate side slope	Qualifying conditions
		Downstream face
Rock fill	1 on 1.5+	
Cohesionless soil	1 on 2.5	Slope protection, grass
	1 on 2.0+	Slope protection, dumped rock
		Upstream face
Rock fill	1 on 1.5+	Center core
	1 on 2.5+	Inclined earth core
Cohesionless soil	1 on 2.5+	Free draining on drawdown
	1 on 2.5+	Seepage vertically downward on drawdown
	1 on 2.5+	Well-graded material where height of capillary rise ≧ height of embankment
	1 on 3.5+	Seepage out of slope on drawdown

Where an embankment is composed of cohesive material, the average side slope depends upon the height of the embankment. The stability number method described in Art. VI, 6, may be used to determine the approximate average allowable slope. Generally in the case of cohesive material it is economical to make the face of the embankment concave, *i.e.*, with steep slopes near the top of the dam and flat slopes near the toe.

Where the foundation material is critical, an embankment with concave faces is generally used irrespective of whether the embankment material is cohesive or cohesionless. An embankment with concave faces produces less severe shearing stresses in the foundation than an embankment of the same height with straight faces.

5. Design Investigations. The following investigations or analyses should be made in connection with the design of an earth dam:

1. Hydraulic analysis to determine the quantity of seepage through the embankment and its foundations and the magnitude and direction of seepage forces.
2. Structural stability analysis of the embankment.
3. Structural stability analysis of the foundation, or of the foundation and embankment in combination.
4. Investigation of slope protection and freeboard requirements.

II. FOUNDATION TYPES

1. General. The choice and design of an embankment section depend to a considerable extent on the foundation conditions at the site. Whereas the properties and distribution of materials composing the embankment are generally known within reasonable limits, the properties and distribution of foundation materials are likely to be erratic and somewhat uncertain. This is especially the case because subsurface conditions in valleys are generally the result of numerous geologic processes involving at least a sequence of erosions and depositions. Stratification of the subsurface soils or the presence of isolated pockets or lenses of material of different properties often adds to the complexity of foundation analysis of dams to be built on soil deposits. Rock foundations may have similar complexities. The success or failure of a dam frequently depends upon the discovery and correct handling of the peculiarities of the foundation. Consequently the following simplified description of typical foundation types should be studied with the understanding that in most cases the foundation is

not homogeneous but consists of various materials, all of which must be analyzed separately as well as in combination.

2. Rock. In the case of a rock foundation, a careful examination must be made to locate open faults, seams, fissures, or caverns through which leakage might endanger the structure or in part defeat its purpose. The presence of these leakage paths may be determined by water-pressure testing of the bore holes.

Sections where any considerable leakage is encountered should be sealed by grouting under pressure with water and cement, or cement and some admixture such as sand, beet pulp, or sawdust, or with clay or bentonite. Each grouting project should be carefully studied before deciding upon the depth of grout holes, the type of grout and pressure to be used, and the setup of the plant. Ordinarily the pressure used should not appreciably exceed the overburden load at the elevation being grouted, particularly on horizontally stratified rocks. A procedure which has been used successfully consists of grouting a row of holes put down at moderate spacing, then putting down intermediate grout holes and testing them with water pressure to ascertain the effectiveness of the first series of holes. This procedure can be repeated until pressure testing indicates that the grouting is fully effective.

Occasionally two or more rows of grout holes are used to increase the size of the grout curtain. Where the topmost rock is difficult to grout, excavation of a trench into it and backfilling the trench with concrete provide a solution. The concrete filling the trench can act as a grout cap. When such treatment is not required, it is usually expedient to place 10 to 20 ft of embankment and to drive the grout pipes through it to the rock, or to use a key wall and grout through it.

In former years it was common practice to construct a concrete key wall to prevent development of seepage at the interface between the earth embankment and the rock foundation and at times to increase the resistance to sliding of the embankment on the foundation. Recent opinion is that where a reasonable area of contact exists between earth and sound rock, key walls are not necessary. However, where an upper zone of fractured rock is cut off with a grout curtain or with a trench filled with concrete, a short path for seepage may exist at the top of the cutoff, and in this case it is desirable to extend the cutoff into the embankment by means of a concrete wall. Detailed discussions of grouting techniques may be found in Refs. 11 and 13.

An alternate to a grout curtain or concrete cutoff trench to sound rock is a wide trench backfilled with impervious embankment material.

3. Pervious. Pervious foundations generally consist of sand and gravel strata which may be relatively shallow or tens of feet deep. Generally no serious structural problem is involved in the design of stable dams on them, provided of course that no especially compressible layer is encountered underneath or that the sand is not unusually loose. The loss of water through the foundation may become a problem, in which case one or several of the following measures may be taken to reduce this loss to a reasonable amount: A clay blanket, 5 to 10 ft thick, may be installed upstream in order to lengthen the path of seepage. When the depth of permeable material is not too great, a cutoff may be constructed by using curtain walls of concrete or impervious soil, or by using sheet piles. Sheet pile cutoffs are effective only in reducing leakage in highly pervious soils, and even in these soils they may not work if boulders are present to damage the interlock. Grouting of the foundation with cement, clay, bituminous material, or bentonite may also come into consideration; and there is the possibility of applying chemical grouting to the foundation. Such treatments have not been used extensively however. Economic and technical considerations will determine the choice for each particular case. Further discussion of blankets and cutoffs is given in Art. V, 7.

In connection with seepage losses, attention is called to the possibility of seepage

at the junction of the dam with the abutments. Investigations should always be made at these points, and precautions taken to prevent excessive loss of water.

4. Impervious over Rock. Impervious foundations generally consist of fine-grained soils, such as silts and clays or well-graded till. This type of foundation presents little difficulty from the standpoint of seepage, but where silt or clay is present it may be structurally unsafe and should be carefully investigated for stability. Methods of stability analysis involving composite surfaces of sliding have been developed for this case and are described in Art. VI.

5. Impervious over Pervious. This type may present the same foundation difficulties as the impervious over-rock type described above. In addition there is the danger of boils or of piping at the downstream side of the dam whenever the thickness of the impervious stratum becomes insufficient to resist the hydrostatic uplift which develops at its base. Very little head is lost in flow through the pervious stratum so that practically the full hydrostatic head of the free-water level at the upstream side of the dam develops at the base of the impervious layer. To relieve this pressure, a relief trench or a series of relief wells may be installed to penetrate down into the pervious stratum. The relief measures may be located at the downstream toe of the dam and empty into a rock toe, or if the dam has a drainage blanket, they may be located underneath the dam. Relief trenches are usually more economical to construct where the depth required is less than about 10 ft. For greater depths, drainage wells are more economical. The design of relief drains is discussed in Art. V, 6. In the case of dipping strata, the previous stratum may outcrop at places downstream, in which case concentrated flow may cause flotation of soil and piping. To prevent this, relief drains should be installed at such places.

6. Stratified. Where a foundation consists of alternate thin layers of fine sand and silt and clay, excessive pore pressure may develop in the more pervious material due to consolidation of the clay or silt under the weight of the embankment. Discussion of this type of foundation may be found in Ref. 8, particularly pages 1416–1421.

III. EMBANKMENT TYPES

1. General. The type of dam as well as the method of its construction depends primarily on the nature of the materials available from excavation and borrow as well as on the nature of the foundation. Sufficient impervious material should be placed in the dam to reduce the seepage to the permissible limit for the given case, and sufficient pervious material should be included to afford stability with economical side slopes. Hydraulic analysis (Art. V) can be made for the flow of water through a dam, and the uppermost line of seepage can be determined. To protect the downstream slope against sloughing, the line of seepage should be kept well within the dam. Methods of analysis are also available for determining the destabilizing effect of water seeping out of the upstream portion of the embankment during drawdown of the reservoir. Features should be incorporated in the dam to minimize this effect.

In the three following paragraphs, different types of earth-dam sections for various proportions of impervious and pervious materials are discussed starting with the case where only impervious material is available and ending with the case where only pervious material is available.

2. Only Impervious Material Available at Site. It sometimes happens that practically all the material available at a site is relatively impervious; in levee construction, uniform material is the rule rather than the exception. Pervious material has to be imported or processed at the site, and for economy a minimum amount of such material should be used in the dam.

For the case of a dam constructed of a single material, seepage of water out of the downstream slope would normally occur, and to prevent this condition, rock-fill toes

or drainage blankets are installed. Breakout of the line of seepage on the downstream face of a dam is illustrated in Fig. 3. Rock toes are usually located inside the dam section and may extend into the foundation where required for drainage as shown in Fig. 4. Where pervious material is not available at the time of construction of the lower portion of the embankment, or where an old dam or levee is to be stabilized,

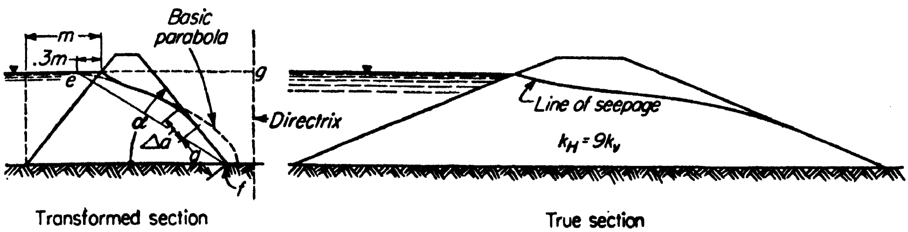


FIG. 3.—Position of line of seepage in uniform embankment on impervious foundation.

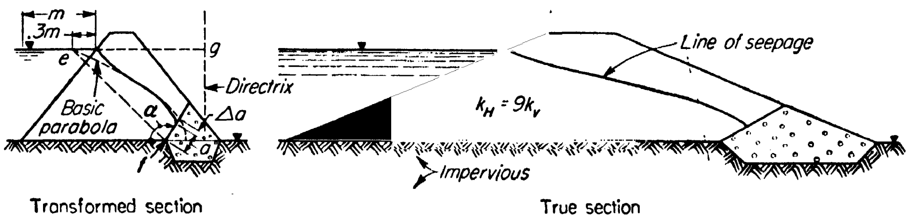


FIG. 4.—Position of line of seepage in uniform embankment with rock-fill toe.

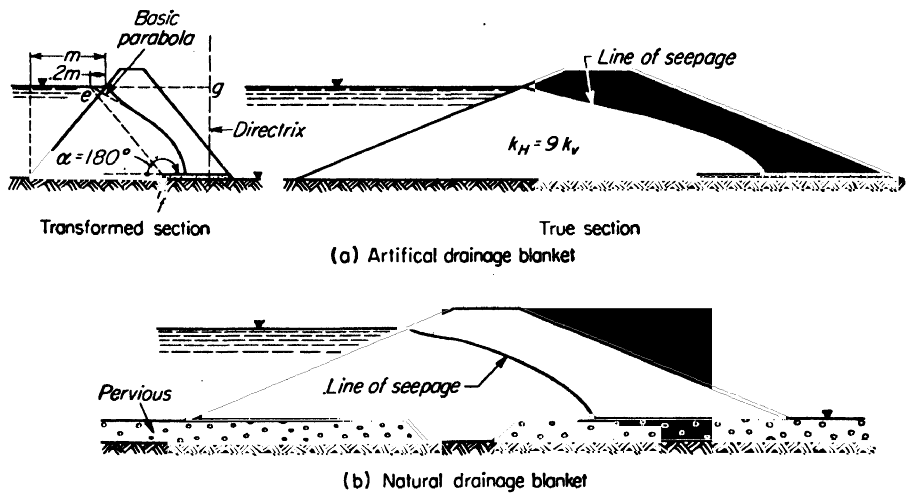


FIG. 5.—Position of line of seepage in uniform embankment with drainage blanket.

a section of pervious material is sometimes placed on top of the downstream face of the impervious material. A filter should be located between the rock-fill toe and adjoining earth embankment to prevent migration of fines from the embankment. A similar filter should be placed between the rock-fill toe and the foundation where required. Design of such filters is discussed in Art. V, 4. Where a high degree of stratification of the embankment may occur, the rock-fill toe has an advantage over the drainage blanket described below in that it intercepts more layers of the embankment. Drainage blankets, Figs. 5*a* and *b*, are usually several feet thick and extend

from 25 to 100 per cent of the distance from the downstream face to the center line of the dam. Where an impervious embankment is placed on a foundation with an upper pervious layer, a natural drainage blanket may be available. In many instances, a drainage blanket will keep the line of saturation farther away from the downstream face than a rock toe and thereby permit design of a section with slightly steeper or more stable downstream slope. Because drainage blankets extend farther into the embankment than rock toes, a shorter seepage path is available to water percolating through the embankment, and somewhat greater leakage results. However, for impervious embankment materials, the amount of leakage in either case is usually tolerable. A drainage blanket also has the advantage that it provides a means of drainage for the foundation. Where the foundation is heterogeneous, collection of underseepage over the large area of the blanket is important. Where consolidation of the foundation will occur owing to the embankment load, the drainage blanket will shorten the path of water being squeezed out of the foundation and thus accelerate consolidation. To minimize the amount of pervious material used, the drainage blanket may be constructed as a strip parallel to the axis of the dam plus a series of drain pipes or drainage strips to remove the water. Determination of the top seepage line for design of rock toes and drainage blankets is described in Art. V, 5.

Occasionally in flood-control dams and levees seepage may not reach the downstream face because water will not be against the upstream face long enough to develop the steady seepage case. Theoretically no rock toe or drainage blanket is required for such structures, but usually nominal drainage measures are installed. For levees where seepage at the downstream face will occur only occasionally and for short periods, no drainage measures need be used, provided the downstream face is flat enough so that the exit gradient of the seeping water is small, say, 0.2.

The upstream portion of a dam composed of impervious material as silt and clay will have to be quite flat in order to be stable upon drawdown. If the foundation is pervious or if a drainage blanket is installed underneath the upstream portion of the dam, seepage forces during drawdown will be vertically downward and a steeper slope will be possible. Figure 5*b* illustrates this type of foundation condition. If both upstream and downstream drainage blankets are used, sufficient impervious material must be located between the upstream and the downstream blankets so that leakage will not be excessive. When the upstream portion of a dam is composed of well-graded impervious material such as certain glacial tills which are relatively incompressible and have a height of capillary rise approximately equal to the height of the embankment, the stability of such an embankment at the end of drawdown may be greater than before drawdown. An upstream drainage blanket would be of no advantage for such soils (Ref. 29, page 249).

3. Nonuniform Materials. Where materials of various permeabilities are available, a better opportunity is presented to obtain an economical design. Usually the impervious material is placed in a central core, but dam sections wherein the entire upstream portion is impervious material, or where an inclined impervious core is placed upstream of the center line, also are advantageous under certain conditions. Either pervious soil or rock fill is suitable for pervious shells or zones. For the downstream shell, the material should be at least 10 times as pervious as the impervious material, preferably more so. For the upstream shell, the material should preferably be so pervious that water will drain out of the shell as fast as drawdown occurs. More economic design usually results if, in addition to the zones for impervious and pervious materials, zones for random fill are provided. Such random zones may be composed of either more or less uniform material intermediate in permeability between that of the pervious and the impervious zones, or it may be composed of various unselected materials which will produce a high degree of stratification. Where the amount of

pervious material is sufficient for only one shell section, usually a downstream shell is constructed and the entire upstream portion of the dam is made of impervious material. Santiago Dam, shown in Fig. 43 is of this type. Such a section is advantageous for certain types of storage dams where leakage must be kept to a minimum and where rapid drawdown of the reservoir will not occur. If design must be made for the rapid drawdown condition, the measures described above for the impervious upstream section of the dam of uniform material apply.

Typical sections of the central impervious core type of dam are shown in Fig. 6. The relative size of core and shells depends upon the relative amounts of impervious

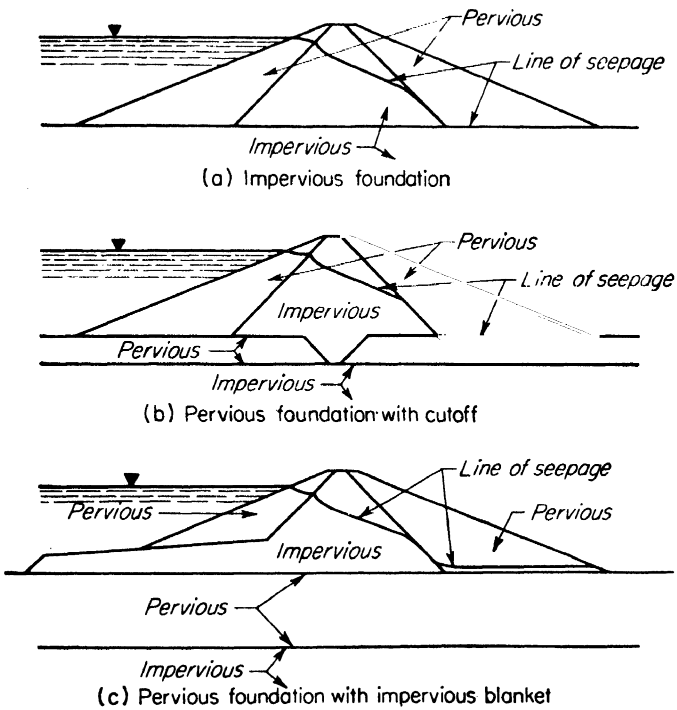


FIG. 6.—Core-type embankments.

and pervious materials available. A minimum width of 8 ft is required for operation of compaction equipment, and the core must have sufficient width for the watertightness required. Mud Mountain Dam shown in Fig. 46 and Watauga Dam shown in Fig. 47 are of the central core type. Where the section rests on pervious foundations, either a cutoff to impervious material (Fig. 6*b*) or an upstream blanket of impervious material connecting to the core (Fig. 6*c*) should be used. The cutoff may be located centrally as shown in Fig. 6*b*, or it may be placed under the upstream portion of the core. By placing the cutoff upstream, seepage conditions are less severe in connection with stability of the downstream slope. Zones of random fill may be placed between the core and the shells, as shown on cross section of Mohawk Dam (Fig. 45). Where the foundation is strong and the embankment material is critical in determining the slopes, the central core type of section usually results in a section of minimum yardage. Central core types of dams have been constructed on rock foundations with rock shells with slopes as steep as the angle of repose of the rock (1 on 1.2 to 1 on 1.4). For such

slopes to satisfy the criteria for the factor of safety given in Art. VI, 2, the angle of internal friction has to be appreciably greater than the angle of repose.

In recent years, a composite dam section has been developed with impervious material in a core inclined upstream. Nantahala Dam shown in Fig. 48 is believed to be the first one built with this type of section. It was built under the direction of J. P. Growdon, chief hydraulic engineer of the Aluminum Company of America. Such a section permits placement of the downstream pervious zone before placement of the impervious material. The impervious material is placed on the upstream slope of the downstream section with proper filter protection between it and the pervious material. A thin shell of pervious material is placed on top of the impervious zone with a proper bedding layer. Where the strength of the foundation is critical in determining the slopes of the dam, this type of section may be the one of minimum yardage since pore pressures in the foundation and the obliquity of the resultant force on the foundation are less than in a central core type of dam. Because of the thin upstream pervious shell, drawndown conditions are usually critical in connection with the design of the upstream slope. Where pervious soil is used instead of rock fill in the downstream section, considerable stratification can be permitted because of the large size of the section. Alternately, a large portion of the pervious section can be used for a random zone.

4. Special Watertight Cores and Facings. Where no sufficiently impervious soil is available, an impervious membrane of concrete, wood, or steel may be considered. When such material is used in connection with earth dams, careful consideration should be given to uneven movement within the embankment due to consolidation and seepage pressure. Cracking of the membrane would lead to a dangerous concentration of flow in the area of the crack, followed by excessive seepage forces and piping.

For the above reason, core walls should be carefully articulated to allow for settlement; also the embankment should be brought up evenly on both sides to prevent unequal loading of the wall. Empirically, the minimum thickness of a reinforced-concrete wall should be about 18 in. at the top with a batter of about 1 to 30 on each face, whereas an unreinforced concrete core wall should be 3 to 5 ft at the top with a batter of about 1 to 20 on each face.

An analysis of the stress condition in a core wall is theoretically possible. On the assumption that the embankment on either side of a core wall is constructed uniformly, the forces acting on the wall should be balanced and the wall should be subject to little stress. However, when the water load comes on one side of it, a new distribution of forces comes into effect which tends to push the wall downstream until sufficient pressure is developed in the downstream fill to balance these forces less the load taken by the wall itself. This movement may be sufficient to cause the usual type of concrete wall to be overstressed and cracked near its base and at other places, particularly if the fill is not placed uniformly. The repeated effect of reservoir filling and emptying may destroy the concrete near the base and render the wall ineffective. The ideal membrane would be a flexible one and might be produced by building a wall from stone, dense brick, or concrete block laid in asphalt, and having a vertical asphalt membrane in the center. A concrete core wall would also be improved if it were provided with an asphalt membrane either on the upstream face or in the center. This would improve the imperviousness of the wall and prevent leaching of the concrete by continual seepage. Also, in the event of moderate cracking of the concrete, the asphalt would yield and retain the seal. Such membranes utilizing asphalt are still in the developmental stage. If a concrete core wall is to be used, consideration should be given to construction of an inspection gallery with the wall.

Concrete, timber, or steel facings of dams are sometimes used for sealing fills of pervious materials. Since these structures are even more subject to movement than

core walls, extreme care must be exercised in the design of joints and seals, especially at the junction with the foundation; their use is not recommended when other alternatives are available. Further discussion of facings is contained in Sec. 6.

IV. METHODS OF CONSTRUCTION

1. Preparation of Foundation. It is advisable that the impervious zone of a dam or levee extend into the foundation in the form of a cutoff trench. Such a trench is valuable for exploring the foundation for forgotten pipe lines or drains in settled areas and for locating pockets of undesirable material as soft swamp deposits or pervious streambed sands and gravels. This trench and any test pits or exploration holes should be carefully filled with compacted impervious embankment material.

Special treatments for earth foundations have occasionally been used or considered. At Franklin Falls Dam, explosives were used to compact a loose sand and incidentally to reduce its stratification (Ref. 18). Vibroflotation has also been considered for compaction of loose sand foundations (Ref. 9). Chemical and electrical treatments to improve the characteristics of a foundation are in the developmental stage.

The foundation under the dam should be cleared, grubbed, and stripped of topsoil and any other undesirable material. Under rolled fills, earthen foundations should be moistened if necessary and harrowed or lightly plowed, then rolled as for a layer of the dam. If the foundation is wet, dewatering by means of well points, deep wells, sheeted sumps, etc., should be used so that impervious and semipervious rolled fill can be placed in the dry. Cohesionless material can be placed in water and compacted by passes of heavy crawler tractors or by vibroflotation in deep fills. The submergence in water facilitates compaction of such materials. Rock fill can also be placed in the wet state. Care must be exercised in draining a wet foundation so that seepage forces do not loosen and weaken the foundation soils.

For reasons of economy, it is frequently desirable to construct cofferdams for dewatering so that later they can be incorporated in the dam. Because sand and gravel fills and rock fills can be satisfactorily placed in water, they are particularly adapted to such cofferdams. An impervious blanket can be placed on the upstream side of the cofferdam to make it watertight. Where necessary, the blanket is removed after the cofferdam has served its purpose.

When earth fill is placed on rock foundations, care should be exercised that all crevices are plugged. Also knobs of rock which would interfere with proper operation of compaction equipment should be removed. If the upper zone of rock is broken or jointed, grouting is commonly used, as described in Art. II, 2.

2. Embankment Compaction. As a soil is made more compact, its shear strength increases and its compressibility and permeability decrease. With increase in the compactive effort, the degree of compaction, and the shear strength, compressibility and permeability of the soil approach asymptotically their maximum or minimum possible values as the case may be. Theoretically, the compactive effort should be increased as long as the cost of an additional increment of compaction is less than the value of the benefits derived from the corresponding incremental improvement in soil properties. For example, on a strong foundation where the shear strength of the embankment is critical in determining the embankment side slopes, an increase in shear strength would permit an increase in steepness of side slopes and thereby a saving in cost. However, where the foundation is critical in connection with determination of side slopes, an increase in shear strength of embankment material is not apt to be significant in steepening the side slopes. Cohesionless silts and fine sands should be compacted to greater than 50 per cent relative density in order that they will not liquify when subject to earthquake shock or vibration.

Where embankments are placed on weak foundations, usually only light compaction should be used in the lower portion of the embankment. The use of heavy equipment and many repeated passes of equipment tend to develop cracks or slickensides in the foundation and embankment and may result in excessive spreading of the foundation and failure.

With regard to compressibility of embankment materials, it is desirable to compact the materials to such a degree that settlement of the embankment due to its own compressibility is negligible. Some expansive types of clay, if compacted according to usual procedures, will swell upon saturation under light loads. If it is necessary to use such material, cognizance must be taken of such swelling in designing the embankment section.

The reduction in permeability that can be effected by compaction is relatively small. Usually much more variation in permeability will exist from point to point in a particular zone in a dam than the variation in permeability that can be effected by compaction. Thus it is not usual to increase the compactness of a soil just to decrease its permeability.

3. Rolled Fills. Both impervious embankment materials having plasticity and cohesionless sands and gravels are placed in an embankment in layers and compacted by rolling. However, the method of rolling and the optimum moisture content for compaction are quite different for the two types of soils. Embankment materials are frequently excavated by scrapers or by power excavators. Power shovels are good for obtaining a mixture of materials from top to bottom of the excavation cut. Scrapers are good for stripping a soil that occurs as a shallow layer. However, by cutting at an angle to stratification, a mixture of strata can be obtained with a scraper. Transportation of material to the embankment site may be by scraper if that is used for excavation or by truck or large bottom-dump wagons. The scrapers and bottom-dump wagons spread the material on dumping although bulldozers are also required for spreading. If the embankment material contains a large percentage of boulders over 4 to 6 in. in diameter, they may be removed by passing the material through grizzlies (steel rails or sections spread 2 to 3 ft. apart) and screens. A small percentage of boulders can be hand-picked from the material after spreading on the embankment, or the boulders may be pushed to the side by means of a bulldozer equipped with a rake-type blade. It is preferable not to waste the boulders but rather to use them for slope protection or in rock-fill sections. Since the supply of material for a particular zone of a dam will not be perfectly uniform, care should be exercised to minimize the effect of the variability. In the impervious section of a dam, the most impervious material should be placed in the center of the zone and the less impervious at the sides, thereby avoiding the stratification which would result if the materials were placed in alternate layers, and at the same time making a zone less susceptible to migration of fines during seepage. The materials in pervious zones should grade from the finer, next to the impervious zone, to the coarser, next to drains or rock-fill sections.

Impervious materials are spread in layers 4 to 8 in. in thickness and compacted by 6 to 8 passes of a sheep's-foot roller. The pressure on the feet of the rollers vary from 250 to 450 psi depending upon the size of roller and how it is weighted. The water content at which maximum compaction can be obtained with, say, six passes of a particular roller is termed the optimum water content, for that soil, roller, and number of passes. Laboratory compaction tests can be performed as described in Art. IX, 3, to determine approximately the optimum water content for compaction. The optimum water content is less for heavy rollers than for light rollers. When greater density is required than that obtained with 6 to 8 passes of a light roller, increasing the weight of the roller should be resorted to instead of additional passes. It is better to compact a soil at somewhat below optimum water content than above. In the

field, a rough test for determining whether a soil is above or below optimum is to roll a piece of it into a thread. If the thread breaks when rolled down to ¼ in., the soil is about at its optimum water content. If the water content of a soil when spread on the fill is too low, it can be increased by sprinkling. If the water content is too high, it can be reduced by harrowing. When borrow pit material is too wet due to high ground-water table, drainage trenches can be constructed in the pit area. The water content of even impervious soils can be reduced several per cent in this manner. Soils which occur at high water content and which cannot easily be reduced to the optimum water content may be placed and compacted above the optimum, provided the soil is not too wet to be trafficable. If the lightest sheep's-foot roller will not work satisfactorily, rubber-tired wobbly-wheel rollers may be used. These rollers are advantageous in compacting wet silts and have a tendency to squeeze out excess water by their kneading action. Recently heavy rubber-tired rollers have also been used as an alternate to sheep's-foot rollers under ordinary conditions. Tire loads of 25,000 pounds or more and tire pressures of 85 pounds per sq. in. or more are used and the soil is compacted in layers about 15 in. in thickness loose measure. In all cases, the embankment design should be based on the soil properties for the soil compactness achieved.

Sands and gravels are best placed in 6 to 12-in. layers and compacted by passes of a heavy crawler-type tractor. The vibration of the tractor contributes an important part to the compaction. Flooding the sand and gravel with water during compaction is essential for best results. Several small vibration machines and at least one roller-type vibration machine are available for compacting sand and gravel.

4. Rock Fills. Rock fills may be placed in layers or by dumping from great height. The layers may vary from 5 to 35 ft or more in thickness. The method of placing rock in layers 35 ft or more in thickness grades into the method of placing by dumping from great height. For rock fill placed in layers, water in an amount equal to two to four times the volume of rock should be used and should be directed at the rock from two nozzles to sluice all material completely. The sluicing should remove all soil and fine rock particles from the points of bearing of the rocks on each other. In the method where the rock is dropped from considerable height, sharp points of bearing are broken off, thus avoiding excessive settlement from the crushing of such points of contact under subsequent overburden weight and water load. Also, the vibration set up by the impact of the falling rock is beneficial for compacting the rock fill. The rock may be transported to the dam by trucks, or large rubber-tired wagons, and then end-dumped in layers if the layer method is used, or end-dumped starting with full height at the abutments. Cableways have also been used for placing rock fill, and in several instances rock fill has been blasted down from abutments. Since it is much easier to place the rock fill with a side slope equal to the angle of repose of the rock, it is advantageous to step rock-fill slopes in about 30-ft lifts. Each step is placed at the angle of repose of the rock, and the width of berms at each lift is adjusted so that the average slope is the design slope. Usually no limitations are put on the gradation of rock particles for rock fill. The largest sizes are those which can be conveniently handled, and there should be enough small sizes to permit construction of a filter between the core material and the rock fill without undue difficulty. Although quarry-run rock is acceptable, and it is not worth while to process the rock to improve its gradation, still the quarry operations should be set up where possible to obtain the best graded rock possible. Quarrying should be perpendicular or parallel to joint planes in order to produce a maximum of bulky-type particles. Flat and elongated particles bridge between other rocks and produce a loose fill subject to settlement especially upon breaking of the flat bridging pieces. Flat-shaped rocks, if found in the fill, should be broken into smaller bulky pieces. Also any bridging should be destroyed.

Where rock fill is placed in thin surface layers for slope protection, sluicing and dropping the rock from considerable height are not necessary.

5. Hydraulic Fills. In the western part of the United States, at the end of the nineteenth century, the hydraulic method of handling earth, which was developed in placer mines, was applied to earth-dam construction. It rapidly gained favor because (1) of its cheapness in the days before truck and tractor haul was developed and (2) it permitted a gradation of material from fines in the central core to the coarsest material at the outer slopes. With the recent developments in hauling and compacting equipment and methods, the hydraulic-fill method of construction is not so widely used, but in many cases where the size of dam is large, it is still the best and most economical method.

The hydraulic fill is best suited to sandy and gravelly materials such as glacial deposits where the whole material does not contain sufficient fines for relative impermeability, or where a well-graded material from gravel or sand to silt or clay is available

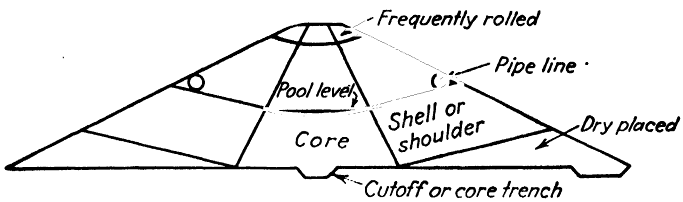


FIG. 7.—Hydraulic-fill construction.

but can be placed more cheaply by the hydraulic method. Materials containing high percentages of impervious material are not suitable for hydraulic fills. The grain size of typical shell and core materials resulting from placement by hydraulic fill is shown on Fig. 2.

The method of constructing hydraulic fills is illustrated in Fig. 7. In modern practice, the base and lower portion of the dam are prepared by rolled-fill methods. Features, such as an impervious upstream blanket, are constructed by rolled fill as are also the initial pervious shoulders which drain toward the proposed pool. The pipe line or sluiceway is carried along the shoulders near the outer slopes where the material is discharged so that the coarse material will settle at the outside and the fines will be carried into the pool. In the past, the width of the core has frequently been made equal to the height of the dam above the horizontal section. However, in the interest of stability and economy, the core should be made as narrow as possible consistent with leakage requirements. The size of the core pool is controlled by spillways. The size of the pool and the quantity of water used should be adjusted so that the desired width of core is obtained and material finer than about 0.01 mm is wasted. By wasting material finer than 0.01 mm, the rate of consolidation of the core is enhanced. The maximum rate of increasing the height of a hydraulic-fill dam is controlled by the rate of consolidation and increase in shear strength of the core. The size of plant should be chosen so that, when operating at optimum capacity, the rate of construction does not exceed the rate allowable by the consolidation of the core. During construction, samples should be taken of both core and shell to check the gradation of materials and, in the case of the core, the degree of consolidation. Care should be taken that tongues of pervious material do not extend beyond the minimum limits set for the core. If probes indicate the presence of such tongues, they should be broken up with water jets or other means. Likewise, clay balls or lenses of impervious material should not be present in the shell proper.

V. HYDRAULIC DESIGN

1. General. An earth dam should be stable under all conditions of seepage, and the seepage losses through the dam and foundation should be kept within the limits prescribed by use of the reservoir and by economic considerations. The pattern of flow of water through an embankment and its foundation is an important factor affecting the stability of the dam and is the basis for computation of leakage quantities. In the following paragraphs methods for determining the pattern of flow, or flow net, and computing the quantity of seepage are described first. The phenomenon of piping and its control by filters and drains is then considered. Finally the effects of various embankment features, such as rock-fill toes, drainage blankets, and impervious blankets, on the pattern of flow and the design of such feature are discussed.

The flow of water through all soils except the coarser ones is of the laminar type and is in accordance with Darcy's law:

$$Q = kiAt$$

where Q = quantity of flow in time, t.
k = coefficient of permeability.
i = hydraulic gradient expressed as head lost per unit length of flow path.
A = superficial area of flow (total cross-sectional area for flow, not merely cross-sectional area of soil pores).

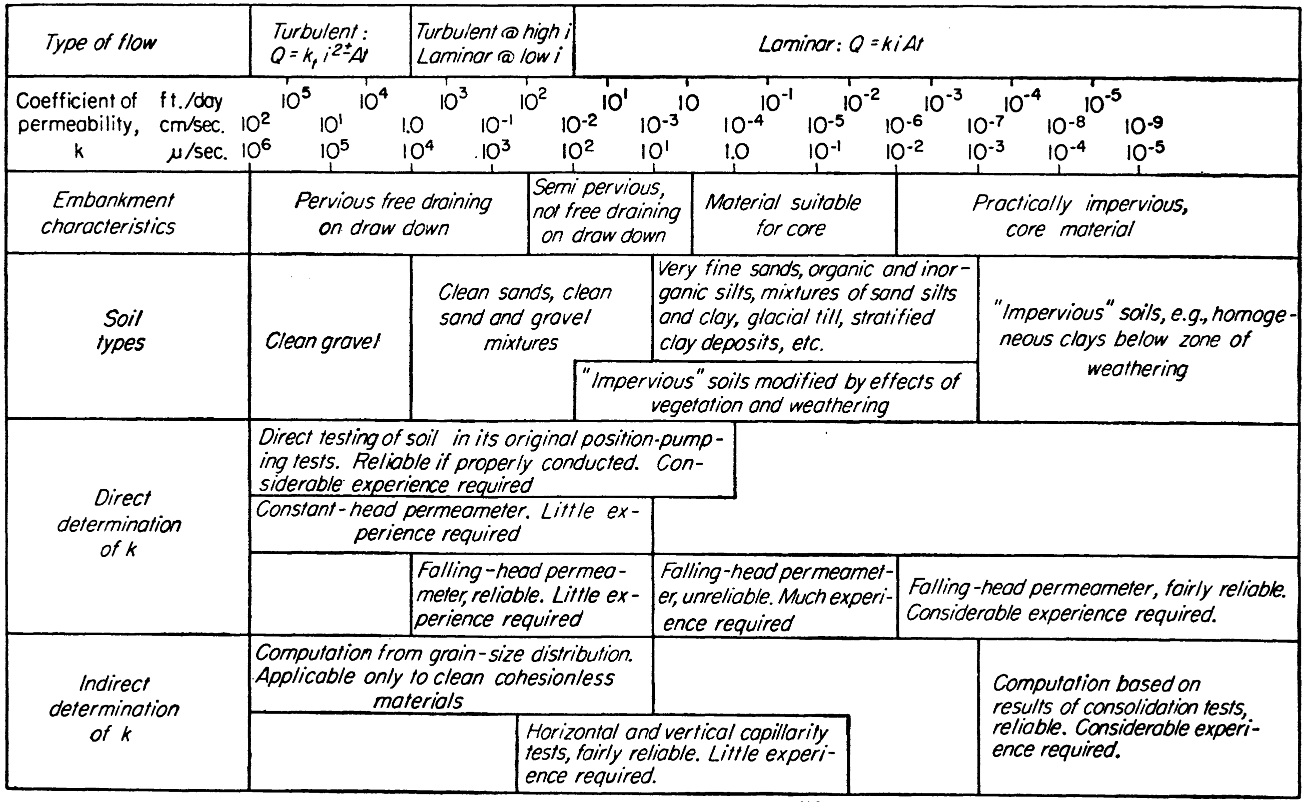
The coefficient of permeability may be determined in the laboratory by tests on undisturbed or remolded samples, as described in Art. IX, 4, or by field seepage tests as described in Art. VIII, 5 and 7. Also various means are available for making rough estimates of the coefficient of permeability which are adequate in some instances. Figure 8 summarizes the permeability of the entire range of soil types. Figure 9 gives the results of a large number of tests on undisturbed foundation materials using the 20 per cent diameter as the comparable size. It will be noted that actual results vary considerably from the mean curve because other factors, such as grain-size gradation and shape and soil structure, have important effects on the permeability of a soil as well as the D_{20} size. Hazen's expression for permeability in terms of the 10 per cent size, $k = CD_{10}^2$ is also frequently used for estimating the permeability of coarse silts and sands. Using cgs units in the equation, the range in C values reported by Hazen for tests on uniform fine sands is 41 to 146. Recent tests reported by Burmister for a wider range of cohesionless soil types indicate a range in C values from 20 to 600. In both cases, however, a mean value of about 100 was obtained for C.

Natural soils almost invariably occur in a stratified condition with the result that the permeability of the soil varies from a maximum in the direction of stratification to a minimum perpendicular to the stratification. Even rolled-fill embankments of selected material will have some stratification due to the inevitable variation in material from layer to layer. To assist in estimating the maximum and minimum permeabilities for an anisotropic soil, the following formulas are of assistance:

$$k_{\max} = \frac{k_1d_1 + k_2d_2 + \cdots k_nd_n}{d_1 + d_2 + \cdots + d_n}$$

$$k_{\min} = \frac{d_1 + d_2 + \cdots + d_n}{\dfrac{d_1}{k_1} + \dfrac{d_2}{k_2} + \cdots + \dfrac{d_n}{k_n}}$$

where k_1, k_2, d_1, d_2, etc., are the corresponding permeabilities and thicknesses of the strata or layers composing the soil. Casagrande (Ref. 5) suggests that for rolled fills of slected material the ratio of $k_{\max}$ to $k_{\min}$ is probably at least 9:1.



(After A. Casagrande and R. E. Fadum)

FIG. 8.—Permeability and drainage characteristics of soils.

2. Flow Nets. The pattern of flow for steady seepage of an incompressible fluid through a porous soil (at constant pore space) can be expressed mathematically by a Laplacian equation. For the case of seepage through a compressible soil undergoing

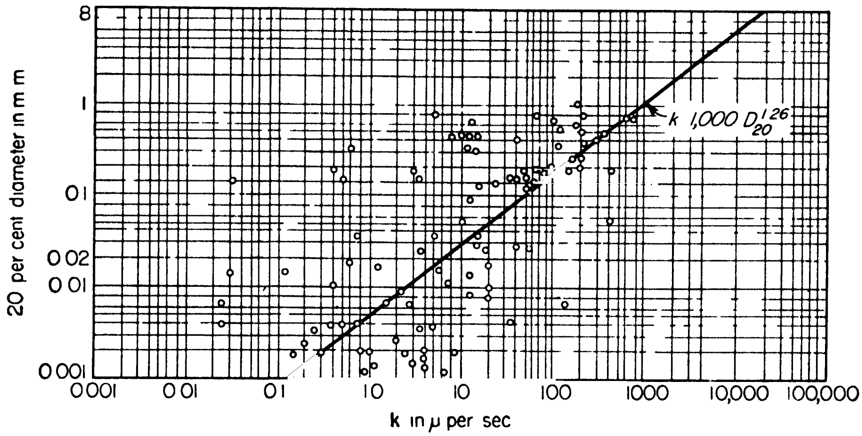


FIG 9 Permeability curve

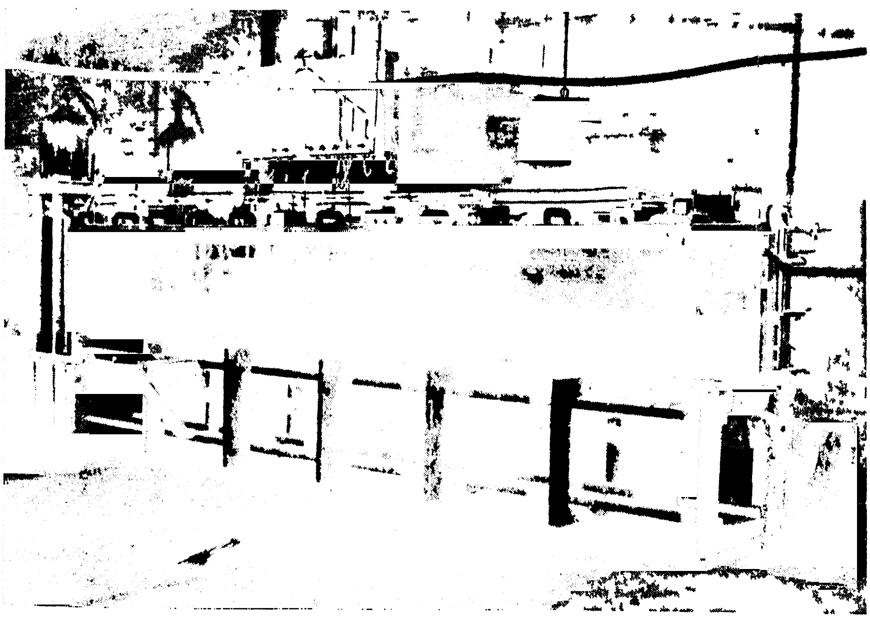


FIG 10—Determination of flow net from model of dam in a flume

consolidation or expansion, the pattern of flow may be determined by superposition of the steady seepage flow and the flow caused by consolidation Direct solution of the Laplacian equation for the steady seepage case is very difficult and is available for only a few cases as for the case of seepage underneath an impervious sheet pile bulkhead. For two-dimensional flow, a trial-and-error graphical solution known as a flow net has been proposed by Forchheimer and is widely used (Ref 5 and Ref. 25, Chap.

9). Difficult flow nets can be determined from models or by methods of relaxation proposed by Southwell (Ref. 24). A picture of a model flume is shown in Fig. 10. In the model, the dimensions and permeabilities bear the same relationship to each other as in the prototype. Dye lines are used to trace the flow lines. Model flumes are expensive and cumbersome, and they have been more or less displaced by electrical analogy models and models consisting of a viscous fluid flowing between two closely spaced plates.

A typical flow net for an isotropic embankment on an impervious foundation is shown on Fig. 11. The net consists of flow lines and equipotential lines which intersect at right angles. For any particular problem there exist an infinite number of

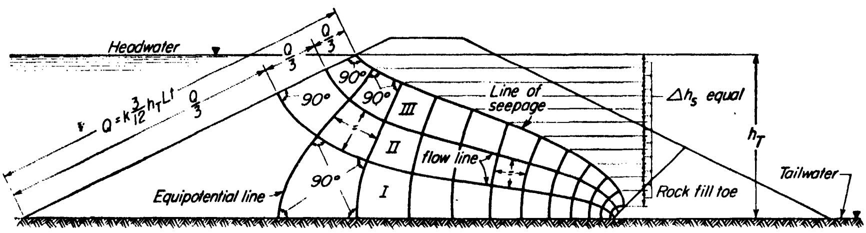


FIG. 11.—Flow net for isotropic embankment on impervious foundation.

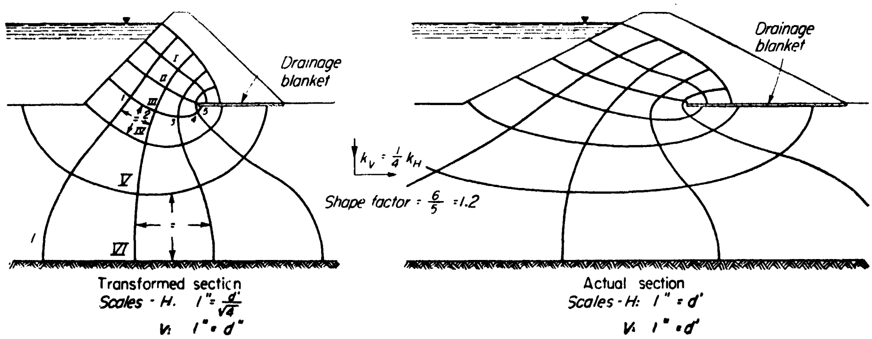


FIG. 12.—Flow net for stratified embankment and foundation.

possible flow lines and equipotential lines. In constructing a flow net, the lines are chosen so as to form squares. Two adjacent flow lines of a flow net form a flow channel, and the quantity of flow through each channel of the net is the same. The equipotential lines connect points of equal total head and intersect the line of seepage (top flow line) at equal increments of elevation. Instructions for sketching flow nets are given in Refs. 5 and 25. In sketching, it rarely happens that the number of flow channels and that of equipotential drops both come out to be full numbers. Usually there will be one fractional flow path or one fractional equipotential drop, as shown in Fig. 18. For the sake of appearance, many of the nets shown in this book have been specially chosen to have an even number of channels and drops.

In the case of stratified material, the flow net of squares is drawn on a section transformed in such a manner that dimensions in the direction of maximum permeability are reduced by dividing them by the square root of the ratio of k_{max} to k_{min} (or the dimensions in the direction of minimum permeability can be increased by multiplying them by the square root of the ratio of k_{max} to k_{min}). After the flow net has been drawn on the transformed section, the flow lines and equipotentials can be transposed back to the actual section. This process is illustrated in Fig. 12.

The following procedure is valuable in connection with determining the position of the top flow line and is illustrated by Figs. 3, 4, and 5*a*. A parabola, called the basic parabola, is constructed through point *e* with point *f* as a focus. For flat upstream slopes, the coefficient by which the distance of the figures must be multiplied to obtain point *e* is equal to ⅓, for steep upstream slopes ¼, and for very steep slopes less than ¼. The directrix of the parabola is located at a distance, *ge*, from *e*. The distance *ge* equals the distance *fe*. Every point on the parabola is equidistant of the focus and the directrix by definition. For a dam with horizontal drainage blanket, $\alpha = 180°$, Fig. 5*a*, the basic parabola gives the location of the exit point of the top flow line directly. For dams with rock toes and dams with ordinary downstream slopes, as shown in Figs. 4 and 3, respectively, the location of the exit point of the top flow line is determined by applying the correction $\frac{\Delta a}{a + \Delta a}$ to the basic parabola.

A chart of values of $\frac{\Delta a}{a + \Delta a}$ slope angle has been prepared by Casagrande and is presented in Fig. 13.

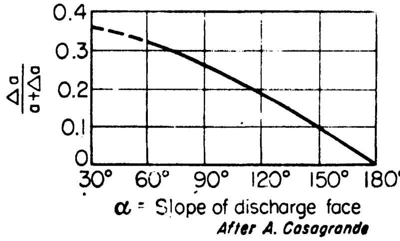


FIG. 13.—Basic parabola correction.

3. Quantity of Seepage. The quantity of seepage can be computed directly from a flow net or, in certain instances, from charts or equations without the construction of a flow net. The equation for computation of the quantity of seepage from a flow net is as follows:

$$Q = k \frac{n_f}{n_d} h_t L$$

where Q = quantity of seepage in length of dam under consideration.
k = effective coefficient of permeability which is $\sqrt{k_{\max} k_{\min}}$.
n_f = number of flow channels of net.
n_d = number of equipotential drops of net.
h_t = head difference between headwater and tail water.
L = length of dam to which the flow net applies.

The ratio n_f/n_d is termed the shape factor of the flow net.

Both graphical and analytical methods are available for computation of the quantity of seepage through an earth dam on an impervious foundation. The results of such computations are presented on Fig. 14 as a family of curves. The chart is entered with the values of d/n and α as defined on the key sketch, and the seepage equation with the proper shape factor coefficient determined by interpolation from the family of curves. The above-mentioned chart can also be used for estimating the quantity of seepage through an embankment based upon a foundation of approximately the same or lesser permeability.

Approximate solutions for the quantity of seepage underneath an impervious embankment founded on a pervious foundation have been developed by Terzaghi. The definitions of terms for the equations are as given before or as defined in Fig. 15.

Case I. When $I > 2U$:

$$Q = \frac{hk}{0.88 + \frac{I}{U}} L$$

Case II. When $I < 2U$:

$$Q = \frac{hk\left(\frac{2U}{I} - 1\right)^{1/2}}{2} L$$

In both the chart and the solution by Terzaghi, the dimensions used should be those of the transformed section described above, and permeability should be the effective permeability.

4. Piping and Filters. Piping occurs when the force exerted on the soil by seeping water exceeds the resistive force offered by the soil. Filters are provided at places

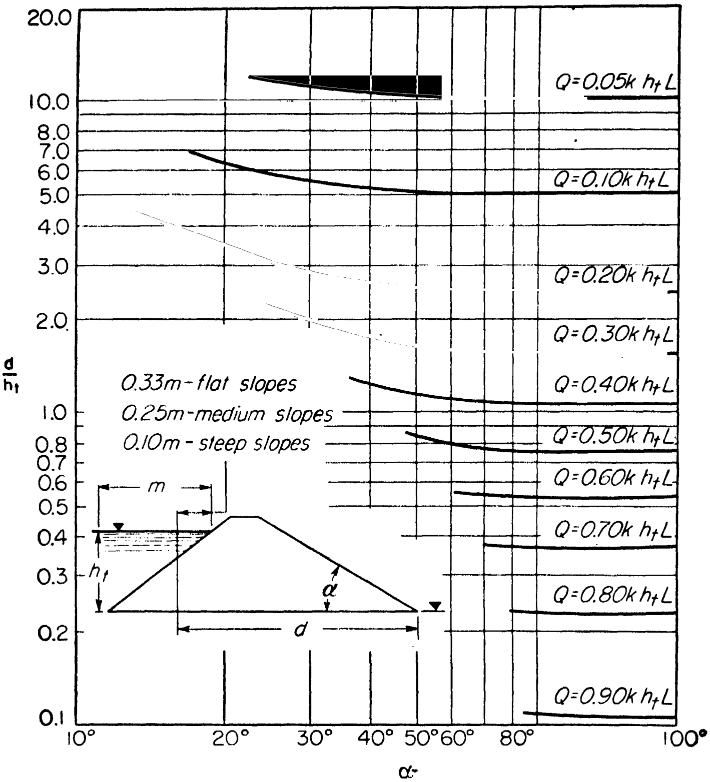


FIG. 14.—Seepage through embankment on impervious foundation.

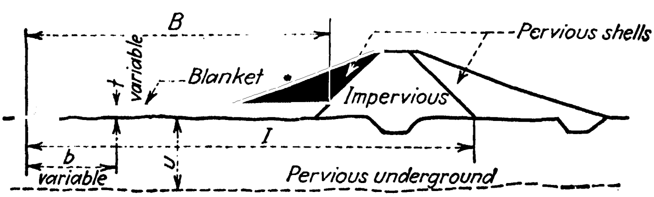


FIG. 15.—Blanket diagram.

where seeping water emerges from the soil when the resistive force of the soil does not have sufficient factor of safety against migration of fines. An earth dam and its foundation form a barrier that maintains a differential head between the water surface in the reservoir and the tail water below the dam. The potential energy represented by this difference in head is dissipated as frictional loss as the water flows through the soil. The seepage force, J, exerted on the soil by the water, is equal to the unit weight of water, γ_w, times the hydraulic gradient, i, and is always in a direction per-

pendicular to equipotential lines. For either an isotropic or stratified soil, the hydraulic gradient is determined by dividing the head lost between two equipotential lines by the perpendicular distance between the lines measured on the true scale drawing. Where water flows into the upstream face of an embankment, the seepage force has a stabilizing effect, but where seepage is out of an embankment the seepage force has a destabilizing effect. When upward flow exists in a cohesionless soil and the head loss per foot of length exceeds the submerged effective weight of the material, movement of the soil particles will take place. For example, if the weight of a foot

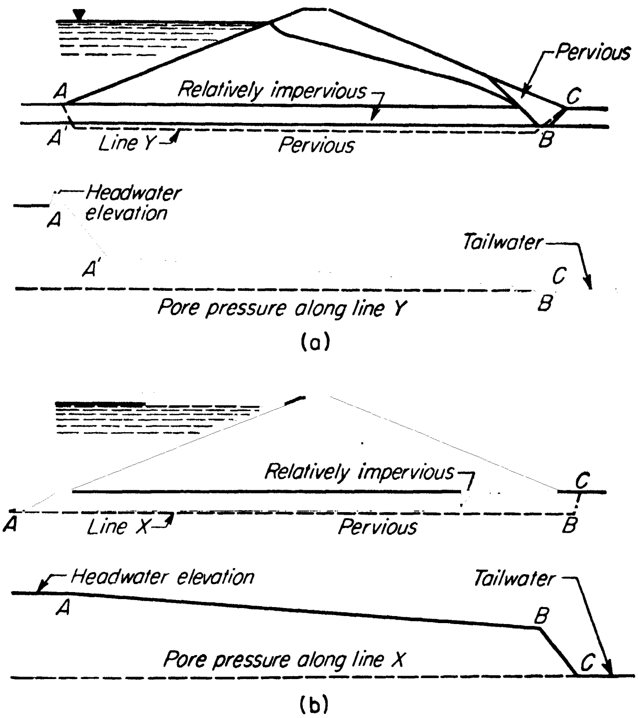


FIG. 16.—Blanket diagrams with hydraulic grade line.

cube of saturated cohesionless soil is equal to 125 lb, the submerged weight will be 62.5 lb; hence if there is an upward hydraulic gradient of 1, the seepage force will be equal to 62.5 lb and the material will be in a condition of unstable equilibrium, termed a quick condition. Figure 16*b* illustrates an embankment and foundation combination for which a quick condition or boils are likely to develop at the landside toe. Such conditions are sometimes alleviated by ringing the boils with sandbags to raise the tail water and thus reduce the hydraulic gradient. Proper design in this case requires a drain at the landside toe similar to the one for the case shown in Fig. 16*a*. The drain reduces the pore pressure at point *B* to practically tail-water pressure, and the hydraulic gradient and seepage force in the toe area to practically zero.

Whenever water flows from a material to a more pervious one, the possibility of migration of fines or piping should be considered. Even a very minor washing away of fines at the downstream side of a dam is serious. As soon as some fines are washed away, the resistance to seepage along the path of seepage is reduced and increased flow results. Owing to the increased flow, the rate of washing away of fines is increased.

By such progressive action a large tunnel-like opening can develop over a period of years, or decades, and ultimately cause failure of the dam. Development of such a tunnel is called piping. If the discharge point of the tunnel is located underneath rock fill, the piping may never be noticed until too late. A filter layer should therefore be placed between two adjacent materials whenever the sizes of the pores of the material into which the water is flowing are larger than the sizes of the coarser particles of the material from which the water is coming. On the basis of research reported in Refs. 3 and 36, the following procedure for designing filters is recommended. The D_{15} size of the filter should be (1) no larger than 4 to 5 times the D_{85} size of the material to be protected and (2) preferably at least 4 to 5 times larger than the D_{15} size of the material to be protected. The first requirement is to prevent migration of fines and the second to ensure adequate permeability of the filter. Usually the shape of the grain-size distribution curve for the filter material is made similar to the grain-size distribution curve for the material being protected. When the filter is to be placed between impervious earth fill and rock fill, the D_{15} size of the filter is determined as above, but in addition the D_{85} size of the filter must be no less than one-fourth to one-fifth the D_{15} size of the rock fill adjacent to the filter. The design of such a filter is illustrated in Fig. 17. Sometimes the resulting filter gradation covers too wide a range in particle sizes. In such cases, the filter can be constructed in two or more layers, each layer satisfying the above criteria with respect to materials adjacent to it. A two-layer filter for the above case will be more pervious than a single-layer filter. The thickness of the filter layers varies from 12 to 18 in. per layer in drainage trenches and at rock toes to over 8 ft (horizontal measurement) for filters between the core and the shell sections of a dam. In the latter case, the main reason for the 8-ft width is to provide enough room for proper operation of compaction equipment. Adequate compaction of filters between core and shell sections is essential during construction to prevent excessive settlement and adjustment of the fill. Where filters are placed on natural soils, it is usually more economical to design the filter to protect most of the soil and then place an additional layer between the regular filter and the finer pockets of foundation soils where such pockets occur.

5. Rock Toes and Drainage Blankets. The use of rock toes and drainage blankets for keeping the line of seepage from breaking out on the downstream slope is described in Art. III. The extent of the drainage blanket or toe should be such that the top flow line is well within the downstream face. For an embankment on impervious foundation, the position of the top flow line can be determined by the corrected basic parabola method, illustrated in Figs. 3, 4, and 5*a*. Where the foundation is not impervious, a flow-net study may be required. In determining the line of seepage, a conservative assumption regarding stratification should be made and allowance should be provided for capillary rise in the soil. Rock toes and drains should be properly designed with filters as necessary to prevent piping.

6. Drainage Wells and Trenches. Whenever the foundation conditions consist of a stratum overlain by a less pervious stratum, drains should be installed downstream from the center line of the dam to relieve the pressure in the lower stratum. If the weight of the upper stratum is not sufficient to resist the pressure, boils or blowups will occur at the downstream toe. Even if the weight of the upper stratum is sufficient, the danger of piping through the upper stratum exists. In the first case, the drains should extend into the lower stratum. In the second case, the drains should extend deep enough to ensure that practically all flow will be to the drain. As stated in Art. II, 5, shallow drains are usually trench drains, and deep drains are a series of relief walls. In both cases, they should be designed to prevent migration of fines. The quantity of water flowing to a trench drain can be estimated by constructing a flow net. The design of the spacing, size, and depth of drainage wells is discussed in

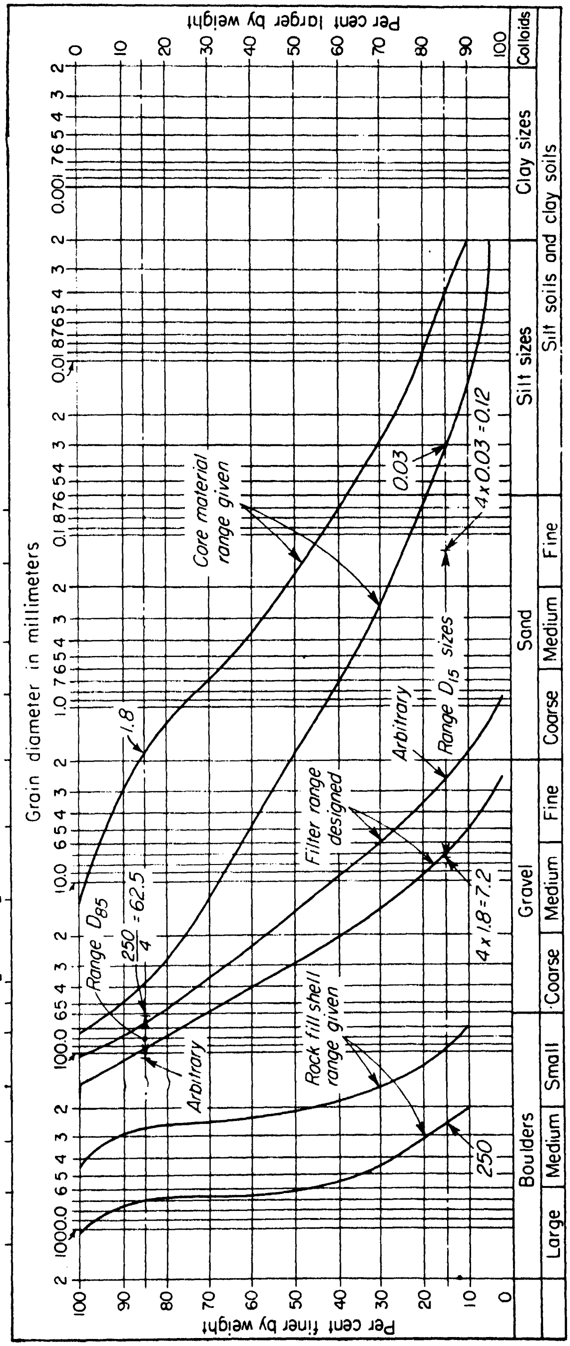


FIG. 17.—Design of filter.

Ref. 35. Valuable discussions of the control of underseepage are contained in Refs. 19, 33, and 34.

7. Impervious Blankets and Cutoffs. Impervious upstream blankets and cutoffs are measures used to reduce the amount of seepage through a dam foundation. An impervious clay blanket placed upstream of a dam and connected to the impervious section is a convenient way of effecting moderate reductions in amounts of seepage.

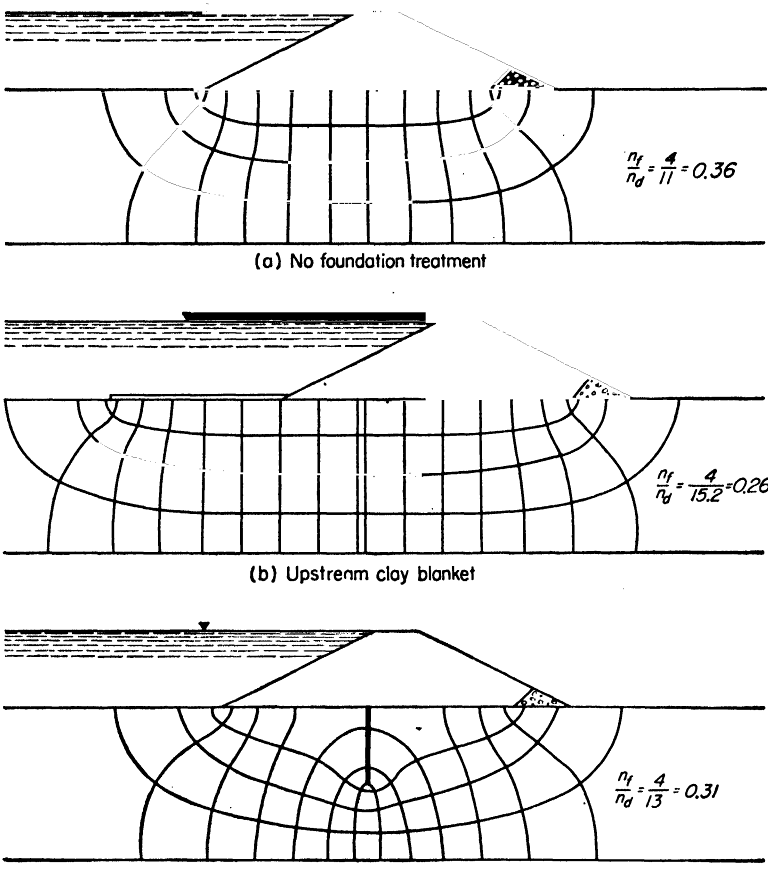


FIG. 18.—Effect of foundation treatments on flow through pervious foundation

A comparison of Figs. 18*a* and *b* shows that, by increasing the total length of impervious material to 1.65 times the length of the base of the impervious section of the dam, the quantity of seepage is reduced to 1/1.47 of the amount without the blanket. The quantity of seepage is thus somewhat less than inversely proportioned to the total length of impervious material. The effectiveness of a blanket depends upon the proportionate increase in number of equipotential drops that result from its addition to the dam. A blanket is advantageous only when an appreciable lengthening of the path of flow can be effected by a blanket of reasonable length as for the case of a dam with narrow impervious section founded upon a foundation material with practically no stratification. Blankets should be composed of material at least 100 times less

pervious than the foundation material. They are usually 5 to 10 ft in thickness; the blanket may taper in thickness away from the dam. The design of impervious upstream blankets is discussed in Ref. 2.

A comparison of Figs. 18*a* and *c* shows that only a slight reduction in seepage is effected by a partial cutoff through a pervious foundation. Partial cutoffs usually are advantageous only when the foundation has high stratification. The efficacy of partial cutoffs for controlling underseepage is discussed in Ref. 37.

Blankets and partial cutoffs have limited application because the reductions in seepage quantities which they affect are relatively small. The range in permeability of soil is tremendous; from 10,000 ft per day for gravels to $\frac{1}{1,000,000}$ ft per day for clays, and the effect of small variations in soil structure and particle sizes on the permeability of a soil is great. Thus estimates of the average permeability of a foundation usually have no better than 1,000 per cent accuracy. To be significant, a blanket or cutoff should thus effect at least about a ten-time reduction. Where the quantity of seepage is a critical consideration in the design, a complete cutoff is required.

VI. STRUCTURAL DESIGN

1. General. The structural design of an earth dam comprises a stability analysis and a settlement analysis of the embankment and its foundation. Whenever the shearing force along any surface through the embankment and/or foundation exceeds the shearing strength along that surface, a stability type of failure results. The failure may consist of sloughing of material near the surface of the embankment in a localized area, or of a deep circle type of failure involving a large volume of the embankment and foundation. Stability analyses are made to ensure that the embankment is designed with adequate factors of safety against such types of failure. Settlement analyses are made to determine the magnitude and rate of occurrence of settlement due to consolidation of the embankment and its foundation. Such analyses are used for estimating the additional height that has to be added to an embankment so that after settlement has occurred the top of the dam will be on grade and also for designing conduits passing through or underneath the embankment. In the case of poorly draining compressible soils, stability analyses are dependent upon settlement analyses in that the shearing strength of the soil depends upon the percentage consolidation of the soil.

2. Stability Analysis Consideration. Two approaches are available for analyzing the stability of an embankment. In one approach a surface of sliding is assumed, and the stability of the sliding mass is analyzed. For purposes of analysis, the trace of the surface of sliding on a cross-sectional view may be assumed to be a straight line, a circle, a logarithmic spiral, or a composite of such lines. In the second approach, methods of stress analysis are used to determine the stresses throughout the embankment and its foundation. The ratio of shearing stress to shearing strength is then checked at critical points and also along possible surfaces of sliding. The first approach is preferred since the subject of stress distribution throughout soils is a controversial matter at this time.

In either of these approaches it is convenient to express the shearing strength of a soil by the following equation:

$$s = c_e + \sigma \tan \phi_e$$

where s = shearing strength.

c_e = effective cohesion.

σ = soil pressure normal to shear surface, usually the intergranular soil pressure before shear occurs.

ϕ_e = effective angle of internal friction.

The cohesion and angle of internal friction of a particular soil vary, depending upon the drainage permitted the soil during shear, the rate of shear, and other factors. The expression for shear strength used in a stability analysis should represent the shear strength of the soil under the particular conditions of shear in the embankment. The word effective is used with the cohesion and angle of internal friction to designate those values applicable to the particular conditions under which the soil is sheared. Methods for determining c_e and ϕ_e of a soil are described in Art. IX, 4. Usually the values obtained from a consolidated quick test conservatively represent field conditions.

The factor of safety against stability failure is defined as the ratio of the average shearing strength along the critical sliding surface to the average shear stress acting on that surface. The factor of safety may also be expressed as the ratio of the total shearing resistance to the total shearing force. The commonly used factors of safety are shown in the accompanying table. The lower factors shown in case 1 should be

	Minimum factors of safety	
	Case 1	Case 2
Upstream slopes, dry	1.5	2.0
Downstream slope, steady seepage	1.5	2.0
Upstream slope, rapid drawdown	1.25	1.5

used only when the embankment is composed of cohesionless soils or soils of very low plasticity whose shear strength is well defined. The higher factors listed for case 2 should be used when the embankment is composed of plastic soils. The above factors of safety apply to analyses where reasonable assumptions are used in connection with determining seepage forces. The factors of safety for the drawdown condition are less than those for the steady seepage condition because failure after drawdown of the reservoir is not so disastrous as failure of the dam under full reservoir. Sometimes very conservative assumptions are made regarding the rapid drawdown case, and a factor of safety less than 1.25 (about 1.0) may be used.

3. Cohesionless Material, Simple Cases. The analyses of the stability of an embankment composed of cohesionless material under (1) condition of no seepage and (2) condition of seepage parallel to the face of the slope are direct and simple. The critical surface for sliding is the face of the embankment or a surface parallel to the face at shallow depth. Computation of the factors of safety for the two cases may be made using the following expressions:

Condition 1, no seepage,

$$F = \frac{\tan \phi_e}{\tan i}$$

where i = angle of face of slope.

Condition 2, seepage parallel to the face of the slope,

$$F = \frac{\tan \phi_e}{\tan i} \times \frac{\gamma_{subm}}{\gamma_{sat}}$$

where γ_{subm} = unit weight of material submerged.

γ_{sat} = unit weight of material saturated = $\gamma_{subm} + \gamma_{water}$.

If seepage is at an angle intermediate between the horizontal and the angle of the slope and at a gradient equal to the slope angle, the stability of the embankment

will be less than that for the condition of seepage parallel to the face. Analyses for such cases of seepage, and for the case where free draining slope protection counterweighs the slope, should be made using the critical circle or composite surface methods described below.

4. Critical Circle Method. Investigations of slide failures of embankments and their foundations indicate that the trace of the failure surface on a section normal to the slide can be closely approximated by a circle. Based on this fact, a trial-and-error solution has been developed wherein a series of trial circles through the embankment and foundation are analyzed. The circle with the lowest factor of safety against stability failure is considered to be the critical one. Analysis of the stability of each trial circle can be made by the friction circle method or the slices method. The slices method is the more common method of solution and is described in the

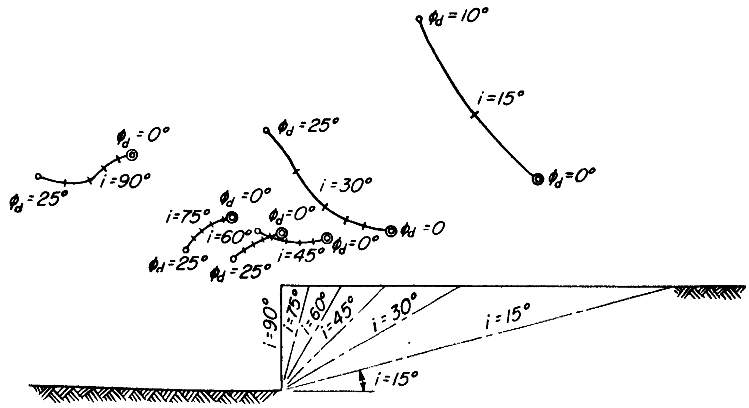


FIG. 19.—Influence of value of developed friction angle, ϕ_d, on position of center of critical toe circle for different values of slope angle, i.

following paragraph. Discussion of the friction circle method may be found in Ref. 25, pages 441–448. For the case of an embankment and foundation of homogeneous material, Taylor has prepared a stability number chart expressing the results of many friction circle solutions. The chart greatly simplifies the stability analysis of slopes to which it is applicable. Taylor's chart as modified by Terzaghi is presented in Fig. 23 and its use discussed in paragraph 6.

5. Slices Method of Stability Analysis. The first step in the slices method is to choose a trial circle. The three possible types of trial circle are illustrated on Fig. 23 and are termed (1) slope circle, (2) toe circle, and (3) mid-point circle. The slope circle may be critical when a firm stratum limits the bottom point of the circle or where the stability of a limited height of slope in an area of destabilizing seepage forces is to be investigated. Toe circles or circles approximating them are frequently critical. A chart showing the location of centers of such circles for various values of developed friction angle, ϕ_d, and slope angle, i, is presented in Fig. 19. The location of centers of mid-point circles is usually governed by the depth to a firm stratum as rock in the foundation. Also the critical circle usually passes close to the intersection of the crown of the dam and the face of the dam opposite to the face being analyzed. Critical circles pass through as much of the weaker material in the dam or foundation as possible.

The second step is to divide the circular segment into an arbitrary number of vertical slices and to determine the forces acting on each slice. Figure 20 shows a

trial circle divided into eight slices; generally six to ten are adequate. The complete free-body diagram of forces for slice 4 of Fig. 20 is shown in Fig. 21. The following discussion of the treatment of the forces acting on a slice is based on an unpublished paper given by Taylor at the annual A.S.C.E. Meeting in New York, January, 1949. The weight force, W, is the total weight of soil particles and water in the slice. The forces U_l, U_r, and U_b are the resultant water thrusts on the left side, the right side, and the base of the slice, respectively. Their directions are normal to the surface on which they act and, where steady seepage occurs through the embankment, their magnitude can be determined by utilizing a flow net. The lateral earth thrusts, E_l and E_r, are indeterminate in magnitude and direction but, as shown in solution B on Fig. 20, reasonable values can be determined for them by a trial-and-error procedure. The resultant earth force on the base of the slice can be represented by a tangential force C_d which is termed the developed cohesion force and an earth thrust P, at an angle of ϕ_d. The developed cohesion and developed friction angle are defined as

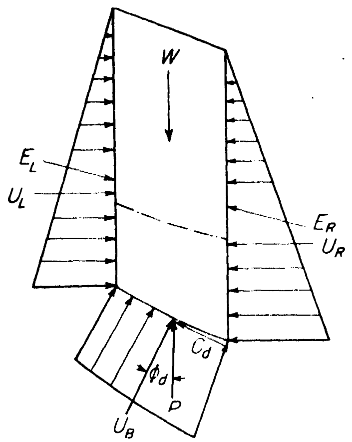


FIG. 21.- Free-body diagram of slice 4 of Fig. 20.

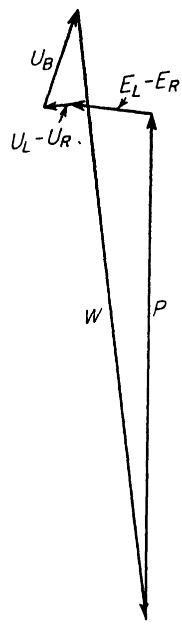


FIG. 22.-Force diagram for slice 4 of Fig. 20.

$$c_d = \frac{c_e}{F}$$

and

$$\phi_d = \text{arc tan}\left(\frac{\tan \phi_e}{F}\right) = \frac{\phi_e}{F}$$

A force diagram for the forces acting on the slice is shown in Fig. 22. The net lateral earth and water thrusts are used in this diagram, and the normal and tangential components of the resultant earth thrust on the base of the slice are indicated.

Two methods of solution are shown in Fig. 20. In method A, the earth thrusts, E_l and E_r, on each slice are assumed to counterbalance each other. The solution is somewhat simplified thereby, and for ordinary cases the accuracy of solution is satisfactory. An alternate assumption that can be made in solution A is that the water forces, U_l and U_r, as well as the earth thrusts E_l and E_r, counterbalance each other. In solutions of the above type, the factor of safety is computed by the following equation:

$$F = \frac{\Sigma N \tan \phi_e + c_e \hat{L}}{\Sigma T}$$

The N and T forces are the normal and tangential components of the earth thrusts P acting on the bases of the slices. The term $\hat{L}$ is the length of the failure arc.

In method B, reasonable directions are assumed for the earth thrusts E_l and E_r, and solution is made by trial and error so that the forces acting on each slice are in equilibrium and the summation of the $(E_l - E_r)$ thrusts for all the slices equals zero.

In methods where the E_l, E_r, U_l, and U_r forces are omitted, the force diagrams for several slices are entirely unreasonable. The obliquity of the P force varies from slice to slice; on some it is much less than ϕ_d and on others much greater. The reason that solutions of type A have satisfactory accuracy is that the T forces, due to compensating effects, are determined exactly; also the errors made in the individual N forces are small and somewhat compensating.

When an embankment is subject to steady seepage, determination of the force U_b on the base of the slice may be determined from the piezometric line obtained from a flow net, as shown in Fig. 20. In compressible embankment materials such as clay, the force U_b is computed using the proper pore pressure.

$$u = \sigma_T - \bar{\sigma}$$

where u = pore pressure.

σ_T = total normal pressure (overburden pressure) on a horizontal plane at point in question after drawdown.

$\bar{\sigma}$ = total normal intergranular pressure on above plane.

For the case of an embankment or foundation undergoing consolidation, the intergranular normal stress is determined for the proper percentage consolidation. For the case of rapid drawdown, consolidation under the increment of load caused by the drawdown is usually assumed to be zero; the intergranular normal stress is thus the same as it was before drawdown. Frequently the face of a slope closely approximates the piezometric line for the rapid drawdown case.

6. Stability Chart. The stability factor chart presented in Fig. 23 affords an accurate and very simple method for determining the factor of safety against stability failure for a homogeneous dam and foundation in (1) the dry condition, (2) the completely submerged condition, (3) the condition of seepage parallel to face of slope, (4) the condition of complete rapid drawdown from crown to toe for embankment composed of compressible material. Approximate solutions can also be made for nonhomogeneous dams and for the condition of seepage not entirely parallel to the face of the slope and for the condition of rapid drawdown over part of the slope. The chart gives the relationship between factor of safety F, height of slope H, angle of slope i, developed friction angle ϕ_d, and developed cohesion c_d. The effect of a firm stratum at shallow depth in the foundation and the effect of counterweights a short distance away from the toe of slope are also included by means of the terms, n_D and n_x, defined in Fig. 23.

The steps used to solve two typical stability problems are outlined below. The solutions are for embankments and foundations in which no seepage or consolidation is occurring. If the slope is above the free water surface, the total unit weight is used for γ. If the slope is completely submerged, the submerged unit weight is used for γ. Modifications to the given solutions to solve other problems are also given.

Problem A: **Determine the permissible slope for an embankment of prescribed height to satisfy a prescribed factor of safety.**

1. Determine $c_d = \frac{c_e}{F}$, and $\phi_d = \text{arc tan}\left(\frac{\tan \phi_e}{F}\right) \approx \frac{\phi_e}{F}$.
2. Compute $N_s = \frac{\gamma H}{c_d}$.
3. Determine n_x, and the minimum of the actual value of n_D and the value of n_D from Fig. 23b.

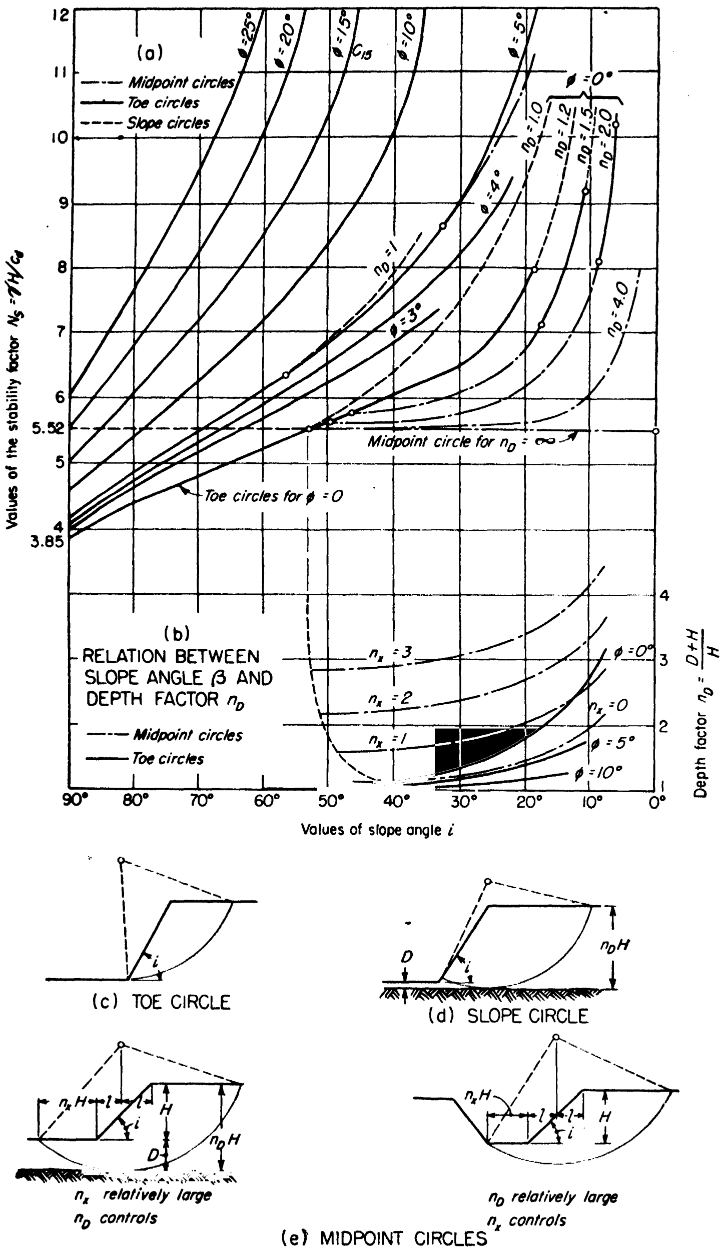


Fig. 23.—Stability chart.

4. Enter chart with N_s and ϕ_d using the proper n_D line when $\phi_d < 5°$ and $i < 53°$, and determine the slope angle, i.

Problem B: Determine the factor of safety for a slope of prescribed height.

1. Assume a value for the factor of safety.
2. Compute ϕ_d = arc tan $\left(\frac{\tan \phi_e}{F}\right) \approx \frac{\phi_e}{F}$.
3. Compute n_x, and determine the minimum of the actual value of n_D and the value of n_D, from Fig. 23b.
4. Enter chart with i and ϕ_d, using the proper n_D line when $\phi_d < 5°$ and $i < 53°$, and determine the stability factor, N_s.
5. Check the assumed factor of safety against F computed from $F = \frac{c_e}{c_d} = \frac{c_e N_s}{H}$.
6. Repeat this procedure until the assumed and the computed F agree.

As an alternate to problem B, the factor of safety may be prescribed, and it may be desired to determine the permissible height of embankment. The solution is similar to that of problem B. For the third and fourth cases above where the face of the slope approximates the piezometric line for pore pressure along the critical circle, the three problems may be solved as indicated, except that $\phi_d = \frac{\gamma_{\text{subm}}}{\gamma_{\text{total}}} \times$ arc tan $\left(\frac{\tan \phi_e}{F}\right)$. Also γ_{total} is used in computations for N_s. When the embankment and foundation are composed of more than one material, an approximate solution can be made by obtaining weighted average values for ϕ_e, c_e, and γ. The following steps may be followed in the solution:

1. Assume average values for ϕ_e, c_e, and γ.
2. Solve the problem as described above and determine ϕ_d and i.
3. Draw the critical circle; either the proper mid-point circle or the toe circle defined by Fig. 19 for ϕ_d and i, whichever applies.
4. Check the assumed average values; repeat the solution if necessary with more accurate average values.

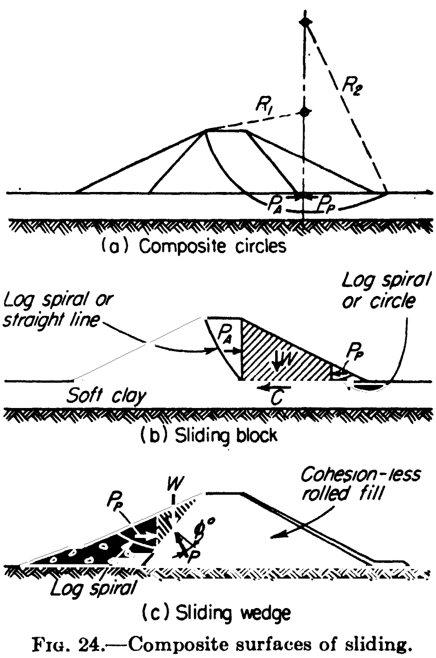


FIG. 24.—Composite surfaces of sliding.

7. Composite Surfaces of Sliding. At times, the critical surface for sliding may be better approximated by a composite of several types of surfaces. The commonest elements used for making the traces of composite surfaces are circles of various radii, straight lines, and logarithmic spirals. Three types of composite surfaces are shown in Fig. 24. In computations involving composite surfaces, it is usually convenient to represent part of the surface by the equivalent active or passive pressure as indicated on Fig. 24. In determining such active and passive pressures, ϕ_d and c_d should be used.

8. Fluid Core and Pervious Shell. In the design of hydraulic- or semihydraulic-fill dams, the assumption is sometimes made that the core is a heavy liquid. It follows that the shell should be designed to withstand the pressure of this liquid. Gilboy has developed a method of calculating the stability of dams with liquid cores (Ref. 10).

For a factor of safety of 1.0, the following equation applies:

$$\sqrt{r} = \frac{(c - a)\sqrt{1 + b^2} + \sqrt{c - a}\sqrt{c - b}\sqrt{1 + a^2}}{(1 + c^2) - (c - a)(c - b)}$$

where a = contangent of angle of core slope with horizontal.
b = contangent of angle of internal friction of shell material.
c = contangent of angle of outer slope with horizontal.
d = ratio of unit weight of core to unit weight of shell.

It should be noted that the slopes are independent of height. The assumption of a liquid core is conservative; actually some consolidation of the lower portion of the core occurs during construction. If estimates of this consolidation and the corresponding shear strength of the core can be made, then a critical circle or composite sliding surface type of analysis can also be made.

9. Stress Analysis. Stability analyses utilizing stress analysis methods are generally limited to the case where an embankment is founded on clay and are at the present time best adapted to indicating zones of overstress. Equations given by

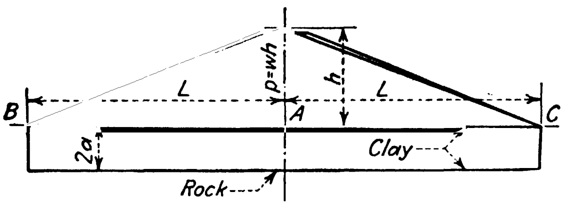


FIG. 25.—Diagram for Jurgenson's equations.

Jurgenson in Ref. 14 are frequently used to indicate maximum shear stresses in a foundation. These equations are derived on the assumption that the resultant load applied to the foundation at any point along the base of the dam is equal to the weight of fill directly above that point. Actually the resultant load on the foundation must have a tangential component. Also, the weight of the fill above any point only roughly approximates the normal component of the resultant as reported by Taylor in Ref. 27. The equations given below for Fig. 25 are therefore quite approximate. For the case where a is greater than P, the equation for maximum shearing stress τ_m is

$$\tau_m = 0.256p$$

where p is the load applied at the center of the embankment. This maximum stress theoretically occurs at a depth of one-fourth the base width of the dam below the center line of the embankment. For the more usual case where a is much less than L (a is less than $L/10$), the equation for the maximum shear stress is

$$\tau_m = \frac{pa}{L}$$

and occurs at the (rough) rock boundary. Stability analyses should always be made by the critical circle or composite surface type of analysis. The stress analysis method should be used only where that method is applicable to supply auxiliary information.

10. Settlement Analysis. In the design of an embankment and its appurtenant structures, it is important to estimate the ultimate settlement and rate of settlement. Computation of these two items is termed a settlement analysis. The steps involved in such a computation are described below.

When a compressive load is applied to a confined soil, the total volume of the soil decreases. For all practical purposes, this volume change is only a change in the void volume of the soil; the volume of soil particles remains constant. When the voids of a soil are filled with water, the rate at which the void volume can change depends upon the rate at which the water content of the soil can be changed.

When an increment of pressure is applied to a saturated soil in which the pore pressure is the natural hydrostatic pressure, the entire increment of pressure is at first carried by an increase in water pressure which is termed hydrostatic excess pressure. At the moment of application of load, the soil is considered to be at zero

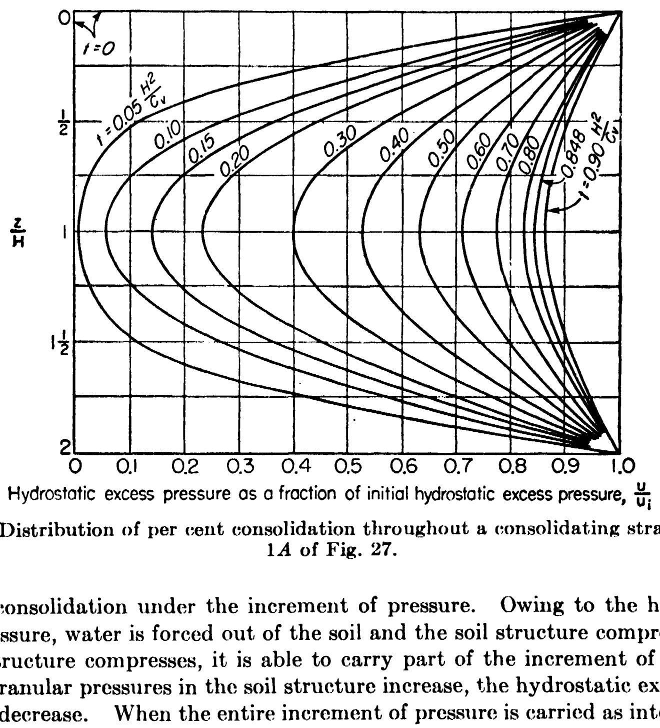


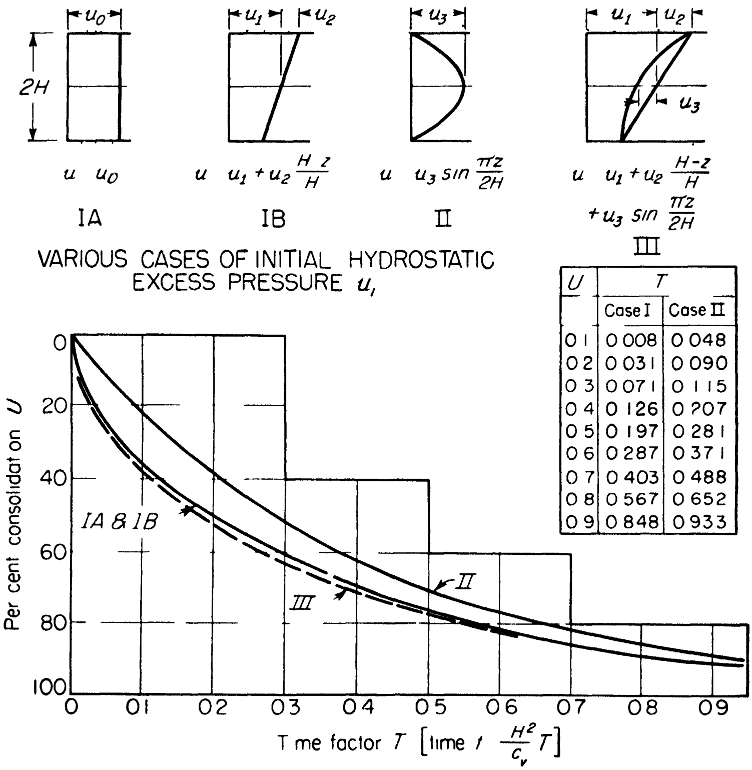
FIG. 26.—Distribution of per cent consolidation throughout a consolidating stratum, Case 1*A* of Fig. 27.

per cent consolidation under the increment of pressure. Owing to the hydrostatic excess pressure, water is forced out of the soil and the soil structure compresses. As the soil structure compresses, it is able to carry part of the increment of load. As the intergranular pressures in the soil structure increase, the hydrostatic excess water pressures decrease. When the entire increment of pressure is carried as intergranular pressure, the soil is considered to be 100 per cent consolidated under the particular increment of load. The portions of a soil stratum nearest the drainage faces drain first because the hydraulic gradients created by the hydrostatic excess pressure are greatest there. The variation with time, t, of the per cent consolidation U_z at various depths, z, in a stratum of thickness $2H$ with drainage at top and bottom and for the case of hydrostatic excess constant with depth at time of application of load, is given in Fig. 26. The average consolidation throughout the stratum is given on Fig. 27, for three different types of initial hydrostatic excess which may be encountered. The hydrostatic excess diagram for curve I*A* is that for the case described above. Cases I*B*, II, and III represent other types of possible field loading.

The soil properties required for settlement analysis computations are determined by means of the consolidation test described in Art. IX, 4. The two required properties are the index of compressibility, C_c and the coefficient of consolidation, c_v.

The ultimate settlement is determined as follows: At several depths throughout the stratum the changes in void ratio of the soil due to the increment of load are deter-

mined. The increase in stress reaching a particular depth is determined using the theory of elasticity. Either the Boussinesq or Westergaard formula for distribution of a vertical stress may be used depending on whether the soil is homogeneous or varved. The distribution of vertical compressive stresses σ_z due to a point load is given in Fig. 28. By dividing the area of distributed load into squares whose sides are no larger than one third the depth to the point at which the stress is being computed and by summing the increments of stress due to each area, the total increment of stress



U	T	
	Case I	Case II
0.1	0.008	0.048
0.2	0.031	0.090
0.3	0.071	0.115
0.4	0.126	0.207
0.5	0.197	0.281
0.6	0.287	0.371
0.7	0.403	0.488
0.8	0.567	0.652
0.9	0.848	0.933

FIG. 27. Rate of consolidation curves.

due to a distributed load can be computed for any point in the foundation. The decrease in stress due to release of load by excavation can be computed in a similar manner and should be considered where applicable. References 14, 30, 20 and 21 contain the results of the integration of various types of distributed loads in tabular or chart form. The change in void ratio Δe at a particular depth is expressed by the equation

$$\Delta e = \frac{0.435 C_c \, \Delta p}{p_{av}}$$

where C_c = index of compressibility

Δp = net increase in vertical compressive stress at a point in foundation

p_{av} = initial vertical compressive stress at point, in foundation plus $\Delta p/2$

The ultimate settlement, ρ_{ult} is computed from the equation

$$\rho_{ult} = \frac{H}{1 + e}, \text{ drainage at either top or bottom of stratum}$$

$$\rho_{ult} = \frac{2H}{1 + e}, \text{ drainage at both top and bottom of stratum}$$

where H is the thickness for drainage in one direction.

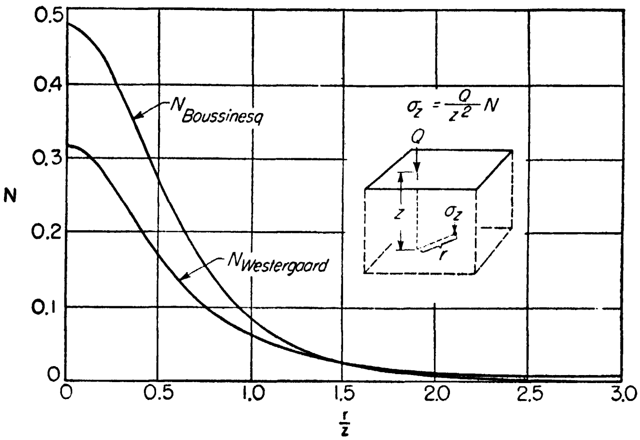


FIG. 28.—Distribution of vertical stresses due point load on surface of semiinfinite elastic solid.

The time rate of settlement is obtained by adjusting the scales of the proper curve of Fig. 27. Each per cent of consolidation value is replaced by the ultimate settlement times that per cent of consolidation. The time factor, T, scale is converted to a time scale using the proper values of c_v, and H. An example of a settlement analysis as well as a detailed discussion of consolidation theory is given in Ref. 28. References 10, 30, and 25 also contain a discussion of the theory of consolidation and settlement analysis.

VII. SLOPE PROTECTION

1. General. The slopes of earth embankments must be protected against erosion by wave action, rainwash, frost, and wind action. Protection cannot economically be provided against the erosion which would occur if the dam were overtopped. A major cause of earth-dam failures has been overtopping; therefore, care must be exercised that freeboard and spillway capacity are ample to prevent it. A detailed discussion of slope protection is given in Refs. 22 and 38. Unfortunately, only meager data are available for a basis of design of slope protection. The procedure described below represents best practice in 1950.

2. Wave Action. The height and velocity of waves are functions of fetch and wind velocity. Fetch is the clear-water distance from the dam to the opposite shore. Generally the straight-line distance is used. However, where slight bends in the measured line will lengthen the fetch, such bends may be incorporated according to judgment. The Molitor-Stevenson equation for the height of waves, h_w, is as follows:

For a fetch greater than 20 miles,

$$h_w = 0.17(VF)^{0.5}$$

For a fetch less than 20 miles,

$$h_w = 0.17(VF)^{0.5} + 2.5 - (F)^{0.25}$$

where h_w = height of wave from trough to crest, ft.
V = wind velocity, mph.
F = fetch, statute miles.

For wave heights between 1 and 7 ft, the velocity, v, in feet per second is given approximately by the formula

$$v \approx 7 + 2h_w$$

where h_w is in feet.

3. Freeboard. Freeboard (sometimes termed net freeboard) is the vertical distance between the maximum water surface of the spillway design flood and the crown of the dam. This vertical distance should be ample to prevent overtopping of the dam due to wind setup and wave action.

Wind setup may be computed by the Zuider Zee formula:

$$S = \frac{V^2F}{1{,}400D} \cos A$$

where S = setup above pool level, ft.
V = wind velocity, mph.
F = fetch, miles.
D = average depth of water, ft.
A = angle of incidence of waves.

The height of waves above pool level plus the run-up of waves on the face of the dam may be computed by the formula:

$$\text{Height of wave action} = 0.75h_w + \frac{V^2}{2g}\, 1.5h_w$$

The relationship between height of wave action and fetch for wind velocities of 50, 75, and 100 mph is shown graphically on Fig. 29.

The distance generally used for freeboard is the sum of the setup plus the height of wave action. In some instances, however, an additional foot or so may be added according to judgment. The freeboard distance so computed is usually greater than 5 ft and thus usually greater than the depth of frost penetration. If the depth of frost penetration is greater than the above computed distance, it should be used for the freeboard distance.

4. Slope Protection. The types of slope protection used on earth dams and levees vary from sod to several feet of dumped riprap. The choice of slope protection for a particular slope depends upon the type of erosive action against which the slope is to be protected and the frequency of attack. The discussion below takes up the various types of slope protection which may be considered for the upstream face of a dam, for the downstream face of a dam, and for levees.

The upstream slopes of dams are commonly protected with dumped riprap or its equivalent. The thickness of the riprap may vary from 1.5 to 5 ft, depending upon the severity of wave action. Suggested values for thickness of riprap for various fetches and wind velocities are given in Fig. 29. The riprap should be founded on a filter designed as described in Art. V, 4. The thickness of the filter should be about half the thickness of the riprap. The stone used for riprap should be hard and durable and be able to resist long exposure to weathering. Stone suitable for riprap generally should be able to pass the standard soundness tests used for concrete aggregate. However, occasionally rock which casehardens will not pass soundness tests and yet be satisfactory for riprap. Fifty per cent of the stone by weight should have diameters about equal to the proposed thickness of riprap. The remaining 50 per cent should be graded downward as quarry conditions permit to enable reasonable design of the underlying filter bed. The riprap should extend from the top of the dam to about

5 ft below the lowest normal pool level. To prevent raveling at the lower end, the riprap is usually abutted against a large stone embedded in the embankment. Also a berm is frequently located at the lower end of the riprap to assist in preventing raveling and to facilitate construction of the riprap. Where rock is expensive, a layer of hand-placed riprap about half the thickness of the dumped riprap may prove more economical, the cost of the additional labor required being compensated by the saving in quantity of rock used. Run-up of waves on the relatively smooth face of hand-placed riprap is greater than on dumped riprap. Also, dumped riprap can accommodate itself somewhat better than hand-placed riprap if any settlement of the dam should occur. Other alternatives which may be considered where rock is expensive are cast tetrahedron blocks of concrete, or ceramic blocks. Also, slope protection consisting of continuous concrete pavement has been used in several instances, *e.g.*, McKay Dam in Oregon and Lake Babcock in Nebraska. L. F. Harza suggests that the concrete pavement be 8 in. thick and have steel reinforcement at mid-depth of slab consisting of bars running parallel to the dip and strike of slope, each at 0.5 per cent. No filter blanket is recommended, although if cracks occurred leaching would result. If an articulate concrete pavement or one with joints is used, a filter bed is required. Porous concrete is not suitable for slope protection. Since only limited experience is available in connection with concrete slope protection, careful study and great care must be exercised if its use is contemplated.

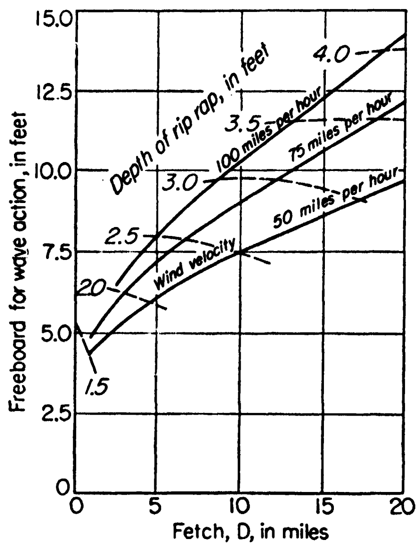


FIG. 29.—Relationship between fetch, freeboard for wave action, and thickness of dumped riprap for slope protection.

The downstream face of an earth dam is usually protected with sod where the climate is suitable. In arid climates or other climates where sod cannot grow, a thin layer of stone or gravel riprap may be used. To develop a sod, 4 to 6 in. of topsoil is spread on the downstream face and the face seeded and strip-sodded. The grass or vine used for making the sod will depend on local conditions. To prevent the accumulation of large quantities of rainwater at the bottom of the slope of high dams, berms which intercept the runoff may be located on the slope. Care must be exercised in designing and maintaining the berms to see that the water which they intercept is safely conducted away. Culverts are considered more satisfactory for collecting and removing this water than pipes. Where riprap is used on the downstream face, its thickness and permeability should be ample to conduct safely the rain water flowing down the slope. The stability of such riprap against sloughing caused by the seepage force of the rainwash can be analyzed in a manner similar to the method discussed in Art. VI, 3. Where the downstream slopes of earth dams are subject to tail water, they should be protected by riprap.

Both the riverside and landside slopes of levees are commonly protected with sod. Where river currents would cause erosion of the sod, as at the outside of bends, dumped riprap slope protection or its equivalent may be used. Other types of slope protection that have been used on levees include articulate concrete mats, asphalt pavement, and willow mats.

Slope protection accounts for an appreciable percentage of the cost of earth dams and levees. **Further study and careful field observations are urgently required to permit closer design of slope protection, especially that used for protection against wave or current action. New and cheaper ways of constructing slope protection should be explored.**

VIII. SUBSURFACE INVESTIGATIONS

1. General. Many methods are available for determining the nature of subsurface conditions at a dam or levee site. Methods for both soil and rock exploration are discussed in this article. The methods for soil exploration consist of these types: (1) indirect methods wherein no or only practically worthless samples are obtained, (2) methods wherein representative but disturbed samples are obtained, methods in which undisturbed samples are obtained. Seepage tests in soil and pressure tests in rock are also used. A general discussion of the program of subsurface investigation for an earth dam or levee (the type, spacing, depth, sequence of drill holes, etc.) is given in Art. I, 3. A thorough discussion of subsurface exploration programs and methods is given in Ref. 12. Only the more common and important methods of exploration are described below.

2. Indirect Methods. The *seismic method* is the most extensively used of the various geophysical methods in which no samples are obtained. It is described in detail in Refs. 31 and 23. It consists essentially in observing the time required for shock waves, set off by small explosive charges located at various distances from a detector, to reach the detector. The waves travel either entirely through the soil overburden, or down through the overburden to rock, through the rock, and then up through the overburden to the detector. From observations of the time required for the shock waves to reach the detector by the two paths, the depth to rock can be computed. The figure so determined frequently checks the depths obtained by drilling, to within 1 or 2 ft. In some instances determination of the depth to ground-water table, or to the top of an underlying soil stratum definitely different from the overlying soil, can be made.

The *electrical resistivity* method is similar to the seismic method except that electric current is used instead of shock waves. It is described in Ref. 31. The method has been used mainly for locating underlying gravel beds.

3. Representative but Disturbed Sample Methods. *Auger or posthole diggers* are frequently used for soil explorations of shallow depth particularly in borrow areas. Due to the augering action the natural structure of the soil is completely disturbed. However, the water content and relative proportion of the various soil constituents remain unchanged. The auger or posthole digger method is one of the few economical methods where samples can be obtained at their natural water content.

Drive (or dry) sample borings are very commonly used in dam and levee subsurface exploration work. The drill hole in which drive samples are obtained is generally put down by wash boring methods, although at times auger or churn drill methods are also used. Casing is used as required to maintain the walls of the drill hole. The drive sample is obtained by driving a thick wall sampler 1 to 3 ft into the natural soil below the bottom of the drill hole. Samples may be taken continuously or at each change in soil strata, but at intervals no greater than 5 ft. A split barrel sampler similar to the one shown in Fig. 30 should be used. The sampler should be equipped at the top with a check valve and, when required to sample loose cohesionless soils or soft cohesive soils, it should be equipped at the bottom with a trap. The sampler may vary from 1⅜ in. I.D. and 2 in. O.D. to 2½ in. I.D. and 3 in. O.D. The larger size is desirable especially for sampling soils containing gravel. Record should be kept of the hammer weight and drop and the number of blows required for driving the sampler. As

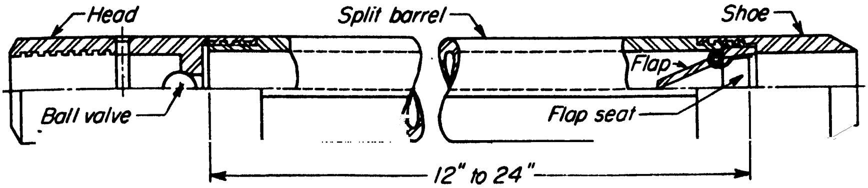


FIG. 30.—Split barrel sampler.

described on pages 294 and 300 of Ref. 29 and page 71 of Ref. 4, such data may be used to infer the relative density of cohesionless soils and the relative consistency of cohesive soils. Continuous-drive sample boring is advantageous for exploring borrow areas where gravel in the soil prevents the use of auger borings. Such drill holes should be put down exclusively by the process of continuous sampling. Wash boring methods should be avoided so that samples of material above the groundwater table may be obtained at their natural water content.

In connection with auger or continuous-drive sample borings in borrow areas, it is desirable to install observation wells and note the variation in groundwater table level over a complete cycle of seasons.

4. Undisturbed Soil Sampling. The *Denison double-tube core barrel*, shown in Fig. 31, is excellent for obtaining undisturbed samples of soils, especially those in a relatively compact state. The Denison sampler can obtain specimens of practically all soils except gravelly uncemented soils. The sampler consists of an inner core barrel and liner which remain stationary, and an outer core barrel and toothed bit which revolve. The length of bit used can be varied so that the cutting edge of the inner core barrel can follow, be flush with, or lead the bit as required to obtain the best sample. At the top of the inner core barrel is a cage check valve and at the bottom a basket spring trap. The diameter of the sample obtained is 6 in. and therefore ample for all laboratory testing. The sample is shipped to the laboratory in the liner. The drill hole is put down by coring with the sampler, and continuous soil cores are obtained. Drilling fluid instead of clean water is used in the drilling to minimize the use of casing which is expensive to handle in the large sizes required.

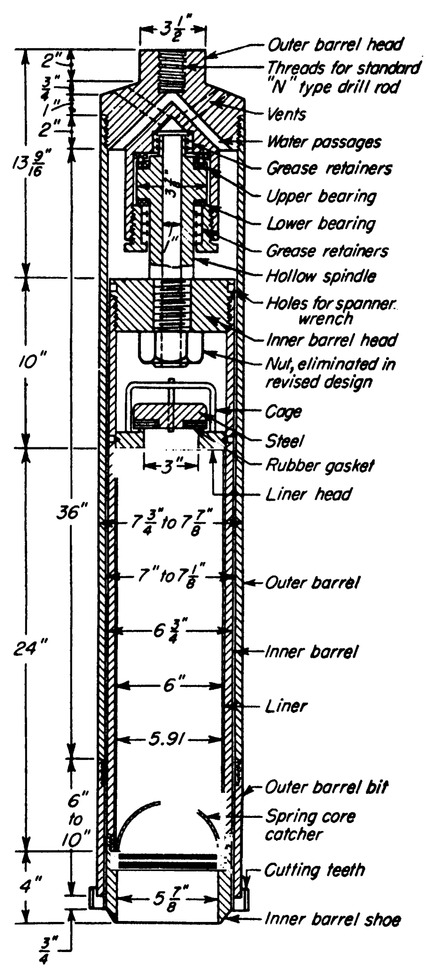


FIG. 31.—Denison double-tube core barrel.

The *stationary piston-type thin-wall sampler*, shown in Fig. 32, is suitable primarily for sampling soft cohesive soils and also some fine sands. Generally, soils which can be sampled with it are not suitable for dam foundations, although by use of sand drains such soils may conceivably be consolidated and made suitable. Levees, because of their lower height, may be founded on such soils, and in this case this type of sampling is very valuable. As in the case of drive sample borings, the drill hole is put down by the wash boring process. Before sampling, the drill hole is carefully cleaned of all disturbed soil. The sampler is lowered to the bottom of the drill hole

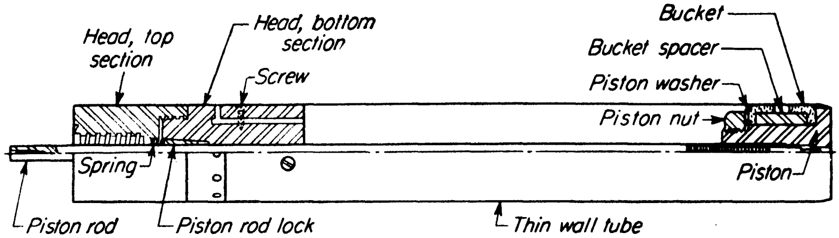


FIG. 32.—Stationary piston-type sampler.

with the piston held fixed at the bottom of the sampler. The piston rod is then held stationary with respect to ground surface and the thin-wall sample tube forced into the natural soil at a rate of about 1 fps. The piston is then again made fixed relative to the sample tube and the sampler withdrawn. After capping the sample tube, the sample is shipped to the laboratory. The sample is suitable for all laboratory testing. For average soil conditions, the cutting edge of the thin-wall tube should be about 0.7 to 1.5 per cent less in diameter than the inside of the tube in order that friction between the sample and walls of the tube may be reduced.

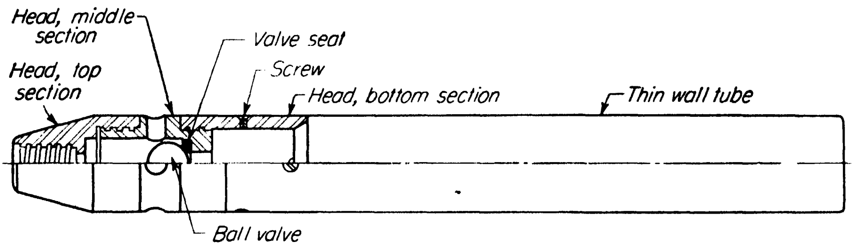


FIG. 33.—Thin-wall (Shelby tube) sampler.

The *thin-wall tube* (*Shelby*) *sampler*, shown in Fig. 33, is used for sampling soils similar to those sampled by the fixed-piston type of sampler. It is not so elaborate as the piston-type sampler and is usually smaller in diameter, and somewhat less undisturbed samples are obtained. Its cutting edge should be about 0.7 to 1.5 per cent less in diameter than the inside diameter of the tube. Also a check valve should be located at the top. The samples obtained are satisfactory for unconfined compression tests and triaxial shear tests, but are rather small for direct shear and consolidation tests.

Undisturbed samples from test pit or trench are obtained by careful carving from a bench at the bottom or side of the pit or trench. A detailed description of the procedure is given on pages 372–383 of Ref. 12. This method is about the only one suitable for obtaining undisturbed samples of soils with gravel or boulders. Practically any size or shape of sample can be obtained, but the depth to which the method can be used is limited by the depth of the test pit or trench.

5. Field Permeability Tests. These tests are of particular value in determining the in-place permeability of granular soils for which it is difficult, if not impossible, to obtain undisturbed samples. Two general types of seepage tests may be performed. One type is performed by utilizing a single bore hole. In the other a central bore hole and several surrounding observation holes or wells are used.

In the single-hole type of seepage test, casing is driven to the bottom of the bore hole and the hole cleaned out flush with the bottom of the casing. Any of three procedures may be used: the rising, the falling, or the constant-water-level procedure. In the rising-water-level procedure, the water level in the bore hole is lowered by pumping or bailing to below the ground-water table, and the natural rate of rise of the water in the bore hole is observed. If the material at the bottom of the bore hole becomes quick in such a test, the falling-water-level procedure should be used. In this case the water level in the drill hole is raised above the ground-water table and the rate of fall of the water level observed. In the latter case, care must be exercised to see that no fine material is suspended in the water in the drill hole. Upon settling, such material would form a relatively impervious skin at the bottom of the drill hole and vitiate the results of the test. The equation for computing the permeability from either of the above types of observations is as follows:

$$k = 1.8 r_0 \frac{\log_{10} h_1 - \log_{11} h_2}{t_2 - t_1}$$

where r_0 = inside diameter of casing.
h_1 and h_2 = difference in elevation between water level in casing and natural ground-water table at times t_1 and t_2, respectively.

Where the permeability of the soil being tested is high and the rising- or falling-water-level procedures described above would not be satisfactory because of the fast rate at which the water level would change, the constant-water-level procedure may be used. In this procedure, observation is made of the rate at which water must be added to the bore hole to maintain the water level at constant level, such as the top of the casing. The coefficient of permeability is computed from the following formula:

$$k = (0.25 \text{ to } 0.5) \frac{q}{h r_0}$$

where r_0 = inside diameter of casing.
h = difference in elevation maintained between water level in casing and natural ground-water table.
q = rate of pumping into or out of casing to maintain constant water level in casing.

In the method wherein observation wells in a radial pattern from a central bore hole are used, water is pumped either into or out of the central hole at a constant rate. The central hole should extend to the full depth of the pervious stratum, also the casings for the observation wells should be perforated. Observations are made of the water levels in the observation wells and, when flow conditions become steady, the permeability is computed by the following formula:

$$k = \frac{q \ln \frac{r_2}{r_1}}{\pi(z_2^2 - z_1^2)}$$

where q = rate of pumping.
z_1, z_2 = heights of water above bottom of pervious stratum in observation wells at radial distances of r_1 and r_2, respectively, ($r_1 < r_2$).

The observation well method gives the average permeability of the material between the wells and is the more reliable. Methods of utilizing only a single bore

hole give the permeability of the material in the immediate vicinity of the bottom of the drill hole.

6. Rock Drilling. *In subsurface investigations for a dam, it is essential to determine the depth to bedrock and the nature of the bedrock.* Whenever rock is encountered, it should be core-drilled to determine whether it is actually bedrock or boulders. Usually 2⅛-in. (NX) cores obtained with a diamond bit are satisfactory. Occasionally smaller diameter cores may be used. In some instances, 24 or 30 in. diameter holes have been put down by calyx drill methods. These latter holes are large enough to permit a man to be lowered into them for inspection of the rock *in situ*. Double-tube core barrels with core catcher and check valve should be used. Also a drill with hydraulic feed is preferred to one with screw feed. The rate of progress of a drill under a particular pressure is an indication of the nature of the rock. The percentage of core recovery as well as the location of any faults such as soft pockets should be carefully noted.

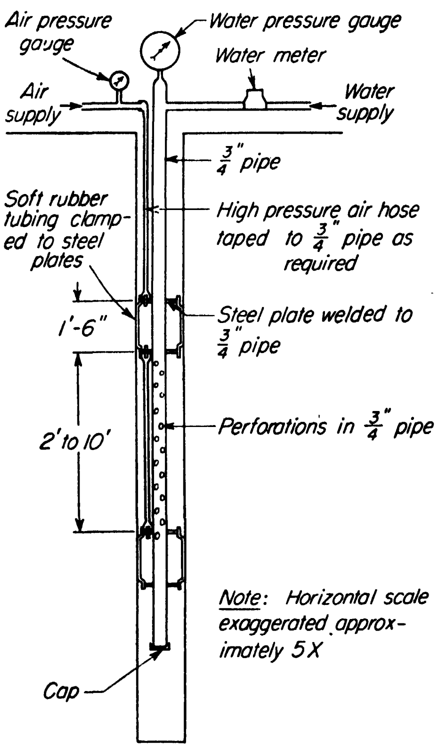


FIG. 34.—Pressure testing apparatus.

7. Pressure Tests. Information on the soundness of rock *in situ* may be obtained by pressure testing. The apparatus used and the methods of testing depend upon the peculiarities of each site. A simple apparatus recently developed is shown in Fig. 34. A more elaborate apparatus is described on page 35 of Ref. 16. In analyzing the pressure test results, it is helpful to have data on the leakage at various pressures, such as 10, 20, and 30 or 15, 30, and 45 psi above natural ground-water pressure.

Besides testing at one of the series of pressures mentioned above, it is also valuable at times to test at much greater pressures to check the possibility of opening plugged seams. The number of pounds per square inch of applied pressure, however, should not exceed the number of feet of overburden. As a rough indication of the order of magnitude of permissible leakages, for NX-size drill holes under 30 psi excess pressure, a loss of 2 gpm for a 5-ft test section would indicate a foundation intermediate between pervious and impervious.

IX. LABORATORY INVESTIGATIONS AND TESTS

1. General. There are three general types of laboratory tests which are performed on earth-dam materials: classification and index tests, compaction tests, and physical property tests. The classification and index tests are simple and are performed on both foundation and borrow materials. They are used to amplify and to impart exactness to the visual classification of materials. Also, they are used to indicate roughly the range in variation of the physical properties of the soil. On the basis of

index tests, samples can be chosen for the more elaborate physical property tests so that the range in variation in the properties of a soil can be covered with a minimum of testing. The index property tests can also be used for determining weighted average soil properties. The compaction tests are used to determine the optimum moisture content for placing fill materials and for predicting the unit weight of such fill. Physical property tests are performed on undisturbed samples of foundation materials and on compacted samples of borrow materials. Generally the design of an embankment is based upon the results of the physical property tests, although at times, values for some of the properties can be assumed with sufficient accuracy without tests. It is essential that physical property tests performed in the laboratory represent field conditions as closely as possible. In this connection it should be noted that it is very difficult to saturate laboratory compacted test specimens as completely, as the embankment material will ultimately be saturated in the field. Also, it is impossible to take undisturbed samples without some amount of disturbance. Thus judgment has to be used in choosing from the test results the proper values for the physical properties to be used in the design.

Standard test procedures have been set up for the classification tests and compaction test. The physical property tests are not standardized but are generally performed in about the same way by the various soil-testing laboratories. It is difficult to standardize the property tests because research is continually developing improved techniques for testing and because projects frequently require special modifications for the particular soil or conditions encountered at the site. Because soil property tests are not standardized, it is essential that the designer know the particular test procedure used in each case so that he can properly apply the results of the tests. Because of space limitations, only the more common and important soil tests can be mentioned in this section.

2. Classification Tests. *Natural Water Content.* The water content of a soil is expressed as a percentage and is defined as the ratio of the weight of water in the soil specimen (difference between the natural weight of the soil specimen and the oven-dry weight) to the weight of soil particles (oven-dry weight) times 100. The natural water content of a soil is of general assistance in soil classification and is useful as an index test for the shear strength of normally consolidated clays. The relation between the natural water content of a soil and the optimum water content for compaction is a major consideration in choice of borrow materials.

Unit Weight. The unit weight of a soil may be determined by any of several methods wherein the weight and volume of a particular mass of soil are determined. The natural unit weight is frequently useful in classification of foundation soils. The unit weight of soil particles (alternatively, the specific gravity of soil particles) is required for void ratio or porosity determinations, and is at times useful in connection with classification. Knowledge of the unit weights of embankment and foundation soils in the dry, drained, saturated, and submerged conditions is required in structural and hydraulic design.

Grain-size Distribution. The grain-size distribution of a soil is determined by sieves for particles larger than about 0.07 mm and by hydrometer or some type of sedimentation analysis for particles finer than 0.07 mm. Standard test procedures are given on pages 45–55 of Ref. 1. The results of grain-size analyses are usually presented as a plot of grain sizes vs. per cent finer by weight as shown on Figs. 2 and 17. Grain-size analyses are useful for classification of cohesionless soils but rarely for plastic soils. They are required for filter design.

Liquid and Plastic Limits. The liquid and plastic limit tests are performed on plastic soils and comprise the standard method of classification of such soils. The liquid limit, w_L, is the water content of the soil at an arbitrarily defined borderline

condition between the semiliquid and plastic states. The plastic limit is the water content of the soil at an arbitrarily defined borderline condition between the plastic and solid states. Standard procedures for test are given on pages 56–60 of Ref. 1. *The plasticity chart, Fig. 4, page 919 of Ref. 6, is generally used as a basis for classification. The liquid limit has been found (Ref. 29, page 66) to be directly related to the* compressibility of normally consolidated ordinary clays. Normally consolidated clays are those which have never been subjected to a pressure greater than their present overburden pressure. The liquid limit is related to the compressibility by the equation:

$$C_e = 0.009(w_L - 10\%)$$

Chemical Analysis. At present, chemical or mineralogical tests are performed on earth-dam materials to determine the presence of soluble minerals or of highly swelling clay minerals as montmorillonite. With the further development of the chemistry of soils and the chemical treatment of soils, it is likely that more tests of a physical chemical nature will be made in the future.

Relative Density. The relative density test is a classification and index test used for cohesionless soils. The relative density, D_R, is defined as follows:

$$D_R = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

where e = ratio volume of voids to volume of soils.

For determining the maximum void ratio, the soil, in oven-dry condition, is gently poured from a funnel at low height into a container of known volume. The minimum void ratio is determined according to the modified A.A.S.H.O. procedure mentioned in the following paragraph, or by some other procedure, perhaps utilizing vibration, which effects a greater density. In reporting relative density results, the procedure used for determining the minimum void ratio should be stated.

3. Compaction Tests. Laboratory compaction tests are performed to determine the optimum water content for compaction of a particular soil. The tests are generally performed either by the Proctor procedure described on pages 73–75 of Ref. 1 or by the modified A.A.S.H.O. procedure described below. In both instances, the tests are performed on only the portion of the soil passing the No. 4 (4.76-mm opening) sieve and the optimum water content refers to the water content of this material. If the percentage of material coarser than the No. 4 sieve is small and if the coarser material occurs as plums in a matrix of the finer material, the unit weight determined by the compaction tests may be adjusted to take the plums into account. However, if the percentage of material coarser than the No. 4 sieve is large and if the coarse particles do not occur as plums, other means of tests have to be devised. In both the above-mentioned standard tests, the soil is rammed in layers into a cylindrical mold of $\frac{1}{30}$ cu ft. In the Proctor test, three layers are compacted with 25 blows of a 5-lb rammer falling 12 in., in the modified A.A.S.H.O. five layers are compacted with 25 blows of 10-lb rammer falling 18 in. The modified A.A.S.H.O. procedure generally gives results more in agreement with the field results of 1950 field compaction equipment. A comparison of the results of the two procedures is presented in Fig. 35. The greater compactive effort of the modified A.A.S.H.O. procedure produces a greater density at a lower optimum water content. The results of a penetration test performed with the standard proctor needle are presented in Fig. 35. Where the earth fill does not contain stones, the proctor needle provides a simple empirical means for checking the field compaction. A discussion of the correlation between laboratory and field compaction and the effect of compaction, above and below optimum moisture content on soil strength, is given in Ref. 32.

4. Physical Property Tests. *Permeability.* As indicated on Fig. 8, the permeability of coarse-grained soils is generally determined by constant head permeability tests and of fine-grained soils by falling head permeability tests. Indirect determination of the coefficient of permeability for clay soils may be obtained by computation from consolidation tests and from the horizontal or vertical capillarity tests mentioned in the next paragraph. In the constant head type of permeability test, the head difference between the ends of the sample is maintained constant and the quantity of flow measured. The permeability is computed from the equation given in Art. V, 1. In

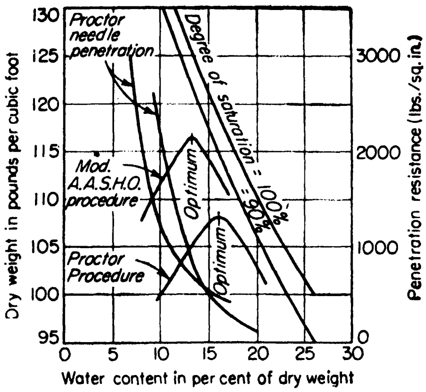


FIG. 35.—Results of compaction tests.

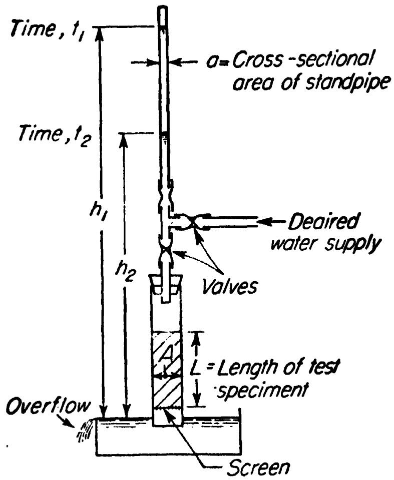


FIG. 36.—Schematic sketch of variable head permeameter.

the variable head test, the sample is connected to a headwater standpipe, as shown in Fig. 36. The coefficient of permeability is computed from the following equation for which the terms are defined in Fig. 36:

$$k = 2.303 \frac{aL \log_{10} \dfrac{h_2}{h_1}}{A(t_2 - t_1)}$$

In determining the permeability of saturated soils, care must be exercised so that air bubbles are not entrapped in the test specimens. By using air-free water prepared by boiling and by having the water decrease in temperature while flowing through the soil, discrepancies due to entrapped air in the sample can be kept to a minimum. A general discussion of permeability testing is given on pages 119–122 of Ref. 25.

Capillarity. The capillary head of a soil may be determined by the Beskow procedure described on page 200 of Ref. 1 or by the horizontal or vertical capillarity tests described on pages 146–151 of Ref. 25. The height of capillary rise is used in connection with determination of the breakout point of water seeping through an embankment and in connection with rapid drawdown analysis of fine-grained soils.

Unconfined Compression Test. The unconfined compression test is a simple compression test performed on undisturbed or compacted cylindrical samples of cohesive soil. It is used to determine the approximate shear strength of the soil for the particular condition of the soil at the time of test. The shear strength is approximately equal to $0.5p$, where p is the unconfined compressive strength. The unconfined com-

pression test may also be used as an index test for choosing samples for the more elaborate triaxial shear tests.

Triaxial Shear Tests. The triaxial shear tests furnish the most complete information on the shear strength of soils and the changes in volume and pore pressures that occur in soils during shear. As illustrated in Fig. 37, the test is essentially a compression test on a cylindrical specimen of soil which is encased in a rubber sheath and subjected to an all-round pressure. By increasing the compressive load, shear failure can be brought about on planes making angles of roughly 60 deg with the horizontal plane. Many different procedures of test are possible. Either full consolidation, partial consolidation, or no consolidation may be permitted the specimen under the applied all-round (chamber) pressure, and full, partial, or no drainage may be permitted the specimen during shear. Pore pressures that develop may or may not be observed. The three most common tests used for earth-dam analysis are the con-

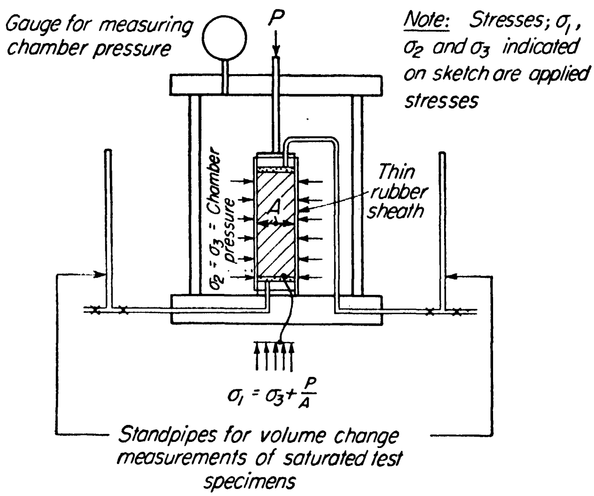
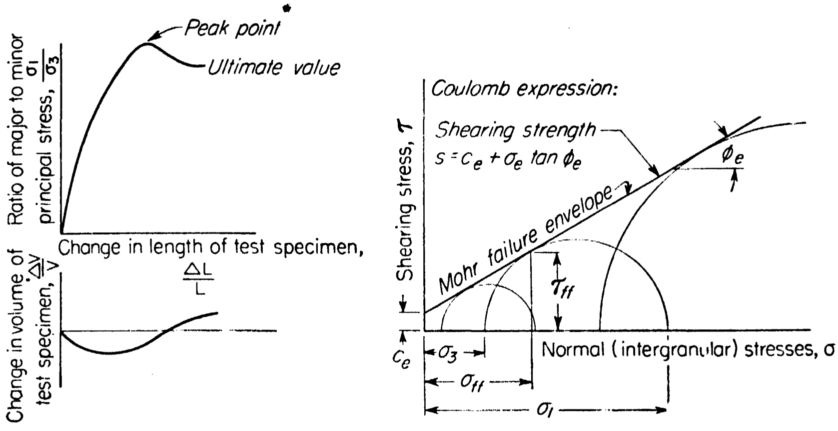


FIG. 37.—Schematic sketch of apparatus for triaxial shear test.

solidated drained test (consolidated slow), the consolidated undrained (consolidated quick), and the partially consolidated, undrained test (unconsolidated quick). The consolidated drained tests are used for saturated soils which will be free draining in nature and for dry or partially saturated soils in which pore pressures will not develop. The results of such a test are shown in Fig. 38. In design, the Coulomb equation $s = c_e + \sigma \tan \phi_e$ is used to express the shear strength. For dry or free-draining saturated cohesionless materials, c_e equals zero. Consolidated drained tests on clayey soils, such as compacted embankment soils containing air or saturated preconsolidated foundation soils under conditions which permit drainage, would yield results where c would have a value greater than zero. In this case the term σ_e equals σ_{ff}, the normal stress on the failure surface. This relationship is shown on the Mohr diagram, Fig. 38*b*.

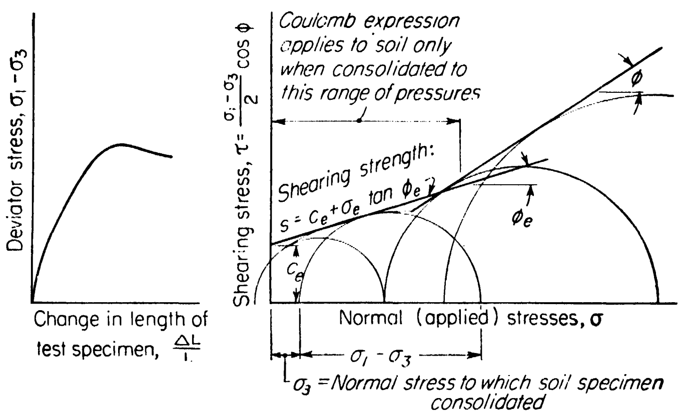
The consolidated undrained test is used for saturated soils which are consolidated under the normal stresses applied to them but which cannot drain during shear. The Coulomb expression for shear strength derived from such tests can be used for stability analysis of dams which have been standing for sufficient time so that the embankment and foundation are both completely consolidated. The results of a consolidated undrained test are given in Fig. 39. In this case, τ must be computed from the expres-

sion $\tau = \frac{\sigma_1 - \sigma_3}{2} \cos \phi$ where the value of ϕ may be obtained from consolidated drained tests or may be assumed (28 to 30 deg for silts and clay soils). The normal stress at a point on the assumed failure plane approximately equals the average of the three



(a) DATA OBTAINED FROM TEST ON SINGLE SPECIMEN (b) MOHR DIAGRAM FROM TESTS ON THREE SPECIMENS

Fig. 38.—Consolidated drained triaxial shear test.



(a) DATA OBTAINED FROM TEST ON SINGLE SPECIMEN (b) MOHR DIAGRAM FROM TESTS ON FOUR SPECIMENS

Fig. 39.—Consolidated undrained triaxial shear test.

principal stresses acting at that point, *i.e.*, it equals approximately the average pressure causing consolidation. In the laboratory, the test specimen is therefore consolidated to a uniform all-round pressure which about equals the average field consolidation pressure, the only difference being that in the laboratory the pressure is applied uniformly. This approximation and difference in application of pressure are believed to have negligible effect on the applicability of the shear strength results. In this case σ_e equals σ_c, the all-round pressure at which the soil is consolidated.

The partially consolidated undrained test is used to determine an expression for shear strength for the case of a foundation consolidated under its original overburden, but not consolidated under the additional load of the embankment. In this test, the soil specimen is consolidated to an all-round pressure equivalent to the pressure to which it has been consolidated in nature, then the all round pressure is increased and the sample sheared undrained. The shear stress τ is computed as for the consolidated undrained test. The plot of τ is made against the all-round pressure used for the

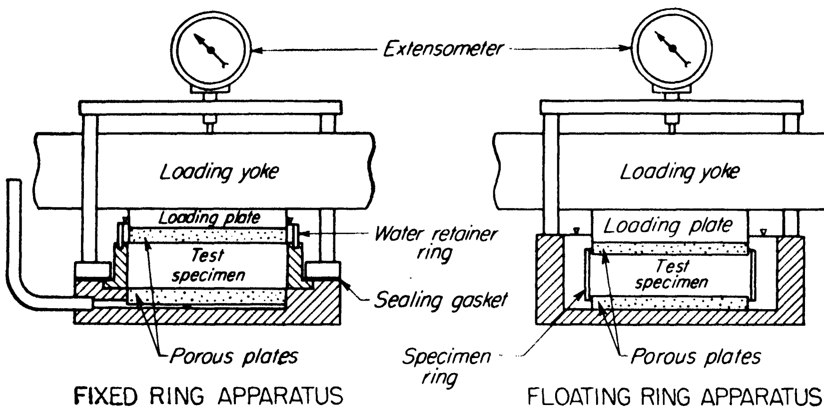


FIG. 40.—Consolidometers.

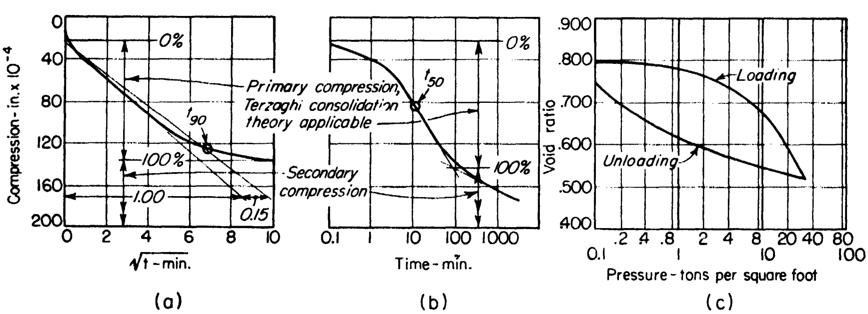


FIG. 41.—Consolidation test data.

shear test, and it is this pressure which is used for σ_e in the Coulomb expression for shear strength. For use in stability analysis σ_e of the Coulomb expression should be the normal stress on the failure plane. Thus some error, probably more than in the consolidated undrained case, is introduced in assuming that $\sigma_e = \sigma$ all-round. As an alternate to the use of the Coulomb expression for the partially consolidated undrained shear strength, a procedure employing the *in situ* shear strength may be used as described on pages 395–399 of Ref. 25.

Direct Shear Test. In this test a thin specimen of compacted or undisturbed soil is sheared along its mid-plane in a shear box, which consists of an upper and a lower frame. A normal load is applied to the specimen across its mid-plane, and then shearing is effected by relative displacement of the upper and lower frames. Because it is impossible in this test to control drainage during shear and to make reliable measurement of volume changes during shear, this test has been more or less replaced by

the triaxial shear test. However, it is much simpler than the triaxial shear test and does have advantages in certain tests, such as drained shear tests on clays.

Consolidation Test. This test is used to determine the consolidation and compressibility characteristics of a soil for use in settlement analyses. The soil to be tested is placed in a consolidometer, as shown in Fig. 40. Compressive loads are applied to the specimen in a sequence about as follows: 0.125, 0.25, 0.5, 1.0, 2.0, 4.0, 8.0, 2.0, 0.5, 0.125 kg/cm^2. Each load is generally maintained for one day and the time rate of compression under each load determined. Typical time rates of compression curves are shown plotted to a square root of time scale and a semilog scale Fig. 41*a* and *b*. A pressure void ratio curve plotted to semilog scale is shown in Fig. 41*c*.

X. FIELD OBSERVATIONS DURING AND AFTER CONSTRUCTION

Several types of field observations should be made during the construction of an earth dam and after its completion. One series of observations consists of controls to confirm that the fill is placed so that it has the density and physical properties required by the design. Field density measurements are made at regular intervals, one for about each 3,000 cu yd in the fill. Since field compaction is somewhat different from laboratory compaction, undisturbed samples of the fill should also be taken and the actual physical properties of the fill determined.

When a dam or foundation contains soil which will consolidate under the weight of the embankment, the rate of construction of the embankment frequently has to be controlled so that the rate of consolidation and the corresponding rate of increase in shearing strength of the soil are not exceeded by the rate of load application. In such instances piezometers or other pore-pressure-measuring devices may be installed in the consolidating soil and the rate of consolidation observed by the rate of decrease in hydrostatic excess pore pressure.

When a dam undergoes settlement due to consolidation of a soil in the foundation or embankment, allowance has to be made for this settlement in the height to which the embankment is constructed. To check the predicted rate of settlement, observation of the actual settlement in the field should be made. For the case where most of the settlement is caused by consolidation in the foundation settlement disks may be set at the top of the foundation and the settlement of the top of the foundation observed. The settlement caused by the foundation and enbankment may thereby be separated. Such observations also permit more accurate computation of the actual yardage placed in the embankment.

Failures in earth embankments or foundations are preceded usually by movements of such magnitude as to indicate clearly the probability of failure. Careful daily inspections of the embankment should be made and, if the possibility of failure is suspected, hubs should be set in the critical areas and systematic observation made of their horizontal and vertical movements. In cases of foundation failure, a rapid increase in settlement is evident immediately after initial overstressing of the foundation, and soon afterward the ground near the toe of the embankment commences to heave. Embankment failures are indicated in advance by bulging of the slopes. When there is any question as to the safety of the embankment, raising of the embankment should be immediately stopped and berms added or the slopes flattened for stabilization. Careful analysis of the situation and perhaps redesign are required before construction of the dam is resumed.

Great economic benefits are possible from development of more accurate methods of design of earth dams. The accuracy of any design methods depends entirely upon the accuracy with which the assumptions and theories used in the design represent field conditions. In essence, design consists of methods of predicting field behavior

and then apportioning the structure so that it behaves as desired. Observations of actual field behavior of an embankment and foundation are therefore essential for checking present design methods and for developing better methods. Whenever possible, opportunity should be taken to observe the stresses, strains, and seepage conditions in an embankment and its foundation. Such observations to be of value must be carefully planned, executed, and analyzed. The program of observations thus extends from the design stage through the construction stage and some time into the period after construction; they are best carried out by an organization having control of all three stages. Methods of making and analyzing observations of this type are discussed in Ref. 27.

XI. TYPICAL DESIGNS OF EARTH DAMS

1. Germantown Dam, Miami River, Ohio. This dam is illustrative of good practice in hydraulic-fill dams, with favorable materials available for construction. The foundation is largely a glacial till well graded from gravel to silt except for somewhat more pervious materials in the valley bottom. A cross section of this dam is shown in Fig. 42.

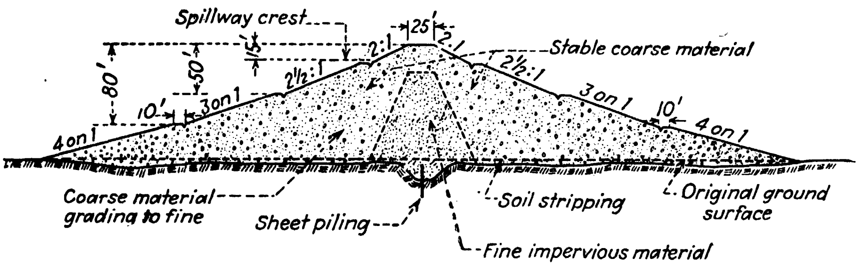


FIG. 42.—Cross section of Germantown Dam.

The core thickness at any elevation is equal to the height of dam above that elevation. The outlets were founded on rock in one abutment, and the spillway was carried through a saddle some distance from the dam.

2. Santiago Dam, Orange, Calif. This dam consists of a rolled fill made with favorable materials. Overburden in the foundation consisted of a surface layer of 5 ft of silty material underlain by 30 ft of gravel. Bedrock was sandstone and shale. The overburden material on the hillsides produced a well-graded and structurally strong impervious material. The river bed supplied an ample quantity of good pervious gravel. The design of this dam is illustrated by Fig. 43.

Since water was very valuable in this case, the impervious section was carried to rock. The impervious section, because of its structural stability, could safely be placed on the upstream side, which in turn permitted easy connection to a blanket necessary on the valley wall. Under the downstream pervious section, the natural blanket was stripped to pervious material to assure drainage of the foundation and thereby permit a downstream slope of 1 on 2.

3. Piedmont Dam, Stillwater Creek, Ohio. This dam illustrates most unfavorable conditions. The foundation consisted of 40 ft of poorly consolidated silt and clay weak in structural strength. Further, except for a limited quantity of sandstone rock, the only material available for the embankment was weathered shale and clay overburden. A cross section of this dam showing the flat slopes necessary to ensure the stability of the embankment and foundations is presented in Fig. 44. The outlets

were carried in a tunnel through one abutment, and the spillway was cut through a saddle remote from the dam.

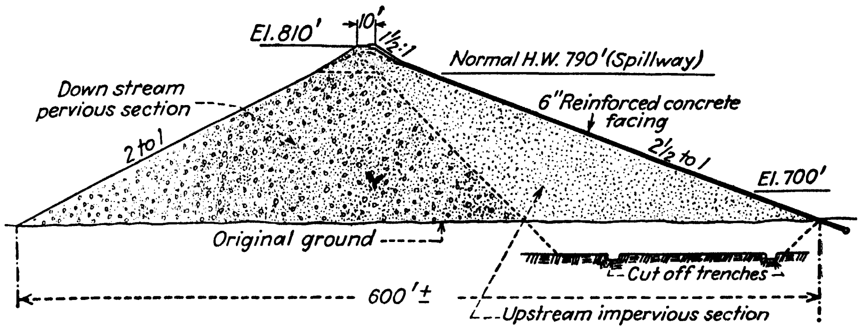


FIG. 43.—Cross section of Santiago Dam.

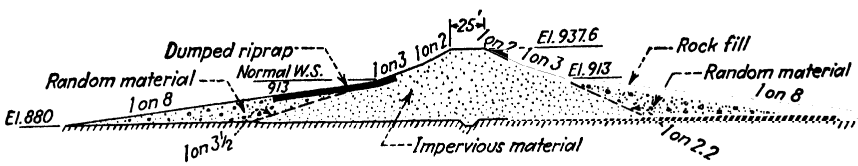


FIG. 44.—Cross section of Piedmont Dam.

4. Mohawk Dam, Walhonding River, Ohio. This illustrates a high dam on deep pervious foundations (Fig. 45). The depth to bedrock across the valley bottom is 150 ft; the overburden consisted mostly of sand and gravel with a thick blanket of silt except in the stream bed. The central section of the dam is composed largely of structurally strong impervious material. An impervious blanket connecting with the impervious section extended 625 ft upstream from the center line. The purpose of this blanket was to reduce seepage by lengthening the path; an impervious cutoff of 150 ft to rock was not justified economically.

On the downstream side, the design was directed toward controlling the large seepage quantities anticipated, 30 to 50 cfs. This was handled by a pervious downstream shell section and berm on the downstream side. Care was taken to strip the natural impervious blanket to the underlying gravel, before placing these sections. This design was developed to relieve, without danger of piping, the hydrostatic pressures under the dam and to avoid any uplift pressures downstream from the toe. At the present time, drainage wells would be considered for controlling such underseepage and pressures.

The outlets for this dam were in a tunnel, and the spillway was through a natural saddle which had to be greatly enlarged by excavation. All the rock fill and a large part of the earth fill came from required excavations.

5. Mud Mountain Dam, White River, Washington. This dam is noted for being the highest rock-fill dam built up to 1950 and for having an impervious core made of processed borrow material. The dam is 425 ft high and is located in a narrow box canyon. The crest length of the dam is 700 ft whereas the base width is 1,600 ft. A cross section of the dam is shown on Fig. 46. The yardages in the dam are 427,000 cu yd of rolled impervious earth core, 143,000 cu yd of transition material, and 1,539,000 cu yd of rock fill. Available earth borrow consisted of an upper stratum of blue

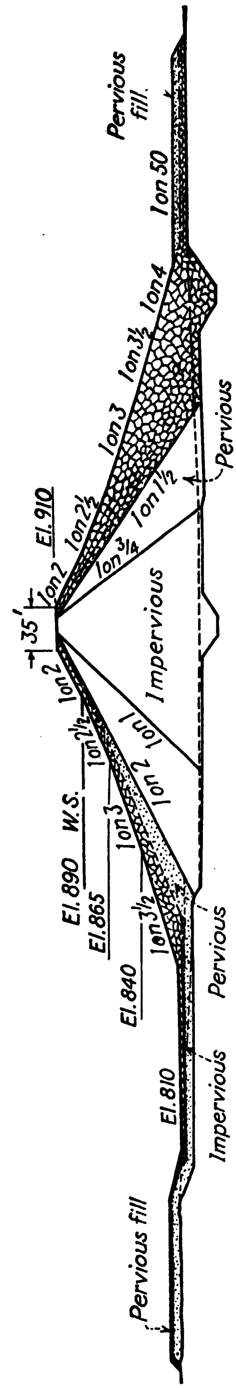


FIG. 45.—Cross section of Mohawk Dam.

glacial till underlain by sand and gravel with clay. The natural water content of the borrow material was above the optimum for compaction and could not be reduced sufficiently by drainage to permit proper construction. A drying and mixing plant was set up to produce a mixture at about the optimum water content consisting of sand and gravel with about 20 to 40 per cent of the glacial till. To prevent wetting of the fill during the rainy season, the earth fill operation was performed under a huge tent covering the canyon. The impervious core and the transition zones flanking it were founded on bedrock whereas the rock shells were founded on the natural stream-bed material. The transition zones grade from fine next to the impervious core to coarse next to the dumped rock fill. It was specified that each cubic yard of dumped rock

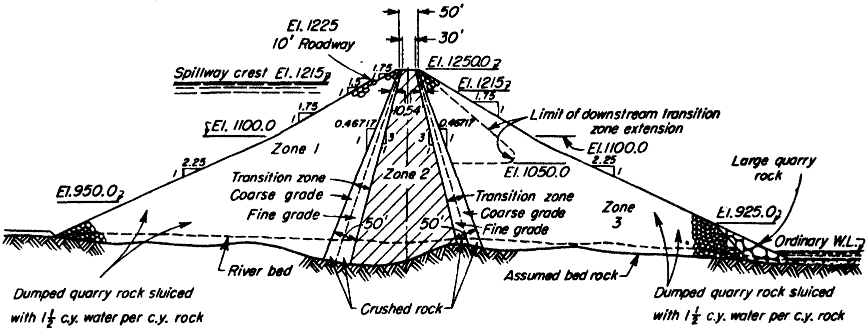


FIG. 46.—Cross section of Mud Mountain Dam.

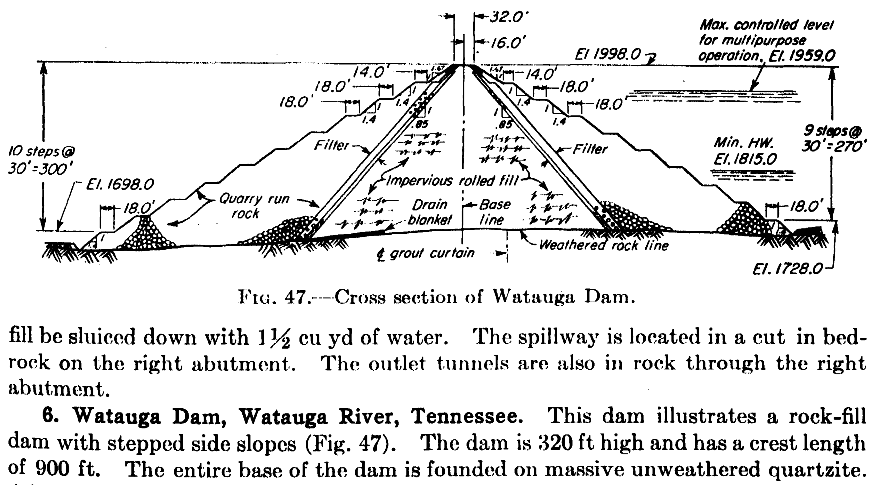


FIG. 47.—Cross section of Watauga Dam.

fill be sluiced down with 1½ cu yd of water. The spillway is located in a cut in bed-rock on the right abutment. The outlet tunnels are also in rock through the right abutment.

6. Watauga Dam, Watauga River, Tennessee. This dam illustrates a rock-fill dam with stepped side slopes (Fig. 47). The dam is 320 ft high and has a crest length of 900 ft. The entire base of the dam is founded on massive unweathered quartzite. A large fault passes under the upstream rock fill. The top 30 ft of the foundation was treated with simple low-pressure grouting. The dam is composed of 1,500,000 cu yd of impervious rolled fill, 250,000 cu yd of crushed-stone filter, and 1,750,000 cu yd of quarry-run rock. The borrow material used varied from clay to sand and occurred in both residual and alluvial deposits. Because of the location of the borrow pits, the high water table, and the capillary action, the moisture in the natural ground was well above optimum. It was necessary to open four large borrow pits and move from one to another as the material in each became satisfactory for use in the fill. Plowing and then aerating the earth followed by removal in 18-in. cuts proved to be the best procedure for obtaining material at the proper water content. Because of the hardness

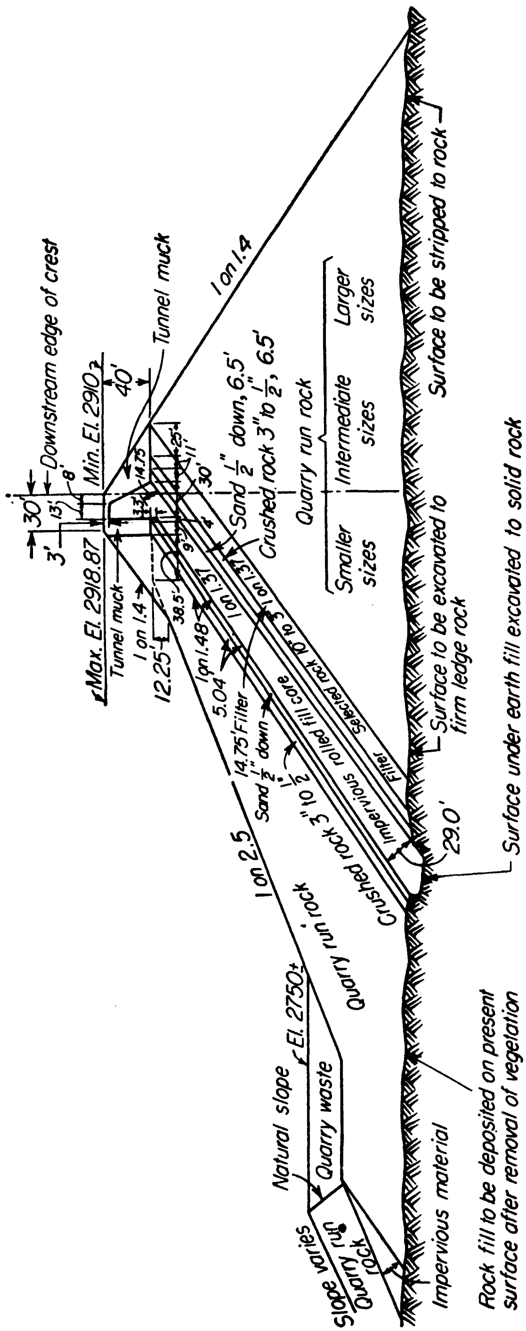


FIG. 48.—Cross section of Nantahala Dam.

of the quartzite and the difficulty of drilling blast holes, coyote blasting tunnels were used. In one instance 527,000 lb of dynamite were exploded in a single blast to produce over 800,000 cu yd of rock. An uncontrolled morning-glory spillway is located in the right abutment. A sluiceway tunnel connects with the spillway tunnel. A power intake and tunnel are located in the left abutment. Further description of this project may be found in Ref. 17.

7. Nantahala Dam, Nantahala River, North Carolina. This dam exemplifies a rock-fill dam with sloping core. A cross section of the dam is shown in Fig. 48. The maximum height is 260 ft and the crest length, 1,200 ft. The foundation is rock. All rock for the dam was obtained from the spillway excavation, and the rolled earth fill for the impervious core was obtained from a nearby borrow pit. The following quantities of material were used: 1,692,000 cu yd of quarry-run rock, 350,000 cu yd of processed rock for filters and 223,000 cu yd of rolled earth fill. The maximum settlement at the end of 2 years was 1 ft and after 8 years, 2½ ft. The seepage through the dam is approximately 2.75 cu ft per min under full reservoir. The spillway is constructed in a side hill cut in the right abutment. A diversion tunnel and a power tunnel are located in rock in the left abutment.

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SECTION 6

ROCK-FILL DAMS

By I. C. Steele

The design and construction of rock-fill dams originated in California soon after the discovery there of gold in 1848. The early structures, built for mining purposes, were the result of necessity and economic expediency in that unlimited amounts of useful material were available at or near the dam sites, situated in the mountains remote from railroads and highways. They were the outgrowth of the rock-crib dam and generally of small proportions daringly designed. French Lake Dam, 64 ft high and completed in 1859, is believed to be the first true rock-fill dam ever constructed.

The use of stored water was later extended to the irrigation of foothill farm lands and to the early generation of hydroelectric power. A large number of rock-fill dams, then unprecedented in size and design, were constructed during one period 1900–1920.

In recent years, further advances in the field of engineering and construction have resulted in the building of successful rock-fill dams, in California and elsewhere, well within the range of the high dams of the world. Advances in the art of quarrying and in the manufacture of loading and transportation equipment have made it possible for the rock-fill dam to compete favorably with high dams of other types. Settlement and other data obtained in recent years have led to a more scientific knowledge concerning their design, construction, safety, and behavior. Their relative merits, advantages, and disadvantages can now be given definite consideration in selecting the type of dam to be built at any site that offers ample quantities of suitable rock with which to make the fill. Modern practice in rock-fill design generally is typified by the views of Salt Springs Dam in California, shown by Figs. 1*A* and 1*B*; the details of the structure are shown by Figs. 4 and 10.

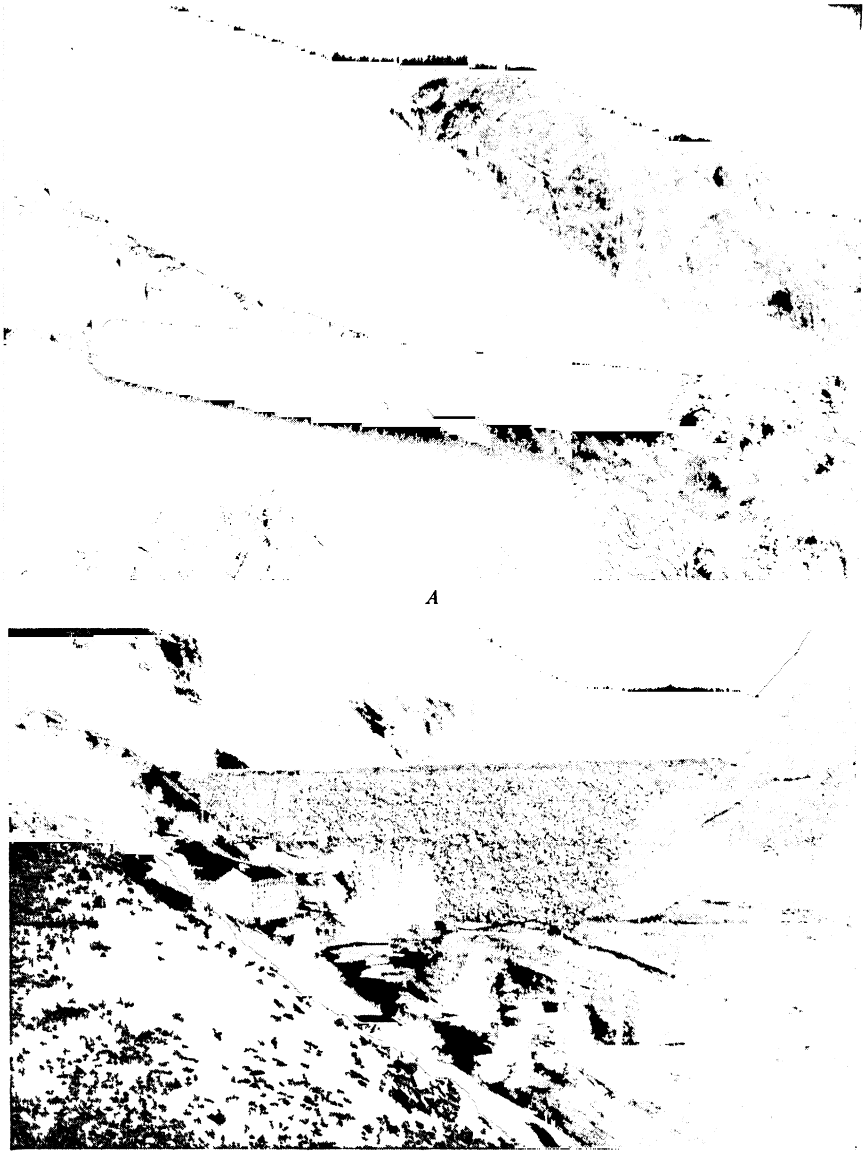
I. CHARACTERISTICS OF ROCK-FILL DAMS

Rock-fill dams are characterized through the placement of fragmentary rock to form the embankment support necessary to withstand reservoir water pressure and other forces exerted against or within the structure. Watertightness is obtained through the medium of an impervious membrane, generally on the upstream face of the dam, connected to a cutoff wall extending into the foundation material.

Cross sections of rock-fill dams usually lie intermediately between gravity and earth-fill dams and can generally be classified as one of the four types shown in Fig. 2.

Type *A* cross section is characteristic of the earlier rock-fill dams constructed principally in California and of some rock-fill or stone dams in foreign countries. (A Russian publication "Rock-fill and Dry Rubble Dams," by S. N. Moisseiff, Moscow, 1935, contains many excellent references to and illustrations of dams built in this and in foreign countries.) Great variation occurs in upstream and downstream slopes and in thickness of dry-rubble sections. The interior fill generally consists of loose dumped rock which, in many of the older California dams, contains much dirt and small rock (see Fig. 9). Typical of this cross section are first Bowman,

completed in 1876 and dismantled in 1927, first Bear River, Meadow Lake, first Fordyce, all in California (Table 6), and Vannino and Devero dams in Italy (Table 7)



A

B

FIG. 1 —Salt Springs Dam, Pacific Gas and Electric Company California (*A*) Completed structure showing spillway at left center (*B*) View of downstream face

Type *B* cross section consists of a massive upstream section of dry rubble or masonry and a supporting fill of loose rock The Relief Dam in California (Table 6) and the Malpaso Dam in Peru (Table 7) are of this type.

Type *C* cross section contains a relatively thin section of dry rubble on the upstream face, laid with a slope steeper than the natural angle of repose for loose rock and supported on a loose-rock fill of major proportions. The dry-rubble section acts as a retaining wall to support the loose fill and as a foundation for the impervious membrane. Structures of this type are the North Bowman, Bonito, Dix, Fordyce (Table 6), and Ghrib dams (Table 7).

Type *D* cross section is representative of several dams constructed in recent years. The main body of fill is constructed of loose rock having a downstream slope equal to or flatter than the natural slope and an upstream slope that approximates the angle of repose and supports a dry-rubble wall of moderate proportions. This type is especially adaptable in regions where severe earthquakes may occur. Probably for dams 100 ft or higher, it is the easiest and cheapest type to build. The most notable dam of this type is the Salt Springs Dam in California. This structure rises

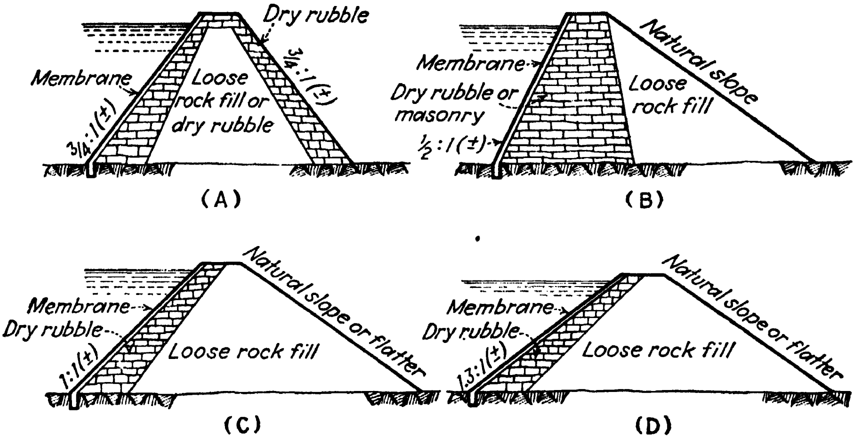


FIG. 2.—Typical sections of rock-fill dams.

328 ft above foundation. Other structures of this type are the Strawberry, Buck's Creek, San Gabriel 2 (Table 6) and Prins dams (Table 7).

II. GENERAL REQUIREMENTS FOR ROCK-FILL DAMS

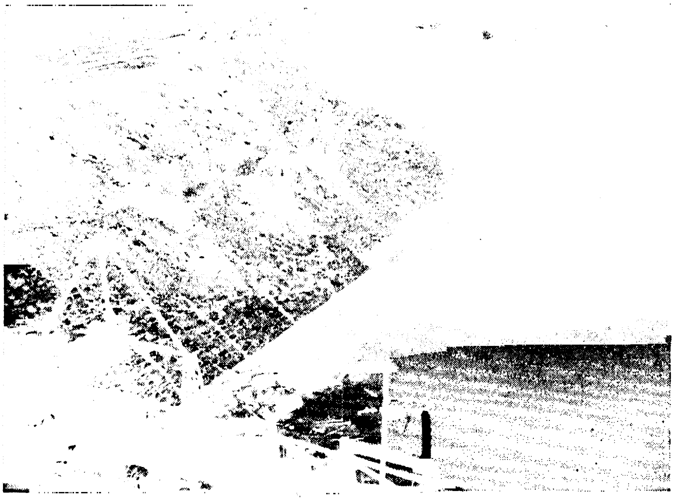
Several factors influence the selection of the rock-fill type of dam for any given site and purpose. These include the physical requirements, which can be determined, and certain less tangible features.

1. General Considerations. High and important rock-fill dams demand special consideration of certain inherent factors. After completion, settlement of high fills is due in large measure to water pressure, so it is desirable though not essential that the reservoir level be lowered to permit occasional inspection and possible repair of the impervious membrane; otherwise, leakage, if it develops, will remain a source of water loss, and, eventually, more serious damage to the membrane may result. Such phenomena might some time impair the stability of the dam, even though rock fills properly designed and constructed will safely pass fairly large volumes of water.

During the construction of Dix River Dam in Kentucky, a flood occurred which caused the reservoir level to rise well above the tops of the concrete facing slabs. E. W. Brown[1] states that the maximum rise occurred on Dec. 8, 1924, at which time the recorded difference between reservoir and tail-water levels was 35.7 ft and that the average dimensions of the rock-fill area exposed to reservoir water were 750 ft

[1] Vice-president, Kentucky Utilities Company.

horizontally and 50 ft vertically (slope measure). Figures 3A and 3B, respectively, show an upstream view and a portion of the water emerging from the loose fill at the toe of the dam. An estimate furnished by Brown and based on probable total flood



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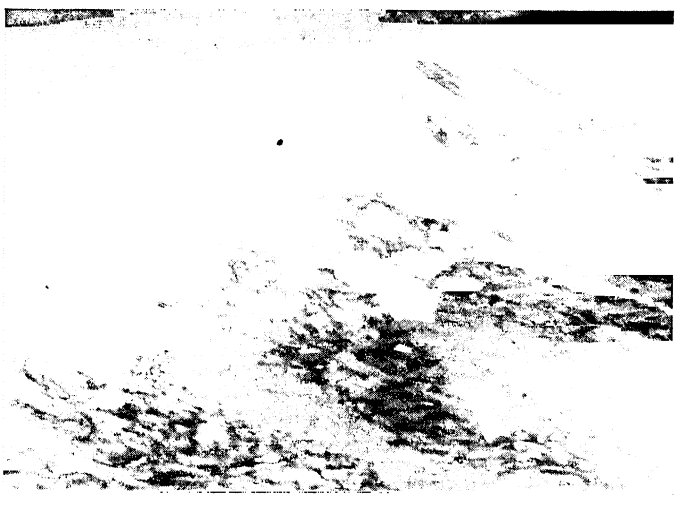


B

FIG. 3.—Dix River Dam during construction. (A) View showing flood-water level above completed concrete membrane on upstream face. (B) View showing flood water emerging at downstream toe of rock fill.

and diversion-tunnel discharge indicates that the flow through the rock fill may have exceeded 3,000 cfs. No sloughing of fill at the toe was noticed.

2. Availability of Suitable Material for Embankment Fill.[1] Ample rock of suitable character with which to make the embankment fill must be reasonably available if

[1] Credit is due Chester Marliave, consulting geologist and licensed civil engineer, for valuable assistance in the preparation of sections relating to geology.

the dam is to be a successful structure. It may be procured from quarry operations or from natural deposits, or both. The fill material must be sufficiently sound and of such geological composition as to resist erosion, undue weathering, excessive breakage and crumbling under the loads to be applied thereon. Quarries frequently yield rock that is much inferior, either in quality or quantity, to that anticipated from preliminary observations. This is especially true of rock that is badly sheared or jointed, for it is then susceptible to deep weathering, causing it to crush readily under concentrated loading. Quarry (and dam) sites for important rock-fill structures should therefore be thoroughly prospected and carefully examined by competent geologists prior to final location and construction unless solid rock, unquestionable in character and quantity, is exposed or evident.

The ideal rock should be hard, dense, massive, and free of incipient cracks that would allow it to crush under loading. The rock should not soften under saturation nor should its constituents be soluble. It should be free of unstable minerals that would weather mechanically or chemically, causing the rock to disintegrate. Almost all the plutonic rocks, such as granites, diorites, and gabbros, are suitable for rock-fill purposes. Among the volcanic rocks the basic types, such as the basalts and andesites, are generally preferable to the more acid rhyolitic types. In the volcanic series, the vesicular lavas, because of their lightness, are generally to be avoided, as well as the accompanying tuffs and breccias which are apt to be of low specific gravity and poorly cemented. In the sedimentary classification, the hard massive sandstones are generally suitable as are also the well-cemented conglomerates. Occasionally chemical rocks such as limestone may be used if they are reasonably insoluble.

The structural characteristics of a rock are also important. Platy cleavages such as are found in slates generally make the rock undesirable, but a gneissic structure which is common in granitic rocks offers little objection. Rocks having schistose structures are generally unsuitable for high dams.

3. Foundation Requirements. Foundation requirements for rock-fill dams are less severe than for gravity-type concrete dams but more severe than for earth dams. Except for the area in the vicinity of the cutoff wall, the foundation rock need only be of such character that it will support the overlying rock fill without major yielding or being displaced. It should not soften or be eroded by abutment percolation which might carry the material into the voids of the fill, causing settlement of the dam. The location of all faults, slips, shear zones, soft seams, and other structural weaknesses should be ascertained and the dam so located as to avoid or otherwise best accommodate to these features.

The foundation area should be cleared of silt, earth, and sand layers or deposits. River-bed excavation of water-deposited debris consisting of boulders, gravel, and sand is sometimes optional, depending on the characteristics of the deposit. Frequently such deposits are more compact and less yielding under load than the rock fill of the dam. If utilized for the foundation, they should permit reasonable drainage and be adequately resistant to erosion and piping. The downstream half of the rock fill of the Salt Springs Dam was placed on river boulders and gravel (see Fig. 4). Examination disclosed varying sizes of water-worn boulders, rocks, and gravel, so deposited that the faces of each rock particle contacted those of surrounding particles. Exploration to bedrock disclosed no underlying soft material. The river bed upstream from the axis of the dam and all abutment areas were excavated to solid rock. The Strawberry, Swift, Bucks Storage, and other dams were founded on river gravels across stream beds. The central section of Malpaso Dam,[1] Peru, rests on 100 ft or more of a mixture of boulders, gravel, sand, and some clay (see Art. IV).

[1] Courtesy of Sanderson & Porter, Engineers.

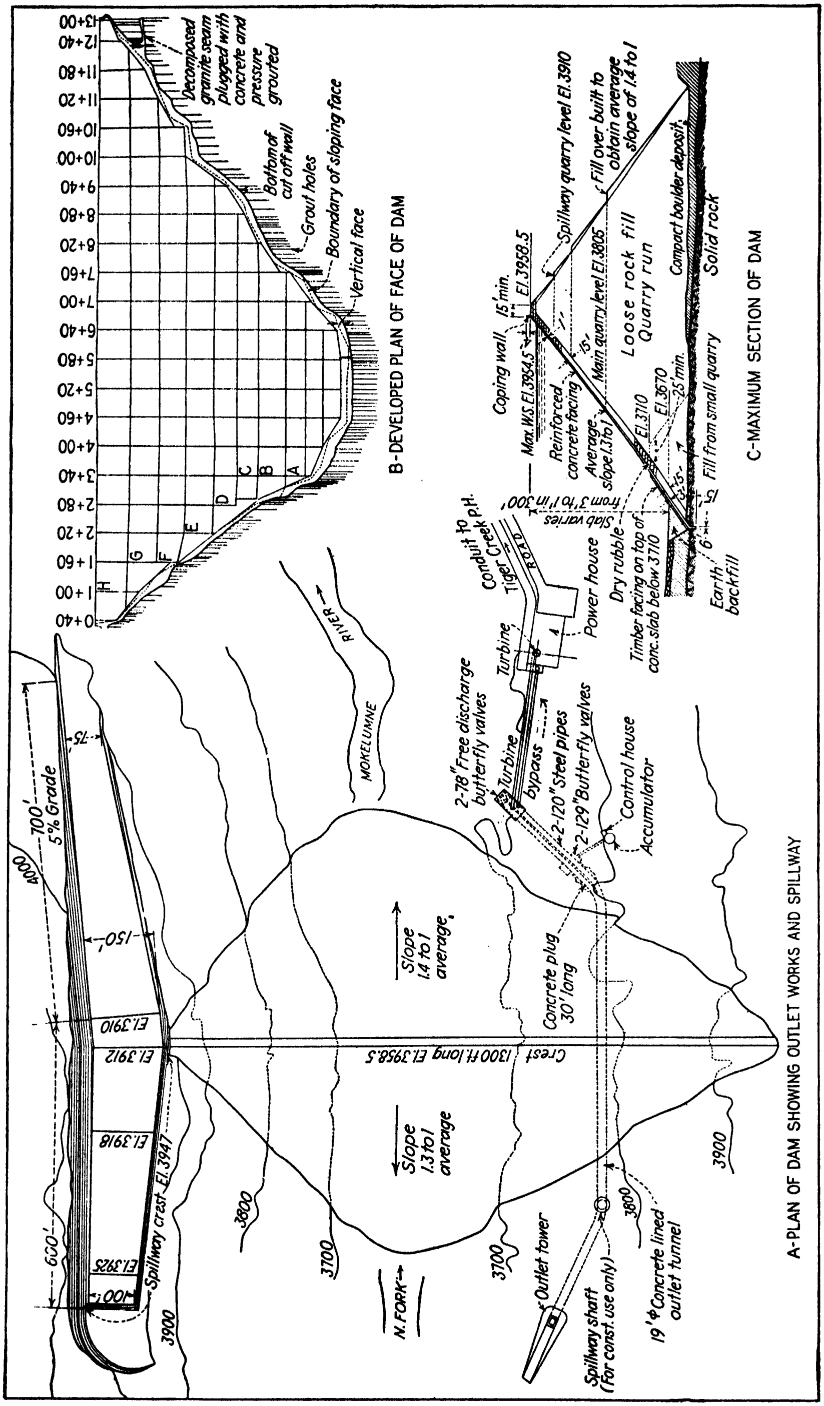
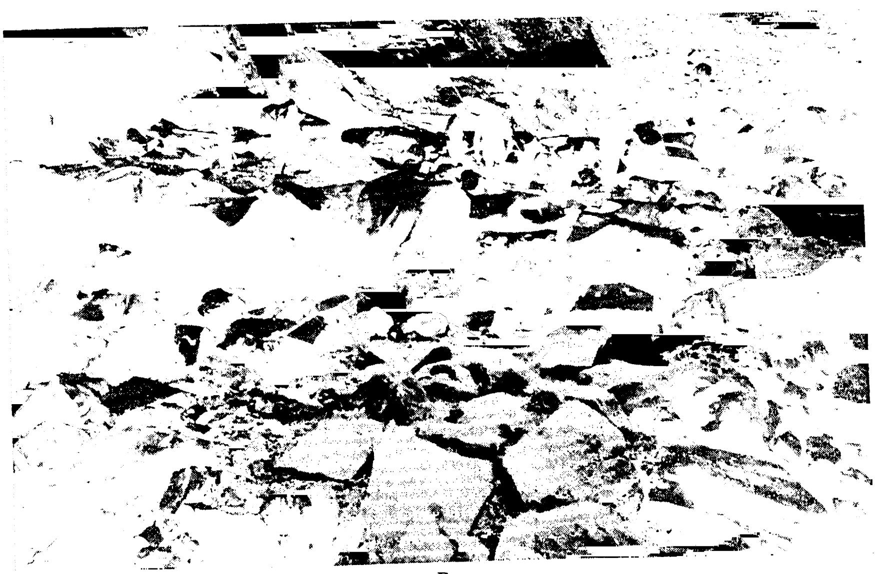


Fig. 4.—Plan, cross section, and developed plan of upstream face, Salt Springs Dam.

A



B

FIG. 5.—Malpaso Dam during construction, Cerro de Pasco Copper Corporation, Peru. *A*. View of upstream face, showing cutoff wall. *B*. View showing washing of gravel and sand into voids of dry rubble. (*Courtesy of Sanderson & Porter, Engineers.*)

The foundation in the vicinity of the cutoff wall should be capable of resisting jet action in the event of cracks developing in the face. Solid foundation upon which to place the dry rubble and into which the cutoff must be built is necessary.

4. Necessity for Separate Spillway and Outlet Works. The safety of a rock-fill dam requires adequate spillway facilities independently placed with respect to the rock-fill structure. Under no circumstances should the spillway be embodied within or upon the fill. Nor should the waste channel be located as to permit spill water against or dangerously near the toe of the dam.

A separate outlet tunnel is advisable for high and important dams. Conduits may be successfully placed on a solid-rock bench or within a deep trench beneath the fill of low or moderately high dams (see Art. III, 9).

5. Earthquakes. The rock-fill dam, properly designed and constructed with conservative upstream and downstream slopes, is one of the most satisfactory types of dam to resist earthquake shock. The flexible character of the impervious membrane and the supporting fill permits considerable movement without serious effect.

The Department of Irrigation, Chile, in recent years has endeavored to promote a more general use of earth and rock-fill dams because of the frequency of earthquakes in that country.[1] Malpaso Dam, Peru (Figs. 5*A* and *B*; Table 7), is in a region of severe earthquake shocks.

Sanderson & Porter, Engineers, who planned the dam, made the following statement:

"It has been subjected to a number of earthquakes, the two which occurred on October 11, 1938, being the most severe experienced in the district in the last ten years. The more severe of these shocks, together with one other heavy quake on the same day, caused movements at the crest of the dam no greater than about two inches downstream and 1¼ inches vertically. The loose fill settled vertically about 2¾ inches. The reservoir level was about 12 feet below the crest of the dam. These earthquakes have caused no damage whatever to the dam." (See Table 7.)

The severe shock of Apr. 7, 1943, caused the fill in Cogoti Dam to settle 1.38 ft. The damage done was insignificant, and leakage did not increase.

A dry rubble and concrete thrust block section with an intervening sand wedge extending from near the crest to an apex at a point about 175 ft below the crest was adopted by Binnie, Deacon, and Gourley, chartered civil engineers of Great Britain, for the 275-ft Shing Mun Dam in China, in preference to a concrete gravity section, because of probable seismic disturbance in the region.[2]

6. Cost. Careful consideration should be given the following factors in preparing cost estimates for rock-fill dams:

1. Railroad and highway transportation facilities.
2. Costs of foundation preparation and excavation.
3. Location, distance, and elevation of quarry sites with respect to the proposed rock-fill dam.
4. Necessity for separate spillway and outlet works.
5. Costs of labor, materials and equipment.

Accurate estimates of transportation costs, including construction railroad and railroad equipment, roads, road maintenance, hauling, warehousing, and camp expense, are necessary because these costs usually are large and varied for different types of dams. Generally, the rock-fill dam becomes increasingly favorable with increased distance from railroad terminal.

Continued and substantial reductions in the cost of truck haul, quarry operations, loading, and distributing and placing rock have been occasioned in recent years

[1] Ensamble con el Cerro de la Cortina del Rock-Fill de Cogoti, Departmento de Riego, Chile, Paper D-65, Second Congress on Large Dams, Washington, D. C., 1936.

[2] BINNIE, W. J. E., Study of the Facing of Masonry and Concrete Dams, Paper D-14, Second Congress on Large Dams, Washington, D. C., 1936.

largely owing to important improvements in the design and manufacture of power equipment adaptable to economic handling of large masses of loose material.

Rock-fill dams in the United States, because of high labor costs, are mainly built of loose rock with minimum thickness of dry rubble. The loose fill of Salt Springs Dam comprises 93 per cent of the 3,200,000 cu yd of material in the dam. In many foreign countries, cost of labor is low. This condition has prompted the engineers of those countries to construct dams wholly or in large part with dry rubble or dry rubble and masonry.

III. CRITERIA FOR THE DESIGN OF ROCK-FILL DAMS

Dependable criteria governing individual features of rock-fill dams and the stability of the structures as a whole could be established if controlling factors were reasonably similar in all cases. Three principal factors govern these criteria, namely, character of fill material, methods of construction, and settlement. These factors are interrelated and at variance for separate projects.

1. Crest Width. Crest widths of rock-fill dams range from about 4 to 28 ft (see Tables 6 and 7). Required stability of the section beneath the crest and construction methods govern minimum crest widths for low dams, unless limited by statute. High dams usually are more conservatively built and have crest widths ranging from about 15 to 20 ft. Crests, wider than would otherwise be necessary, are sometimes used to accommodate roadways. In such cases, it may be desirable to build guard railing along both edges of the crest of the dam. Minimum requirements for crest widths of rock-fill dams might be considered about as follows:

Dam Height, ft	Crest Width, ft
Under 100	8
100–150	10
150–200	12
Over 200	15

The section immediately beneath the crest of the loose-fill type of dam is frequently constructed entirely of dry rubble in order to avoid construction difficulties or for esthetic reasons (see Fig. 2*A*).

2. Face Slopes. Many early rock-fill dams were built with least possible yardage of material because quarry and placing methods then used were slow and costly and rock masons were plentiful and good. Steep slopes of well-laid dry rubble were adopted, with small loose rock and much debris in the central portions of the dam. A cross section of French Lake Dam as surveyed in 1937 by William Durbrow, Jr., shows upstream slopes of 0.42:1 and 1:1, and downstream slopes of 1/10:1 and 0.35:1. Fordyce Dam prior to enlargement in 1926 was 93 ft high and had an upstream slope of 1:1 and a downstream slope of 1/4:1. Other successful early dams have slopes as steep as 1/8:1. Such steep slopes are no longer considered advisable or economic.

Upstream Slope. In this country, minimum allowable upstream slopes with present methods and costs of construction might be considered about as follows:

Dam Height, ft	Upstream Slope
Under 50	1/2:1
50–100	3/4:1
100–150	1:1
Over 150	1.3:1

Some sloughing of fills has occurred due to inadequacy of dry-rubble sections with slopes as flat as 1:1. Unfilled horizontal keyways encourage this action.

Relatively high dams have been built in recent years with upstream slopes as steep as ½:1 in the case of Relief (140 ft high) and Malpaso dams (255 ft high), ½:1 and ¾:1 in the case of North Bowman Dam (167 ft high), and as flat as 1.35, 1.3, and 1.2:1 at San Gabriel Dam 2 (280 ft high). Dix River Dam (275 ft high) has slopes of 1.2:1 and 1.0:1. The upstream slope of Salt Springs Dam averages 1.3:1. A slope approximating the angle of repose for the loose fill will permit easy, safe, and rapid construction of all features of the dam proper. It allows use of high lifts for placement of loose fill well in advance of the dry-rubble facing and impervious membrane (see Art. IV, 1).

Downstream Slope. Under present practice, the downstream slopes for loose fills of rock-fill dams of all heights are no steeper than the angle of repose. Flatter slopes are impracticable and unnecessary except for dams in areas of seismic disturbance and for desired conservatism in connection with high and important dams. Natural slopes vary within individual fills; the range being from about 1.2:1 for low lifts to about 1.4:1 for high lifts and the lower levels of high dams. Predominant minimum, maximum, and average slopes obtained at Fordyce Dam were 1.3:1, 1.45:1, and 1.35:1, respectively. The fill consisted of hard granodiorite quarry-run rock dumped from high lifts.

Rock in some earlier dams was placed by means of cableways and skips and rehandled where necessary with derricks. This method, used in building Strawberry dam,[1] permits a slope somewhat steeper than can be had if rock is dumped from large-capacity trucks or cars. Predominant slopes here are 1.2:1 minimum and 1.4:1 maximum with an average of 1.3:1. Rock in other dams was dumped from small-capacity cars and trestles. A slope of 1.25:1 was thus obtained at Crane Valley Dam (composite dam with rock-fill downstream from central diaphragm). To ensure an average slope of 1.4:1 at Salt Springs Dam, the benches were overbuilt as indicated in Fig. 4*C*. Slopes here range from 1.3:1 minimum to over 1.4:1 maximum. High lifts and large-capacity dump cars were used in placing the major portion of the fill.

3. Dry-rubble or Masonry Section. The function of the upstream dry-rubble (or masonry) section is to support the face membrane, retain the loose fill if on a slope steeper than the angle of repose, and transfer the water load to the mass of loose fill in the dam. The character of material and workmanship is extremely important. Sound rock, selected as to size and shape, is essential. Well-bonded large rock should predominate. Rock-to-rock contact throughout with thorough bedding and chinking of voids is desirable to ensure a dense, compact mass.

The average percentage of voids in the dry rubble of the latest Algerian dams varies from 25 to 32.[2]

In building Malpaso Dam, clean river gravel and sand were washed into the voids after chinking in order to increase the bearing and contact areas within the rock mass and the density of the rubble section (see Fig. 5*B*). [This also was done in placing the loose fill (see Art. III, 4.)]

The facing layer of rock may be laid either horizontally or normal to the upstream slope. The first method permits good bonding of face rock with interior stones, ease of construction, and, with irregularly shaped rock (see Fig. 6), more thorough bedding. The latter method (see Fig. 7) is advantageous with stones of regular shape with which it is possible to obtain a greater compact thickness in a direction normal to the face membrane.

Unchinked voids in the upstream face of the dry-rubble section permit deep bonding of the concrete face with the underlying rock, but may cause sagging of concrete and loss of mortar during construction of the membrane. This largely

[1] HOWSON, GEORGE A., Unique Method Used to Build Rock-fill Dam, *Eng. News*, **75**, 604, 1916.
[2] GUTMANN, I., Algerian Dams of Placed Rock-fill, *Eng. News-Record*, **119**, 889, 1937.

may be avoided by adoption of a reasonably low water-cement ratio and thorough concrete-placement methods. Chinked voids reduce bonding but cause minor loss of concrete and mortar. Several instances of sagging of the freshly poured concrete were noticed at Salt Springs. Repairs later disclosed several areas of honey-

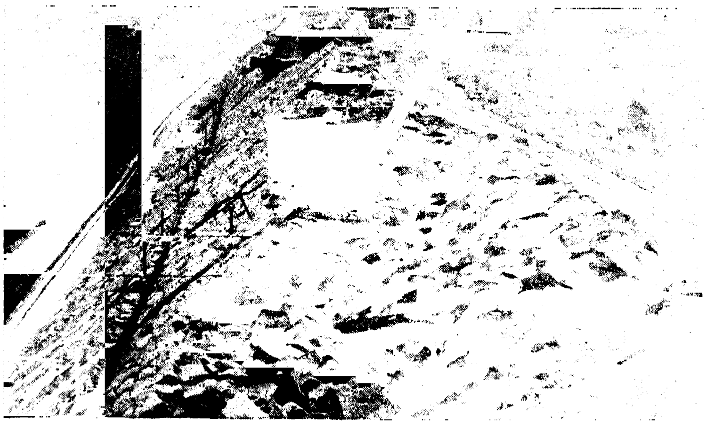


FIG. 6.—View showing dry-rubble section of irregularly shaped rock. Salt Springs Dam.

comb concrete in the under half of a 2-ft-thick slab. This is believed to be in part responsible for fractures shown in Fig. 24*A* and *B*.

Thickness requirements depend upon the upstream slope, height of dam, character of adjacent loose fill, size and character of available rock, and type of equipment used in placing the rubble. Slopes steeper than the angle of repose require sufficient

FIG. 7.—View showing dry-rubble section of regularly shaped rock. Dix River Dam.

thickness throughout to support and retain the loose fill with reservoir empty. Dry-rubble sections of most dams taper from a minimum thickness at the crest to a maximum thickness at the base. A thickness normal to the slope of 5 to 10 ft at the crest, depending on the height of the dam, and increasing about 5 ft for each 100-ft depth of fill, is generally considered adequate. Advantages and reasons for adopting a tapered section are as follows:

1. Settlement at or near the crest is not serious because water and rock loads are light.

2. Total settlement at or near the upstream toe of deep fills owing to water pressure is relatively small but critical.

Large differential movement from zero at bedrock to substantial or moderate amounts within short distances therefrom may cause membrane rupture.

3. Face ruptures near the stream-bed level permit leakage under heavy water pressure and high spouting velocity. High velocity, though quickly dissipated, may cause disintegration of concrete and foundation material near points of rupture.

4. The bottom area of the face membrane, especially across the stream bed, generally is the most difficult of access for making repairs.

5. During construction the stability of the dry-rubble walls may be maintained as the height of the fill increases.

Figures 17 and 18 illustrate many of these principles.

Areas distant from river-bed and abutment foundations often experience greater total settlements but theoretically require less thickness because increment changes in movement usually are more uniform.

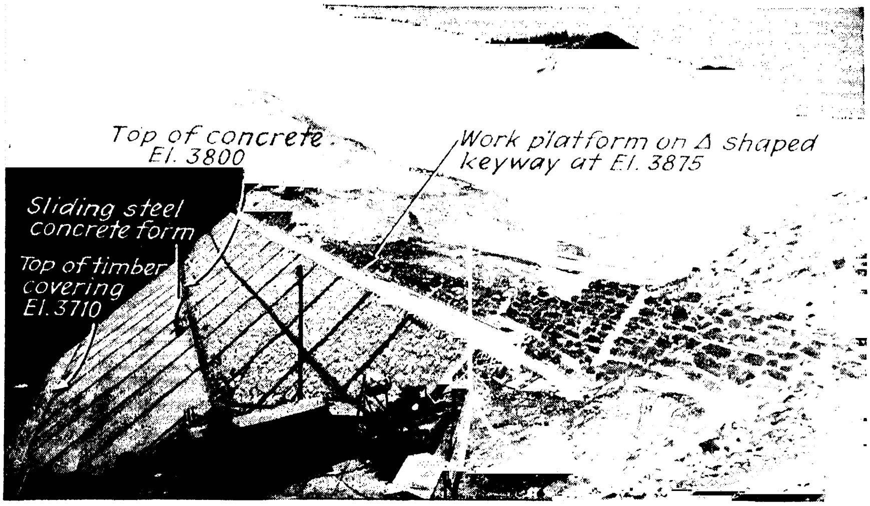


FIG. 8.—View of upstream face during construction. Salt Springs Dam.

Wet-masonry sections sometimes are used in lieu of dry rubble, as instanced at Vargno Dam, Italy, and Piana-Grechi Dam, Sicily. A concrete thrust block was adopted at the Shing Mun Dam, China. Heavy concrete blocks formed the support for the concrete facing of Tepuxtepec Dam.

During construction of Salt Springs Dam, it was found that settlement of the fill made it somewhat of a risk to build the rubble face too far above the elevation of the concrete face. In a few cases, the tremendous forces transmitted from the loose fill to the more rigid rubble face caused large face rock to crack or become slightly displaced when the rubble wall was carried about 100 ft above the concrete face. A view of the upstream face during construction of the Salt Springs Dam is shown in Fig. 8.

Dry rubble on the downstream face of rock-fill dams is necessary only if the slope is to be steeper than the angle of repose. It may be justified if low labor costs warrant the adoption of a section of dam with reduced volume of fill or for esthetic reasons. In no case should it be depended upon for spillway use. Its presence might prevent or delay failure from overtopping, depending on the duration and extent of such action. Bear River Dam (Alpine County, California) safely withstood for several hours overtopping to a maximum depth of 15 in. The dry-rubble wall, one stone thick

and laid on slopes of ½, ¾, and 1:1, gave way in a small area adjacent to one abutment. Continued or greater overpour might have caused failure of the dam. Subsequently, loose rock fill was placed on the downstream side, and the spillway was greatly enlarged. Again the ability of a well-built wall of thin section to retain loose fill is illustrated by Fig. 9, which shows a mass of small rock and fines in the first Bowman Dam retained on a 1:1 downstream slope by one layer of well-laid rubble.

4. Loose Rock Fill. The loose fill of a rock-fill dam usually affords the main support for the upstream face structure and reservoir pressure. The character, size, and grading of rock should be such as to permit reasonably free drainage continuously. The size of rock used in existing structures varies from spawls to stones over 50 tons in weight. Rock-to-rock contact throughout the fill is important. The loose fill of most successful dams has been made with quarry-run rock, *i.e.*, rock produced by normal quarry operations and unsegregated as to size, which gives a dense fill with many rock contacts at minimum cost. Quarry sites should be stripped of overburden and debris. This material, together with soft or disintegrated rock, should be wasted. About 7 per cent of the total yardage removed from Salt Springs Dam quarries was so wasted. No criterion, governing permissible amounts of fines in rock-fill dams, can be justly and universally established. Fines can do no harm unless they prevent drainage or rock-to-rock contact. Washed gravel and sand were deliberately hauled to the site and sluiced into the voids of the loose rock fill of Malpaso Dam (see Art. III, 3).

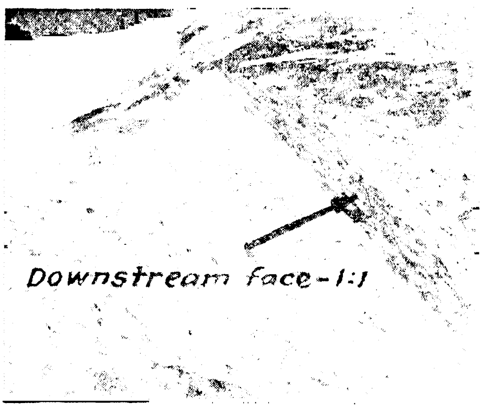


FIG. 9.—View of first Bowman Dam during dismantling process (see text), Nevada Irrigation District, California.

5. Impervious Membrane on Upstream Face. *Concrete Face.* Three general types of reinforced-concrete membranes have been used. These are the monolithic, sliding, and laminated types.

The monolithic type, most frequently used, consists of a single thickness of concrete poured against the rock. It may be of either rigid or flexible construction. Both have proved successful and are adaptable to all usual conditions of stress and exposure. The rigid type contains no expansion joints. Reinforcing steel is continuous in both directions. No very high dam has been constructed with a rigid-type face. Bucks storage dam, 122 ft high, has a rigid-type face 12 to 19 in. thick, which contains but a few hair cracks after 20 years of service. Reinforcement was provided in amounts of about 0.55 per cent vertically and 0.7 per cent horizontally.

Most dams have been built with a flexible monolithic slab, as shown by Fig. 10, consisting of rectangular panels formed by tight horizontal joints and vertically inclined expansion joints. Reinforcing steel does not pass the joints (see Fig. 10*b* and Fig. 23). A minimum of 0.5 per cent of reinforcing steel in each direction is considered good practice. This amount was provided at Salt Springs and Dix dams. Dix Dam has horizontal expansion joints (see Fig. 23*A*). The flexible monolithic face at Malpaso Dam (see Fig. 19*A*) embodies the principle of the wing type used at Tepuxtepec Dam (see Fig. 23*E*). There are no horizontal joints above the cutoff level. Vertically inclined panels (40 ft wide above the continuously submerged portion of the face) weie poured integral with 4- by 3-ft inclined concrete anchors

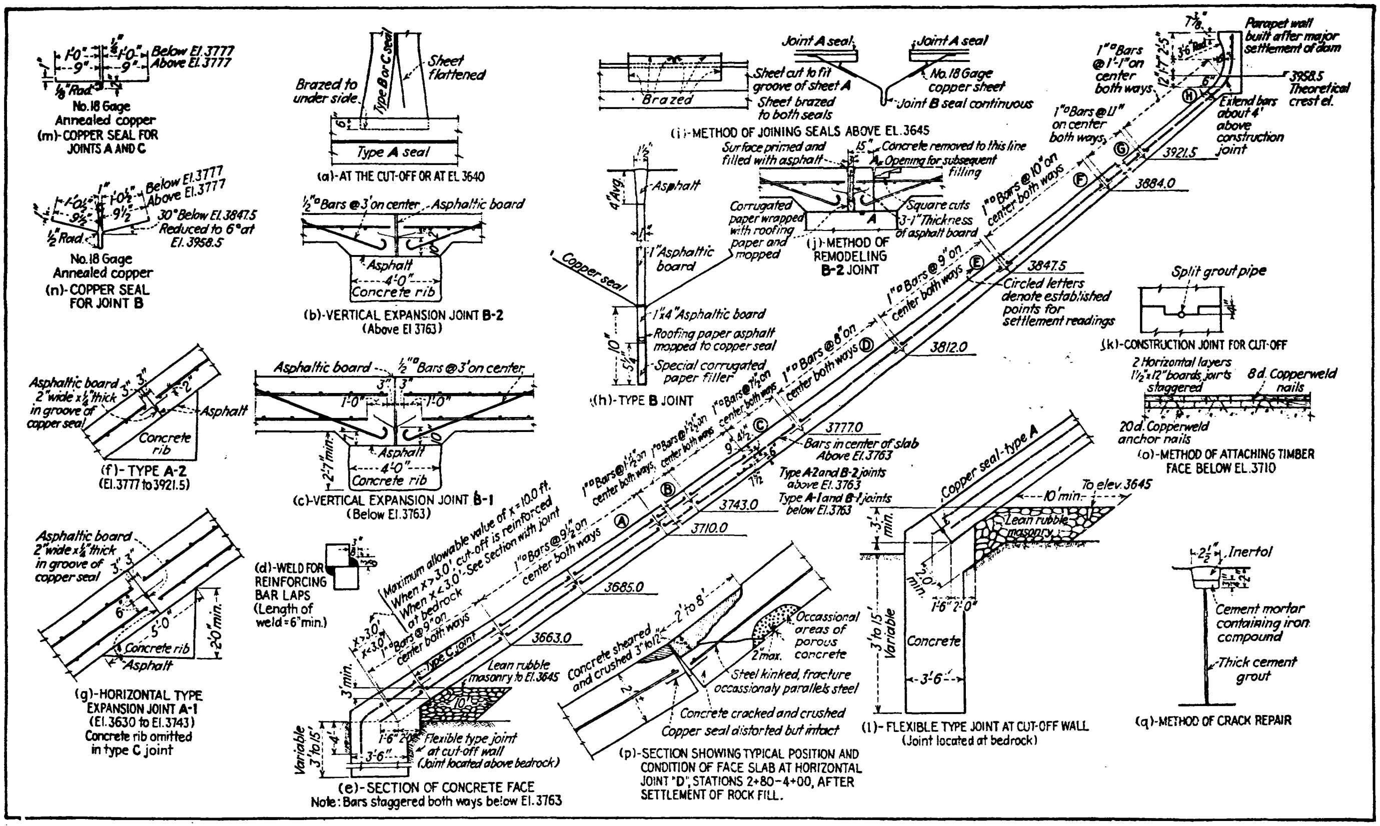


FIG. 10.—Details of face membrane, joints, and cutoff wall. Salt Springs Dam.

located along their vertical center lines. Contraction joints, 1 in. in width, extend parallel to and midway between anchors and are provided with copper seals and soft wood fillers. Joint arrangement and other features are shown and described in *Compressed Air Magazine*, vol. 44, No. 7, p. 5916.

The sliding-type membrane usually consists of a thin, smooth, unreinforced subslab poured against the face rock with asphalt or oil-coated top surface against which is laid a heavier reinforced-concrete top slab. The top slab is free to move independently from the body of the dam unless prevented by water pressure. Vertically inclined expansion joints are spaced. Horizontal joints are tight. Construction inequalities and settlement during construction may cause irregularities in the supporting face and defeat the purpose of the design. Theoretical forces acting parallel to a sliding-type face of uniform slope may be computed from the formula

$$Fe = W(\sin a + f \cos a)$$

and

$$Fc = W(\sin a - f \cos a)$$

where Fe = force under expanding conditions.
Fc = force under contracting conditions.
W = weight of slab.
a = angle of face slope with horizontal.
f = coefficient of friction.

Examples of the sliding-type face are the Strawberry[1] and Swift[2] dams. The face of Strawberry Dam contains some narrow cracks, but there is little leakage.

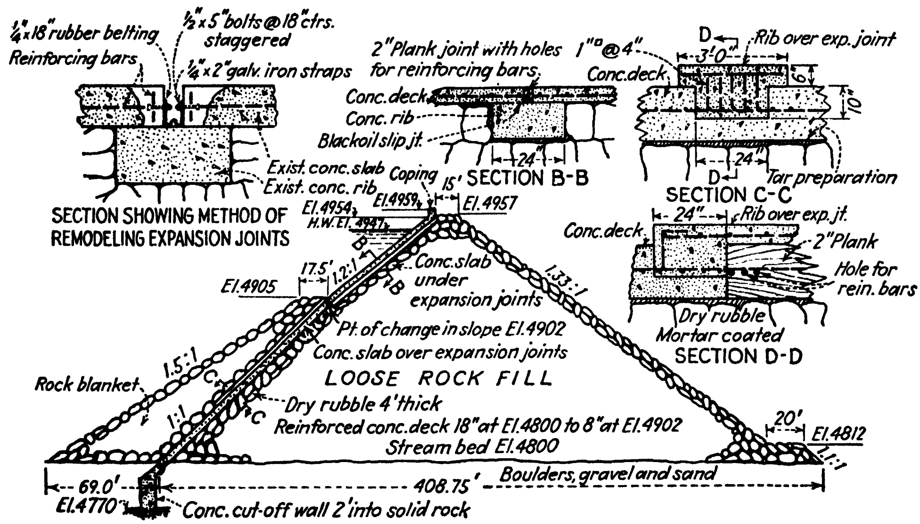


FIG. 11.—Section and joint details. Swift Dam, The Valier-Montana Land and Water Company, Montana.

The face design of Swift Dam (see Fig. 11), completed in 1914, was changed during construction owing to some sloughing of the face structure. The subslab, built by mortar-pointing the face rock, was covered with a tar product. Reinforcing steel, continuous past all joints, including vertical expansion joints, was provided in minimum amounts of about 0.3 per cent horizontally and 0.35 per cent vertically for the

[1] *Eng. News*, **75**, 604, 1916.

[2] Courtesy of C. E. Atwood, General Manager, Montana Western R. R. See also HERON, KENNETH A., The Valier-Montana Irrigation Project, *Eng. News*, **73**, 241, 1915.

section on a 1:1 slope. Horizontal steel for the 1.2:1 slope exceeded 0.6 per cent.

The laminated membrane is illustrated by the unsuccessful shingle type (gunite slabs) of San Gabriel Dam 2 and a successful wing type used at Tepuxtepec Dam in Mexico, as shown by Fig. 23*E*.

Each unit of the laminated shingle-type membrane with horizontal expansion joints has a horizontal anchor. This anchor, with joints open, is subjected to a force which may be analyzed in a manner similar to that shown for the sliding-type face. This force is transferred to the face rock. The total force increases toward the base of the dam owing to the normal settlement plus the forces from the slabs above, and the lower anchors receive the accumulated downslope thrust until expansion joints become closed. Construction imperfections may permit intermittent joint closure, uneven sliding surfaces, and eccentric loading on individual, thin, unbonded slabs. Hydrostatic uplift from water trapped behind slabs may cause failure. This type

FIG. 12.—View showing displacement of gunite top slabs. San Gabriel Dam No. 2, Los Angeles County Flood Control District, California.

of membrane is costly and of questionable success unless adequately supported on a heavy, unyielding subslab, or unless the concrete keyways are adequately supported on rigid struts extending upward from the cutoff in the plane of the face.

The large settlement of rock fill (see Art. IV, 1) prior to completion of San Gabriel Dam 2 was the principal cause of the failure (see Fig. 12)[1] of the uncompleted laminated-type membrane, consisting of a 6-in. minimum thickness underslab placed integral with horizontal anchors at 30-ft centers and four thicknesses of 6-in. reinforced-gunite top slabs at the toe, these decreasing in number to two at the crest. Top slabs, 30 ft square with open spaces at all edges, were lap jointed in plan and suspended in series to the horizontal anchors. Surfaces between slabs were asphalt coated. Joints containing copper water stops and asphalt were placed only in the top slab. In 1935, nearly all slabs above the subslab were removed and a temporary timber face installed pending more complete settlement of fill.

Tepuxtepec Dam was constructed with a wing-type laminated face (see Fig. 23*E*) laid against thick concrete blocks with shaped vertical keyways. There are no

[1] Another view of this failure is shown in *Trans. A. S. C. E.*, **104**, 26, 1939, in the discussion by Cecil E. Pearce of the paper, The Design of Rock-fill Dams, by J. D. Galloway.

[2] *Eng. News-Record*, **114**, 343, 836, 1935.

horizontal anchors. All rock fill in the dam was hand placed. G. R. G. Conway, president, The Mexican Light and Power Company, Ltd., stated in 1947 that the dam and membrane then were in excellent condition. Leakage past the dam reached a maximum of but 3 l/sec with full reservoir.

Thickness requirements of reinforced-concrete membranes depend upon type of membrane, height of dam and water pressure, unsupported span through which membrane may act, resistance of concrete to percolation, minimum temperature if below freezing point, and settlement of rock fill. J. D. Galloway states in his paper, The Design of Rock-fill Dams:[1] "It is believed that a slab of thickness of 1 per cent of the head of water will provide ample thickness. It is also believed that the slab should not be less than 12 in. thick at the top and preferably thicker where climatic conditions may lead to disintegration from frost."

Minimum thicknesses may be applied to dams constructed wholly or largely of dry rubble, refacing of old dams, or replacement of temporary membranes when evidence indicates little future settlement. Minimum top thicknesses for monolithic slabs on new dams may be to the order of 5 in. for low dams, 8 in. for medium-height dams, and 12 in. for high dams. Top and bottom thicknesses for existing dams are shown in Tables 6 and 7.

Sound concrete of not less than 3,000-lb strength at 28 days and designed for maximum density is the best insurance against percolation and ultimate disintegration of concrete membranes. Concrete in many older membranes was of poor quality, especially at pour joints, and considerable repair expense has been incurred. Sound, dense concrete, old and new, generally has been satisfactory.

Waterproofing, or surface treatment, of concrete membranes is seldom done in the United States. Several dams in foreign countries have been built with elaborate face structures, including drainage systems and waterproofed surfaces. Among these are the Vargno and Vannino dry-masonry dams in Italy and Piana-Grechi dry-masonry dam in Sicily. Sections, details, and descriptions of these and many other dams appear in a Russian book by S. N. Moisseiff, the translated title of which is "Rock-fill and Dry Rubble Dams," Moscow, 1935.[2]

Gunite Face. Gunite is usually more dense and impervious than concrete. It is especially adaptable for the membrane on dams of low or moderate height where settlement is small, for replacement of temporary membranes, and for repair of concrete slabs. Minimum-thickness requirements are generally less than for concrete.

The timber face of Meadow Lake Dam, Alpine County, California, burned in 1929. The dam, 7,800 ft above sea level, is 81 miles from the nearest railroad terminal. 38,000 sq ft of reinforced-gunite face of 4½ in. average thickness (see Fig. 13*B*) were placed in 6 weeks' time with one cement gun. One part cement to 3¾ parts of local sand was used for the mix. An antifreeze admixture (5 per cent by weight of cement) was added during cold spells (minimum temperature was about 20F). About 0.3 per cent of steel reinforcing was provided in each direction. After 18 years, inspection showed this membrane to be sound and practically free of cracks except for one vertical opening at a construction joint. The reservoir is emptied each year so that water does not stand against the face in freezing weather. Gunite slabs generally are too thin to withstand repeated severe ice action.

Masonry Face. Wet masonry as an impervious membrane has been used on a few rock-fill dams. It is seldom used on high dams because of slowness of construction and difficulty in making face repairs if cracks develop. Watertightness is more difficult to obtain than with concrete, gunite, or steel membranes. The wet-masonry face on Morena Dam, 6 ft thick at the bottom, is backed with a dry-rubble wall 50 ft

[1] *Trans. A. S. C. E.*, **104**, 20, 1939.

[2] See GUTMANN, I., Algerian Dams of Placed Rock-fill, *Eng. News-Record*, **119**, 889, 1937.

thick at the base and 16 ft thick at the top. The mortar consisted of 1 part cement to 2½ parts of sand.

Wet masonry is more frequently used to support other types of impervious membranes (see Art. III, 3).

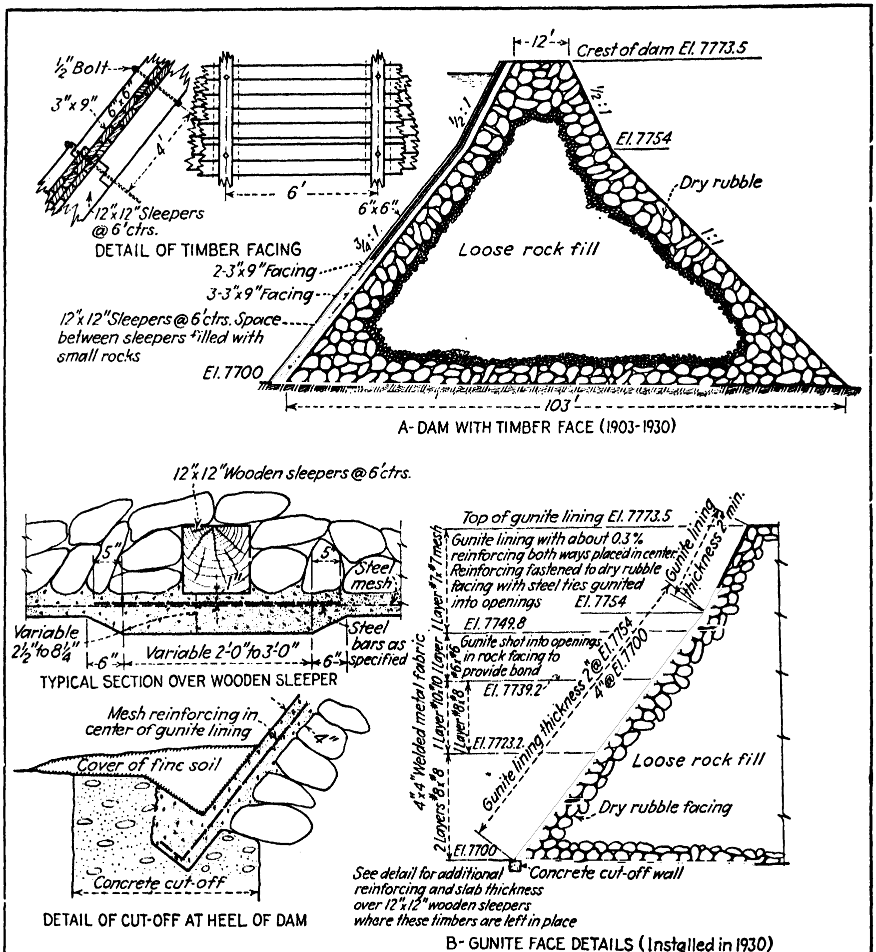


FIG. 13.—Meadow Lake Dam, Alpine County, California, Pacific Gas and Electric Company. (*A*) Section of dam showing timber face, 1903–1929. (*B*) Details of gunite face, 1930.

Timber Face. Timber is satisfactory as a temporary type of membrane and as a permanent covering to increase watertightness on concrete membranes below low-water level. It may temporarily be installed on dams in lieu of the permanent membrane if unusually large settlements of rock fill are expected. If so used, all keyways (see Art. IV, 2) should be filled with concrete or wet masonry. Details and descriptions of the temporary timber face installed for this reason on San Gabriel Dam 2 are given in *Engineering News-Record*, volume 114, pages 343 and 836.

Details shown in Fig. 13*A* are typical of timber faces successfully used on many of the older California dams. Timber faces are usually anchored to the dry rubble to prevent flotation due to buoyancy. Original cost and maintenance expense are usually not great. Principal objections to them are danger of loss by fire, if the reservoir is empty, and leakage. The timber face on Meadow Lake Dam was almost

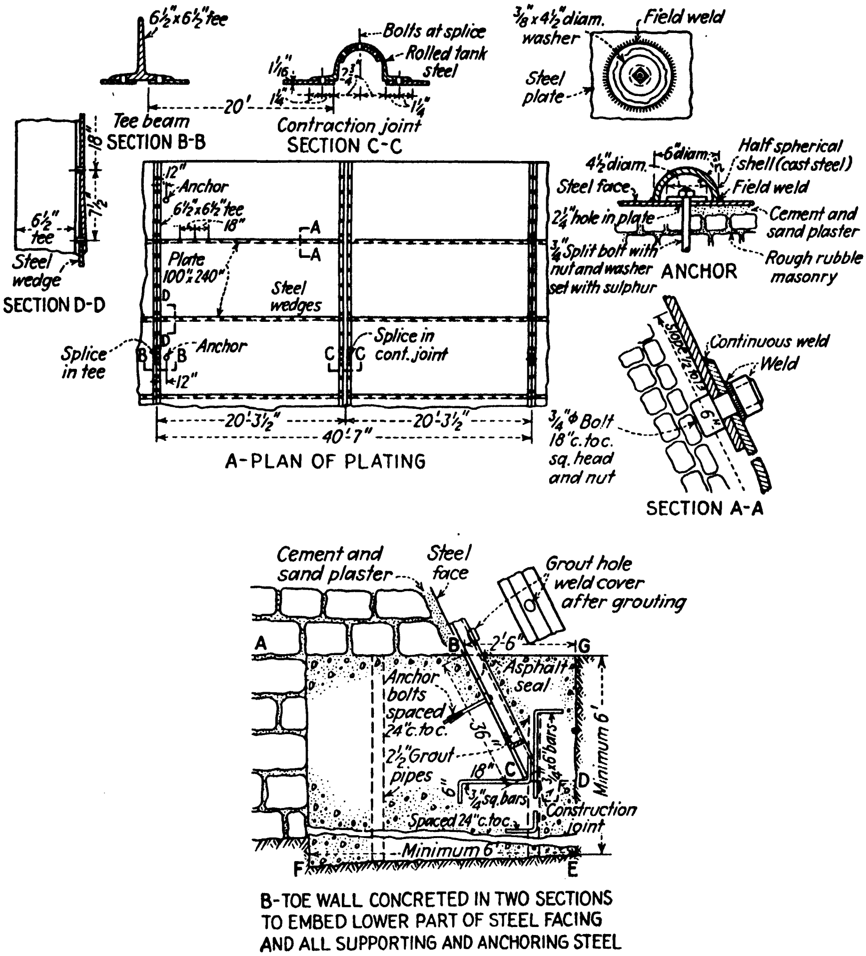


FIG. 14.—Steel-face details, Penrose-Rosemont Dam, Broadmoor Hotel, Colorado.

completely destroyed by fire, and that of Bear River Dam, Alpine County, California, was damaged.

Steel Face. Impervious membranes built of structural-steel shapes and plates have been successfully used on rock-fill dams.

The Penrose-Rosemont Dam,[1] Colorado, is 100 ft high above the bottom of the toe wall. Figure 14 shows typical face details. Copper-bearing steel plates, 240 by 100 in. by ⅜, 5/16, and ¼ in. form a complete covering over a 2-in. cement plaster

[1] REID, H. I., Steel Plates with Welded Joints Seal Rock-fill Dam, *Eng. News-Record*, **108-I**, p. 761.

subslab. *Plates at horizontal seams are bolted every 18 in. and lapped, downward* welding being thus permitted. All bolt heads and seams were electrically welded.

The steel face of Skaguay Dam (see Table 6), completed in 1901, has proved satisfactory. The back sides of the plates were not painted. A bituminous-paint coating was applied to the face side, and later in 1926 a coat of asphalt paint was applied. Inspection[1] in 1932 indicated that damage caused by corrosion was negligible.

El Vado (gravel) Dam,[2] New Mexico, 175 ft high, is faced with welded steel plates 100 in. wide by 24 ft 5 in. long supported on light steel horses embedded in a $1\frac{1}{2}:1$ gravel slope. Steel V-shaped troughs at 25-ft centers form inclined expansion joints.

Earth Blanket. A rock-fill dam may be made impervious to harmful passage of water by building a suitable earthen fill or blanket on the upstream slope. Such dams are composite structures, described in Sec. 5.

Face Drainage. Water from seepage, leakage, rain, or sluicing may accumulate behind face membranes due to presence of fines in the fill or river-gravel foundations, or if the face slab rests against a tight wall such as concrete or masonry. Uplift pressures thus created may cause rupture or failure of the membrane. Artificial drainage should be provided if conditions indicate this possibility and other corrective means are not feasible.

Uplift pressure from sluicing water partially displaced a portion of heavy concrete facing of Salt Springs Dam during an early stage of construction. Temporary drainage holes were drilled through the concrete and the affected area filled with cement grout. No subsequent uplifting developed, though occasional seepage spots are evident above receding reservoir levels.

The Shing Mun Dam in China[3] was designed and constructed to permit face drainage. The impervious membrane is supported on buttresses at 12.5-ft centers, these being part of a concrete thrust block of large proportions. This arrangement provides a series of vertically inclined inspection galleries useful in collecting leakage. Special openings were provided to lead this water into the downstream dry-rubble section.

Face-drainage systems were provided at Vannino Dam, Italy, and Piana-Grechi Dam, Sicily. The membranes of these structures are backed by wet masonry.

6. Impervious Diaphragms (Core Walls). Membranes placed at or near the center of rock-fill dams are inaccessible, and their condition cannot be determined until a bad leak develops. Their use is considered inadvisable. Before they can function, water under reservoir pressure must penetrate the upstream fill. Thus only that portion of the dam downstream from the core wall is effective in resisting water pressure. Difficulties were experienced at Oved-Kebir,[4] Tunis, and Crane Valley (composite type), California, dams because of central core walls.

7. Safety against Sliding. Rock-fill dams generally are of sufficient cross section and weight to preclude failure from sliding. The factor of safety against sliding for most of them (excluding those with central core walls) is greater than for any other except earthen-type dams. Water load creates the sliding force. Resisting force results from weight of the rock and the vertical component, if any, of water load on the impervious membrane. Reduced weight due to buoyancy should be used in calculating sliding factors for poorly drained fills or where the dam is partly submerged owing to backwater downstream. Well-drained fills containing the lowest percent-

[1] HOVEY, OTIS E., "Steel Dams," p. 79, American Institute of Steel Construction, New York, 1935.

[2] SEGER, CHARLES P., Steel Used Extensively in Building El Vado Dam, *Eng. News-Record*, **2**, 211, 1935.

[3] See paper by W. J. E. Binnie, Study of the Facing of Masonry and Concrete Dams, Second Congress on Large Dams, World's Power Conference, Washington, D. C., 1936.

[4] NOETZLI, F. A., Corewall in Rock-fill Dam Tilts When Reservoir Fills, *Eng. News-Record*, **109**, 529, 1932.

ages of voids afford the greatest safety against sliding. Table 1 shows computed sliding factors on a horizontal plane at the base of a section of dam 1 ft long, 200 ft high, with a crest width of 15 ft, an upstream face membrane with reservoir at crest level, and face slopes as shown. Rock fill is assumed to weigh 100 lb/cu ft in place.

TABLE 1.—SLIDING FACTORS AT THE BASE OF A ROCK-FILL DAM 200 FT HIGH WITH IMPERVIOUS MEMBRANE ON UPSTREAM FACE

Face slopes		Rock load, 1,000 lb	Water load, 1,000 lb		Total vertical load, 1,000 lb	Sliding factor
Upstream	Downstream		Vertical	Horizontal		
½:1	1:1	3,300	625	1,250	3,925	0.318
¾:1	1:1	3,800	937	1,250	4,737	0.264
1:1	1:1	4,300	1,250	1,250	5.550	0.225
1:1	1.3:1	4,900	1,250	1,250	6,150	0.203
1¼:1	1.3:1	5,400	1,562	1,250	6,962	0.179
1.3:1	1.4:1	5,700	1,625	1,250	7,325	0.171
1.3:1	1.5:1	5,900	1,625	1,250	7,525	0.166

Coefficients of sliding for loose rock on rock or loose rock on loose rock vary considerably and probably range from about 0.6 to unity. Sliding factors increase for foundations having downstream inclination.

In high dams with upstream face membranes, the upstream area of rock fill immediately overlying the foundation is subjected to light loading prior to reservoir filling. Sliding could occur here if the sliding factor under water load exceeds the coefficient of friction of loose rock on rock and the rock fragments are insufficiently compacted to withstand, in compression, the maximum applied loads. The late D. C. Henny described this action as local sliding in his discussion[1] of a paper, Salt Springs Dam, by O. W. Peterson.[2]

Sliding along horizontal planes formed at the tops of construction lifts is unlikely. Reasonable precaution only need be taken to remove or displace fines in order to obtain rock-to-rock contact.

8. Cutoff Wall. Some form of cutoff wall is required along the lower boundary of the impervious membrane to prevent harmful leakage at or below foundation level. This usually consists of a concrete-filled trench excavated vertically into solid rock or other impervious foundation material. It may be placed at some other inclination along the abutment slopes. Trench width usually is governed by construction requirements. Trench depth depends upon the desired length of the path of percolation. Governing factors are reservoir pressure and character of foundation. Conservatism is desirable because the cutoff wall and face membrane are the only water barriers and because leaks here are difficult to locate and repair. A length for the path of percolation of 10 to 25 per cent of the dam height generally should be sufficient in solid rock or impervious gravel foundations. Pervious gravels and foundations containing faults, open fissures, or cavernous rocks may require deeper cutoff walls and other special treatment. The cutoff wall of Dix River Dam[3] extends 25 to 30 ft into a noncavernous, fairly solid limestone rock foundation and provides a path of percolation equal to nearly 25 per cent of the dam height. The path of percolation in solid granite

[1] *Proc. A. S. C. E.*, August, 1930, p. 1326.
[2] *Ibid.*, p. 1319.
[3] HOWSON, GEORGE W., World's Largest Rock-fill Dam Built on Dix River, *Eng. News-Record*, **94**, 548, 1925. See also *Eng. News-Record*, **94**, 1058, 1925.

with occasional seams at Salt Springs Dam is about 10 per cent of the dam height. The cutoff wall of Swift Dam (see Fig. 11) penetrates about 30 ft of river gravel and 2 ft of rock and that at Malpaso Dam extends 140 ft below the stream bed through glacial drift into bedrock (see Fig. 19*A*). Several low dams have been constructed with cutoff walls resting on rock.

Precipitous foundation irregularities which may cause large differential settlements of rock fill in the vicinity of the cutoff wall may be removed by excavation of high points and ledges and filling of low areas with lean concrete or other materials in order to provide a beneficial alignment for the cutoff wall.

Sections of cutoff walls extending above the foundation level must resist reservoir pressure and, with empty reservoir, the downslope thrust in the face membrane. Concrete gravity sections were used at San Gabriel Dam 2 and Strawberry and Buck's storage dams. Salt Springs Dam has a reinforced-concrete section resting on a gravity masonry block (see Fig. 10*E*).

Horizontal construction joints in the cutoff should be avoided or otherwise should be provided with suitable water stops. Vertical construction joints generally are provided with water stops and grouted.

Foundations underlying cutoff walls of important dams usually are drilled and pressure grouted. Depth and spacing of grout holes and grouting pressure are dependent upon factors governing depth of cutoff trench and upon the manner in which they take grout. About 18,500 lin ft of grout-hole drilling was done at Dix River Dam. Holes were 200 to 250 ft deep. At Salt Springs Dam, 161 holes averaged 42 ft in depth (see Fig. 4) and took an average of 2,600 lb of cement per hole under pressures of 50 to 130 psi. In addition, an average of 820 lb of cement was forced into open spaces at construction joints and around the cutoff wall through each of 90 piped holes. The cutoff wall of Tepuxtepec Dam was carried 125 ft below stream bed through a badly fractured area. Grout holes, drilled at about 15-ft centers and to depths corresponding to reservoir depths at each point, took from one sack to 20 tons of cement each at pressures of 50 to 1,500 psi.

9. Outlet Works. Tunnel outlets through solid abutment formation and separated from the dam have generally been adopted for high and important rock-fill dams. River diversion tunnels usually are constructed to suit permanent outlet requirements. These may be lined with plain or reinforced concrete or gunite and pressure grouted to minimize cracking, or they may surround steel-pipe outlets suitably anchored and sealed against reservoir pressure. Steel pipe in reinforced-concrete culverts and steel pipe encased in concrete have been successfully used beneath low and moderately high rock fills. Such conduits should be conservatively designed and fully protected against rupture from construction operations and forces created by weight and movement of rock fill. Danger of displacement and rupture may be minimized by placing the conduit in a deep trench excavated in solid foundation material.

Outlet tunnels under reservoir pressure should have ample overburden of solid character reasonably free from laminations or folds which might permit progressive failure owing to hydraulic-jack action unless the tunnel lining is self-contained.

Leakage of about 107 sec-ft under a static head of 162 ft developed at the lower end of the outlet tunnel at North Bowman Dam.[1] About 75 per cent of this leakage was observed to come from under a rock slab which lifted about 0.2 ft. Permanent repairs were made, and no subsequent trouble of consequence has developed.

No essential difference exists in requirements for upstream or downstream control works from those applicable to other types of dams, except that unwatering facilities should be of ample capacity to permit occasional lowering of reservoir level.

[1] TIBBETTS FRED H., Repair of Breaks in Outlet Tunnel of Rock-fill Dam, *Eng. News-Record*, **102**, 904, 1929.

10. Spillway. Design requirements of spillways are fully covered in Sec. 7. Conservatism in capacity allowance and certainty of operation under the highest possible intensity of watershed runoff are essential because overtopping is an outstanding hazard to the safety of a rock-fill dam. Material excavated to form large spillway channels is generally useful in making the rock fill of the dam at but slightly greater cost than for rock procured from established quarries.

11. Residual Freeboard. There exists no universally accepted rule establishing freeboard dimensions of rock-fill dams. Administrative authorities in some states have formulated rules and regulations that include minimum allowances for dams of various types. The Arizona code prescribes a minimum residual freeboard of 3 ft for rock-fill dams and permits accomplishment by the use of parapet walls.[1] It recommends adoption of residual freeboard based on the formula

$$H = 1.5(1.5d^{0.5} + 2.5 - d^{0.25})$$

where H = vertical height of freeboard, ft,
d = distance in miles from the dam to the most remote point along the shore of the reservoir, measured across open water.

The California law prescribes no minimum, but rules and regulations recommend adoption of residual freeboard based on the preceding formula.

Residual freeboard should be greatest for dams located in earthquake areas or for dams that otherwise may settle abnormally.

12. Parapet Wall. A parapet or coping wall along the upstream edge of the crest of a rock-fill dam forms a protective railing and may have esthetic value. Large

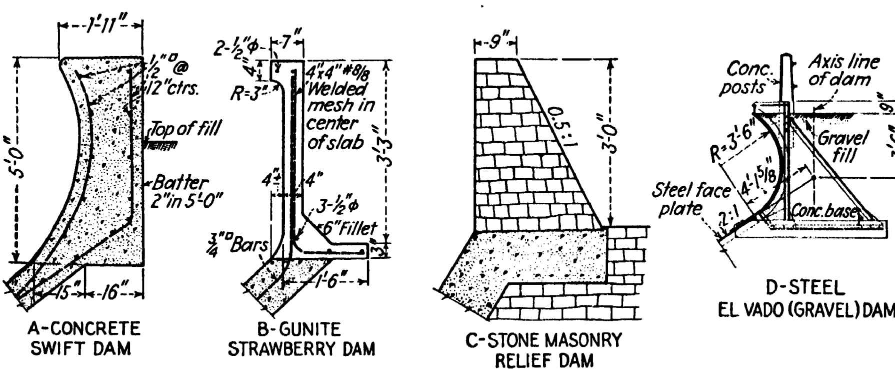


Fig. 15.—Typical sections of parapet wall and wave deflectors.

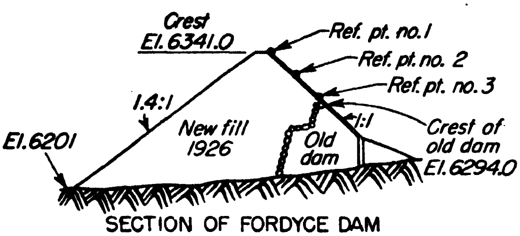
increases in spillway capacity without sacrifice of reservoir storage space have been inexpensively provided by building parapet walls on several of the older dams in California. Two types of coping wall constructed for this purpose are shown in Fig. 15*B* and *C*. The spillway channel at Salt Springs Dam was purposely enlarged to accommodate a possible flood discharge with reservoir level at the top of the rock fill. Construction of the parapet was postponed until 1947 when radial-type spillway gates were installed to provide 9,400 acre-ft of added reservoir capacity. A flow equal to nearly twice the maximum anticipated flood can be accommodated without encroaching upon the 4-ft free board provided by the parapet wall.

[1] Fraps, J. A., Arizona Adopts a Code for Dams, *Civil Eng.*, **3**, 195, 272, April, May, 1933.

IV. THE SETTLEMENT OF ROCK-FILL DAMS

Rock-fill dams of all types settle. Settlement results from compaction of fill and foundation under loads from weight of rock and reservoir pressure. All loads are carried through the fill to the foundation or transferred thereto through shearing resistance of the rock mass. Extent of total settlement is somewhat proportional to the amount of applied loads and the distance in the direction of the resultant forces through which they act.

Vertical settlement results from weight of rock and vertical components of water pressure. Vertical settlement at any point on the face due to rock load only should vary as the square of the vertical distance from face to foundation and can be expressed as $X(h/H)^2$, in which h is the height of point above foundation and H the height corresponding to settlement X. With a constant coefficient of settlement, the vertical displacement due to water load can be expressed by the factor $S = Ph/K$, in which P is the vertical component of water pressure and K is a modulus of settlement similar to Young's modulus for an elastic substance. Lines of equal values of K were thus determined for Salt Springs Dam and graphically illustrated by I. C. Steele and Walter



RECORDED MOVEMENTS IN FEET FOR MAXIMUM SECTION							
Point	El. orig. setting	1 Year		2 Year		10 Year	
		Hor.	Vert.	Hor.	Vert.	Hor.	Vert.
1	6342	0.59	0.59	0.67	0.68	0.88	0.96
2	6321	0.18	0.38	0.24	0.41	0.34	0.64
3	6302	0.21	0.34	0.22	0.36	---	---

FIG. 16.—Section and settlement of rock fill, Fordyce Dam, Pacific Gas and Electric Company.

Dreyer in their discussion[1] of J. D. Galloway's paper, The Design of Rock-fill Dams. Values of K for separate projects vary and are dependent upon rock characteristics and methods of construction. They likewise may vary for a single project as instanced at Salt Springs (see Art. IV, 2).

Horizontal or downstream movement is principally caused by horizontal components of water pressure against the impervious membrane.

Lateral movement is caused by settlement of rock fill from abutment foundations toward the deeper central sections of fill. This is graphically illustrated by J. D. Galloway in his paper, The Design of Rock-fill Dams.[2]

Rock fills are made up of solid rock fragments and voids of many shapes and sizes. Rock-to-rock contact, especially in loose fill, largely consists of points and edges which bear on or against adjacent stones. As load is applied, rock points and edges crush and weak, fractured, and seamy rocks break. Many broken fragments drop into the voids. Each failure permits movement of the overlying or adjacent mass in the general direction of the applied force. There develops a gradual readjustment of rock particles within the fill until sufficient bearing area is produced to establish equilibrium. This process is rapid during the construction period and for several months after substantial water load is applied. It continues indefinitely but to a lesser degree after the dam is subjected to full reservoir pressure for a short period of time.

Table 2 shows periodic vertical and downstream crest settlements of several dams. These generally are for points at sections of maximum depth of fill and do not include movements prior to initial setting of points.

[1] *Trans. A. S. C. E.*, **104**, 61, 1939.
[2] *Trans. A. S. C. E.*, **104**, 10, 1939.

TABLE 2.—APPROXIMATE MAXIMUM CREST SETTLEMENTS OF ROCK-FILL DAMS

Dam	Approx depth of fill, ft	Approx total time		Total movement, ft		Source of information, remarks
		Years	Months	Vert.	Horiz.	
Bonito (before enlargement in 1943)	92	0	4	0.48	0.20	W. H. Kirkbride, chief engineer, Southern Pacific Co.
		1	0	0.52	0.26	
		6	0	0.59	0.33	
		10	5	0.63	0.36	E. E. Mayo, chief engineer, Southern Pacific Co.
Bowman (built 1876, dismantled 1926)	96	10	..	0.46	....	J. D. Galloway, consulting engineer. Measurements by Waggoner, Warnecke, and Armitage
		30	..	0.97	....	
		39	..	1.16	....	
		45	..	1.28	....	
Dix River	270	0	2	0.3	0.45	L. F. Harza, consulting engineer. E. W. Brown, vice-president, Kentucky Utilities Co. Geo. W. Howson
		2	0	1.6	1.2	
		3	0	1.8	1.4	
		11	0	2.4	1.9	
		13	0	2.5	2.0	
		14	11	2.8	2.1	
		16	11	2.9	2.3	
		20	5	3.2		
		24	1	3.5	2.7	
Salt Springs	328	0	6	0.37	0.13	Pacific Gas and Electric Co. Measurements at Sta. 5 + 20 Pt. H (5 ft below top of fill)
		1	..	0.60	0.26	
		2	1	1.27	0.64	
		5	7	1.75	0.91	
		9	10	2.10	1.07	
		14	6	2.45	1.17	
Strawberry	140	0	3	0.73	0.42	Pacific Gas and Electric Co. Figures do not include settlement during a 2-month period following readings taken 1 year 3 months after completion
		1	3	0.94	0.95	
		7	2	1.70	1.21	
		13	4	1.85	1.43	
		19	0	2.11	1.60	
Swift	125 (Sta. 2 + 00)	0	5	0.16	0.16	C. E. Atwood, general manager, Montana Western R. R. Settlement of fill exceeds higher values shown by as much as 0.5 ft
		0	11	1.30	1.46	
		1	9	1.77	1.87	
		3	2	2.13	2.13	
		10	0	2.48	2.45	
		21	10	2.93	3.10	
		31	11	3.14	3.20	
Cogoti	172	4	3	1.34	....	Miguel Montalva C, Departo de Riego, Chile. Includes 1.38 ft settlement caused by severe earthquake of Apr. 7, 1943
		8	3	2.88	....	
Bakhadda	146	After final filling		1.2	0.9	GUTMANN, I., *Eng. News-Record*, **119**, p. 889.
Ghrib	233	..	..	1.8	....	GUTMANN, I., *Eng. News-Record*, **119**, p. 889. Exclusive of foundation settlement
Malpaso	....	2	5	0.89	1.82	Sanderson & Porter, Engineers
		10	6	1.26	2.53	

Figure 16 shows a section of the enlarged Fordyce Dam and recorded settlements at points above the crest of the original structure, which was 93 ft high and contained much small rock and fine material. The height was increased 47 ft in 1926 to form the present Fordyce Dam.

Little information is available on settlement of face membranes below the crest. The most complete data are those assembled subsequent to completion in 1931 of Salt Springs Dam. The concrete face of this dam was built in 60-ft square panels formed by horizontal and vertical joints. A bronze plug was set in one corner of each panel, and accurate measurements thereof were taken at frequent intervals. Figure 4*B* shows the arrangement of panels and point designations. Points lettered *A* are in the lower left-hand corners, and points lettered *B* to *H* are in the upper left-hand corners of the panels. Figure 17 shows settlement of points for the period 1931–1946. The top rows of figures denote approximate extent of opening or closing of joints as measured near the crest of the dam 2¾ and 14½ years, respectively, after completion thereof. Table 3 shows the number and date of observations plotted in Fig. 17.

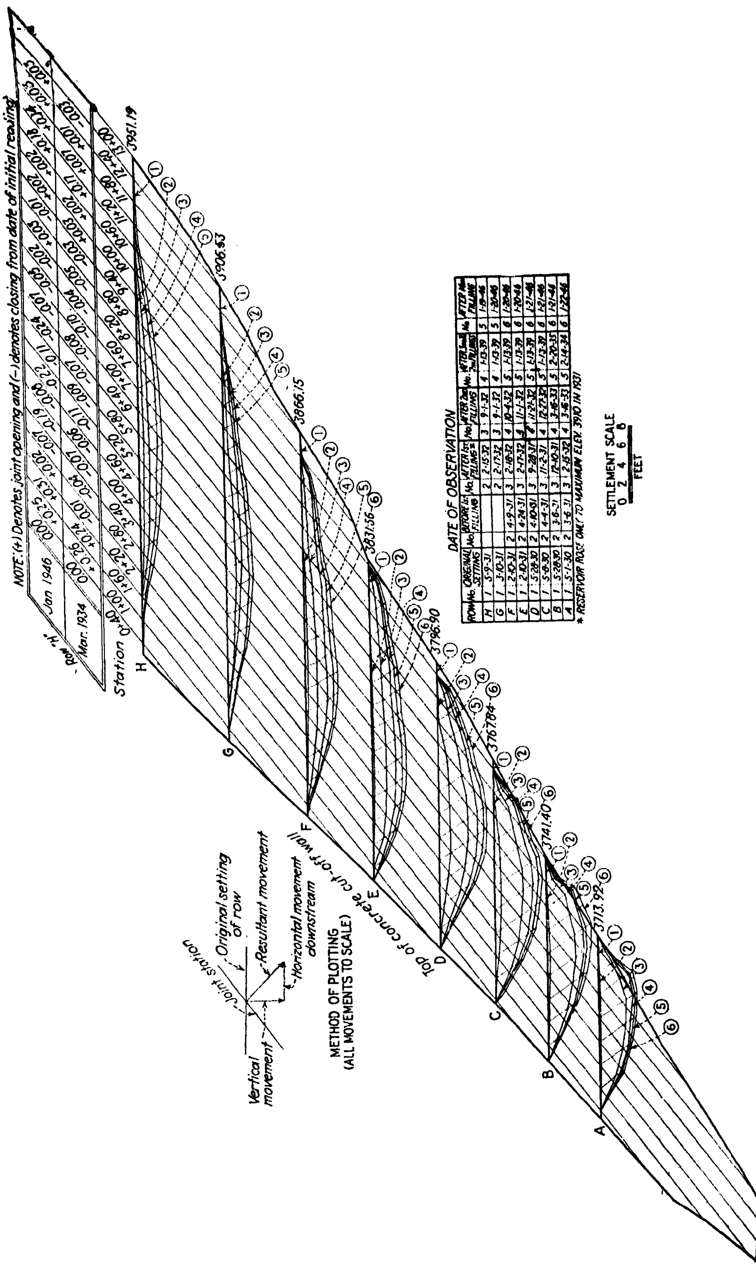
TABLE 3.—NUMBER AND DATE OF OBSERVATIONS PLOTTED IN FIG. 17

Row	Original setting		Before first filling		After first filling		After second filling		After third to seventh fillings		After fourteenth filling	
	No.	Date	No.	Date	No.	Date	No.	Date	No.	Date	No.	Date
H	1	May, 1931	..	....	2	Feb., 1932	3	Sept., 1932	4	Jan., 1939	5	Jan., 1946
G	1	Mar., 1931	..	....	2	Feb., 1932	3	Sept., 1932	4	Jan., 1939	5	Jan., 1946
F	1	Feb., 1931	2	Apr., 1931	3	Feb., 1932	4	Oct., 1932	5	Jan., 1939	6	Jan., 1946
E	1	Feb., 1931	2	Apr., 1931	3	Feb., 1932	4	Nov., 1932	5	Jan., 1939	6	Jan., 1946
D	1	May, 1930	2	Apr., 1931	3	Sept., 1931	4	Nov., 1932	5	Jan., 1939	6	Jan., 1946
C	1	May, 1930	2	Apr., 1931	3	Nov., 1931	4	Dec., 1932	5	Jan., 1939	6	Jan., 1946
B	1	May, 1930	2	Mar., 1931	3	Dec., 1931	4	Mar., 1933	5	Feb., 1935	6	Jan., 1946
A	1	May, 1930	2	Mar., 1931	3	Feb., 1932	4	Mar., 1933	5	Feb., 1934	6	Jan. 1946

TABLE 4.—DEPTHS AT WHICH MAXIMUM MEASURED SETTLEMENTS OCCURRED, SALT SPRINGS DAM

Sta.	Point of max settlement	Approx depth spill crest to point, ft	Approx total depth spill crest to foundation, ft	Relation depth of point to total depth, %
2+20	*F*	82	151	54
2+80	*E*	117	187	63
3+40	*D*	153	235	65
4+00	*D*	152	265	57
4+60	*D*	152	300	51
5+20	*C*	182	312	58
5+80	*C*	181	312	58
6+40	*D*	151	292	52
7+00	*D*	152	267	57
7+60	*D*	152	240	63
8+20	*D*	152	182	83[1]
8+80	*F*	63	167	38
9+40	*F*	82	147	56
				Average 56

[1] Not included in the average.



ROW	No.	ORIGINAL SETTING	No.	BEFORE 1st FILLING	No.	AFTER 1st FILLING*	No.	AFTER 2nd FILLING	No.	AFTER 3rd FILLING	No.	AFTER 4th FILLING
H	1	5-9-31			2	2-15-32	3	9-1-32	4	1-13-39	5	1-19-46
G	1	3-10-31			2	2-17-32	3	9-1-32	4	1-13-39	5	1-20-46
F	1	2-10-31	2	4-9-31	3	2-18-32	4	10-4-32	5	1-13-39	6	1-20-46
E	1	2-10-31	2	4-24-31	3	2-17-32	4	11-1-32	5	1-13-39	6	1-20-46
D	1	5-28-30	2	4-10-31	3	9-28-31	4	11-21-32	5	1-13-39	6	1-21-46
C	1	5-8-30	2	4-4-31	3	11-2-31	4	12-27-32	5	1-13-39	6	1-21-46
B	1	5-28-30	2	3-6-31	3	12-10-31	4	3-16-33	5	2-20-35	6	1-21-46
A	1	5-1-30	2	3-6-31	3	2-15-32	4	3-16-33	5	2-14-34	6	1-22-46

FIG. 17.—Diagram showing settlement of upstream face, Salt Springs Dam.

Table 5.—Lateral Movement of Observation Points in Face Membrane Salt Springs Dam

Point	Observation period, years	0+40	1+00	1+60	2+20	2+80	3+40	4+00	4+60	5+20	5+80	6+40	7+00	7+60	8+20	8+80	9+40	10+00	10+60	11+20	11+80	12+40
		Lateral movement, ft.																				
H	7.8	−0.02	+0.29	+0.54	+0.54	+0.47	+0.34[2]	+0.22	+0.16[3]	−0.07	−0.07[2]	−0.23	−0.33	−0.38	−0.39	−0.41	−0.40	−0.40	−0.14	−0.05	−0.02	−0.05
G	7.9	......	+0.31	+0.50	+0.51	+0.47	+0.45	+0.37	+0.24	+0.15	+0.08	−0.04[2]	−0.12	−0.13	−0.12	−0.12	−0.10	−0.07	+0.11	+0.13	+0.13	
F	8.0	......	......	+0.66	+0.70	+0.65	+0.58	+0.50	+0.49	+0.43	+0.29	+0.26	+0.17	+0.10	+0.15	+0.16	+0.13	+0.19	+0.16			
E	8.0	......	......	......	+0.29	+0.27	+0.24	+0.20	+0.16	+0.07	+0.02	0.00	−0.05	−0.07	−0.05	−0.04	−0.08					
D	8.5	......	......	......	......	+0.25	+0.20	+0.14	+0.13	+0.04	−0.06	−0.09	−0.13	−0.27	−0.13	−0.13	0.00					
C	8.5	......	......	......	......	+0.06[1]	+0.26	+0.20	+0.16	+0.08	+0.04	−0.06	−0.09	−0.11	−0.14	−0.23						
B	4.8	......	......	......	......	......	+0.07	−0.05	+0.01	+0.01	+0.03	−0.06	−0.07	−0.11	−0.12							
A	3.9	......	......	......	......	......	+0.25	+0.20	+0.12	+0.07	+0.03	−0.03	−0.07	+0.04								

See Fig. 5*b* for location of points.

\+ Denotes movement toward right abutment.

− Denotes movement toward left abutment.

[1] Observation period 6.5 years.

[2] Observation period 5.8 years.

[3] Observation period 4.5 years.

Figure 18 shows periodic settlements at selected stations.

Table 4 shows depths at which maximum measured settlement occurred at selected sections and their relation to total depth of fill below spill crest. Settlements at two points, C and D, were about equal for each of three stations, 5+20, 5+80, and 6+40.

Table 5 shows lateral movement of points. Maximum lateral movement took place at point F, Sta. 2+20. This point moved 0.73 ft toward the center of the dam in 4.6 years after initial reading thereof. Of this amount, 0.25 ft occurred prior to completion of the dam. Slight reverse movement occurred here and elsewhere during the period of record. In general, points to the left of Sta. 5+80 moved to the right and points to the right of Sta. 6+40 moved leftward. Maximum left-hand movement was 0.41 ft at point H, Sta. 8+80. Reasons for greater settlements near the left abutment are stated in Art. IV, 1, Methods of Construction.

Measurements at San Gabriel Dam 2, prior to completion of repairs[1] in 1935, indicated a crack 14 in. wide in the dry-rubble section. This opening adjoined a heavy concrete block immediately below the crest of the dam and near the right abutment where loose rock fill about 100 ft in depth had been rapidly placed. The extent of this movement is not typical of rock-fill structures.

Malpaso Dam, a cross section of which is shown in Fig. 19A, was founded on a glacial drift consisting of large and small boulders, gravel, sand, and silt. Measurements of a monument at the base of the fill about on the axis of the dam and 114-ft downstream from the heel indicate possibly 3.6 ft of foundation compaction, principally during the construction period. Observations during construction were taken about at points A, B, C, D, E, and F on the face of the wet-masonry wall (see Fig. 19A). Concerning the horizontal movement of these points, Sanderson & Porter, Engineers, who designed the dam, state:

"Between May, 1935, and March, 1936, a 10-months period, point 'A' moved downstream 0.12 feet. By the first of October, 1936, it had returned upstream .09 feet, thus leaving it when submerged at the latter date .03 feet downstream of its position on May 1, 1935.

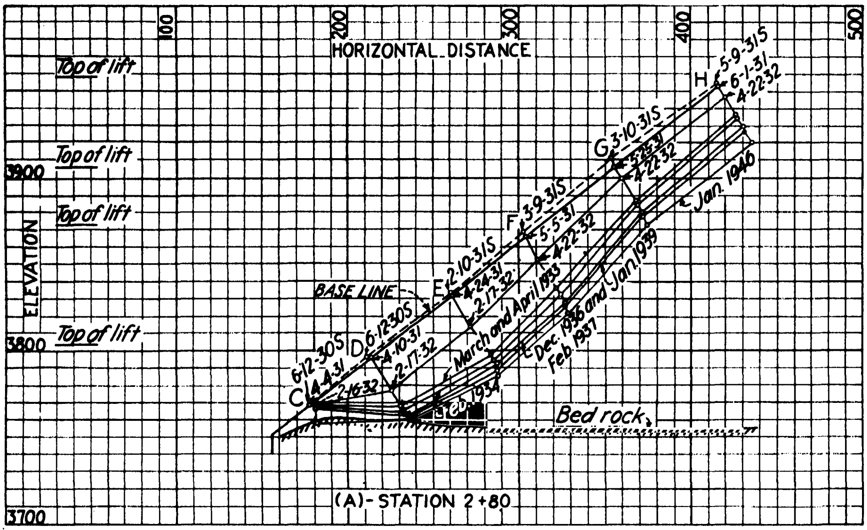
"Between October 14, 1935, and February 6, 1936, point 'B' moved downstream .32 feet and returned 0.17 feet by October 1, 1936, when it was submerged permanently. Between February 6, 1936, and June 8, 1936, point 'C' moved downstream .3 feet, returned .1 foot in the next 32 days and again moved slowly downstream, so that on October 6, 1936, it was 0.3 feet downstream from its original position.

"Point 'D' had a steady downstream movement from April 15, 1936, when it was set, to November 30, 1936, at which time it was 1.2 feet downstream of its original position. Point 'E' moved downstream without a recession from May, 1936, to November 30, 1936. At the latter date it was 1.62 feet downstream from its original position. Point 'F' moved similarly to point 'E', a total of 1.61 feet between July 6, 1936, and November 30, 1936."

Movements shown in Table 2 for Malpaso Dam were of a bolt set in the apron crest on the dam axis and apply to the period, Oct. 3, 1936, to Mar. 25, 1947. About half the indicated motion occurred during the four months, October, 1936, to January, 1937, during which the reservoir was filling for the first time. Movement was proportionately less toward the side walls.

In order to permit downstream motion independent of the cutoff, the 5-ft-thick apron base rests in a groove 3 ft 6 in. deep and of variable width formed in the crest of a 32- by 23-ft concrete arch ring at the top of the cutoff wall. Figure 19B diagrammatically illustrates the apron-base details. A constant 6-in. width of space for upstream motion extends between abutments. Width of space for downstream

[1] Construction, Subsidence and Repair of San Gabriel Dam No. 2, *Eng. News-Record*, **114**, 343, 1935.



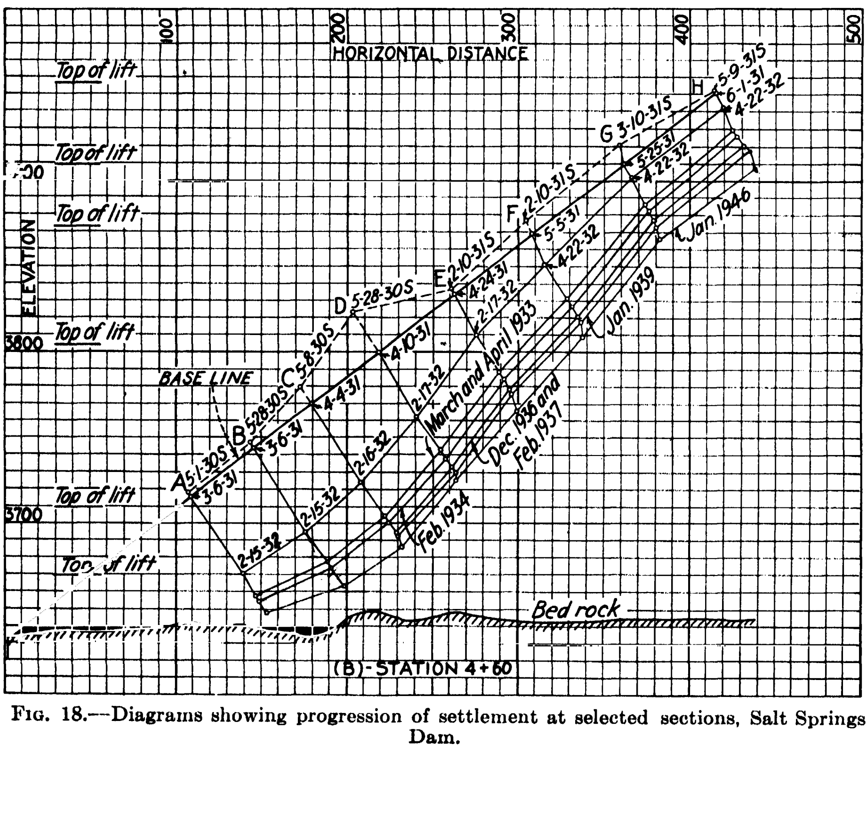


FIG. 18.—Diagrams showing progression of settlement at selected sections, Salt Springs Dam.

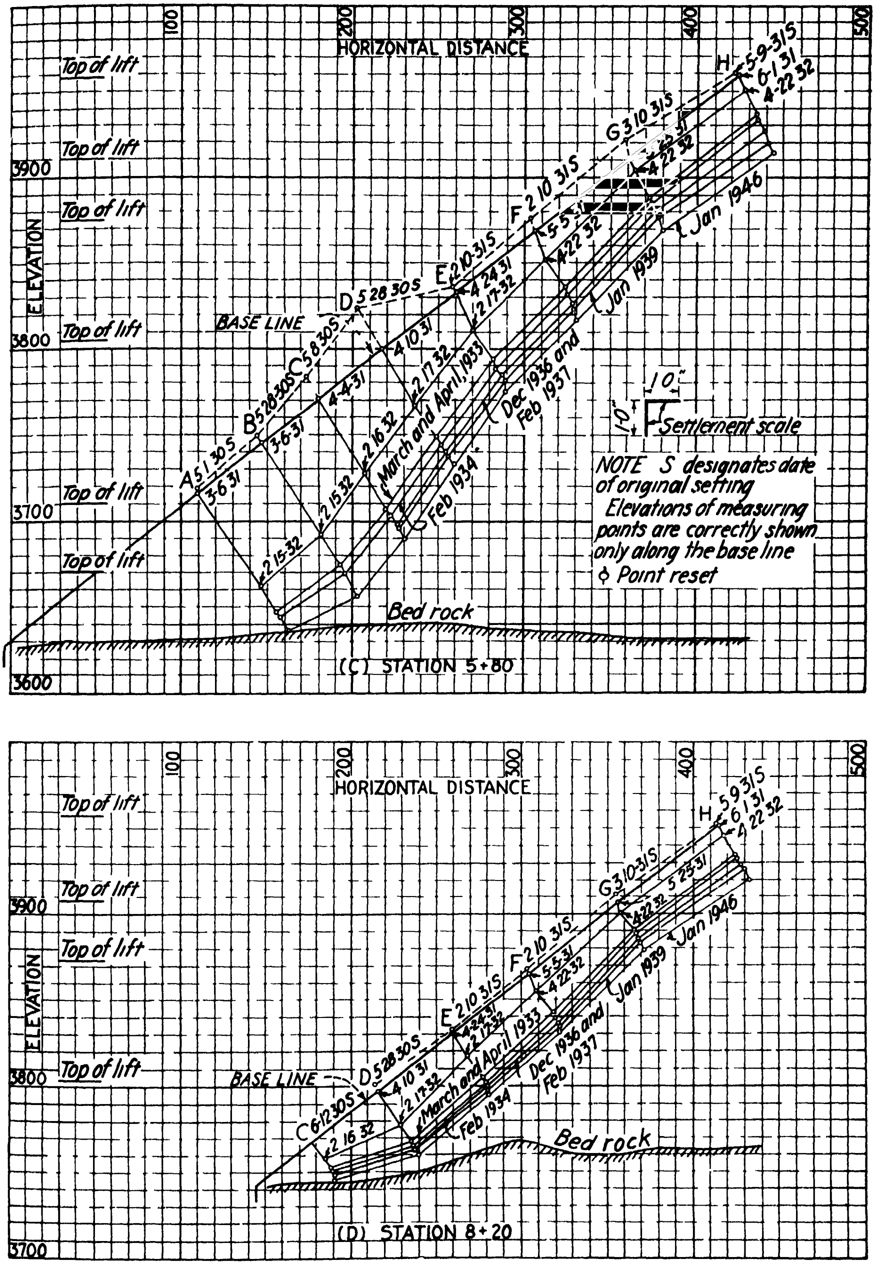


FIG 18 — (*Continued*)

motion varies from 18 in. at the dam axis to 4 in. at the side walls. Observations between Oct. 3, 1936, when pouring of the apron was completed, and Jan. 29, 1937, when the lake was first full, indicated downstream movements near the apron base of

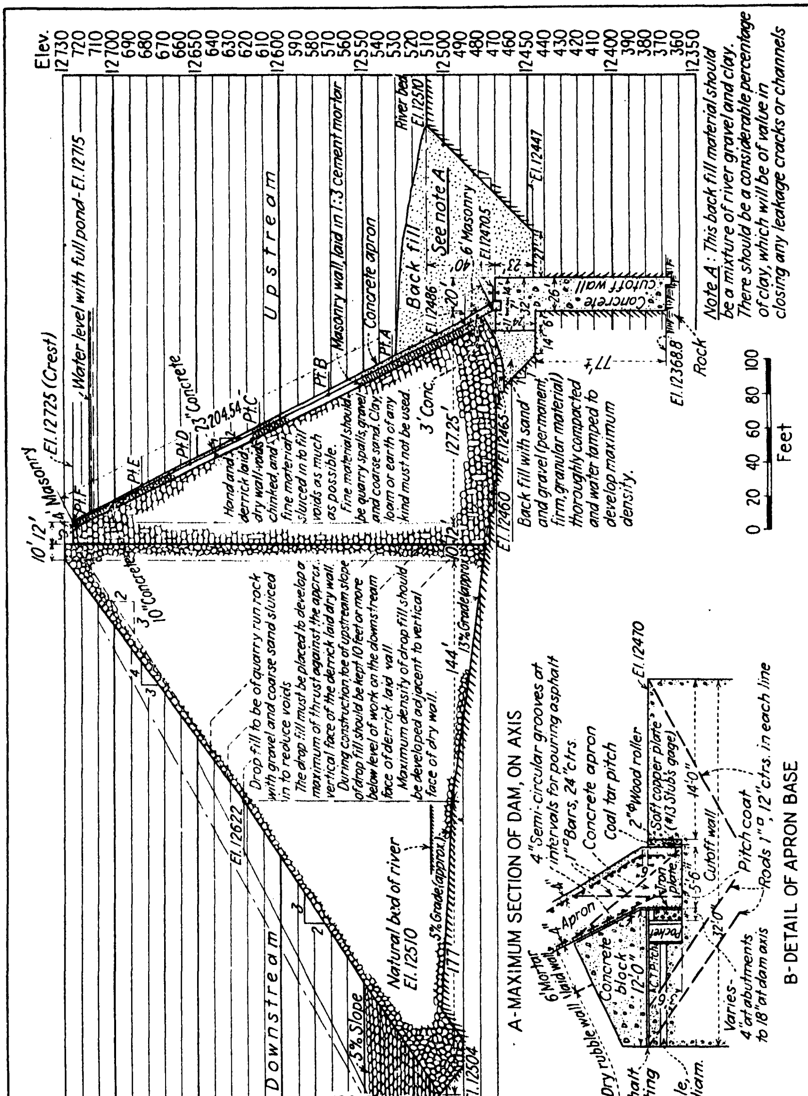


Fig. 19.—(A) Cross section through axis; (B) details of apron base. Malpaso Dam, Peru. (Courtesy of Sanderson & Porter, Engineers.)

6, 7⅛, and 7¼ in., respectively, at points 3 ft and 45 ft to the right and 37 ft to the left of the dam axis. By autumn of 1938, these increased to 7, 9⅜, and 10¼ in., respectively.

1. Factors Contributing to or Affecting Settlement. *Height and Cross Section of Dam and Water Pressure.* Maximum loads from weight of rock occur at the bottom of the highest column of rock within the structure. Rock load increases during construction, reaching a maximum upon completion of the dam, at which time major settlement under the highest points of the dam has probably taken place. The shallower fills along the face slopes and those overlying the abutments are not heavily loaded, and the upstream face becomes vulnerable to further settlement under water load. This principle is illustrated at points *D* (Fig. 18*C*) and *A* to *E* (Fig. 18*B* and *D*). Vertical components of water pressure are additive to the rock load and increase with flattening of the upstream slope. Theoretical resultant forces change in direction and magnitude as illustrated in Fig. 20. Data and this analysis indicate that

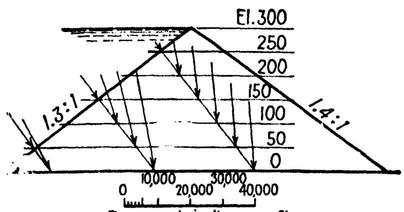


Fig. 20.—Diagram showing direction and amount of resultant forces due to water pressure and vertical rock load at any point within a rock-fill dam.

1. Flat upstream slopes settle more than do steep slopes.
2. No great settlement of fill will occur in low dams because neither rock load nor water load is of large magnitude.
3. Major crushing and readjustment of rock in the bottom zones of deep fills takes place before water load is applied.
4. The partially compacted rock zones beneath the upstream face of high dams are especially subject to additional settlement under water pressure.

Character of Foundation and of Rock in Fill. Rock-fill dams should not be built on foundations that permit important settlement under anticipated loads (see Art. II, 3). The dry-rubble type of dam was selected for the Bakhadda, Ghrib, and Bon-Hanifia cyclopean masonry dams in Algeria, partly because of compressibility of foundation ground.[1] Heights of these dams above foundation are about 146, 233, and 180 ft, respectively. One point in the foundation of Ghrib Dam settled about 6 in. during construction operations. Foundation settlement of 3.6 ft at Malpaso Dam, previously described, is believed to be the greatest of record.

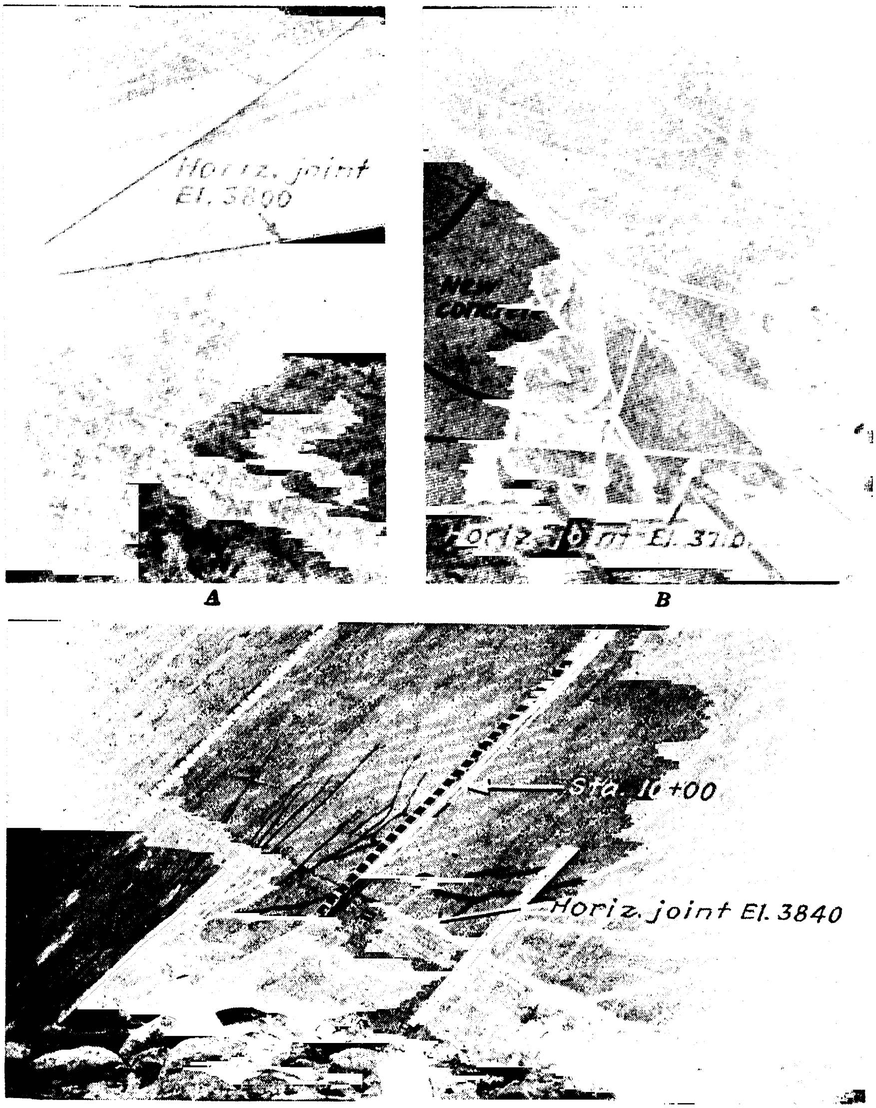
Rock-fill materials should be reasonably free of weaknesses that permit unusually large settlement (see Art. II, 2, 3). Height of dam and methods of construction may govern their use.

Shape and Slope of Canyon Walls. The total vertical and horizontal movement of rock fills under load is not materially affected by either the shape of the canyon or the slope of the abutment walls. Differential settlement is decidedly influenced thereby and may cause the formation of cracks and local rupture of the face membrane. Canyons having side walls that are flatly and uniformly sloped limit differential settlements to smaller magnitudes than do those with steep slopes. Box-shaped and stepped canyons may cause harmful disturbance in narrow zones immediately adjoining the immovable abutments. Special treatment should be given to the membrane at these points.

Measurements at Salt Springs Dam show greater lateral movement near the left abutment (see Table 5), the general slope of which is 35 deg, than near the right abutments, where the average slope is 26 deg. The right abutment contains the higher steps or cliffs. Face rupture here extends farther from the cutoff wall. More vertical cracks as contrasted to diagonal cracks appear in this region. Examples of these

[1] Martin, M. M., and Drouhin, Barrages en enrochements arrimes d'Algerie, Second Congress on Large Dams, Washington, D. C., 1936; also Gutmann, I., Algerian Dams of Placed Rock-fill, *Eng. News-Record.* **119,** 889, 1937.

effects are shown in Fig. 21. Lateral movements, noted previously, indicate general compaction of fill toward the right. The steep left slope is thought but partly respon-



A B

C

Fig. 21.—Repaired cracks in concrete membrane, Salt Springs Dam. (*A*) View along left abutment (1932). (*B*) View along right abutment (1932). (*C*) View at change of foundation slope at right abutment (1937).

sible for this action (see Methods of Construction). The major portion of fill in the dam was placed from right bank quarries.

Methods of Construction. The technical press adequately describes construction plants, equipment, economy of construction, and methods used in procuring and

placing rock-fill material and in building other features of rock-fill dams. Comment herein therefore is confined largely to construction methods and procedure affecting settlement of dams built principally of loose rock fill.

Preferred construction methods are those which reasonably and safely produce maximum compaction of fill during construction and provide minimum settlement in the completed structure at least cost. Controlling factors in obtaining this result, about in order of importance, are use of quarry-run rock in the loose fill, adoption of high lifts, liberal use of sluicing water, avoidance of rapid rises in depth of fill, and maximum feasible delay in constructing the permanent face membrane.

Advantages of quarry-run rock are stated in Art. III, 4.

High lifts with liberal use of sluicing water contribute much toward compaction of fill during construction and are superior to low lifts for the following reasons: rocks dumped from high lifts roll or slide through greater distances, causing more complete breakage of the softer and structurally weak rock; rock points and thin edges are broken or ground off; rock piles that accumulate on the slope are released by impact of other rock and create extensive slides, causing further breakage and compaction; voids are more nearly filled with broken fragments and with the smaller sound rock; and with adequate sluicing the desired rock-to-rock contact more nearly obtains throughout the fill. Lifts 75 to 150 ft in height are recommended for high dams.

More large rock lodge near the base of high lifts than low lifts, but the resultant segregation of sizes within the fill usually is satisfactory.

The success of high lifts can best be attested by comparing crest settlements of existing dams. About one-half the yardage in Salt Springs Dam was placed from a bench level 165 ft above foundation. Fill from the spillway quarry reached 240 ft below the level of the dump (see Fig. 22). Other lifts ranged from 35 to 84 ft. Maximum vertical crest settlement for a 328 ft depth of fill was 1.3 ft 2 years after completion (about 0.4 ft per 100 ft depth of fill) and 2.4 ft 15 years after completion. The rock is a medium-grain hornblende granite (quartz monzonite, 2.70 sp gr) and has a crushing strength of about 19,000 psi.

High lifts also were employed in constructing Dix River Dam, the maximum being about 120 ft. Maximum vertical crest settlement for a fill depth of about 275 ft was 1.6 ft 2 years after completion (about 0.6 ft per 100 ft) and 2.8 ft 15 years after completion.[1] Dense limestone rock (smaller in average size than the granite rock in Salt Springs Dam) constituted the fill. Its crushing strength ranged from 12,500 to 15,000 psi.

Eighty per cent or more of the loose fill in Strawberry Dam was dropped from cableways (the drop generally being less than 30 ft) and partly rehandled with derricks. The vertical settlement for a 140-ft depth of fill measured 1.3 ft 21 months after completion (equal to settlement of Salt Springs Dam, over twice as high) and 2.2 ft 20 years after completion. The rock, a medium-grained hornblende granite (2.68 sp gr), is generally tougher and offers greater resistance to weathering than the rock in Salt Springs Dam.

At San Gabriel Dam 2, 25-ft lifts were specified and employed. Hard gneissic rock, graded to eliminate small rock, was used to make the fill. Sluicing was prohibited. Considerable settlement of the 265-ft fill was noticed as the crest of the dam was approached. Heavy rains occurred, and settlement rapidly increased. Cross sections plotted from field notes indicate a vertical crest settlement at one place of 8.6 ft from Dec. 12, 1933, to Jan. 16, 1934, and a total of 14.7 ft from Dec. 12, 1933, to Aug. 9, 1934. Downstream movement for the former period was 4.4 ft.

Sluicing water preferably is applied at the point and time of dumping the rock fill but may be effectively applied during any stage of work and to other parts of the fill.

[1] Courtesy of L. F. Harza, consulting engineer.

High nozzle velocity, usually necessary for delivery of the desired flow, is desirable but not essential because it is quickly dispelled when the stream strikes the embankment. Large volume at each sluicing point is important. The amount and rate of application varies with the character and speed of placement of rock fill. At Salt Springs, the total amount of water applied averaged twice the volume of the fill. A higher average would have been beneficial. Rate of application was lower during early stages of construction. Later as many as six 2- to 3-in. streams, each delivering about 600 gpm, were employed. The hard, dense-grained granodiorite rock in Fordyce Dam was quite free of fines, and no sluicing was done.

Excessive fines usually accumulate on and near bench levels, especially if high lifts are used. These areas require special treatment to minimize settlement and to provide good bond with the next fill. Scarifying and sluicing have given satisfactory results. At Salt Springs, shallow pits were dug at 6- to 10-ft centers over the bench

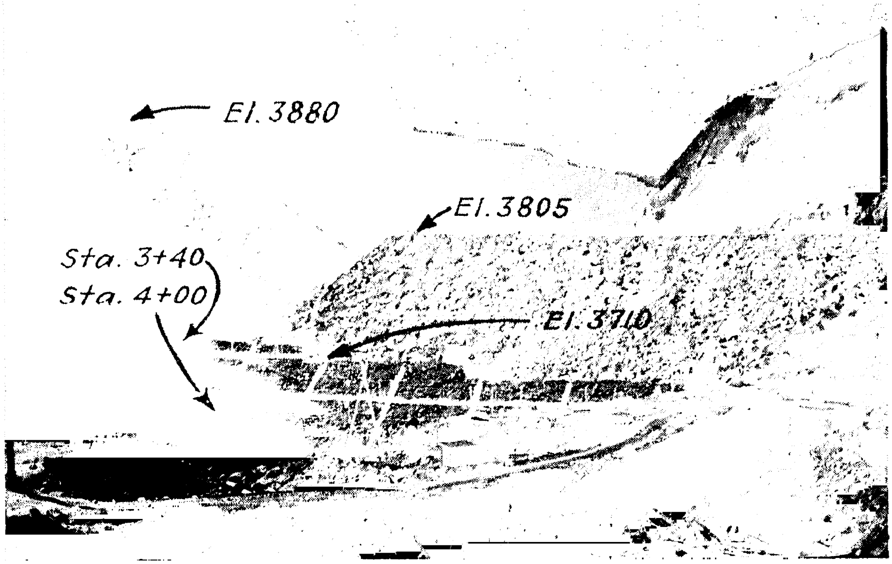


FIG. 22.—View showing zone of closure with high lifts of loose rock fill, Salt Springs Dam.

areas. Fines were loosened with low-strength powder placed in the base of these pits and washed into voids in the underlying fill, an exposed surface of reasonably clean, fragmentary rock and some large stones being left.

Rapid rises in depths of loose fills encourage settlement and should be avoided. Fig. 22 shows the closing zone of two high fills at Salt Springs in the vicinity of Sta. 4+00 where settlement subsequent to construction of the face was relatively greater than elsewhere in the dam (see Fig. 18*A* and *C* and Table 5). The 240-ft lift at the left abutment contained a greater percentage of small rock and fines and was less thoroughly sluiced than that at most places in the dam. Ill effects of these conditions are illustrated in Fig. 24*A* and *B*.

2. Provisions for Settlement. *Crown and Face Curvature.* It is customary and desirable to construct rock-fill dams with liberal allowances for vertical and downstream settlement. The crest usually is crowned above theoretical level in amounts proportional to estimated vertical settlements or from zero at the abutments to some maximum over the deepest section of fill. Otherwise, additional fill, or a coping wall, may later be required to maintain intended freeboard.

Upstream camber (in plan) ensures lateral compression in the face membrane as a whole during all stages of downstream and vertical settlement. Otherwise, tension cracks may develop and vertical expansion joints may open beyond desirable limits.

The face of Salt Springs was constructed concave to reservoir water in vertical planes to prevent buckling due to outward components of downslope thrust. Horizontal offsets for laying out face curvature are shown in *Engineering News-Record*, **104,** Table II, p. 95. Settlement records show that face concavity would have resulted had the face been constructed along a straight line. Concavity does not prevent face rupture due to either local convexities or eccentric loading within the face occasioned by differential settlements. Figure 18 shows the necessity of building each section of dry rubble to established grade, rather than along lines continuous with preceding slopes.

Joints in Impervious Membranes. Comment to follow is confined to reinforced-concrete membranes. Principles involved are applicable to other types of face membranes.

Face membranes, subjected to heavy water pressure, must follow the settlement of, and generally remain in contact with, the underlying fill. Stresses occasioned through settlement and temperature change must be provided for if serious face rupture is to be prevented. Hazard of rupture usually is reduced through division of the membrane into panels bounded by joints along which movement may take place. Maximum joint spacing is limited by temperature requirements and, in general, may be adopted for that large area which is subjected to reasonably small and uniform increments of differential settlement. Ample strength and flexibility here usually are obtainable with any reasonable type of membrane and joint arrangement.

Zones adjoining the cutoff wall, and certain other locations, however, are subject to larger differential settlements and require more careful attention. Low and moderately high dams generally have experienced little or no face rupturing. In high dams where settlement is proportionately large, tensile, compressive, and shearing stresses from all causes may exceed the resistive strength of the membrane regardless of its character and proportions. This condition may be prevented or minimized through the use of suitable joints opportunely placed. Examples of successful types of horizontal and vertical joints for concrete are shown in Figs. 10, 11, and 23.

The monolithic concrete face of Salt Springs Dam is about 9 acres in area. The portion affected by visible cracks with reservoir level 250 ft below the crest is about 5 per cent of the area of the exposed face. Tightness of the entire face is attested by records of leakage past the dam. Maximum leakage of 31 sec-ft occurred when the reservoir was first filled. This was reduced to 11 sec-ft, with full reservoir, after visible cracks were repaired.

With some exceptions flexure cracks lie within 60 ft of abutment walls. They are especially characteristic of the 20-ft strip along and roughly parallel to and about 10 ft removed from (see Fig. 21) the canyon walls. The lean masonry section downstream from the cutoff wall (see Fig. 10) probably influences the position of the lower limit of the damaged zone. The uppermost fracture is about 60 ft vertically below the crest of the dam. As many as nine cracks closely spaced appear at some elevations. Breaks are through the heavy concrete slab and usually at right angles thereto. They range in width from hairline cracks to a maximum of perhaps 1½ in. in one or two places where lateral movement is evident. The majority are about ⅟₁₆ in. wide. These cracks were repaired in conformity with Fig. 10*q*.

Other regions requiring special attention are those within which differential settlements permit concentration of heavy compressive forces at the top or bottom of the face membrane or where cumulative compressive stresses caused by settlement of the rock fill have been communicated by bond to the face membrane to such an extent as

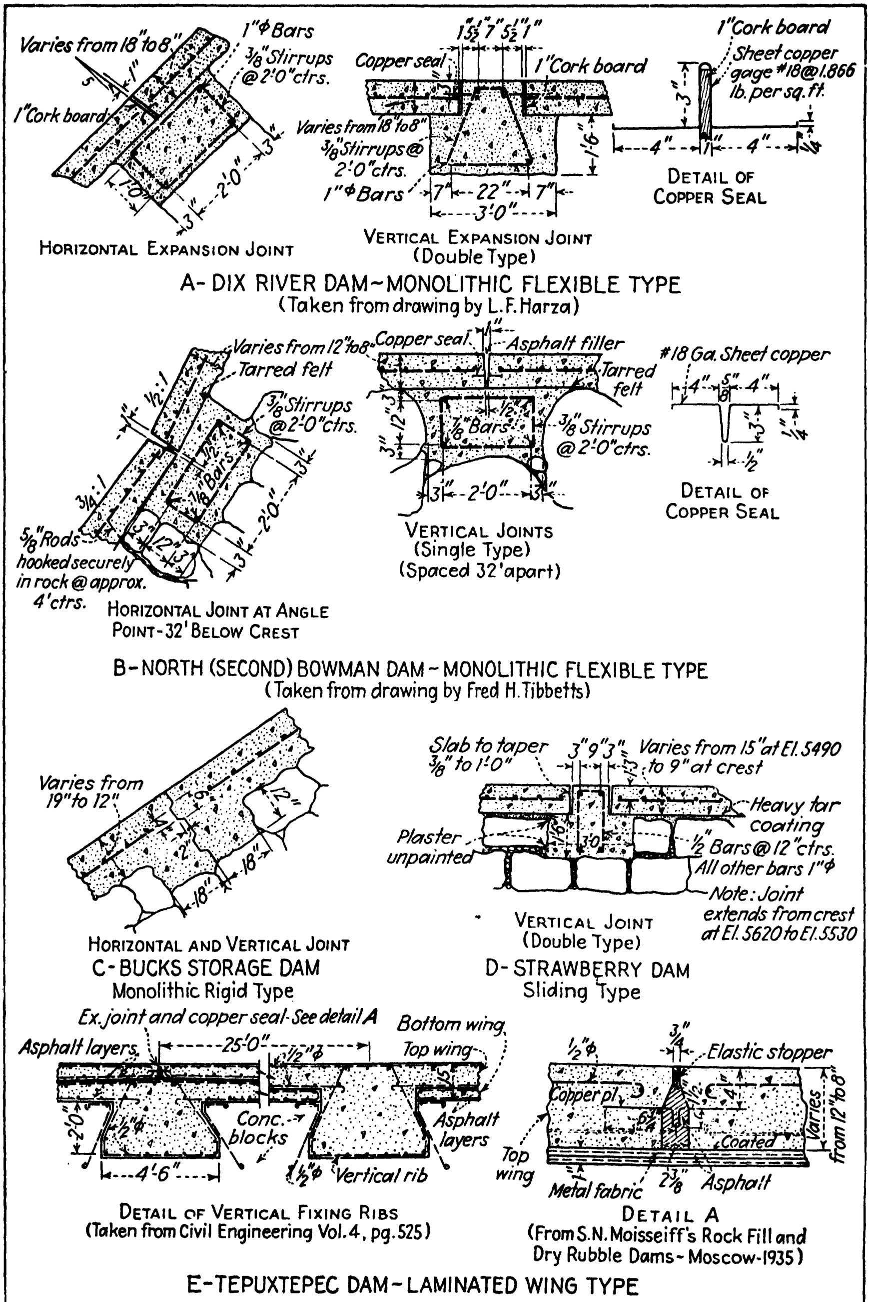


FIG. 23.—Typical joint details of concrete face membranes of rock-fill dams.

to cause failure by crushing or shearing. Performance at Salt Springs illustrates these points. Major settlement created heavy compression in the top of the slabs adjacent to joint *D* (Fig. 10*e*). Concrete along 420 ft of joint *D* Stas. 2+80 to 7+00 crushed first at the top and later throughout the thickness of the slab (see Figs. 10*p* and 24*A*). The slope length of the face between joints *D* and *E* decreased a maximum of 2 in. in the region noted above. Repair work consisted of complete removal

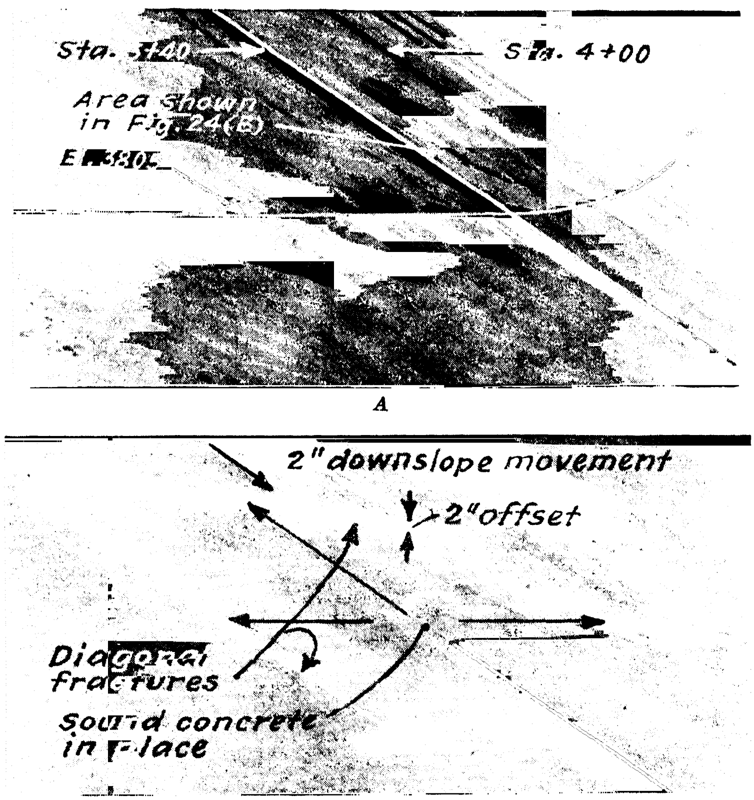


FIG. 24.—Fractures in concrete face membrane in vicinity of joint *D*, Sta. 2+80 to Sta. 4+00, Salt Springs Dam. (*A*) Typical fractures along horizontal joint. (*B*) Fracture at Sta. 3+50.

of all broken and crushed concrete, squaring the edges of the damaged area normal to the face of the slab, and replacement with new concrete or gunite. Thick cement-sand grout was injected through drilled holes above the repaired areas to fill the cavities formed underneath the displaced slabs due to the outward movement. Occasional areas of concrete were found to be of an inferior character because of loss of mortar during placement. Figure 24*B* shows a section of sound concrete bordered by an area of crushed and sheared concrete above it.

Horizontal joints in concrete membranes usually are of simple design. Square cold joints with copper water stops have proven satisfactory. Asphalt or other suitable material placed on the exposed edge of the lower slab is helpful in preventing leakage and bonding of concrete surfaces. Concrete ribs are frequently poured into keyways

TABLE 6.—LIST OF IMPORTANT ROCK-FILL DAMS IN THE

No.	Name of dam	Owner	Location	Year completed	Approx contents, cu yds	Approx height above foundation, ft	Crest width, ft	Crest length, ft	Thickness at base, ft
1	Bear River (first)	Pacific Gas & Electric Co.	California	1900	56,000	75	13	748	133
2	Bear River (enlarged)	Pacific Gas & Electric Co.	California	1932	85,000	80	7	755	164
3	Beaver Park	Mosca Irrig. Dist.	Colorado	1914	....	87[1]	16	370	...
4	Bonito (enlarged)	Southern Pacific R. R. Co.	New Mexico	1943	186,000	113	12.5	480	278
5	Bowman (first)	....	California	1872-1876	55,000	96	..	425	...
6	North Bowman	Nevada Irrig. Dist.	California	1927	370,000	168[1]	15	680	...
7	Bucks Storage	Pacific Gas & Electric Co.	California	1928	347,000	122[1]	12	1,220	320
8	Cucharas	Pueblo-Rocky Ford Irrig. Co.	Colorado	1911	195,000	125[1]	20	550	...
9	Dix	Kentucky Utilities Co.	Kentucky	1925	1,747,000	275	20	1,032	712
10	Drew's	The Goose Lake Valley Irrig. Co.	Oregon	1915	45,000	65	10	610	170
11	Escondido	Escondido Mutual Water Co.	California	1895	37,000	76	10	380	140
12	Fordyce (first)	Pacific Gas & Electric Co.	California	1873-1881	....	93	5	800	139
13	Fordyce (enlarged)	Pacific Gas & Electric Co.	California	1926	417,000	140	10	1,060	335
14	French Lake	Nevada Irrig. Dist.	California	1859	....	67	4	204	85
15	Meadow Lake	Pacific Gas & Electric Co.	California	1903	40,000	74	12	775	103
16	Morena	Morena Mutual Irrig. Co.	California	1912	324,000	167	16	520	389
17	Penrose-Rosemont	Broadmoor Hotel Water & Power Co.	Colorado	1932	....	100[2]	22	580	206[4]
18	Relief	Pacific Gas & Electric Co.	California	1910	137,000	140	11	505	290
19	Sabrina	California Elec. Power Co.	California	1909	47,000	70[1]	10	1,065	150
20	Salt Springs	Pacific Gas & Electric Co.	California	1931	3,200,000	328	15	1,300	905
21	San Gabriel Dam 2	Los Angeles Co. Flood Control Dist.	California	1935	1,200,000	280	18	600	785
22	South Lake	California Elec. Power Co.	California	1910	75,000	80[1]	10	650	170
23	Skaguay	Southern Colorado Power Co.	Colorado	1901	42,000	70	20	405	148
24	Strawberry	Pacific Gas & Electric Co.	California	1916	330,000	140[1]	15	612	382
25	Swift	Valier-Montana Land & Water Co.	Montana	1914	....	157[1]	15	465	478[5]

[1] Height, stream bed to low point of crest.
[2] Height, above bottom of toe wall.
[3] Horizontal thickness.

UNITED STATES WITH PRINCIPAL CHARACTERISTICS OF EACH

Face slopes, ratio horiz. to 1 vert.		Thickness upstream dry rubble, ft		Impervious membrane			Authority	Remarks
				Type	Thickness, in.			
Up-stream	Down-stream	Top	Bot-tom		Top	Bottom		
½, ¾	½, ¾, 1	8	16	Timber	2-1½ pl	......	Owner	Enlarged 1932
½, ¾	½, 1.35	8	16	Timber	2-1½ pl	......	Owner	
½	½, 1.5	5	5	Reinforced concrete	12	24	*Eng. News*, 74 p. 660	Monolithic rigid-type face
1.17, ¾	1.25	11[6]	20[6]	Reinforced concrete	8	12	W. H. Kirkbride E. E. Mayo	Facing coated with asphaltic material
1	1	6	18	Timber	3	9	August J. Bowie, Jr., "Practical Treatise on Hydraulic Mining"	Log crib 1872. Enlarged 1876. Dismantled 1926
½, ¾	1.4	6[3]	25[3]	Reinforced concrete	8	12	*Western Const. News*, Oct. 10, 1926, p. 83	
1.4	1.5	3	7	Reinforced concrete	12	19	Owner	Monolithic rigid-type face
1	1.5	30[3]	30[3]	Reinforced concrete	18	18–24	*Eng. Record*, Nov. 4, 1911, p. 538	Dry rubble chinked and clay puddled
1, 1.2	1, 1.4	4	14	Reinforced concrete	8	18	*Eng. News-Record*, vol. 94, p. 548	Timber cover over lower 160 ft of concrete
½	1.5	5	16	Timber	4¾	4¾	*Eng. News*, vol. 77, p. 100, and Hubert Koons	Refaced 1930 with 2 layers 1¾ in. creosoted fir
½	1, 1¼	5	15	Timber	2-1½ pl	2-3 pl	J. D. Schuyler	Timber face backed with concrete 2 in. thick
1	¼, ½	...	....	Timber	....	....	Owner	Enlarged 1926
1	1.35	4	6	Reinforced concrete	12	18	Owner	Monolithic flexible-type face
0.42, 1	0.1, 0.35	...	....	Reinforced gunite	2	2	Survey of 1937. William Durbrow, Mgr., and Charles T. Law, Assistant Engineer, for owner	Crest raised 3 ft and timber face replaced with gunite in 1937.
½, ¾	½	6	7	Reinforced gunite	2	4	Owner	Original timber face burned 1929
½, 0.9	1.5	16	50	Masonry	3 ft	6 ft	*Trans. A. S. C. E.*, 1912, p. 37	Raised 5 ft 1916–1917 and 10 ft in 1922–1923. 12-in. reinforced concrete face over masonry to point 42 ft above stream bed
½	1.4	4[3]	12½[3]	Steel	¼	⅜	*Eng. News-Record*, vol. 108, p. 761	Steel face rests against rubble masonry wall 2 to 8½ ft wide faced with 2-in. mortar
½	1.5	11	108	Reinforced concrete	12	18	J. D. Galloway Owner	Concrete face 18 to 36 in. in bottom 27 ft. Entire face tied to 2 thickness of masonry
¾	1¼	5	6	Timber	5	9	Owner Walter L. Huber	Facing renewed 1929 with redwood planks on 8- by 8-in. creosoted fir sleepers
1.3 (av.)	1.4	15	15	Reinforced concrete	12	38	Owner	Monolithic flexible-type face
1.2, 1.3, 1.35	1.5	6	16	Reinforced concrete	8	24	*Eng. News-Record*, 114, pp. 343, 836, and H. E. Hedger	Timber face replaced 1947–1948 with reinforced-concrete sliding-type face on concrete subslab below spillway level and reinforced-gunite slab or concrete buttresses above spillway level
¾	1¼	5	6	Timber	5	9	Owner Walter L. Huber	Facing renewed 1930 with redwood plank on 8- by 8-in. creosoted fir sleepers
0.58	1.2	...	....	Steel	¼	½	*Eng. News*, Jan. 1, 1903, p. 2	Dry rubble and layer small rock back of steel plates
1, 1.2	1.35	4	18	Reinforced concrete	9	15	Owner	Sliding-type face over 3-in. concrete subslab
1.2, 1.5	1⅓	4	4	Reinforced concrete	6	24	C. E. Atwood and Ford, Bacon, and Davis	Sliding-type face over 2-in. subslab

[4] Downstream side strengthened with 200-ft thickness of material 35 ft high.
[5] Includes 20-ft berm 12 ft above stream bed on downstream slope.
[6] Horizontal thickness of first dam completed in 1931.

TABLE 7.—LIST OF IMPORTANT ROCK-FILL DAMS IN FOR-

No.	Name of dam	Location	Approx year completed	Approx contents, cu m	Height above foundation, m	Crest width, m	Length, m	Thickness at base, m	Face slopes: Horizontal to 1-vertical: Up-stream	Face slopes: Horizontal to 1-vertical: Down-stream
1	Bakkadda	Algeria	1933	320,000	45	5	220	108	0.86, 1	1.25
2	Bou-Hanifia	Algeria	....	670,000	55	5	360	125	0.8, 1	1.33
3	Fum-El-Gueiss	Algeria	....	130,000	23	3	250	63.5	1	1.25
5	Ghrib	Algeria	1936	670,000	72	5	270	146.5	Varies 0.67 to 1	1.25
5	Caritaya	Chile	1935	....	38	5	156	121	1.5	1.5
6	Cogoti	Chile	1940	....	84.5	8	160	250.5	1.4 avg	1.6
7	Shing Mun	China	1936	....	84[1]	10	212 at water level	170	0.27	1.5
8	Alpe Cavall	Italy	1926	229,000	41.4	4.2	180	.....	0.7	1.33
9	Devero	Italy	1921	54,000	22.6	4	115	51	0.5, 1	1
10	Vannino	Italy	1924	....	25.5[1]	4.5	115	.....	0.8	0.8, 1
11	Vagno	Italy	1919	....	23	3.85	114	37.1	0.7	0.7
12	Patjel	Java	1932	136,000	37.4	6.0	..	110	1, 1.3	1.2, 1.5
13	Taxhimay	Mexico	1934	....	41	4	233	92	0.75	1.4
14	Tepuxtepec	Mexico	1929	108,000 cu yds	123 ft[1]	15 ft	900 ft	240 ft	0.7	Varies 0.7 to 1
15	Manuherikia	New Zealand	1935	....	110 ft	14 ft	520 ft	350 ft	....	......
16	Tarlebo	Norway	...	....	17	2.9	150.5	.....	0.675	1
17	Malpaso	Peru	1936	....	255 ft[2]	22 ft	497 ft	470 ft	0.5	1.33, 1.5
18	Piana-Grechi	Sicily	1922	115,000	39	4.85	279.7	.....	0.7	0.7
19	Malungushi	So. Africa	1935	175,000 cu yds	117 ft[1]	.....	......	.....	.....	....
20	Prins	So. Africa	....	....	34.4	6.1	427	.....	1.25	1.5
21	Oued Kebir	Tunisia	1925	....	115 ft	25 ft	1,140 ft	.....	1, 1.5	1.5
22	Karachunovskia	Ukraine	....	100,000	22.3	7	202	.....	1, 1.1, 1.25	1.5

[1] Height above stream bed.

EIGN COUNTRIES WITH PRINCIPAL CHARACTERISTICS OF EACH

Character of rock-fill in main body of dam	Construction under face membrane	Impervious membrane	Remarks
Dry rubble	Dry masonry	Reinforced concrete, 2 layers 0.4 m and 0.3 m thick over gunite-coated rubble	M. M. Martin, and Drouhin, Barrages en enrochements arrimes d'Algerie. Paper C-9, Second Congress on Large Dams, Washington, D. C., 1936. *Eng. News-Record,* vol. 119, p. 889, vol. 120, p. 749
Dry rubble	Wet masonry 3.5 m thick	2–6-cm layers bituminous concrete between 2 layers porous concrete	
Dry rubble	Wet masonry	Reinforced concrete, 1 layer 0.2 m to 0.35 m thick	
Dry rubble	0.8 m wet masonry over dry masonry 3 to 10 m thick	2-6 cm layer bituminous concrete between 2 layers porous concrete	
Loose fill..................	Dry rubble	Double layer reinforced concrete slabs separated with asphalt compound 0.02 m thick. Top slabs 4 m square by 0.1 m thick. Subslabs 7.5 m square by 0.25 m to 0.13 m thick	Eduardo Reyes Cox, del Departamento de Riego, Santiago, Chile. All edges of slabs at Caritaya Dam are separated by asphalt compound joints 0.01 m to 0.02 m thick
Loose fill	Dry rubble 2.0 to 4.65 m thick	Reinforced-concrete slabs 10 to 2.5 m square and 0.8 to 0.25 m thick over concrete subslab	Miguel Montalva C, Departo de Riego, Chile
Dry rubble with wet-masonry facing to retain sand wedge	Concrete thrust block with ribs forming drainage channels	Reinforced concrete 0.9 m to 1.8 m thick	W. J. E. Binnie, Study of the Facing of Masonry and Concrete Dams, Paper D-64 Second Congress on Large Dams, Washington, D. C., 1936. *Eng. News-Record,* vol. 117, p. 677.
Dry rubble	....	Reinforced concrete	S. N. Moisseiff
Dry rubble and loose fill with dry-rubble downstream slope	....	Wet masonry covered with asphalt	S. N. Moisseiff
Dry rubble	Reinforced concrete	Reinforced concrete with drainage system	S. N. Moisseiff
Dry rubble	Wet masonry	Reinforced concrete sliding type with sheet iron and impregnated felt covering	S. N. Moisseiff
Dry rubble	....	Reinforced concrete	S. N. Moisseiff
Loose fill	Dry rubble	Reinforced concrete monolithic flexible type	Chas. P. Williams, Mexico, D. F.
Dry rubble (hand packed) with berms	Heavy concrete blocks	Reinforced concrete laminated-wing type	*Civil Eng.,* 4, p. 524. G. R. G. Conway, president, The Mexican Lt. & P. Co., Ltd.
Loose fill	....	Reinforced concrete 2 ft thick	The Commonwealth Engineer. Nov. 1, 1935 (courtesy of Robert A. Sutherland)
Dry rubble	....	Wet masonry	S. N. Moiseeiff
Dry rubble upstream section 12 to 136 ft thick. Backed by loose fill. Voids in both sections reduced by sluiced coarse and fine gravel and some sand	Wet masonry 4 to 5 ft thick	Reinforced concrete. Thickness varies from 5 to 3 ft in bottom 18 ft and from 36 to 10 in. in upper 237 ft	Sanderson & Porter, Engineers. A detailed description appears, in *Compressed Air Mag.* **44,** No. 7, p. 5916
Dry rubble	Wet masonry	Reinforced concrete with drainage and corrugated-iron covering (coated)	S.N. Moisseiff
....	....	Bitumastic concrete 4 in. to 12 in. thick	*South African Mining Rev.,* Je., 1924
Loose fill	Thin layer dry rubble and layer of small rock	Reinforced concrete 0.23 m to 0.76 m thick	S. N. Moisseiff. Rock-fill and Dry Rubble Dams, Gosstrojisdat, Moscow, 1935
Loose fill and dry rubble	....	Hollow rigid reinforced concrete corewall, 6.5 to 30 ft thick	*Eng. News-Record,* **109,** p. 529. Partial failure 1929 Repaired 1931
Loose fill	Wet masonry	Reinforced concrete	S. N. Moisseiff. Partial failure 1925

[2] Height above top of cutoff wall.

built in the rock fill to strengthen the support under the edges of the face slabs. Dix River Dam has 1-in. cork-filled expansion joints, 70 ft apart, supported on rectangular-shaped reinforced-concrete ribs with tarred-felt covering. Reinforcing steel crosses only construction joints.

Three types of horizontal keyways have been used. Rectangular keyways have been most common and are satisfactory. Deep keyways as used at San Gabriel Dam 2 (see Fig. 25) invite downface movement and displacement of face rock. The right-triangular keyway (see Fig. 10*f*), believed to be first used at Salt Springs Dam, and

FIG. 25.—View of horizontal keyway, San Gabriel Dam No. 2.

later at Taxhimay Dam in Mexico, proved satisfactory and has certain advantages over other types. Rocks forming the horizontal and vertical legs of the triangle are well bonded into the dry-rubble section and minimize downslope movement. The horizontal leg forms a useful walkway and platform for runways during construction. This keyway permits greater and safer delay in placing the face membrane with consequent lesser settlement of fill thereafter. The concrete face of Salt Springs Dam was poured in four stages with vertical distances beginning at the bottom of 80, 90, 75, and 83 ft, respectively. Slight displacement of face rock at keyways occurred at several places.

Vertically inclined settlement and expansion joints of either the single or double type are usually supported on ribs of concrete poured into rectangular-shaped keyways left in the underlying face structure. The single-joint type is preferable because of its simplicity and reduced leakage hazard. Copper water stops and ample provision for lateral movement due to compression or tension in the concrete slab are important requirements. The 1-in. space provided between slab edges at Salt Springs proved in some places to be inadequate. Joints at Stas. 2+80 to 7+00 have completely closed. A few small areas of concrete adjoining three of these joints spawled. These were in the two top panels only and did not exceed 3 in. in depth. Joints near the abutments opened as noted in Fig. 17. The copper sheet was designed to safely withstand in tension an opening of 4 in. with maximum depth of water. The anchorage bend is upward to prevent trapping of air during concreting operations. At no place where investigations or repairs were made was there discovered any rupture of either the copper sheet or the brazed joints.

Figure 21*C* illustrates advantages obtainable with diagonal joints in the zones adjoining the cutoff wall of high dams. The few joints provided at or near the top of the cutoff wall at Salt Springs Dam, together with horizontal and vertical joints and the lean masonry support shown in Fig. 10*e*, were effective in reducing the number of cracks and in confining them to an area where they are readily accessible for repairs. Two or more diagonal joints roughly parallel to the abutment slopes and closely spaced would undoubtedly have prevented formation of many of the cracks.

Ample protection against serious rupture of the face membranes of high dams generally should result with the following procedure in the arrangement of joints:

1. Place horizontal and vertically inclined joints at 40- to 60-ft intervals over the general area of the face.
2. Place additional horizontal joints at reasonably close intervals where maximum total settlements may occur in the general area of the face after the placement of the membrane. [See, for example, point *D* (Fig. 18*A* and Fig. 18*C*), point *B* (Fig. 18*B*, and Fig. 17), and points *B*, Stas. 7+00 and 7+60.]
3. Place one or more joints at closer intervals parallel to and immediately above a joint at or near the top of the cutoff wall and extending across the streambed and along the abutment slopes.

The foregoing procedure together with judicious application of the principles and general requirements previously stated concerning foundations, dry rubble, and loose fill should result in the satisfactory performance of rock-fill dams.

SECTION 7

SPILLWAYS AND STREAM-BED PROTECTION WORKS

By Emory W. Lane and Calvin V. Davis*

The great majority of reservoirs formed by damming natural valleys require spillways to act as safety valves in case the inflow becomes so great as to endanger the safety of the dam by causing water to flow over the top of it or to prevent the reservoir from filling to a level sufficient to cause damage to adjacent property. The best design for a spillway in any given case depends principally upon the discharge capacity required, the topography and geology of the dam site, and the design of the dam itself.

SPILLWAY DISCHARGE CAPACITY

Design Flood. In most cases, it is necessary to provide sufficient spillway capacity to discharge an inflow as large as experience has shown can be expected ever to occur in that locality. A discussion of flood flows will be found in Sec. 25. A preliminary approach to estimating expected flood peaks is to plot an envelope curve embracing all known floods which have occurred in the general region in which the spillway is located. Typical of such curves is Fig. 1 upon which is plotted logarithmically all great floods which have occurred in Washington and Oregon on the west slope of the Cascade Mountains.

Also plotted on Fig. 1 is the envelope curve for all great floods which have occurred in the United States according to Creager.[24]† Creager gives the equation for this curve in the general form as

$$Q = 46CA^{(0.894A^{-0.048})} \tag{1}$$

or

$$q = 46CA^{(0.894A^{-0.48})-1} \tag{2}$$

in which Q = estimated maximum flood peak, cfs.
q = corresponding flood expressed in cfs per square mile drainage area.
A = drainage area, square miles.
C = a coefficient depending on the drainage area.

Creager found that the enveloping curve for $C = 100$ embraced all but a few of the known great floods which have occurred in the United States. This coefficient will vary from region to region. Figure 1 indicates that a curve for $C = 80$ would be satisfactory for general use in the Pacific Northwest, west of the Cascades. East of the Cascades this coefficient would not apply.

In preparing preliminary estimates of flood peaks by this experience curve method, it is essential to make a separate investigation for each project. It is not satisfactory to accept at face value the data shown by any such chart as Fig. 1 without first bringing it up to date to show all recent flood occurrences.

A second, and more advanced, approach to the problem of estimating flood peaks is by transposing great storms, which have been known to occur in the region, over the drainage area under consideration. The resulting flood hydrographs are then determined by rational methods.[25] Figure 2[26] illustrates a design flood hydrograph, pre-

* The principal additions to this section are the responsibility of the Editor.
† Superior numbers refer to the Bibliography on pp. 288 and 299.

pared by such methods, which was prepared for a proposed damsite on the Cowlitz River in the State of Washington. The peak discharge of this hydrograph is plotted on Fig. 1 to show general agreement between the two methods.

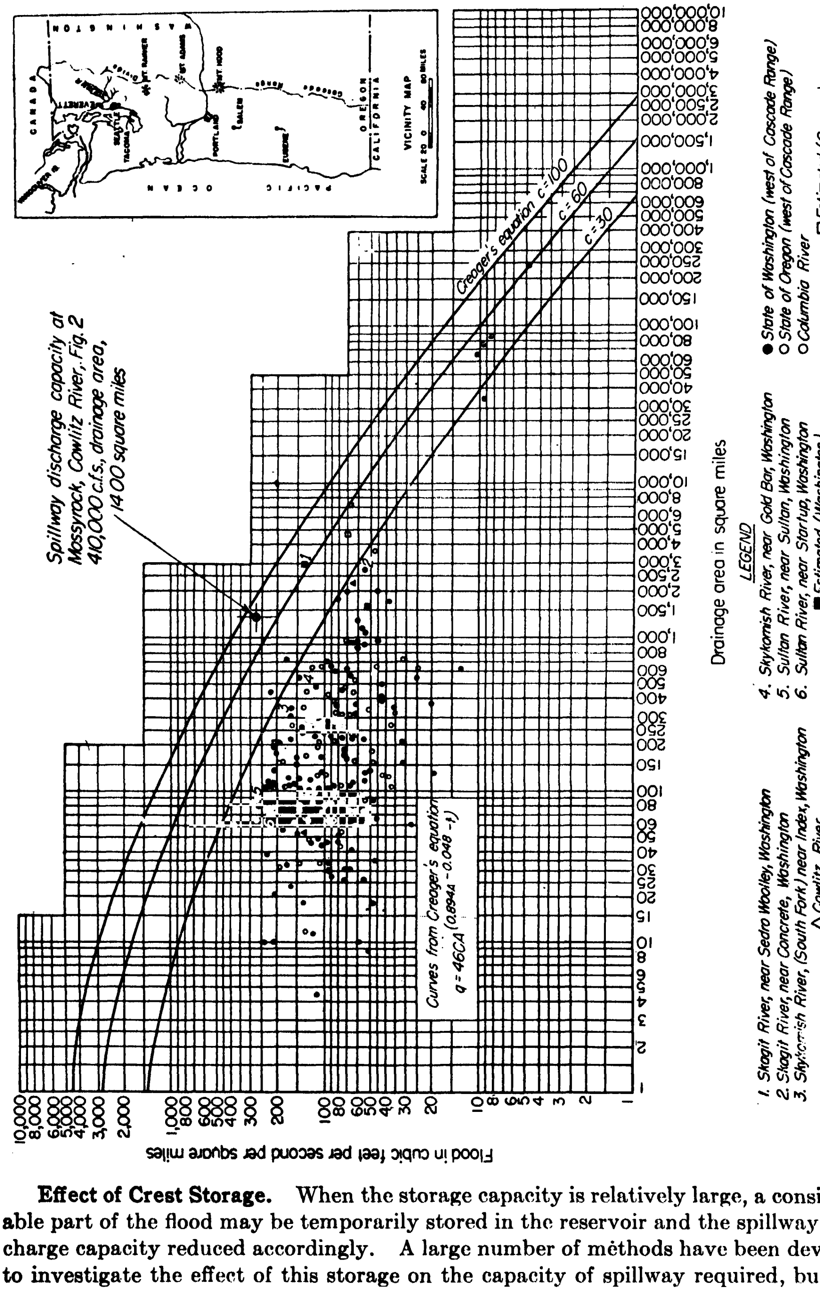
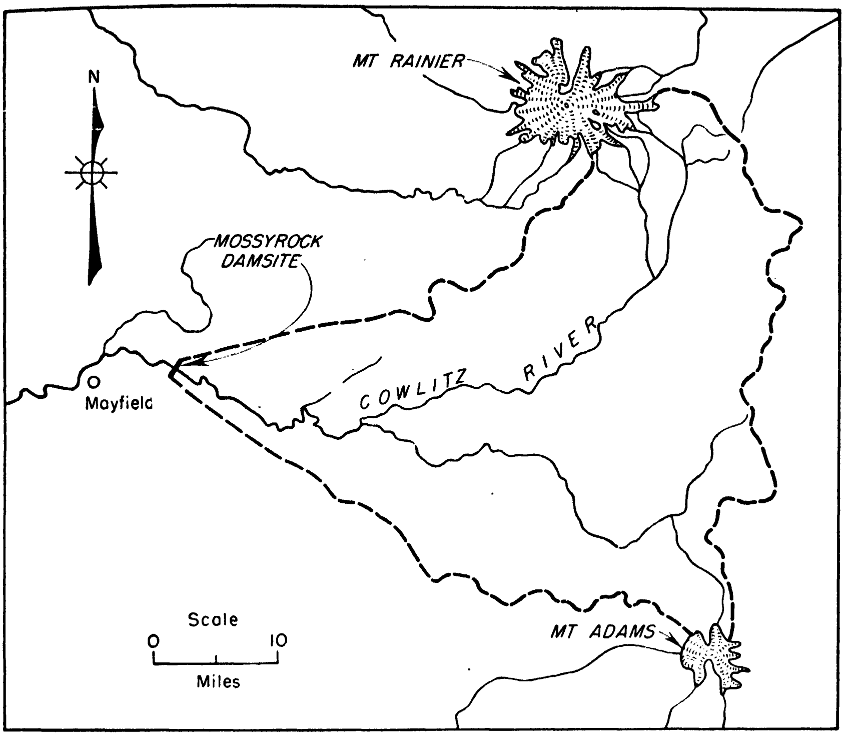


Fig. 1.—Flood discharges of record, Washington and Oregon, west of Cascades.

Effect of Crest Storage. When the storage capacity is relatively large, a considerable part of the flood may be temporarily stored in the reservoir and the spillway discharge capacity reduced accordingly. A large number of methods have been devised to investigate the effect of this storage on the capacity of spillway required, but no



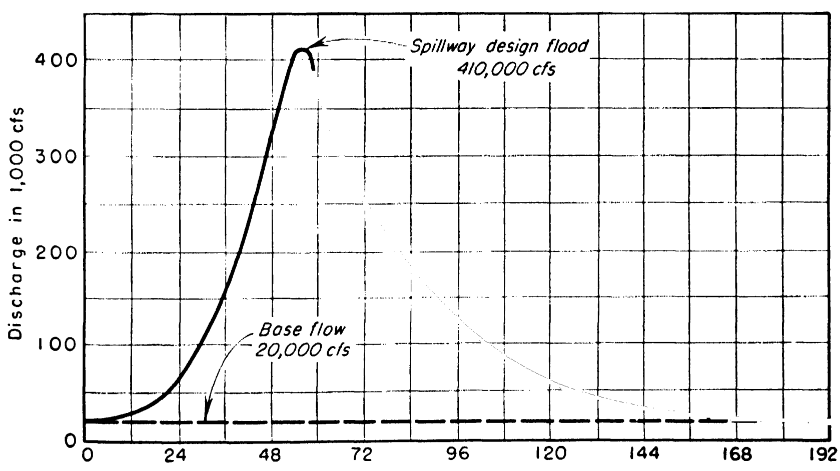


Fig. 2.—Drainage-area map and spillway design flood, Mossyrock Project, Cowlitz River, Washington.[26]

single one is best for all conditions. The general principles of flood routing have been summarized in Sec. 1. In this section will be found three other applications of these principles: the Woodward method, the cut-and-try method, and the Sorensen graphical method, which are more applicable to the problem of spillway design. The Woodward method is based on a mathematical solution involving four assumptions: (1) that the net inflow rate is constant, (2) that the reservoir area in which the storage takes place is constant, (3) that the spillway discharge varies as the three-halves power of the head, and (4) that the reservoir is filled to the spillway crest when the flood begins. The solution of this problem is presented in Fig. 3, which gives the ratios of the total net inflow volume to storage volume above the spillway for various ratios of maximum spillway discharge to net inflow rate. The term *net inflow volume* is used to indicate that the volume of flow which passes the dam through any other route than over the

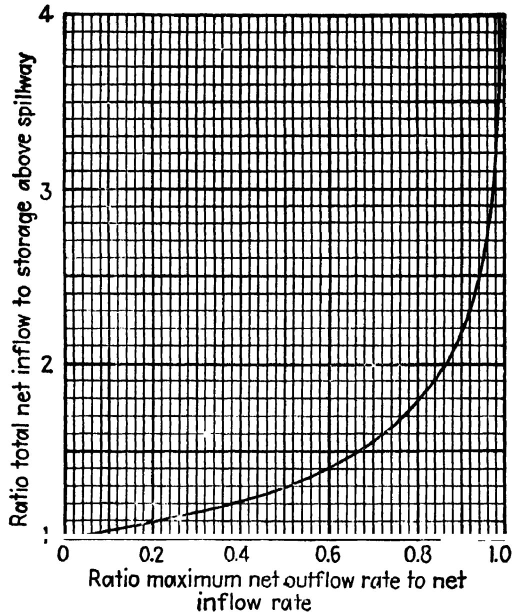


FIG. 3.—Curve for estimating spillway capacity required by Woodward method.

spillway (as, for example, through the powerhouse) must be subtracted from the total inflow volume before using the diagram. A similar correction must be made to obtain the net inflow rate. To illustrate the use of this diagram, suppose a certain reservoir has a capacity between the spillway crest level and the maximum safe level to which it can be permitted to fill of 100,000 acre-ft and that the reservoir is filled to the spillway level at the beginning of a flood of 200,000 acre-ft which enters with an average flow rate of 50,000 cfs. Suppose also that no discharge takes place except over the spillway. The ratio of the total net inflow volume to the storage volume above the spillway is therefore $\frac{200{,}000 \text{ acre-ft}}{100{,}000 \text{ acre-ft}} = 2.00$. Figure 3 shows that for this condition the ratio of the maximum spillway discharge rate to the net inflow rate is 0.865. Since the net inflow rate is 50,000 cfs, the maximum spillway discharge rate must be $50{,}000 \times 0.865 = 43{,}250$ cfs. Of course, the conditions of an actual case never exactly fit the assumptions on which the solution is based; the inflow will probably not be constant, the area of storage is unlikely to be uniform throughout the height, and

the spillway discharge rarely varies exactly as the three-halves power of the head. However, by using the total volume of storage above the spillway and the average rate of inflow, surprisingly close agreement is usually obtained. In computing the dimensions of the spillway required for a given maximum discharge, a weighted average value of the spillway discharge coefficient should be used. Since most of the outflow takes place near the maximum stage, the coefficient should be less than but close to that which would exist at maximum discharge. Since the relations on Fig. 3 are non-dimensional, any system of units may be used if care is taken that the two values used in obtaining any ratio are in the same units.

Final designs should always be checked by a more exact method. The easiest of these methods to understand is the cut-and-try or trial-and-error method. From the design of the spillway and the data on the reservoir and the flood considered, three curves as follows should be prepared: (1) a rating curve showing the total discharge of spillway and outlets plotted against reservoir elevation, (2) a reservoir volume curve showing the volume available for the reduction of flood crests plotted against reservoir elevation, and (3) an inflow hydrograph during the flood period. The spillway discharge throughout the flood can then be determined by the following procedure: the time period is first divided up into short increments, and the total inflow volume during each period is determined from curve (3). Begin with the first period, and make a trial estimate of the level in the reservoir at the end of the period. The discharge rate at the end of the period corresponding to this elevation is then determined from curve (1). The average discharge rate for the period is next computed by averaging the rates at the two ends of the period, and this multiplied by the duration of the period gives the total outflow volume. This volume subtracted from the inflow volume during the period should give the storage volume during the period. If the elevation at the end of the period is correctly assumed, this storage volume should equal that between the surface level at the beginning of the step and the assumed level at the end of the step, as indicated by curve (2). If it does not, the assumed elevation at the end of the step should be revised upward or downward, depending on whether the computed storage at the end of the step was greater or less than the storage indicated by curve (2), until agreement is secured. The process is then repeated by using the next step until the entire flood is routed through the reservoir.

Several variations of the cut-and-try process are possible, for several quantities can be assumed at the end of the step (for example, the outflow rate) and computations made to see if the value assumed fulfills the required conditions, but extensive experience has shown that the labor involved in all these methods is practically the same; hence, only one is given.

The Sorensen graphical method involves the solution of the differential equation:

$$dV = (I - Q)\,dT \tag{3}$$

where V = reservoir volume.
I = rate of inflow.
Q = spillway discharge (outflow).
T = time.

This equation is not amenable to a mathematical solution as I is a function of T and Q is a function of V.

Equation (3) can be approximated as

$$\Delta V = V_f - V_i = \left(\bar{I} - \frac{Q_i + Q_f}{2}\right)\Delta T \tag{4}$$

Where $\bar{I}$ is the average inflow for the period ΔT and the subscripts i and f refer to the initial and final conditions of the period ΔT. Equation (4) in the transposed form,

$$V_f + \frac{Q_f \Delta T}{2} = V_i - \frac{Q_i \Delta T}{2} + \bar{I} \Delta T \tag{5}$$

can be solved for V_f by arithmetic methods or by use of a special slide rule.[34]

A graphical solution is possible with Eq. (4) transposed to read

$$V_i + (\bar{I} - Q_i)\frac{\Delta T}{2} + (\bar{I} - Q_f)\frac{\Delta T}{2} = V_f \tag{6}$$

For any particular reservoir, curves 1 and 3 of Fig. 4 can be plotted. Curve 3 is reservoir volume plotted as a function of reservoir elevation, and curve 1 is reservoir

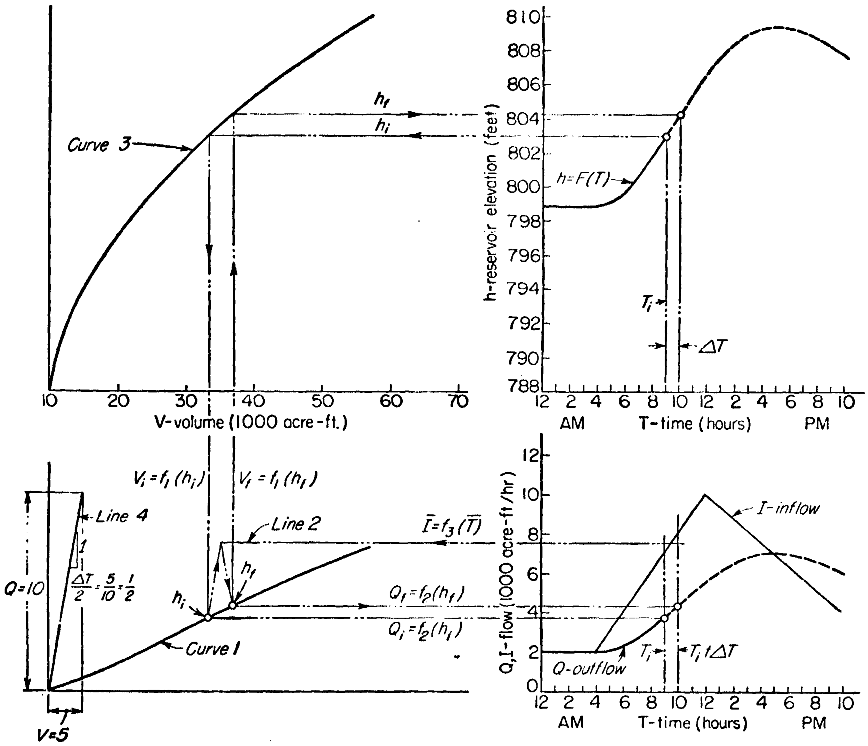


Fig. 4.—Sorensen graphical method for routing floods.

volume plotted as a function of uncontrolled spillway discharge. Adjacent to curve 1, the inflow hydrograph is plotted to the same discharge scale as curve 1. To the left of curve 1, line 4 is drawn at a slope equal to $\Delta T/2$.

Figure 4 will serve to illustrate the graphical solution of Eq. (6). At some time, T_i, the reservoir is known to be at elevation h_i; and spillway discharge to be Q_i. The steps required to determine Q_f and h_f at the end of time ΔT are

1. Draw line 2 horizontally from $\bar{I}$ (average inflow for period ΔT).
2. From initial point, h_i on curve 1, draw a line at slope $\Delta T/2$ to intersect line 2.
3. From this intersection draw a line at reverse slope $\Delta T/2$ to intersect curve 1 at h_f.
4. Project horizontally from h_f to time $T_i + \Delta T$ to give the extension of the outflow hydrograph.

5. Project vertically from h_f to curve 3, and from there horizontally to time $T_i + \Delta T$ to give the extension of the reservoir elevation curve.

6. Use the value of h_f thus found as h_i for the next period ΔT and repeat the foregoing steps.

The final results obtained are the reservoir inflow and outflow hydrographs superimposed and the reservoir stage hydrograph. If the latter is not desired, the upper half of Fig. 4 and use of curve 3 can be omitted.

It is believed that the graphical solution has certain advantages over other methods in the form of presentation of final results and in the reduction of potential interpolation errors inherent in methods requiring the reading of values from graphic scales. For general proof of this method see Appendix B.

TYPES OF SPILLWAYS

Nearly all spillways fall into one of six types or are made up of a combination of them: (1) overfall, (2) trough or chute, (3) side channel, (4) shaft or glory hole, (5) siphon, and (6) gate type. The overfall type is by far the most common and is adapted to masonry dams that have sufficient crest length to provide the desired capacity and where the foundation will withstand or can be protected sufficiently to withstand the scour of the overfalling water. Trough or chute spillways are commonly used for earth dams. Side-channel and shaft-spillway types are most frequently found in narrow canyons. The siphon spillway is usually used to provide automatically a nearly constant headwater level under varying flow, and sometimes where the crest length of the dam is restricted. The gate type of spillway is used where it is desirable to remove the effects of the dam during high water to prevent excessive flooding. The trough or chute type is often combined with one of the other types, the trough sometimes taking the form of a tunnel through the abutment of the dam. Overfall and siphon spillways are usually located in the main dam and the trough, side-channel, and shaft types near or in the abutments.

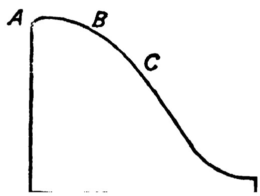


FIG. 5.

Overfall Spillway. The usual form of overfall spillway has a rounded crest with an ogee face as shown in Fig. 5. The crest is formed to fit the shape that the overflowing water would take under conditions of maximum discharge, if the dam were replaced by a weir, with a sharp crest at the junction of upstream face of the dam with the rounded top surface. For example, the curve ABC would be shaped to fit the underside of the nappe of a weir with an upstream face the same shape as that of the dam and the sharp crest at point A, when the weir is discharging the same quantity as the spillway under the maximum head that is expected to flow over it. If the upstream face of the dam is vertical, the crest shape will be that for a vertical-face weir, and if the upstream face of the dam is sloped, the crest will follow the shape of the nappe of a weir having a face of similar slope. The shape of the nappe from a sharp-crested weir was first determined by Bazin.[3] Other work has been done by Scimemi[4] and the U.S. Bureau of Reclamation.

Figure 6 shows the shape of the upper and lower surfaces of the nappe for various slopes of the upstream face and Fig. 7 the shape, over a greater range, for a weir with a vertical face. Table 1 gives these data more exactly. This table was derived from an analysis based on studies of the U.S. Bureau of Reclamation and the results of Bazin and Scimemi. These curves are expressed in terms of the head on the highest point of the rounded crest, the velocity of approach being included in the head. They are developed for low velocities of approach, and where high velocities are used, small corrections are necessary. To use these curves in design, the maximum head (includ-

ing velocity head) of overflow of the spillway is first determined and the shape of the crest curve can then be designed as shown on the curves, using this head as the unit of distance. For example, if the head is 10 ft, all values taken from the curve should be multiplied by 10 ft to give the crest curve dimensions. The curves for the upper nappe surface will be useful in designing the clearance for spillway bridges and training walls at the ends of the spillway.

If the form of nappe for a vertical-face dam is applied to the ordinary nonoverfall cross section of a dam, it will be found that it is necessary to thicken the dam some-

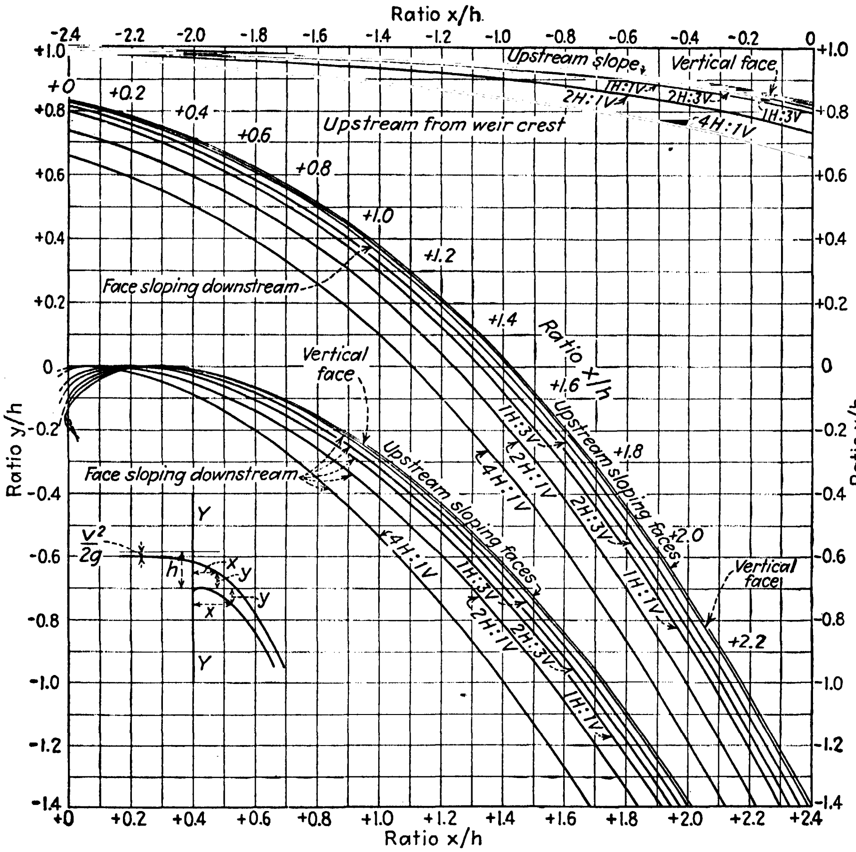


FIG. 6.- Shape of upper and lower nappe for weirs with faces of various slopes.

what as shown in Fig. 8a. The excess of the overfall section over that of the nonoverfall can be considerably reduced by moving the upstream face and the crest curve of the overfall section in an upstream direction until the crest curve becomes tangent to the downstream face of the nonoverflow section. The junction of crest curve and upstream face of the overflow section then extends upstream from the upstream face of the nonoverflow section. For most of the height of the dam, the face for both sections can be made to coincide by corbeling out the curved crest, as shown in Fig. 8b. The saving resulting is the difference between the area of this corbel in Fig. 8b and the excess of the overflow over the nonoverflow in Fig. 8a. Experiments have shown that if the vertical face portion of this corbel has a height of three-tenths the maximum

head on the crest, the nappe shape and the discharge coefficient are practically the same as if the vertical face extended for the full height.

Some dam designers have considered it desirable, in order to avoid the possible formation of vacuum beneath the nappe, to shape the spillway somewhat larger than would be indicated by the weir nappe shape, but experiments by the U.S. Bureau of

TABLE 1.—NAPPE COORDINATES FOR SPILLWAY DESIGN

Lower Nappe

Horizontal coordinates x/h	Vertical coordinates y/h								
	Vertical	Weir face slope							
		Downstream					Upstream		
		$1H:3V$	$2H:3V$	$1H:1V$	$2H:1V$	$4H:1V$	$1H:3V$	$2H:3V$	$1H:1V$
0.0	− 0.125	− 0.098	− 0.066	− 0.045	− 0.012	− 0.004	− 0.159	−0.176	−0.191
							− 0.136	−0.136	−0.136
0.05	− 0.066	− 0.051	− 0.032	− 0.021	− 0.002	− 0.000	− 0.080		
0.10	− 0.033	− 0.026	− 0.015	− 0.008	0.000	− 0.003	− 0.044		
0.15	− 0.014	− 0.011	− 0.005	− 0.001	− 0.002	− 0.010	− 0.022		
0.20	− 0.004	− 0.003	− 0.001	0.000	− 0.007	− 0.020	− 0.009		
0.25	0.000	0.000	− 0.001	− 0.003	− 0.014	− 0.033	− 0.002		
0.30	0.000	− 0.001	− 0.004	− 0.009	− 0.023	− 0.049	0.000		
0.35	− 0.004	− 0.005	− 0.010	− 0.018	− 0.038	− 0.068	− 0.003		
0.40	− 0.011	− 0.012	− 0.019	− 0.030	− 0.053	− 0.090	− 0.010		
0.45	− 0.021	− 0.022	− 0.031	− 0.045	− 0.072	− 0.115	− 0.020		
0.50	− 0.034	− 0.035	− 0.046	− 0.062	− 0.093	− 0.143	− 0.033		
0.55	− 0.049	− 0.051	− 0.064	− 0.082	− 0.117	− 0.173	− 0.048		
0.60	− 0.066	− 0.069	− 0.084	− 0.104	− 0.141	− 0.206	− 0.065		
0.65	− 0.085	− 0.090	− 0.106	− 0.128	− 0.168	− 0.240	− 0.084		
0.70	− 0.106	− 0.113	− 0.131	− 0.154	− 0.197	− 0.277	− 0.105		
0.75	− 0.129	− 0.138	− 0.158	− 0.182	− 0.228	− 0.315	− 0.128	Same as 1:3 upstream slope	Same as 1:3 upstream slope
0.80	− 0.157	− 0.165	− 0.187	− 0.212	− 0.261	− 0.356	− 0.153		
0.85	− 0.185	− 0.195	− 0.218	− 0.244	− 0.297	− 0.400	− 0.180		
0.90	− 0.216	− 0.227	− 0.251	− 0.278	− 0.333	− 0.443	− 0.210		
0.95	− 0.249	− 0.261	− 0.286	− 0.314	− 0.373	− 0.490	− 0.242		
1.0	− 0.283	− 0.297	− 0.323	− 0.352	− 0.413	− 0.538	− 0.277		
1.1	− 0.358	− 0.375	− 0.403	− 0.434	− 0.501	− 0.641	− 0.351		
1.2	− 0.441	− 0.461	− 0.491	− 0.524	− 0.597	− 0.752	− 0.433		
1.3	− 0.532	− 0.555	− 0.587	− 0.622	− 0.701	− 0.871	− 0.523		
1.4	− 0.631	− 0.657	− 0.691	− 0.728	− 0.813	− 0.998	− 0.621		
1.5	− 0.738	− 0.767	− 0.803	− 0.842	− 0.933	− 1.133	− 0.727		
1.6	− 0.853	− 0.885	− 0.923	− 0.964	− 1.061	− 1.276	− 0.841		
1.7	− 0.976	− 1.011	− 1.051	− 1.094	− 1.197	− 1.427	− 0.963		
1.8	− 1.107	− 1.145	− 1.187	− 1.232	− 1.341	− 1.586	− 1.093		
1.9	− 1.246	− 1.287	− 1.331	− 1.378	− 1.493	− 1.753	− 1.231		
2.0	− 1.393	− 1.437	− 1.483	− 1.532	− 1.653	− 1.928	− 1.377		
2.2	− 1.711	− 1.761	− 1.811	− 1.864	− 1.997	− 2.302	− 1.693		
2.4	− 2.061	− 2.117	− 2.171	− 2.228	− 2.373	− 2.708	− 2.041		
2.6	− 2.443	− 2.505	− 2.563	− 2.624	− 2.781	− 3.146	− 2.421		
2.8	− 2.857	− 2.925	− 2.987	− 3.052	− 3.221	− 3.616	− 2.833		
3.0	− 3.303	− 3.377	− 3.443	− 3.512	− 3.693	− 4.118	− 3.277		
3.2	− 3.781	− 3.861	− 3.931	− 4.004	− 4.197	− 4.652	− 3.753		
3.4	− 4.291	− 4.377	− 4.451	− 4.528	− 4.733	− 5.218	− 4.261	Same as 1:3 upstream slope	Same as 1:3 upstream slope
3.6	− 4.833	− 4.925	− 5.003	− 5.084	− 5.301	− 5.816	− 4.801		
3.8	− 5.407	− 5.505	− 5.587	− 5.672	− 5.901	− 6.446	− 5.373		
4.0	− 6.013	− 6.117	− 6.203	− 6.292	− 6.533	− 7.108	− 5.977		
4.2	− 6.651	− 6.761	− 6.851	− 6.944	− 7.197	− 7.802	− 6.613		
4.4	− 7.321	− 7.437	− 7.531	− 7.628	− 7.893	− 8.528	− 7.281		
4.6	− 8.023	− 8.145	− 8.243	− 8.344	− 8.621	− 9.286	− 7.981		
4.8	− 8.757	− 8.885	− 8.987	− 9.092	− 9.381	−10.076	− 8.713		
5.0	− 9.523	− 9.657	− 9.763	− 9.872	−10.173	−10.898	− 9.477		
5.2	−10.321	−10.461	−10.571	−10.684	−10.997	−11.752	−10.273		
5.4	−11.151	−11.297	−11.411	−11.528	−11.853	−12.638	−11.101		

TABLE 1.—NAPPE COORDINATES FOR SPILLWAY DESIGN.—(*Continued*)
Upper Nappe

Horizontal coordinates x/h	Vertical coordinates y/h								
		Weir face slope							
	Vertical	Downstream					Upstream		
		$1H:3V$	$2H:3V$	$1H:1V$	$2H:1V$	$4H:1V$	$1H:3V$	$2H:3V$	$1H:1V$
-2.4	0.989	0.988	0.985	0.983	0.980	0.973	0.990		
-2.2	0.987	0.986	0.981	0.977	0.972	0.957	0.988		
-2.0	0.984	0.983	0.977	0.971	0.964	0.940	0.985		
-1.8	0.980	0.979	0.971	0.964	0.955	0.922	0.981		
-1.6	0.975	0.974	0.965	0.957	0.945	0.904	0.976		
-1.4	0.969	0.968	0.958	0.949	0.934	0.885	0.970		
-1.2	0.961	0.959	0.950	0.941	0.921	0.865	0.962		
-1.0	0.951	0.948	0.939	0.930	0.904	0.842	0.953		
-0.8	0.938	0.935	0.926	0.917	0.883	0.817	0.940		
-0.6	0.921	0.918	0.908	0.899	0.858	0.788	0.923		
-0.4	0.898	0.895	0.885	0.875	0.826	0.754	0.900		
-0.2	0.870	0.865	0.853	0.841	0.786	0.712	0.872		
0.0	0.831	0.826	0.811	0.796	0.737	0.659	0.833		
0.05	0.819	0.814	0.798	0.783	0.723	0.643	0.822		
0.10	0.807	0.802	0.785	0.768	0.708	0.627	0.810		
0.15	0.793	0.788	0.770	0.752	0.692	0.610	0.796		
0.20	0.779	0.774	0.755	0.736	0.675	0.591	0.782		
0.25	0.763	0.758	0.739	0.719	0.657	0.572	0.766	Same as 1:3 upstream slope	Same as 1:3 upstream slope
0.30	0.747	0.742	0.721	0.700	0.638	0.550	0.750		
0.35	0.730	0.724	0.702	0.680	0.617	0.528	0.733		
0.40	0.710	0.704	0.681	0.659	0.596	0.504	0.713		
0.45	0.690	0.683	0.659	0.636	0.572	0.480	0.693		
0.50	0.668	0.661	0.637	0.613	0.549	0.452	0.671		
0.55	0.646	0.638	0.613	0.588	0.523	0.424	0.650		
0.60	0.621	0.612	0.587	0.562	0.497	0.394	0.625		
0.65	0.596	0.586	0.560	0.535	0.470	0.363	0.600		
0.70	0.568	0.558	0.531	0.505	0.439	0.330	0.572		
0.75	0.539	0.529	0.501	0.475	0.408	0.298	0.543		
0.80	0.509	0.498	0.470	0.442	0.375	0.261	0.513		
0.85	0.478	0.466	0.438	0.409	0.340	0.223	0.482		
0.90	0.444	0.431	0.402	0.373	0.303	0.183	0.449		
0.95	0.410	0.395	0.366	0.337	0.264	0.141	0.415		
1.0	0.373	0.358	0.327	0.297	0.223	0.098	0.379		
1.1	0.295	0.278	0.245	0.214	0.135	0.005	0.302		
1.2	0.210	0.191	0.156	0.124	0.039	-0.096	0.218		
1.3	0.118	0.097	0.060	0.026	-0.065	-0.205	0.127		
1.4	0.019	-0.005	-0.044	-0.080	-0.177	-0.322	0.029		
1.5	-0.088	-0.115	-0.156	-0.194	-0.297	-0.447	-0.077		
1.6	-0.203	-0.233	-0.276	-0.316	-0.425	-0.580	-0.191		
1.7	-0.326	-0.359	-0.404	-0.446	-0.561	-0.721	-0.313		
1.8	-0.457	-0.493	-0.540	-0.584	-0.705	-0.870	-0.443		
1.9	-0.596	-0.635	-0.684	-0.730	-0.857	-1.027	-0.581		
2.0	-0.743	-0.785	-0.836	-0.884	-1.017	-1.192	-0.727		
2.2	-1.061	-1.109	-1.164	-1.216	-1.361	-1.546	-1.043		
2.4	-1.411	-1.465	-1.524	-1.580	-1.737	-1.932	-1.391	Same as 1:3 upstream slope	Same as 1:3 upstream slope
2.6	-1.793	-1.853	-1.916	-1.976	-2.145	-2.350	-1.771		
2.8	-2.207	-2.273	-2.340	-2.404	-2.585	-2.800	-2.183		
3.0	-2.653	-2.725	-2.796	-2.864	-3.057	-3.282	-2.627		
3.2	-3.131	-3.209	-3.284	-3.356	-3.561	-3.796	-3.103		
3.4	-3.641	-3.725	-3.804	-3.880	-4.097	-4.342	-3.611		
3.6	-4.183	-4.273	-4.356	-4.436	-4.665	-4.920	-4.151		
3.8	-4.757	-4.853	-4.940	-5.024	-5.265	-5.530	-4.723		
4.0	-5.363	-5.465	-5.556	-5.644	-5.897	-6.172	-5.327		
4.2	-6.001	-6.109	-6.204	-6.296	-6.561	-6.846	-5.963		
4.4	-6.671	-6.785	-6.884	-6.980	-7.257	-7.552	-6.631		
4.6	-7.373	-7.493	-7.596	-7.696	-7.985	-8.290	-7.331		
4.8	-8.107	-8.233	-8.340	-8.444	-8.745	-9.060	-8.063		
5.0	-8.873	-9.005	-9.116	-9.224	-9.537	-9.862	-8.827		
5.2	-9.671	-9.809	-9.924	-10.036	-10.361	-10.696	-9.623		
5.4	-10.501	-10.645	-10.764	-10.880	-11.217	-11.562	-10.451		

Reclamation indicate that this is unnecessary. Although small vacuums may form near the crest owing to the difficulty of accurately fitting the nappe shape, the area covered is small and does not extend much beyond the crest. Even for heads somewhat exceeding the design value, the vacuum is not severe over a large area.

Piers are frequently used on the crest of spillway dams to support the gates. To cause as little obstruction as possible, their upstream ends should be rounded or pointed like the bow of a ship.

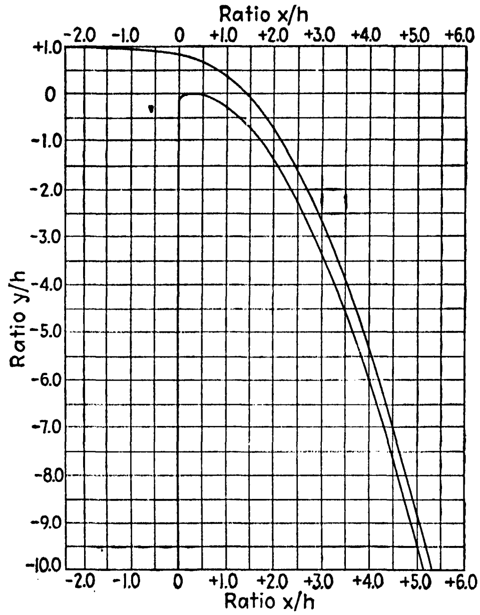


FIG. 7.—Shape of upper and lower nappe for weir with vertical face.

The pier end may be rounded by making it in the form of a semicylinder (Fig. 9*A*), or for greater efficiency it may be pointed by having cylindrical surfaces tangent to the sides of the pier, the radii of these surfaces being greater than half the thickness of the piers as shown by Fig. 9*B*.* With such forms, the reduction of discharge when computed by using the net crest length is probably not over 2 per cent (see Fig. 18). A good form is obtained by having the radii of the cylindrical surfaces equal to the pier thickness. For this form, the reduction would probably not exceed 1 per cent. The preceding reduction values are for piers with noses at or upstream from the dam face. Somewhat greater discharge seems to be obtained by placing the pier noses downstream from the dam face, but other considerations than discharge capacity usually control the position of the pier point.

The form of the downstream end of the pier is usually not so important. By making the piers thinner at the downstream edge, the flow from adjacent openings between the piers spreads somewhat over the space behind the pier and improves the conditions for the dissipation of energy below the weir. The thinning of the pier should be done very gradually, or the water will not follow the pier sides. Where drum gates are used on the crest, the downstream ends of the piers must be formed so that a channel is open through which the air can get to the underside of the nappe when the drum gates are in a partly open position.

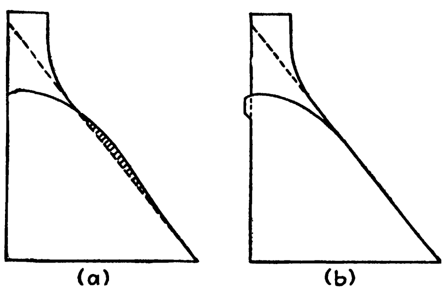


FIG. 8.

On arch dams of thin cross section, the shape of the nappe is sometimes obtained on the downstream side by having the top of the dam overhang upstream a considerable distance. In other cases, no attempt is made to fit the dam to the overflowing

* See also pp. 271 and 272.

water, but the nappe is permitted to spring free from the dam and fall without touching it. On high dams of the Ambursen type, the downstream face is formed to fit the nappe shape, with ample air venting as a safety factor, but on medium height dams the downstream face may be omitted near the base of the dam and in low dams left out entirely. The water in these cases falls freely into the stilling pool, as extensive experience has shown that this can safely be done. In all cases where the nappe falls freely, ample opportunity must be provided for air to reach the underside of the nappe, for the quantity necessary to prevent partial vacuum is surprisingly large.

The reverse curve joining the downstream face of the dam to the apron may be of any convenient radius, but if made too sharp the pressure required to change the direction of the flowing water may be so great as to give undesirably high pressures in the joints at the downstream side of the dam and thus reduce the stability of the structure.

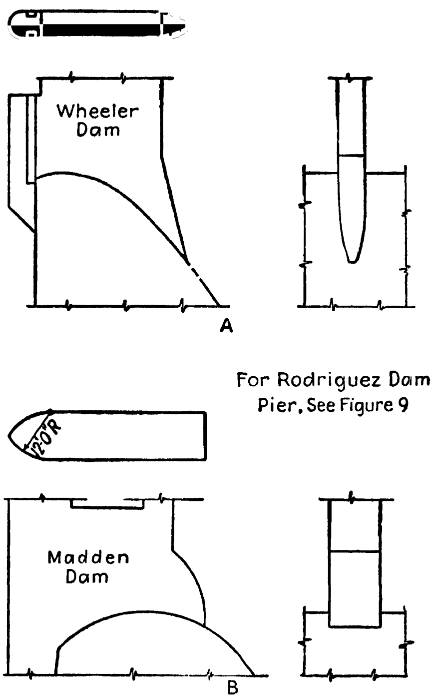


FIG. 9.—Typical pier shapes.

Trough or Chute Spillways. A trough or chute spillway consists of an open conduit conducting the water from the reservoir to the waterway downstream from the dam. Frequently there are gates at the upstream end to control the flow. Ogee or side-channel spillways often are built with a chute to carry away the water. Tunnels through the abutment rock form a special class of chute spillways. Except where tunnels are used, the chute spillways are practically always straight, because of the difficulty of changing the direction of water flowing at supercritical velocities.

The spillways of the Rodriguez Dam (Fig. 10) and the Morris Dam (Fig. 11) are typical of this type.

Where chute spillways are formed on earth, great care should be exercised to provide adequate subdrainage in order to prevent heaving due to frost or displacement due to water pressure. Should displacement occur such that the downstream block projects above the upstream one, as shown in Fig. 12*A*, the velocity head of the water striking the projecting edge would be converted into pressure head which might lift the downstream slab, causing the destruction of the spillway. If the lapping of the slabs is made with the one downstream extending under the one upstream (Fig. 12*B*), such a projection is less likely to occur. It is very important that the downstream block at a joint does not project above the upstream one when they are lying in their undisturbed position.

The simplest form of trough spillway is of uniform width throughout, such as that shown by Fig. 10. Changes of width or curved alignment lead to complications which can be solved only by means of laboratory experiments. In contracting or curved sections, stationary waves are apt to be set up which may strike and overtop the side walls. A contraction can sometimes be worked out in a spillway with gates at the upstream end by constructing a stilling pool just downstream from the gates with a narrower chute leading away from it. If expanding sections are used, the expansion should not be too abrupt, or the water will not follow the side walls.

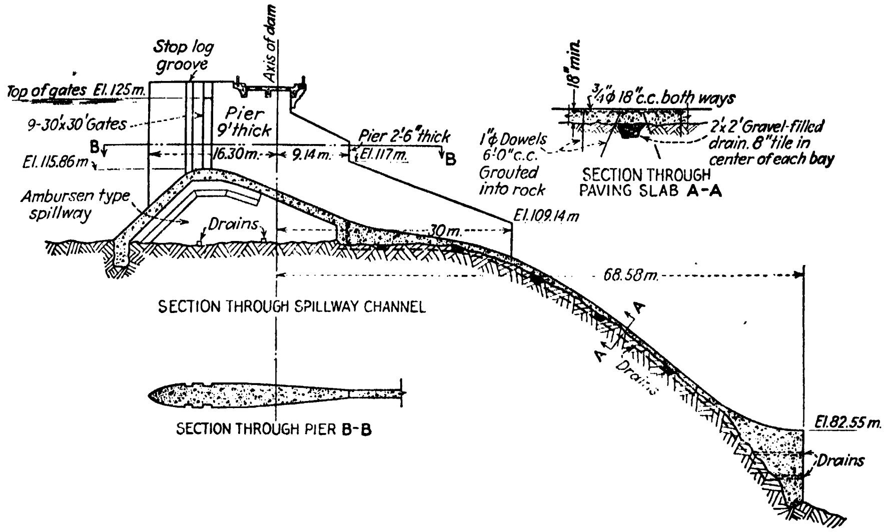


FIG. 10. Spillway of Rodriguez Dam.

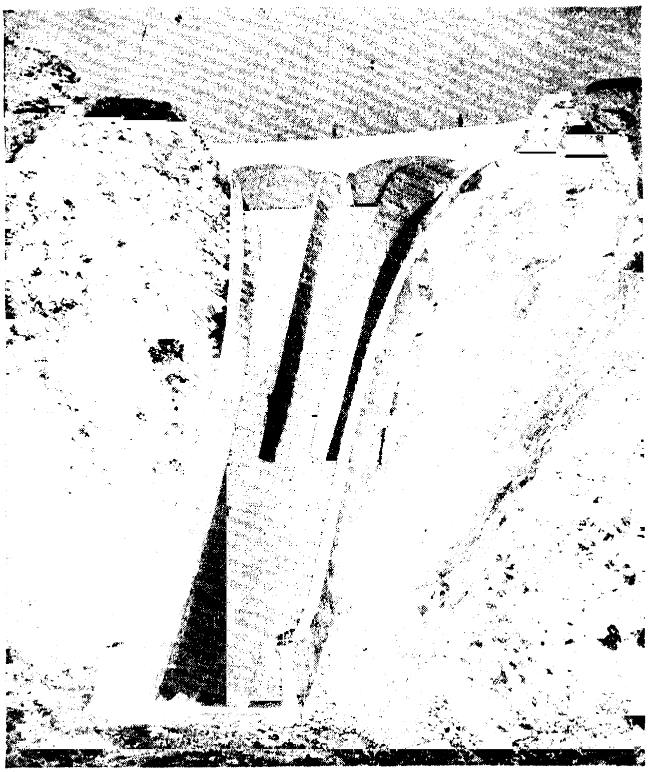


FIG. 11.—View of the Morris Dam Spillway.

Side-channel Spillways. The side-channel spillway is commonly used in sites where the sides are steep and rise to a considerable height above the dam. In this form, the water falls over the spillway crest into a channel, in which the flow is parallel to the crest, which leads eventually to the stream below the dam. Space does not permit a discussion of the hydraulics of the flow in this channel. A complete explanation of it is given by Hinds.[6] This analysis is based on the assumption that all the energy of the overfalling water is dissipated in turbulence, and the slope in the side channel must be sufficient to accelerate the overfalling water in the direction of flow down the channel. Observations on many spillway models have confirmed the essential accuracy of this analysis. In the spillways of Hoover Dam (Fig. 13), a cross weir was constructed at the downstream end of the overpour section, to provide a considerable depth of water in the channel at all flows, in which the energy of the overflowing water could be dissipated, without causing excessive turbulence in the tunnel that carried the water back to the river. In this case, it was very desirable to avoid turbulence in

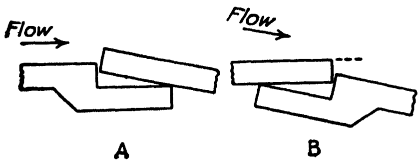


Fig. 12.

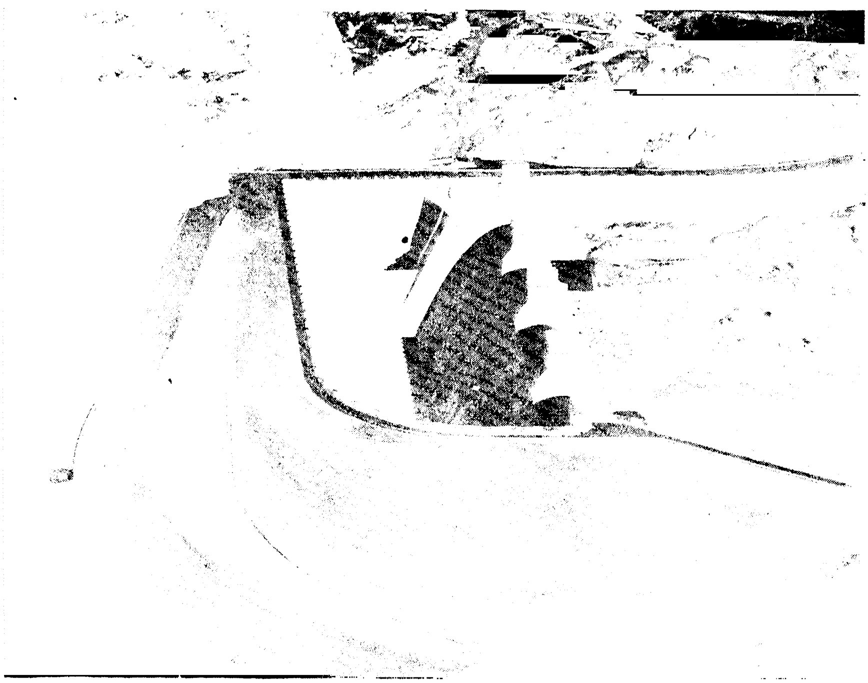


Fig. 13.—Hoover Dam spillway.

order that the water might flow through the long tunnels without undesirable effects due to entrained air. The increase in the size of channel required with this weir was negligible. Extensive model experiments[7] on the Hoover[30] Dam spillways indicated that turbulence in the side channel can probably be reduced at less expense by such a device than by any form of baffles that might be used. Attempts to divert the over-

falling water in a downstream direction, the velocity of flow in the channel being thus increased and the necessary size decreased, were unsuccessful in evolving any practical method. The tests demonstrated that the flow conditions in the channel downstream from the overflow section will be improved if the channel is narrowed, downstream from the overflow section, by offsetting the side toward the dam in by an amount equal to the thickness of the stream of water falling over the weir.

In connection with the design of the spillways of the Hoover Dam, extensive studies were made of crest shapes and discharge coefficients. These are much more extensive than are feasible to give in this book. Detailed information will be found in the Bureau of Reclamation report.[28]

Model tests have demonstrated the hydraulic feasibility of building the side channel in very steep canyons in a tunnel in the bank, the inflowing water coming through side openings from the reservoir into the tunnel. A spillway of this type was worked out by the U.S. Bureau of Reclamation for the Moon Lake Dam,* but was not constructed because of unsuitable character of the rock. In certain cases, channels with discharge from both sides have been constructed, but the fundamental analysis is the same as for flow over one side.

Glory-hole or Shaft Spillways. In glory-hole or shaft spillways (Fig. 14), the water flows over the lip of a funnel-shaped spillway and discharges down a shaft or tunnel. This form of spillway is adapted to narrow canyons where room for a spillway is restricted. A disadvantage of this type is that the discharge beyond a certain point increases only slightly with increased depth of overflow and therefore does not give so great a factor of safety against underestimation of flood discharge as do most other forms.

The glory-hole type has been tested extensively in models, but so far as known, it has not been subjected to high discharges on the actual structures. Because in ordinary model tests the air-entraining effects cannot be reproduced to scale, for the surrounding air pressure is not reduced to a magnitude corresponding to the model size, the degree of agreement of model tests with the prototype action is uncertain. The Davis Bridge spillway[8] is typical of the form. Since the spillway is placed on the side of the hill, it is usually provided with channels leading to it from both sides. Unless these channels are very deep, the water does not flow over the spillway crest in a radial direction, but, owing to the tangential component of the water as it approaches the weir, it is more or less deflected from a radial path in the direction of the path of approach. It tends to pass over the lip, therefore, with a component toward the bank side of the spillway, which results in a concentration of flow about the middle of the bank side and causes an unequal flow down the spillway shaft, which gives rise to considerable turbulence. This undesirable condition can be largely eliminated by placing piers on the crest to guide the water, as was done at Davis Bridge.

Considerable turbulence occurs at the bend at the bottom of the vertical shaft from this form of spillway. For comparatively low heads this is probably not serious, but the action under high heads is uncertain. For high dams, it would seem to be advantageous to begin to incline the tunnel as short a distance as possible below the intake and provide ample access of air to the inclined section.

The form of the spillway is largely controlled by the discharge to be accommodated and the depth of overflow permitted, for the length of crest must be sufficient to provide for the required discharge at the maximum head permitted. Thus large discharges and small depths of overflow give rise to large diameters of the intake section. The size of the outlet tunnel is determined by the discharge and fall and is commonly constructed so that the tunnel will flow full throughout its length but not cause a backwater action on the spillway crest under conditions of maximum discharge.

* *Western Construction News*, **10**, 96, April, 1935.

Gate- or Barrage-type Spillway. Gate-type spillways consist of a series of gates separated by masonry piers, with floors between the piers to prevent scour of the river bed, and cutoff walls extending into the river bottom to prevent undermining. The principal problems of such spillways are the design of the gates, which is treated in Sec. 8, and stream-bed protection, which is discussed later in this section.

Spillway Tunnels. Tunnels are frequently used as the outlet channels for various types of spillways. They are lined with concrete, and the cross sections are usually

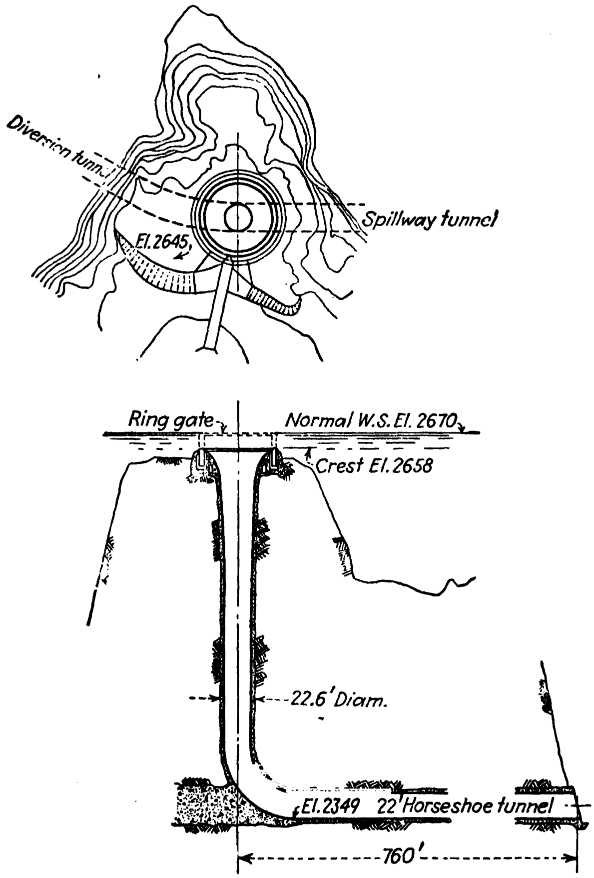


FIG. 14.—Owyhee glory-hole spillway.

circular but not necessarily so. The ability of the concrete to withstand high velocities of flow seems to be amply demonstrated where the water is clear and flows parallel to smooth walls. If the water contains fine silt in suspension, it will probably not cause excessive scour unless the surfaces are rough or there are projections that cause eddies or whirls, which tend to project the abraiding silt particles against the concrete. Coarser silt will probably cause greater cutting action than fine silt. When reservoirs behind dams become filled with material so that the bed material, sometimes of heavy gravel or even boulders, passes over the spillway, severe abraiding of the concrete is apt to occur. If there are projections or sharp corners in the tunnels, vacuums tend to form behind them, and if the velocities are high enough, severe cavitation sometimes

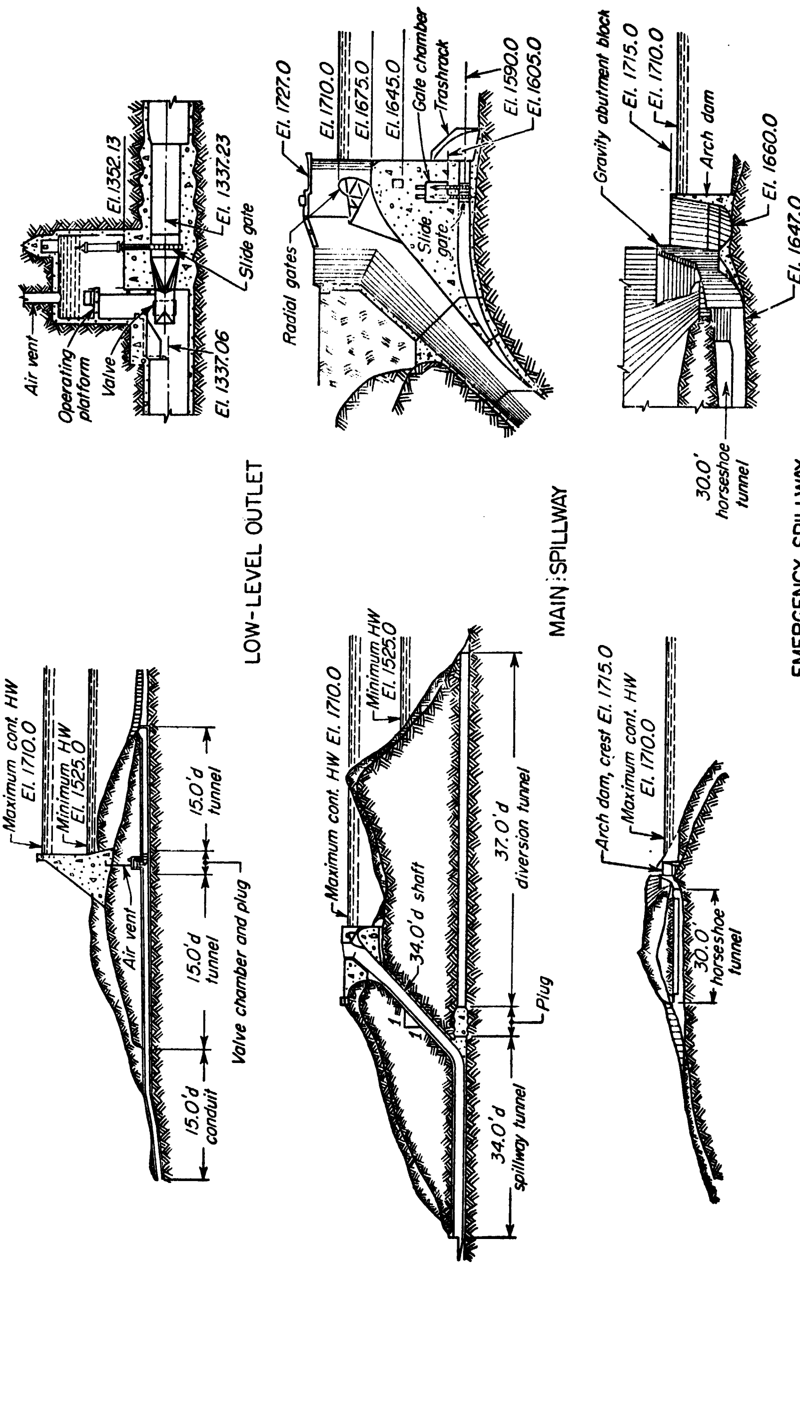


FIG. 15.—Spillway and outlet works, Fontana Dam, TVA.

takes place. Especial effort is therefore necessary to secure smooth surfaces and accurate alignment for all tunnel spillways. This precaution is equally important in other forms of spillways where velocities are very high.

The cross section of the tunnel generally should be designed to have ample air space above the water surface at all flows. The swift flowing water tends to drag the air above it along, and if ample space is not present throughout the tunnel length, the air may collect at certain points and burst out at intervals. Under other conditions, vacuums may form causing undesirable hammering. Models cannot be relied on to duplicate the prototype action in this respect.

Spillway tunnels are usually straight but may be curved if the radius of the bend is not too small. With high velocities and small radii, the water flowing swiftly along the bottom of a straight circular tunnel will rise on the outside of a curve and, if the curve is not too abrupt, return to the bottom again after passing it, with perhaps a tendency to swing from one side to the other beyond the bend. If the bend is abrupt, the water may rise on the side so rapidly that it is carried completely around the tunnel in a spiral movement. Such action would probably produce undesirable air conditions in the tunnel.

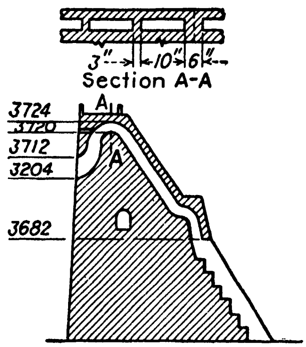


Fig. 16.—Siphon spillway in O'Shaughnessy Dam.

The Fontana project in North Carolina (TVA) offers an interesting example of a tunnel spillway as shown by Fig. 15. Planning studies for this project showed that an economical over-all arrangement could be obtained by making permanent use of the tunnels built for diversions during construction. These tunnels are designed to discharge approximately 100,000 cfs each under design flood conditions. For more detailed information on the projects, reference should be made to the various TVA[29] reports on the Fontana project.

Siphon Spillways. Where the available space is limited and where the discharge is not extremely large, siphon spillways are often superior to other forms. They are also useful in providing automatic surface-level regulation within narrow limits. Because the siphon spillways prime rapidly and bring into action their full capacity, they are especially useful at the powerhouse end of long power canals with limited forebay capacity where the power may go off and the turbines shut down rapidly, the provision of a considerable discharge capacity being made necessary within a very short time in order to avoid overflow of the canal banks.

Figure 16 shows a cross section of one of 18 siphon spillways in the O'Shaughnessy Dam (as initially constructed), which have a combined capacity of about 20,000 cfs. Siphon spillways are often built with a basin at the lower end so that the discharge end will be submerged, but this is not necessary, as an ejector action can be introduced by placing a bend or lip in the downstream leg, which deflects the water when flowing over the crest at a slight depth to the opposite side of the siphon barrel. The lower end of the siphon barrel will thus be sealed, and the flowing stream, by carrying along bubbles of air from the inside of the siphon, will reduce the pressure inside enough to cause the siphon to start.

After the siphon action is started, unless the siphon is vented, the upstream water level will be drawn down to the level of the entrance before flow ceases. The magnitude of the drawdown can be controlled by means of a vent, as shown in Fig. 16, through which the air enters the siphon and destroys the prime as soon as the water level has fallen below the vent.

The siphons should be made with gradually contracting entrances and curves of as

large radius as possible. The capacity can sometimes be increased by gradually expanding the downstream section of the tube. Where ice is likely to form, the entrance should be placed at considerable depth. Siphons should be so proportioned that a vacuum approaching 34 ft is not formed at sea level, and this limit must be reduced about 1 ft for each 850 ft of elevation above sea level.

It is not possible to compute accurately the action of siphon spillways. Much better results can be obtained by model tests, but care should be exercised, as the model results do not always agree with the prototype action in all particulars.

SPILLWAY DISCHARGE COEFFICIENTS

Free Overfall. The discharge for an overfall spillway of nappe shape varies as the following equation:

$$Q = C(L - KNH)(H + h_v)^{3/2} \tag{7}$$

in which Q = total discharge over the spillway, cfs.
L = net length of the spillway, ft.
h_v = velocity head of approach, ft.
N = number of complete pier contractions (two per pier).
K = pier contraction coefficient.
H = head on the spillway crest, ft.

Recent tests have shown that the value of K, usually given as 0.03 to 0.04 for a pier with a semicircular shaped nose in the same plane as the upstream face of the

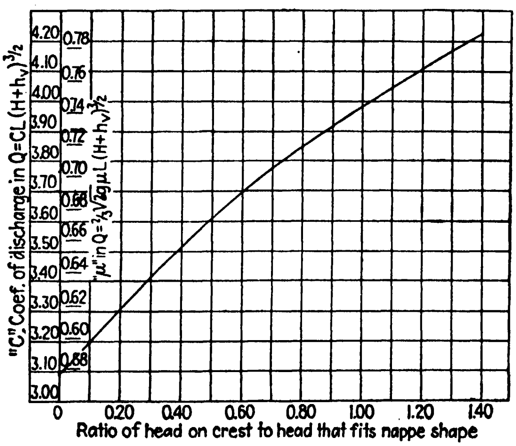


FIG. 17.—Discharge coefficients of dams with nappe-shaped crests.

spillway, is too large. Tests conducted without piers, for large gates, say 30 to 40 ft high, first, to evaluate the coefficient of discharge, C, and then repeated with the piers, revealed that the average value of K varies from 0.01 to 0.02 for fairly high heads, 20 to 40 ft, and up to 0.04 for heads of a few feet.[30]

The discharge coefficients for an overfall spillway of nappe shape varies with the head starting at about 3.10 and reaching approximately 4.0 at the design head. The coefficients as functions of the relation of the head to the design head are shown in Fig. 17. This diagram is based upon a study made by E. W. Lane and W. M. Borland of the results obtained upon many models and actual dams. Coefficients for dams with rounded crests other than nappe shape, but not too widely different, will

approximate closely those which would occur on that nappe-shaped crest which most nearly agrees with the upper part of the rounded crest.

Figure 18 shows a comparison of the curve of coefficients on Fig. 17 with the coefficients obtained from recent model tests for a large dam in the Pacific Northwest having piers with a clear spacing of 40 ft-0 in. and values of H varying from 25 to 60 ft. It will be noted that the two curves are very close together, showing that properly designed piers have relatively little effect on the discharge coefficients.

In the operation of reservoirs during the passage of a flood, it is necessary to determine in advance the discharge capacity of gates at partial gate openings. An

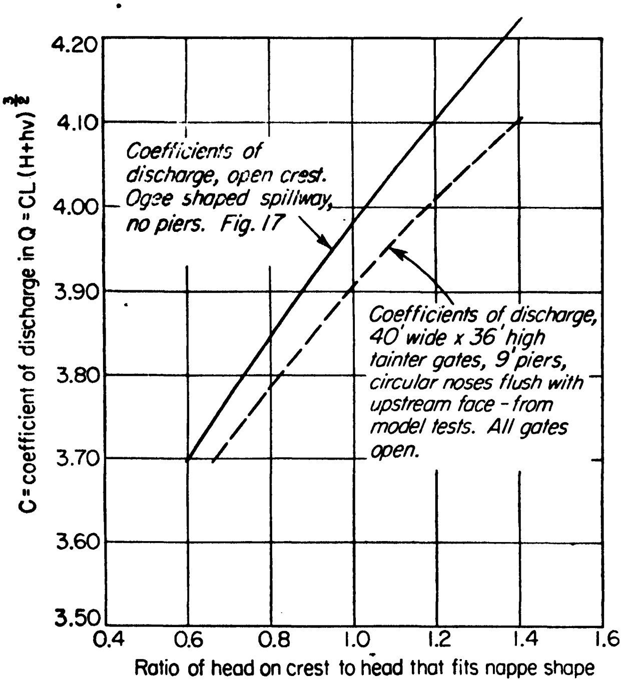


FIG. 18.

accurate knowledge of discharge coefficients at partial gate openings may best be obtained from model tests. In making preliminary studies, however, it is useful to give consideration to measurements taken for both models and prototypes. Typical studies follow for tainter, vertical leaf, and drum gates.

Figure 19 shows the results of a model study to determine a rating curve for a 40 ft wide by 38 ft high Tainter gate. The dotted curves show the discharge for a single bay with adjacent gates closed. The solid curves show the discharge for a single bay with adjacent gates open. Discharge curves are plotted both for varying heads and for varying gate openings. As the discharge increases, it follows the free flow curve until the water touches the gate lip, whereupon an abrupt decrease occurs. This decrease is followed by a more gradual rise in pool level along the curves for flow representing orifice discharge. In falling stages, no abrupt change in discharge occurs. The curves for orifice discharge in Fig. 18 are essentially the same for both rising and falling stages.

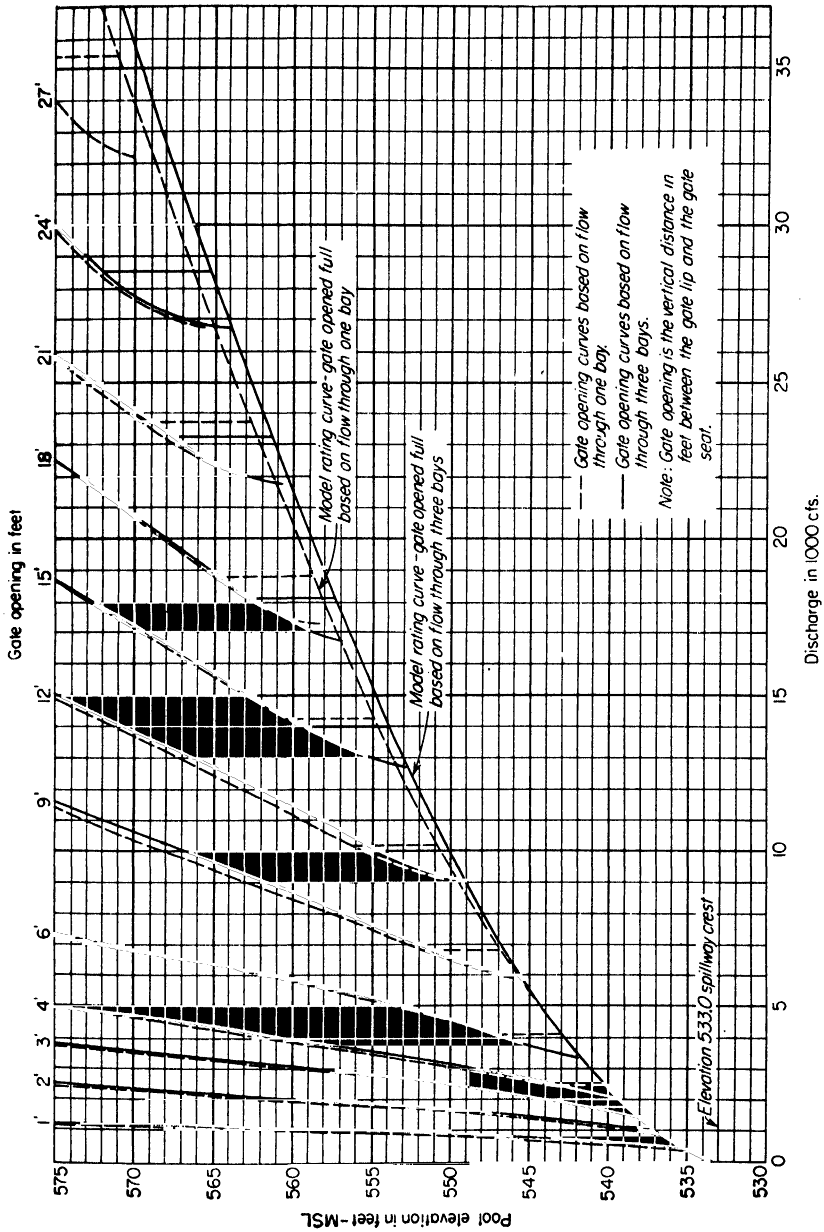


FIG. 19.—Discharge rating curves from hydraulic-model study of one 40- by 38-ft. Tainter gate. (*Courtesy of U.S. Army Engineers, Vicksburg, Miss.*)

The basic formula for spillway discharge with a vertical leaf gate operating at partial opening is

$$Q = mL(h_1^{3/2} - h_2^{3/2}) \tag{8}$$

in which h_1 = head on upper edge, ft.
h_2 = head on lower edge, ft.
L = net length, ft.
m = coefficient of discharge.

m in Eq. (8) will not be identical with C in Eq. (7) except at points where there is free overfall.

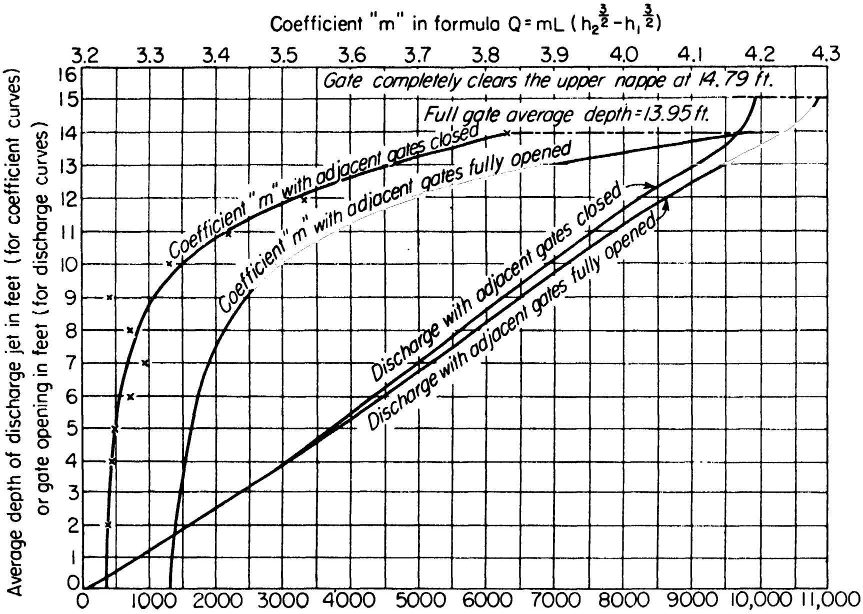


FIG. 20.—Spillway rating curve, Wilson Dam.[32]

Interesting experiments were conducted at Wilson Dam, Ala., to determine the values of m.[32] These measurements were made at the dam with various heights of gate openings and with various combinations of gates in operation.

The spillway section of the Wilson Dam consists of a gravity overflow dam with the permanent crest about 80 ft above the stream bed. Flow is controlled by 58 gates 18 ft high by 38 ft wide. The piers are 8 ft thick. The pier noses are circular in form and flush with the upstream face of the dam.

Figure 20 shows the rating curve and discharge coefficients for two operating conditions: (1) with adjacent gates completely closed and (2) with adjacent gates completely open. Both sets of tests were made with the pool surface held constant at 18 ft above the permanent crest. The value of h_1 should not be taken as the distance from the pool level down to the point where the gate clears the water, but rather it should be equal to the head on the crest minus the average depth of the discharging jet. The jet will curve slightly as it approaches the piers.

Interesting model experiments, later largely confirmed by measurements at the prototype, were made for Norris Dam (TVA), Norris, Tenn., to determine the coeffi-

cients of discharge for a drum gate type spillway.[33] Figure 21 shows the dimensions of the crest of this structure. Three hydraulically operated steel drum gates are installed on the concrete spillway crest. The spillway crest is at elevation 1,020 and is

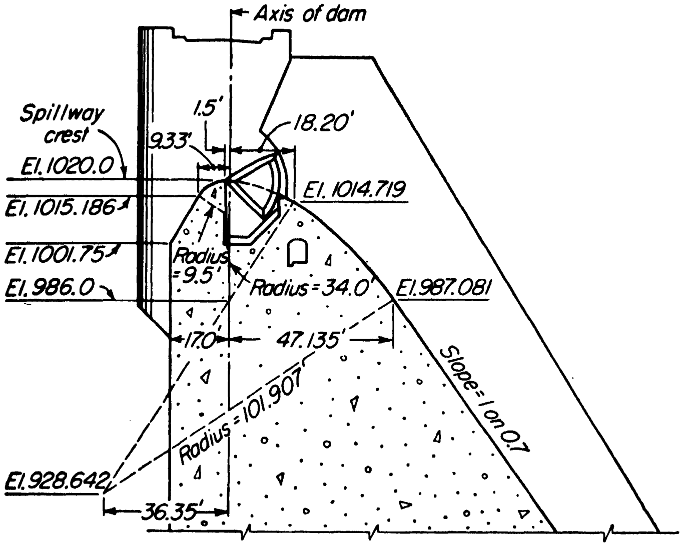


FIG. 21.—Details, spillway crest, Norris Dam.[33]

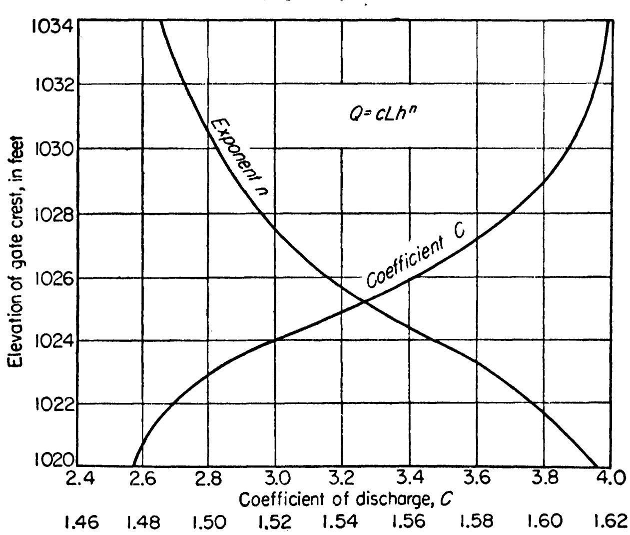


FIG. 22.—Spillway discharge coefficients, Norris Dam.[33]

divided into three bays each 100 ft long. The gates may be raised to any elevation between 1,020 and 1,034 to retain flood peaks.

Discharge coefficients for the Norris spillway were measured by the Bureau of Reclamation on a 1:72 scale model. It was found that, because of the shape of the drum gate crest, which changed with the elevation of the gate, C in Eq. (7) was neither

a constant nor a function of the head. Figure 22 shows the results of the model tests with a constant reservoir level at elevation 1,034 and a varying crest elevation.

Actual coefficients were calculated for the gates from records of their performance during the flood of January and February, 1937, with the results obtained from this model. This comparison indicated that the coefficients predicted by the model were 3.9 per cent larger than those actually measured on the dam during this period.

Submerged Flows. The most complete available treatment on the coefficients of discharge for submerged flows will be found in "Studies of Crests for Overfall Dams."[28] Space limitations do not permit a full treatment in this section. Some of the most important results of these tests will be found summarized in Appendix A of the bulletin. The discharge coefficients commonly used for submerged flows are also given in Appendix A. Aside from these studies, few reliable data are available on the effects of submergence. The information in Appendix A should be used only as a guide in making preliminary investigations. Model tests should furnish the basis for any final design.

SCOUR PROTECTION BELOW OVERFALL DAMS

Unless proper precautions are taken, the velocity of the spillway discharge may erode the stream-bed material and undermine the dam until failure occurs. The hydraulic jump is usually the most effective way of preventing this erosion as it quickly reduces the velocity of the water to a point where it is incapable of damaging the stream bed.

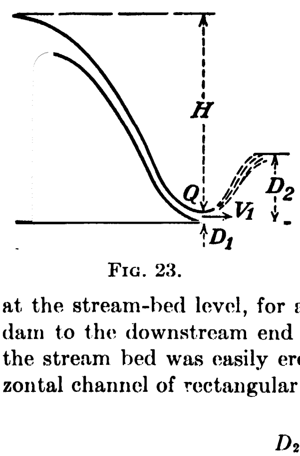


FIG. 23.

In cases where a complete jump cannot be obtained, investigations for the hydraulic jump still serve a useful purpose as these may be used as a basis for determining the best method of affording stream-bed protection.

The simplest kind of protection could be used if a jump would form at all discharges on a horizontal floor at the stream-bed level, for a simple paving on the stream bed extending from the dam to the downstream end of the jump would be all that was necessary, even if the stream bed was easily erodible. The formula for the hydraulic jump in a horizontal channel of rectangular section is

$$D_2 = -\frac{D_1}{2} \pm \sqrt{\frac{2V_1^2D_1}{g} + \frac{D_1^2}{4}} \tag{9}$$

where D_1 and D_2 are the depths upstream and downstream from the jump, respectively, and V_1 and V_2 are the corresponding velocities. Consider an ogee dam (Fig. 23) discharging 100 cfs/ft of length of crest with a drop H, from the headwater surface to the upper face of the high-velocity stream below the dam, equal to 50 ft. The velocity V_1 (friction being neglected) would then be $\sqrt{2g \times 50} = 56.8$ ft/sec, and the depth D_1 would be $\frac{100}{\sqrt{2g \times 50}} = 1.76$ ft. Substitution in the preceding formula gives a value for D_2 of 17.9 ft, which is the height of tail water required to form a perfect jump at the toe of the dam.

The height of the tail water for each discharge seldom corresponds to the height of a perfect jump. Frequently the elevation-discharge curve of the tail water or the tail-water rating curve at the downstream side of the dam is as shown in Fig. 24, but the curve of elevation that would be required to form the perfect jump on an apron at river-bed level is as shown by the jump-height curve AB. This shows that, for dis-

charges of less than 20,000 sec-ft, the tail-water height is greater than that required to form a perfect jump, but at greater discharges the tail-water height is too low.

The relations between the positions of these curves fall into the four following classes:

Class 1. Jump-height curve always above the tail-water rating curve.
Class 2. Jump-height curve always below the tail-water rating curve.
Class 3. Jump-height curve above the tail-water rating curve at low discharges and below at high discharges.
Class 4. Jump-height curve below the tail-water rating curve at low discharges and above at high discharges.

The best form of protection depends largely upon which of these four conditions exists.

Class 1. If the jump-height curve is always above the tail-water rating curve, the jump will not form near the toe of the dam on an apron at stream-bed level but will sweep across the apron at high velocity and attack the bed downstream. This happened at the Wilson Dam,* where the conditions fall in this class.

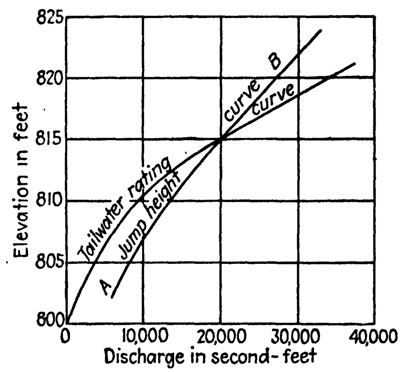


FIG. 24.—Tail-water rating curve.

Cases in the first class frequently occur when a dam is placed at the head of a rapids or sudden drop in the stream bed. Under these conditions, the tail-water level is low and less than the height required to form the jump. Usually too, the bed is of solid rock, which will withstand considerable scour. Under these conditions, an upward curving apron or bucket is frequently put on the dam, which throws the stream of high-velocity water passing over the dam upward so that it travels some distance from the toe of the dam before it falls back and strikes the stream bed. Here the energy is dissipated by impact of the water on the river bottom and adjacent water, and although some scour takes place, it is too small or too far from the dam to endanger it. This form of protection has been used on the Holyoke, Conowingo, Safe Harbor, and Bull Run dams (Fig. 25). When the tail-water depth is nearly sufficient to cause the jump, baffles or sills may be successfully used, but they often receive so much impact from drift or ice that they are expensive to build with a sufficiently strong anchorage. Baffles or piers do not dissipate so much energy as many engineers believe, and they are therefore not so effective as the hydraulic jump† if the ratio of H to D_1 is large. In a number of instances baffles have been subject to attack, which in some cases is probably cavitation and in others may be abrasion. Until further experience has accumulated to indicate the limits of safe practice, it will be advisable not to place baffles in water flowing at very high velocity. Where they are used in the stilling pools of high dams, they should be located downstream from the hydraulic jump.

On rock foundations where the tail-water level is nearly high enough to cause the jump to form, experience indicates that if no protection is added downstream from the bucket of the dam the bottom will be scoured out until sufficient depth is provided to permit the jump to form, and no further scour will take place.[16]

If the tail-water level is not high enough to form a perfect jump, it may be raised by building a low secondary dam below the main dam with sufficient height to cause the jump to form at the foot of the main dam for all conditions of discharge. This

* *Eng. News-Record*, **98**, 100, 1927.
† *Eng. News-Record*, **97**, 800, 1926.

method has been extensively used for dams on earth foundations. Such a form, developed by the Fargo Engineering Company and used on the Junction Dam, is shown in Fig. 25. This method has also been used for a rock foundation at the Martin Dam of the Alabama Power Company. Considerable light on the dimensions required for such a pool are shown by the experiments of C. M. Stanley.[15]

Another method which may be suitable in some cases is to excavate a pool downsteam from the dam to provide a depth sufficient for the formation of the jump. In this case, the tail-water level is not changed, but the depth required to form the jump is provided by the lowering of the channel bottom rather than by raising the water surface. This method was used in the Norris and Wheeler dams as shown in Fig. 25.

The efficiency of energy dissipation in these stilling pools can usually be considerably increased by placing a section of sloping floor between the main dam and the

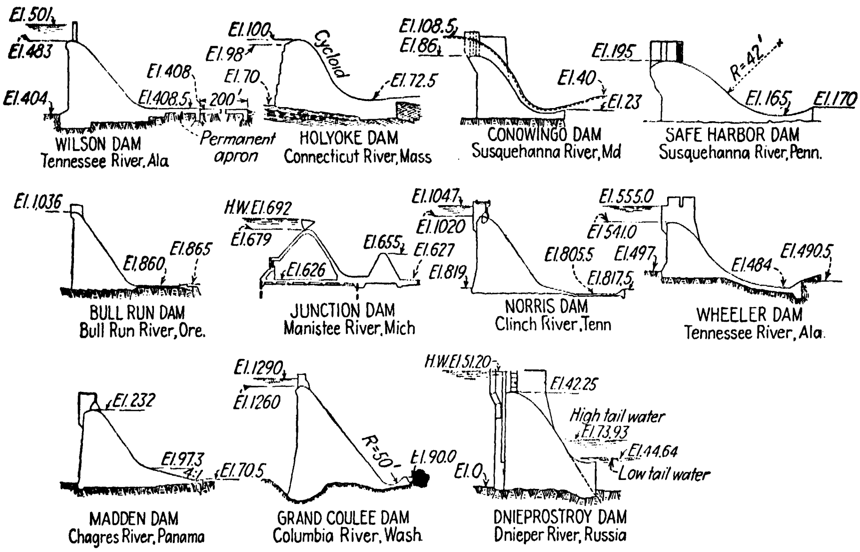


FIG. 25.—Cross sections of dams showing various methods of protection against scour.

bottom of the stilling pool, both for pools formed by secondary weirs and those formed by excavation. The stream of high-velocity water falling over the dam will flow down this sloping apron until an elevation is reached at which the depth and velocity of its flow is just sufficient to cause a hydraulic jump of the height required to raise the water level to the tail-water elevation. At this point, the jump will take place. By making such a slope extending from the highest elevation of pool floor which will form a jump reaching tail-water height at any discharge to the lowest elevation which will form the jump, an efficient jump can be formed at all discharges. For cases in Class 1 the sloping apron would be entirely below the stream-bed level. The slope should preferably be 4 horizontal to 1 vertical or flatter, although 3 to 1 gives fairly good results. Experiments have shown a rapid decrease in efficiency as slopes become steeper than 3 to 1.

An excellent example of such a sloping apron is shown by Fig. 26, a section through the Petenwell spillway constructed on the Wisconsin River near Necedah, Wis. The Petenwell spillway is constructed entirely on a foundation of medium sand. The sloping apron gives complete protection against erosion under all flow conditions.

The required length of the stilling pools depends upon the erodability of the mate-

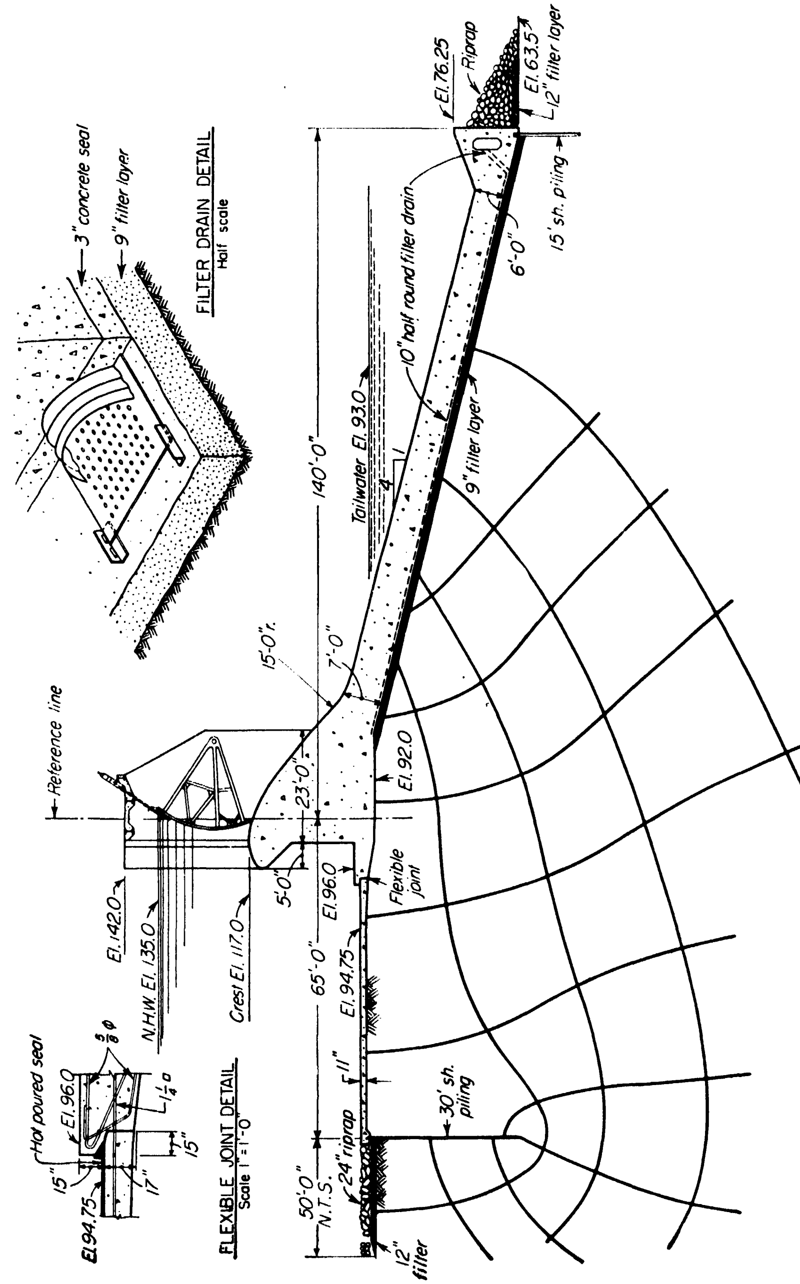


FIG. 26.—Cross section of Petenwell spillway, Wisconsin River, near Necedah, Wis. (Courtesy of Harza Engineering Company.)

rial at the end of the pool. If this is resistant, the pool length may be shorter than if it is easily scoured. Few reliable data are available, but a length of five times the net height ($D_2 - D_1$) of the jump is probably sufficient in most cases if a sill and riprap is used at the downstream edge of the basins on erodable foundations.

Class 2. Dams in the second class are apt to occur where the foundation rock is at a considerable depth and where the tail-water surface at all discharges is therefore higher than necessary to form the jump. Under these circumstances, if an ogee dam with conventional bucket is used, the water flowing down the face of the dam dives under the tail water and travels at high velocity a long distance along the bottom, forming only a very imperfect jump. The more nearly the tail-water depth corresponds to the depth required to form the jump, the shorter the distance the high velocities extend downstream. A perfect jump could be formed for any single discharge by building a level raised apron below the dam to give just the correct depth for the formation of the jump, but this apron would probably not be at the correct elevation for other discharges.

By using a sloping apron, as previously discussed, it is possible to cause an efficient jump at all discharges. For this class, the apron would lie entirely above stream-bed level. Probably, the outstanding example of the use of the sloping apron for dams in the second class is the Madden Dam (Fig. 25) built to store water for the Panama Canal. The form used there was developed as the result of extensive model tests.[17,18] The slope and elevation of the apron were determined by trial and that apron selected for which the curve giving the relation between discharge and height of tail water necessary to form a jump at the toe of the dam most nearly agreed with the natural tail-water curve.

Another method for protection against scour with class 2 conditions is to place the apron as low as possible and curve it up sharply at the end. The Grand Coulee Dam (Fig. 25) is the outstanding example of this type of protection.[19] The overfalling water follows around the bottom of the curved apron and rises to the surface, mixing with the water moving around in the roller held in by the bucket. As this mixed water rises to the surface, part falls back into the bucket and part flows downstream near the surface, without attacking the bottom. When the upward moving sheet of water passes the end of the apron and comes in contact with the tail water, it mixes also somewhat with the tail water and carries much of it along, drawing the water near the bed of the stream toward the dam. The current thus produced has considerable transporting power and will carry materials up the downstream face of the bucket. If this material is of a nature such that it will abrade the concrete, it is necessary to excavate the stream bed far enough downstream to remove all material that will be transported. If the tail-water depth is too shallow, the water that would otherwise fill in the space above the bucket will be swept out and the swiftly flowing stream will rise upward, often considerably above the level of the tail water, and fall back into the stream again at the surface, where it will cause no scour. This form also sets up currents along the bottom toward the dam. The Dnieprostroy Dam in the U.S.S.R. (Fig. 25) falls into this class. The tail-water conditions may in some cases produce an action like that at Dnieprostroy at some discharges and like that at Grand Coulee at others.

Class 3. The third class occurs where the tail-water depth at low discharges is insufficient to cause a jump to form on a horizontal apron at river-bottom level but is more than sufficient at high discharges. This is a common case, and the solution consists in artificially creating enough water depth to make the jump form on the apron at low discharges. This may be done by a low secondary dam or sill across the channel near the downstream end of a level apron. This secondary dam must be high enough to cause the hydraulic jump to form upstream from it for all flows for which

the natural tail-water depth is insufficient. The depth required may also be secured by depressing the apron.

If the velocity of the water is not too great, this third case is often favorable to the use of baffle piers or some form of dentated sill near the end of the apron, as these tend to break up the high-velocity flow at low discharges and also to raise the tail water, both of which actions promote the formation of the jump. These baffles would also be advantageous at high flows, for then the depth of tail water is greater than is required to form the jump, and the nappe over the crest tends to dive down to the bottom of the river and flow along the apron at high velocity and is broken up by the sills or piers. The sloping apron can also be applied to this class. To fit these conditions, the upper end of the apron is above bed level and the downstream end is depressed below bed level.

Class 4. The fourth class, where the tail-water depth is sufficient at low flows but too small at high flows, can be solved by increasing the depth of tail water sufficiently to cause the jump to form for the maximum discharge contemplated, either by a secondary dam or an excavated pool. With these facilities of the magnitude required for the maximum flow, the tail-water depth would be more than sufficient to cause the jump to form on the apron at the lower flows. In many cases, this condition will not be objectionable for low flows, but if it is, a sloping apron can be used to provide the correct depth.

MISCELLANEOUS CONSIDERATIONS IN ENERGY DISSIPATION

Where the ratio of H to D_1 (Fig. 23) is large, the proportion of the energy of the falling water which is used up in the jump is great, but where the values are small, the jump is not so effective as an energy dissipator. Stevens[23] has given the following formula for the percentage of energy dissipated:

$$\text{Energy dissipated (per cent)} = \frac{6.25\left(\sqrt{1 + 16\dfrac{H}{D_1}} - 3\right)^3}{\left(1 + \dfrac{H}{D_1}\right)\left(\sqrt{1 + 16\dfrac{H}{D_1}} - 1\right)} \tag{10}$$

from which the following have been computed.

Values of H/D_1	Per Cent Energy Loss
0.5	0
1	1.4
2	9
5	28
10	44
20	58
30	65

When the ratios of H/D_1 are low, baffle piers can be used to advantage to assist in dissipating the energy of the overfalling water. In most cases, low values of the ratio result from small values of H, and therefore the velocities are relatively small, and baffles are not subject to great shocks. In many cases, such as at Bonneville and some of the Tennessee River dams, low values of the ratio result from high values of D_1 and securely anchored baffle piers are used in spite of the high velocity. These baffles take many shapes, some of which are shown on Fig. 33.

To make the foregoing analysis as simple as possible, it has been assumed that the stream flow was uniformly distributed over a fixed length of the dam, and therefore for a given discharge there would be a fixed condition of overflow and a fixed tail-water elevation. By the proper selection of crest length, however, it may be possible to

secure a closer agreement between the tail-water rating curve and the jump-height curve than is secured by an arbitrarily chosen length. If the first assumed crest length produces a class 1 condition, better agreement will be secured by increasing the length. Similarly, a class 2 condition would be improved by decreasing the length. To select the best arrangement, it is necessary to estimate the cost of the various designs giving equal scour protection and select the least expensive.

Use of Crest Gates. With gates on the crest of dams, the discharge at any point along the crest may vary considerably for a given tail-water elevation. To be entirely safe, the protection should be designed so that it will be sufficient with any possible condition of gate openings and flow. As a practical matter however, where the structure is subject to intelligent supervision, it may be assumed that reasonable judgment will be used in operating the gates and sufficient protection provided so that no severe damage will result with the undesirable conditions acting for a limited time such as might occur, for example, in the case of the failure of a gate to close when desired. In most cases, it will be found best to have the crest gates designed to be capable of

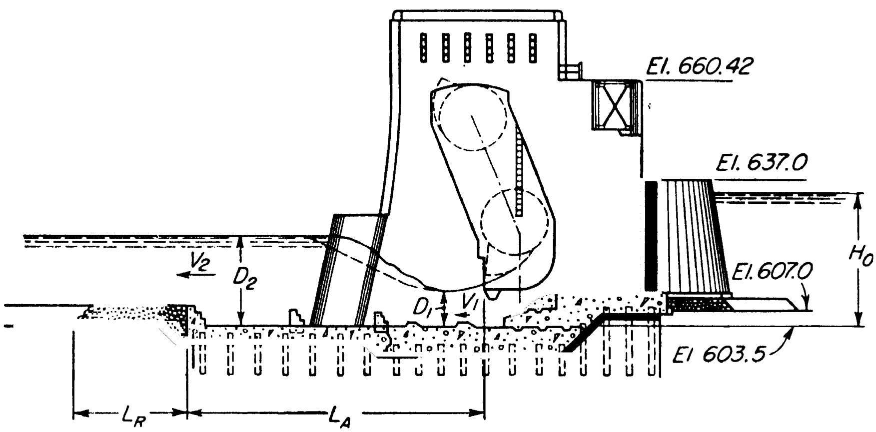


FIG. 27.—Typical cross section of a roller-gate dam on the upper Mississippi River.

operating partly open, in order to distribute the water uniformly over the crest instead of permitting only an entirely open or entirely closed condition. Various types of protective works below gates are shown in Figs. 27 to 30.

Ends of the Overflow Spillway. Walls are usually necessary at the sides (or ends) of the hydraulic-jump pools of the overflow sections of spillway dams to prevent the tail water from flowing from the side into that portion of the jump the surface of which is below tail-water level. Where this action is permitted to take place, a very imperfect form of jump results, and high velocities persist for considerably greater distances downstream than if this flow is prevented. Walls are also often necessary on the face of overfall dams leading to the ends of the stilling pool to prevent the spreading of the water after it passes the crest. The face of these walls should usually be a vertical plane at right angles to the spillway crest as other arrangements are apt to cause a concentration of flow into the stilling pool which will cause excessive scour.

Basin Erosion. Care should be exercised to be sure that no loose gravel, rocks, or other solids are left in stilling pools, since such material will be rolled around on the pool bottom by the flowing water and may seriously erode the concrete. Currents of water drawn into the stilling pools of overflow dams at the ends of the overflowing nappe may bring in gravel or rock, which will have a severe abrasive effect. All loose material on the stream bed downstream from the dam which would be subject to this

action, when flow is passing over all or part of the length of the spillway, should be removed.

Tail-water Rating Curves. Enough has been given in the foregoing to show the importance of the tail-water rating curve in the solution of spillway problems. One of

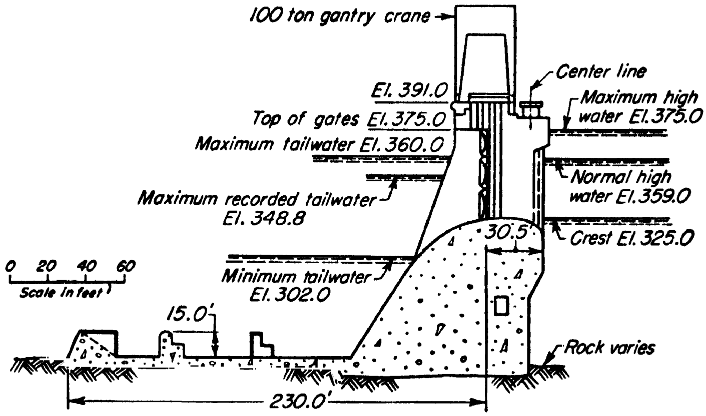


FIG. 28.—Section through Kentucky Dam spillway.

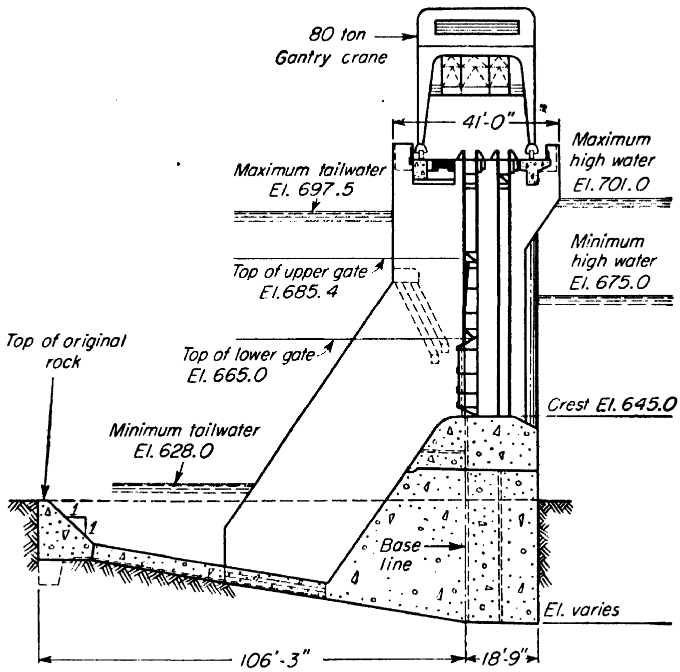


FIG. 29.—Section through spillway at Chickamanga Dam.

the first steps in attacking the problem at any site is to determine the tail-water rating curve, either by observing the actual levels for a wide range of discharges or by computation. The former method should be used if possible, but fairly satisfactory results may be secured by determining the tail-water elevation for various discharges by means of backwater curves for these flows. Since the water levels at downstream

points for these discharges are probably also unknown, two backwater curves for each discharge may be started at considerably different assumed elevations far enough downstream from the dam that the computed curves reach practically the same elevation at the dam, thus showing that the elevations reached at the dam are independent of the starting elevations assumed.

Conservatism should be used in determining the tail-water curve to be used in design, unless it is accurately known and not likely to be changed. In many cases, it will be desirable to use two curves, one as high and the other as low as there is any probability of the curves reaching. The design should perform satisfactorily within these limits. In dams built on streams with movable beds, allowance should be made for the retrogression that usually occurs below such structures.[20]

Energy Dissipation for Chute Spillways. There is a wide difference in the precautions taken in providing for the dissipation of the energy at the end of chute spillways. In the case of the Ft. Peck Dam, the spillway, which is a considerable distance from the dam, provides for a discharge of 255,000 sec-ft. with a velocity of 100 fps and dis-

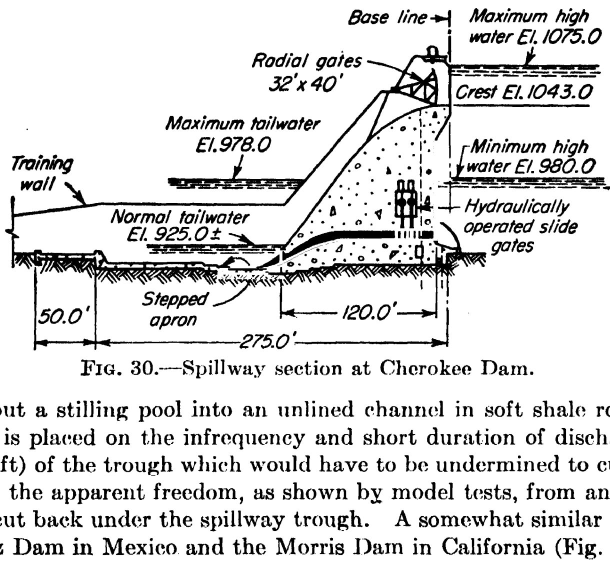


FIG. 30.—Spillway section at Cherokee Dam.

charges without a stilling pool into an unlined channel in soft shale rock.* In this case, reliance is placed on the infrequency and short duration of discharge, the long length (5,200 ft) of the trough which would have to be undermined to cut back to the reservoir, and the apparent freedom, as shown by model tests, from any tendency of the water to cut back under the spillway trough. A somewhat similar case is that of the Rodriguez Dam in Mexico and the Morris Dam in California (Fig. 10) where the end of the spillway chute is turned up and discharges the water with an upward inclination which carries it some distance from the end of the spillway before it strikes the sides or bottom of an unlined canyon in easily eroded rock. In other cases, elaborate stilling pools are provided for lower dams and smaller discharges in comparatively sound rock. Obviously, chute spillways founded for their entire length on earth should be provided with adequate stilling pools at the end, and in all cases very conservative designs should be provided where the failure of the dam would lead to loss of life or the destruction of valuable property.

Because of the cheapness of construction, stilling pools with sloping sides appear to be very attractive, but model tests indicate that the dissipation of energy in the hydraulic jump in such channels is much less complete than in a pool of the same length when the sides are vertical. In most cases, vertical or nearly vertical walls seem justified, in spite of higher cost. It will usually be found advantageous at the end of the floor of chute stilling pools to have some form of sill. The remarks made elsewhere regarding sills apply to these also.

* *Civil Eng.*, **6**, 741, November, 1936.

Figures 31 and 32 show an interesting arrangement of energy dissipators used at the outlets of the Fontana spillway tunnels (see also Fig. 15). Figure 31 shows these to be clamshell-shaped buckets or curved deflectors, designed to spread the spillway discharge over a wide area. Figure 32 shows this spillway in action.

Stilling Pools for Gate Dams. In gate dams, the gates are usually operated to distribute the flow nearly equally across the stream to reduce the scour and in some

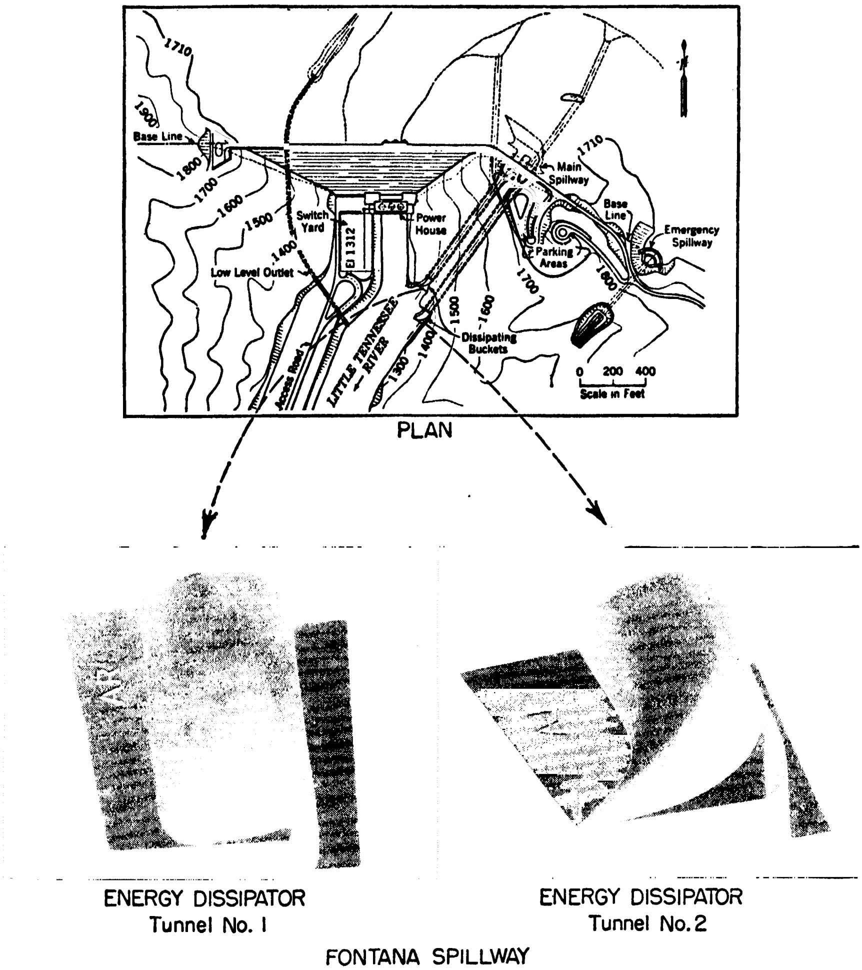


FIG. 31.—Energy dissipators of Fontana spillway tunnels. (*Courtesy of Tennessee Valley Authority.*)

cases to improve navigation conditions downstream. Under these circumstances, this form of dam usually operates under class 3. At low flows, the gates are only slightly opened and the water flows beneath them at comparatively high velocity, but in small volume. A slight lowering of the apron is usually sufficient to form a jump. With increased discharge, the tail water usually rises more rapidly than required to form the jump and at medium flows the jump is drowned out. For this condition, baffles and a sill at the lower end of the apron are helpful. At higher discharges, the gates are

raised to sufficient height that the flow under them does not reach the critical depth, and hence no jump is possible. Under these conditions, the baffles on the floor and sill are also helpful. The protection required for this form of dam is therefore usually a slightly depressed apron with baffles on it and a sill at the lower end. Figure 27 shows a typical case.

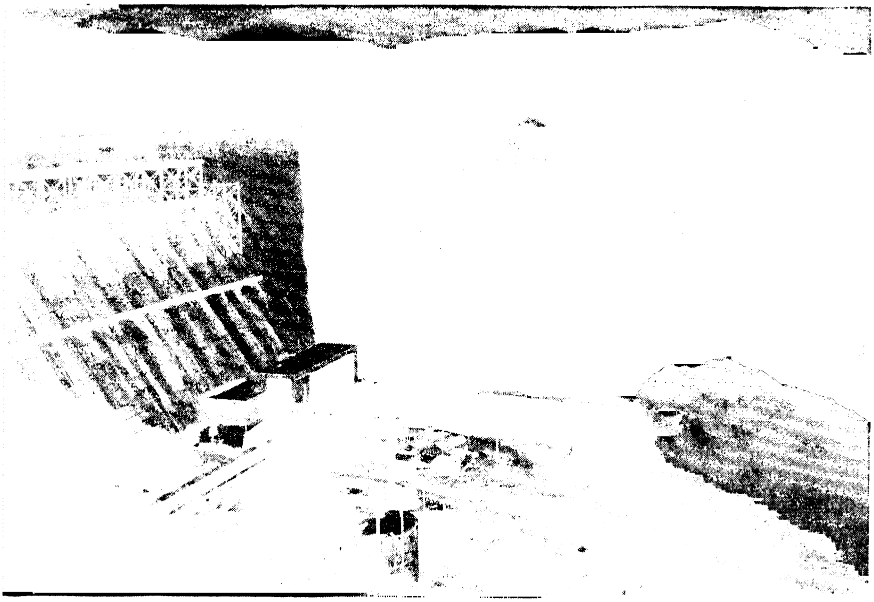


FIG. 32.—Fontana spillway in action. (*Courtesy of Tennessee Valley Authority.*)

As a result of extensive model studies by the U.S. War Department at the state University of Iowa,[22] the following relations have been worked out for the length of apron and riprap required:

For apron without baffle piers,

$$\frac{D_2}{H_0} = \frac{1}{\left(\frac{L_A}{V_2^2}\right)^{0.6}} \tag{11}$$

For apron with baffle piers,

$$\frac{D_2}{H_0} = \frac{1}{\left(\frac{L_A}{V_2^2}\right)^{0.7}} \tag{12}$$

Length of riprap,

$$\frac{D_2}{H_0} = \frac{0.65}{\left(\frac{L_R}{V_2^2}\right)^{2/3}} \tag{13}$$

The meaning of these symbols is shown in Fig. 27. Extensive model research in India has also usually led to the use of a wide apron sloping downstream from the gate, with baffle piers mounted on it.

MISCELLANEOUS CONDITIONS

Weirs on sandy rivers have sometimes been made of great quantities of loose rock piled across the stream, with a long slightly sloping paved floor on the downstream

side. To protect this form of weir against scour, it should be designed so that the jump will occur high enough up on the sloping floor that it causes no scour of the river bed. At low discharges, the jump could be farther down the slope than at high flows, without causing excessive scour. To attain these conditions, it may be necessary to extend the floor somewhat below the river-bed level or to regulate the length of the weir, as previously discussed.

It will usually be found advisable to place a sill across the downstream edge of a stilling-pool floor or dam apron, for such a sill tends to set up upstream currents on the bottom immediately downstream from the sill, which draw the bed material up toward

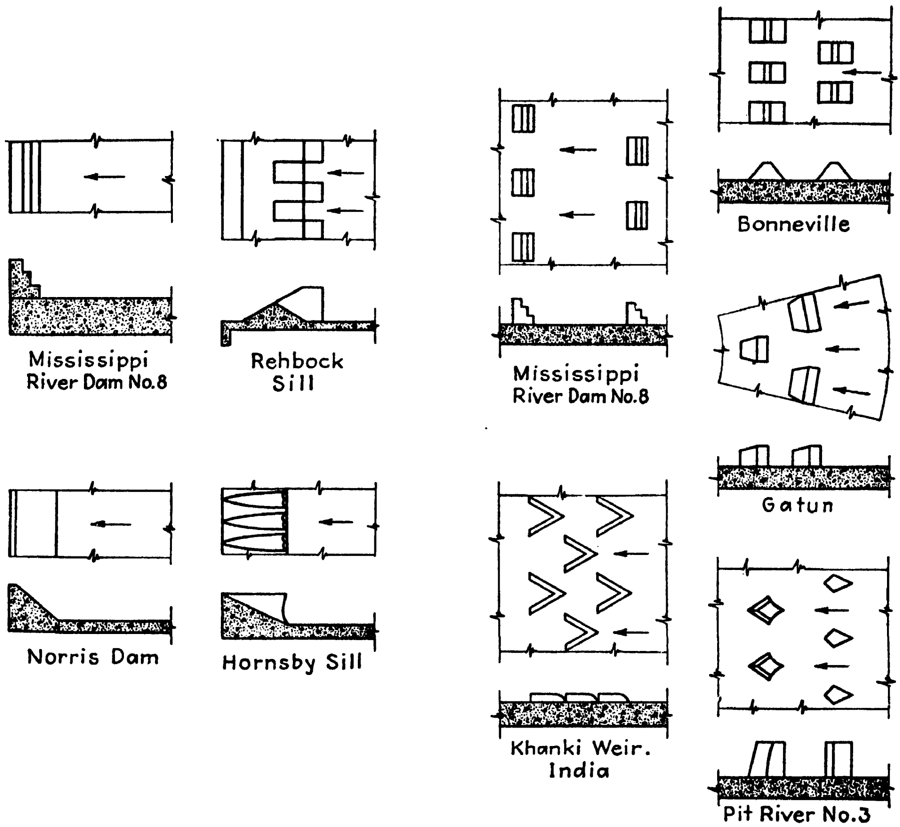


FIG. 33.—Typical forms of baffle piers and sills.

the dam instead of carrying it away, and thus tends to prevent the dam from being undermined. This is especially necessary on structures founded on earth but is often desirable for rock foundations also. Many forms of sill have been used. One of the most successful is the dentated sill of Dr. Rehbock, but there are many other forms that work nearly as satisfactorily. Several forms of sill are shown on Fig. 33. In general, it is desirable also to drive a row of steel sheet piling across at the downstream edge of the apron of a dam on erodible foundation, to prevent the scour from undermining it.

The curved form of an arch dam tends to cause the overfalling water to converge into a small space and thus makes the dissipation of its energy difficult. Where the nappe follows the dam face, some form of hydraulic-jump pool is a probable solution,

but where the nappe springs free a deep tail water is often built up by means of a secondary dam, into which the water can fall, the depth being made great enough to prevent serious erosion to the river bed. In the case of the Calderwood Dam, a curved bucket was constructed to deflect the falling jet from a downward to an upward direction and thus protect the river bed.

Trouble has been experienced on overfall dams on erodible foundations where the apron or stream bed was higher at the sides of the stream than in the center, because at low flows the water, after passing beyond the end of the apron, tended to flow at high velocity from the sides to the center of the stream along the downstream edge of the dam, undesirable scour being thus produced. For this reason, on erodible beds the apron and stream bed should usually be at the same level across the stream for the entire width of the spillway.

HYDRAULIC MODEL TESTS

Although the foregoing discussion will enable preliminary designs to be drawn up with considerable accuracy, hydraulic model tests should be made for every important spillway. The model should cover the stream for a short distance upstream and a longer one downstream, in order that its action on the stream bed for some distance below may be studied. The action of the water at the ends of the spillway is often important, and should be studied in its relation to adjacent structures. As the cost of model tests increases rapidly with the size of the model when large-size models are used, preliminary tests are frequently run on smaller models, and large sizes used as a check only when the design is fairly well worked out. In general, discharges ranging from 2 sec-ft for smaller spillways to 7 cfs for large ones will be ample for all but the most important structures.

Spillway models should always be operated under the tail water and other conditions that will be found in nature; a failure to make the tail-water conditions duplicate those which will occur in the prototype will almost certainly lead to unsound conclusions. The model should be run under every condition that can happen in the prototype, not simply the way in which it is planned to operate it, for unexpected circumstances may arise. In testing models with high falls, care should be used in the investigations of the effects of baffle piers, for it is possible to secure vacuums at various points on the piers in the models that correspond to prototype vacuums of more than 34 ft. Under such conditions, the prototype may not perform as satisfactorily as the model will, and cavitations and vibrations may occur in the prototype which do not appear in the model.

ACKNOWLEDGMENTS

In preparing this chapter, the authors have drawn heavily on experimental data collected by the U.S. Bureau of Reclamation. It is expected that a detailed report on these experiments will be issued by them later. Sincere thanks are due for their permission to use these data in advance of their publication. The authors are also indebted to D. C. McConaughy, M. E. Nelson, C. J. Posey, I. A. Winter, and numerous others for valuable assistance, data, and suggestions.

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SECTION 8

SPILLWAY CREST GATES

By James S. Bowman[1] and Joseph R. Bowman[2]

Introduction. However advantageous a fixed-crest free overfall spillway may be from the operating standpoint, the great value of property and improvements in many reservoir sites prohibits the backwater stages resulting above such a dam during flood. At the same time, the desirable storage or head requirements on projects concerned with the conservation of water or control of floods may often approach the limit that can be obtained at the feasible dam sites.

The solution of this problem lies in some form of movable crest, by means of which increases in flood stage above the dam may be materially lessened within the limits of backwater effect, without seriously curtailing the low water storage or head normally available.

Many types of movable crests are in successful operation. However, relatively few of these many types may be suitable or economical for a given situation. The problem of the engineer is to select and design the proper type and size of crest gate that will pass the maximum flood without appreciable damage to the dam and other structures and in which there is proper economic balance between the cost of structures and operation and the costs due to the resulting flood stages upstream.

The degree of success with which this problem is solved depends on knowledge of the local conditions under which the spillway gates must be operated and an appreciation of the features of any type which are fundamentally suitable or unsuitable for these conditions.

Conditions under Which Gates Must Be Operated. Generally speaking, the local physical, hydrological, climatic, and operating conditions must be properly evaluated in selecting the most suitable size and type of spillway gate. Consideration must be given as to whether:

1. The foundation is susceptible to serious erosion or is very resistive
2. Surface turbulence downstream is objectionable
3. The stream carries heavy drift or logs
4. Floods are flashy or rise gradually
5. Floods are frequent throughout the year or are confined to a few occurrences at definite seasons
6. Intense floods occur frequently or only at long intervals
7. The stream is subject to severe ice floes
8. The gates will freeze in during the winter and, if so, whether they may be expected to be frozen or clear at the time of the first spring floods
9. The pool level must be closely regulated
10. Water can be wasted or must be conserved
11. The spillway must be operated and maintained by other personnel or will have its own operating force
12. Some operating force will be available at all times or only occasionally
13. The reservoir must be lowered rapidly to provide flood-control storage

The study of many of these conditions falls under a special subject, such as hydrology, and therefore will not be treated in detail in this section.

From the fact that a spillway usually concentrates the discharge in a much narrower width below the dam than was occupied by flood flow under natural conditions, every consideration should be given to the possible consequences of this change in

[1] Chief Water Control Planning Engineer, Tennessee Valley Authority.
[2] Hydraulic Engineer, Harza Engineering Company.

regimen. This fact argues for as long a spillway as feasible. Particularly where foundations are susceptible to serious erosion, lengthening of the crest, with corresponding reduction in depth of overflow, reduces the energy to be dissipated per unit length. The economics of this problem may be approached by comparing the cost of various spillway lengths with the cost of providing the required stilling pool or other protection downstream. The persistence of surface turbulence for considerable distances downstream is often objectionable from the standpoint of navigation or because of serious bank erosion. Although this is largely a problem of spillway design, the effects may often be minimized by proper consideration of the size and arrangement of the gates.

Heavy runs of ice or drift necessitate long gates from the fact that piers must not offer sufficient obstruction to cause jams. If the ice goes out with a moderate rise, a number of gates of the overflow type or a length of open spillway should be provided. However, on many large reservoirs, the ice melts in place and but little passes the spillway. A study of this condition should be made on adjacent existing reservoirs.

Frequent flashy or intense floods require gates with mechanism that can be readily and conveniently operated at any time with minimum operating labor. When gates must be constantly available for use in cold climates, a type should be selected that will reduce the leakage to a minimum and avoid the dangers of freezing which are inherent in some types. The design should be such that various methods of heating and ice removal can be readily provided.

Where pool levels must be closely maintained or fluctuated within certain limits, it is advisable to provide a number of gates designed particularly for this service as well as for other purposes. These gates should be arranged for close adjustment and may be operated from fixed hoists controlled from the powerhouse or other point if frequent attention is required. However, if the incremental volume of the reservoir is large, compared with the variation in discharge, small inaccuracies in gate adjustment are usually of no consequence. Automatic operation is largely related to this question by way of the attendance required to maintain constant pool levels. However, a considerable amount of attendance and a high degree of maintenance is required for any so-called automatic gate, and the additional cost of this equipment is rarely justified by any actual saving in operation.

It is fortunate that proper provision can usually be made to meet any of these conditions without detriment to others. Great ingenuity is often required, however, to meet an abnormal condition in a small development where the more elaborate provisions possible on a large development cannot be justified either as to investment or cost of operation. The question often arises as to how far provision against extremely improbable circumstances or combinations of circumstances should be carried. Except where loss of life or great property damage might result from possible inadequacy, this question may often be resolved by comparing the additional charges with the possible damage, such as overtopping of masonry structures or the flooding of land, that would occur at an estimated frequency. Such estimates should, however, be used only as a guide in the exercise of the soundest judgment in each case and not as a justification for inadequate works. Frequency studies do not fix the time of occurrence, and the possible consequence of serious damage during the first few years of operation must be considered.

FLASHBOARDS, STOP LOGS, AND NEEDLES

Flashboards,[1] stop logs, and needles are the simplest and probably the oldest types of movable-crest devices. Where the size and type of installation is such that they

[1] In this section, flashboards are distinguished from stop logs, or other devices, as not being supported in grooves at the ends or having a permanent hoisting mechanism attached.

can be readily handled by the operating force available, they are efficient and economical. However, where the installation is large or where frequent freshets require continual manipulation, the operation becomes laborious and hazardous. They may be often adopted with considerable saving in cost for portions of the spillway that will be in use only during the most extreme floods. One example of this use is at the Wyman Dam on the Penobscot River in Maine. New England is replete with ingenious adaptions of both flashboards and stop logs.

Flashboards. *Pin Supported.* The most customary type of flashboard is that supported on edge by vertical pins set in pipe sockets along the crest of the dam. This form is usually limited to a maximum height of about 3 ft, the length of the board being reduced as the height increases, so that the weight of a panel is not excessive for handling. Usually the ends of the boards overlap at each alternate pin, every other panel of single-thickness boards being reversed so that the cleats will bear against the intermediate pins. Double-thickness boards are built with an offset at each end so that adjacent panels will have a single-thickness overlap of several inches. These two arrangements are shown in Fig. 1.

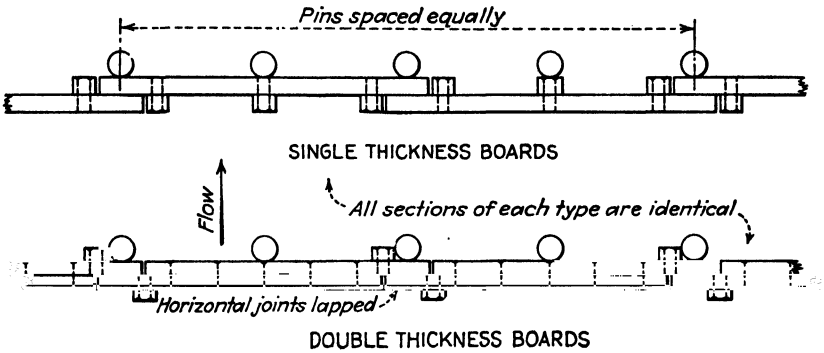


FIG. 1.—Pin-flashboard arrangements.

Pins may be made from either steel rods or pipes designed so that the elastic limit will be exceeded and the pin will bend over when the board is overtopped to a predetermined height. Some pins are made with a circumferential groove immediately above the top of the socket and tempered at this point to give a high elastic limit so that they will break off rather than bend and thus leave a more unobstructed crest. The size and spacing of pins is determined by height and thickness of the boards. The interrelation of these factors will determine the most economical layout.

Braced. When a flashboard is so high that pin supports alone are not feasible, or when a ready means of tripping is desired, the pins or stanchion member may be supported by a knee brace extending down to the crest of the dam. It may be arranged so the whole support, or horse, will overturn when the headwater reaches a predetermined level, or the braces may be tripped by a cable extending the length of the dam and passing through holes near the foot of each brace. The cable can be pulled at an angle from the bank downstream, or with buttons attached, it can be pulled lengthwise to trip the braces individually.

In designing braced boards, care should be taken that the resultant of all forces does not fall above the foot of the brace. High boards will usually require that the foot of the stanchion be anchored to avoid overturning. Stanchion timbers which are notched to receive the brace or for anchorage at the foot should be reinforced on account of the uncertainty of shearing resistance along the grain in carrying forces due to the impact of floating material. It is rather essential to set an accurately

located steel angle along the crest of the dam for supporting the foot of the braces. This allows a standard brace to be used without special cutting and lowers the frictional resistance which must be overcome in tripping.

Hinged. Hinged flashboards have the advantages that they are not lost when tripped, and, with the proper facilities, they can be raised against considerable head.

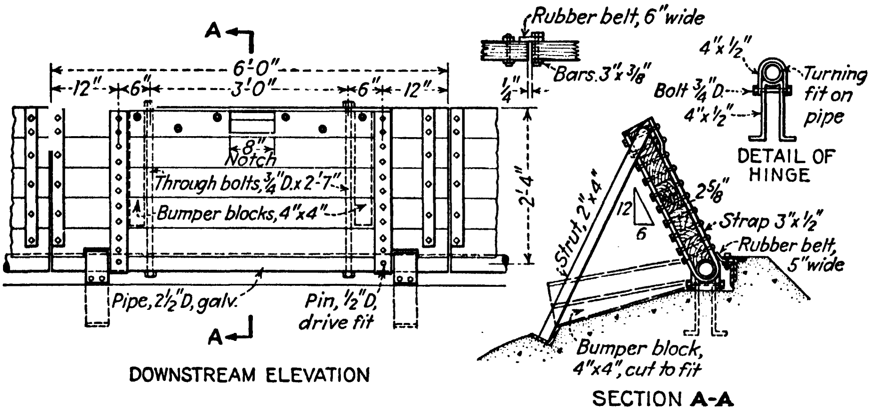


FIG. 2.—Two-foot hinged flashboards. (*Designed by Fargo Engineering Company.*)

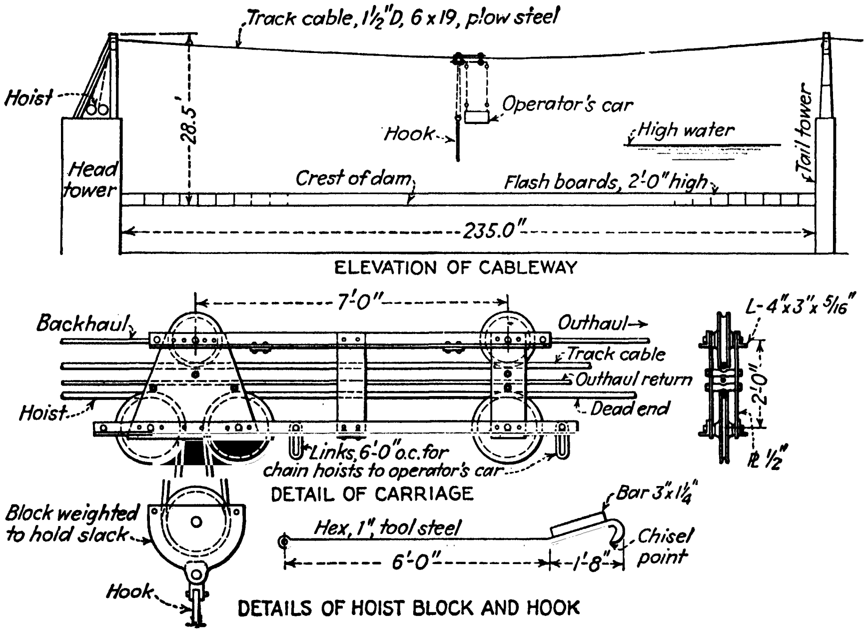


FIG. 3.—Cableway for operating hinged flashboards.

They may be built simply or as elaborately as the head or prevention of leakage may require. Figure 2 shows a board of this type, which may be considerably varied in design to meet local conditions. One end of each panel should be left rough and with a slight overlap when constructed, so it can be sawed to match the finished end of the adjacent panel at the time of erection.

An overhead cableway similar to that shown in Fig. 3 has been frequently used for operating flashboards, especially those of the hinged type, where a bridge is not available. This requires a track cable for the carriage, from which is suspended the lifting hook and operator's car; a travel cable; and a hoisting cable. A standard two-drum hoist is required for the travel and hoisting lines. Braces are handled with a pike pole or special tongs. Chain hoists of ample capacity must be provided to keep the operator's car above the water as the hoisting load varies.

Flashboards have the advantage of providing an unobstructed crest when lowered. Where possible, a regulating gate, of sufficient capacity to pass the normal flow, should be provided so that the headwater may be temporarily lowered to allow the flashboards to be set up. This may conserve water and head for a considerable period after the recession of high water, during which it might otherwise be impossible to restore the flashboards. Such a gate may also be utilized for passing small rises which would otherwise trip the boards. Automatic flashboards have been a fertile field for invention. However, the wear and corrosion of moving parts with the accompanying change in frictional resistance, fouling with trash, and similar troubles have in time spelled the failure of many of these schemes that were alluring on paper.

Stop Logs. The customary stop logs are dimension timbers spanning horizontally between vertical grooves in adjacent piers. They are built up one on another, a vertical bulkhead being formed from the crest of the spillway to the headwater level. They may vary in size from short lengths, which can be handled by one man, to sizes limited only by the span and the capacity of a power winch to raise them. A means of handling is provided by cutting a longitudinal slot vertically through the timber near each end. A bolt is then passed transversely through this slot on the horizontal center line of the timber. A pike pole having a special hook with a line attached is lowered down the groove in the pier until the hook enters the slot and engages the bolt, after which the line may be hoisted by hand or by a power winch.

Since the logs must be handled through overflowing water, it is imperative that the grooves in the piers be made amply deep to protect the hoisting device from the current during the fishing operation. This depth is usually deeper than the allowable bearing stress of the timber requires and, for any considerable depth of overflow. should be not less than 12 to 16 in. The groove should not be wide enough to permit the timber to turn sufficiently to bind. The outer downstream corner of the groove should be protected by a continuous steel angle to provide ample bearing area, to give greater watertightness, and to minimize frictional resistance in moving the timber.

In proportioning the timbers for bending moment, allowance should be made for deterioration in strength during their normal life. This will vary widely, depending on the resinous nature of the timber. Where large timbers are not readily available, a hollow steel cylinder of welded construction may be more economical and also lighter. A short section at each end must be squared up by plate and angle construction to provide a proper bearing and sealing surface.

Stanchion Type. Where a portion of the spillway is used infrequently, during excessive floods, the stanchion type of stop log is economical and is easily operated. As shown in Fig. 4, the opening between piers is divided into several short bays by vertical beams which are hinged to the runway at the top and seated in the spillway crest at the bottom. These bays are closed by small stop logs. By jacking up the beams slightly, they are unseated at the bottom and swing free; this action releases the stop logs.

Stop logs may prove an economical substitute for more elaborate gates where relatively close spacing of piers is not objectionable and where variations in flow require the removal of only a few logs, except at infrequent intervals. They are adaptable to

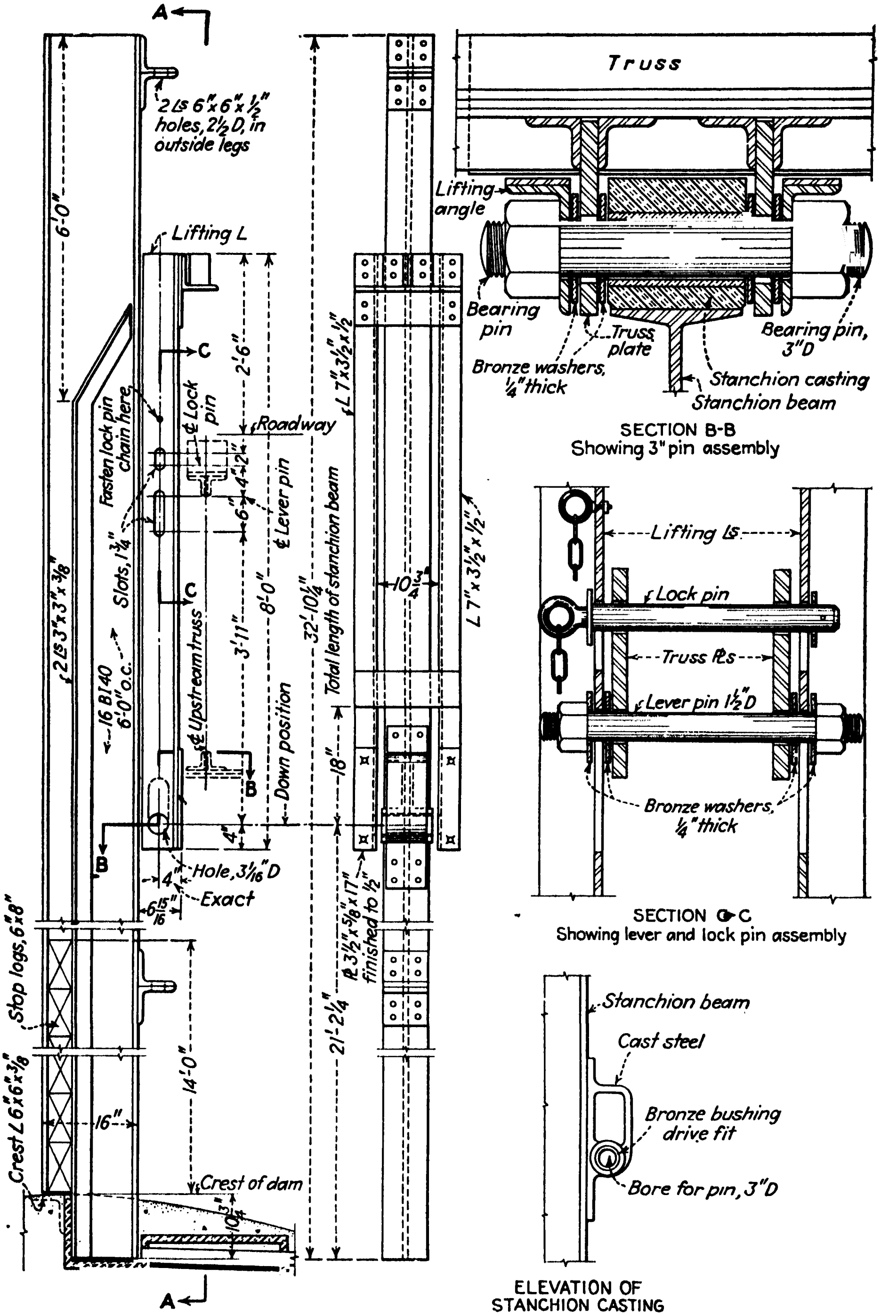


FIG. 4.—Stanchion sections, Highfalls Dam. (*Designed by Harza Engineering Company for Algoma District Power Company, Ltd.*)

deeper openings than are flashboards, and where a bridge is available, shallow stop logs are often substituted for flashboards on account of the greater facility of operation.

Needles. Needles are set on end side by side to close an opening. They are supported at the top by a runway, from which they are handled, and are supported at the bottom by a ledge on the sill or spillway crest. They are usually made of dimension timbers. The most outstanding example of their use is at the Bonnet Carre spillway on the Mississippi above New Orleans.

Needles are somewhat difficult to place in swift water of considerable depth. Hence, their use is largely confined to emergency spillways where they are seldom raised and where they can readily be replaced after flood waters have receded. For emergency bulkhead construction in still water, needles are preferable to stop logs for the reason that they sink readily and can be staunched as placed. On the other hand, it is difficult to hold stop logs down and force them into proper sealing contact without differential head. After the entire bulkhead has been placed and a differential head established, the frictional resistance may become so great that the logs cannot be forced into position to close the leaks between them.

TAINTER GATES

The conventional form of Tainter gate consists of a skin plate formed to a segment of a cylinder, the vertical elements being circular arcs, which is supported by a framework of horizontal or vertical purlins and stiffeners. The purlins in turn may be supported by vertical or longitudinal girders from which two or more radial struts converge downstream to horizontal shafts or pins that are anchored in the piers and carry the entire thrust of the water load. The skin plate is made concentric to this pin, and hence the resultant of the water pressure passes through the pin, creating no moment to be overcome in hoisting the gate. This is the fundamental principle of the gate. The hoisting load consists solely of the weight of the gate, the friction between the side seals and the piers, and the small force necessary to overcome the moment of the frictional resistance at the pins. Figure 5 shows an example of this type of gate.

In the north woods where this gate originated, the early examples were constructed entirely of timber. Modern construction is almost universally of steel, although creosoted timber has been used for the face in recent instances. There are several variations of the conventional design which have been used to a limited extent. In one of these, the pin is placed upstream and connected to the gate by tension members which can be lighter than required for compression struts. However, very little weight is saved by this arrangement, for the struts are a small part of the weight of the gate. This method is open to the objections that the piers must be usually extended upstream for pin anchorage, trash and ice accumulate in the tension arms, the gate is not held rigidly and is apt to jam, and a vertical water load accumulates on the horizontal members since the skin plate of necessity must be on the downstream side. Another variation is to make the curvature of the skin plate concentric with some point above the pin so that the moment of the resultant water pressure assists in hoisting the weight of the gate. Since this device has little effect when the gate is partly raised and complicates the design and fabrication, it has been little used. In cases where flood stages are so high that a gate rotating about a fixed pin would be submerged, resort to a movable pin has permitted the use of a Tainter gate. In this device, the pin is anchored to a block which can be hoisted in a vertical slot in the pier face. Formerly, many of these gates were designed with several main girders or trusses set vertically and supported by radial struts extending to a shaft resting in pillow blocks on a heavy girder extending from pier to pier. This practice leads to a multiplicity of structural details and is now rather obsolete.

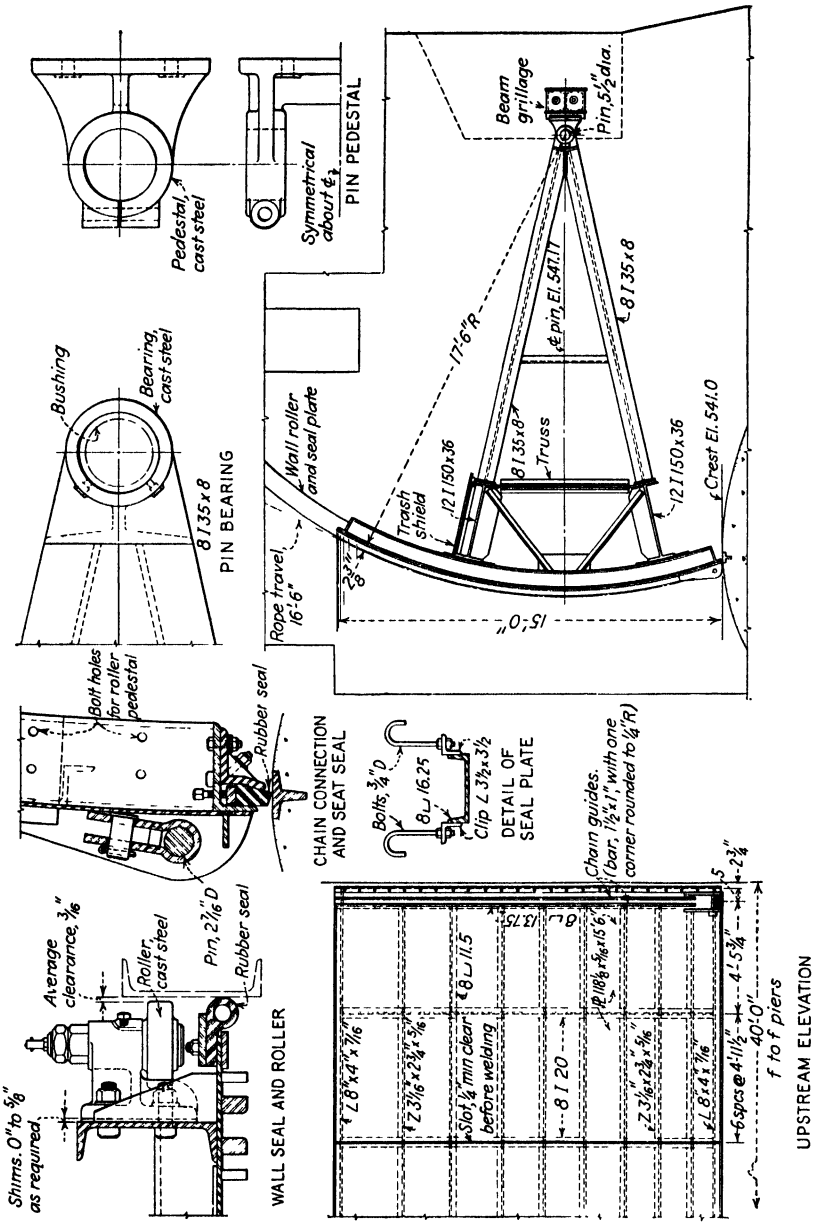


FIG. 5.—Tainter gates, Wheeler Dam, T.V.A. (*Designed by U. S. Bureau of Reclamation.*)

The general layout of a Tainter gate is a matter of trial and judgment since there is no fixed relation between the length of the arms and the height of the gate. The pins should be above ordinary high water, and the bottom of the gate when raised should clear floating material carried by the extreme flood. The shorter the radius of the face or the greater the inclination of the chord from the vertical, the heavier the gate will be. The longer the radius and the higher the pin, the higher the elevation required for the runway. Except when necessary to provide flood clearance, the pin is normally located within the elevation of the lower half of the gate.

The proper arrangement of purlins and stiffeners and thickness of skin plate, where most of the weight is involved, can be determined only by comparative studies of cost. To allow for corrosion, the skin plate should have a minimum thickness of 1/4 or 5/16 in. Although there is a slight arch action due to curvature, this plate is normally designed as a flat plate fixed on four sides at the purlins and stiffeners. Buckle plates have sometimes been used for the face on large gates to reduce the weight, as the metal may be figured in tension. The purlins may be of equal strength with variable spacing, according to the water load, or they may be spaced about equally with variable strength. If the stiffeners can be spaced to form approximately square panels between the purlins near mid-height of the gate, a minimum weight of skin plate will usually be obtained. For gates up to 25 ft long by 12 ft high, channels may be used throughout for purlins and the arms riveted directly to these; for large gates, I beams will normally be used for purlins with a vertical girder interposed between them and the arms. Since most joints in the face are subject only to shear or compression, welded construction can be readily and economically used.

The arms should be far enough from the piers not to become encased in ice formed on the piers by water leaking past the seals or splashing over the top. On small gates, the plane of the arms may be swung inward from the pins to intersect the face approximately at the outer fifth-points. This arrangement effects considerable reduction in the weight of the horizontal members by converting them into one fixed and two cantilever beams of equal bending moment and at the same time removes the arms from ice danger. Where the arms are attached to vertical girders for supporting the purlins, the number of arms will be determined by the most economical construction, both arms and girder being considered. In smaller gates, the arms are riveted to a pin plate near their common intersection. This plate is built up as required for bearing, and after shop erection the pinholes are located at the true center of curvature of the face and the plates are bored to receive a bronze bushing which is press fitted and keyed. On large gates where a pin plate does not afford sufficient bearing, a steel bearing block is made with an integral sector to which the arms may be riveted. After the gate is bolted up in the field, the bearing blocks are centered on the pins and babbitt bearings poured in place. Means for grease lubrication should be provided for the bearings to reduce frictional resistance and to inhibit corrosion.

The pins are made from cold-rolled shafting, finished at the ends to receive the bearings. They customarily are extended through each pier so that one shaft carries the adjacent ends of two gates. An anchorage to carry half the water load of each gate must be provided as near each face of the pier as possible to relieve the concrete of crushing stresses and to reduce the cantilever moment on the pin. The shaft may either be cast in the concrete when the pier is poured or be provided with an adjustable anchorage in a cutout which is grouted after the gate is erected.

Rubber side seals have long been used on Tainter gates, the most usual type being of rubber belting 6 to 8 in. wide. One edge is attached to the end of the gate around the upstream face of the skin plate, and the other edge is bent back so that a considerable portion of the width is in contact with the pier face, or it may be inserted in a concentric groove, in the pier, the downstream face of which is flush with the skin

plate. In either case, the sealing surface at the pier should consist of an accurately formed steel shape against which the belting can lie. Although this gives slightly higher frictional resistance than rubber on concrete (1),[1] it affords a tighter seal and prevents surface spalling from freezing at this point where the concrete is saturated. Probably the most satisfactory seal is the music-note seal originated by the Bureau of Reclamation. This in contact with the back of a curved channel embedded in the pier is probably the tightest and most permanent of any. The skin plate should extend to within about 1 in. of the pier, but it is often advisable to obtain this close clearance with an extension plate concentric with the skin plate and attached to its upstream face by means of bolts in slotted holes for close adjustment. The rubber seal is clamped to the gate by a round-edged retaining bar with brass bolts, closely spaced to assure a watertight fit. The lower end of the seal should be trimmed accurately in place to fit the spillway crest.

Timbers set in the spillway crest or bolted to the bottom of the gate to receive a knife-edge attached to the other have been used, but these are difficult to fit and are subject to decay and damage by drift. A preferable scheme is to use a heavy steel channel with the back downward for the bottom of the gate. A recess is left along the spillway crest, and after the gate is erected this is filled with stiff grout, the gate lowered to a restrained bearing on it, and the grout allowed to set in contact. Another successful device is to attach a music-note seal to the lower edge of the upstream face and projecting slightly below it. This seals under compression on a steel member embedded in the spillway crest.

Tainter gates are hoisted by two chains or cables attached to the upstream face near the bottom by a shackle and extending to the drum of a hoist overhead. If a movable hoist is used, provisions for dogging the chain or cable are made on the runway deck.

Where the span requirements are not too great, and where objectionable submergence by high water does not occur, Tainter gates are probably the simplest and least expensive of any type of gate for moderate heights. Spillway gates of this type as large as 64 ft long by 30 ft high (2) and 50 ft long by 37 ft high (3) have been installed on high dams in the United States; Tainter gates nearly 79 ft long and 36 ft high are in service in Spain (4), and designs are under way for gates about 41 ft square for a dam in Central America. Submersible Tainter gates up to 80 ft long and 25 ft high are installed at upper Mississippi River navigation projects. Gates 35 ft long and 15 ft high are not exceptional, and gates 25 ft long and 12 ft high are in common use. They can be maintained reasonably tight and are widely used under severe ice conditions. They are not suited to the passage of floating material unless fully open, which may involve waste of water.

HINGED-LEAF GATES

This type of gate is simply a rigid flat leaf hinged at bearings along its lower (upstream) edge. In its raised position, the leaf slopes upward and downstream at an angle of between 20 and 30 deg from the vertical. When lowered, it lies approximately in a horizontal position. In Europe where several forms of this gate are built, it is known as the *shutter weir*. The skin plate is supported on purlins framing into the bearings at the upstream end and into a heavy longitudinal girder, extending from pier to pier, at the downstream end. The position of the leaf is controlled by hoisting attachments near the ends of the main downstream girder. The hoisting force is provided by a conventional mechanical hoist or by a counterweight device so that the leaf rises or falls automatically with slight incremental changes in headwater level, to pass the required discharge.

Figure 6 shows a mechanically hoisted gate of this type installed at the High Falls Dam on the Michipicoten River in Ontario. It is used for log sluicing and also releases

[1] Figures in parentheses, thus (1), correspond with numbered items in the Bibliography at end of this section.

sufficient water to float the logs down over the falls. The ends of this gate are recessed into the piers to prevent damage to the end seals and suspender bars. At the bottom of the gate, an accurately formed segment of circular plate is attached to the ends of the purlins, concentric with the bearing pins. Accurately fitted steel plates clamp a strip of braided jute against this curved surface. The ends are fully sealed, in the raised position only, by compressible rubber seals. The gate is simple and rugged in construction throughout and is well adapted to the purpose for which it is used.

Figure 7 shows different arrangements of an automatic counterweighted gate built under patents of Stauwerke A. G., of Zurich, Switzerland. With gates of this type,

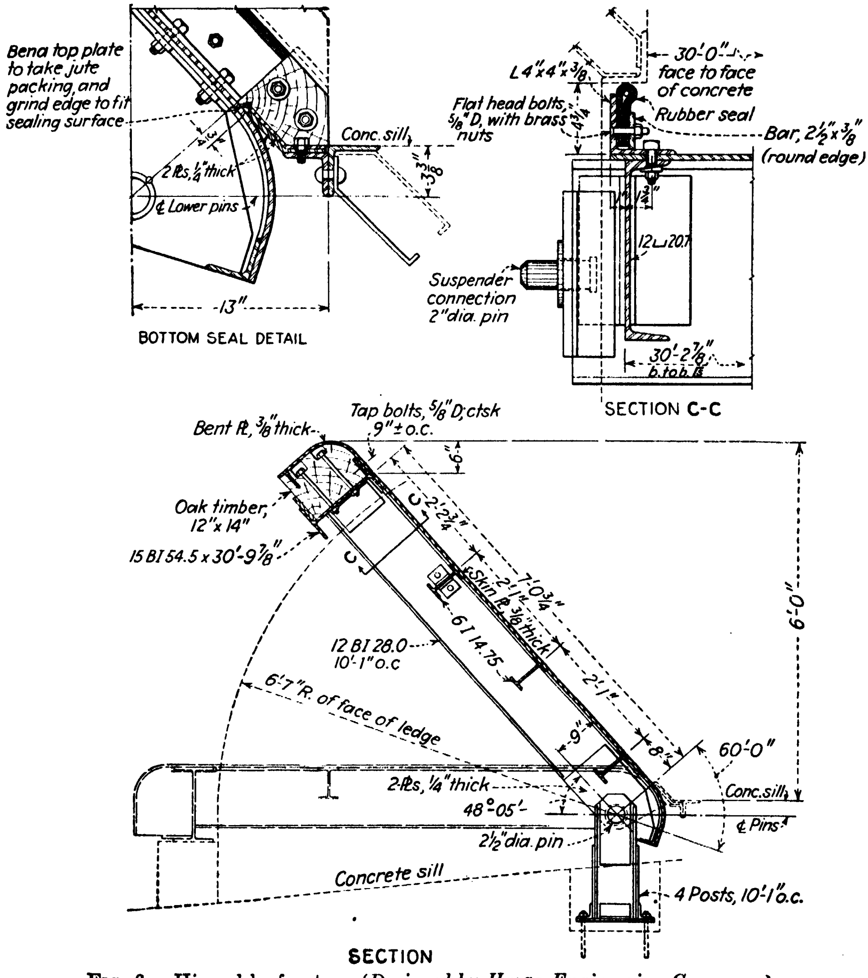


FIG. 6.—Hinged-leaf gate. (*Designed by Harza Engineering Company.*)

the moment of the water pressure increases as the leaf is lowered. This is compensated for, however, by mounting the fulcrum of the walking beam on a segment with heavy gear teeth which advance on a rack as the gate falls, the lever arm of the counterweight being increased so that equilibrium is reached with a small movement of the gate. This simple deadbeat device can be very perfectly designed. Small incremental increases of pool level are required to actuate each downward movement of the

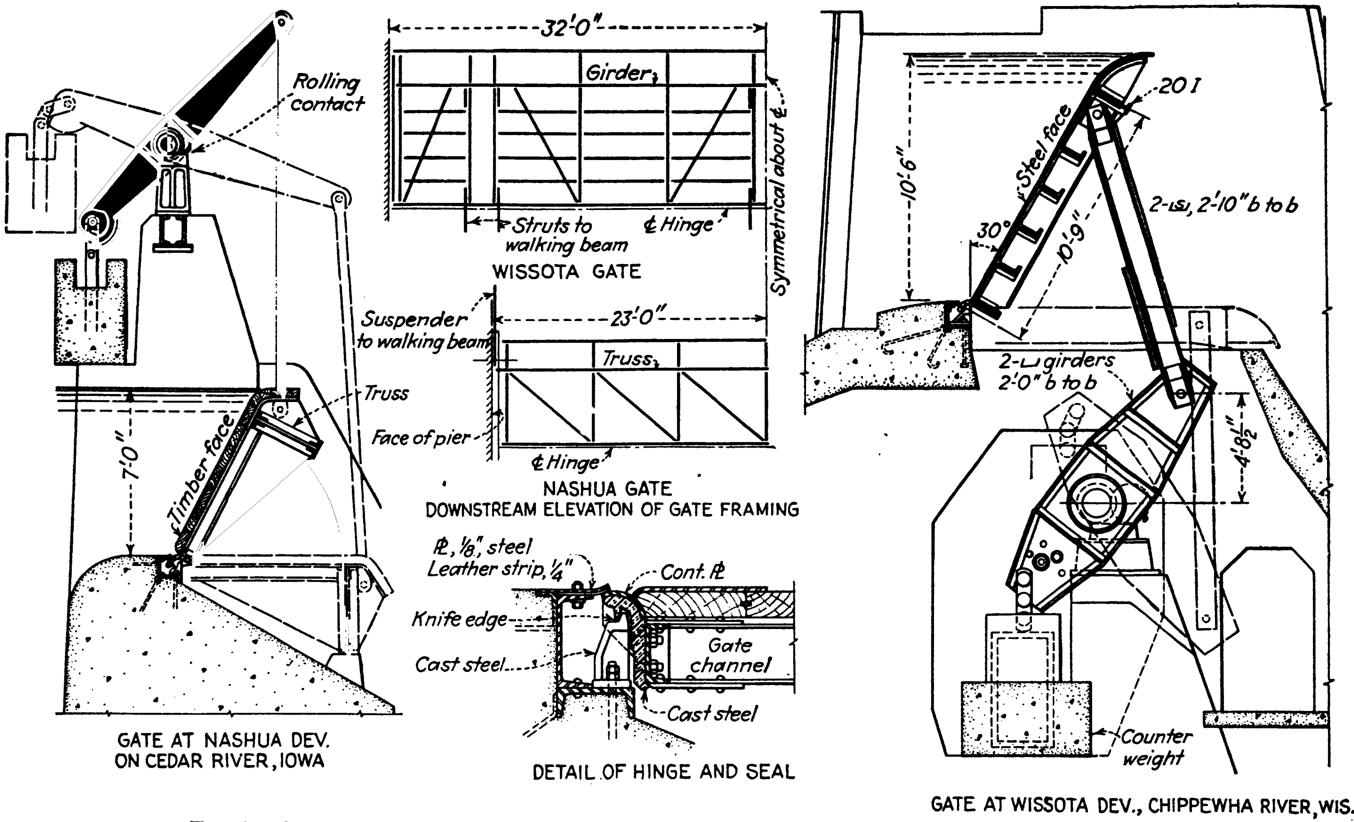


Fig. 7.—Stauwerke automatic counterweighted gates. (*Fargo Engineering Company, Engineers.*)

gate after equilibrium is established, and corresponding decreases are required to raise the gate as the discharge becomes less. The downstream edge of the gate is supported by a heavy girder to which the suspenders are attached, and at the upstream edge the purlins frame into a member to which the hinge is attached. The hinges consist of tool-steel knife-edges, on the underside of a segmental casting attached to the gate at each purlin, which rock in a bearing supported on the spillway masonry. The bottom seal is formed by a strip of leather attached to a flexible plate and bearing on a continuous circular plate which is concentric with the knife-edges and is bolted to the hinge castings. The side seals consist of flexible-steel plates about 6 in. wide extending upstream along the pier from the end of the gate with a strip of bronze, wood, or leather attached to the outer edge and making watertight contact with the pier face. The face may be of timber or plate steel.

Two of these gates 46 ft long and 7 ft high, of the overhung type, were installed on the Cedar River at Nashua, Iowa, and 13 gates 64 ft long by 10.5 ft high, of the underhung type, were installed on the Chippewa River near Eau Claire, Wis.

For automatic operation, the knife-edge bearings and fulcrum must be very carefully and accurately constructed. Corrosion has serious effect on the sensitivity of these parts, so great in some instances that several times the calculated variation in pool level has been required to move the gates after some length of service.

Many variations of the automatic-leaf gate have been installed. At Tallulah Falls in Georgia, a cylindrical counterweight rolls upward on a curved track as the gate is depressed. For by-passing water with normal pool level, a hoist must be provided to raise the counterweight on this type of gate. If tail water rises so high during extreme flood as to submerge the leaf, provision must be made for blocking the counterweight in position to prevent the leaf from rising, owing to loss of weight. The downstream side of the leaf must be effectively aerated to prevent decrease in atmospheric pressure under the overflowing sheet. In locations subject to low temperatures, the hinge design should provide against the knife-edges being forced off their bearings, should the gate start to lower with an accumulation of ice beneath.

The *bascule gate* (5), manufactured by the S. Morgan Smith Company exclusively, is essentially a hinged flashboard. It offers the advantages of automatic headwater regulation and a long unobstructed spillway for passage of ice or floods. The gate consists of an all-welded gate leaf attached to a torsion cylinder which extends beyond the full gate length into gate-operating chambers in the piers. The cylinder is supported by intermediate bronze-lined bearings along the crest and by end bearings located in the piers. Molded rubber seals are mounted on plates attached to each end of the gate leaf, making wiping contact with steel plates embedded in the piers. The longitudinal seals consist of rubber sealing strips attached to the fixed metal in the dam crest, making a wiping contact with the torsion cylinder. Hydraulic (oil) cylinders located in the operating chambers turn the torsion cylinder through an angle of about 90 deg to raise or lower the gate. The first gate of this type was installed in 1939 (6), and approximately half a dozen have been installed since that time. A bascule gate 100 ft long and 6 ft high is in service at the Wood Falls hydro development in Vermont.

Hinged-leaf gates are admirably suited for regulating gates and for passing heavy drift, as the length to which they can be built is limited only by the stiffness of the downstream girder and the capacity of the hoisting facilities. They may be fully lowered onto a substructure consisting merely of a sill and slab, where many other surface spilling types require a recess of considerable depth. The side seals are somewhat of a problem, for these must move transversely across the face of the pier and it is difficult to obtain the required flexibility to give watertightness and stiffness to resist buckling as the gate is raised. This difficulty may be overcome to some extent

by facing sealing surfaces on the piers with very smooth plate. All the seals are difficult of access for maintenance, and facilities for stop logs or needles should be provided above the gate.

VERTICAL-LIFT GATES

The designation vertical-lift gates is here used to include all rectangular gates supported by vertical guides in which the gates move vertically in their own plane. The hoist is usually mounted on a runway overhead, and the gate is either raised or lowered, depending on the particular design, from its normally closed position by means of cables or stems. The gate proper consists of a framework to which a skin plate is attached, normally on the upstream face. This presents no unusual difficulties in design, the principal problem being the determination of the arrangement of beams and girders and skin-plate thickness which will result in the most economical construction. However, such features as seals, lifting mechanisms, dogging devices, rollers, guides, and similar appurtenances require meticulous care in design and warrant a careful study of the operating behavior of such features under similar conditions on existing projects.

Sliding Gates. In this type, the frame of the gate bears directly on the downstream guide member, the seal being formed by contact between the two. The coefficient of friction in sliding may vary from 0.5 to 0.9, which requires large hoist capacity not only for raising but for lowering as well, for only in the smaller sizes will the weight of the gate exceed the frictional resistance when the gate is near the position of maximum water loading.

This fact rather definitely limits the size of this type to such dimensions that the additional cost for wheels or rollers to lessen the frictional resistance is greater than the additional hoist capacity. As a practical matter, this limits the use to small spillways, wasteways, log flume inlets and similar purposes, and in these uses it is particularly adapted to being lowered for passing the discharge over the top.

Sliding gates may be built of either timber or steel, depending on requirements of strength and permanence. Timbers used for the water face of gates should have grooved edges into which loose-fitting splines are inserted. The timbers are not fitted closely together, to allow for swelling; otherwise the gate will warp after submergence and the sealing contact at the edges will be destroyed. On steel gates which normally remain closed, either the guide or the sealing strip on the gate should be of bronze to avoid rusting shut, which will occur in a steel-to-steel contact.

Fixed-wheel Gates. This type differs from the sliding gate in having a series of fixed wheels mounted along each end to carry the water load to a vertical track on the downstream side of the gate groove. The wheels substitute rolling friction for sliding friction, which allows the gate to be self-closing under its own weight. The wheel shafts are located between the main horizontal girders and are either supported by two vertical members of the frame or in pedestal bearings bolted to the frame. At Bonneville, this type was used for gates 50 ft wide by 50 ft high. The gates at several of the Tennessee Valley Authority projects are 40 ft wide by 40 ft high.

In heights of 20 to 50 ft, for which this type of gate is adapted, considerable difficulty is experienced in providing for surface passage of trash and for the great headroom required for hoisting the gate above the water surface. Two general methods have been used for overcoming these difficulties:

1. To split the gate horizontally into two leaves of approximately equal height. These two leaves travel in the same guides and the upper sets directly on top of the lower. The upper section may be further divided to provide a suitable height for surface spilling or regulation. The upper sections are raised and removed from the guides, and the lower section is grappled and raised to the required position. This method also considerably reduces the hoisting load.

2. To divide one gate horizontally but arrange it so that the upper section can be depressed alongside of the lower section. The two can then be raised together in this relative position.

At the Safe Harbor development on the Susquehanna River, the spillway gates are Stoney gates 48 ft long by 33.5 ft high. However, four of these gates are split into two leaves for regulating purposes (7). The top leaf is the fixed-wheel type 15.5 ft high, the wheels rolling on a track downstream from the roller path of the bottom leaf, and can be raised or lowered independently on the bottom leaf, and when lowered the tops of the two are at the same elevation.

In Europe, the Maschinenfabrik Augsburg-Nurnberg (MAN) has designed and built a large number of double-leaf gates, some as large as 82 ft long by 39 ft high (8). The design seems to impose a limitation of the height of the upper leaf in these large sizes to between one-third and one-fourth of the total height of the gate. By ingenious design, both sections travel on the same track, and a single hoist may be used. An interesting feature is the mounting of the wheels in a truck frame which carries the thrust of the gate through a rocker bearing.

Figure 8 shows the fixed-wheel spillway gate at Dix Dam. The size of this gate (35 ft long by 10 ft high) is such that no unusual conditions are imposed on the design. The wheels are cast steel with bronze bushings. The side seals are spring bronze with a rubber sealing strip bearing on the upstream guide angle. The gate is of welded construction except at the principal connections. Pockets were provided in the piers for the track and guide members which were later accurately erected on anchor bolts and grouted in.

One of the spillway gates at Pickwick Landing Dam on the Tennessee River is shown in Fig. 9. This gate is 40 ft long by 40 ft high and is divided horizontally into two equal sections. The load on the lower section is so great as to require heat-treatment of the wheel treads and rails to provide safe bearing strength. The wheels are rolled high-carbon steel. Double-tapered roller bearings are used on the lower section, but pressures on the upper section are sufficiently low to permit self-closing with bronze bearings. Provision is made for grease packing all bearings from the downstream side of the gate.

The gate is hoisted by a horizontal lifting beam, the ends of which travel in the gate guides. Automatic grappling hooks engage lifting hooks on the upper corners of each section. A follower suspended from the lifting beam is used for grappling the lower section through the water. By providing retractable dogs at suitable points along the slot to engage the wheels on the gate, either section can be held in intermediate positions.

All seals are of the rubber music-note type, installed so that both compression and water pressure are effective in making them watertight. The principal embedded parts for attaching the rail and guide members were accurately supported in place on a heavy steel tower which was embedded in the pier. Details of the wheels and seals are shown in Fig. 10.

Stoney Gates. The fundamental difference between Stoney and fixed-wheel gates is that in the former a moving train of rollers is substituted for the fixed wheels in the latter. The roller train, composed of horizontal rollers held in position by shafts bolted into continuous vertical bars on each side, is attached to neither the gate nor the guide, but rolls vertically between the two as the gate is moved. As the rollers transmit the entire load from a bearing strip on the gate to a roller path on the guide, there is no axle friction, and only rolling friction is developed. Since the gate moves on the roller diameter while the roller revolves on its radius, the roller moves only one-half the distance of the gate movement and the bottom of the roller train always lags the gate by one-half the distance the gate is opened.

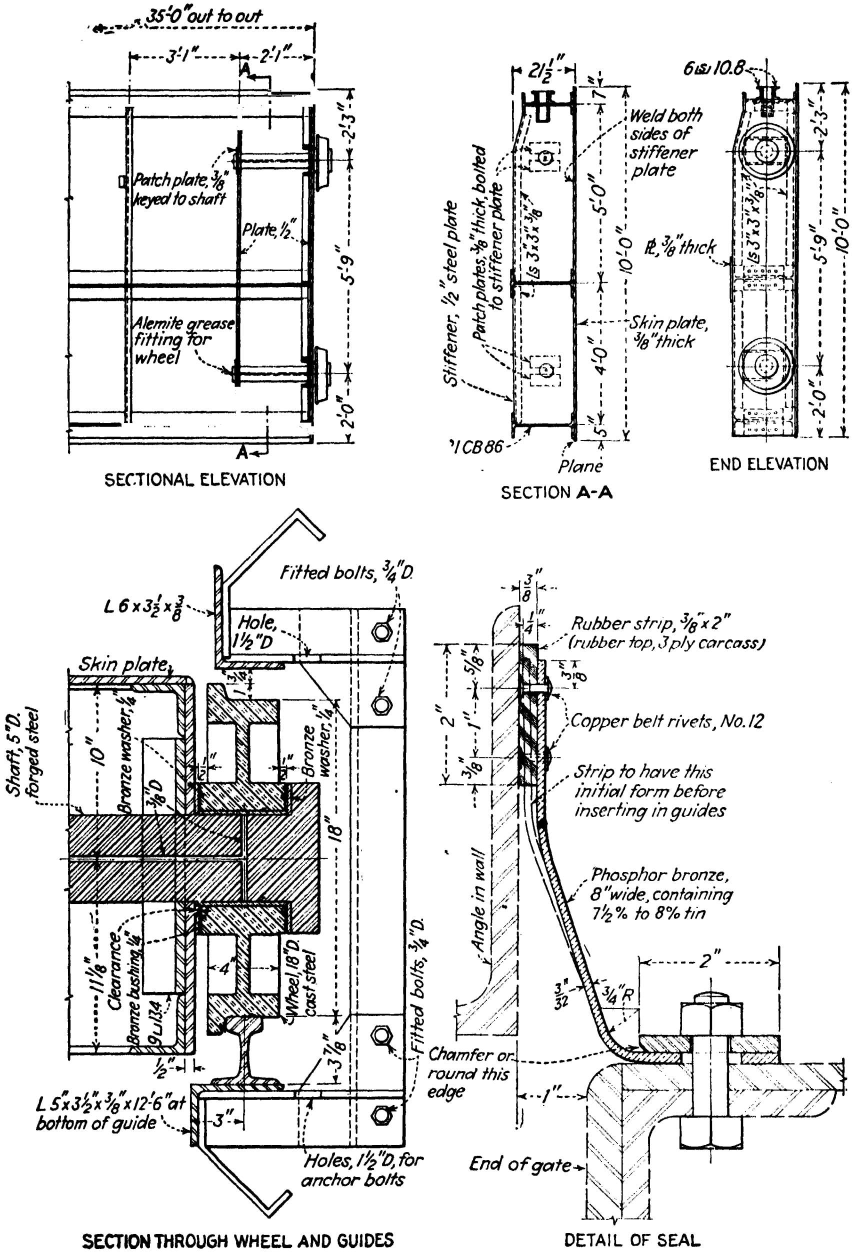
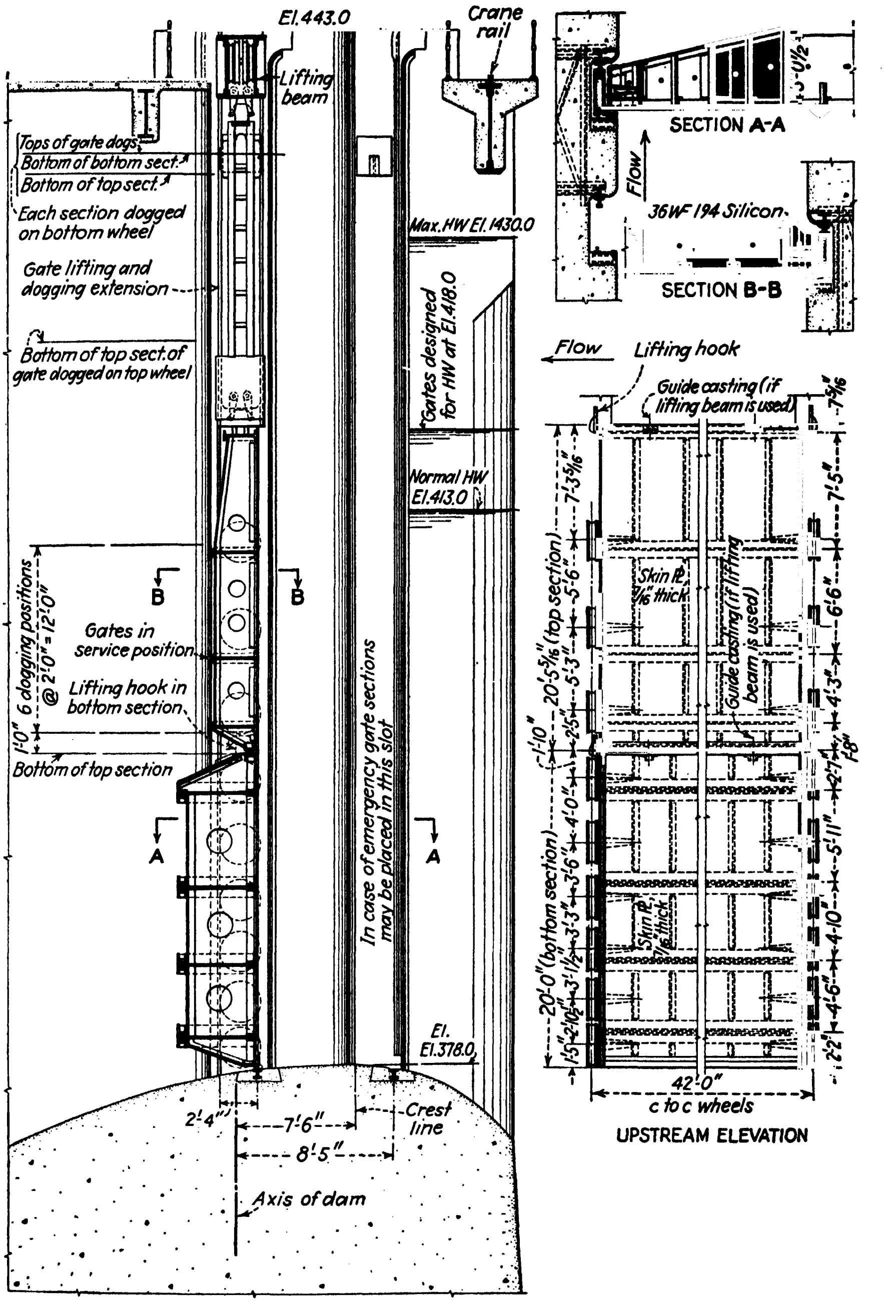


Fig. 8.—Crest gates, Dix Dam spillway. (*Designed by Harza Engineering Company for Kentucky Utilities Company.*)



SECTIONAL ELEVATION

FIG. 9.—Spillway gates, Pickwick Landing Dam, T.V.A.

Figure 11 shows details of the end of a Stoney crest gate and the roller train. The gate leaf is constructed much the same as any other vertical-lift single-leaf gate.

The side bars of the roller train are made of steel angles with the short legs turned inward to fit recesses in the sides of the roller path. This detail is essential to afford lateral support for the train against high velocities. The roller shaft is made up of a steel through bolt on which is fitted a bronze sleeve which forms a spacer between the

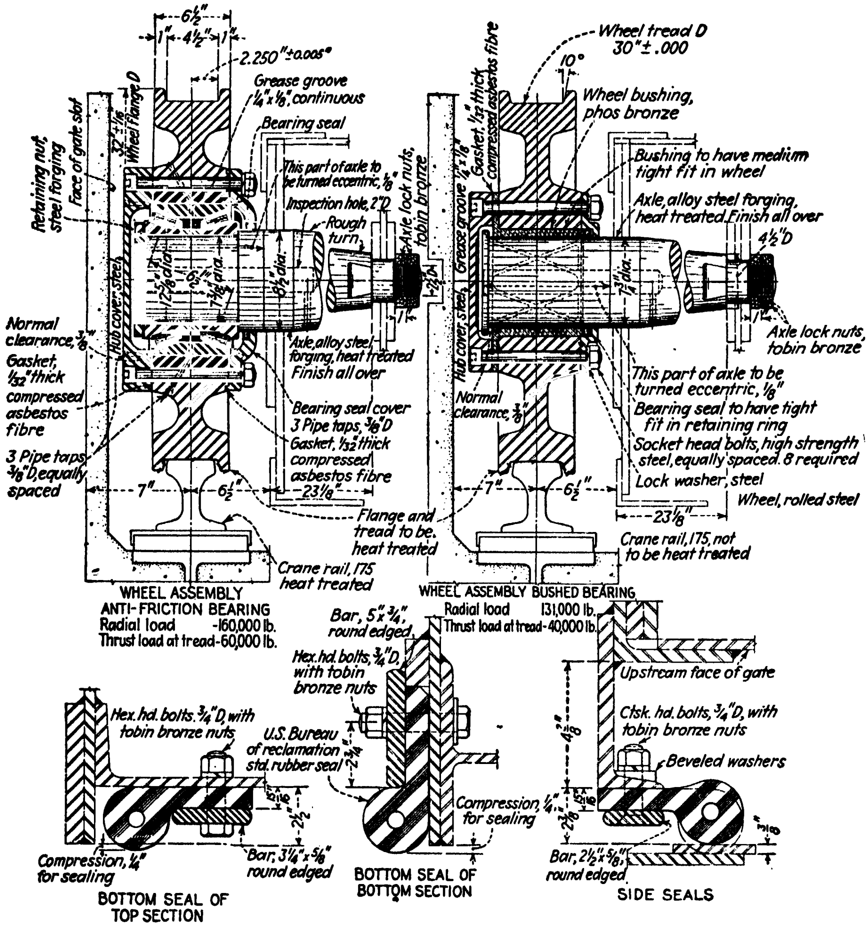


FIG. 10.—Wheels and seals for spillway gates. Pickwick Landing Dam.

side bars as well as providing a noncorrosive bearing for the roller. This construction allows any individual roller to be removed without dissembling the entire train. Thin bronze washers should be inserted between the ends of the rollers and the side bars. The rollers are made of cold-rolled steel with finished faces and are spaced to carry equal load in the lowest position.

The roller paths are made from rolled or forged steel, as required for bearing and strength, planed to remove local unevenness, and set to as true alignment as possible. The downstream path may be built up of a wide section on the surface with a narrower section beneath to provide a recess for the angle leg on the roller-train bar. On

account of the deflection of the gate, the upstream path must have a slight rocking movement to assure uniform bearing on the rollers. This is provided by allowing the web of the end girders of the gate to protrude beyond the flanges of the girders. This arrangement is accurately planed to match a groove in the back of the path. The path is fastened to the girder angles with shouldered bolts having sufficient play to permit a slight rocking effect. The downstream path should stop somewhat short of the spillway crest in order that debris in the corner will be sluiced out.

The side seals are customarily flexible round bronze rods held loosely in place at the edge of the skin plate by retainers in such manner that the water pressure will force them over the space between the skin plate and the upstream guide angle. Rubber-belting seals have also been used successfully for this purpose but are subject to deterioration. The bottom seal can be of any of the types suited to vertical-lift gates.

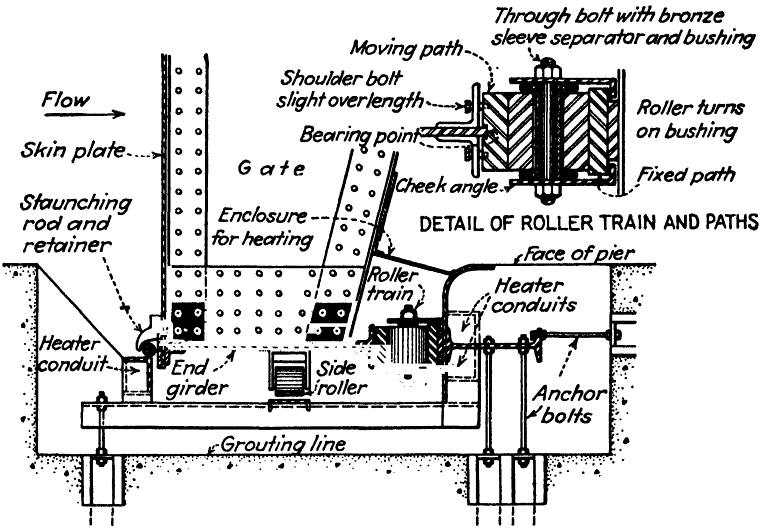


FIG. 11.—End section of Stoney gate. (*From F. Newell, Dominion Bridge Company, Ltd.*)

A roller on each side of the gate as near as possible to the bottom is required to prevent excessive lateral movement, on account of the roller path and seals. This roller should be adjustable so that it can be set to the minimum feasible clearance with its path on the inner face of the gate groove. To prevent the roller train from slipping downward with continual operation, a sheave is usually mounted at the top of the train. A two-part line is passed through this sheave, one end being fixed to the runway and the other end to the top of the gate, so that the exertion of a pull equal to the slippage maintains the train in its normal position. At the Calderwood (9) development of the Aluminum Company of America on the Little Tennessee River, a pinion mounted at the top of the roller train and engaging a rack on the gate performs the same function.

On account of the low coefficient for rolling friction, the Stoney gate has been used extensively in the past for large self-closing gates. It still has some advantage in this respect, although the development of low-friction roller bearings has made it possible to substitute the more simple fixed-wheel gate in many cases. It is difficult to split a Stoney gate to save headroom or for surface regulation, and the rolling and drum gates have largely replaced it for lengths exceeding 50 ft. The Stoney gate has also

been popular where extremely low winter temperatures prevail because of its tightness and the ease with which the ends can be enclosed for heating.

Caterpillar Gates. Caterpillar gates have been used for spillway crest gates, but the type of construction is more expensive than justified for the conditions under which crest gates normally operate.

General. The major design problems in large vertical-lift gates center on the enormous loads that must be distributed with substantial uniformity through the wheels or rollers to the rail or path on which they travel without local overstresses which might result in cumulative failure. Fairly recent developments in heavy-duty antifriction bearings, alloy steel, and heat-treatment, which are commercially economical, have greatly increased the feasible size of gates of this type.

The track and its embedded supporting members must be accurately and rigidly fixed in such manner that the supporting concrete in the piers will not be overstressed. The wheel shafts may be cantilevered from the gate, supported at both ends in a vertical box girder construction at the ends of the main girders, supported in pedestal bearings attached to the gate, or set in a carriage forming a truck for a pair of wheels which supports the main girder on a rocker bearing. There appears to be an advantage in permitting some degree of flexibility in the end of the gate and its connection to the wheels to minimize concentrations of load that would result from great rigidity and unavoidable inaccuracies. Where the construction does not allow shimming for accurate alignment of wheels, the fixed ends of the axles may be turned slightly eccentric with the bearing surface. By rotation of the axle before it is keyed, the wheel may be accurately aligned.

Wheels may have either flat or double-flanged treads. The material may range from cast iron to rolled high-carbon steel, depending on the loads to be carried. The design should be symmetrical and shaped to avoid local stress concentration. The bearings may range from a bronze sleeve to antifriction roller types, depending on the pressures and the allowable coefficient of friction to make the gate self-closing. Ample provision should be made for lubrication from accessible points. The more elaborate bearings should be set in a watertight hub with provision for grease injection. Where a bar section is used for the roller path, it should be permanently attached to a rigid supporting member before planing. A tee-rail section can be attached mechanically to the embedded parts in the field and has the further advantage that the head may be planed with a slight curvature in cross section which will allow the wheels to respond to a reasonable deflection of the gate without undue concentration of stress. Experience indicates that for properly designed wheels, rollers, and track a factor of safety of not less than four applied to the ultimate strength of the material is satisfactory. The wheel treads and rail may be heat-treated where necessary to increase the allowable working stress.

On account of its durability and flexibility in conforming to uneven sealing surfaces, the music-note rubber seal appears to be the most satisfactory type yet developed. It is much more permanent and more resistive to tearing than rubber belting, less subject to damage than spring metal plates, and more flexible than bronze rods.

With variations in detail, the vertical-lift gate can be adapted to any size installation from the smallest up to the largest normally required. Lengths that are feasible allow pier spacing that offers but little obstruction, and the relatively great heights to which this type can be built permit a very low weir where flood stages are extremely high and a large portion of the channel must remain unobstructed. The major objection to this type is the difficulty of providing for surface regulation and passage of debris when a height of 15 or 20 ft is exceeded.

The vertical-lift type of gate can readily be enclosed at the ends and conduits for electrical space heaters built in back of the seal plates and guides so that the gate can be operated under the most severe winter conditions.

BEAR-TRAP GATES

A bear-trap gate consists essentially of two leaves, an upstream leaf hinged and sealed along its upstream edge and a downstream leaf hinged and sealed along its downstream edge. When the gate is lowered, the leaves are in horizontal position with one leaf lying on top of the other. The two leaves have a sliding seal or hinge at their juncture and are sealed against the piers at each end. When pressure from headwater is applied in the chamber underneath, the gate can be raised to any desired height so long as the two leaves remain in contact. The water pressure under the gate is regulated by an adjustable weir or by the setting of inlet and outlet valves in a

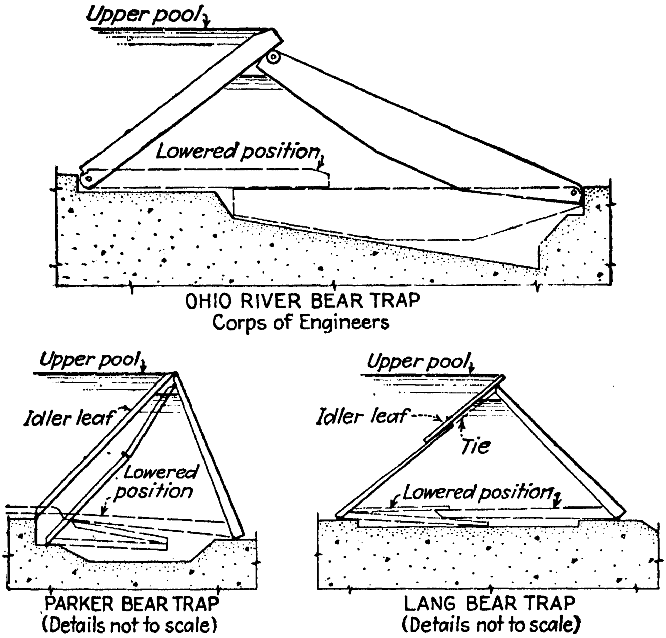


FIG. 12.—American bear-trap gates.

control chamber in the abutment of the spillway. This was probably the first gate involving the principle of the application of headwater pressure for its operation.

Bear-trap gates have been used in the United States for over a hundred years as regulating gates in movable navigation dams and for log-sluicing operations. An improved form of bear trap has been developed and widely used in central Europe as an automatic spillway crest, and a number of gates of this type have been installed in the United States.

American Bear-trap Gates. In American engineering practice, a large number of variations from the simple bear-trap form have been designed. The principal forms of these are shown in Fig. 12. The purposes of these variations have been to eliminate friction between the leaves, to close the reentrant angle to trash accumulation, to economize on substructure width, and to make it possible to raise the gate with small differential head. The type now most widely used in this country is that installed by the Corps of Engineers on the Ohio River improvement. Figure 13 shows the latest development of this type.

This gate has a length of 91 ft and a height of 17 ft from lower sill to upper pool level. The downstream leaf is of heavy plate girder construction with a buckle-plate

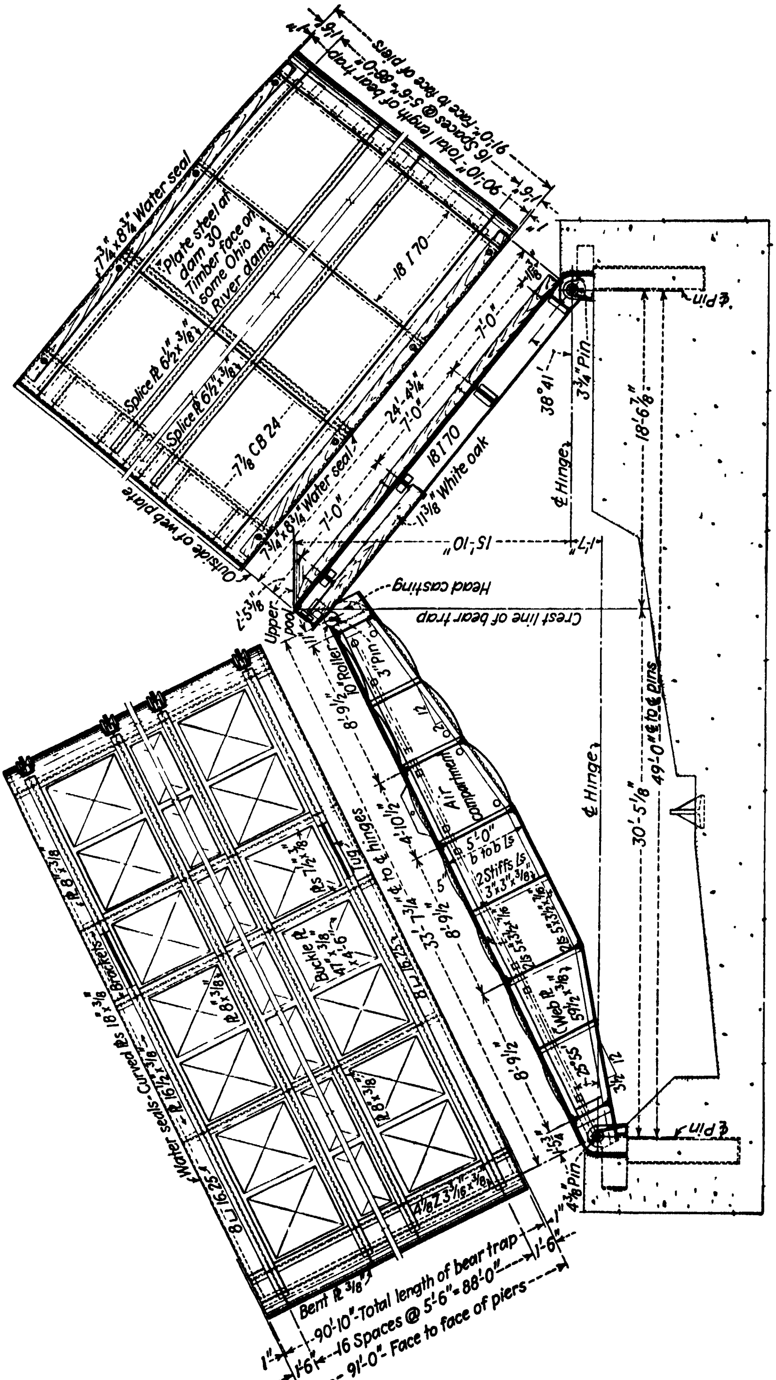


Fig. 13.—Bear-trap weir. Lock and Dam No. 30, Ohio River. (Corps of Engineers, U. S. A.)

face, whereas the upstream leaf is composed of a heavy I-beam frame with plate-steel or timber face. The cross girders on both leaves are spaced 5 ft 6 in. apart and hinged at the sills in bearings attached to I-beam anchorages; at the peak the girders in the upstream leaf rest on, and form a path for, rollers along the edge of the upper leaf. The gate can be held in raised position when empty by connecting the two leaves, at the peak, with links before the water pressure is released from underneath.

The upstream and downstream seals consist of ⅜-in. convex circular plates which are attached to the edge of the leaves, concentric with the bearing pin, and fit tightly against similar concave plates anchored to the concrete sills. The end seals on the upstream leaf are timbers bolted to the leaf with a small clearance to the face of the piers. Curved steel plates are hinged to the ends of the downstream leaf in such fashion that they maintain contact with the pier and the leaf, a seal being thus formed against internal pressure.

Water is introduced under the gate by means of 8 inlet ports leading from a longitudinal conduit in the upper sill and is released through 16 outlet ports leading to a conduit in the lower sill. These conduits connect with water passages in the adjacent pier through sluice valves which are so arranged that the pressure beneath the gate can be controlled to such portion of the differential between upper and lower pools as may be necessary to hold the gate in any desired position. The lower leaf is completely enclosed with plates so that compressed air may be introduced to increase the buoyancy when the differential head is low or when a load of silt has accumulated on the leaves in lowered position.

These gates are used not only to pass large floods but also to regulate the upper pool level during minor variations in flow and avoid the necessity of constantly maneuvering the wickets that compose the navigable pass section and a considerable length of the remainder of the dam.

Since these bear traps are installed on the river bed, Poirée dams are provided upstream and downstream to facilitate unwatering for maintenance. Not less than two bear traps are included in a dam, so at least one will be in service while one is undergoing repairs.

European Bear-trap Gates. The most prevalent European type of bear trap is that developed by Huber and Lutz of Zurich, Switzerland. This type, known as the *roof weir*, is shown in Fig. 14. By adding a vertical lip to the upstream leaf that carries rollers bearing on the downstream leaf, by reducing friction and eliminating the reentrant angle, and by introducing curvature in the downstream leaf, the objections to the original bear trap have been eliminated.

The internal water level required to sustain this gate in any position is practically constant, being slightly below the top of the downstream leaf in raised position. The framework is entirely of steel, either rolled shapes or light trusses according to requirements of strength and stiffness. The facing is timber, provided with splines and fitted in place between the main framing members. The main girders are usually spaced about 5 to 6.5 ft centers and are exactly opposite in each leaf, the rollers supporting the upstream leaf traveling directly on the upper flanges of the downstream girders. The girders are drilled near their ends to receive a bronze bushing which turns on a steel pin supported by structural posts embedded in the masonry. Links between the girders of the two leaves restrain them from rising above normal position. They can be maintained in the raised position while the water is drawn from underneath, by lowering a strut that is hinged to the upstream girder so that it engages the upper end of the downstream girder. The upstream leaf is designed for full headwater pressure with the chamber empty, whereas the downstream leaf is designed for full internal pressure with the gate raised.

The upstream and downstream seals are leather strips supported by flexible metal

plates attached to the sill angles. These bear on segmental steel plates bolted to the ends of the girders so as to be concentric with the pins. The former seals against external pressure, whereas the latter is arranged to seal against internal pressure. The side seals consist either of a spring-metal plate with leather sealing strip or of heavy leather strips bent against the pier and reinforced with riveted brass strips to prevent buckling. Gates of this type have been built as long as 85 ft and as high as 13 ft. The largest single gate seems to be 100 ft long and 12 ft high.

Timber faces are generally used on bear-trap gates, for the weight is usually less that with all-steel construction. The lifting force underneath is also increased by the

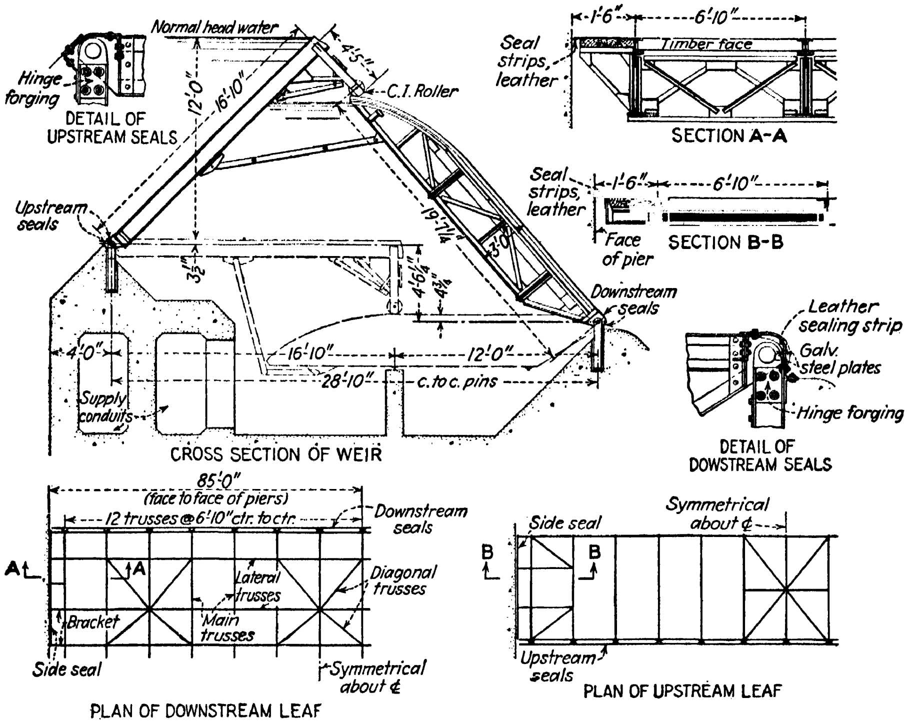


Fig. 14.—Huber and Lutz roof weirs for Texas Hydro-Electric Corporation. (*Fargo Engineering Company, Engineers.*)

thickness of the timber, but where the leaves will be entirely submerged by high tail water, the design should be checked to assure that the buoyancy is not sufficient to float the leaves. Wooden faces protect against ice formation in the interior and eliminate the formation of anchor ice on the downstream leaf when a thin sheet of water is passing over. The timbers are usually well saturated, but a higher degree of permanency may be obtained by the use of creosote.

The design must provide adequate longitudinal stiffening of the lower leaf to compensate for unequal frictional resistances at the ends and along the juncture of the leaves. Warping of the leaf in any direction must be prevented by carrying the stresses from the longitudinal truss to the bearings by means of lateral and diagonal trussing. On short, broad leaves, the stiffness of connections may be adequate, but in long leaves of any considerable width, trussing is imperative to prevent warping.

In designing a bear-trap gate, great care should be used in determining the external water load in various positions, and calculations should usually be checked by experi-

ments. This fact is of particular importance in determining (1) the differential head required to raise the gate and (2) the water level underneath required to establish equilibrium when the gate is partly raised. For automatic operation, it is essential to determine the conditions of equilibrium for all positions.

Large logs and trees may cause severe damage to a partly lowered bear trap, because when part way over the crest the head end may fall to the downstream leaf with sufficient impact to break the planking and the rear end will drag over the crest timbers. To meet this condition, heavy transverse skid timbers may be bolted to the surface of the downstream leaf at about 2-ft intervals and a heavy steel angle should be anchored to the ends of the purlins to cover the edge of the upstream leaf.

The accumulation of silt under bear traps set on the river bed has been a source of considerable trouble and expense. The U.S. Engineer Department has now developed methods for removal of this silt by sluicing.

Bear-trap gates are well suited for surface regulation and passing drift and ice. They can be built in almost any length usually required, and as no overhead structure is needed, they offer little obstruction to flood flow. In low-head dams, there is considerable saving in the cost of substructure owing to elimination of a deep recess or gate chamber. They are not seriously affected by ice, for the leaves drop away from ice accumulation on the piers when being lowered and the overflowing sheet cuts the ice away as the gate is raised.

The major objection to a bear-trap gate is the low coefficient of discharge resulting from the broad flat crest when the gate is lowered. This is, of course, not objectionable in low dams with the sills practically at river-bed level.

ROLLING GATES

The conventional rolling gate consists of a cylindrical plate-steel roller, approximately as large in diameter as the height of opening to be closed and spanning between

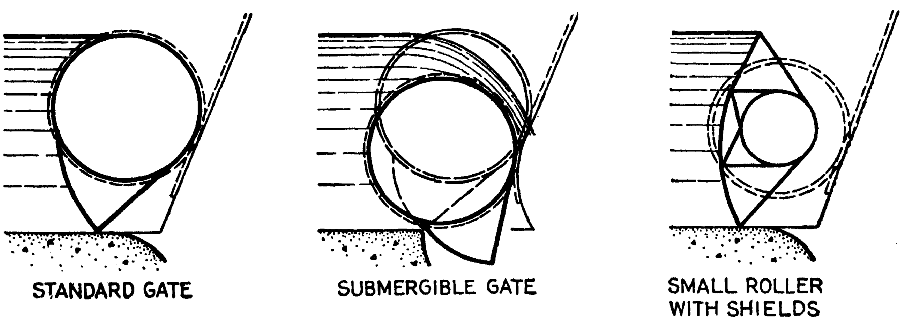
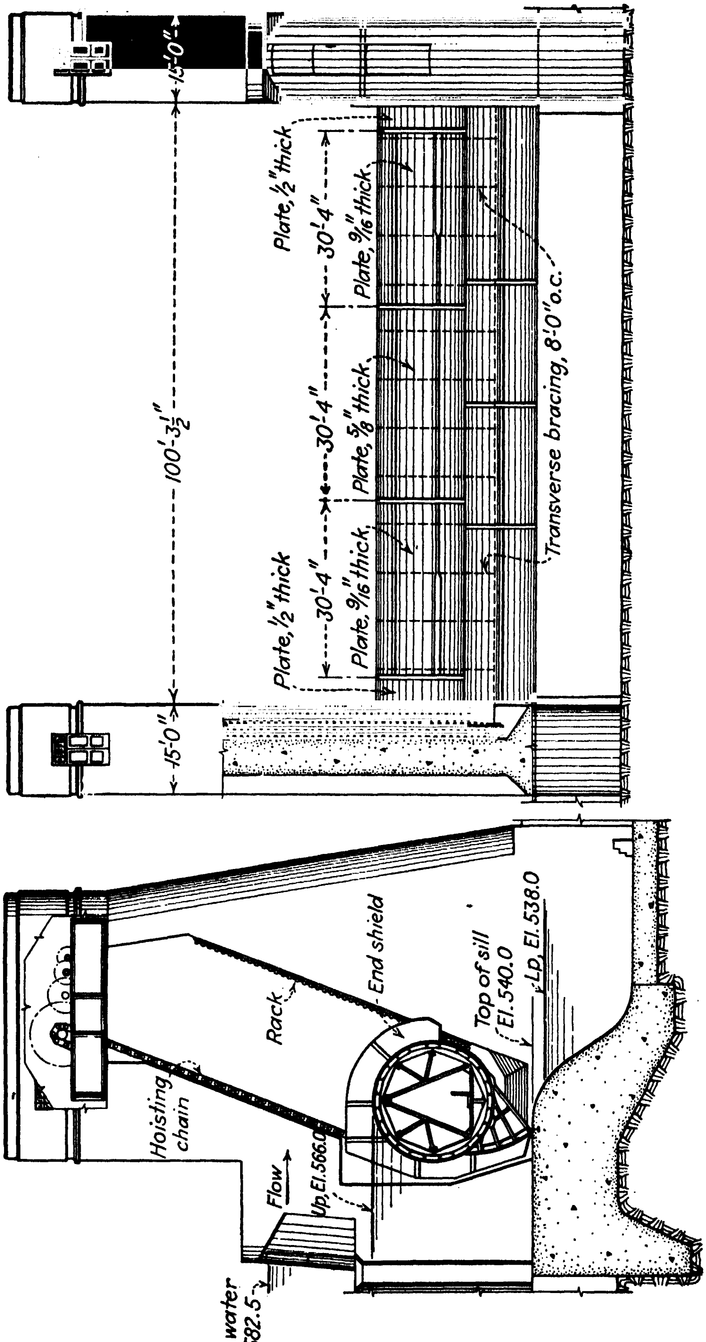


Fig. 15.—Types of rolling gates.

piers. Encircling each end of the roller is a heavy annular rim casting with peripheral teeth and a bearing surface which transfer the loads to similar teeth and a bearing surface on a sloping rack supported by a ledge in the piers. The gate is raised or lowered along this rack by means of a heavy chain which winds around and over the top of the gate at one end and pulls upward, parallel to the rack. This gate was developed in Europe, but a number of large installations have been made in the United States. The largest gate of this type reported is that on the Glommen River near Raanaasfos, Norway, which is 147.3 ft long by 21.3 ft high.

There are two principal variations of the conventional type: (1) where the roller is greatly reduced in size and merely forms the rolling member on which the water face, composed of a sector of considerably greater height and radius, is supported, and (2) the Greisser gate in which light trusses span between load disks with toothed rims of



SECTIONAL ELEVATION

UPSTREAM ELEVATION

Fig. 16.—Rolling gate at Lock and Dam No. 1, Kanawha River. (*Corps of Engineers, U. S. A.*)

the required circumferential length and support the curved sector of the water face. The conventional type may be also arranged so it can be lowered a small distance for passage of drift, or an ice shutter may be hinged on top for the same purpose. Figure 15 shows these various arrangements diagrammatically.

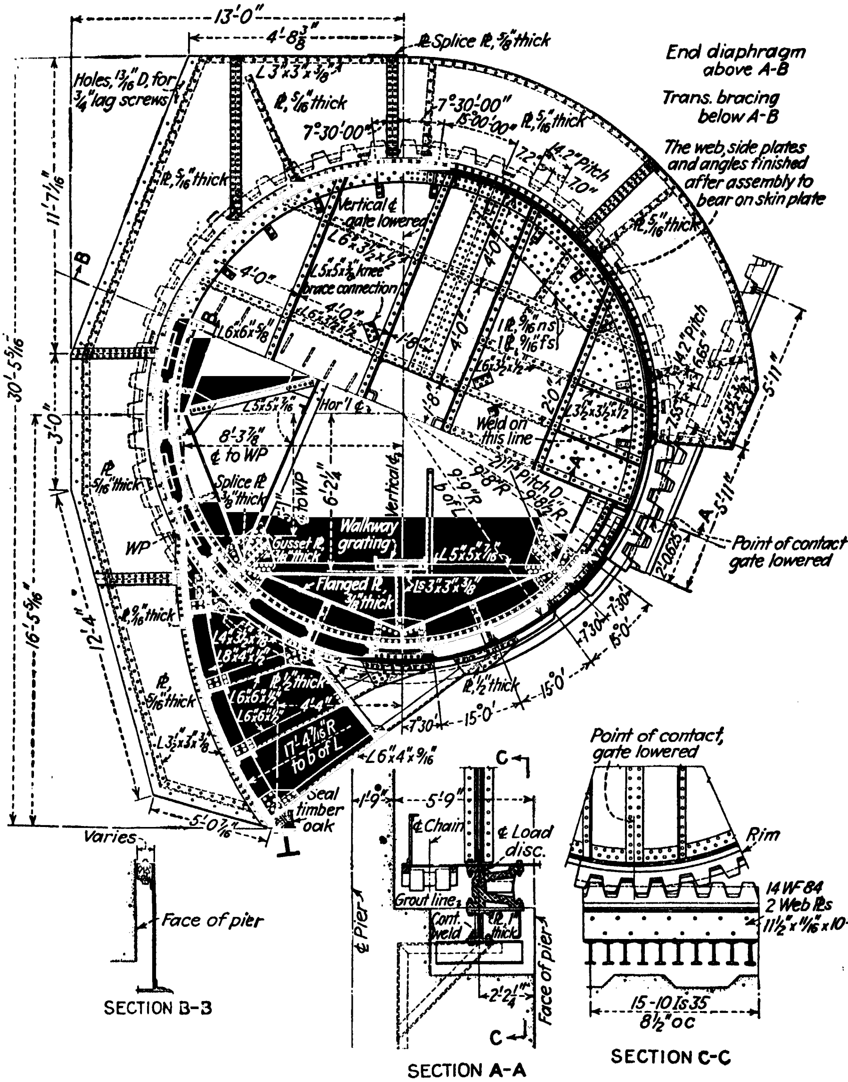


FIG. 17.—Details of rolling gate at Lock and Dam No. 1, Kanawha River.

The rolling gate has been extensively used by the Corps of Engineers on the upper Mississippi and the new Ohio River improvements. Figures 16 and 17 show the gate installed at Lock and Dam 1 on the Kanawha River. This gate has a total height of 26 ft, the roller being 19.5 ft in diameter and the lower lip being 6.5 high, and may be considered as the conventional type, for the roller carries directly over one-half the

horizontal water load and a lip is always required to support the bottom seal. The transverse stiffening consists of three segmental trusses, the inner chords of which intersect at the main splice points forming an extremely effective and economical arrangement. These support longitudinal purlins at an angular interval of 15 deg. The internal diameter of the roller is constant throughout, but the shell thickness varies from end to end as required by the bending and torsional moments.

The cylindrical rolling gate is almost invariably driven from one end by a chain of the Gall or MAN type. A guard chain which is anchored to the top of the pier and winds around the gate from the downstream side is occasionally used on the undriven end to prevent the gate from dropping, should the teeth on the rim jump the rack.

The load disk is built up of a diaphragm plate, reinforced by angles in checkerboard pattern where the rim is in contact with the roller or in radial pattern where the roller is much smaller than the rim. Additional stiffeners and patch plates are used as required on the downstream half of the diaphragm to distribute the heavy concentration of loading from the point of contact of the rim on the rack. Two heavy angles are riveted to the periphery of the diaphragm with sufficient surplus metal to permit finishing to exact diameter to receive the shell. The rim and rack castings are of medium cast steel, each sectionized in equal lengths and bolted together. The teeth and bearing path are formed side by side on the same casting, the latter conforming to the pitch line and being centered over the load disk on the rim. Changes in length of the gate due to temperature is taken up by transverse sliding of the rim on the rack, the movement being limited by the side contact of the bearing path on one against the teeth on the other, for which sufficient clearance must be provided. Since the bearing surfaces are only in full contact at the mean length of the gate, care must be taken to provide sufficient width so the allowable bearing stress is not exceeded on the area remaining in contact in the extreme positions.

The bottom seal usually consists of a timber bolted to the bottom of the lip and resting on a steel beam embedded in the spillway crest. The side seals consist of a timber bolted to the outer edge of the end shields and bearing against armor on the pier face. These seals are set to have a slight wedging effect when the gate is closed. A lip on the underside of the gate is essential for establishing a definite point of normal contact for the bottom seal and also for eliminating the negative pressures that would be caused under the gate by spouting velocities along the shell at partial openings. Its relation to the roller should be such that it will lift vertically away from the crest when the gate is raised, rather than having to scrape through accumulated debris.

The rack is usually installed on a slope not flatter than 2.5 on 1, in order that the resultant of the horizontal and vertical forces will lie sufficiently below the point of contact to assure closing of the gate.

As indicated by the wide range of design in the various elements of existing gates of this type, there seems to be no established practice for determining the most economical relation of such features as diameter to total height or shell thickness to diameter. It is probable that these vary with the length for gates of equal height. The fact that there is considerable variation in published weights for gates of the same size indicates that the most economical design has not always been used.

The cylindrical rolling gate has been built almost twice as long as any other type of lift gate for the intermediate heights, which is of decided advantage in streams subject to severe ice floes. It is not affected by sheet ice pressure, and heating facilities can be readily provided for the seals and end mechanism. There are relatively few major streams on which stages are so high that the limiting depth of these gates will not afford sufficient discharge capacity to control the headwater elevation. The use of a submergible roller or an ice shutter on the conventional type allows surface spilling to considerable depth.

The principal disadvantages of this type are the comparatively high cost and the thickness required for the piers, which usually lead to its installation only in cases where the controlling conditions are imperative.

Greisser Gates. In the Greisser gate (1), the roller wheel and the face are both sectors instead of complete circles. The face is not concentric with the wheel but is concentric with the point of contact between the wheel and rail when the gate is nearly closed. This is essentially a Tainter gate rolling upward on a segment circumscribed upon the pin and the top of the face and of which the upper arm forms the chord. Steel members span across the opening between these points, forming the tension chords of top and bottom trusses supporting the gate face. The area between the segments at the ends of the gate is closed by a stiffened plate diaphragm. The toothed sector of the rim and rack have a length sufficient to raise the gate above headwater. Beyond this point, the bearing rail continues to take the horizontal thrust but the vertical reaction is carried by a cable which is anchored to the top of the pier and is wound under the sector as the gate is raised by the main hoisting chain.

This gate was developed by V. H. Greisser of the Washington Water Power Company, and several have been installed, the largest of which is 100 ft long by 12.5 ft high. It is well adapted to sizes beyond the range of the Tainter gate and is considerably lighter than the full roller gate. However, it does not have the torsional stiffness of the latter and must be driven at both ends. It possesses advantages over the Tainter gate in that the entire gate can be raised above high tail water and the reaction at the piers is relatively low with a downward slope.

DRUM GATES

The drum gate fundamentally is an acute circular sector in cross section, formed by skin plates attached to internal bracing. It is hinged at the center of curvature, which may be either upstream or downstream, in such manner that the entire sector may be

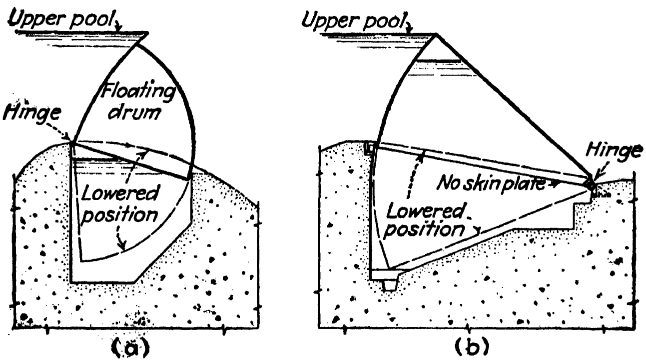


Fig. 18.—Drum gates.

raised above the masonry crest or may be lowered so the upper surface becomes coincident with the crest line. These gates are controlled by the application of headwater pressure underneath, in the same manner as bear traps. Figure 18 shows the two principal arrangements of this type of gate. Section *a* in the figure shows that developed by the U.S. Bureau of Reclamation, which is hinged on the upstream side and is enclosed on all three faces and at the ends to form a watertight vessel. This gate has been used on many projects designed by the Bureau of Reclamation and has been built as large as 135 ft long by 28 ft high. Figure 18*b* shows the type that is hinged on the downstream side and that usually is enclosed only on the upstream and downstream surfaces. In Europe, gates of this type as large as 165 ft long by 16 ft high have been

constructed. A gate 48 ft long by 12 ft high was built on the Chicago Drainage Canal at Lockport, Ill., and two similar gates were built on the Genesee River at Rochester, N. Y. At Coolidge Dam in Arizona are six concrete sector gates 50 ft long by 12 ft high.

Bureau of Reclamation Drum Gate. The location of the hinge along the upstream side simplifies the sealing problems as the hinge and the upstream face, which moves transversely on the pier, can be sealed against the external pressure resulting from this design. Likewise the seal against the curved face is more effective against internal pressure and can be so applied with this face downstream. With this hinge location, the large lowering moment added by water pressure on the upstream face is compensated for by the displacement obtained by closing the bottom face and ends, so the sector becomes a watertight float. Whereas this arrangement requires additional metal, it contributes materially to the stiffness required to prevent warping of the structure.

Figure 19 shows the design of one of these gates 100 ft long by 14 ft high. The internal bracing is composed of plate girders, conforming to the cross section of the gate and spaced 2 ft 4 in. on centers. Hinge castings are anchored along the upstream edge of the gate chamber opposite each girder and support a 4-in. bronze pin. The web plates of each girder are extended between the ears of the castings to form the bearing on the pin. The downstream seal is attached to the downstream seat casting which also forms the stop for the gate in raised position. This casting is sectionized into 4-ft lengths which are bolted together to form a continuous member except at contraction joints in the masonry where the sections are doweled to preserve alignment and to allow longitudinal movement. The last two frames at each end of the gate are omitted and replaced by the girder beams and stiffeners that support the buckle-plate bulkhead as shown in section *AA*. This bulkhead is placed at sufficient distance from the pier face to allow access to the end seal supports which are bracketed from the bulkhead by gusset plates.

The girder connections are riveted up and the skin plate attached in the field. Roundabout skin-plate joints occur at the center of every third girder, spaced both ways from the center of the gate, and the seam is welded full length. The joint where the faces intersect is also welded continuously. The hinge seal is composed of two steel plates, bolted to the sill angle and the hinge casting, and bearing on a segmental steel shield which is attached to the gate, concentric with the pins. The end seals are composed of 10 gage spring brass, to the water side of which is riveted a sheet of rubber $\frac{3}{16}$ in. thick. These are built up in lengths $11\frac{7}{8}$ in. long with the rubber overlapping the joint between the brass sheets, and since the seal has curvature in two planes, the free end of the brass sheet is slotted down to the flange at 4-in. intervals before bending, and the rubber sheet attached afterward. The downstream seat seal is of similar construction, except that the curvature being only in one plane, it is not slotted and is installed in $23\frac{7}{8}$-in. lengths with a rubber sheet used only at the joints. Special seals must be used to form the junctions between the main seals at the ends of the gate, the greatest difficulty occurring at the downstream corner. At this point, a bronze casting is hinged to the pier and held against the end ship channel by a spring. The downstream seat seal is stationary against one end of this casting, whereas the other end forms an intersection with the downstream end seal which slides across it. All seals must be installed so that with the maximum clearances due to temperature or inaccuracy there will be residual tension.

The entire area of the pier against which seals bear is armored with a finished semisteel face, $\frac{7}{8}$ in. thick backed by ribbed construction. This is sectionized rectangularly into 4- or 5-ft sections, the outer flanges of which are finished and bolted together.

The seepage that accumulates in the drum is led off through a 4-in. suction hose

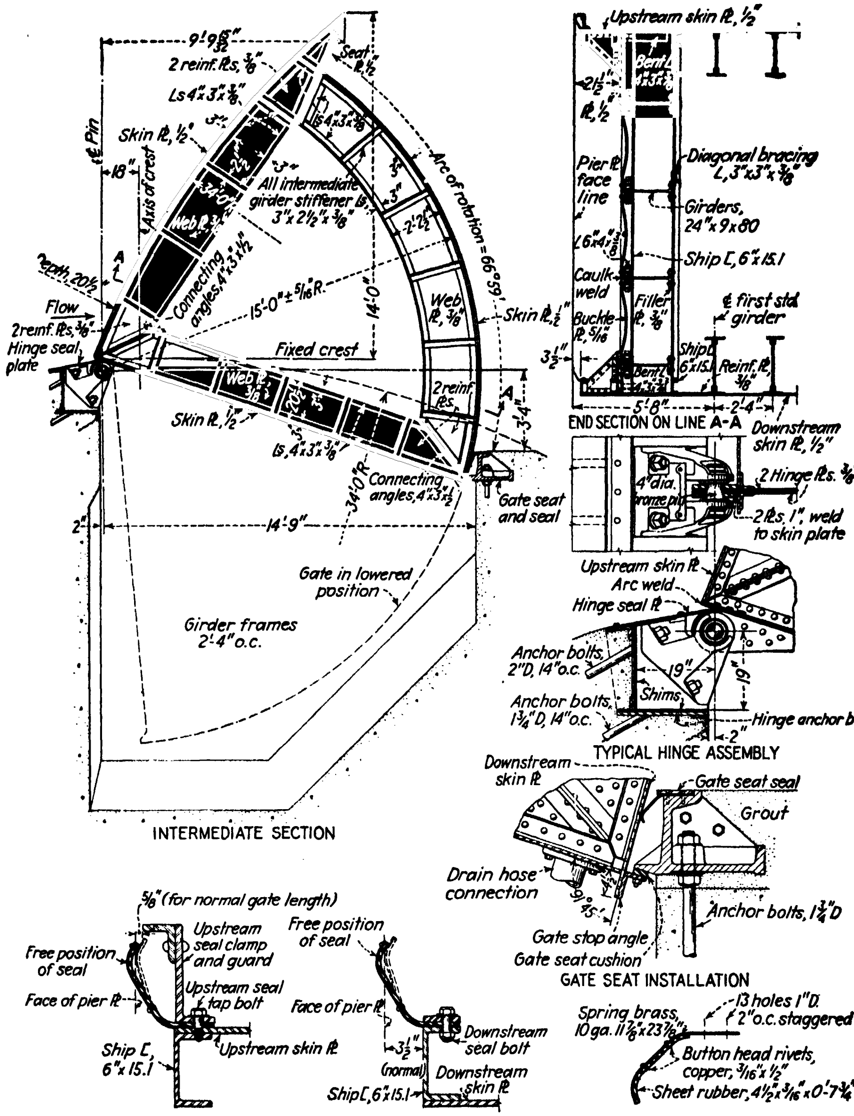


FIG. 19.—Drum gate designed by U.S. Bureau of Reclamation.

attached to the lower corners of the gate and leading to an outlet at the center of the recess.

The triangular trusses, composed of plate girders, are designed as rigid frames, the upstream members being designed for full water load in any position including the maximum surcharge above normal to which the headwater may rise. The bottom members must likewise be designed to carry full water load in the position that results in maximum moment and the downstream members as struts to carry the thrust from

the other two faces and as beams to carry water load when the gate is lowered into the recess. The allowable working stress in the latter must be such that variations in deflection will not interfere with the seat and seal clearances.

Gates with Downstream Hinge. Although this type of sector gate, shown in Fig. 18*b*, has been used to some extent in the past, it has a number of obvious disadvantages when compared with the type developed by the Bureau of Reclamation. The only apparent advantage lies in the fact that the upstream face is concentric with the hinge and consequently headwater pressure causes no moment that must be balanced by pressure in the chamber underneath. However, the differential head is so small when the gate is lowered that compressed air or floats must frequently be introduced underneath to start the gate upward. The introduction of the skin plate on the underside overcomes this difficulty by forming a float, which likewise makes the Bureau of Reclamation gate workable.

The downstream hinge in this gate is composed of series of seat castings and castings attached to the gate, both of which partly surround and make contact with a continuous pin, thus forming the seal. The upstream seal is a flexible metal-plate bearing on the circular face and should be set in a recess to allow the gate to drop flush with the crest of the weir. Ordinarily, the top surface of the gate when lowered forms a broad crest with a slight slope downstream, but in order for this face to conform to the nappe surface to give maximum discharge, the width of gate must be about 20 per cent greater than that of the Bureau of Reclamation type.

Drum gates are not adapted to low dams having low substructures on account of the deep excavation required and the liability of flooding of the recess in which the gates float. The operating mechanism for controlling water pressure beneath the gates may be placed in the abutments, or in the piers with access through a gallery, the necessity of an overhead bridge being eliminated and the pier height being reduced to a minimum. They are very well adapted for surface regulation and, on account of the lengths to which they can be constructed, are suitable for passing debris. Sheet ice causes no damage because of the upstream slope, and the seals can be easily protected against freezing under usual winter conditions. With extremely low temperatures, special provision would have to be made for insulation and heating of the downstream seals. A very reliable automatic regulating device has been developed for the Bureau of Reclamation gate.

MISCELLANEOUS TYPES

The principal types of moveable weirs used in the improvement of the Ohio River and its tributaries by the Corps of Engineers will be briefly discussed. These types have been rarely used except for canalization by low-lift dams and particularly in connection with the navigable passes and regulating weirs in the original Ohio River improvement. Detailed descriptions of these weirs will be found in various papers of the Corps of Engineers and in "The Improvement of Rivers," by Thomas and Watt.

The requirements governing the use of these structures were (1) a wide opening of full river depth through which tremendous tows could run downstream in high water, (2) limited contraction of the stream and consequently negligible drop through the passes, and (3) manipulation against several feet of drop without resort to fixed overhead works.

These structures are the trestle dams, which include the Poirée and Boulé, the A-frame and the wicket dams, which include the Chanoine and Bebout.

Poirée Dam. The Poirée dam consists of a curtain of timber needles supported at the bottom by a sill and at the top by a beam carried by steel trestles. The trestles are spaced from 3 to 8 ft centers across the stream, the two legs being hinged at the base in bearings in such fashion that when the trestles are lowered they nest together

in a recess in the base slab. The trestles are raised or lowered by a chain from a winch on the abutment, and the needles are handled by a derrick boat on the upstream side. The application of this type at present is largely confined to use as cofferdams below roller and bear-trap gates and at lock entrances.

Boulé Dam. In the Boulé dam, the trestles are very similar in construction to those of the Poirée dam. However, instead of vertical needles, horizontal stop logs or panels are used which span from trestle to trestle and slide in guides on the upstream legs. This type has not been extensively used.

A-frame Dam. The A-frame dam is composed of a series of A-shaped frames, the upstream and downstream legs of which are hinged at the bottom in the same manner as the Poirée trestles. Instead of using needles or stop logs, however, the hinge spacing is made only slightly greater than the width of the leg so that when the frames are in vertical position they form a reasonably tight dam. When lowered, they nest together in a recess between the sills and form a smooth flat crest. The frames are raised, by a chain or cable, successively out from the abutment, each being latched to the one previously raised as it reaches vertical position. A-frames about 13 ft in height have been used on the Ohio River. Recently, several of the fixed dams on the Cumberland have been raised 2 ft in height by means of A-frames installed on the original crest. This type of dam can be readily and safely maneuvered by a small crew and is not particularly susceptible to damage from overflow or drift.

Chanoine Wicket. The Chanoine wicket dam consists of a curtain formed by timber leaves or wickets about 3 ft 8 in. in width and inclined somewhat downstream. The butt of the wicket is supported against a sill along the upstream side of the base slab, and the upper reaction is carried by a prop. The upper end of this prop is hinged to the wicket somewhat above the normal center of pressure, and the foot slides in a grooved casting, or hurter, embedded in the base slab. A horse is hinged on the same axis as the wicket and prop, and the two legs are hinged to the base slab, the axis being thus supported when the prop is raised. The wicket is lowered by pulling upstream on the top until the foot of the prop disengages from a notch in the hurter, the horse being allowed to fold forward to a horizontal position on the base slab with the wicket lying over it. In raising, the butt of the wicket is pulled upstream until the horse is raised and the foot of the prop is seated; the butt is then depressed into the current until the pressure below the pivot rights the wicket and forces the butt against the sill. These operations are carried on from a maneuver boat lying upstream along the raised wickets.

The Chanoine wickets were extensively used on the early Ohio River improvement for navigable pass and regulating weir sections of the navigation dams. These wickets pivot automatically on the horses with a rise of the upper pool, a certain degree of regulation being provided by passage of water underneath. However, in practice, the wickets go on the swing when rise of tail water raises the center of pressure or when drift causes impact on the tops. Under these circumstances, it proved extremely difficult to restore the wickets without an undesirable drop in the upper pool and the standing horses accumulated quantities of drift. These conditions led to gradual adoption of the Bebout wickets for the regulating weir sections in the later dams. The Chanoine wickets are still used in the navigable passes, where they do not serve for regulating purposes.

Bebout Wicket. The Bebout wicket has all the component parts of the Chanoine wicket, but with differences in detail that result in essential differences in operation. The legs of the horse, instead of being rigid, have a knee joint, and the prop consists of two parallel members that are hinged to the base slab instead of a single member sliding in a groove. As long as the center of pressure on the wicket is below the common axis of the wicket, horse, and prop, the butt of the wicket is in contact with the

sill, the horse is in tension, and the wicket is stable. However, when the center of pressure rises above the hinge, the butt of the wicket moves upstream away from the sill and the compressive force on the horse bends the knee. The downward component of the water pressure on the wicket then folds the entire contrivance flat on the base slab.

This wicket has the automatic feature of the Chanoine and has the further advantage that it folds flat when tripped automatically instead of going on the swing. For this reason, the Bebout is more satisfactory for use on the regulating weirs.

Both the Bebout and Chanoine wickets have a clearance of about 3 in. between panels to avoid interference in operation. These spaces are closed by needles whenever water must be conserved to maintain the upper pool.

CREST GATE HOISTS

Hoists operated exclusively by hand are generally used for only the smallest of crest gates, although many motor-operated hoists for larger gates have provision for emergency manual operation. Crest gate hoists may be either the fixed or the traveling type, regardless of the type of gate handled, except in the case of the roller gate, which is ordinarily operated by a fixed hoist. Traveling hoists or gantry cranes can be used economically to handle a large number of gates, whereas fixed hoists are usually limited to handling but a few gates. Hoists of the fixed type are particularly adaptable to remote-control requirements, or to situations in which simultaneous operation of all crest gates is mandatory. Combinations of both fixed and traveling hoists are employed at many stations, the former type being used to operate the regulating gates and the latter to handle the less frequently used flood gates.

The hoisting speed of any crest gate should not exceed 3 fpm, since mechanical difficulties may suddenly arise from many potential sources during the hoisting of the gate, and the hoist should be stopped before serious trouble develops. The hoisting speed for heavy gates should not exceed 2 to 2.5 fpm.

Gates may be hoisted by either chains or cables, but cables should not be used unless special precautions are taken to minimize deterioration due to submergence. Hoist chains should be specially fabricated to fit the hoist drums or sprockets, otherwise operation will be unsatisfactory. Stresses should be moderate under the heaviest pull to be expected, or stretching and rapid wear will result. An alloy-steel dredge chain with stud links is especially resistant to deformation and wear. Hoist chains should be proof-tested at the fabricating shop. Cables are more satisfactory than chains on fixed hoists where dogging of chains is not necessary to hold the gates in raised position. Cables also should be only moderately stressed under the heaviest loads to be expected.

The starting pull on a gate may be considerably in excess of that required after the gate is in motion. This may be due to freeing of the seals, stiffness in bearings, corrosion of metal surfaces, and similar causes and may be appreciable where a gate is operated infrequently. This heavy initial pull can usually be applied by a motor having a starting torque from 50 to 100 per cent greater than the normal running torque. This high torque should not overstress chains or cables, but a somewhat lower factor of safety than normal can be permitted in most of the mechanical parts of the hoist under this load.

In order to prevent the load on the hoist from backing off when power is disconnected, a self-locking worm, solenoid brake, or mechanical brake should be used, depending on the motive power, the operating conditions, and the service requirements. Hoists should be provided with a manual operating device or with an auxiliary source of power, especially where spilling is entirely dependent on gate operation. Where movable hoists are used, not less than two hoists should be provided, the total

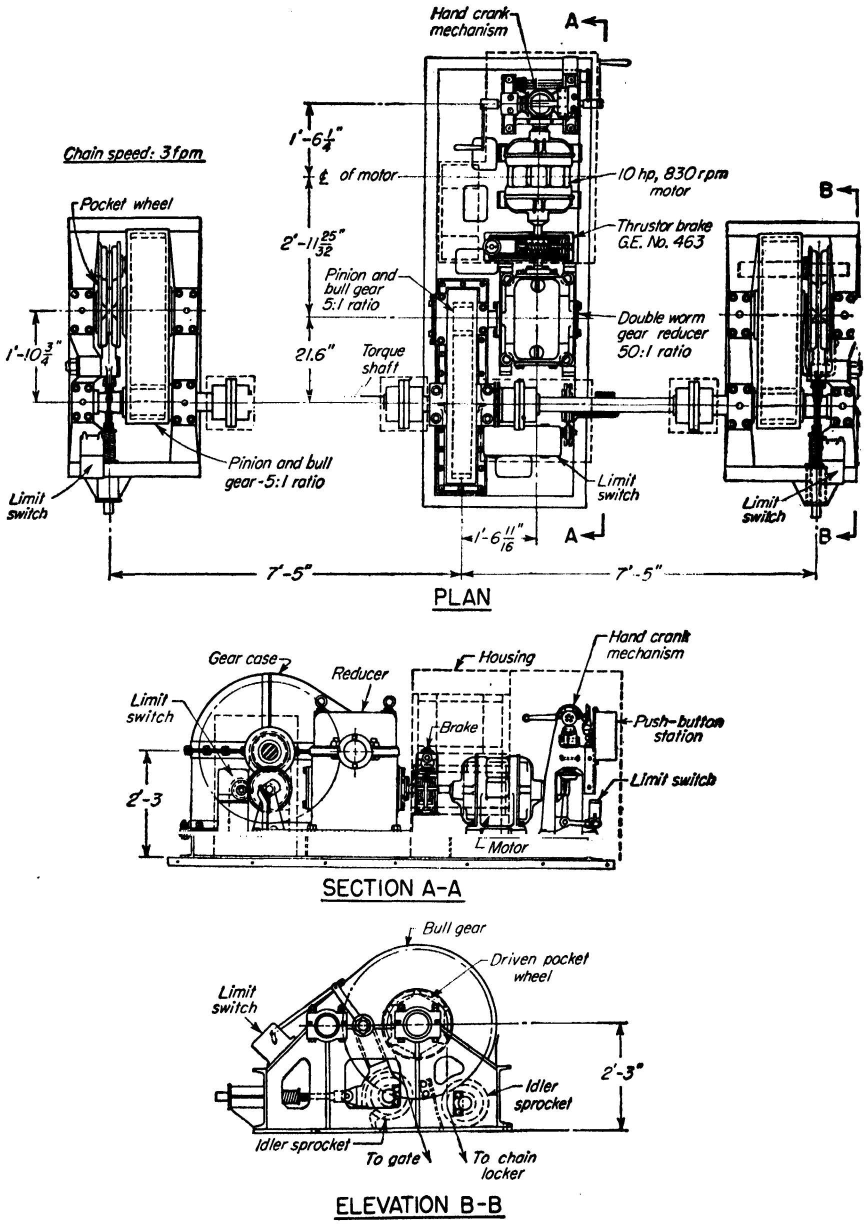


FIG. 20.—Forty-ton fixed hoist for Tainter crest gate, Apalachia Dam, T.V.A.

number being determined by the length of time allowable for adjusting all gates under the most adverse conditions.

Tainter Gate Hoists. Nearly every type of hoist is adaptable to the Tainter gate. Gates of this type usually are handled by two-drum hoists, but many small Tainter gates are handled by single chains or by single sets of cable falls. Many hoists use low-ratio speed reducers in the initial stage with a moderate or high-speed torque shaft connecting the two hoisting assemblies. In other hoists, the drums are mounted directly on a low-speed torque shaft driven by a motor and reducer either at the center

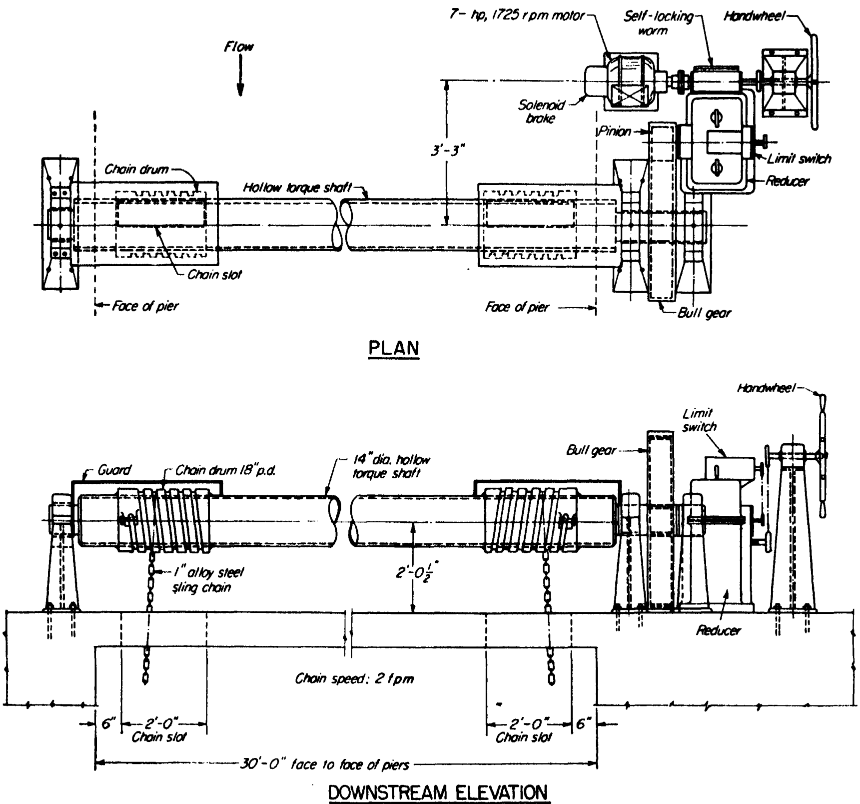


FIG. 21.—Twenty-ton fixed hoist for Tainter crest gate. (*Harza Engineering Company.*)

or on one end. Typical adaptations of these basic arrangements to fixed hoists are shown in Figs. 20 and 21. Figure 22 shows a Tainter gate hoist of the former design mounted on a truck for traveling. The traveling mechanism for this hoist is driven by a gear and jaw clutch mounted on the torque shaft and consists of a friction winch which winds a cable attached to both ends of the runway. In other designs of traveling mechanisms the axle of one pair of wheels may be geared to a special motor. If the runway is on a curve, the complication of a differential may be avoided by driving only the wheels on one side of the truck. Either fixed or traveling hoists may be housed if desired.

The 32- by 23-ft Tainter crest gates at the T.V.A. Hiwassee Dam are normally operated by a dual-purpose traveling gantry crane which also handles coaster-type emergency gates for the low-level sluiceways; a traveling Tainter gate hoist powered

by a gasoline engine is provided for emergency service. At several hydro plants in Sweden each spillway Tainter gate is raised in a single stroke of a hydraulic cylinder.

A pair of hydraulic (oil) cylinders operates each of 15 spillway gates 30 ft long by 18 ft high at the Petenwell development of the Wisconsin River Power Company. The gates at this plant are raised simultaneously about 15 in. per stroke, thus spreading the flow uniformly across the spillway apron; at the end of each stroke, it is necessary to dog the gates and reset the piston rods for the next stroke. Tests made under operating conditions show that all 15 gates can be opened fully in less than 2 hours. Seventeen gates at the Castle Rock plant of the same company are operated in an identical manner. Both plants were designed by the Harza Engineering Company.

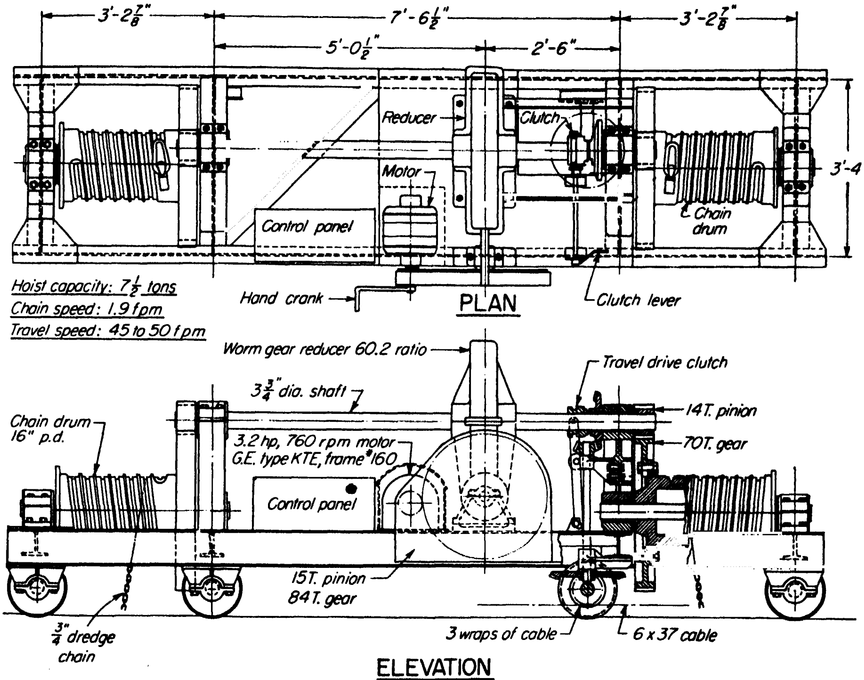


FIG. 22.—Traveling hoist for Tainter crest gates. (*Fargo Engineering Company.*)

Vertical-lift Gate Hoists. Clearance for vertical-lift gates generally requires the hoist to be located some distance above the spillway deck. Thus traveling gantry cranes are frequently employed to handle a large number of gates, whereas a few gates may be handled by fixed hoists mounted on elevated platforms. Fixed-wheel or Stoney gates are usually hoisted by one or two sets of cable falls, depending on the size and relative proportions of the gate. Where high frictional resistance is encountered in lowering, rigid threaded stems are attached to the gates and are driven by screw hoists. Arrangements of this sort have been used extensively for large Stoney and slide gates, and preference for the screw hoist in handling fixed-wheel gates is in evidence on many Canadian projects where heavy ice flows are expected. Screw hoists also can be used where cables or chains would be fouled by debris, as in the case of a gate lowered to pass trash.

Inasmuch as it is necessary to handle a Stoney gate as a single leaf, traveling gantries designed for this service are high structures of large capacity, such as the

cranes installed at Calderwood Dam in Tennessee (9). Traveling gantries have become increasingly popular with the more extensive use of fixed-wheel gates, which readily lend themselves to sectionalizing, thereby materially reducing the required height and capacity of such cranes. Gantries of the type shown in Fig. 23 handle two-section gates 40 ft square at the T.V.A. Chickamauga, Guntersville, and Pickwick

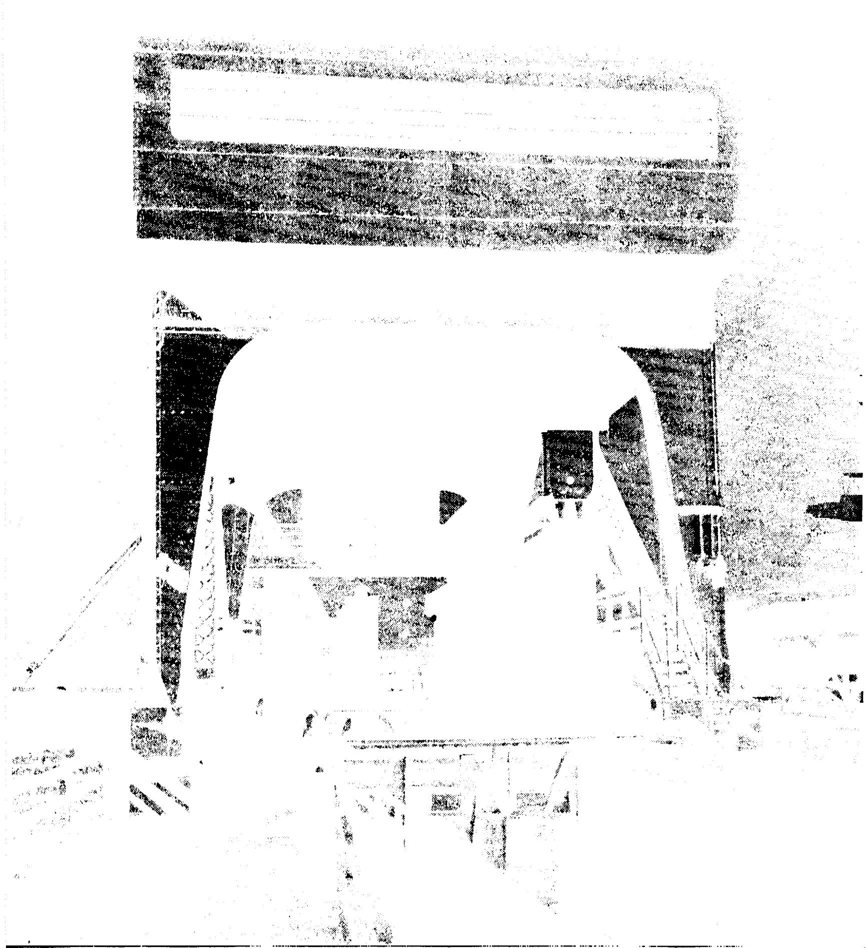


FIG. 23.—Traveling gantry crane for fixed-wheel, vertical-lift crest gates. (*Tennessee Valley Authority.*)

Landing projects. Similar cranes are in service at the T.V.A. Kentucky Dam, handling three-section crest gates 40 ft long and 50 ft high. These cranes are equipped with trolleys for moving the gates between service and storage positions, and are provided with auxiliary hoists for performing other services.

Fixed hoists mounted on elevated platforms have been used extensively for lifting single-leaf Stoney and fixed-wheel gates, and they are particularly adaptable to remote-control operation. Such hoists usually are equipped with counterweights to reduce the required capacity and cost of hoisting machinery. Hoists of this type handle the

50- by 50-ft spillway gates at the Parker and Bartlett Dams of the Bureau of Reclamation (13).

Rolling Gate Hoists. It was stated earlier in this section that the rolling gate is almost invariably driven from one end. Likewise, rolling gate hoists are nearly always of the fixed type, in part because their use results in the elimination of an expensive, long-span, heavy-duty service bridge. The gate cylinder is hoisted by a bar-and-pin (Gall or MAN) type of chain wrapped partway around one end of the gate, the opposite portion of the chain passing over a multiple sprocket and onto a storage rack. The hoist itself is a compact set of heavy-duty reduction gearing usually driven by an electric motor. A typical setting of a rolling gate, together with its lifting chain and hoist, is shown in Fig. 16.

ESTIMATING DATA—GATES

Convenient means for estimating the weights of several types of crest gates and hoists are given in Figs. 24 to 27. Weights of gates are expressed by the general

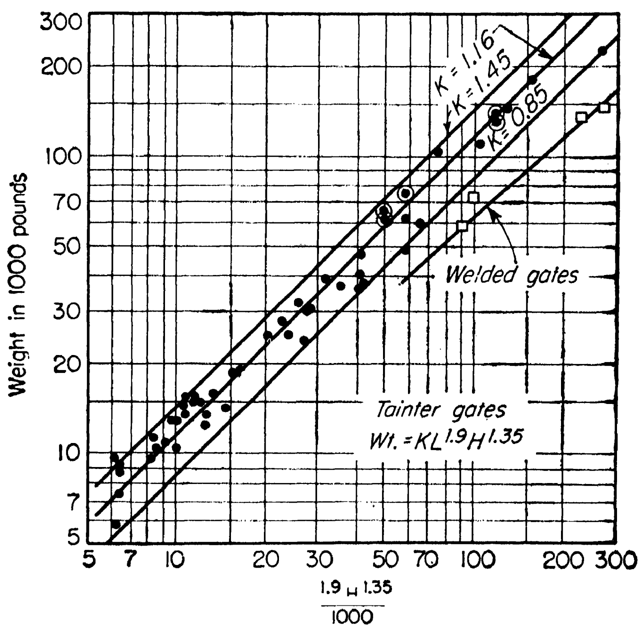


FIG. 24.—Weight of Tainter crest gates.

equation,

$$W = KL^mH^n$$

wherein W = weight, lb.
K = weight coefficient.
L = clear width of gate opening, ft.
H = nominal height of gate, from spillway crest to normal water level on gate, ft.
m and n = exponential powers, constant for a given type of gate.

Tainter Gates. The weight of the moving portion of a Tainter gate may be determined graphically from Fig. 24 or directly by means of the equation, $W = KL^{1.9}H^{1.35}$, wherein K has a mean value of 1.16 but varies from 0.85 to 1.45. Most of the gates shown include an allowance for corrosion added to the computed skin plate thickness.

The gates indicated by encircled points have a corrosion allowance of $\frac{1}{16}$ in. added to their skin plates and principal frame members. The weight of embedded parts, such as trunnions and anchorage, sills, and seal plates, may be taken as 35 per cent of the weight of the moving portion of the gate, although this percentage varies from 10 for small gates to 50 for a few of the larger ones. Tainter gate costs are generally slightly higher than for other types of structural steel work, the differential probably being accounted for by the large amount of continuous, watertight welding in the skin plate assembly, and by the precise alignment work required for installing seals and embedded parts.

Vertical-lift Gates. Gate leaf weights, including wheels or rollers, for both fixed-wheel and Stoney gates, may be obtained from Fig. 25 or from the equation,

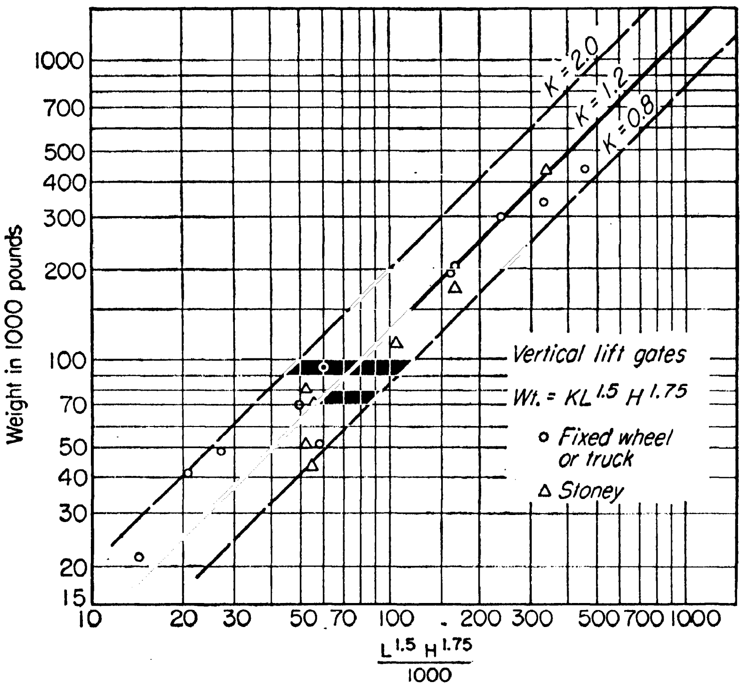


FIG. 25.—Weight of vertical-lift crest gates.

$W = KL^{1.5}H^{1.75}$, where values of K vary from 0.8 to 2.0, with a mean value of 1.2. The weights of guides, including wheel and roller paths, sills, and seal plates, vary from 10 to 50 per cent of gate leaf weights, with the lower percentages associated with the smaller gates. Unit costs frequently run higher than Tainter gate costs because of the high cost of precise field alignment of guides. The erection costs of large gates may be reduced by increasing the inherent stiffness of the guides. This practice naturally leads to higher first costs, but it can result in appreciable savings in over-all costs, particularly when a large number of gates is involved.

Rolling Gates and Hoists. The weight of the rolling cylinder portion of a rolling gate may be determined from Fig. 26, or by the equation, $W = KL^{1.5}H^{1.67}$, using a mean value of 2.85 for K, although K may vary from 2.40 to 3.40. The weight of fixed parts, including racks and safety chains, may be computed as 20 per cent of the cylinder weight. A fixed-hoist unit, complete with lifting chains, will weigh from 25

to 40 per cent of the weight of the cylinder, depending on the degree of submergence of the gate and other loading conditions, but a figure of 30 per cent of the cylinder weight may be used for average conditions.

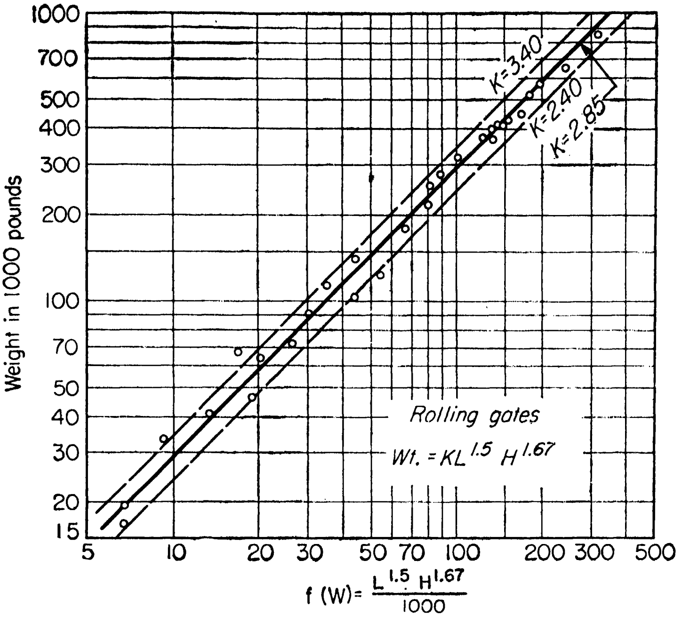


FIG. 26.—Weight of rolling crest gates.

Average unit prices, at 1950 cost levels, for rolling gates installed are as follows: (1) gate cylinders and fixed parts, 35 to 40 cents, (2) hoists with lifting chains, 60 to 70 cents, and (3) gates and hoists complete, 40 to 50 cents per pound.

Bascule Gates. Sufficient data for determining the weights of gates of this type are not yet available. However, the S. Morgan Smith Company indicates the cost per square foot of net damming area of the bascule gate, complete with operating mechanism, to be about the same as for rolling gates.

Drum Gates. The weights of drum gates, complete with embedded parts, operating mechanisms, and piping, may be estimated from Fig. 27, or by means of the equation $W = K(LH)^{1.33}$, where K varies from 26 to 35, with a mean value of 31. Most of the gates shown are hinged upstream, and a generous addition of plate thickness for corrosion is included because of the general inaccessibility of this type of gate for frequent painting. Average unit prices for drum gates installed vary from 30 to 35 cents per pound at 1950 cost levels.

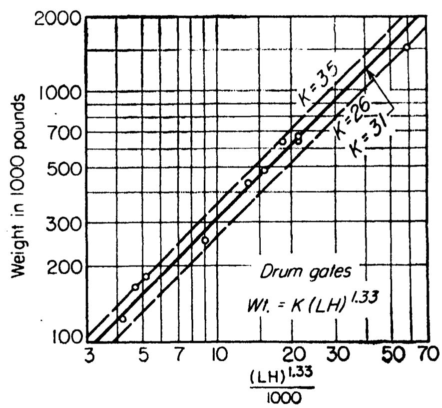


FIG. 27.—Weight of drum crest gates.

ESTIMATING DATA—HOISTS

The following paragraphs give the means for estimating the capacities and weights of fixed and traveling types of hoists for Tainer and vertical-lift gates. Means for estimating the weight and cost of rolling gate hoists were included earlier with estimating data for rolling gates. No such data are given for drum gates because that type of gate requires no external mechanical lifting device. Unit prices for installed equipment may vary from 60 cents per pound for larger hoists to $1 or more per pound for hoists of smaller capacities; gantry crane prices may be scaled somewhat lower because of the larger amount of structural steel work involved. Manufacturers' estimates, or precedents for equipment similar to that proposed, usually form the better guides for estimating the cost of gate hoists.

Tainter Gate Hoists. There is no well-established relationship between the weight of a Tainter gate and the weight of its hoist. Hoist capacities may vary from 75 to 150 per cent of the weight of the gate leaf, depending on the construction and relative dimensions of the gate and its general setting on the spillway crest.

For *fixed-type* hoists,

$$\text{Weight (lb)} = K \times \text{capacity (tons)}$$

where values of K vary from 400 to 800 for two-drum hoists; with 650 as a fair general average. The weight of single-drum hoists may be as little as half the weight of two-drum hoists.

The weight of hoists of the *traveling type* may be estimated by means of equation,

$$\text{Weight (lb)} = K \times (\text{capacity in tons})^{1.33}$$

The coefficient K varies from 250 to 470, but a value of 370 is fairly representative.

Vertical-lift Gate Hoists. The capacity of a hoist for a vertical-lift crest gate should exceed the weight of the gate and lifting beam, if used, plus frictional resistance. The weight of the gate may be determined from Fig. 25. A lifting beam weighing 10 to 20 per cent of the gate weight will be necessary when a traveling hoist is considered. Combined friction of seals and wheels, rollers, or sliding surfaces, may be estimated as a percentage of the hydraulic thrust on the gate, as follows:

Fixed-wheel gates (wheels)	3 to 5 per cent
Stoney gates (rollers)	10 per cent
Sliding gates	35 per cent

Friction encountered in starting the gate from its closed position may reach values twice as high as those given above; however, this condition is of relatively short duration, and in many cases the resultant larger hoist pull may be safely absorbed by the natural overload capacity of the hoist.

The approximate weight of machinery and bedplates of *fixed hoists* without counterweights may be estimated by means of the equation,

$$\text{Weight (lb)} = K \times \text{capacity (tons)}$$

using the following values for K:

Two-drum hoists	$K = 500$
Single-drum hoists	$K = 150$

(The weight of supporting structures is not included.) It should be remembered, however, that variations in hoist arrangement can result in considerable departure from the values given.

The weight of *traveling gantry cranes* for crest gate service can be determined from the curves in Fig. 28, crane weight being plotted against a parameter,

$$f(W) = \frac{CS}{1,000}\left(1 + \frac{A + B}{2S}\right)$$

where C = maximum hoist pull, tons.
S = span of runway rails, ft.
A and B = respective lengths of upstream and downstream legs of crane (ft), measured from runway rail to hoist platform or trolley rails.

The lower curve gives the weight of open utility-type gantries, whereas the upper curve is representative of enclosed gantries of the T.V.A. type, shown in Fig. 23.

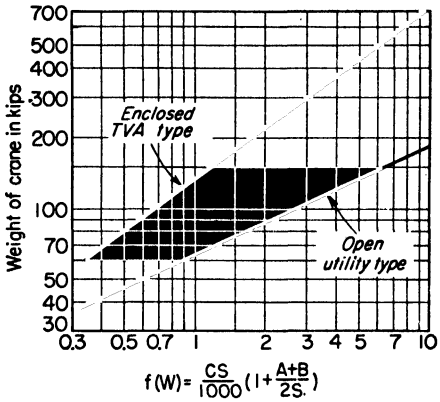


FIG. 28.—Weight of traveling gantry cranes for crest gates.

ACKNOWLEDGMENTS

The greater portion of the material presented in this section of the first edition (Sec. 10 therein) was accumulated by its author, James S. Bowman, during his association with several well-known engineering organizations. Experience of the same nature was invaluable to the authors in the preparation of the new article dealing with crest gate hoists. In preparing the articles on estimating data, it was necessary to draw heavily on the work of H. G. Gerdes, F. L. Boissonnault, the Tennessee Valley Authority, and the Bureau of Reclamation, as well as on personal experience; the authors are especially grateful to the D. J. Murray Manufacturing Company and the S. Morgan Smith Company for their generous contribution of data on gate hoist installations.

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SECTION 9

HIGH-PRESSURE OUTLET WORKS

By Phillip A. Kinzie

High-pressure outlets may be considered as such when they control and regulate the release of water for irrigation, domestic, and power demands from dams, reservoirs, and conduits under heads in excess of 75 ft when the quantity Q involved is not less than 150 cfs discharge when operating at maximum capacity. Conventional slide gates, gate valves, and similar equipment commonly employed for release and regulation of flow under lower ranges of head are usually found to be inadequate when used for the more severe conditions of operation under heads in excess of this value.

Starting with the equipment first developed to cope satisfactorily with the higher heads demanded by irrigation practices, we find that the most prominent are as follows:

STANDARD HIGH-PRESSURE GATES

A standard high-pressure gate consists of two rectangular heavily ribbed castings which form the water passage upon the upstream and downstream sides of the leaf for a longitudinal distance approximately equal to that of the horizontal width of this passage. These two castings, known as the *gate frames*, are bolted together where their adjacent mating vertical-flanged faces are brought into watertight engagement. These frames are arranged to form a vertical slot or recess in the walls on each side of the water passage, within which the gate leaf is received, supported, and guided in its opening and closing movements. The leaf is usually provided with bronze seal bars on its downstream face which rest, when the gate is closed, upon similar bronze seat bars of a different composition attached to the downstream sides of these slot walls. The full water load against the upstream face of the leaf is carried to its seal bars and transferred into the seat bars on the gate frames, and in consequence whenever the leaf is moved upward to open the gate or downward to effect closure, the leaf seal bars act in similar manner to the runners on a sleigh and are thus forced to rub or slide upon the mating surfaces of their supporting seat bars attached to the gate frames. It is from this characteristic that the term *slide gates* derives its origin. An extensive series of tests were made by the Bureau of Reclamation to determine those bronzes best suited to carrying heavy loads under sliding contact and capable of resisting abrasion under these conditions. These tests and experiments led to the development of class C and class D bronzes. These two different compositions, when used in rubbing or sliding contact one against the other, are capable of sustaining 3,000 psi load without seizure when water lubricated, but for design purposes the bearing pressures where sliding occurs should not exceed 700 psi (see page 344).

The slot recesses in the gate frames are continued upward in the envelope or bonnet castings whose lower flanges are bolted to a flange formed on the top of the frame castings. The gate leaf when raised to its open position is received within this bonnet, the water passage being left free. The gate frames and bonnets are strongly ribbed both vertically and horizontally to distribute the heavy reactions set up by the hydraulic hoists in the opening and closing cycles of operation, but they are not usually

designed to take the internal water pressures to which they are subjected in service. This is cared for by circling these parts with hoops of reinforcement bars and then encasing the gates in concrete up to the joint between the top bonnet flange and the bonnet cover.

The bonnet cover is designed to resist the maximum internal water pressure and is bolted to the upper flange of the bonnet (see Fig. 1). This cover is heavily ribbed and converges into the circular flange that forms the lower head of the hydraulic hoist cylinder which rests upon and is bolted to it. The hoist cylinder continues upward

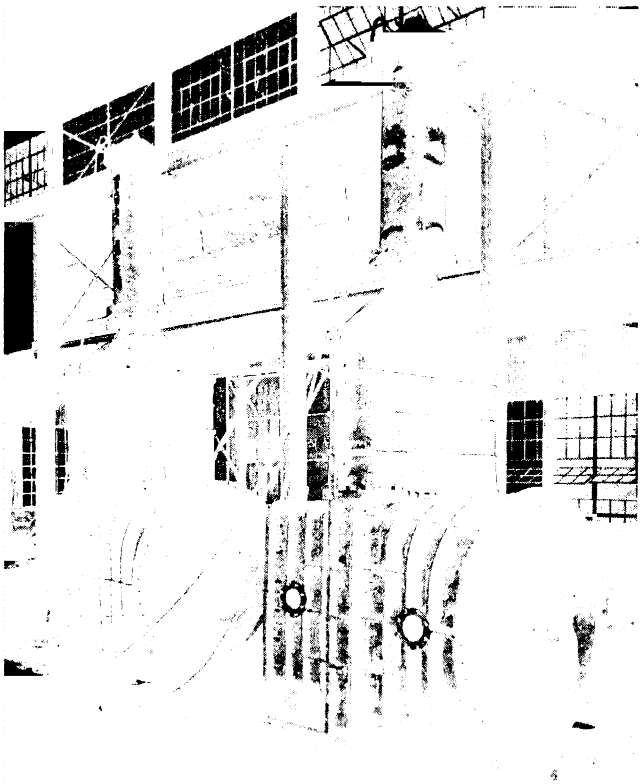

Fig. 1.—Five- by five-foot high-pressure gates with built-in transitions from square to round in downstream frames. Arranged to receive short-body 60-in. needle valves, as shown in Fig. 22.

and terminates in a similar upper flange to which the cylinder head is bolted. Within the cylinder is a semisteel or bronze piston attached to the upper end of the gate stem. The stem extends downward through stuffing boxes in the bonnet cover which prevent the escape of oil from the bottom of the cylinder and the escape of water from the interior of the bonnet below. The lower end of the stem continues into the gate leaf and is fastened to it by a suitable nut. First-grade packings of graphited flax or of the built-up chevron type are used in the stuffing boxes for these stems.

The piston in the hydraulic cylinder is made oil-tight by means of a series of four to six conventional metal piston rings closely fitted into mating slots machined in the periphery of the piston. The hydraulic hoists are designed for pressures ranging from

750 up to 1,500 psi, according to requirements, and are operated by either triplex oil pumps or high-speed nitrided-steel gear pumps, with capacities ranging from 5 to 30 gpm, depending upon gate sizes and operating time-cycle requirements. A light-grade inexpensive lubricating oil, such as S.A.E. 10 or Navy Specifications 2110-Saybolt Universal 90 to 120 sec at 130F with a viscosity index of approximately 70, is used in the piping systems. These consist of pressure delivery and exhaust return lines fitted with air chambers, pressure-relief valves, and high-grade straight-way lever-operated plug valves for controlling the gate operations.

The general arrangement of the main parts of these gates ensures easy assembly, facilitates maintenance, and provides a completely self-contained structure capable of withstanding the reactions produced by the hydraulic cylinder which may, in some instances, exceed 1,000,000 lb.

In the earlier installations, the importance of avoiding every possible abrupt change in the water-passage surfaces, such as recesses and bolt heads, and of the prime necessity for the admission of an ample air supply on the downstream side of the gate leaf was as yet unknown; consequently these earlier installations had been in service but a relatively short time when it was discovered that the metal adjacent to the downstream side of each recess, bolt, or other departures from continuity of the water-passage surfaces was being eroded away at an alarming rate. The gravity of this condition was accentuated at that time by the fact that its cause was unknown, and, in consequence, preventative measures were experimental and consequently of a very doubtful character. Among the first of the gates of this general character to be put into service under relatively high heads were the two sets of three gates each, 5 by 10 ft in size, installed in the Roosevelt Dam, Salt River project, Arizona, in 1908. These gates were intended for service under a maximum head of 220 ft. They were operated by hoist cylinders in wells extending to the top of the dam from an elevation 33 ft above the sluiceway floor. These gates were put into service while the reservoir water surface was still much less than its maximum, but trouble soon developed, and when an inspection of their condition was made it was revealed that both they and the tunnel below them had been seriously damaged.

The concrete and metal linings were found to have been loosened, eroded, or washed out, the bolts and fastenings of the gates themselves had become loosened and in some cases were missing, the bronze seats damaged by blows from the loosened parts, and the bronze roller trains used behind the gate leaves either broken or carried entirely away. The concrete in the roof, floor, and walls of the outlet downstream from the gates was badly eroded by the high-velocity flow, and holes 4 to 6 ft deep had been torn in the living rock of the tunnel floor.

After the damage had been repaired, these gates were again put into service, but continued release of water through them damaged them still further, finally forcing their abandonment after it was found that they could no longer be safely used. The tunnels were accordingly plugged with concrete, and two 38-in. needle valves, guarded by bronze slide gates, were installed.

Four gates of this earlier type were installed at Pathfinder Dam, North Platte project, Wyoming, in 1909. These gates were 44 in. broad by 77 in. high and were installed in the north tunnel which is cut through solid granite.

When the discharge from these gates was sufficient to nearly fill the tunnel below, reverberating and hammering sounds made themselves evident at the tunnel outlet. The intensity of these increased as the flow through the gates increased, until these hammering, pounding reverberations resembled an approaching thunderstorm, varying from low and indistinct rumblings in the incipient stages to sharp peals and explosions during the periods of maximum discharges and attaining such violence as to cause the dam and canyon walls to tremble. A draftsman's triangle hanging on a nail

on the wall of the operator's house, located some distance back from the canyon rim, was observed to dance and quiver in synchronism with the more violent explosions.

When the water was shut off, it was found that large masses of rock had been torn out of the solid granite walls, roof, and floor of the tunnel, portions of the concrete below the gates were damaged or destroyed, the ¾-in. plate-steel linings were torn as if they had been made of paper, anchor bolts were sheared off in some instances and their nuts stripped from the bolts in others. The gates had suffered but little injury, due primarily to their having been operated in their wide-open positions.

An inclined air shaft was then cut through the roof of the tunnel immediately downstream from the gates, and the damage in the tunnel was repaired. When water was again turned through the gates, it was found that the air shaft prevented recurrence of the phenomena that had caused the previous damage.

The experience derived from this and other earlier installations demonstrated that it was essential to admit air in large quantities into the water passages closely adjacent

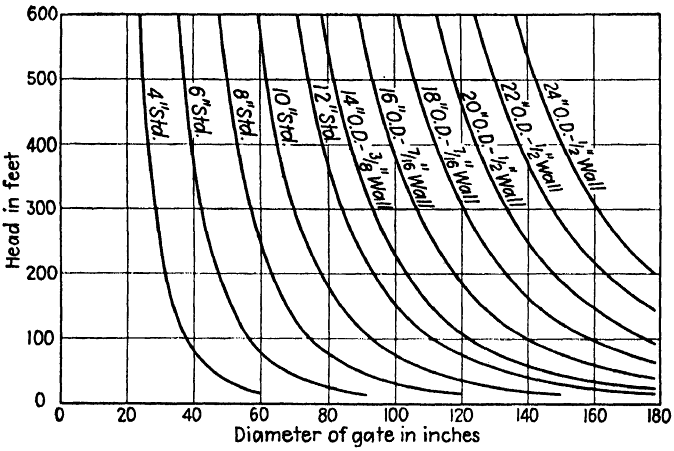

FIG. 2.—Diagram for determining air-vent pipe sizes for ring follower, paradox, and ringseal gates.

to the downstream leaf faces and to keep the walls of the water passages free from cavities or projections. Continued observations disclosed that when these precautions were followed the vibration-implosion phenomena and the destructive erosive-pitting effects later to become known as *cavitation* were practically eliminated even when operating this equipment under heads considerably in excess of those at which these troubles were first encountered in earlier installations. As the result of data derived from many later installations, the pipe sizes required for different sizes of gates under varying heads were determined and are given in Fig. 2. This chart can be used for square or rectangular gates by finding the equivalent cross-sectional area of a round gate on the chart.

Experience derived from many installations under widely diversified conditions of operation, and over a considerable period of time, quite conclusively proves that although gates of this type can be successfully operated in their fully opened positions under heads as high as 300 ft, they are not satisfactory for continuous operation at partial openings for heads in excess of 75 ft. In other words, it is not recommended that they be used for regulation of water released under heads higher than 75 ft, as it has been found that their use at partial openings, even with ample air admitted to their downstream faces, is accompanied by rapid deterioration of their

principal members, probably caused by the high-velocity water passing beneath them, setting up acute vibrations in the leaf and supporting parts.

When conditions of this character arise, the usual procedure is to place either a needle valve or a tube valve below the gate and then regulate the discharge with one or the other, the gate valve being left wide open as a guard or emergency gate for shutting off the flow should the regulating means (needle or tube valve) require servicing or become inoperative. If the requirements are such that close control at small openings of the valve are required during prolonged periods of time, *i.e.*, if the valve must be operated at a small percentage of its total capacity for long intervals, then a needle valve will be found to be best adapted to these conditions. If the valve will be required to operate at 20 per cent or more of its ultimate capacity, during most of the time it is in service, and if precise refinements of release are not required at openings of 20 per cent or less, then a tube valve will be satisfactory for such service and will afford a more economical installation because of its lower initial cost.[1]

High-pressure gates have a wide field of use. They are used in the outlet conduits of storage reservoirs for outlet and control-regulation purposes where the head is less than 75 ft. An outstanding example of this use is the American Falls Dam, Minidoka project, Idaho, where a battery of twenty 5- by 5-ft high-pressure gates is used to release and regulate the irrigation and power water through the dam under a maximum head of 70 ft. These gates were embedded in the concrete of the dam, with their bonnet covers and hoist cylinders extending upward through the floor into the operating gallery. Two common header pipes extend throughout the length of this gallery, with branches provided with individual segregating valves opposite each gate hoist cylinder, so arranged that by opening the top valve the upper header pipe is placed in communication with the upper end of the cylinder, whereas the bottom header pipe is placed in communication with the lower end of the hoist cylinder by opening the valve on its connecting pipe.

Midway of the galleries' length, a dual set of motor-driven high-pressure oil pumps with tanks, valves, etc., is located in an alcove and so arranged that by opening two valves and closing two others either the upper or the lower header pipe may, at the option of the operator, be connected to the high-pressure delivery line from the pumps, while the pump suction line will likewise be connected to the other header pipe. By this means, the operator can manipulate the valves in the pump chamber either to put pressure in the top header to lower, or in the lower header to raise, any gate or gates he may select to operate.

High-pressure gates are often used arranged in tandem in sluiceways through the bases of high dams. Under conditions of this nature, no attempt is made to use them for regulating purposes, as they are either fully closed when not in service or fully opened when in use. A notable example of such use are the six 5 ft 8 in. by 10 ft 0 in. sluiceways through the base of Madden Dam, Panama Canal Zone. Each of these six conduits is provided with a tandem pair of high-pressure gates. Four gates for two conduits are included in each of the three operating chambers, which are located near the upstream face of the dam. The center of the three chambers is provided with the oil pump, motor, tank, selector valves, etc., with two pipe headers extending to the chambers on either side. The upstream gate of each tandem pair is held in reserve as an emergency gate and is never opened or closed (except in emergencies) until after the downstream gate has been closed and the pressures balanced by means of the by-passes provided for that purpose. Normally it is kept in the raised (open) position, suspended from a semiautomatic gate hanger suspended from a bolt embedded in the concrete roof of the gate chamber. The downstream service gates are each provided with automatic hydraulic gate hangers that are operated by the oil pressure in the lines which causes the hoists to function only after the hanger has released its grip upon

[1] It has been found that hollow-jet valves will satisfy this condition with still greater economy.

the stem extending from the piston through the top cylinder head. These hangers are entirely automatic and require no manipulation by the operator to cause them to function.

Standard designs for square and rectangular high-pressure gates ranging in sizes from 2 ft 9 in. by 2 ft 9 in. to 5 ft by 5 ft, and from 3 ft by 4 ft to 4 ft 9 in. by 12 ft 0 in. have been developed. These gates are arranged for operation under heads up to 90 ft when equipped with cast-iron leaves, 140 ft when supplied with semisteel leaves, and for heads up to 250 ft when cast-steel leaves are used. The hydraulic hoist sizes are increased in similar increments to accommodate these three ranges of head, and the gate bonnet covers are made with three sizes of flanges to receive their respective hoist cylinders. Designs for special service gates in odd sizes such as 6 ft 0 in. by 7 ft 6 in. for 310-ft operating head and 600-ft static head have been developed and built.

RING-FOLLOWER GATES

The ever-mounting demands for larger gates and their operation under higher heads have passed the upper limitations generally accepted for the high-pressure gates just described and have led to the development of what are known as *ring-follower gates* (see Fig. 4). The water passageways through these gates, instead of being square or rectangular in cross section as is the case in the high-pressure gates, are made circular, which has the advantage of eliminating transitions in the conduits from circular to square on their upstream sides and from square or rectangular back to circular again on their downstream sides when installed in circular conduits or pipe lines.

The waterway being circular permits the installation of a bellmouthed inlet at the face of the dam, which improves the over-all efficiency of the conduit installation and prevents the pitting erosion and eating away of the concrete and metal work which has been found to occur adjacent to the entrance face when square or rectangular conduits have been employed under high velocities for prolonged periods of service. As the result of extensive tests in the Bureau of Reclamation laboratories, and later confirmed by tests of the 102-in conduits through Grand Coulee Dam (see Fig. 3), it has been found that a trumpet-shaped bellmouth with elliptically curved profile, made to the equation $\frac{X^2}{(0.5013D)^2} + \frac{Y^2}{(0.15041D)^2} = 1$ with D = diameter of conduit, having a major orifice diameter 1.30 times the nominal inside diameter of the conduit, and reducing to and becoming tangent to the conduit diameter at a distance equal to one-half the nominal inside diameter of the conduit, downstream from the conduit face, gives very satisfactory results.

The elliptical equation given above provides an inlet orifice whose jet will be sufficiently oversize to ensure contact at every part of the resulting converging walls.

This elliptical curve is based upon the shape of a free discharge jet issuing from a sharp-edged circular orifice. The ratio of area of contraction from the orifice area to the minimum cross-sectional area of the jet is 0.592. The ratio of the orifice diameter to the minimum jet diameter is 1.300, and the ratio of the minimum jet diameter (conduit) to the orifice diameter (inlet) is 0.760.

If the inlet face of the dam or structure is on a slope, or if an elbow is required to make the gate portion of the conduit horizontal, then a converging elbow immediately downstream from the bellmouthed entrance is recommended, and in order to avoid forming negative pressures by centrifugal force on the inner radius of the elbow, the elbow radius should be made larger than would otherwise be normally employed. As an alternative, the elbow can be of constant diameter and a taper reducer placed in the horizontal run with its large end connected to the elbow outlet. If the elbow angle is 3. deg, the elbow radius should be approximately $21.2D$, the elbow inlet diameter approximately $1.035D$. The bellmouth diameter should be increased to

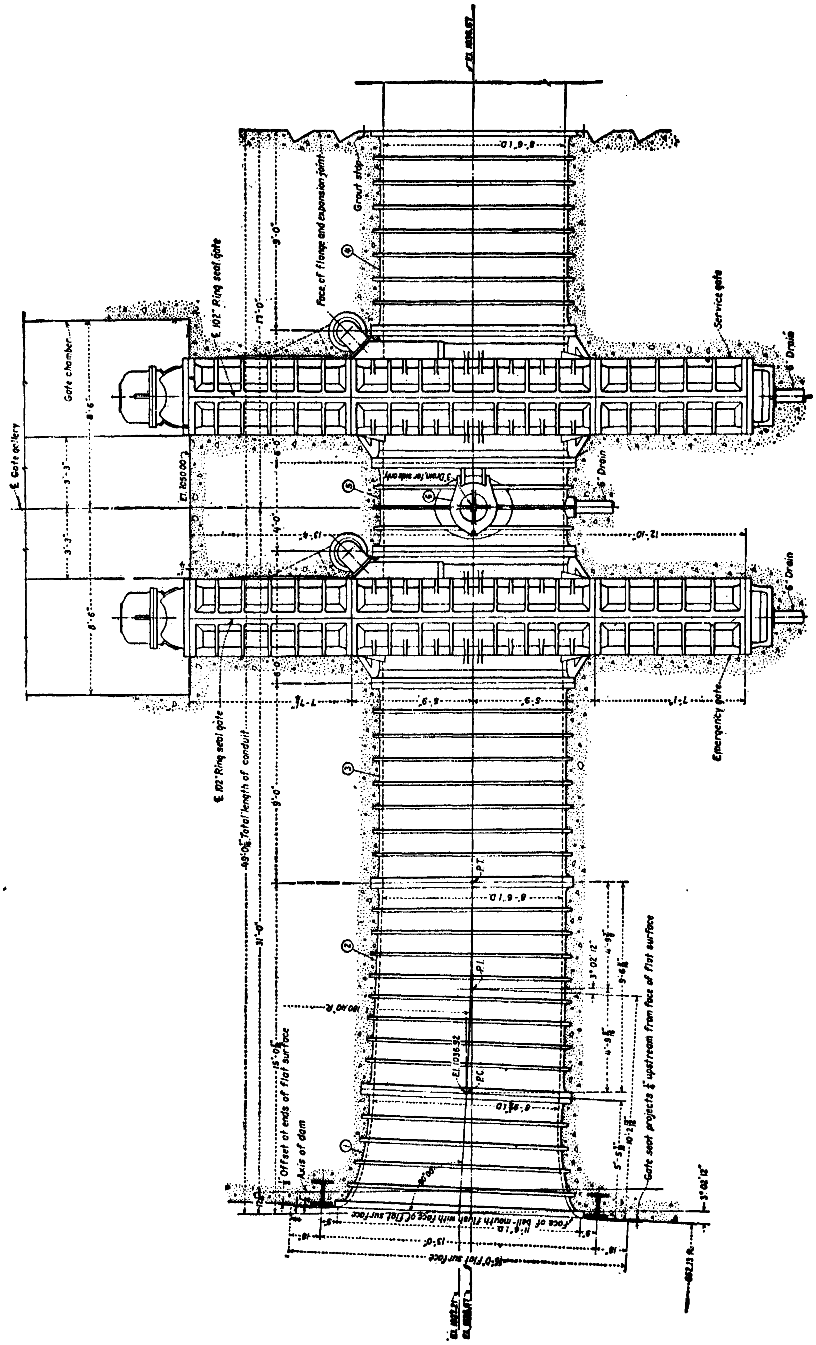

FIG. 3.—102-in. ring-seal emergency and service gates, with bellmouth inlet casting normal to sloping upstream face of dam, and connected to horizontal run of conduit lining by a long-radius elbow. Forty of these gates operate under 250-ft head at Grand Coulee Dam.

$1.35D$, its outlet to $1.035D$, and its length to $0.641D$, which includes a short tangent length of $1.035D$ pipe at its outlet end. This will give an over-all length of bellmouth and elbow of $1.765D$. If the elbow is $8\frac{1}{2}$ deg, the radius becomes $9.675D$ and the inlet diameter $1.04D$. The bellmouthed entrance would become $1.355D$, its outlet $1.04D$, its length $0.641D$, and the over-all length of bellmouth and elbow $2.078D$.[1] For intermediate elbow angles, the preceding values can be interpolated; however, model tests of the new arrangement should be made, if possible.

The importance of these bellmouthed entrances to conduits wherein the velocities of flow are 50 fps or more is just beginning to be appreciated, and much more attention should be devoted toward obtaining satisfactory entrance conditions to such conduits when subjected to long periods of continuous use than has been done heretofore. Besides preventing cavitation, it has been found that these circular bellmouths quite appreciably improve the over-all discharge coefficient of the conduit.

A still further advantage which these ring-follower gates provide is the fact that when they are fully opened they afford an unimpeded passageway that is essentially the equivalent of the conduit within which they are inserted, or, expressing this in a different way, the open vertical slot, common to all conventional high-pressure gates, in the wall on each side of the water passage and exposed to high-velocity flow, is eliminated in the ring-follower gates, whose water-passage walls afford practically unbroken continuity for stream flow, and so avoids the erosive action that occurs adjacent to these slots in high-pressure gates when subjected to operating heads in excess of 250 ft.

This very essential feature of smooth, continuous water-passage surfaces is effected in the ring-follower gate by attaching a metal ring, of the same internal diameter as that of the conduit, to the under edge of the gate leaf and so positioning it that when the leaf is raised to the open position the interior surface of the ring is alined with the circular wall surfaces of the water passage on either side, as is seen in section *BB* of Fig. 4.

Owing to the fact that the ring is attached to the gate leaf and consequently follows it up and down in its opening and closing movements, these gates derive their name *ring-follower* gates.

The presence of the follower ring beneath the gate leaf necessistates a recess or envelope below the conduit to receive the ring when the gate leaf is lowered for closing. This recess is provided by a lower bonnet that is bolted to the underside of the gate frames in the same manner as the bonnet above them, which receives the bulkhead portion of the gate leaf when it is raised to the fully opened position. The lower end of the lower bonnet is closed by a cover bolted to its lower flange face. In this lower cover, one or more drain outlets of ample area are provided to permit draining the interior of the entire gate and to flush out any sediment that may collect in the bottom of the bonnet. In those localities where the water passing through the conduits is heavily laden with silt or sand, care must be devoted to ensure adequate provision for removal of any foreign material that might otherwise collect in this lower bonnet and eventually make the gates inoperative.

An air-inlet manifold is cast integral with the downstream gate frame, around the upper exterior surface of the water passage wall, and a series of $1\frac{1}{2}$ in. diameter cored holes, equally spaced in staggered relationship in four circumferential rows, admit air to the conduit in close proximity to the downstream face of the gate leaf, as may be seen in section *BB* of Fig. 4. Two symmetrically positioned inlet flanges admit air to this distributing manifold from an air-inlet pipe carried to a screened inlet located at an elevation above high water level in the reservoir.

The gate leaf is usually made in two parts, bolted together, consisting of the upper bulkhead section and the lower ring-follower unit. On the downstream face of the leaf

[1] Tests of both the prototypes (made in the field) for the above examples show that for the first one having an over-all length of bellmouth and elbow of $1.765D$, $C = 0.98$ in $Q = C\sqrt{2gh}$ and for second one with over-all length of $2.078D$, $C = 0.97$.

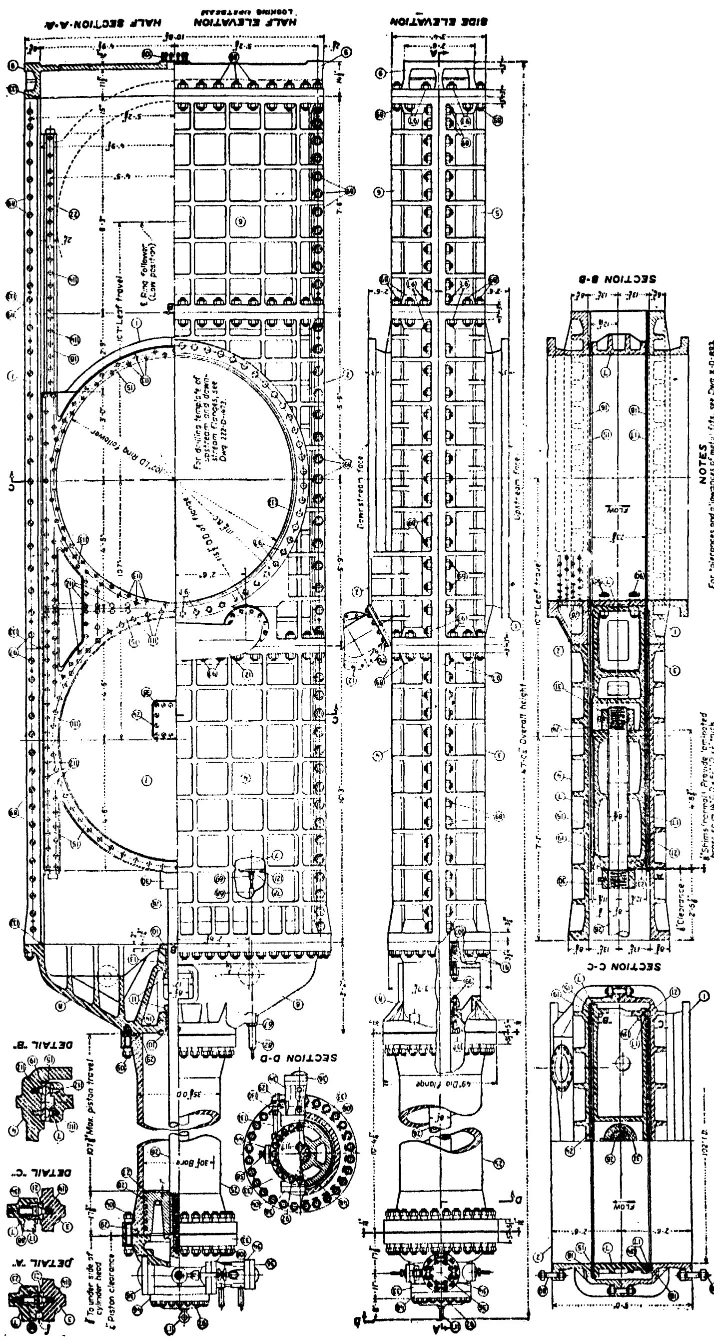

Fig. 4.—102-in. follower emergency gate with automatic hydraulic gate hanger. Twenty of these gates, under 356 ft maximum head, guard the 102-in. paradox service gate (see Fig. 5) at Grand Coulee Dam.

are facing rings and vertical runner strips of class *D* bronze, and similar rings and strips of class *C* bronze are fastened with flat-head bronze screws to the upstream faces of the downstream gate frames and bonnets to form the sliding ways upon which the vertically moving leaf and ring follower are supported and seated. In gates of the larger sizes, difficulty has been experienced in obtaining sound castings of class *D* bronze, and in some instances it has been found necessary to use No. 6 bronze, which has been proved to be quite satisfactory for making the relatively large castings involved. Class *C* bronze is usually used for the seats on the gate frames and No. 6 bronze (Federal Specifications QQ-B-691a) for the seats on the gate leaves. Physical properties and chemical compositions for these bronzes are as follows:

Chemicals	Class C	Class D	No. 6 Bronze
Copper	82.00–83.00	82.00–83.00	85.00–89.00
Tin	6.75– 7.50	4.75– 5.50	7.50–11.00
Lead	4.50– 5.00	7.75– 8.25	0.00– 1.00
Zinc	5.00– 6.00	4.00– 5.00	1.50– 4.50
Ultimate	....	....	35,000 psi
Elongation	....	....	18%

The leaf castings are usually made of high-strength gray iron of about 50,000 psi tensile strength for the lesser heads, of steel castings for higher heads, and of alloy cast steel having a minimum ultimate strength of 80,000 psi and a yield point of not less than 50,000 psi for still higher heads.

The method of attaching the gate stem to the leaf, the general construction of the upper bonnet cover, the hoist cylinder, and the piston are quite similar to those employed in the latest type of high-pressure gate construction previously described.

The upper cylinder head, however, is modified to receive a cylinder-head gate hanger which is enclosed and contains a pair of swinging jaws operating in a horizontal plane that engage the underface of the conically shaped stem extension cap. This cap is retained by a safety stud which is necked to break without injury to other parts of the equipment should malfunctioning of the hanger occur. A small oil-operated cylinder actuates a device that wedges the jaws apart to release the hanger whenever oil pressure is introduced into the lines to operate the hoist.

PARADOX GATES

As a companion to the ring-follower emergency gates, there has been developed by the engineers of the Bureau of Reclamation a service gate for opening or closing off the flow through conduits of sizes up to 102 in. in diameter under heads up to and in excess of 600 ft. Gates of this type when inserted into and used in conduits are known as *Paradox* gates.

Paradox gates have a number of features in common with their companion ring-follower gates inasmuch as they provide circular waterways with practically unbroken flow surfaces in the same manner and by means of similar ring-follower members beneath and attached to their leaves (see Fig. 5). They likewise have bonnets extending beneath their conduits to receive their ring-follower members when their leaves are lowered for closure and are also provided with air inlets in the upper portions of their waterways (see sections *BB* and *CC*) immediately downstream from their gate leaf faces in substantially the same manner as that employed in the ring-follower gates previously described, and here their similarity ends.

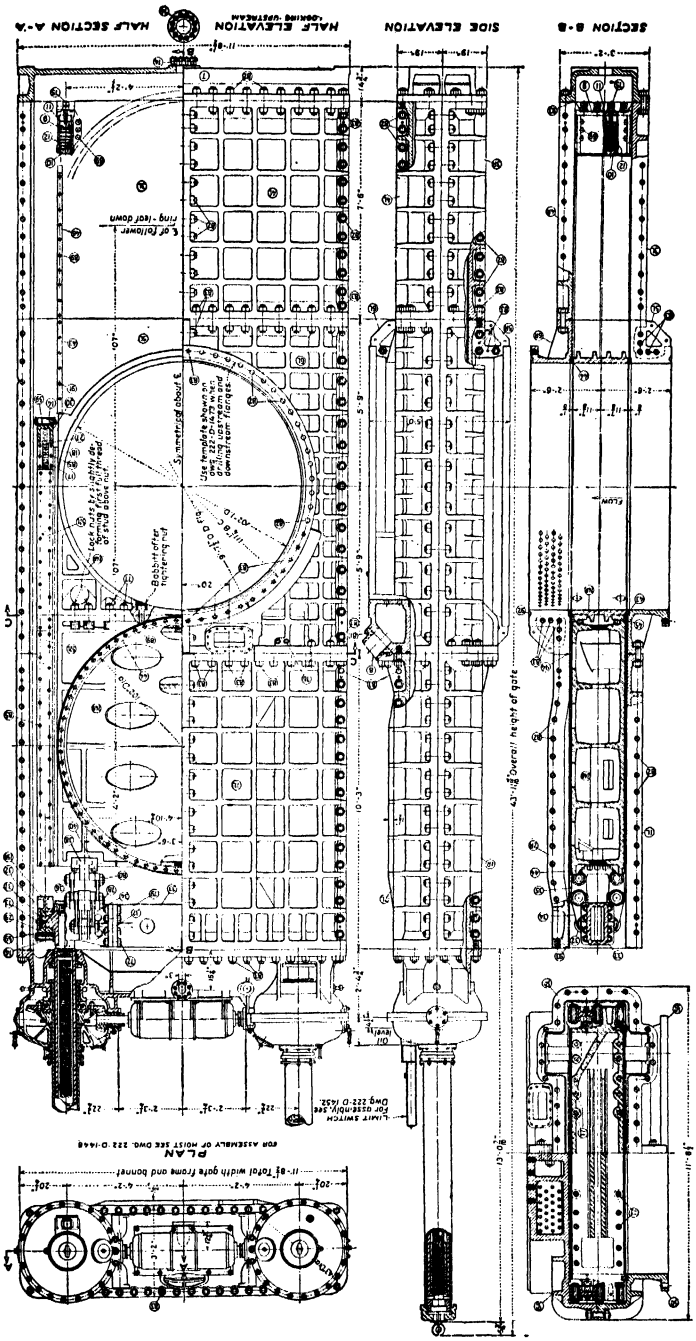

FIG. 5.—102-in. Paradox motor-operated service gate. Twenty of these are installed under 356 ft maximum head at Grand Coulee Dam, in the lowest tier of river outlets.

Paradox gates descend or ascend in closing or opening while supported along either side (see section CC) by vertical endless trains or belts of rollers that extend throughout practically the entire height of the gate leaf and its follower ring. The leaf is lowered in closing until the bronze seal ring affixed to the downstream face of its bulkhead portion is brought into horizontal alignment with the mating bronze seat ring enclosing the water passageway in the adjacent face of the downstream gate frame, whereupon further downward movement of the gate leaf is arrested by lugs on the underside of the follower ring coming into contact with stops arranged for that purpose in the bottom bonnet cover. Meanwhile the roller carriage continues its descent approximately 11 in. further and in so doing withdraws the inclined roller trains interposed between the carriage and the gate leaf and so permits the gate leaf to move horizontally about ⅛ in. downstream until its seal ring contacts the seat ring, so completing closure and allowing the entire water load upon the upstream face of the leaf to be transferred through its seal ring and into the supporting seat ring of the gate frame, thus ensuring watertightness.

In opening the gate, the roller carriages first start moving upward until they engage their inclined roller trains with the downstream inclined surfaces of the leaf, and continued upward movement of the carriages then forces these roller trains to wedge the leaf away from contact with its seat, causing a horizontal separating movement of the leaf in an upstream direction of about ⅛ in. When this has been accomplished, the toggles connecting the leaf and the crosshead are fully extended, whereupon the gate leaf with its ring follower, the two inclined roller trains, and the two roller carriages all move upward as a unit until such time as the ring follower is brought into axial alignment with the water passage or conduit, when limit switches stop the hoist and the upward movement ceases and the opening cycle is then completed.

From consideration of the foregoing description and inspection of the reference drawings, it will be seen that no sliding contact between seating and sealing surfaces ever occurs while these gates are passing through their opening or closing cycles of operation, and in consequence those surfaces are not subjected to the rapid wear and deterioration that occurs when sliding contact under high-intensity loadings is present.

Inasmuch as all movements of the gate leaf occur while it is supported and transported on the roller-train systems as described, the force or power input required to operate it is relatively small.

Paradox gates are not suitable for operation at partial openings. They are used as service gates in the lowermost of the three tiers of conduits at Grand Coulee Dam because the quantities of water handled there are so large that sufficiently close regulation of reservoir release can be had by opening or closing one or more of the battery of twenty 102-in. gates in the lower tier, or of similar gates in the two upper tiers, without necessity of resorting to operation at partial openings of any of the gates.

RING-SEAL GATES

Since the fabrication and installation of the 102-in. Paradox gates at Grand Coulee Dam, the Bureau of Reclamation engineers have further advanced and simplified the design and fabrication of high-pressure conduit gates by inventing, developing, fabricating, and installing eighty 102-in. ring-seal gates in the middle and upper tiers of conduits at Grand Coulee Dam.

From the assembled drawing of this gate shown in Fig. 6 and the assembly sections in Fig. 7, it will be seen that this gate is quite similar to the Paradox gate, just described, in that it is operated by an electric motor hoist with twin screw stems and has a ring follower beneath the leaf. Further inspection, however, will reveal that the hoist stems, instead of extending vertically above the gear cases, are turned downward into the bonnet below, and it will also be seen in section *CC* of Fig. 7 that the endless

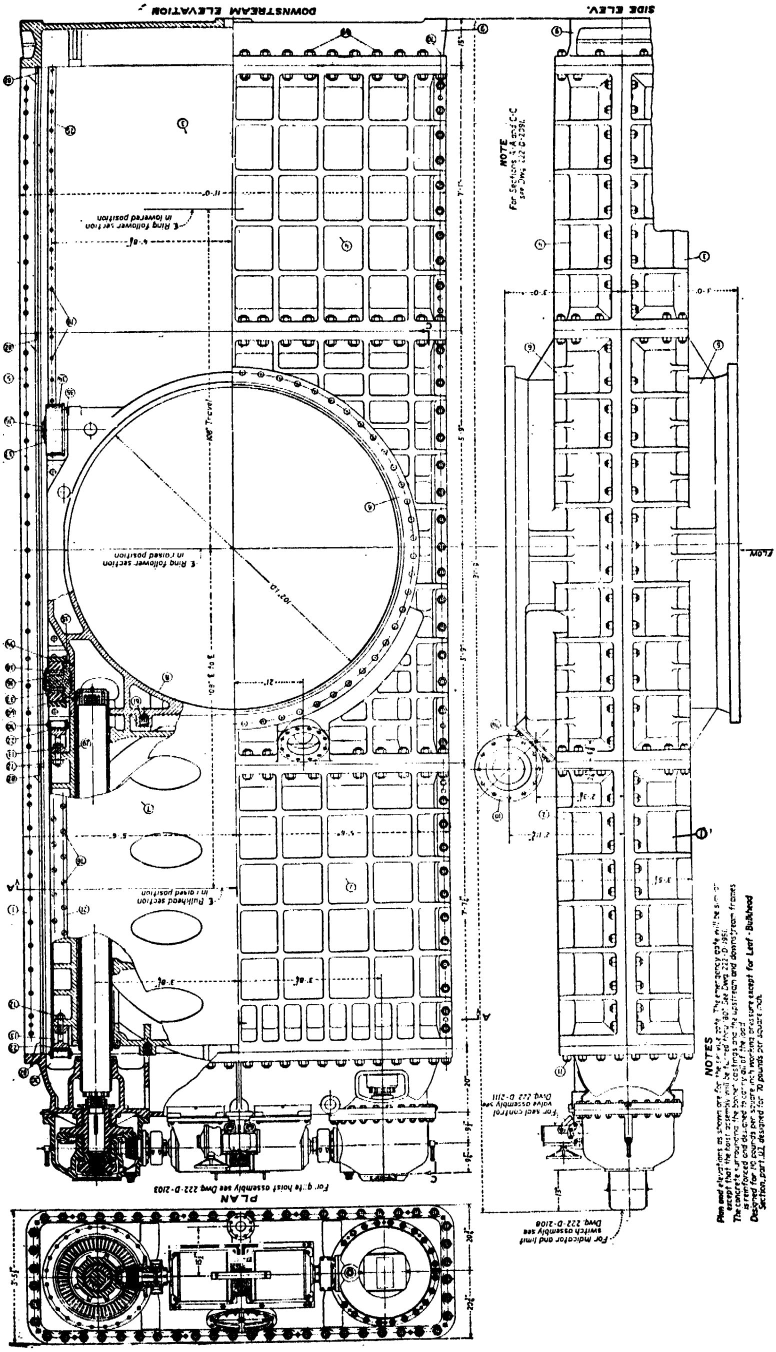

FIG. 6.—102-in. ring-seal electric-motor-operated gate as used under 250-ft head. Note stems inverted into interior of gate structure. See Fig. 8 for general installation.

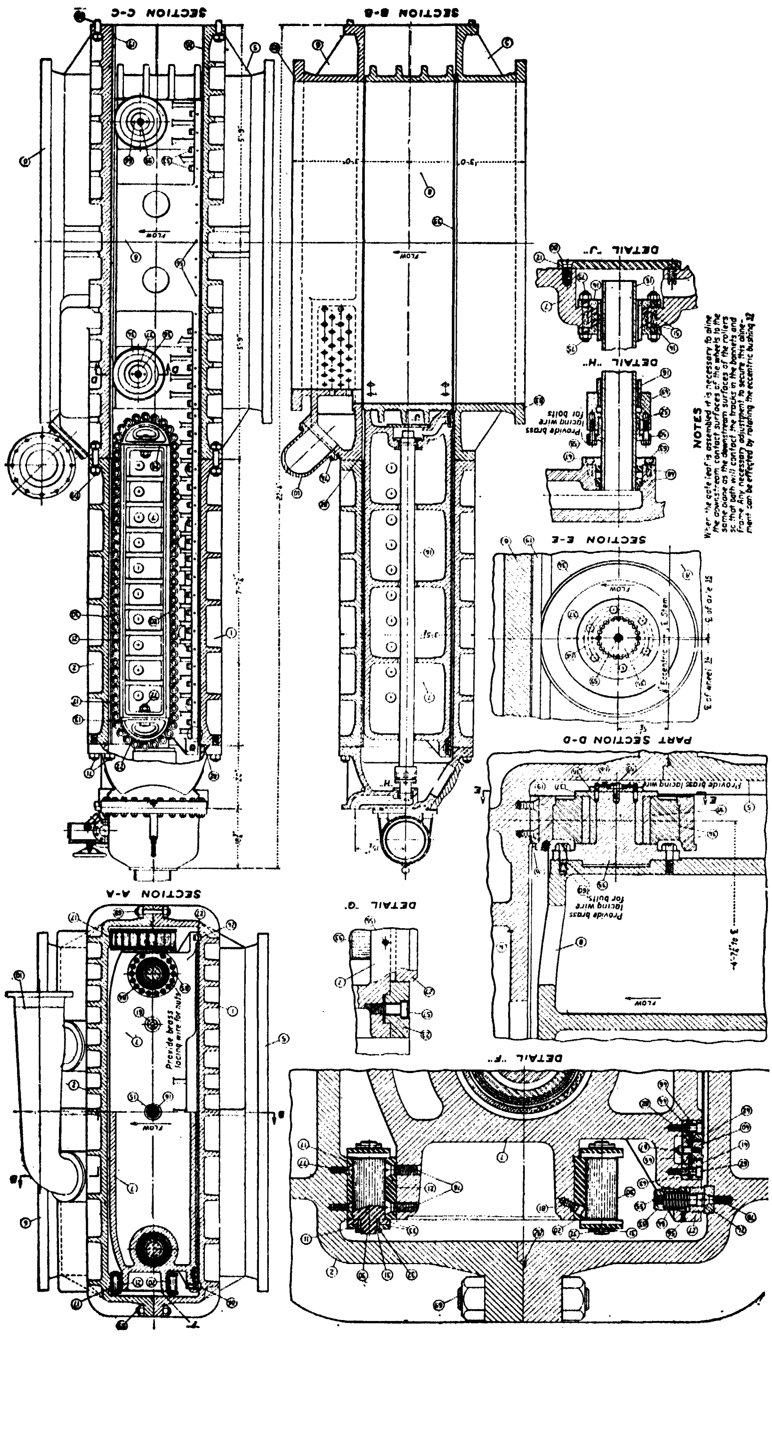

FIG. 7.—Subassemblies and sections of 102-in. ring-seal gate, illustrating hydraulically operated ring seal, eccentric adjustment of ring-follower wheels and telescoping seal pipe.

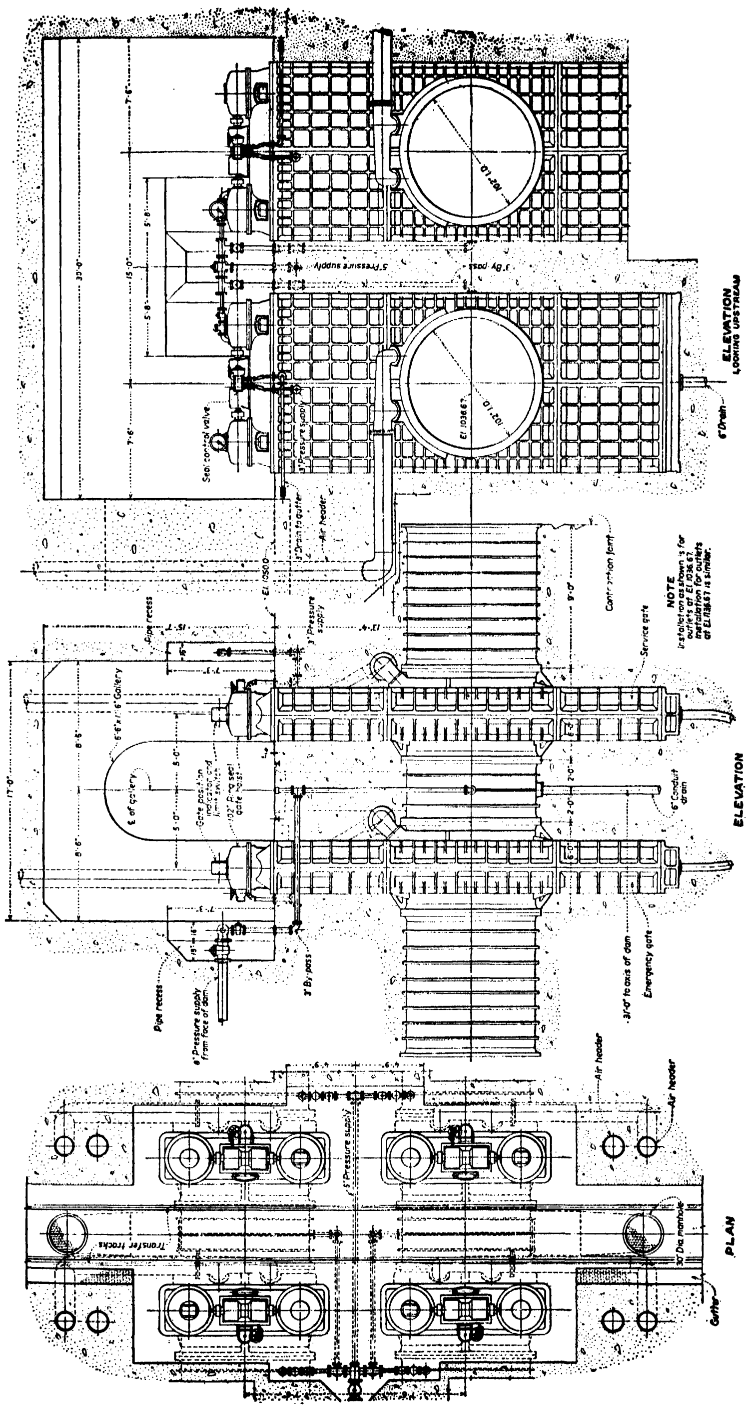

FIG. 8.—Tandem installations of 102-in. ring-seal emergency and service gates as installed in the middle and upper tiers of river outlets of Grand Coulee Dam. Eighty of these motor-operated units are employed there.

roller trains have been shortened so that they support the leaf only, whereas the ring-follower portion is carried by four wheels. It will also be noted that the inclined-wedge roller trains and the crosshead with its toggle mechanism, as used on the Paradox gate, have all been eliminated. Instead of the leaf being moved in the direction of or counter to stream flow to seat and unseat it, as is the case in the Paradox gate, the same office is accomplished by causing the sealing ring carried by the gate leaf (see section *BB* and detail *F* of Fig. 7) to be forced outward (downstream) to seating engagement with the seat ring in the downstream gate frame, by means of hydraulic pressure in the annular chamber formed in the bulkhead (leaf) casting, within which the sealing ring is floatingly mounted, as shown in detail *F*. Before opening movement of the leaf can occur, the hydraulic pressure is released from behind the sealing ring, and the conduit pressure forces it back upstream away from contact with the stationary seat ring, the hoist being thus allowed to raise the leaf vertically while supported on its roller trains and wheels and so open the gate without any sliding action occurring in the same general manner as the Paradox gate.

Inasmuch as the ring-seal gate has no protruding hoist stems and the vertical height of the crosshead and toggle mechanism of the Paradox gate has been eliminated, the over-all height of this gate is quite substantially reduced from 43 ft 11 15/16 in. to 31 ft 9 in., its weight from 186,000 to 148,000 lb; thus more economical to build and install.

Three of these 102-in.-diameter ring-seal gates have been installed in the Seminoe Power Plant at Seminoe Dam, four of the same size in the Hiwassee Dam for Tennessee Valley Authority, and 80 of the same size in the middle and upper tiers of outlet conduits at Grand Coulee Dam. A typical gate chamber with two tandem aligned pairs of gates as installed in the middle and upper tiers at Grand Coulee Dam is shown in Fig. 8, where the low headroom required by this improved design is clearly shown, likewise the arrangement of the air inlet headers, the control and drain piping, and the relative position of the 6 ft 6 in. by 11 ft 6 in. service gallery with its tracks for a specially built servicing car.

REGULATING VALVES

After the rectangular gates at Roosevelt Dam had, by their rapid deterioration, definitely demonstrated the impracticability of reservoir-outlet regulation with that type of equipment in 1908, O. H. Ensign, chief electrical engineer of the Bureau of Reclamation, found a means to overcome this trouble by inventing the Ensign-type needle valves, which were placed at the inlet ends of the conduits through the dams and operated by hydraulic chambers within themselves. These valves opened and closed easily and were in general far superior to anything previously available. They had a number of serious disadvantages, however, for where the water had a corrosive or scale-forming tendency they soon became inoperative, and as they were located on the upstream faces of the dams, it was then necessary to drain the water down to expose them for servicing, which in some cases meant disastrous waste of precious water that was impossible to replace.

From experience derived from the operation and servicing of these Ensign valves, it became increasingly evident that equipment devoted to such service should be placed where it was readily accessible at all times, so that immediate attention and servicing could be performed whenever required, without having to sacrifice irreplaceable stored water or waiting until normal reservoir operations had lowered the water level sufficiently to expose the valves for servicing.

The Reclamation engineers accordingly developed a modified form of the Ensign balanced valves which were placed at the downstream ends of the conduits, where they would be accessible at any time, and these possessed the further great advantage that the enormous energy stored in the water passing through them was released

harmlessly into the open air instead of being expended to a large degree in the destruction of the conduits below the valves, as previously occurred when they were located on the upstream faces of the dams. Valves of this new design became known as balanced needle valves and internal-differential valves.

Balanced Needle Valves. Valves of this type were installed at a number of dams and their superiority over their predecessors was proven by use, leading to gradual improvements in design, fabrication, and methods of control, until in 1928 the Reclamation Bureau engineers developed a new principle for needle-valve construction and operation which resulted in materially reducing the physical dimensions, weight, and costs of free-discharge needle valves over that of the previous valves. These valves are so arranged and constructed that the annular external ring around the needle or piston, commonly known as the *bull ring*, is eliminated, a marked reduction being made possible in the external diameters of the valves accompanied by an even more marked reduction in their over-all lengths. These valves are known as:

Internal-differential Needle Valves. In valves of this design, the interior is divided into three tandem aligned pressure chambers designated *A*, *B*, *C*. These chambers are formed by a fixed diaphragm inside the needle, supported by a heavy diaphragm tube concentric with the axis of the valve, with the rear end terminating in a flange bolted to the upstream conical end of the needle receiving cylinder formed integrally with the valve body. The rear end of the needle is closed off by a hemispherical head, provided with a bushed hub which rides upon the diaphragm tube as the needle and its attached head move back and forth in opening or closing the valve. The exterior cylindrical surface of the needle is telescopically mounted in an enclosing cylinder which is supported by radial ribs extending through the water passage from the walls of the exterior shell.

Chambers *A* and *C* are interconnected by passages formed in the diaphragm tube so that water can readily pass from one to the other in either direction, and in consequence the pressures in these two chambers are always equalized. Pressure introduced into these chambers will cause the needle to move in the closing direction. When pressure is released from these chambers and introduced into chamber *B*, it produces a force upon the inner face of the hemispherical needle head which causes the needle to move in the opening direction.

These valves provide a larger effective opening force and a larger differential closing force over the opening force than any of the previous valves, owing to the fixed diaphragm inside the needle and the three tandem-aligned pressure chambers that are thus made available. This increase in available operating force permits operation under lower heads at the close of the irrigation season when the reservoir levels are drawn down and ensures ability to operate under low conduit pressures. When the valves have been idle for a long time, the friction of the needles is often increased by corrosion or scale depositions, and under such conditions much additional force may be required to operate them.

Because the external bull ring, previously employed to secure opening movement of the needle, was replaced by the enhanced area of the annular interior surface of the needle head, these valves derive the name of *internal-differential* needle valves.

The needles of these valves are prevented from slamming when they reach the limits of their travel in either an opening or closing direction by so arranging the port apertures communicating with the different chambers that, as the needles approach either extreme of travel, their speed is gradually reduced by progressively constricting and covering their ports.

The control stands and air-vent stands for these valves may be mounted either upon a floor above or directly upon the upper side of the valve body to suit individual conditions of installation.

Two 60-in. valves of this type were installed at Gibson Dam, another pair of the same size at Echo Dam, and still another pair at Coolidge Dam. As a result of the

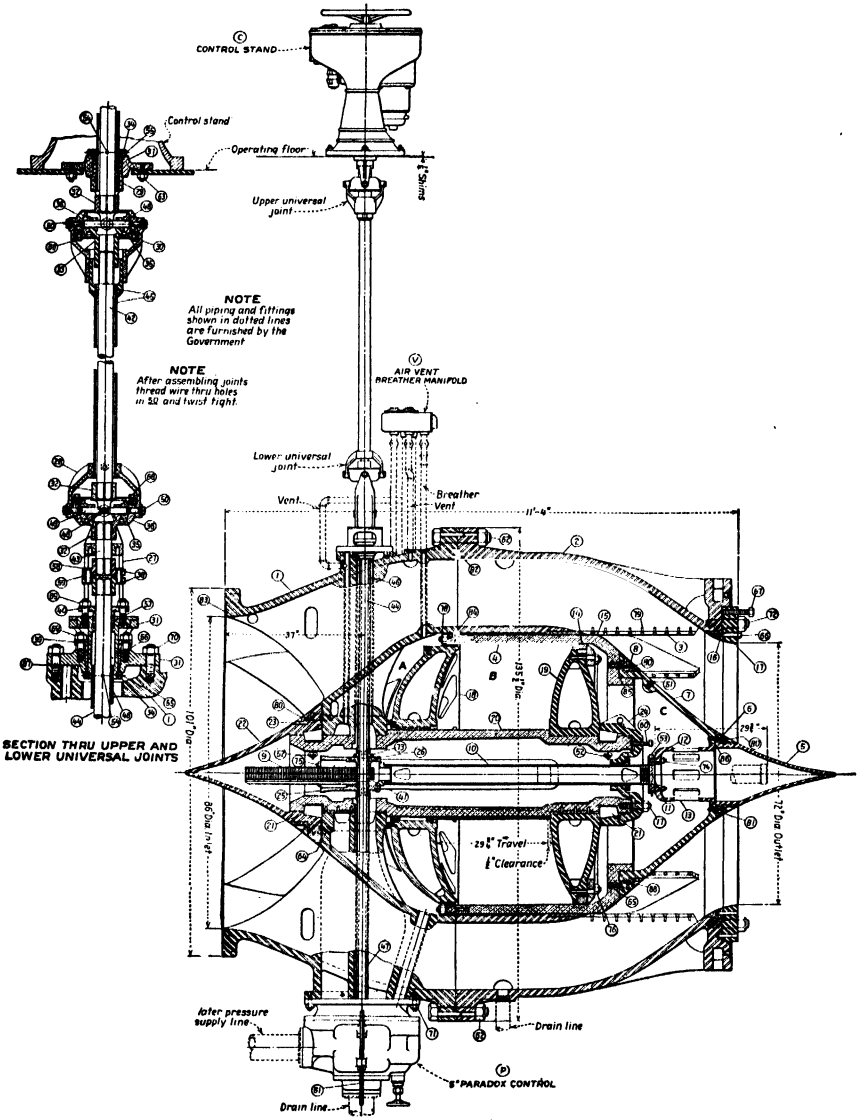

FIG. 9.—Sectional assembly of 72-in. internal-differential needle valve, designed for 610 ft maximum head, as installed at Hoover Dam. The control is arranged for either direct manual operation, or for interlocking remote, electric-motor operation and indication.

experience derived from the operation of these valves, additional improvements and revisions were made upon the valves that followed. Observations made on the action of the improved valves in turn made still further betterments possible.

Much difficulty was experienced in finding a means for preventing the outward or inward flow of water through the clearance between the exterior cylindrical surface of the needle and the bore of the bronze liner in the front end of the cylinder. This condition was finally corrected by changing chamber *A* into a plain cavity with a liberal drain to carry the inflow away before it could build up any appreciable pressure (see Fig. 9) and by discontinuing the interconnecting passage between chamber *A* and chamber *C*. By this arrangement, chamber *B* was used as before to open the valve and chamber *C* was used only to close the valve. Chamber *B* is exhausted to the atmosphere during the closing cycle by means of a new control known as the Paradox control located beneath the valve and placed in communication with chambers *B* and *C* by means of cored passageways formed in the bottom radial rib, as shown in the

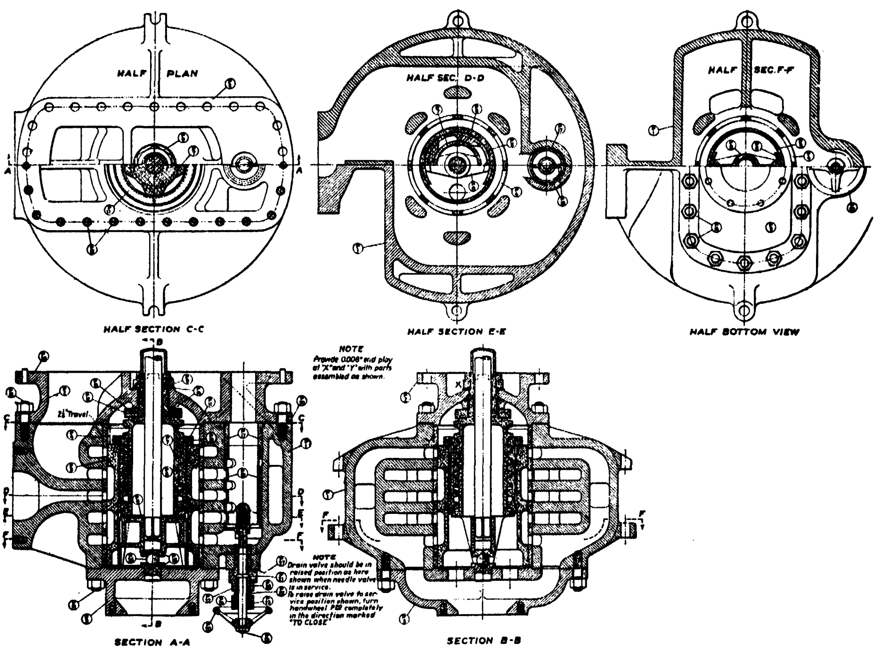

Fig. 10.—Assembly and sections of paradox control with built-in supplemental drain. This control is seen in Fig. 9, bolted to the under side of the 72-in. needle valve as installed at Hoover Dam.

above-mentioned figure. This control utilizes two moving elements consisting of a cylindrical male member with a coarse, rapid-pitch multiple thread upon its external diameter meshing with the female member having a similar thread tapped into the hub of a three-spool valve mounted to travel vertically in the multiported control body bolted to the bottom of the needle valve, as shown in Fig. 9. This Paradox control assembly is shown in various sections in Fig. 10.

Valves equipped with this Paradox control have a number of advantages over their predecessors, for this control system makes it possible to move hydraulically the needles throughout the extreme ranges of travel without requiring the presence of water in the conduits. This feature has proved to be most advantageous, as it makes the routine inspection and scale-removal operations much easier to perform.

Twelve 84-in. valves of this type are installed in the canyon-wall outlets at Hoover Dam, and twelve 72-in. valves of similar construction are installed in the tunnel-plug

outlets. Valves of this type have proved to be economical to build and capable of giving satisfactory and dependable service. The marked reduction in the physical dimensions of these valves over those required for their predecessors of equal capacity makes possible substantial reductions in the size of the control structures within which they are installed and a corresponding reduction in the cost of these structures.

Two 84-in. valves of this type were installed at Madden Dam, Panama Canal Zone. These valves were tested by piezometers in the supply conduits to determine flow and discharge capacities. Results of those tests indicate that the coefficient of discharge for these valves exceeds 75 per cent based on the total net effective head at the inlet flanges and the gross nozzle areas. Pressure-rise tests were likewise made by closing the valves as rapidly as possible. It was found that the maximum pressure rise attainable was less than 2 psi.

FIG. 11.—Shop-assembling and testing two 84-in. interior-differential needle valves. The sprayed-zinc coating on the needle of the near valve, and on the interior surfaces of the nozzle of the far valve, shows clearly here.

Preliminary tests conducted on the 84-in. internal-differential needle valves at Hoover Dam produced results indicating coefficients ranging from 75½ to more than 80 per cent, but these results are questionable. Further tests conducted on the 84-in. internal-differential needle valves indicated a coefficient of discharge of 78 per cent. These valves were tested by means of a calibrated Pitot tube and the piezometers at the outlet conduits. Very little authentic and reliable data, other than this, are available to date on this subject, and much remains to be learned before the ultimate ideal of construction and performance of this type of hydraulic equipment can be closely approached.

In Fig. 12, capacity curves for balanced needle valves based on Tieton tests are given. These values are quite conservative for the later valves with curved lip nozzle orifices.

For valves having an inlet to outlet diameter ratio of 6 to 5 and with a sharp-edged discharge or nozzle orifice, the discharge coefficient becomes approximately 0.65 (see Fig. 13).

The marked scale-depositing characteristics of the waters encountered in some of the Western watersheds made evident the desirability of providing a superabundance of operating force in needle valves installed in such localities to ensure prompt response to the controls, particularly in closing, so as to avoid unnecessary loss of valuable

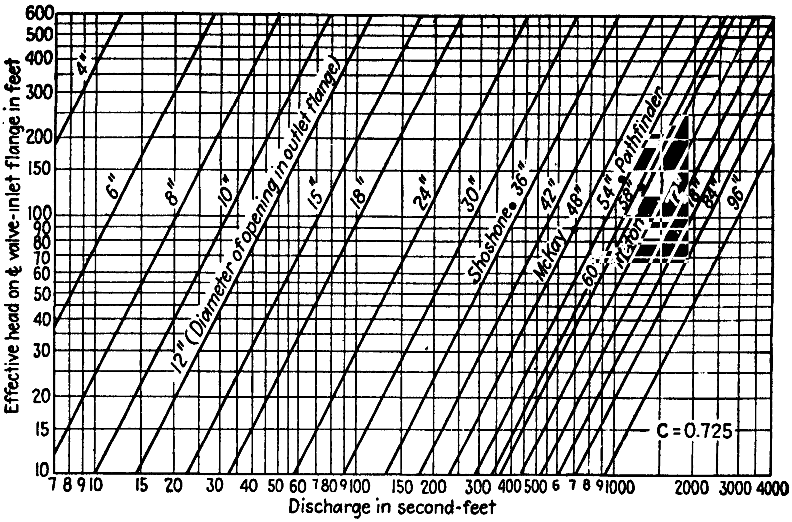

FIG. 12.—Needle-valve and tube-valve capacity curves, for needle valves having outlet diameters 0.833 as large as their inlets, and for tube valves having both their outlet and inlet diameters the same size.

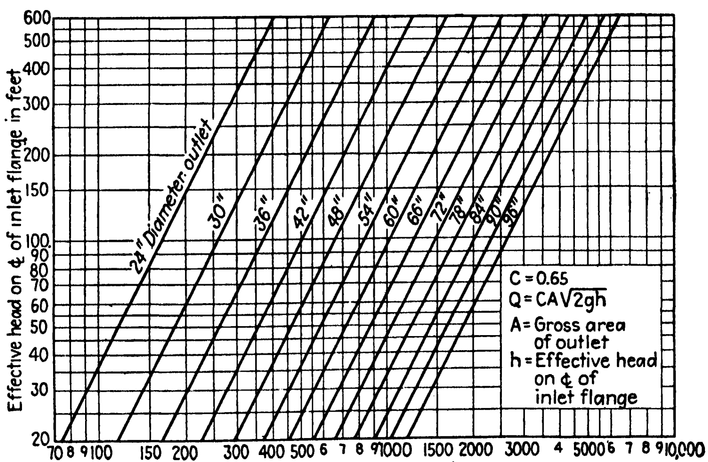

FIG. 13.—Needle-valve capacity curves for valves with outlet diameters 0.833 as large as their inlets, and with sharp-edge nozzle orifices.

stored water. This superabundance of force made it particularly desirable to find some practicable means whereby inward or outward leakage through the clearance space between the exterior of the needle body and the bore of the needle cylinder, prevalent in the first internal-differential needle valves, would be prevented, a closing

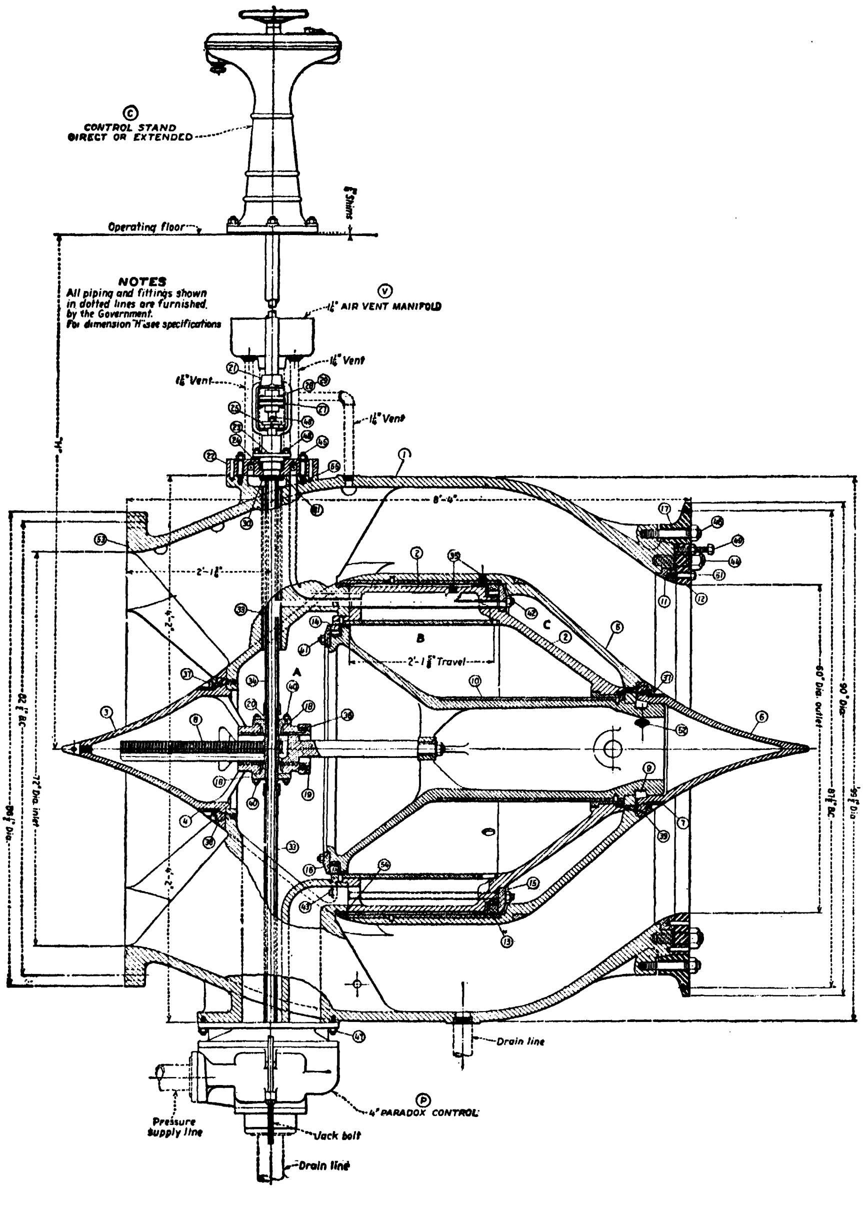

FIG. 14.—Longitudinal sectional assembly of interior-differential needle valve, with control stand.

force being made available which could be produced in chamber A without the attendant disadvantages manifest in those valves.

Much study was devoted to this problem, with only discouraging results, until finally a workable solution was found in a quite marked modification of the internal-differential needle valves. This further improvement and change in needle-valve construction resulted in valves which became known as:

Interior-differential Needle Valves. A longitudinal sectional assembly of a valve of this type is seen in Fig. 14. In this valve, the needle cylinder as previously formed in the body and nozzle of its predecessor has been eliminated, and the equivalent has been provided by forming the cylinder inside of the needle. By this arrangement, the needle, instead of telescoping inside the cylinder as previously occurred, now telescopes over a member fixed to the valve body known as the *body extension.* The exterior diameter of the needle body forms the inner boundary of the annular water passage through the valve. This permits the use of a smaller diameter than before, and in consequence reduces the over-all diameter and length of the valve, the result being a complete valve weight that is nearly 25 per cent less than that of the previous valves of equal capacity.

By this arrangement of the needle mounting, the entrance of the circumferential operating clearance space between the bore of the needle and the cooperating diameter of the supporting body extension is now placed at the upstream end instead of at the downstream end, where the higher velocity of flow tends to produce a strong suction when valves of the previous type are discharging. This rearrangement also made possible the installation of an expanding piston ring, found so effective in preventing leakage between chambers B and C in the previous valves, in the valve-body extension to prevent leakage through that clearance space in either direction, thus making practicable the employment of cavity A as a pressure chamber under all conditions of operation.

The changed construction and rearrangement of the parts in this valve results in chamber A being a complete chamber instead of an annular chamber; chamber B remains unchanged as an annular chamber; and chamber C is changed from a complete chamber to an annular chamber, owing to the diaphragm tube being fastened to the needle.

In these valves, the problems of drainage and air venting of the chambers are made simple and more efficient, for the presence of the valve-body extension makes possible the installation of a direct drainage port from chamber C to chamber A at a location low enough to remove practically all water by gravity flow. A similar port on the upper side allows all the air in A to escape. Chamber B is likewise served by a port in its lowest portion, ensuring complete drainage, and a complementary port of its high side for venting. This arrangement makes possible the elimination of the intercommunicating bleeder ports between chambers C and B and from B into A formerly required, for the ports from chambers A and C and from chamber B to the Paradox control now serve as gravity drains as well.

The Paradox controls used with these valves are essentially the same as those used with the internal-differential valves except that they do not include the separate cylindrical screw-actuated drain valve. This member is not required, for the former cavity A is now made into a pressure chamber and no longer requires a separate drain outlet through the control.

Interior-differential needle valves have been installed in the outlet works of many dams in sizes ranging from 36- to 84-in. outlets and under a large diversity of operating heads and judged from performance, servicing, and lessened fabricating costs are considered to be superior to any of their predecessors.

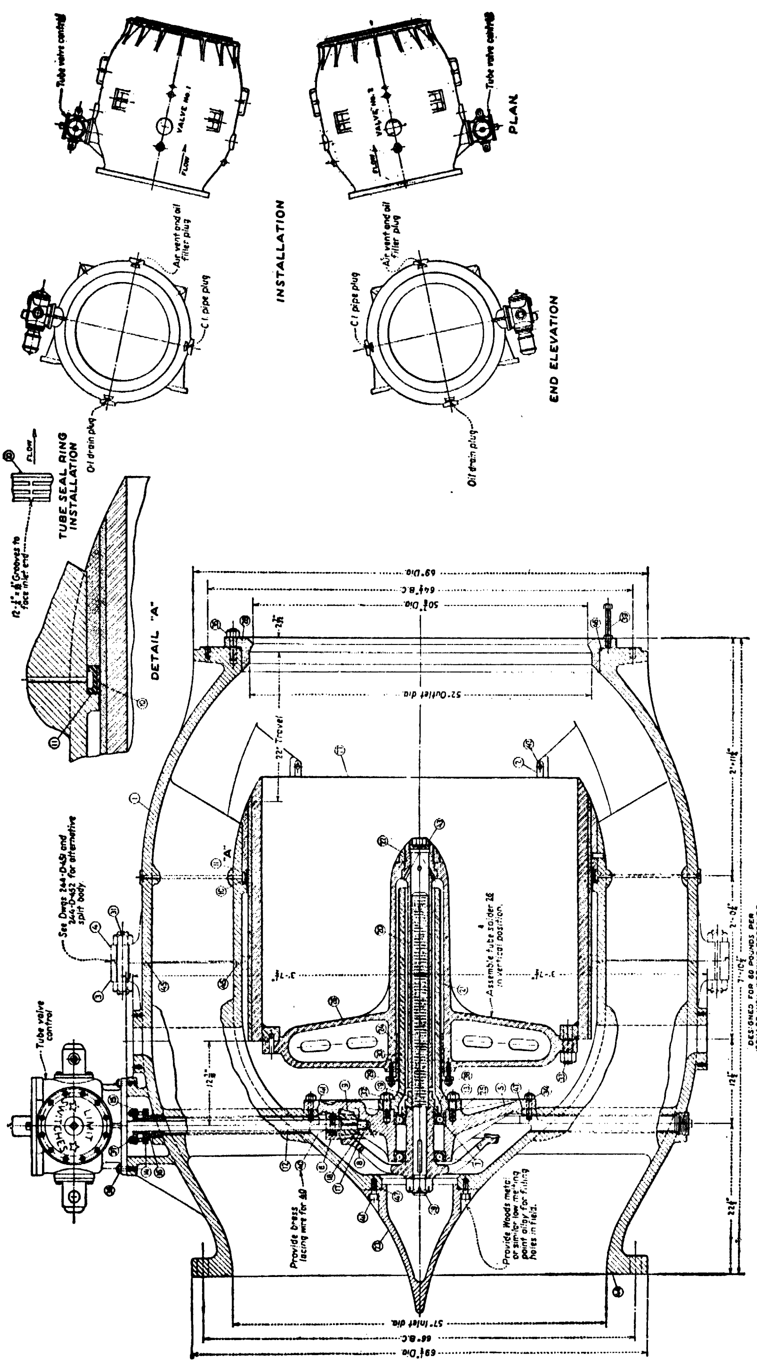

FIG. 15.—Longitudinal sectional assembly of 52-in. outlet by 57-in. inlet tube valve, with direct-mounted control.

Tube Valves. In their efforts to still further improve and simplify the design and fabrication of needle valves, the Bureau of Reclamation engineers invented and developed a new valve known as the *tube valve*, which is in many respects similar to an orthodox needle valve and in other respects markedly different, as will be seen by inspection of Fig. 15 which shows that the valve body, with its radially disposed ribs supporting the centrally positioned receptacle which telescopingly receives the closure member, is essentially the same as that of an ordinary needle valve. Here, however, the similarity ends, for further study of this sectional assembly reveals that the axially moveable closure member, instead of being a needle with chambers for hydraulic operation, is a short length of plain tube with a transverse spider spanning its upstream end and that its movement is accomplished mechanically by means of a screw stem encased in an oiltight sheath and actuated by bevel gearing driven through the shaft extension to the motor-reduction gear unit that is bolted to the external flange of the valve body.

By this unique arrangement and construction of the valve-closure element (the tube), it will be seen that practically all those areas on the moveable parts that are subjected to unbalanced hydraulic forces in a needle valve have been eliminated in this new valve and that, in consequence, it now becomes practicable to mechanically operate valves of the largest size when subjected to high pressures with but a relatively small amount of the energy input that would otherwise be required.

Since there are no hydraulic chambers employed in this new design, the close clearances formerly necessary to prevent leakage therefrom no longer exist, and it is possible to eliminate much of the expensive machine work formerly required and at the same time avoid the necessity of supplying the many parts of bronze and other expensive nonferrous metals formerly essential to the maintenance of such close-working fits when exposed to the scaling or corrosive action of water for long periods of time.

Valves of this design have been fabricated in sizes ranging from 28¼ in. diameter outlets up to 90 in., and larger sizes ranging from 96 up to 102 in. will soon be put into service at Friant and Shasta dams.[1] From the experience derived from the fabrication and installation of these valves, it has been found that they are lighter in weight and more economical to build and install than even their latest predecessors in needle valves and that they do not require the pressure supply and other control piping usually associated with needle valves.

Operation of these valves had demonstrated that for releasing water at rates of 35 per cent, or above, of their ultimate capacities under free discharge conditions their performance compares favorably with that of needle valves; however, their use is not recommended for prolonged periods of release at lesser openings than these, as their jets then become ragged and subject to pulsation effects that would not be desirable for continuous operation under this condition.

In the smaller sizes of tube valves, manual operation alone is usually provided, but in the larger sizes both manual and electric-motor operation are incorporated in a single self-contained control unit (see Fig. 16), which can be bolted directly to a receiving flange on the valve body (as shown in Fig. 15) or be mounted upon a floor or platform at any desired distance above the valve, being connected thereto by a single shaft which drives the operating mechanism inside the valve and also ensures that the indicator mechanism on the control stand will be maintained mechanically in correct synchronism with respect to the varying positions of the closure element (tube) as the valve is operated.

Tests of a 20-in. conduit tube valve conducted at Hoover Dam gave very satisfactory results as a regulating agency in a closed conduit when operating under heads in excess of 350 ft, manual closure and opening and regulation being accomplished without difficulty. During these tests, which were run to verify the correctness of the

[1] It was later decided to install hollow-jet valves at Friant Dam.

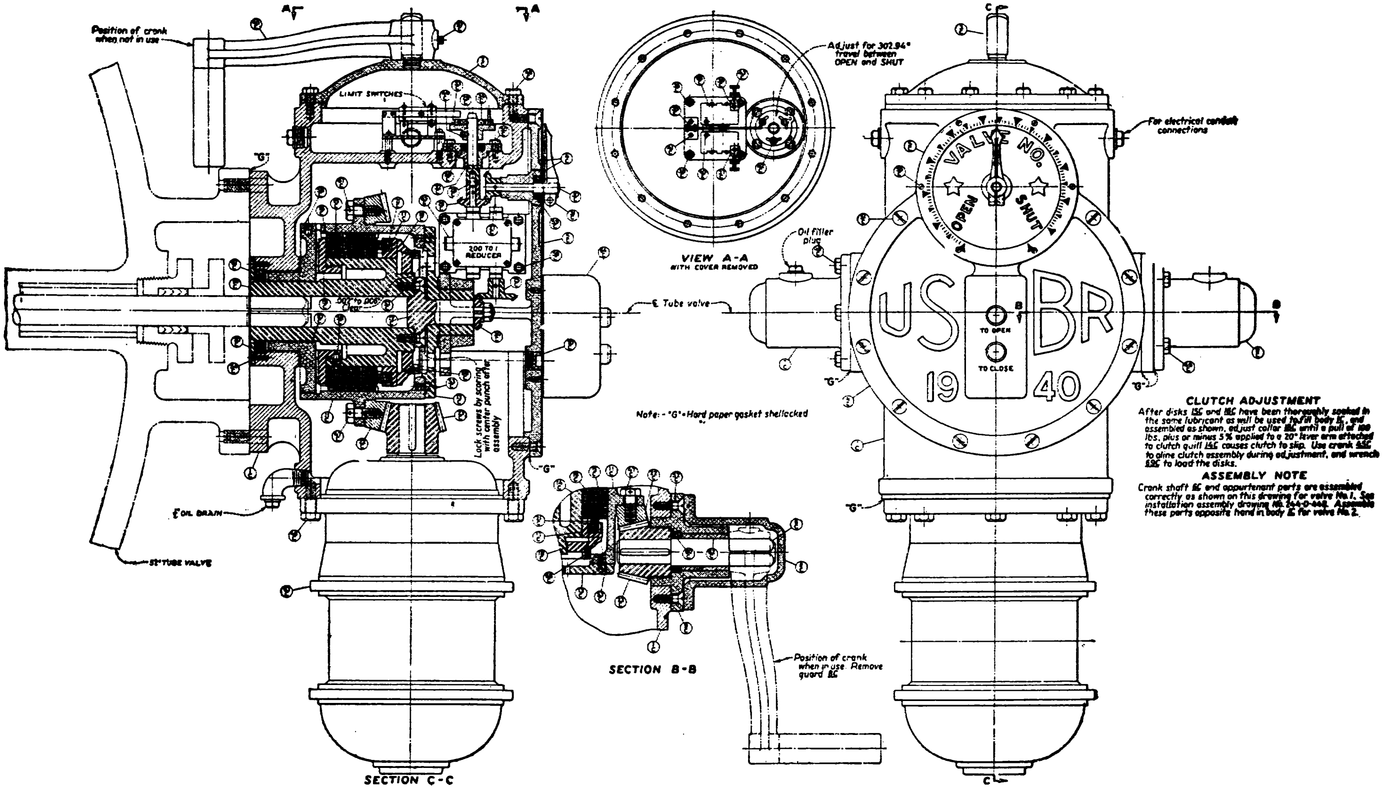

FIG. 16.—Assembly and sections of the combination manual and electric-motor-operated control mechanism and indicator for the tube valve shown in Fig. 15.

designs for the similar conduit tube valves of 102-in. inlet and outlet diameters being installed at Shasta Dam to operate under a maximum head of 323 ft, the anticipated requirement of admitting air to the rear (upstream) end of the open tube within the interior of the valve whenever that closure element is in positions adjacent to closure was very definitely confirmed, likewise the law of similitude of performance between the 6-in. model as tested in the Denver laboratory under heads up to 100 ft with the test results on this 20-in. conduit tube valve at Boulder Dam was found to be substantially true.

The coefficient of discharge of short-body type free-discharge tube valves has been found to be substantially the same (0.605) as that of free-discharge needle valves having sharp-edged nozzle orifices (0.640) but less than free-discharge needle valves with rounded-edge orifices (0.780); however, this last unfavorable comparison is not considered to be serious as rounded-edge orifice needle valves, although having a discharge coefficient in excess of 0.72 per cent, have been found to be subject to serious cavitation on their conical needle surfaces just below the zone where their needles contact their seats when closed. This cavitational tendency has been found to develop whenever their effective heads on these needle valves are in excess of 150 ft and is most rapid when the valves are operated at long periods with their needles in approximately the 40 per cent open position. Newly developed long-body type free-discharge-tube valves, with inlet and outlet diameters made equal, have a coefficient of discharge of 0.76 based on the gross inlet area.

Hollow-jet Needle Valves. This type of outlet regulating needle valve was recently developed in the Denver Office of the Bureau of Reclamation to simplify valve construction and reduce cavitation. Development was begun in 1940 on this type of valve, in which the closure member or needle is moved upstream to close against the inlet end of the valve. This valve, as shown in Fig. 17, became known as the hollow-jet valve because the jet, as it leaves the valve, is in the form of a longitudinally slotted tube or segmental jet having a hollow interior instead of a solid stream as comes from the conventional-type needle valve.

As the water flows through this valve, it leaves the outer circumferential knife-edge formed on the needle seat ring and clears the inner cylinder of the valve to form the hollow jet. This reduces the area of the wetted valve surface in contact with the high-velocity flow. It will be noted that the water is caused to turn through an angle of about 93 deg, in its passage along the median line through the valve.

This valve consists of a valve body having an inlet of the same diameter as the conduit, with an outwardly curving and expanding shell, and an inner cylinder. The inner cylinder is centrally positioned by four equally spaced hollow splitters with their downstream ends left open. This permits free access of air to the interior of the hollow jet through the cavities in the splitters and through the slits in the walls of the jet itself caused by the splitters.

The interior parts of the valves are so arranged that the needle and needle support, together with the operating mechanism, can be assembled as a complete unit and inserted into the inner cylinder of the valve body from the outlet end of the valve. The flange of the needle support is bolted securely to the downstream face of the inner cylinder of the valve body. This facilitates assembly and disassembly in the field and adds to the value of this valve from a maintenance standpoint.

The first models tested were provided with hydraulic operation of the needle through control of reservoir pressure. It became evident that considerable savings in cost could be effected in the prototype design if the hydraulic operation of the needle was replaced by mechanical operation, provided the water load acting against the upstream face of the needle was balanced as closely as possible by pressure within the needle. A location for the balancing port through the face of the needle was found

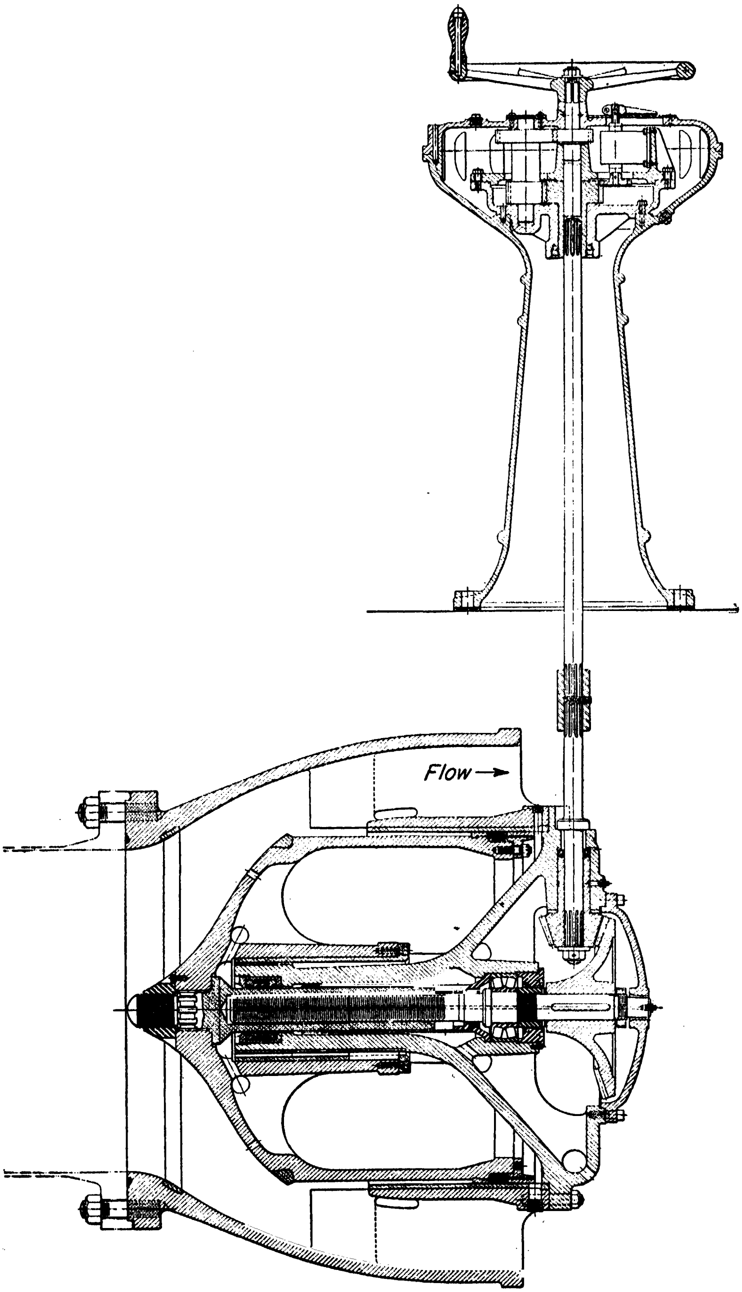

FIG. 17.—Hollow jet valve assembly with manually operated control stand. (*Courtesy of Bureau of Reclamation.*)

that would limit the unbalanced thrust to plus or minus 12½ per cent of the total water load on the needle for any valve opening.

This permits valves of this type to be operated manually in sizes up to about 36 in. in diameter at the inlet. Valves above 36 in. will usually be operated by means of motor-driven controls.

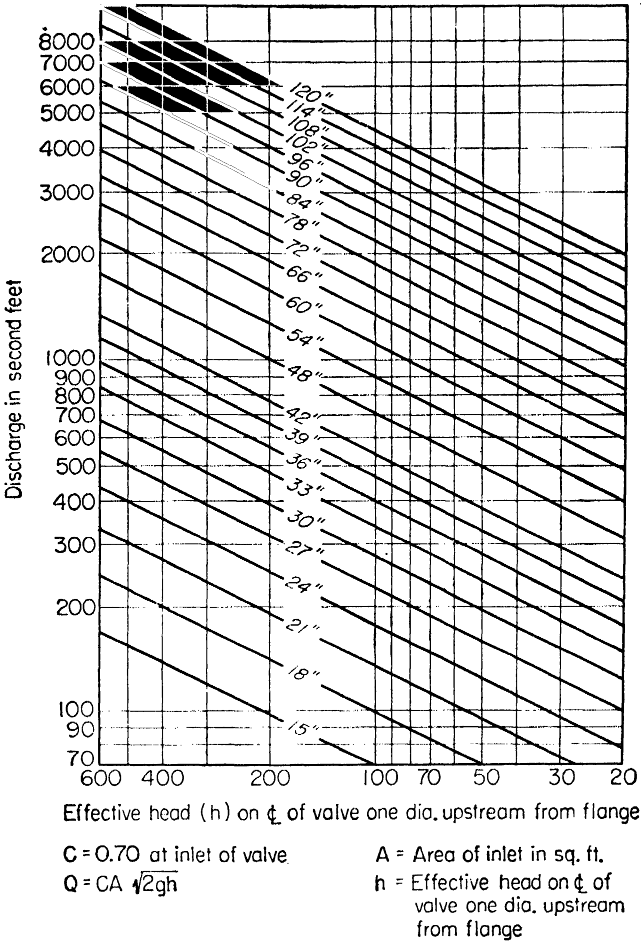

FIG. 18. Hollow jet valve. Discharge capacity curves. (*Courtesy of Bureau of Reclamation.*)

As previously stated, the needle is moved upstream to seat against the inlet. The drive is through a vertical shaft, a bevel gear set, and a threaded stem engaging a nut member attached to the needle tip. In smaller sizes, the vertical shaft is turned by a control handwheel located above the valve (see Fig. 17). For larger sizes, the shaft is turned by a motor-driven gear reducer. The threaded stem and nut are self-locking, and the needle will not creep from any position to which it may be set by the operating mechanism. A mechanically operated position indicator is built into the control mechanism to give the operator the exact valve opening.

This valve should not be used for discharging under water. It can be operated in

a closed conduit provided adequate provision is made for the admission of air at the outlet end of the valve operating under free-discharge conditions. There are no limiting ranges of head or openings through which this valve cannot be operated without cavitation occurring.

Based upon the diameter of the inlet the coefficient of discharge of this valve is 0.7. This can be raised to 0.724 by increasing the normal needle travel 5.5 per cent, without causing negative pressure to occur on either the valve body or the needle. See Fig. 18 for discharge capacity curves.

A 24-in. diameter hollow-jet valve was tested at Hoover Dam in July, 1945, at heads up to 349 ft. It showed no evidence of cavitation. Positive pressures existed on all parts of the valve as indicated by piezometric readings. In order to check further for cavitation, the water passages were coated with glyptal and the valve was operated an additional 16 hr. Examination showed no eroded or worn spots in the painted surfaces. Had there been cavitational erosion present, the paint would have been damaged within an hour by such operation. For installations, see table at end of this section, page 413.

Westinghouse Electric Corporation, Sunnyvale, Calif., is licensed to manufacture these valves for other than governmental use.

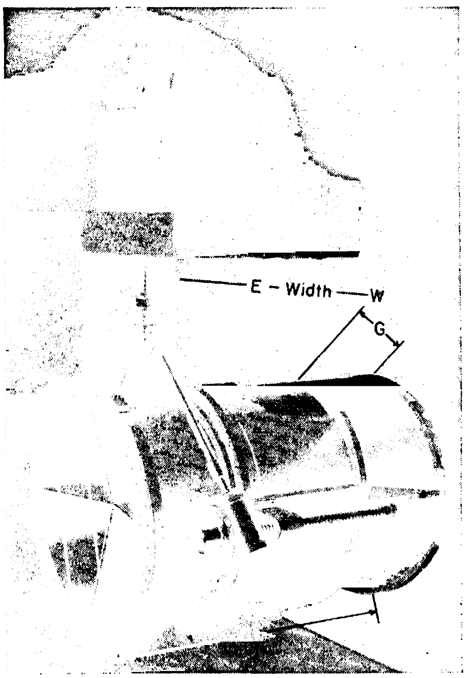

FIG. 19.—Phantom view of Howell-Bunger valve. (*Courtesy of S. Morgan Smith Co.*)

Howell-Bunger Valves.[1] These are free-discharge regulating valves and so are placed at the downstream terminals of their conduits. Each consists essentially of a short body section of conduit, see Fig. 19, having an inlet diameter of the same size as that of the conduit with which it is used. This body section is flanged at its inlet end to provide a bolted connection to its conduit. It encloses radially disposed longitudinal ribs throughout its length, which extend axially on downstream from its discharge end for a distance equal to about one-half its internal diameter, to join and concentrically support a cone-shaped end piece. The apex and axis of this cone are coincidental with the center line of the conduit and valve body, and its apex faces upstream. The base diameter of this cone is made slightly larger than that of the conduit and is provided with a circumferential bronze seat ring against which a similarly shaped bronze ring carried by the downstream end of a longitudinally sliding cylindrical sleeve, internally supported by the outer edges of the radial body ribs, engages. This cylindrical sliding sleeve is circumferentially ribbed at each end for stiffness and is provided with a carbon packing ring at its upstream end which slides upon a bronze bushing encasing the downstream portion of the valve body.

[1] These valves are named after their inventors, C. H. Howell and Howard Bunger, both of whom were formerly employed as engineers in the Denver Office of the Bureau of Reclamation, and who were

To open the valve, the sleeve is moved upstream, it being concentrically supported by the bronze-faced edges of the radial ribs between the cone and the valve body, and by the bronze bushing on the valve body, for a distance of about one-half of the conduit diameter, until its downstream seating end is flush with the downstream end of the valve body. This upstream movement discloses a circumferential cylindrical space between the base diameter of the cone and the end of the valve body through which discharge occurs. The cylindrical area of this space is about twice that of the cross-sectional area of the conduit.

The water passing down the conduit and through the discharge area has its direction of flow changed by the presence of the cone which causes it to expand conically outward at an angle of about 45 deg, from its previous axial travel, and to escape into the atmosphere as a hollow, diverging, cone-shaped jet. This jet has a maximum wall thickness, G in Fig. 19, normal to the surface of the cone and to its path of travel of about 0.4 of the conduit's diameter D. This value of 0.4 multiplied by πD gives a cross-sectional area of the jet at the orifice of a little less than 1.6 times that of the conduit's cross-sectional area. This fact taken into consideration with the fact that the water flowing through the valve is forced to change its direction of travel only once (the axial expansion of flow of 45 deg, caused by passing over the surface of the cone) explains the high coefficient of discharge of 0.9 of the total net head at the valve inlet, which these valves provide. See Fig. 20, discharge capacity curves.

The cylindrical axially sliding sleeve, which constitutes the controlling and closure element of these valves, is subjected to but nominal unbalanced hydraulic forces during the operating cycle, consequently the control mechanism required to accomplish this movement need be of but moderate capacity. A pair of jacking screws on opposite sides of the cylindrical sleeve, driven by gearing from inclined shafts geared to a common vertical shaft extending upward to a hand- or motor-driven back-geared driving head with built-in position indicator mounted upon the operating floor above, has been proved adequate (see Fig. 19). The S. Morgan Smith Company recommends hand operation for valves of 42-in. inlet with a maximum head of 150 ft and for 36-in. size for heads of 200 ft maximum. For higher heads or larger valves, motor-operated controls are in order. The operating gear shown has been proved satisfactory in holding the sleeve at any required opening without creep. Operating experience has shown the necessity of providing ample air inlet space around the valve and between the face of the valve housing structure and behind the adjacent conical surface of the conical jet. Operation under varying heads up to above 700 ft has shown them to be free from vibration, pitting, or cavitation. Their licensed manufacturer, S. Morgan Smith Company, which furnished the drawings and the list of installations, states that they have not been called upon to furnish any repairs or replacements since they started manufacturing these valves in 1937. See table of installations on page 414.

Needle-valve and tube-valve outlets may be divided into six general classes as follows according to their location:

1. On the water face of the dam, as at Arrowrock and Belle Fourche dams, and in the South Tunnel outlet at Pathfinder Dam. Installations of this character were made on some of the earlier dams built by the Bureau of Reclamation but were discontinued after the balanced needle valve was developed for installation at the outlet ends of closed conduits.
2. On the downstream face of the dam, as at Owyhee and Gibson dams.
3. At the outlet ends of the diversion tunnels, as at Tieton and McKay dams.
4. Inside of diversion tunnels, as at Alcova and Hoover dams.
5. At the outlet ends of tunnels or galleries provided for the sole purpose of releasing water from a reservoir under lesser head on the valves than that prevalent at the diversion-tunnel level, as the canyon wall outlets at Hoover Dam.

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later in the employ of J. G. White Co. in Mexico City at the time they invented these valves. Mr. Howell has since reentered the Reclamation Bureau. He is now Project Engineer on the Colorado Big Thompson Project at Estes Park, Colo.

6. Inserted in conduits passing through dams, with operating chambers inside the dams giving access to the tube valves for manipulating, controlling, and servicing them, the outlet ends of their conduits thus terminate flush with the downstream face of the dam, which allows free use of an overflow spillway above and permits the descending sheet of water to pass unimpeded across their conduit outlets, as is done at Shasta Dam.

These six main divisions are susceptible of further subdivision in accordance with the arrangement, type, and location of the service or emergency gates guarding the valves.

For example, under class 2, the 4- by 4-ft high-pressure gates that guard the 48-in. needle valves at Owyhee Dam (Fig. 21) are in a separate gallery near the upstream face

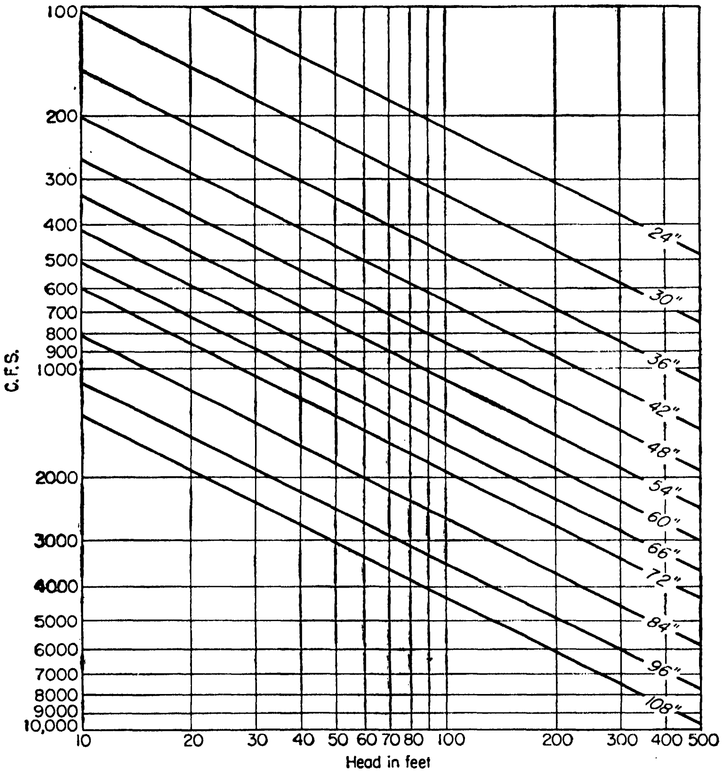


FIG. 20.—Howell-Bunger valves. Discharge capacity curves. (*Courtesy of S. Morgan Smith Co.*)

of the dam, while the 5- by 5-ft high-pressure emergency gates (see Fig. 1) guarding the 60-in. needle valves at Gibson Dam (Fig. 22) have the outlet flanges of the gate frame transitions bolted directly to the inlet flanges of their respective needle valves and are enclosed in the outlet valve house and serviced by the same crane as that employed for maintenance work on the needle valves.

At those dams where diversion of the river is accomplished by means of a tunnel in the rock wall around the dam site, the tunnel in most instances can later be advantageously employed as the outlet for releasing water from behind the completed dam by placing a plug, containing a high-pressure gate chamber, at approximately the point where the axis of the dam intersects the tunnel. One or more plate-steel pipes are provided, extending from the gate chamber to the tunnel outlet portal where suitable

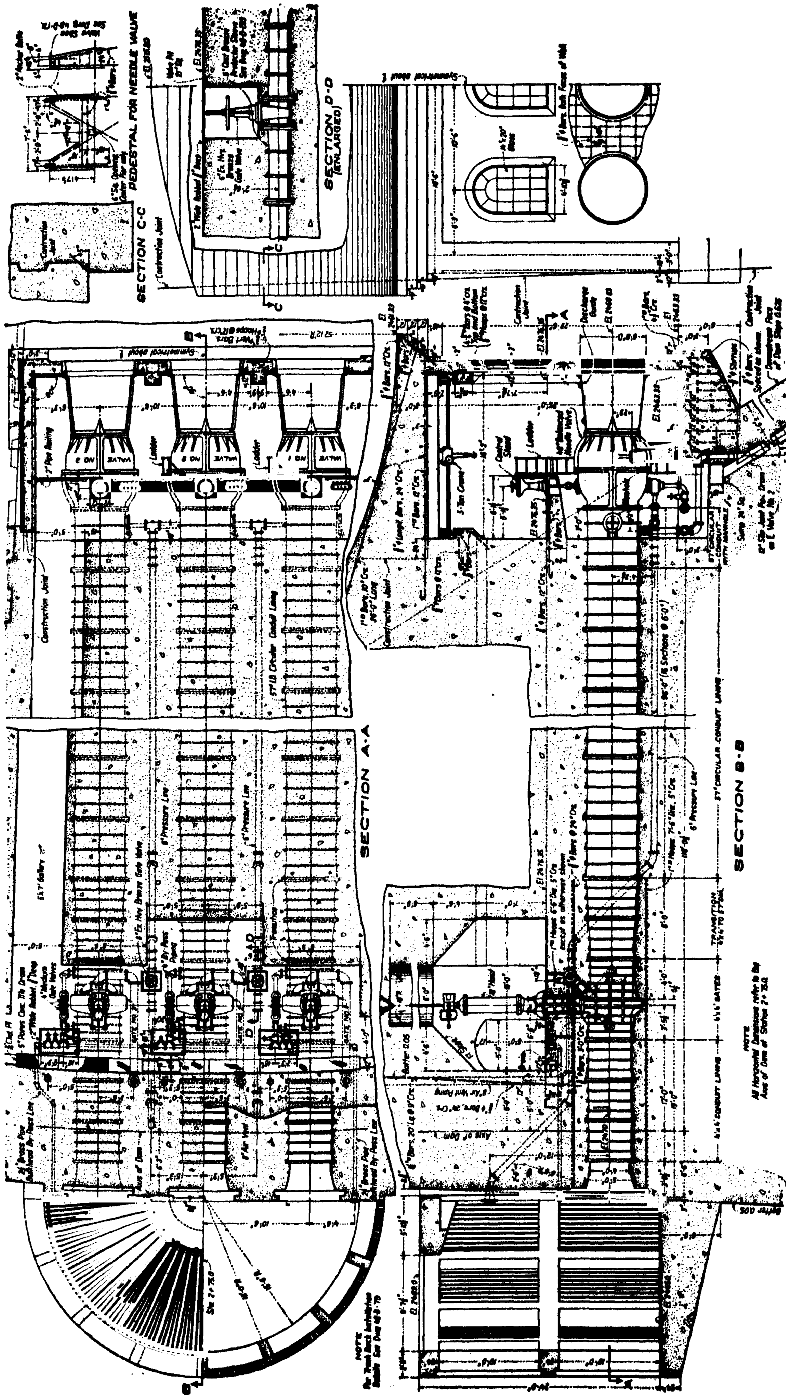


FIG. 21.—Irrigation outlet installation, Owyhee Dam.

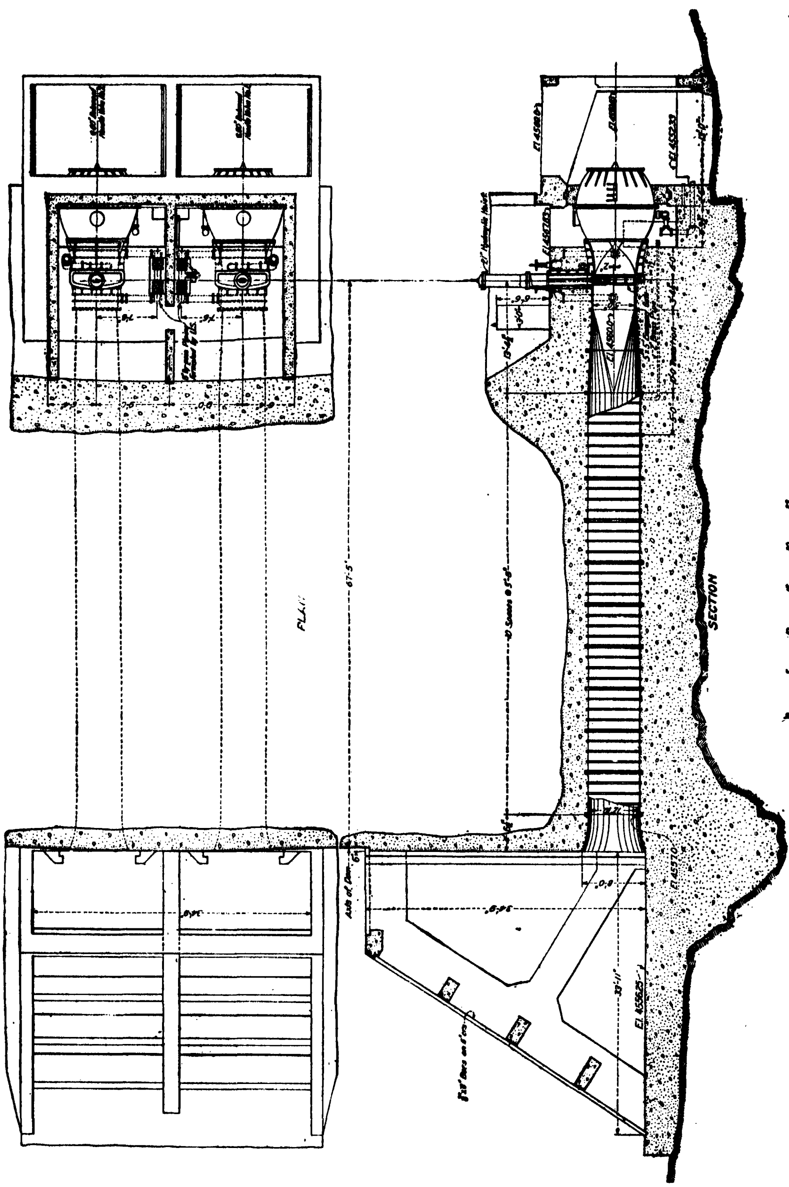


FIG. 22.—Emergency-gate and needle-valve installation for irrigation outlets at Gibson Dam. Typical outlet through a concrete dam.

needle or tube valves are installed in an outlet-valve house and arranged to discharge into the river channel or into a stilling pool below the dam. Tieton and McKay dams (Fig. 23) are typical examples of this type of outlet works, which fall under division 3.

Outlets of this type usually have parrot-cage or plain rectangular trash racks guarding the inlets to the diversion tunnels. If the dam and outlet works are of more than average size and importance, a vertical shaft entending from the emergency gate chambers to the ground surface at an elevation higher than maximum reservoir level is

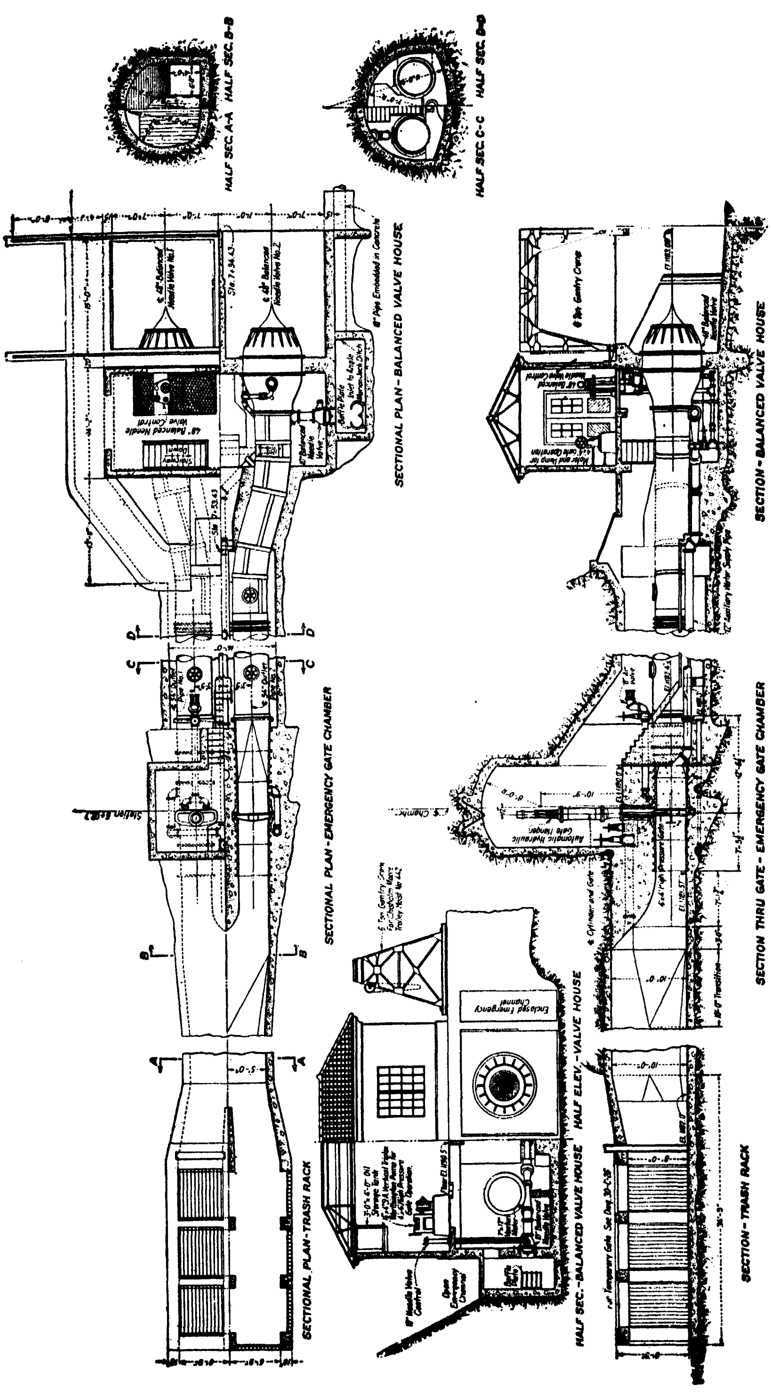


FIG. 23.—General arrangement of complete outlet works at McKay Dam. Typical diversion-tunnel type.

provided for remote-control purposes. A second high-pressure oil pump, motor, and interconnecting oil lines to the high-pressure gates below are so arranged as to permit the closure of the gates by remote control from a house at the top of the shaft, in case a major break should occur in the conduits between the high-pressure emergency gates and the needle or tube valves at the outlet end of the tunnel. The outlet works at Tieton and Echo dams are arranged in this way.

Inasmuch as class 1 needle-valve outlets have already been described in a general way under regulating valves, short descriptions will now be made of typical needle-valve and tube-valve outlets beginning with class 2, the outlets located on the downstream face of the dam.

Class 2 Needle-valve Outlets. The outlet works at Gibson Dam (Fig. 22) are typical of this class. This class might properly be subdivided into those similar to the Gibson outlet, where the emergency gates are immediately behind their respective needle valves, and those similar to the Owyhee outlet (Fig. 21), where the emergency gates are placed in an operating gallery near the upstream face of the dam.

The outlet works at Gibson Dam, a constant-radius concrete-arch-type dam, consist of two 72 in. diameter conduits formed of 5-ft lengths of semisteel castings designed to take full reservoir pressure. The conduits extend through the dam at El. 4559.83 and are provided with single-radius bellmouthed inlets 8 ft in diameter. A reinforced-concrete trash-rack structure common to both conduits and supporting 7/16- by 6-in. bars on 6-in. centers is provided at the upstream end of the conduits. The 72-in. diameter conduits are bolted to transitions 10 ft long near the downstream face of the dam, and the outlet flanges of these transitions are bolted to the upstream flange of two 5- by 5-ft high-pressure emergency gates. The downstream gate frames are formed into transitions from the square to the circular sections and receive the inlet flanges of short body 60-in. internal-differential needle valves, which protrude through and have their body castings embedded in the downstream wall of the needle-valve house.

The emergency gates are embedded in heavily reinforced concrete that merges into the downstream face of the dam, which is likewise heavily reinforced with T-rails in the zone where the two conduits pass through to compensate for the material displaced at that point.

The emergency gates and needle valves are subjected to a maximum operating head of 163.5 ft. The leaves of the emergency gates are of cast steel and are provided with 21 in. diameter hoist cylinders operated by means of a gasoline-engine-driven triplex pump in the valve house. A mast-type jib crane serves both the high-pressure gates and the needle valves. An 8-in. by-pass is provided around each high-pressure gate, so that the needle valve can be closed, the by-pass valves opened, and the pressure balanced on both sides of each gate before it is opened.

At Owyhee Dam (Fig. 21), the needle-valve outlet consists of one semicircular trash rack on the upstream face of the dam, serving three 4- by 4-ft square semisteel conduits with 5-ft square, constant-radius bellmouths formed in the concrete. Three 4-ft lengths of conduit liners immediately downstream from each bellmouth, are bolted to the inlet flanges of the respective 4- by 4-ft high-pressure gate frames. To the outlet flanges of these high-pressure gates are bolted 4- by 4-ft to 57 in. diameter transitions, each made up of two sections 4 ft long. These are connected at their outlet flanges to 57 in. inside diameter semisteel conduit liners that extend through the dam and connect to the entrance flanges of 48-in. balanced needle valves of the split-body type (made necessary by the transportation of all castings and parts to and from the valve house to the top of the dam on a freight elevator inside the dam).

By reference to section *BB* of Fig. 21, it will be seen that the center lines of the 4- by 4-ft high-pressure gates are located 19 ft from the upstream face of the dam in a

gate chamber common to all three gates, having a recess midway of its length on the downstream side, in which the high-pressure oil pump, motor, and control equipment are located. It will also be noted that hoops 7 ft 6 in. in diameter of 1-in. square bars on 5-in. centers are placed around the square bellmouths and all the square conduits from the upstream face of the dam to the downstream flanges of the transitions. The high-pressure gates are provided with automatic hydraulic gate hangers of the stem-extension type. This needle-valve outlet is at El. 2469.83 and is subjected to a maximum head of 200 ft. The high-pressure gates are designed for a 250-ft head, are provided with cast-steel leaves, and are operated by 18 in. diameter hydraulic hoist cylinders.

The 57 in. inside diameter conduit liners connecting the transitions to the needle valves are designed to take the maximum working pressure with a liberal allowance for corrosion.

The needle valves are located wholly within the heavily reinforced-concrete valve house to facilitate handling by means of a 5-ton low-headroom hand-operated crane just beneath the ceiling. The jets pass through removable plate-steel discharge guides that are bolted to the steel orifice liners embedded in the valve-house wall. The roof of the valve house is formed of a massive curving reinforced slab, so designed and shaped as to receive and direct outward any falling water from above in case the dam should be overtopped.

A good example of class 2 outlets as applied to a reinforced-concrete slab and buttress-type dam is found in the needle-valve outlet at Stoney Gorge Dam (shown in Fig. 24). As will be seen in section *BB*, a recess is formed between the buttress walls on the water face of the dam and covered over by ⅞- by 6-in. trash-rack bars with their upper edges flush with the upstream face of the dam. Two 3-ft 6-in. by 3-ft 6-in. square conduits with the earlier type bellmouthed inlets lead to high-pressure gates operated by 12 in. diameter hoist cylinders and housed in a reinforced-concrete enclosure, the upstream wall of which is formed by the outlet side of the trash-rack structure. These gates are designed for 98 ft of head and are operated by a direct motor-driven triplex pump. Bolted to the outlet flanges of the gates are semisteel transitions that connect to 50 in. inside diameter riveted plate-steel outlet pipes supported on concrete saddles. The pipes extend downstream to 42-in. balanced needle valves enclosed in a reinforced-concrete valve house where the pipes are provided with expansion joints close to where they pass through the upstream wall of the needle-valve house. A concrete walkway supported by crossbeams between the buttresses extends from the operating floor of the emergency gate chamber to the operating floor of the valve house. A small hydraulic turbine drives a 25-kw generator that supplies power for operating the oil pump and lights and also charges a 125-volt storage battery for stand-by service.

The two 42-in. balanced needle valves designated as 1 and 2 are identical in so far as their general construction is concerned, but each valve has a control differing from its fellow. Valve 1 has the sleeve type of control, wherein the needle-actuated sleeve is operated by a lever attached to the rear end of the needle so that vertical axial or endwise movement is imparted to the sleeve. Valve 2 has the cylinder type of control, wherein the needle-actuated member is rotated through a rack, attached to the needle, driving a pinion on a vertical shaft, to the upper portion of which a pinion is attached that meshes with and drives an internal gear integrally formed with the cylinder member so rotated by movement of the needle. Both of these controls have been found to work equally well and have proved to be reliable and satisfactory.

The needle-valve outlet at Bartlett Dam is analogous in some respects to the Stoney Gorge installation just described, yet in many ways it is distinctly different. Here two 66-in. needle valves are installed inside a reinforced-concrete valve house

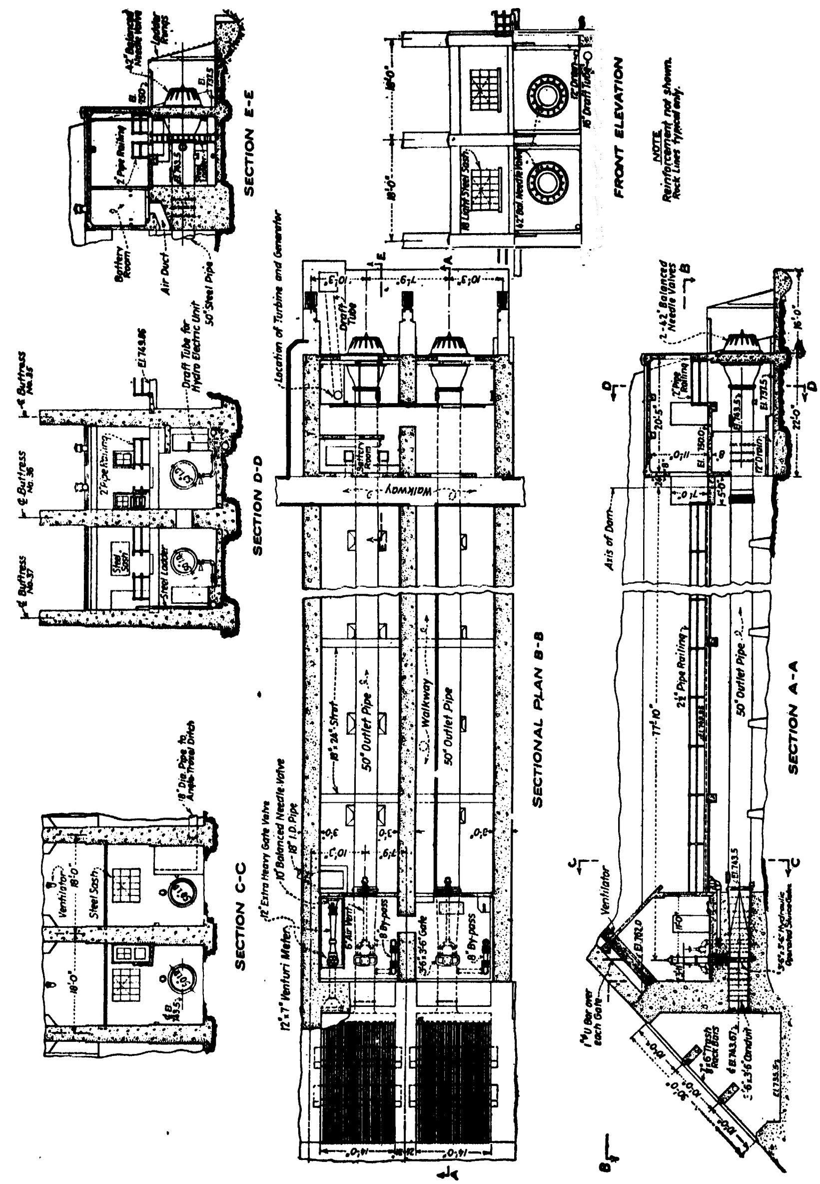


FIG. 24.—General arrangement of complete outlet works at Stony Gorge Dam. Typical outlet in Ambursen dam.

built against the downstream end face of a dual hollow buttress which receives the thrust loads of two adjacent arches that form part of one of the highest multiple-arch dams constructed to date. A vertical slot 8 ft wide is formed between the adjacent buttress walls within which two 72-in. inside diameter ⅜-in. plate-steel outlet pipes, with inside field-welded girth joints, are placed one above the other with the center line of the lower at El. 1,633.00 and the upper 17 ft higher at 1,650.00. These pipes extend from their inlets situated in the crotch formed by the converging surfaces of adjacent

arches at the water face of the dam to the needle-valve house where they are anchored in a heavily reinforced-concrete thrust block and are connected to tapered increasers from 72 in. inside diameter to 79 in. inside diameter whose downstream 79 in. inside diameter outlet flanges are bolted to the inlet flanges of the respective needle valves.

The nozzle flanges of the needle valves are bolted to plate-steel discharge guides that extend through the downstream wall of the valve house. An overhead traveling crane of 15-ton capacity serves both valves, this being made possible by arranging the relative positions of the two valves so that the upper valve is set upstream from the lower valve. By removing the plate-steel discharge guide of the upper valve, the crane can readily serve the lower valve from above.

A cable-operated 8- by 16-ft structural-steel bulkhead gate, operated from a hoist on top of the dam, is provided so that in the event that a flash flood were to occur while either needle valve was rendered inoperative by dismantling for maintenance work this gate can be lowered over the conduit inlet leading to that valve.

Another form of class 2 installation, which has been found to be most advantageous where power plants are built against the downstream faces of the dam, is to place the needle valves inside the power plant, with the emergency gates either in galleries adjacent to the upstream face of the dam or preferably inside the power plant and behind the valves, where the powerhouse crane can serve the entire installation. This was done at Coolidge, Madden, and Seminoe dams.

At Seminoe Dam, two 60-in. interior-differential needle valves are installed in the powerhouse. Each of these valves is protected by a 72-in. ring-follower emergency gate, which is likewise inside the powerhouse and so arranged that by removing steel floor plates they are made accessible to the powerhouse crane. The control stands for these valves are located upon the turbine floor immediately above their respective valves.

Class 3 Outlets. At McKay Dam, the tunnel used for diversion during the construction period was later converted for use as the reservoir outlet. The geological formations were of a nature that made it desirable to line the entire 705-ft length of this tunnel with concrete. The entrance to this tunnel was extended about 40 ft from the portal by a reinforced-concrete transition (Fig. 23) connected to a rectangular reinforced-concrete trash-rack structure 16 ft 6 in. broad by 9 ft 9 in. high, having racks of ⅝- by 6-in. bars placed on 6-in. centers, which extends 24 ft 9 in. farther into the reservoir. The reinforced-concrete plug containing the two 4- by 4-ft high-pressure emergency gates and the operating chamber is located 155 ft upstream from the outlet portal. From these two gates, 54 in. inside diameter riveted ⅜-in. plate-steel outlet pipes, with inside countersunk rivets and outside strap joints, made vitally necessary by velocities of 35 to 45 fps, extend along the concrete-lined horseshoe tunnel (see sections *CC* and *DD*) on concrete saddle supports with a plank walkway between them to the two 48-in. balanced needle valves located in the reinforced-concrete needle-valve house. The needle-valve body castings are embedded in the downstream wall, and the nozzles protrude beyond in the general manner shown in Fig. 23.

The maximum outlet capacity required for irrigation purposes is 800 cfs, a demand that either one of the 48-in. needle valves can readily satisfy when the reservoir water surface is near to maximum elevation. The maximum static head on these outlets measured from the center-line elevation of the needle valves to the maximum reservoir water surface is 138.6 ft. One of these valves, when tested with a net head of 90 ft at its inlet flange, discharged 640 cfs.

The needle valves are installed in a reinforced-concrete valve house located a short distance beyond the outlet portal of the tunnel and are so arranged as to provide ready escape of water from the tunnel without damage to the valve house and its enclosed machinery in the event that a major break should occur in either or both of the outlet

conduits within the tunnel. An operating floor 7 ft 5 in. above the center line of the valves carries the control stands which are located immediately above their respective valves and by shaft extensions operate the cylinder-sleeve controls that are bolted to the upper side of each valve body. These controls have worked well, it being found that regulation of discharge can be held within less than 10-cfs increments. A motor-operated triplex pump and the high-pressure oil-pipe control system for the operation of the high-pressure gates in the tunnel-plug emergency-gate chamber are located on the same floor as the needle-valve control stands. A 6-ton hand-operated structural-steel gantry crane runs along the downstream wall face of the valve house on elevated reinforced-concrete runway beams that span the diaphragm walls on either side of each needle-valve nozzle. These walls are provided with stop-log grooves and are so arranged as to permit enclosure along their tops and in front with planking to prevent the valves from freezing during the winter seasons. The gantry crane makes easy the servicing of the needle valves.

The 4- by 4-ft high-pressure emergency gates in the tunnel plug are designed for a 140-ft head and are operated by 15-in. hydraulic hoist cylinders. The gates are equipped with automatic gate hangers with signal lights for remote control from the valve house. Flap-type air valves with screened intakes, located at the downstream face of the tunnel plug, admit air to the downstream side of each gate leaf. When air is being admitted, or whenever there is no pressure in the conduits, these metal valve flaps hang down, leaving the valves open, but whenever water under pressure comes up into the valves from the conduits, it forces the flaps upward against their seats and thus closes them.

The concrete tunnel plug is keyed into the rock to prevent axial displacement due to full reservoir pressure upon the upstream face. It is heavily reinforced and is provided with open-joint tiles laid in gravel for drainage; the walls of the emergency-gate chamber are likewise provided with tile drains and with weep holes.

Representative examples of class 4 outlets are found in the following outlet works, first at Alcova Dam (Fig. 25) where two 84-in. interior-differential needle valves are installed in a concrete tunnel plug placed about midway of the diversion tunnel's length and arranged so that in the plan view they toe in with their jets converging as they discharge through plate-steel discharge guides into the tunnel beyond.

A large air duct (see sections *AA* and *BB*) admits air, from a shaft extending to the surface above, to the roof of the tunnel just over where the jets from the needle valves emerge from the discharge guides through the wall of the valve chamber. A 102-in. ring-follower hydraulically operated emergency gate is installed in the chamber close behind each needle valve. A 20-ton low-headroom type hand-operated crane serves both needle valves and emergency gates and, when necessary, lifts any of the parts onto a car traveling on rails embedded in the concrete floor of the operating gallery. The car runs to the elevator shaft and onto the platform of the electrically operated freight elevator from where the part can be taken to the surface and loaded onto a truck which can deliver it either to the railroad or to the machine shops at Casper.

The most notable tunnel-plug installations of which we have record are those at Hoover Dam, where in diversion tunnels 2 and 3, located, respectively, on the Nevada and Arizona sides of the river, a total of twelve 72-in. needle valves are installed. Six of these were installed in the Arizona tunnel-plug outlet and six in the Nevada tunnel-plug outlet with their center lines at El. 657.95 and El. 658.89, respectively. This places the valves under maximum operating heads of 571 and 570 ft, being by a considerable margin the highest operating heads with which the Bureau of Reclamation has had to contend to date.

Both of the tunnel-plug outlets are essentially alike in their main features. Each consists of an operating gallery 34 ft 6 in. broad by 114 ft 6 in. long containing six

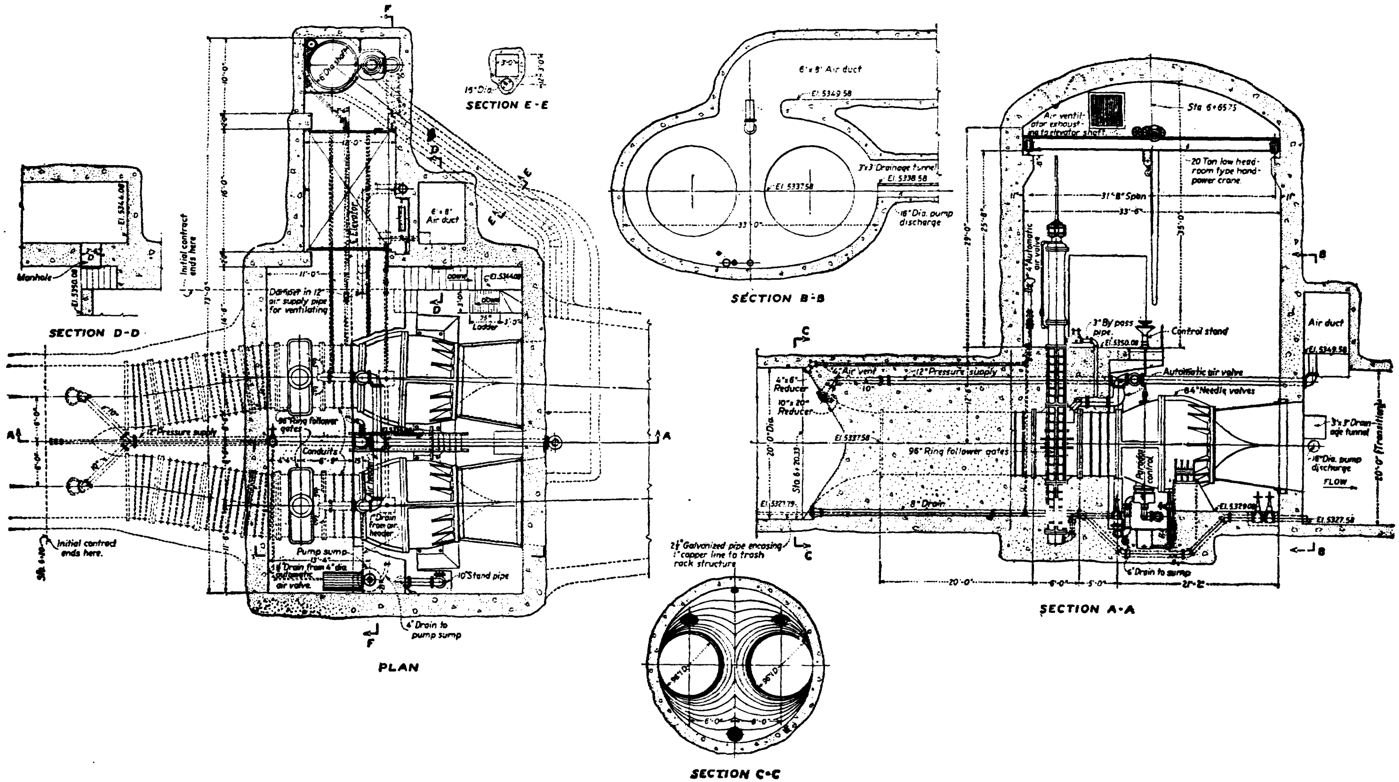


FIG. 25.—Tunnel-plug outlet works at Alcova Dam, consisting of two 84-in. interior-differential needle valves guarded by 96-in. ring-follower emergency gates.

72-in. needle-valves with 86-in. Paradox emergency gates, a 16-in. deep-well type tunnel drainage pump, all the operating and control equipment for the needle valves and the guard gates, and an overhead 20-ton capacity electric traveling crane.

The maximum pressure to which the valves will be subjected, including water hammer, is 265 psi, but they are designed for 300 psi maximum working pressure. When all 12 of the valves are discharging at maximum capacity, they will release in excess of 42,200 cfs. The energy involved is of a magnitude difficult to visualize; for each valve will then be releasing about 230,000 hp, the total for each group of six being equal to 1,380,000 hp, or the equivalent of seven superocean liners each of the same power capacity as that of the "Queen Mary." This energy is all released into a converging oval, on the downstream side of each tunnel-plug outlet, measuring 50 ft high by 82 ft wide which tapers into the 50 ft diameter diversion tunnel beyond.

Much study and model testing were devoted to the purpose of getting the jets from these valves so directed as to pass out through the tunnel beyond with the least possible disturbance. When these two tunnel-plug outlets were tested together to their ultimate discharge capacity, it was found that their jets were correctly aligned and disposed so that their tunnel exits were swept clear, even under high tail-water conditions, the results obtained from the model tests being thus confirmed.

These valves are provided with special control stands electrically operated and interconnected in symmetrical pairs whereby pairs of valves may be set any desired opening from the operating chamber of each tunnel-plug outlet or from the central control room in the power plant by remote control. The control systems of these valves are likewise electrically interlocked with the Stoney gates at the exits of the respective tunnels so that it is impossible to open any of the needle valves if the Stoney gate at the outlet portal of the tunnel into which they discharge is closed.

Access to both tunnel-plug outlet works is by means of adits in the canyon walls, extending from the tunnel plugs to the downstream ends of the powerhouse wings. Rails are laid in the floors of these adits, and a specially built roller-bearing adit car is employed to transport heavy parts along either tunnel. These same tunnels provide communication with the canyon wall valve houses above by means of vertical shafts in which automatic electric passenger elevators operate.

Water is supplied to the six valves of each outlet by a 25 ft 0 in. inside diameter welded plate-steel header, whose lower end terminates in a three-way branch manifold, each branch in turn bifurcating into wyes whose outlets thus form the six conduits required for the six valves. The 25 ft 0 in. inside diameter header, for each outlet works, extends upstream in the diversion tunnel to a point just below the entrance to the last 13 ft diameter penstock, leading to the turbines located in the downstream ends of the powerhouse. A tapered increaser enlarges the header from 25 to 30 ft in diameter, and the header then continues upstream in the diversion tunnel until it reaches the inclined tunnel leading to the base of the intake tower. Here it curves upward through this tunnel and is made fast to the throat liner of the lower cylinder gate in the base of the intake tower at El. 895.00, as shown in Section on Center Line of Tower (Fig. 26).

The manner in which the upper end of the plate-steel header connects into the throat liner of the lower cylinder gate is illustrated in larger scale in section *AA* of Fig. 27, which likewise shows a vertical section through the lower cylinder gate in the closed position with the water passageways through the intake tower walls. In this same figure, the half-sectional plan shows the 12 radially converging water passages with their semisteel conduit linings.

A similar cylinder gate is placed 150 ft higher in each tower at El. 1,045, as shown in Section on Center Line of Tower (Fig. 26). The two gates in each tower are operated by an electrically driven hoist located on the operating floor at El. 1,232.83

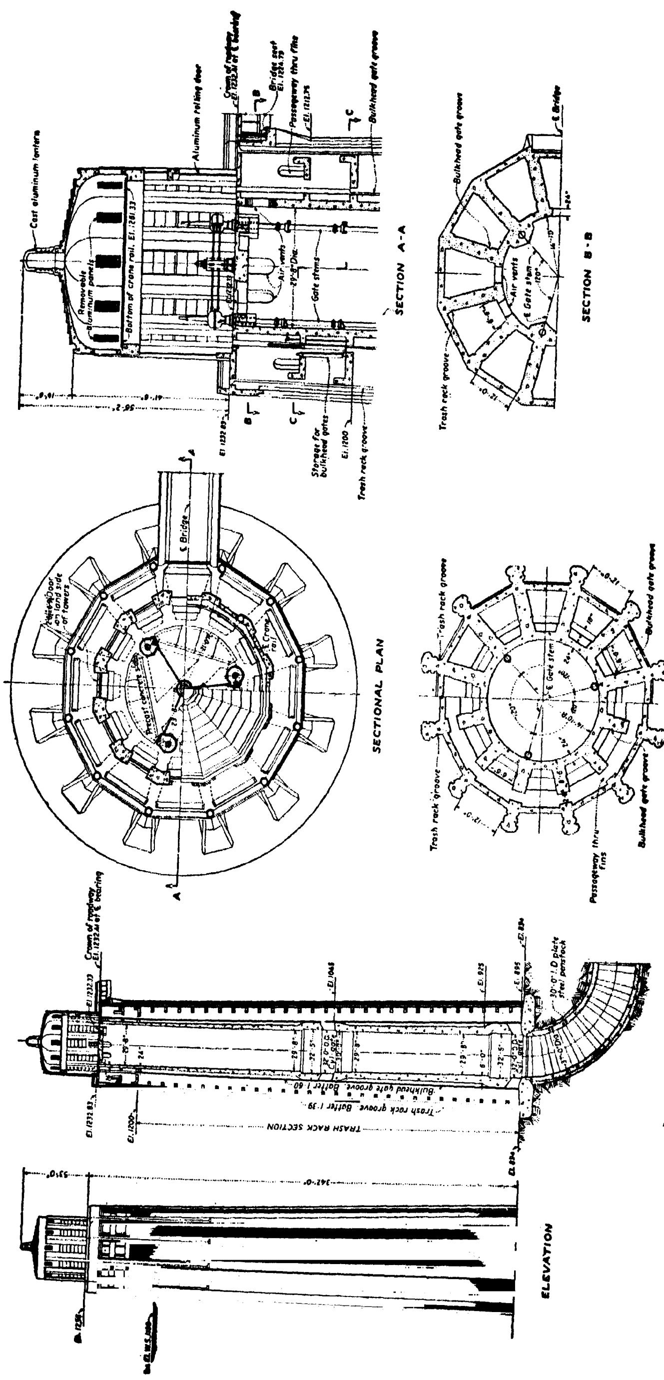


FIG. 26.—Sections and elevations of one of the four similar intake towers at Hoover Dam. All water released from behind the dam must pass through one or more of these intakes.

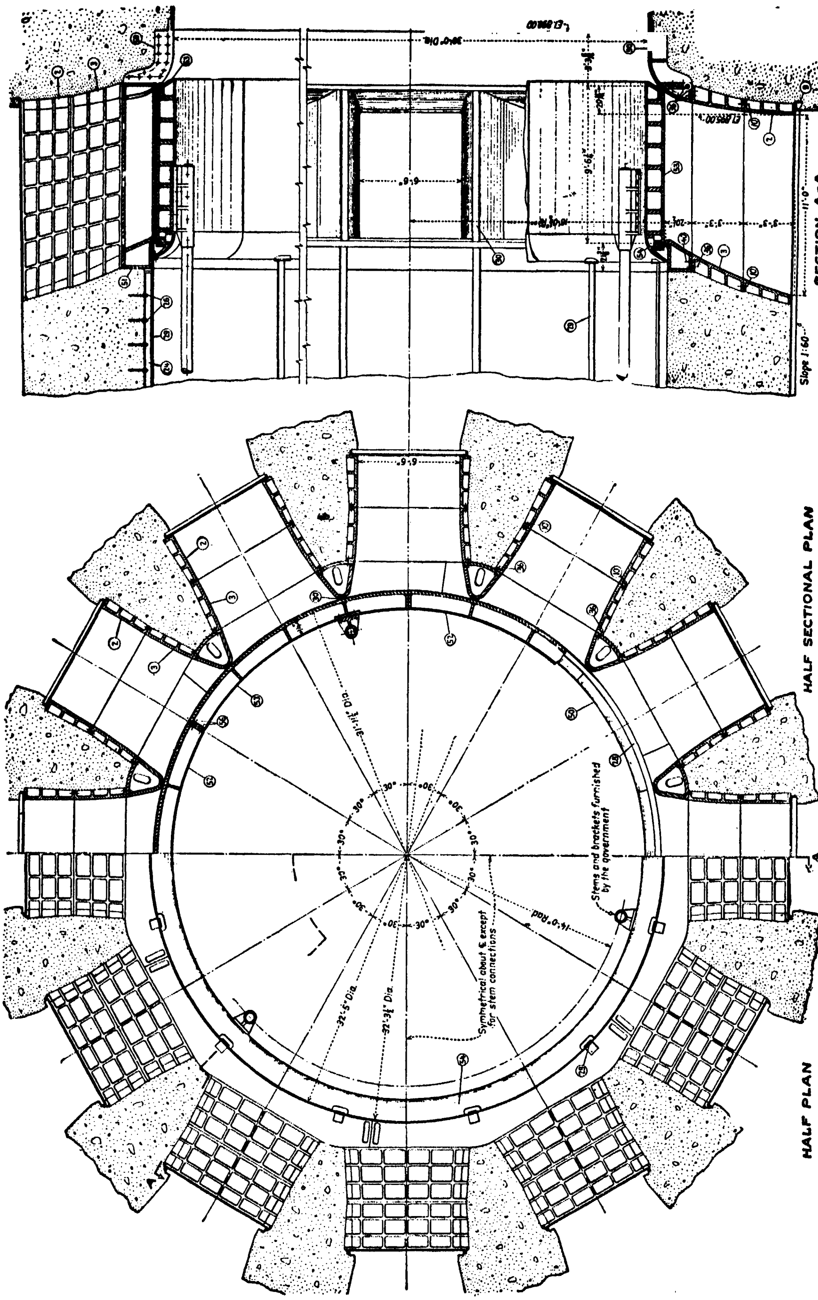


FIG. 27.—General assembly of lower cylinder gate in base of intake tower (see Fig. 26), showing radial water passageways through tower walls.

(see sectional plan and section AA, Fig. 26). The hoist consists of a central dual-drive unit from which three vertically aligned pairs of drive shafts extend radially in a horizontal plane to three dual hoisting units spaced 120 deg apart circumferentially and with vertical center lines on a 28-ft-diameter circle. Each of the three radially extending pairs of drive shafts is coupled to twin worms inside each hoisting unit, which engage and drive worm gears, having vertically disposed hollow hub members to whose extremities threaded lifting nuts are affixed. The lifting nut on the upwardly extending hub, attached to the upper worm gear, engages and supports a solid threaded stem which extends downward to the bottom cylinder gate at the base of the tower below. The lower worm-gear hub-extension lifting nut engages and supports a tubular threaded stem which extends downward, concentrically enclosing the solid stem previously described, and this tubular stem is connected to the upper cylinder gate in the tower below.

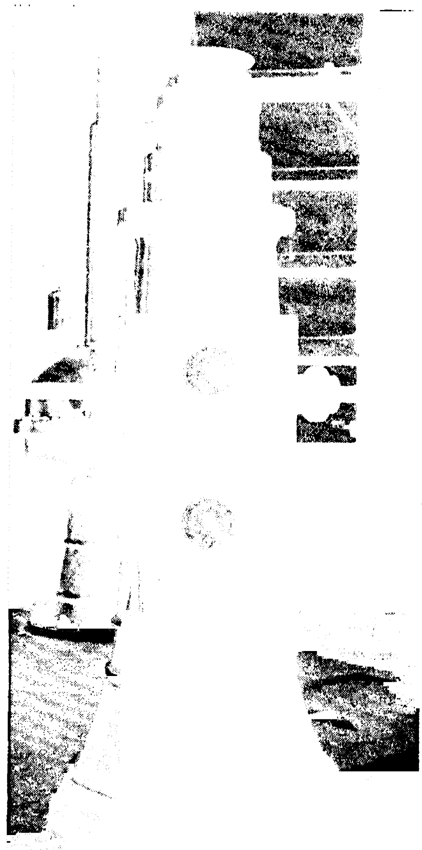


FIG. 28.—Center drive unit, in top of intake tower; see section A-A of Fig. 26. The control panel and indicators for the 32 ft diameter cylinder-gate hoists are shown here.

The center-drive unit has two independent vertically aligned gear motors, the upper motor driving through the upper three radial shafts that are connected with the upper worms and gears in the three hoist units and so operating the lower cylinder gate; the lower gear motor similarly drives the lower radial shafts and worm gears and so operates the upper cylinder gate. In the right-hand view marked "section AA" in this figure, the control panel with the two-gate-position indicator dials is shown. The upper cylinder gates, which are alike in all four intake towers, have a maximum operating head of 184 ft on the bottom sills, and the lower cylinder gates have a maximum of 334 ft on the lower sills.[1]

Both the upper and lower gates have a normal travel of approximately 9 ft, but the hoists are so arranged that when inspection or maintenance is required the gates may be raised an additional 2 ft to provide access to the top seals and seats. The upper or lower gate in each tower may be operated independently or simultaneously by its hoist, the time required for opening being 52 min, whereas closure is effected in 26 min, this being accomplished by the use of two-speed motors.

Two of the four intake towers at Hoover Dam thus afford a third line of defense, by means of the cylinder gates, for the respective batteries of 72-in. needle valves in the tunnel-plug outlet works. The other two intake towers similarly guard and pro-

[1] Recent field tests made with reservoir water surface at El. 1229 show maximum downpulls of 351,000 lb for upper gates, and 875,000 lb for lower gates, exclusive of gate and stem weights.

tect the respective batteries of 84-in. needle valves in the canyon-wall outlet works which are placed in communication with them by means of similar plate-steel headers.

These canyon-wall needle-valve outlets are of the class 5 type and consist of two batteries of six 84-in. valves each, which will normally operate under heads ranging from 160 to 320 ft but are designed for a maximum head of 430 ft. The center lines are placed at El. 820, and the discharge curves show that the combined discharge capacity for all twelve is 47,500 cfs with the reservoir water surface at El. 1,225.00. When this occurs, each valve is controlling in excess of 180,000 hp, making a total for the 12 valves in excess of 2,000,000 hp.

The most outstanding example of valves installed in conduits passing through dams as defined under class 6 is found at Shasta Dam, where eighteen 102-in. conduit tube valves disposed in three tiers of six, eight, and four valves each, with 100-ft vertical distances between each tier, making the bottom tier 332.5 ft below the maximum water surface, are installed in operating chambers formed in the concrete near the upstream face of the dam, with access thereto afforded by longitudinal galleries provided with tracks for special roller-bearing-mounted cars which serve all the chambers on each level and are so arranged that any part of any valve can be removed after a 12- by 12-ft structural-steel coaster gate, arranged for emergency closure, has been lowered down the upstream face of the dam to seal off the bell-mouthed inlet of the conduit leading to the valve to be serviced.

These valves are designed for regulation of their respective conduits from full open down to 35 per cent open for protracted periods. Individual air inlets extending from the horizontal air manifold 78 ft above to annular chambers around the discharge throat and to the open upstream end of each tube element in the interior of each valve are provided for the bottom tier of valves which operate under 323 ft of head.

Studies by electric analogy, later combined with trial runs of 6-in. models of varying shapes and finally confirmed by a 20-in. valve operated under heads exceeding that of the maximum head to be handled at Shasta Dam, were employed to determine the correct shapes and water-passage contours, air requirements, etc., in developing the final designs for these valves, which have a theoretically constant velocity throughout their lengths conforming with that of their 102 in. diameter conduits.

At the downstream face of the dam, the outlet ends of these conduits are curved downward and so shaped as to deliver their discharge as a smooth laterally expanding fan down the face of the dam with the least possible disturbance. This shape was developed from the results derived from numerous model tests and is so effective that when the drum gates discharge from the spillway on top of the dam their descending sheet of water is not appreciably affected either by the presence of these conduit outlets as it passes over them when they are not operating, or when discharging.

These conduit tube valves are electrically operated by push-button control from self-contained control stands immediately above each valve. They are placed in pairs in their operating chambers, which, being located closely adjacent to the upstream face of the dam, make it imperative that the height and length of the chambers normal to stream flow be held to minimum dimensions to avoid unnecessary weakening of the dam in shear and to avoid excessive reinforcement in the upstrean chamber walls. They are provided with special contractible joints at their discharge throats, which permit their easy installation and withdrawal without disturbing the adjacent embedded upstream and downstream conduit castings.

A combination of tube valves and needle valves may be advantageously and economically used where large quantities of water must be released under high heads coupled with the requirement of close regulation for long periods. Friant Dam is an excellent example of this combination of installation. There are two 102- by

110-in. tube valves with two similar size interior-differential needle valves working under a maximum head of 248 ft afford close regulation of the river flow yet afford a combined release in excess of 15,000 cfs when required, while two tube valves and two needle valves of the same size (102 by 110 in.), but working under a maximum head of 114 ft, regulate the flow from small increments up to full capacity of the Friant-Kern Canal.[1]

BUTTERFLY VALVES

Again with reference to the penstock headers of Hoover Dam previously mentioned, four 13 ft diameter penstocks branch off from each of the four 30 ft diameter headers and lead to eight 115,000-hp turbines in the Nevada wing of the power plant and to nine turbines in the Arizona wing, where the 13-ft penstock farthest downstream on that side bifurcates just before it enters the power plant and serves two small turbines of 55,000 hp each. This makes a total of fifteen 115,000-hp turbines and two 55,000-hp turbines in the two wings of the U-shaped power plant. Each turbine is protected and guarded by a butterfly valve located adjacent to the scroll-case inlet and so arranged that either of the two 300-ton powerhouse cranes located in each wing can reach down through removable metal hatchways in the floors above and perform all needful servicing operations, including the setting of the valves as complete units into the conduits or removing them therefrom should such need arise. Special sisterhooks are screwed into tapped holes in the upper ends of the rotor shafts to permit the cranes to lift each complete butterfly-valve assembly in this way.

When all the machinery has been installed in the Hoover power plant, there will then be fifteen 168-in. butterfly valves and two 120-in. butterfly valves in service. The designs for these valves were developed by the engineers of the Reclamation Bureau and were purchased on government specifications, a substantial saving in cost to the government being effected over commercial-type valves available, and provided more compact units that were entirely self-contained, considerably more space being so made available in the power plant for other uses. Both the 120- and 168-in. valves are substantially alike in all essential details, consequently only the 168-in. valves will be described. In Fig. 29, one of these completely assembled valves is shown in longitudinal vertical section AA and in downstream elevation; both of these views include the hydraulic rotor which opens and closes the valve by rotating the leaf 90 deg in the water passage.

The maximum working pressure on these valves, including water hammer, is 271 psi, or 625-ft head. They are designed for 300 psi maximum working pressure, and because of the complex penstock-header systems within which they are installed are arranged for opening or closing operating cycles of not faster than 4 min to avoid setting up harmonics leading to serious pressure rise in these systems. They are designed for safe closure under 300 psi pressure, when interrupting a flow of 8,000 cfs. The waterways through these valves are so shaped as to gradually accelerate the velocity of flow from 14.1 fps at the inlets to 23.2 fps at the outlets when the turbines are developing full power under normal operating conditions.

The 13 ft diameter penstocks leading to the inlets to these valves have tapered increasers expanding to 165⅜ in. where the terminating flanges bolt to the butterfly-valve body castings. Tapered-plate-steel connector pipes of 129 in. inside diameter are bolted to the outlet flanges of the valves. These tapered connector pipes are reduced to 120 in. in diameter at the outlet ends where they are bolted to the inlet flanges of the turbine scroll cases.

Two 12 in. inside diameter symmetrically arranged by-pass lines positioned as

[1] The foregoing program has been changed. Hollow-jet needle valves are now being installed at all outlet conduits in Friant Dam.

shown in downstream elevation are provided around the leaf of each valve, governed by 15½-in. electrically operated and controlled butterfly valves of special design, and so arranged as to automatically open and establish practically equalized pressure upon both sides of the closed main butterfly-valve disk prior to the starting of the

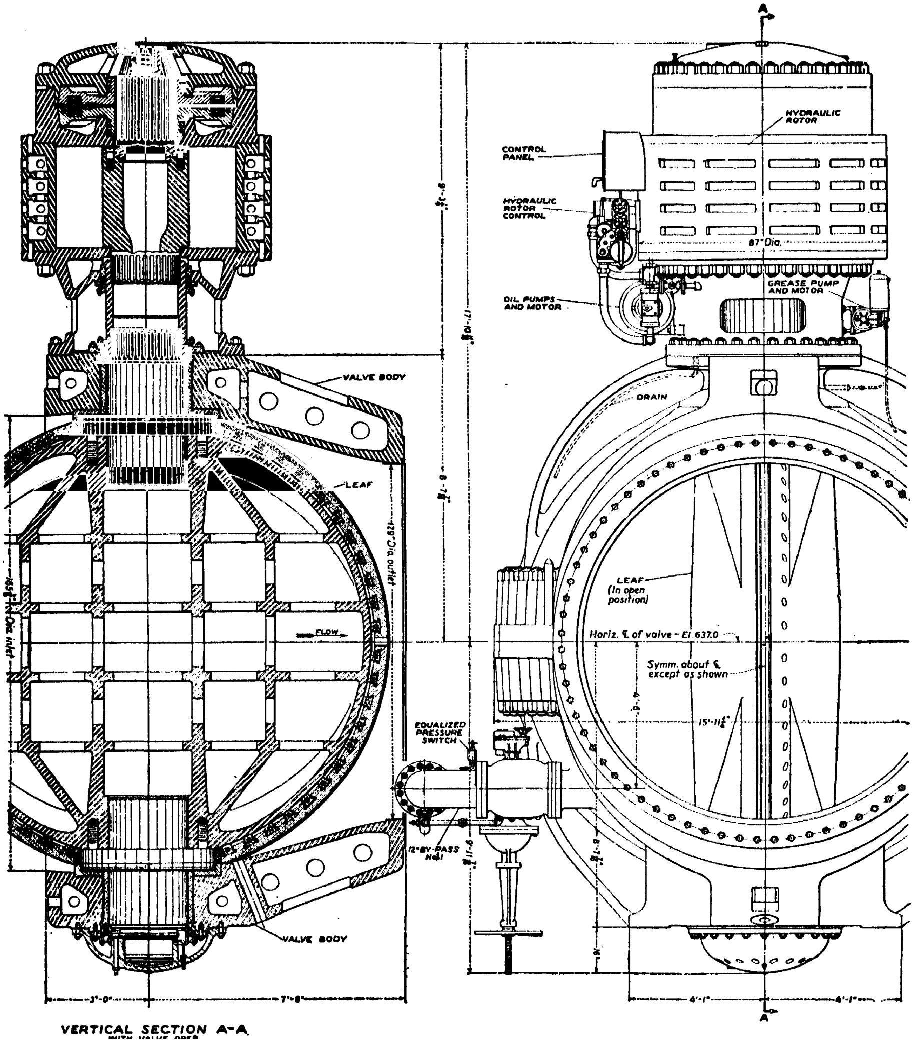


FIG. 29.—Complete assembly and longitudinal section of 14-ft butterfly valve with hydraulic rotor. Designed for 300 psi pressure, one of these valves protects each of the fifteen 115,000-hp turbines of Hoover Dam.

hydraulic rotor in the opening cycle. These by-pass lines have manually operated guard gate valves above the butterfly valves for protection.

The butterfly-valve bodies have two heavy base feet or flanges which rest upon anchor rails carried upon a heavy bifurcated concrete pedestal. The gap in the pedestal allows access to the hydraulically jacked thrust bearing that carries the weight of the rotating parts of each valve, amounting to about 120,000 lb. A high-

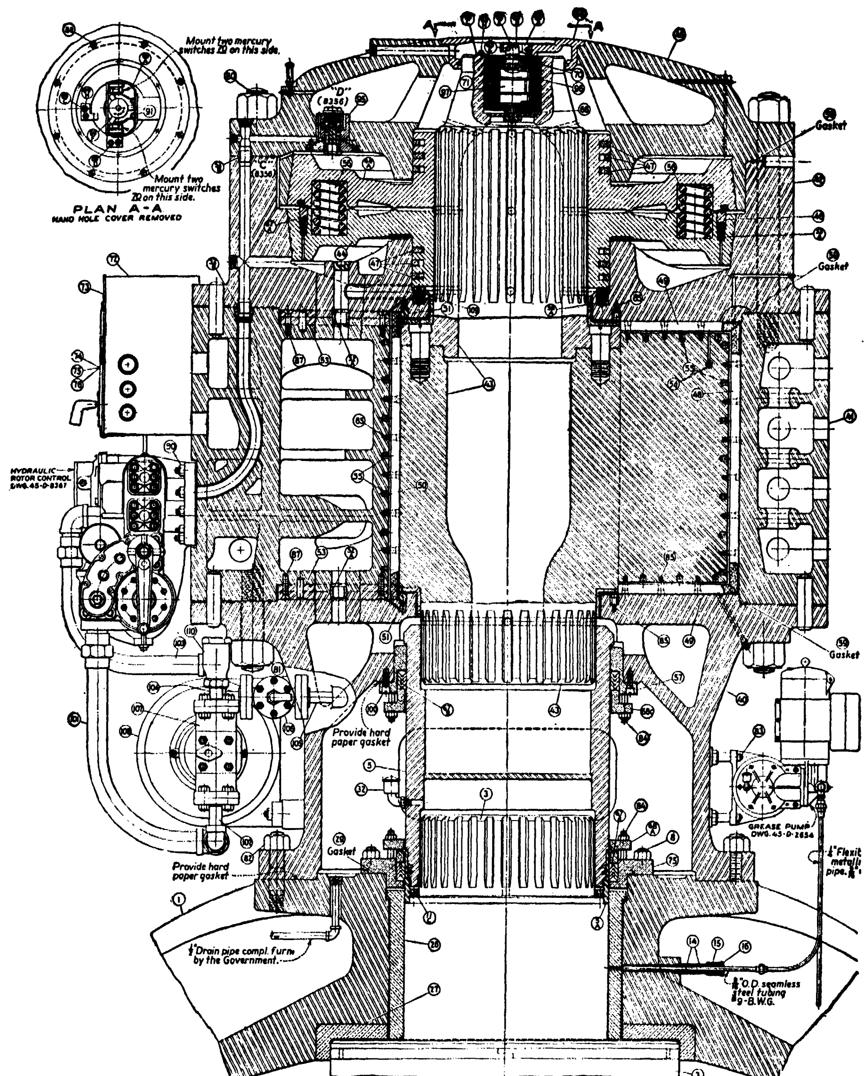


FIG. 30.- Vertical section through the hydraulic rotor which operates the butterfly valve shown in Fig. 29.

pressure Alemite gun is employed to pump oil into the jack, thereby raising the leaf into its correct central position in the valve body, after which a threaded locking collar on the jack shank is pulled up tight, the oil pressure released, and the leaf, stems, etc., are then rotated while supported on the self-aligning roller thrust bearing.

The complete valve consists essentially of two separable units which are the valve proper and the hydraulic rotor, shown in Fig. 30. These two units, which are complete, self-contained, operable entities of themselves, are readily combined into the complete operating assembly shown in Fig. 29 by setting the hydraulic rotor unit (Fig.

30) down onto the receiving flange of the valve unit and then tightening the nuts on studs 82 of Fig. 30 where it will be seen that a counterbore in the bottom flange face of the rotor base receives a matingly turned pilot on the upper face of valve body, which thus automatically ensures the correct concentric alignment of the two units.

Every reasonable precaution has been taken to ensure safety in the operation of these units and to protect the maintenance crews against unauthorized operation of the butterfly valve guarding a turbine within which they may be at work. Removal of the handle and key from the control panel interrupts all electric circuits for that unit so that it is made inoperative either from the panel or by remote control. Removal of the manhole cover from the turbine scroll case automatically cuts the main leads supplying power to the oil pump motor, and, as a still further precaution, the handle on the right-hand side of the rotor-control mechanism can be unlocked by a special key, the handle turned 90 deg, and the key withdrawn after relocking the handle in this position, which leaves two plug valves in the oil circuit closed, so that in the event some short circuit should by any possible chance start the oil pump motor there is no possibility of oil under pressure reaching any of the rotor chambers to start the valve in either direction.

The hydraulic rotor consists of a rotating vane, or rotor 43, having a cylindrical hub with oppositely extending arms 180 deg apart, whose extremities are brought into close proximity to the bore of the cylinder 41 which is provided with two abutments or stators, spaced 180 deg apart, whose inwardly projecting apexes are brought into close proximity to the cylindrical hub of the rotor.

By this arrangement, the interior of the cylinder, when viewed in typical horizontal section (Fig. 31), is divided into four symmetrical pressure chambers, which are interconnected into diametrically opposite pairs by tubular ports through the rotor hub and so arranged that if pressure is introduced into one of the pairs of pressure chambers the force so imposed upon the vertical faces of the opposite arms of the rotor will cause it to rotate in a clockwise direction, or, if the pressure is introduced into the other pair of pressure chambers, then the rotor will be caused to turn in a counterclockwise direction and in so doing will produce similar motion in the splined coupling, whose lower end is splined to the upper stem 3 of the butterfly-valve, which is accordingly rotated to the open position or to the closed position, as the case may be.

In the vertical sectional assembly of the hydraulic rotor (Fig. 30), it will be seen that the rotor 43 has an upwardly extending splined shank on which two oppositely facing brake disks 44 and 44A have matingly splined engaging hubs, these disks being thus caused to rotate whenever the rotor 43 rotates. These brake disks are forced apart vertically, the lower one downward and the upper one upward, by a series of powerful helical coil springs 56 set into pockets in their opposing faces, the conically turned peripheries being thus forced into braking contact with similarly bored conical seats in brake cylinder 42 and cover 45. When these disks are so engaged, the hydraulic rotor and the valve disk in the water passage below are all rigidly locked against rotative movement.

The rotor unit constitutes an entirely self-contained and operable mechanism which can be assembled and tested prior to its installation on the companion valve assembly. Cavities formed in the cover 45 (see Fig. 30), the stators of cylinder 41, the central core of rotor 43, and in the base 40 are all interconnected and so arranged as to form an oil reservoir of ample capacity so that no outside oil-storage tanks or supplemental oil-supplying piping is required. All moving parts operate submerged in oil so that lubrication is ideal. All oil moving from this reservoir to the pumps passes through a large strainer before entering the pumps, and all oil entering the reservoir, when it is being filled, passes through this strainer. All oil and grease pumps, motors, hydraulic rotor-control mechanism, control panel, etc., are mounted directly upon the rotor

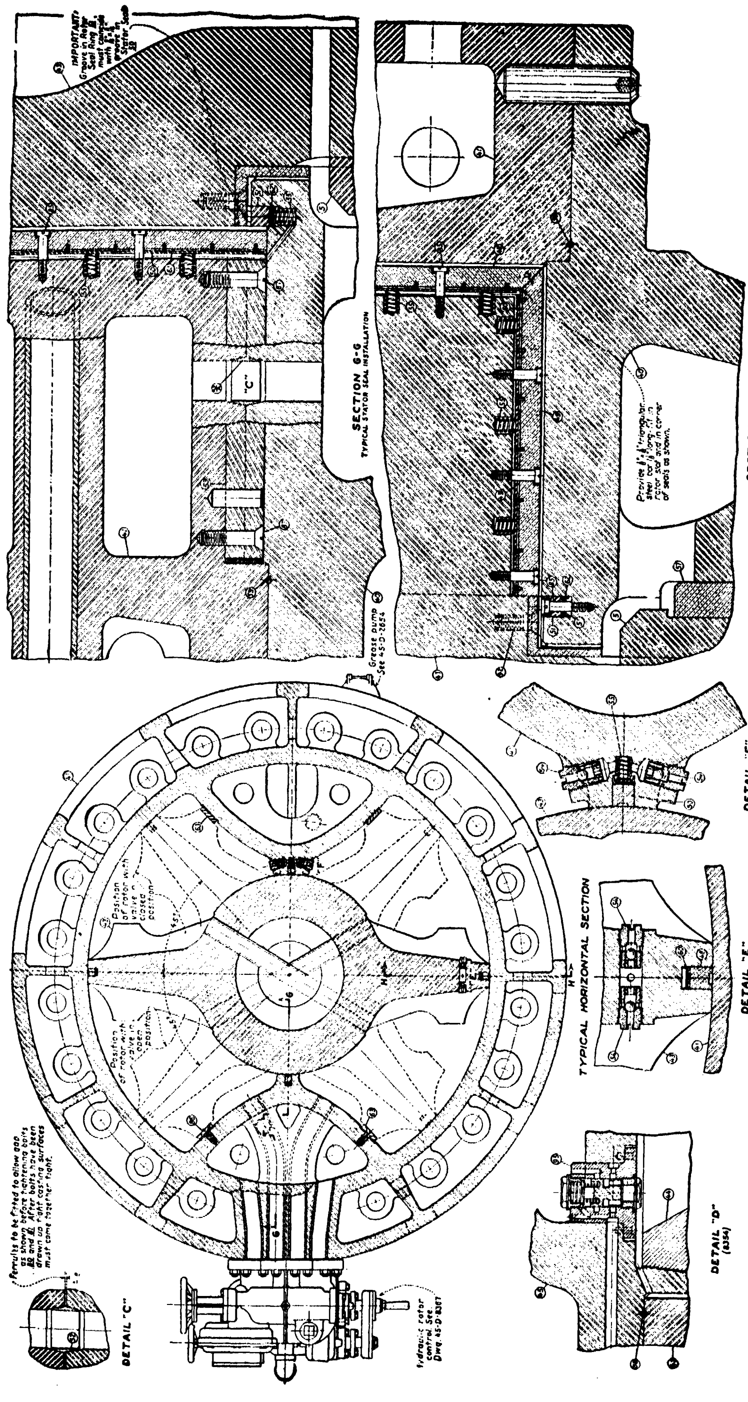


FIG. 31.—Assembled sections of the hydraulic rotor, showing details of construction of the high-pressure oil-sealing mechanisms, and diagrammatic section of rotor in its open, closed, and midway positions.

assembly so that none of these parts need be removed when the rotor unit is completed, tested, and ready to be installed on the valve assembly.

When so installed on the valve, the turning torque which it generates is applied to the valve stem in practically perfect balance so that no bending moment is put into the valve stem when the torque is being applied, whereas with the conventional bell-crank torque mechanisms, much additional bending is imposed upon the stems.

As the rotor unit is bolted directly to the top flange of the valve body, all rotational reactions are self-contained, and no outside anchorages in the walls or floor of the power plant are required to resist the torsional reactions arising from the closing or opening of the valve.

A high-pressure electrically operated grease pump (see Fig. 30) is mounted upon the rotor base on the opposite side from that occupied by the oil pumps and motor. This grease pump, designed to pump against 6,000 psi pressure, is used to lubricate the main valve bearings.

Bronze seals, carried in the periphery of the valve leaf and adjustable under pressure from the downstream leaf face, contact mating bronze seats in the valve body.

The complete valve and rotor assembly (as illustrated in Fig. 29) has an over-all height of 27 ft 10 in., a breadth of 15 ft 11¼ in., an over-all length of 10 ft 6 in., and weighs 380,000 lb.

At the present writing, it is believed that these valves constitute a record for severity of duty to which they are subjected; for under emergency closure conditions they are designed to interrupt a potential energy flow in excess of 600,000 hp. There are other butterfly valves considerably larger than these, such as the 27 ft diameter valves at Conowingo Dam, but they are under low head, and in consequence the operating duties do not closely approach those of the valves at Hoover Dam.

TRACTOR GATES FOR HIGH-HEAD INTAKES

For high-head dams, where large quantities of water are passed through turbines requiring penstocks of exceptional size, the providing of suitable intake gates on the water face of the dam has been a difficult problem to solve, and here again the rapidly increasing demands for larger and still larger installations under higher and still higher heads have necessitated the creation and development of equipment differing in many respects from that previously employed.

The engineers of the Reclamation service, in preparing the designs for Norris Dam for the T.V.A., were faced with the problem of providing intake gates for the two 20 ft diameter penstocks, which have rectangular inlets 16 ft 6 in. broad by 28 ft 6 in. high and are subjected to a maximum head of 184 ft. It was this set of conditions which brought about the development of what are now known as *tractor gates*, now in successful service at Norris Dam (see Fig. 32).

Each of these gates is operated through 28-part lines of steel hoisting rope reeved through sheaves attached to the gate crosshead and multiple sheave blocks carried in the under portion of the frame of the double-drum electric-motor-driven hoist of 300,000-lb nominal capacity, located beneath the parapet on the water face of the dam in enclosed recesses provided to receive them (see sections *CC* and *DD*). The gate-leaf assembly is suspended on the steel cables attached to the hoist above. The cables pass around multiple rope sheaves housed inside the enclosed crosshead member to whose outer ends roller carriages are attached. The roller carriages carry and support each side of the gate leaf through interposed inclined roller trains in similar manner to the usual Paradox gate construction previously described, the arrangement being such that the gate leaf, crosshead, and attached roller carriages with their wedge roller trains all travel as a single entity when they are being lowered from the top of the dam until such time as the leaf comes opposite to the penstock inlet opening, whereupon

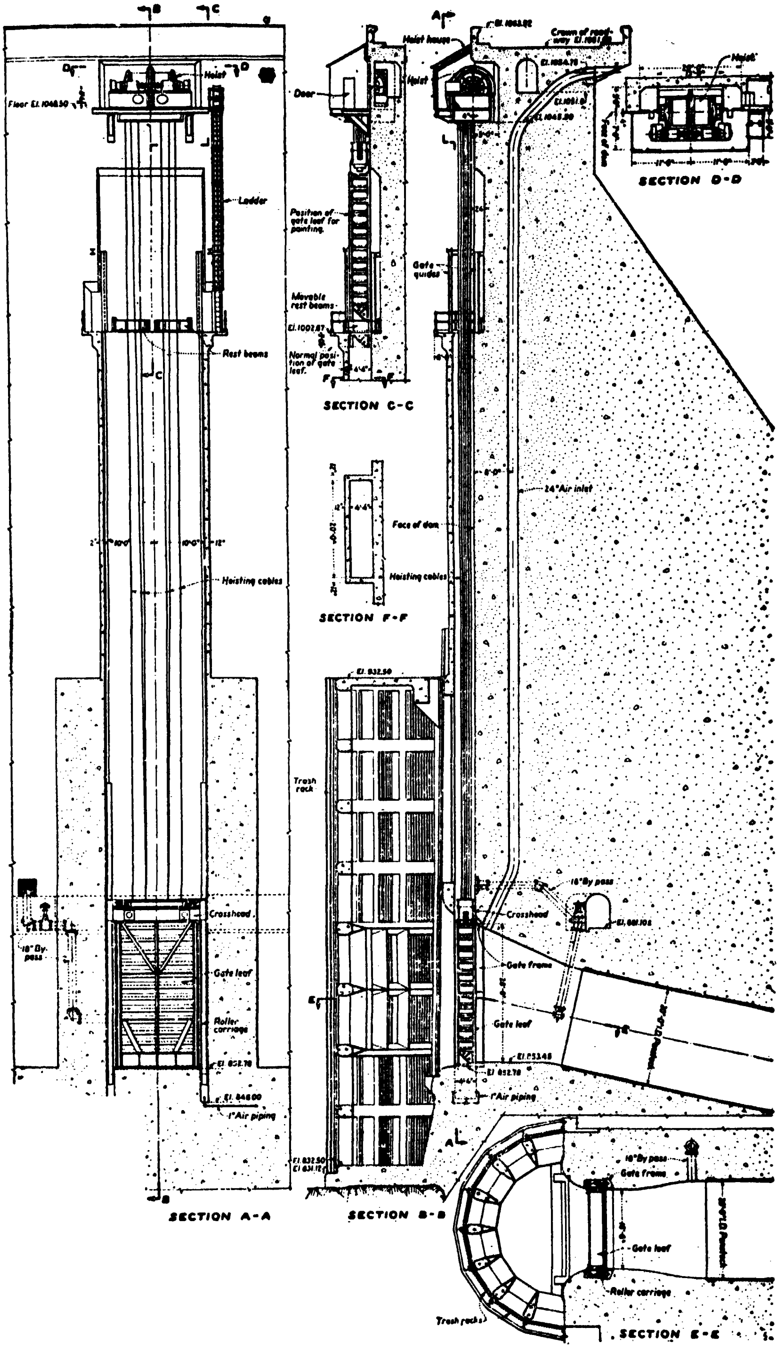


FIG. 32.—16.5- by 28.5-ft cable-operated tractor gate for 20 ft diameter penstock intake under 178-ft operating head.

the further downward travel of the leaf is arrested by the lower crossbeam coming into contact with stops in the gate frame. When this occurs, the leaf is restrained from any further downward movement while the crosshead continues its downward descent about 24 in. farther and so allows the inclined wedge roller trains to gradually withdraw their support from between the roller carriages and the downstream face of the gate leaf, the water pressure prevalent upon the upstream face of the leaf being permitted gradually to force it over horizontally as the wedge roller trains descend until the seal bars on the downstream face come into contact with the mating seat bars on the gate frame, which then receive the entire water load pressing against the upstream side of the gate leaf.

In opening the gate, this process is reversed. The crosshead with its attendant roller trains being drawn upward by the hoist until the cooperating inclined-wedge roller trains engage the sides of the gate leaf, when they then act as rolling wedges which gradually force the leaf horizontally upstream away from sealing contact with the frame seats until these parts are free. The stops on the lower portions of the roller carriages then contact the lower beam of the gate leaf on each side, and the crosshead, roller carriages, inclined roller trains, and gate leaf move upward as a single entity as the hoist performs its duty above.

By turning now to sections *BB*, *EE*, and *FF* of Fig. 32, it will be seen that the entrance to each 20 ft diameter penstock is protected by a trash rack which extends only part way up the face of the dam, and in consequence its top face is usually a considerable distance below the water level in the reservoir.

A concrete closure similar to a chimney, with internal dimensions just large enough to allow the gate leaf and carriage assembly to pass up and down through it, is built against the face of the dam from the top of each trash rack up to the location of the bottom of the gate leaf when in the normal raised position (see sections *AA* and *CC*). By this arrangement, no trash can enter the interior of the trash-rack structures, and all moving parts that would otherwise be required for closing the gate entrance slots are avoided.

A further improvement on penstock inlet gates was worked out by the Reclamation Bureau engineers for use with the 18-ft diameter penstocks leading to the 150,000-hp turbines at Grand Coulee Dam; these gates are known as penstock coaster gates.

PENSTOCK COASTER GATES

These gates are rectangular gates carried on dual tandem-aligned roller trains on either side (see Fig. 36, End Elevation) and are mounted on tracks embedded in the face of the dam in the same general manner as the tractor gates just described. Here the wedge roller trains with cooperating crosshead and synchronizing mechanism are eliminated, however, and seating is accomplished by admitting hydraulic pressure behind the seal bars which are contained within a hydraulic chamber on the downstream face of the gate, which extends around its edges like a rectangular picture frame (see Fig. 36, Downstream Elevation). The assembled seal bars extend around all four sides of the rectangular hydraulic chamber (see detail sections *CC* and *DD*) and are reciprocatingly mounted therein so that when hydraulic pressure is introduced into the chamber behind them it causes them to move downstream for a sufficient distance as to permit their peculiarly grooved faces to make sealing contact with the seat bars in the gate frame embedded in the face of the dam, in substantially the same general manner as the sealing mechanism on the ring-seal gates previously described functions.

Coaster gates may be operated by multiple-cable hoists similar to the tractor gates just described, but this necessitates their being hung above the water when opened, in order to prevent their hoisting cables being rapidly eaten away by corrosion. This

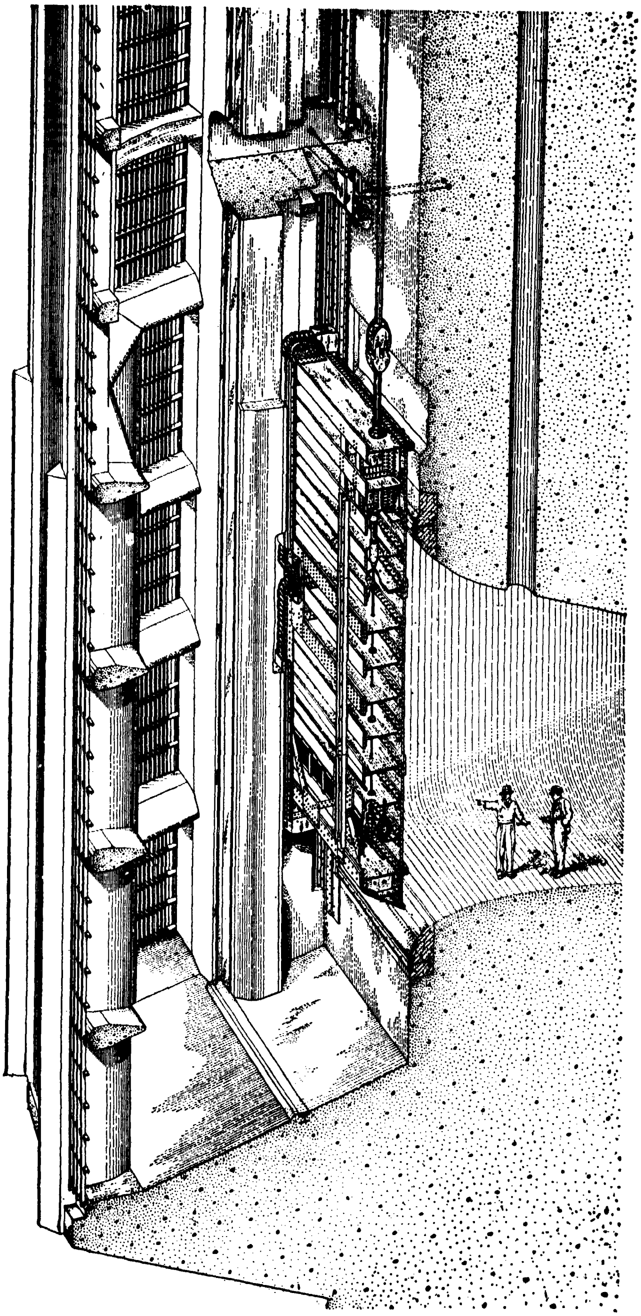


FIG. 33.—15- by 29.65-ft stem-operated coaster gate guarding penstock intake to 150,000-hp turbine at Grand Coulee Dam.

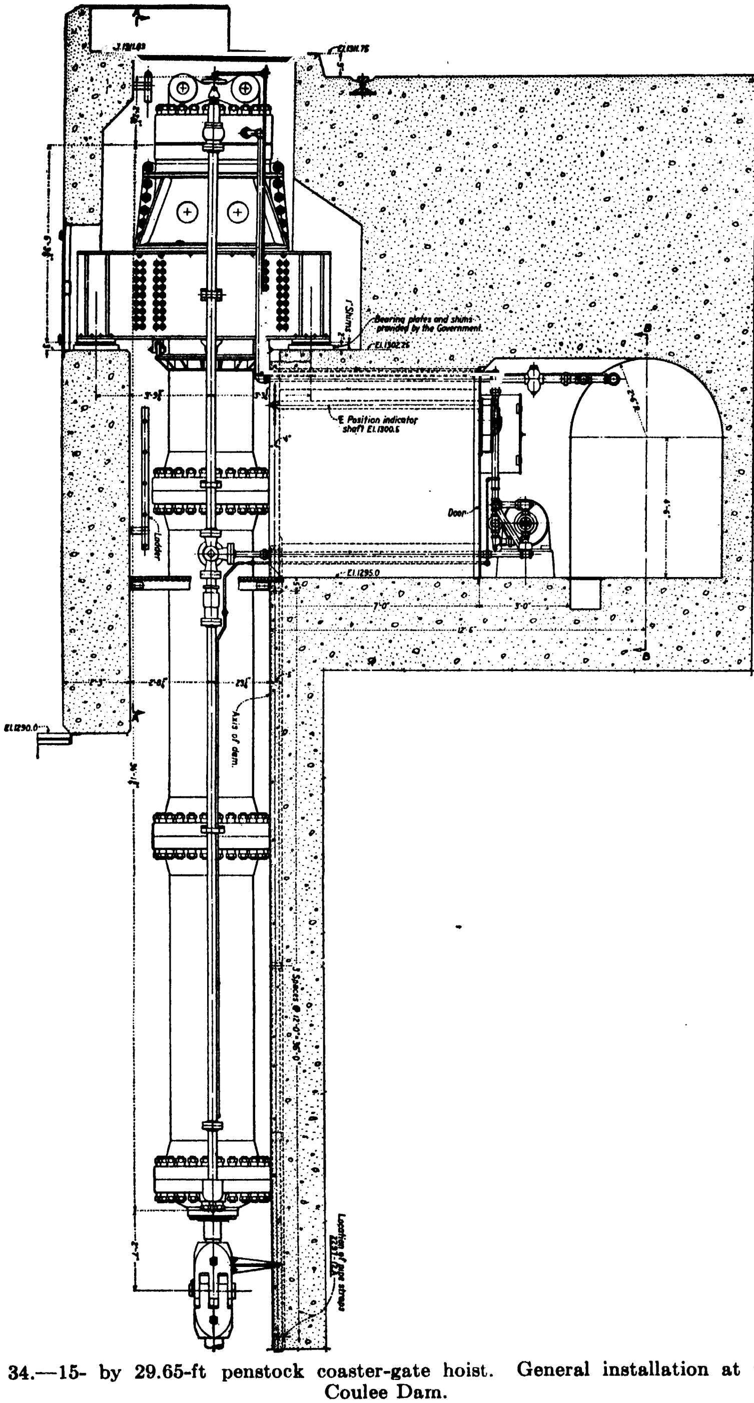


FIG. 34.—15- by 29.65-ft penstock coaster-gate hoist. General installation at Grand Coulee Dam.

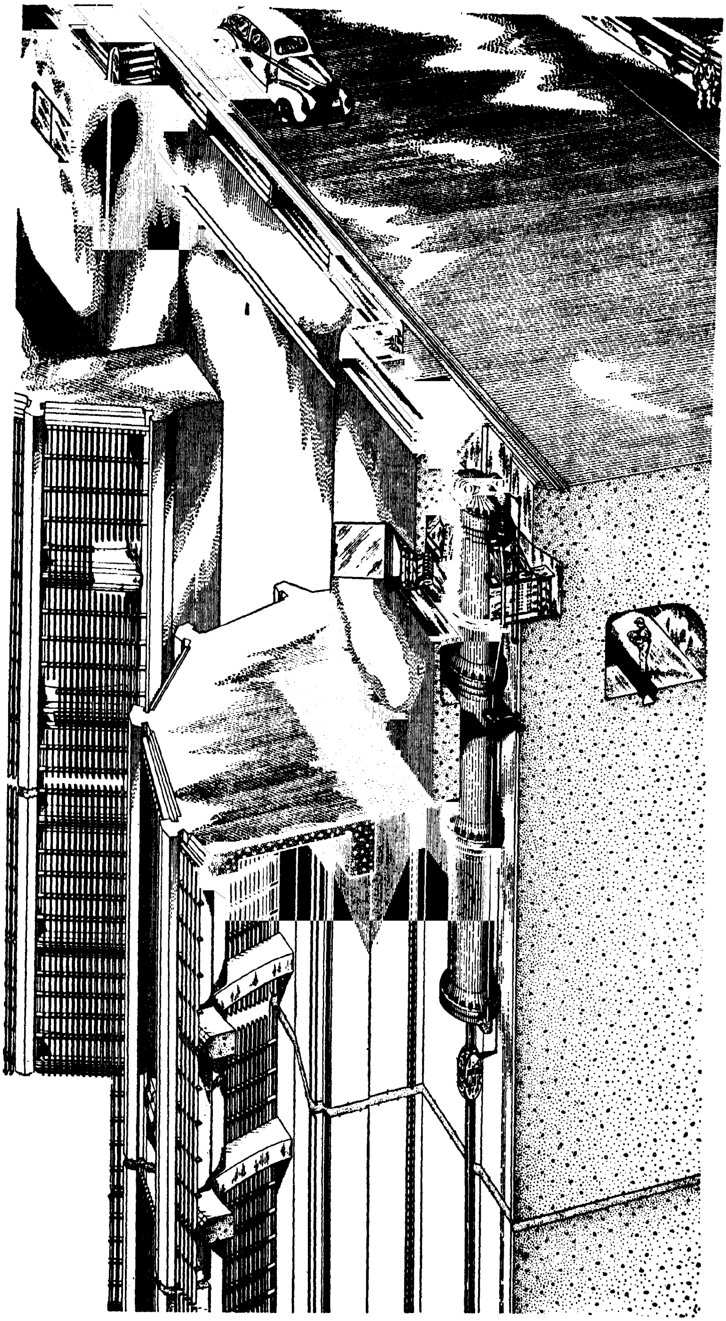


FIG. 35.—Grand Coulee penstock coaster-gate hoist. This perspective view shows how each hoist is entirely concealed beneath the sidewalk floor plates on crest of the dam, and the relative arrangement of the semicircular trash racks, operating gallery, and roadway.

likewise is a serious matter from a time-cycle standpoint when closing under emergency conditions, as 7 to 10 min might be required for the gate to descend to a point where it would begin to shut off flow, whereas complete closure can be effected in 2 min or less if the gate could be left suspended with its bottom edge in continuity with the top of the opening.

By using a hydraulic hoist and knuckle-jointed steel stems 7 in. in diameter (as shown in Fig. 35), this is safely accomplished at Grand Coulee Dam, emergency closure can be effected in a total time interval of less than 2 min if desired. All that is required of the operator is to press the closing button for any gate, whereupon the oil-pump motor automatically starts, releases the automatic gate hanger which is built into the upper hoist cylinder head, and the gate then descends at the predetermined speed to which it has been adjusted to operate.

FIXED-WHEEL GATES

Fixed-wheel gates as usually made of structural steel and consist primarily of a skin plate mounted on horizontal beams that are supported by vertical girders at the gate sides. Wheels attached to the vertical girders transmit the water load on the gate to tracks attached to the face of the main concrete supporting structure. It is this feature of the wheels being fixed and maintained in their relative positions with respect to the gate in similar fashion to those of a railroad car, that gives these gates their name. The tracks are bolted to structural beams, called track bases, embedded and attached to anchor bolts in blockouts or recesses provided in the original concrete of the structure. This arrangement permits the mass of the main concrete structure to be placed first, so creating a rigid, immovable base having embedded anchor bolts with adjusting nuts, see Fig. 37, that makes possible the later installation and alignment of the gate frames, seats, and tracks with a precision and accuracy of adjustment not otherwise possible, and ensures the maintenance of these adjustments while the blockouts are being filled with concrete for final embedment of the parts.

These gates are made in a wide range of sizes, starting at 7.8 by 7.8 ft, thence on up to 50 by 50 ft, and for a correspondingly wide range of heads. They are used for three fundamentally different types of installation:

Surface Type. Located in spillways, etc., where the head is not greater than the gate height. Upstream skin plate and upstream seals are used, with side seals of the L type, while the bottom seal consists of the machined lower edge of the bottom girder and skin plate butting against the matingly machined upper face of a beam embedded in the concrete floor. No downpull occurs in this type. Downpull is defined and described later.

Reference should be made to Sec. 8 for further information on crest gates.

Tunnel Type. Operated in a vertical shaft above a tunnel with heads at any practical height above the gate height, but with no reservoir pressure on top of the gate. Upstream skin plate and seals are used. Top and side seals are of the music-note (compression) type and the bottom seal is of the butt type (similar to surface type). Downpull forces are negligible.

Submerged (or Face) Type. These are located on the upstream face of a dam or intake structure where operating heads are greater than the height of the gate, where the area of the gate in plan is subject to reservoir pressure. This type has the skin plate and the seals on the downstream face. The seals are usually of the music-note (compression) type and are along all four sides of the gate with mitered joints at the corners. Downpull forces occur in this type of gate during emergency closure with flow passing beneath it, as practically static reservoir pressure may then be present on its top area, while the high velocity flow beneath in some instances causes a very large

drop in pressure on its lower surface due to the conversion of pressure head to velocity head.

Skin Plate and Beams. Skin plates are attached to the horizontal beam flanges with rivets in the case of upstream skin plates and with ribbed bolts in the case of downstream skin plates. Recently, welded construction has been used on installations of this type. Advantage is taken of this attachment by providing sufficient rivets or bolts to make the skin plate act as a cover or flange plate to resist beam bending as well as to make the flanges act as haunches to resist skin-plate bending between the beams.

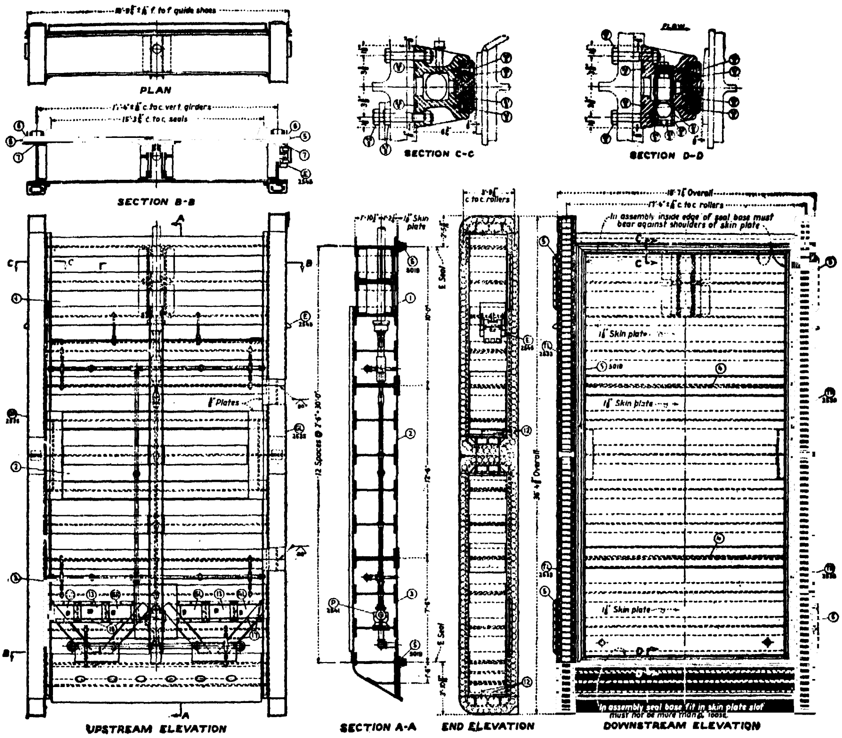


FIG. 36.—Fifteen- by 29.65-ft penstock coaster-gate leaf. Sections *CC* and *DD* show enlarged views of hydraulically operated sealing mechanism, as installed at Grand Coulee Dam.

The longitudinal shear between the skin plate and the beam flanges due to beam bending and to skin-plate bending are combined by the formula $\sqrt{S_b^2 + S_p^2}$, and sufficient rivets or bolts are provided to develop this resultant.

Skin-plate stresses are determined by considering the skin plate as a continuous beam with haunches.

Although the gates are painted to protect them from corrosion, an additional allowance is made in the design when calculating stresses by assuming that $\frac{1}{32}$ in. of material has been removed from all wetted surfaces. This later allowance is assumed to take care of any local corrosion that may occur due to weak spots in the protective coating and to wear from sand blasting or other cleaning processes which will be applied many times during the expected life of the gate.

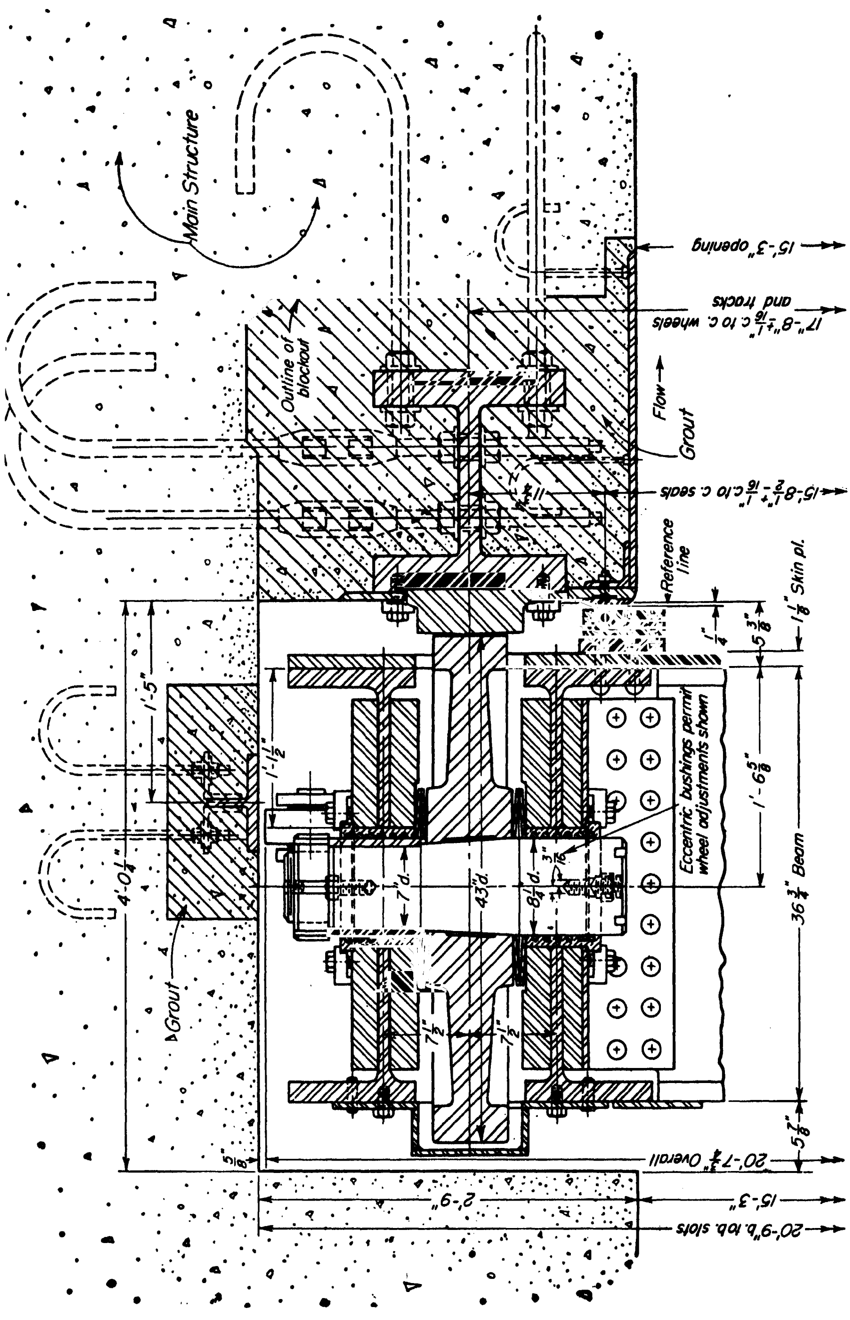


FIG. 37.—Fixed-wheel gate. Slot section assembly showing gate in place on track and frame sections that have been grouted into blockouts formed in the main structure after they have been accurately aligned and held in position by the anchor bolts previously embedded in the main structure concrete. (*Courtesy of Bureau of Reclamation.*)

Beams for the narrower gates, particularly the tunnel and submerged types, are usually WF types. The WF are desirable as their relatively wide flanges assist in resisting the skin-plate bending stresses. For wide gates where the beam depths required are greater than those available in rolled sections, the plate-girder type of cross section is used. For wide gates of the surface type, a fish-bellied girder is used so that the slots in the concrete side walls in which the gates are operated can be kept to a minimum.

Flange plates are sometimes used over either or both beam flanges in addition to the skin plate. The effective width of skin plate as a flange plate has been limited to 0.11 times the beam span. Recent studies have confirmed this 0.11 factor as being very conservative, particularly for short-span beams.

Vertical girders are usually made double in order to receive the wheels between them. Although this makes fabrication more difficult, it gives a more compact arrangement, allows the slots in the concrete side walls to be smaller, and indirectly reduces the downpull on submerged gates as the seal projection can be kept to a minimum, so reducing the area of the gate in plan that is subjected to downward reservoir pressure.

Where practicable, the vertical girders are made in one piece. However, this can be done only where the gates are to be completely knocked down for shipment as is generally necessary for very large gates. Most gates are shipped in fabricated sections as large as can be conveniently handled. This allows a much larger portion of the fabrication to be accomplished in the shop. Where this is done, the outer vertical girders are spliced for field connections, and the inner vertical girders become short pieces fitted between the horizontal beams.

Knowing the water load on the gate, the procedure for wheel design is as follows:

Wheels are usually made of wrought steel, A.S.T.M. Specifications A 244-42, with rims hardened. Tracks are usually made of A.I.S.I. type 410, 13 per cent chrome, stainless iron. This material provides corrosion resistance and can be heat-treated to obtain practically any hardness desired. As the tracks are usually inaccessible after installation, they are made with a Brinell hardness number (B.h.n.) approximately 50 points higher than that of the wheels. This difference tends to make any wear occur on the wheels, which are readily accessible for replacement.

The design of the wheels is governed primarily by the stress in the tread. The required projected area of the tread, *i.e.*, the area represented by the product of wheel diameter and the net face width, is determined by the following formula.[1] Although these tests were made on rollers of relatively small diameters, it has been found that the formula usually gives a conservative design for wheels.

$$\text{Critical stress (psi of projected area)} = (\text{Brinell hardness number}) \times 24.5 - 2{,}200$$

Owing to the necessarily extreme stiffness of the vertical girders of the gate on which the wheels are mounted and to allow for slight misalignment of track surfaces, it is assumed that any one wheel on one side of the gate may not bear on the track for a short distance of travel. This condition will cause an overload on some of the adjacent wheels. This overload is infrequent and of short duration, however it is used in designing the wheel because the maximum allowable shearing stress is somewhat indefinite as will be explained later.

Owing to deflection of the horizontal gate beams when under load and the consequent rotation of the vertical girders in the plane of the gate span, and, considering that it is practically impossible to install the track faces in an absolutely true plane, the track faces are crowned slightly in a plane normal to wheel travel, as indicated in Fig. 38.

[1] NOONAN and STRANGE, U.S. Bureau of Reclamation, Tests on Rollers, *Tech. Memo.* 399, p. 8, Sept. 26, 1934.

This permits the track to receive, at or near the center, the major portion of the wheel loads.

After the diameter and the net tread dimensions have been determined from the above formula, the stresses are analyzed in accordance with the method derived in the University of Illinois Engineering Experiment Station.[1]

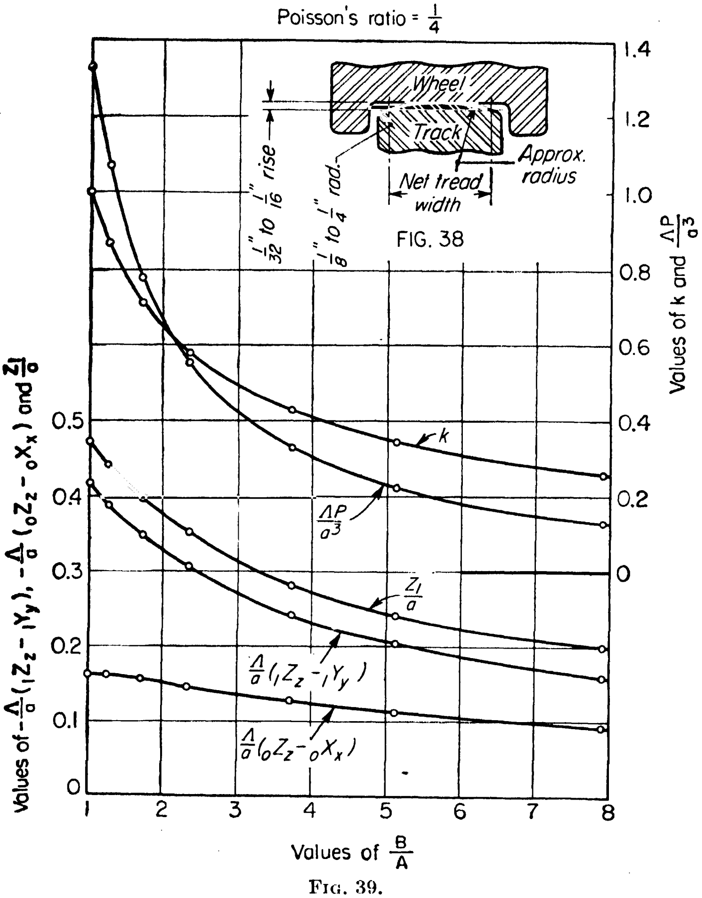


FIG. 39.

FIG. 38.—Detail section through wheel rim and track.

FIG. 39.—Curves showing values of variables used in wheel-loading formulas (3) on page 397. (Graph from *Univ. Illinois Eng. Exp. Sta. Bull.* 212, p. 21.)

The following example illustrates the above procedures:

Example:[2]

Wrought-steel wheel, rim hardened to 255 B.h.n., min.

Wheel load 52,000 lb normal, 80,000 lb max.

Wheel diameter 17 in. max (determined from practical limits).

$$\text{Critical load} = (255 \times 24.5) - 2{,}200 = 4{,}050 \text{ psi, project area}$$

$$\text{Allowable load} = \frac{4{,}050}{3} = 1{,}350 \text{ psi, project area for normal load}$$

[1] THOMAS and HOERSCH, Stresses due to the Pressure of One Elastic Solid upon Another, *Univ. Ill. Eng. Exp. Sta. Bull.* 212, July 15, 1930.

[2] *Ibid.* See pp. 32 and 50 of *Bulletin* for similar examples.

or $= \frac{4{,}050}{2} = 2{,}035$ psi, project area for max load

$$\text{Projected area required} = \frac{52{,}000}{1{,}350} = 38.5 \text{ sq in. for normal load}$$

or $= \frac{80{,}000}{2{,}025} = 39.5$ sq in. for max load

$$\text{Net tread width required} = \frac{39.5}{17} = 2.33, \text{ or, say, } 2\tfrac{1}{2} \text{ in.}$$

P = wheel load = 80,000 lb
R_1 = radius of track = 30.25 in.
R_2 = radius of wheel = 8.5 in.
A = mean of reciprocals of radii in x-direction $= \frac{1}{2R_1}$
B = mean of reciprocals of radii in y-direction $= \frac{1}{2R_2}$
E = moduli of elasticity $= 30 \times 10^6$ psi
o = Poisson's ratio $= \frac{1}{4}$
$\Lambda = \frac{2(1 - o^2)}{E(A - B)}$ = evaluation of elastic properties and shape properties
a = semimajor axis of ellipse of contact
b = semiminor axis of ellipse of contact
z_1 = depth to point of maximum stress difference or point at which maximum shearing stress occurs

$$\frac{B}{A} = \frac{R_1}{R_2} = \frac{30.25}{8.5} = 3.56 \tag{1}$$

$$B + A = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{1}{2}(0.0331 + 0.1177) = 0.07535 \tag{2}$$

From Fig. 39, for
$$\frac{B}{A} = 3.56 \tag{3}$$

$$k = 0.44 \qquad \frac{\Lambda P}{a^3} = 0.34 \qquad \frac{z_1}{a} = 0.28$$

$$\frac{\Lambda}{a}({}_1Z_z - {}_1Y_y) = 0.25$$

$$\Lambda = \frac{2(1 - 0.0625)}{(30 \times 10^6)(0.07535)} = 8.3 \times 10^7 \tag{4}$$

$$a = \sqrt[3]{\frac{\Lambda P}{0.34}} = \sqrt[3]{\frac{8.3(80{,}000)}{(0.34)(10^7)}} = \sqrt[3]{0.1952} = 0.58 \tag{5}$$

$$({}_1Z_z - {}_1Y_y) = 0.25\frac{a}{\Lambda} = \frac{0.25 \times 0.58 \times 10^7}{8.3} \tag{6}$$

$$= 174{,}698 \text{ psi}$$

$$= \text{max difference of stress components}$$

$$\text{Max shearing stress} = \tfrac{1}{2}({}_1Z_z - {}_1Y_y)$$

$$= 87{,}349 \text{ psi} \tag{7}$$

$$z_1 = 0.28a = 0.28(0.58) = 0.1624 \text{ in.} \tag{8}$$

Data on the maximum shearing stress allowable are somewhat indefinite.[1] However, it is generally assumed that a maximum shearing stress of about 90,000 psi is safe. This value is approximately the estimated ultimate shearing strength of a steel having a B.h.n. of 255.[2] In view of the infrequent operation of the wheels and the slow speed, together with the infrequent occurrence of the overload condition on which the stress analysis is made, the 90,000 psi is deemed conservative.

The required rim hardness is specified to penetrate at least $1\frac{1}{2}$ to 2 times the calculated depth, z_1.

The wheels, when assembled in the gate, are each provided with an eccentric pin

[1] *Ibid.*, p. 48.
[2] Concrete and Metals Laboratory, *Report* C-137, p. 3.

(or bushing) adjusting device so that their downstream track-contacting surfaces can all be adjusted to a common plane, either in the shop or in the field.

When the gate leaf is under water load, it becomes virtually impossible to guide its movement into a new course in case it is incorrectly aligned with its opening and frame seats. This is due to the extremely high unit contact pressure between its wheels and the supporting tracks.

To care for this condition the gate is provided with channel-shaped guide shoes at each corner, which engage tongues on the gate guides embedded in the face of the dam. These guide shoes are snubbed-spring mounted, the springs being strong enough to square the gate into correct alignment just before the water load starts to build up against it and so ensures the gate starting to close in its proper position and alignment. To secure this result the gate guides with these tongues are installed from the top of the dam down to about one gate height above the conduit opening with the face to face of the guide tongues held accurately to ¼ in. clearance in each guide shoe. Below an elevation slightly greater than one gate height above the top of the conduit opening, this clearance is reduced to practically zero by making longer tongues on the guides. This causes the shoes to square the gate with the opening just before the water load starts to build up on it. After the gate has been closed and the water pressure against it equalized by the by-pass preparatory to opening, the spring shoes are strong enough to realign it again before the opening cycle is started.

Operating Hoists. Hoisting mechanism may be of the gate stem with hydraulic cylinder type, multiple cable type, or plate link chain type at each side of the gate operated over back-geared chain sprocket hoists, with or without counterweights, or it may be a screw stem at each side of the gate operated by a geared hoist head with a lifting nut on each stem driven by a motor with oppositely extended shafts interposed between the two hoist heads. In some instances, powerful traveling gantry cranes provided for servicing other machinery at the dam or headworks may also be employed to operate one or more such gates at a number of conduit inlets. Where the traveling gantry operates on a bridge spanning the spillway, as at Shasta Dam, it then becomes very desirable to reduce the maximum hoisting load as much as is practicable to avoid the excessive costs for the extra-heavy bridge construction otherwise required. This can be done by designing the gate and cooperating structure to reduce the downpull that is present in all gates of the submerged (face) type, whenever they are emergency-operated with discharge through their conduits. Such a condition arose at Shasta Dam, where the original designs contemplated the operation of the river outlet conduit emergency gate from a barge, and then later it was decided to use the gantry crane carried by the highway bridge over the spillway. This gate is 11.05 by 11.05 ft. It is operated under a maximum head of 323 ft with an equivalent water load of 20,600 psf or a total water load of 2,700,000 lb. Its weight is 83,700 lb plus 10,000 lb for lifting mechanism. (Automatic grappling head.)

Downpull. Model tests have revealed that, when closing gates of the submerged (face) type under emergency conditions (with discharge through the conduit), pressure reduction beneath the gate at any point is equal to the velocity head at that point. The original design for the Shasta 102 in. diameter outlet conduit gate, which is required to operate under a maximum head of 323 ft, provided a bottom sloping down in direction of flow at 45 deg to where this plane cut the downstream gate face. The vertical upstream face of the gate was merged into this bottom slope by a radius of approximately one-fourth the gate thickness. The spring point where the water left contact with this sloping bottom was thus made coincidental with the vertical downstream gate face. The gate seal mechanism extended horizontally downstream 6$\frac{3}{16}$ in. beyond this face. Hydraulic tests of this design on a model with the gate and conduit built on a scale of 1 to 17, showed that the downpull was approximately 75,000 lb

when the gate was wide open, and that this gradually increased to a maximum of 260,000 lb at an opening of 77 per cent, or about 8 ft 6 in. and then gradually decreased to zero at the closed position. At small gate openings, the high velocity jet issuing below the gate impinged upward on the air-vent inlet in the conduit behind the gate and prevented its proper functioning. The high velocity jet discharging into the partially filled conduit below the gate causes a corresponding reduction in pressure through conversion from pressure head to velocity head beneath the gate and explains the unbalanced downpull effect.

A series of hydraulic model tests conducted to improve this condition developed a new shape consisting of a flat bottom gate with a radius of 9¼ in. or 24 per cent of the gate thickness joining the flat bottom with the upstream vertical face of the gate and a vertical lip extension plate extending downward from the downstream edge of the flat bottom for a distance of 14½ in. or 37.6 per cent of the gate thickness, the flat bottom with this lip plate being extended horizontally downstream to provide a clearance of ½ in. between the downstream face of the lip plate and the seat face on the embedded gate frame and with the lower edge of the lip beveled 67 deg from the vertical on its upstream side to form a knife-edged spring point, along its downstream bottom edge (see Fig. 40).[1] With this change the downpull was reduced to 131,000 lb at 55 per cent gate opening, and the objectionable jet impingement on the air-vent inlet was eliminated. This vertical lip was stiffened by triangular gusset plates installed in a vertical plane aligned with stream flow. These extended across and were fastened to the flat gate bottom dying out where the radius joining the vertical upstream gate face comes tangent to the flat bottom surface. The downstream vertical legs of these gussets were attached to the lip plate and extended downward to where they were terminated with their lower corners coinciding with the origin of the 67 deg taper along the bottom of the lip plate. Model tests showed that these gussets did not cause disturbance to flow or any appreciable increase in downpull.

Downpull was still further reduced by a recess formed in the vertical face above the conduit bellmouth. The width of this recess was made approximately the same as that of the bellmouth diameter. The maximum depth of this recess was made 18 in. or about three times the distance that the seal mechanism protrudes from the downstream face of the gate. The lower limit of this recess was made 3 ft 5⅞ in. (or 0.316 of the bellmouth diameter of 11 ft 0⅝ in.) above the point where the bellmouth curve becomes tangent to the vertical face on the vertical center line of the conduit. Starting at zero depth at that elevation, the recess was carried up on an entering taper of constantly increasing depth until its full depth of 18 in. was reached at a height of 4 ft 8 in. farther up, or 0.422 of the bellmouth diameter. This makes a slope of 0.322 to 1.00 vertical. The recess was then continued vertically at constant depth for 6 ft 5 in. (0.58 bellmouth diameter) and from thence was sloped back out on angle of 45 deg to terminate at the face of the dam 18 in. above. This makes the total height of the recess 12 ft 7 in. (1.14 bellmouth diameter). With the recess installed, tests showed that the maximum downpull occurred at 80 per cent gate opening, and that it had been further reduced to 70,000 lb. Tests showed that for gate openings of less than 40 per cent, the presence of a recess with constant depth was undesirable as the downpull force acting upon the top seal having been eliminated by the recess, the increase in pressure against the bottom of the gate which occurred at small openings was large enough to reverse the direction of the net hydraulic forces acting upon the gate. The gate then was subjected to uplift forces greater than that of its weight, and it then refused to close. By tapering off the depth of the recess from full depth to zero in the lower 4 ft 8 in. of its height, this objectionable feature was removed, as this taper

[1] WARNOCK, J. E., and HOWARD J. POUND, Coaster Gate and Handling Equipment for River Outlet Conduits in Shasta Dam, *Trans. A.S.M.E.*, **68**, No. 3, Fig. 3, 1946.

caused the velocity of flow passing between it and the top seal to increase gradually as the gate descended, and so increased the unbalanced downward pressure upon its upper face to compensate partially for the increasing upward force simultaneously occurring below. The downpull then gradually reduced from a maximum of 70,000 lb at 80 per cent opening to about 7,000 lb at 40 per cent opening, then peaked again at about 24,000 lb at 20 per cent opening, then declined to zero at the fully closed position.

While these data are derived from tests made upon a coaster gate, they are equally applicable to a fixed-wheel gate, as the only difference is the substitution of wheels in place of the roller trains, together with the relatively slight difference in weight involved.[1]

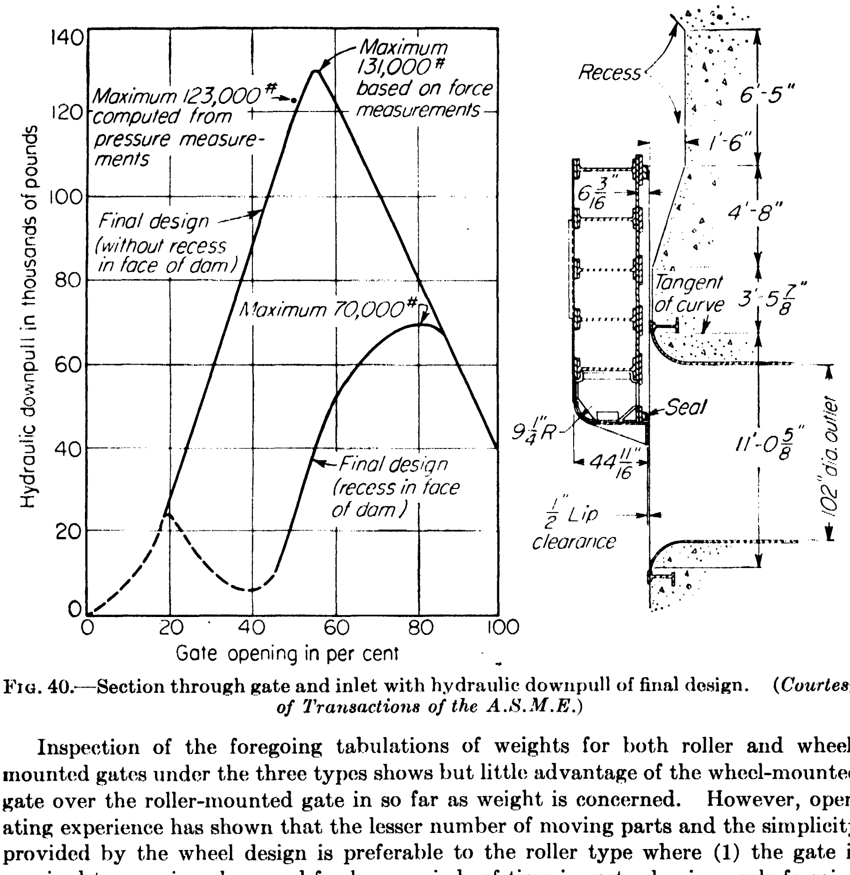


FIG. 40.—Section through gate and inlet with hydraulic downpull of final design. (*Courtesy of Transactions of the A.S.M.E.*)

Inspection of the foregoing tabulations of weights for both roller and wheel-mounted gates under the three types shows but little advantage of the wheel-mounted gate over the roller-mounted gate in so far as weight is concerned. However, operating experience has shown that the lesser number of moving parts and the simplicity provided by the wheel design is preferable to the roller type where (1) the gate is required to remain submerged for long periods of time in water having scale-forming propensities, (2) the gates are operated for regulation purposes as in spillway crests, where they are subjected to the dynamic forces created when high-velocity discharge flow passes beneath them for long intervals, and so subjects their supporting mechanisms to the continuous surging impact of the fountaining discharge.

The five 50- by 50-ft roller-train-mounted spillway gates at Parker Dam are a notable example of this condition. There, the continuous action of the fountaining jets upon the lower portions of the bottom caterpillar roller trains gradually increased

[1] *Ibid.*, pp. 199-206.

No.	Size, ft	Mounting	Head, ft	Weight	Dam and project	Drawing No.
				SURFACE TYPE SPILLWAY GATES		
1	14 × 50	Wheels	50	*G* 87,312	Seminoe Dam	144-D-1413
				F 53,100	Kendrick	
				H 45,856		
2	19.5 × 22	Wheels	22	*G* 34,172	Gila River Crossing	50-D-202
				F 26,600	Gila Project	
				H 22,930		
3	25.8 × 40	Wheels	40	*G* 128,330	Alcova Dam	4-21-36
				F 64,989	Casper Alcova	
				H 86,717		
4	40 × 44.5	Wheels	44.5	*G* 274,463	Horse Mesa Dam	25-D-1267
				F 80,202	Salt River	
				H 134,795		
5	50 × 50	Rollers	50	*G* 400,634	Parker Dam	25-D-1472
				F 118,668	Parker Dam Power Project	
				H 158,400		
6	50 × 50	Wheels	50	*G* 385,000	Keswick Dam	214-D-8948
				F 100,000		
7	50 × 50	Wheels	50	*G* 300,000	Davis Dam	351-D-332
				F 108,000	Davis	
				TUNNEL TYPE GATES		
8	11.25 × 13.67	Wheels	23	*G* 13,124	Minidoka Power Plant	17-D-1195
				F 12,654	Minidoka	
				H 5,250		
9	19.5 × 24.9	Wheels	190	*G* 212,276	Shasta Dam	214-D-7162
				F 236,671	Central Valley	
				H 118,000		
10	22 × 35	Wheels	60	*G* 140,725	Parker Power Plant	231-D-1025
				F 51,918	Parker Dam	
				SUBMERGED (FACE) TYPE GATES		
11	11.05 × 11.05	Rollers	330	*G* 83,700	Shasta Dam	214-D-9436
				F 60,628	Central Valley	
				H Gantry	....	Outlet conduits
12	13 × 20.11	Rollers	108	*G* 78,000	Grand Coulee Dam	222-D-9591
				F 52,500	Columbia Basin	
				H 60,000		
13	15 × 19.05	Rollers	260	*G* 154,300	Shasta Dam	214-D-9188
				F 106,731	Central Valley	
				H 93,441	....	Penstock gate
14	15 × 29.65	Rollers	267	*G* 203,616	Grand Coulee Dam	222-D-2528
				F 140,838	Columbia Basin	
				H 102,700	....	Penstock gate
15	7.8 × 7.8	Wheels	48	*G* 13,100	Altus Dam	258-D-225
				F 14,200	Altus	
16	9.86 × 9.86	Wheels	138	*G* 43,500	Friant Dam	214-D-9694
				F 28,609	Central Valley	
				Gantry	....	Outlet conduit
17	11.92 × 11.92	Wheels	205	*G* 77,480	Friant Dam	214-D-9693
				F 47,813	Central Valley	
				Gantry	....	Outlet conduit
18	15.25 × 30	Wheels	207	*G* 257,270	Anderson Ranch Dam	4-D-521
				F 154,823	Boise	
				H 62,500	....	Outlet conduit
19	17.5 × 34.66	Wheels	127	*G* 193,000	Davis Dam	351-D-174
				F 105,000	Davis	
				H 56,500	....	Penstock gate

G. Gate. *F*. Frame. *H*. Hoist.

the clearance in the roller links, caused the bronze washers mounted upon the pins to revolve, and these, acting like circular saws, proceeded to cut off the pins, so releasing the rollers into the jets, rendering the gates inoperative, and requiring extensive repairs under exceedingly difficult working conditions.

Roller-mounted gates submerged for long intervals in scale-forming waters have been found to have their links encrusted until they cannot articulate to allow their rollers to pass around the ends of their raceway tracks, and so they break and render the gates inoperative.

The simplicity of the wheel-type mounting allows a ruggedness of design which largely overcomes these objections. The smaller number of moving parts are likewise less costly to machine. The tracks on the gate frames for the wheel-type gates are usually less expensive than those required for the roller-type gates. They are likewise easier to install and align.

CYLINDER GATES

At Cle Elum Dam, two external-type cylinder gates arranged in vertical tandem alignment, with the lower gate sill 140.25 ft below maximum reservoir level and the upper gate 70 ft below, regulate the inward flow into the 14-ft 0-in.-diameter central shaft. This shaft extends downward to a curving elbow immediately beneath the lower gate (Fig. 41), whose outlet side communicates with the discharge tunnel located at the same elevation as the inlet tunnel which delivers water from a typical rectangular trash rack in the reservoir at its inlet end through a 14-ft 0-in. horizontally disposed wye, both legs of which are arranged for closure by butterfly valves installed therein, to a vertically disposed annular chamber that surrounds the central shaft as indicated in section *BB*. A 6 ft 0 in. diameter spiral stairway gives access to the butterfly valve operating chamber from the cylinder gate hoist room 126 ft above. This stairway is arranged so that it can be quickly collapsed against the shaft wall to allow material and mechanism to be lowered from the 20-ton crane in the hoist house above. These butterfly-valve bodies are solidly encased in reinforced concrete with their upper stems protruding into the operating chamber to receive their respective turning gear mechanisms and are so arranged that their patented retractable stems may be withdrawn into their leaves, the leaves with the encased stems being thus allowed to be withdrawn through their conduits whenever necessary for repairs, without disturbing their embedded housing or body castings.

Six equally spaced rectangular openings 4 ft 0 in. broad by 6 ft 0 in. high serve as ports through the reinforced barrel (see Fig. 42, section *BB*) of the central 14 ft 0 in. inside diameter shaft, and the impinging jets from these ports when the gates are open serve as a means of dissipating a large portion of the kinetic energy in the water being liberated. The external cylinder gates, each approximately 20 ft in external diameter by 6 ft high, are made up of structural steel plates riveted to internal hoops framed from heavy angles placed back to back to resist the external collapsing pressures. Each gate is made up of six equal segmental sections held together by body-bound bolts at the vertical flanged joints (see Fig. 42) and is raised to open and lowered to close by three stems operated by a hoist similar to those employed to operate the intake tower cylinder gates at Hoover Dam, previously described.

When operating at partial openings of the cylinder gates, it was found that the air ducts supplying air to the interior of the central 14 ft 0 in. inside diameter central shaft were wholly inadequate, and in order to make operation possible, it was necessary to remove the floor plates in the hoist room and leave all the doors and windows open in order to supply enough air to make operation reasonably satisfactory. In an attempt to gage the air velocities inside the operating room, an anemometer was tried, but the air currents were so strong that the rotating vaned wheel element was

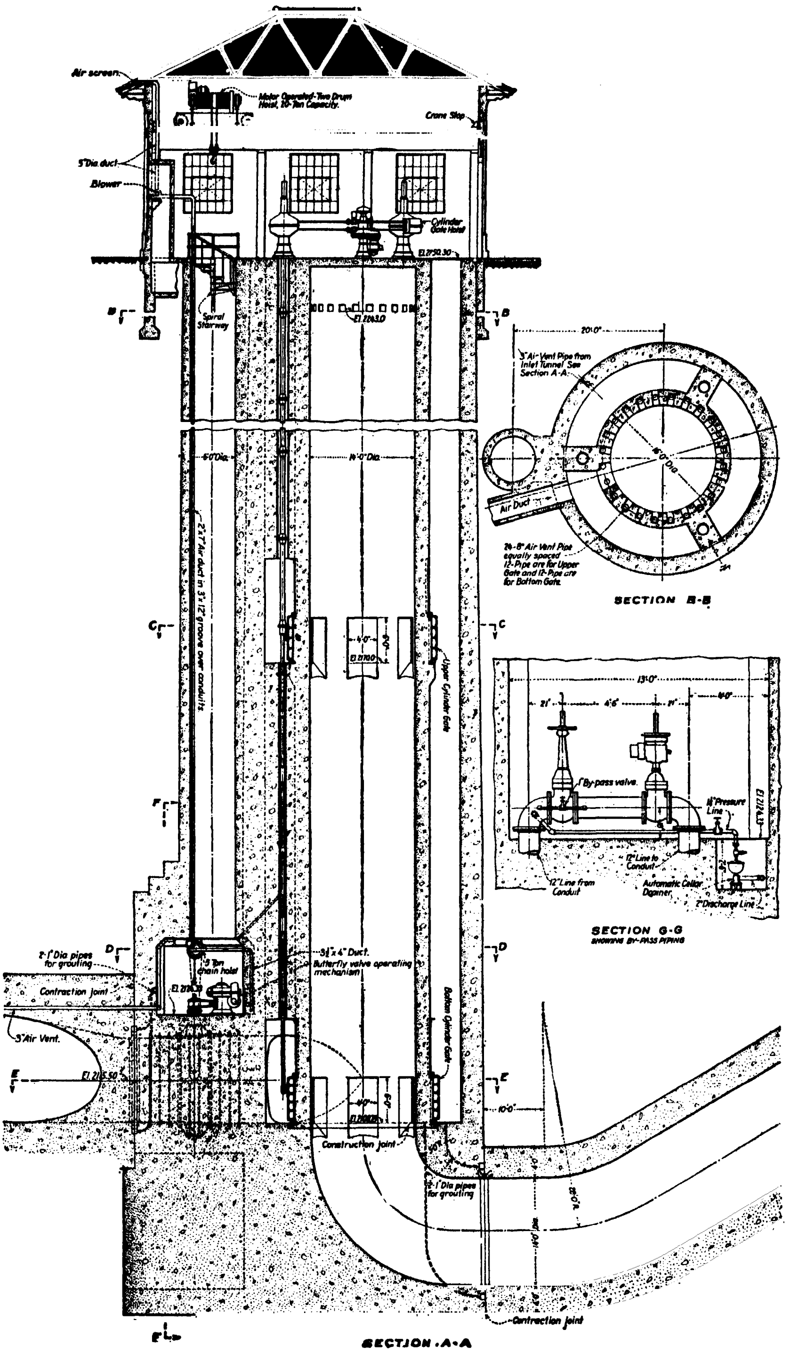


FIG. 41.—Cle Elum Dam outlet works, showing butterfly-valve-controlled inflow to tower and tandem cylinder-gate release to tower's central shaft. Cylinder gates operated by hoists in top of tower.

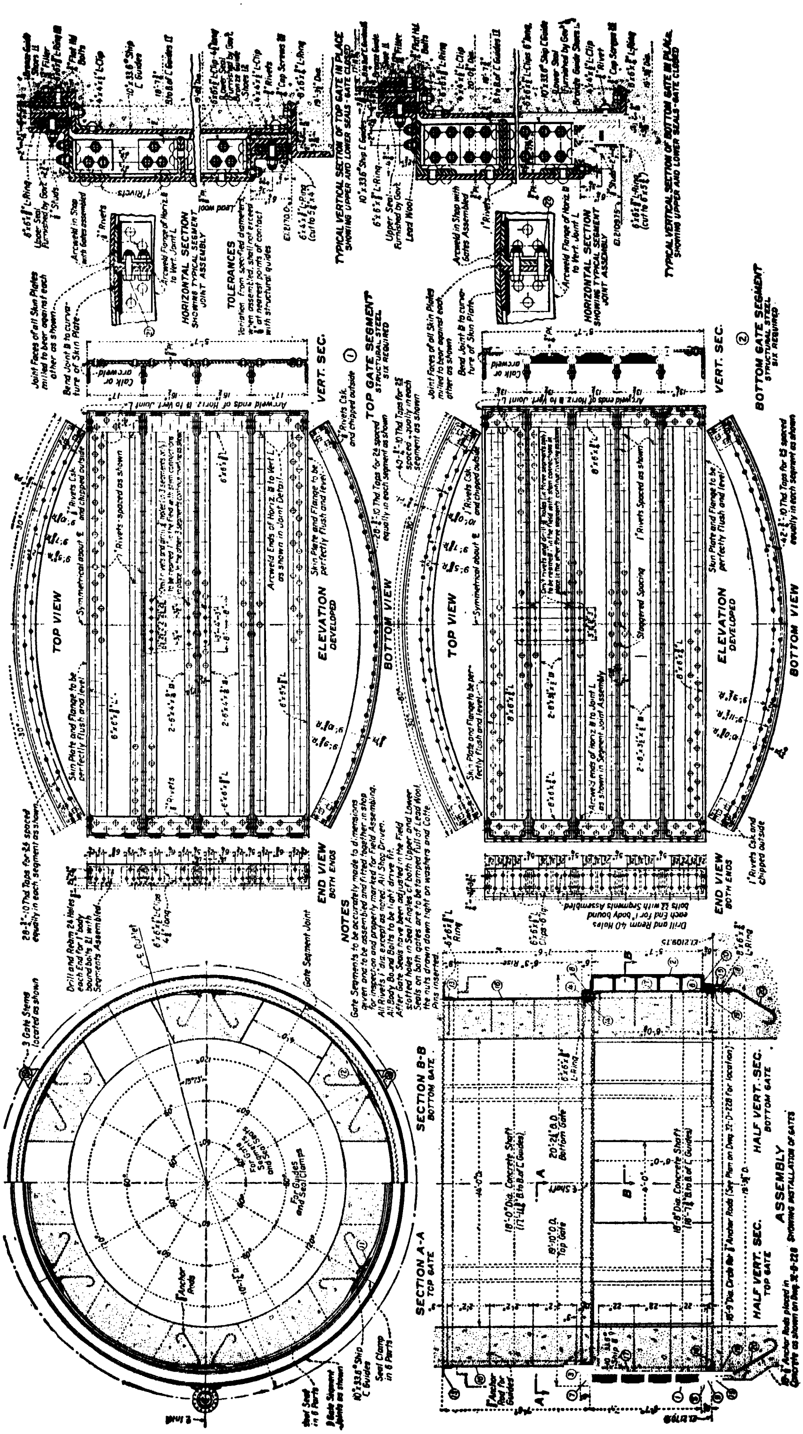


FIG. 42.—Upper and lower cylinder-gate assemblies, gate segments and subassemblies of sealing mechanisms. (See Fig. 41.)

torn from the instrument. The external air ducts leading beneath the hoist-house floor were greatly enlarged with but indifferent success in relieving this condition. Finally after the air ducts had been enlarged from 5 to 100 sq ft and the 6 ft 0 in. diameter ventilating shaft located in the roof of the outlet tunnel 678 ft from the cylinder gate shaft and 219 ft from the outlet portal had been plugged, reasonably good operating conditions were obtained.

The 10 ft 0 in. diameter by 3 ft 4 in. cylinder gate recently installed at Shoshone Dam to release irrigation water through the Shoshone Canyon conduit for the Heart Mountain division is of particular interest as the entire installation is underground, being contained within the rock walls of the canyon immediately downstream from the right abutment of the dam, and the flow through the internal-type cylinder gate is directed upward through a vertical rectangular shaft through rectangular staggered holes in two heavily reinforced-concrete horizontal diaphragms, arranged as shown in Fig. 43. The size, relationship, and positioning of these diaphragms, their rectangular holes staggered for baffling, were worked out by many tests on a series of 1:12 scale models in the Reclamation Bureau laboratory until the energy in the water which is liberated under heads up to 150 ft is so effectually dissipated that there is very little disturbance in the water as it flows away in the conduit to the right.

Six rectangular inlet ports 2 ft 6 in. wide by 3 ft 4 in. high, equally spaced around the external circumference of the gate, direct the jets radially inward and in a downwardly inclined direction so that they impinge and expend much of their energy before the upwardly flowing water reaches the lowermost of the two horizontal diaphragms above, and this in combination with the baffled holes in the diaphragm is so effective that the water surface above is remarkably stable, even when operating under most critical conditions. The rectangular water passages leading to these ports are lined with plate steel embedded in reinforced concrete as shown in Fig. 44. The cylinder gate is guided and held centrally aligned by vertical bronze half-round bars working in similarly shaped split tubes mounted in heavy rubber backing blocks arranged so that the rubber is placed under heavy compression by the mounting bolts (see Fig. 44) which also shows the detail construction of the gate, frames, guides, and seats.

The gate is raised to open and lowered to close by three equally spaced stems operated by a motor-driven hoist unit in the operating room above. This hoist is shown in complete and sectional assemblies in Fig. 45 and is very similar to the Hoover Dam cylinder-gate hoists except for its lesser size and the fact that it is a single gate hoist instead of being a dual hoist.

The normal hoisting speed is 0.333 fpm, and operating time is 10½ min for full travel. The normal hoisting load is 140,000 lb, and the gross lifting capacity of the hoist is 180,000 lb. Dual push-button stations, located in the operating chamber and in the power plant with remote selsyn indication, permit direct and remote control.

This installation is required to give close regulation of the irrigation water released into the Shoshone Canyon conduit in quantities varying between 0 and 1,200 cfs under heads up to 150 ft.

It should be understood that practically every outlet works is a distinctly individual problem requiring its own solution and that what is essential for one installation may not be required or may be wholly inappropriate for another installation. There are so many divergent factors affecting the designs of such structures that no general rule is applicable to all.

Speaking in general terms, the use of free-discharge needle valves or tube valves where their jets are liberated in tunnels or conduits should be avoided where practicable, as air demands vary sharply with different quantities released, and many unexpected phenomena develop that are not easily cared for. Similarly, discharge

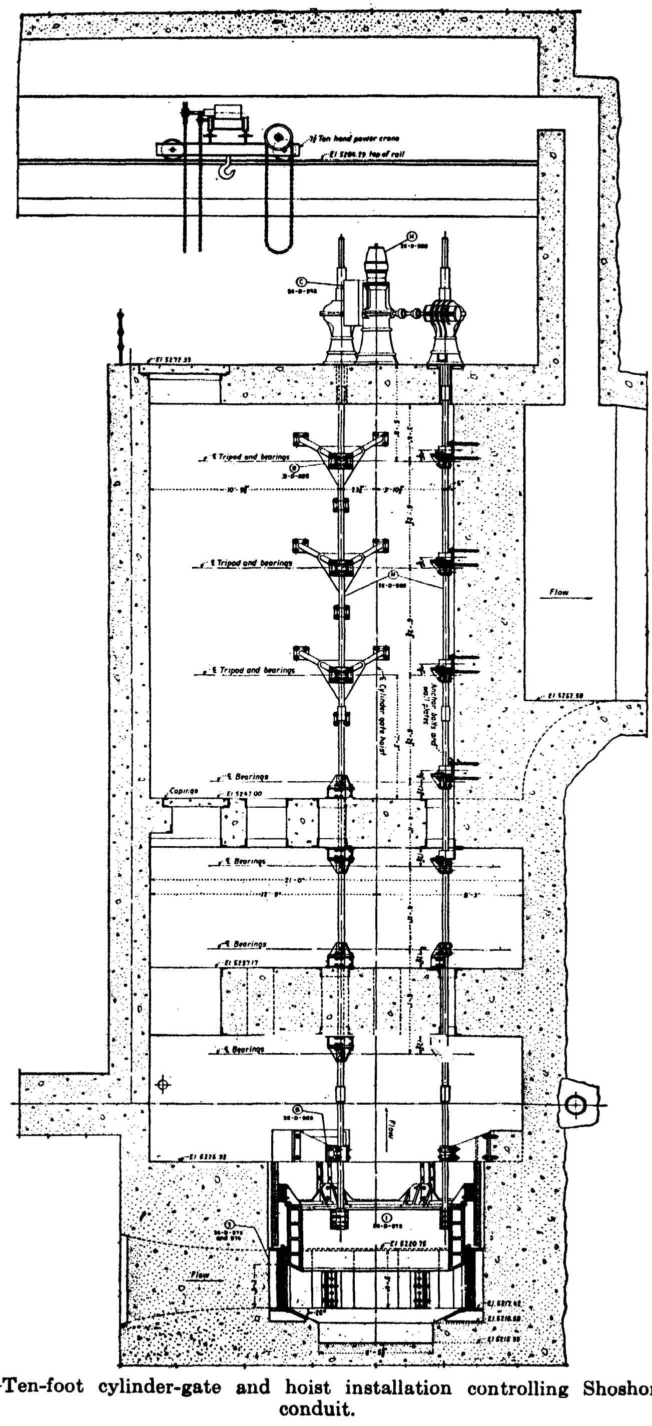


FIG. 43.—Ten-foot cylinder-gate and hoist installation controlling Shoshone Canyon conduit.

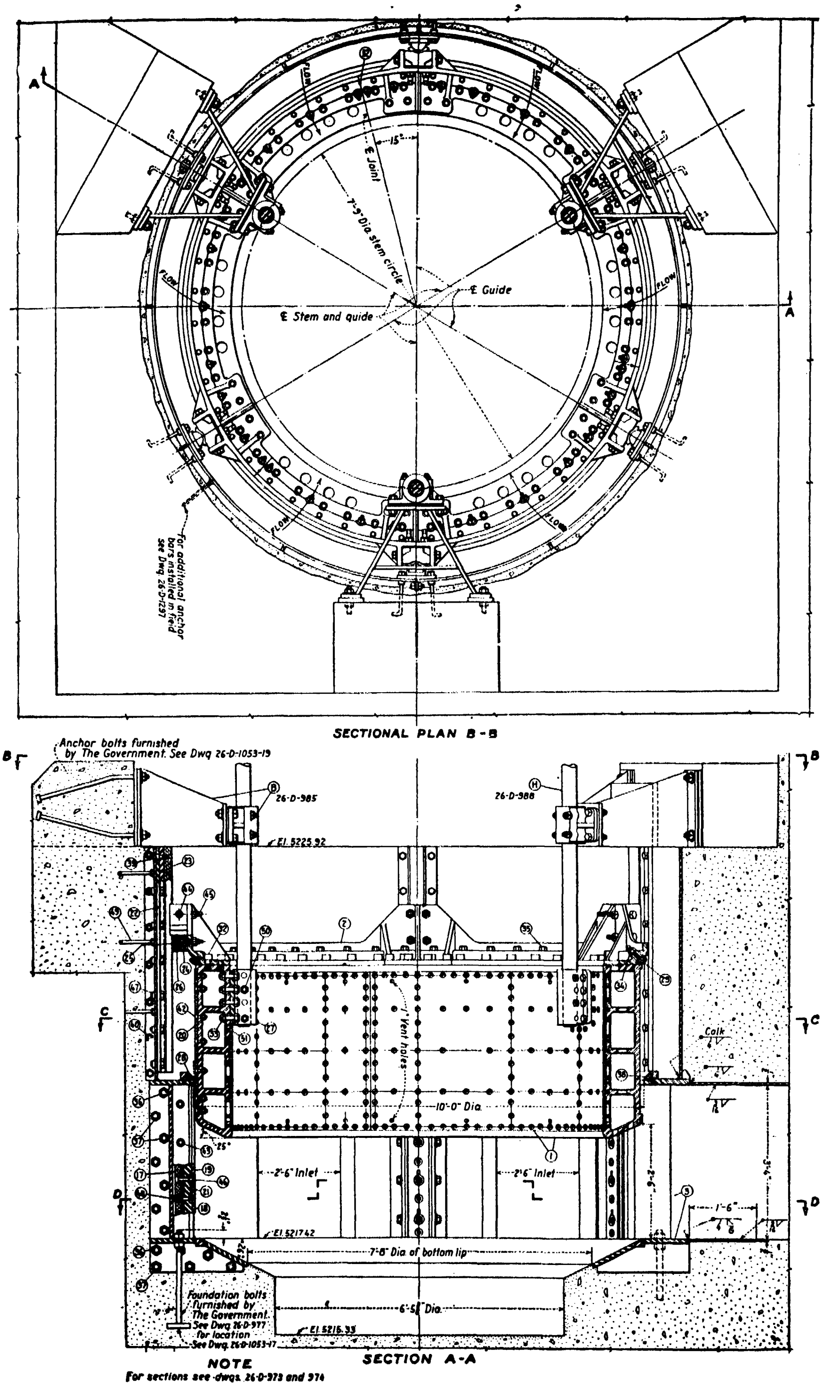


FIG. 44.—Ten-foot cylinder-gate assembly. See lower part of Fig. 43 for location of this mechanism.

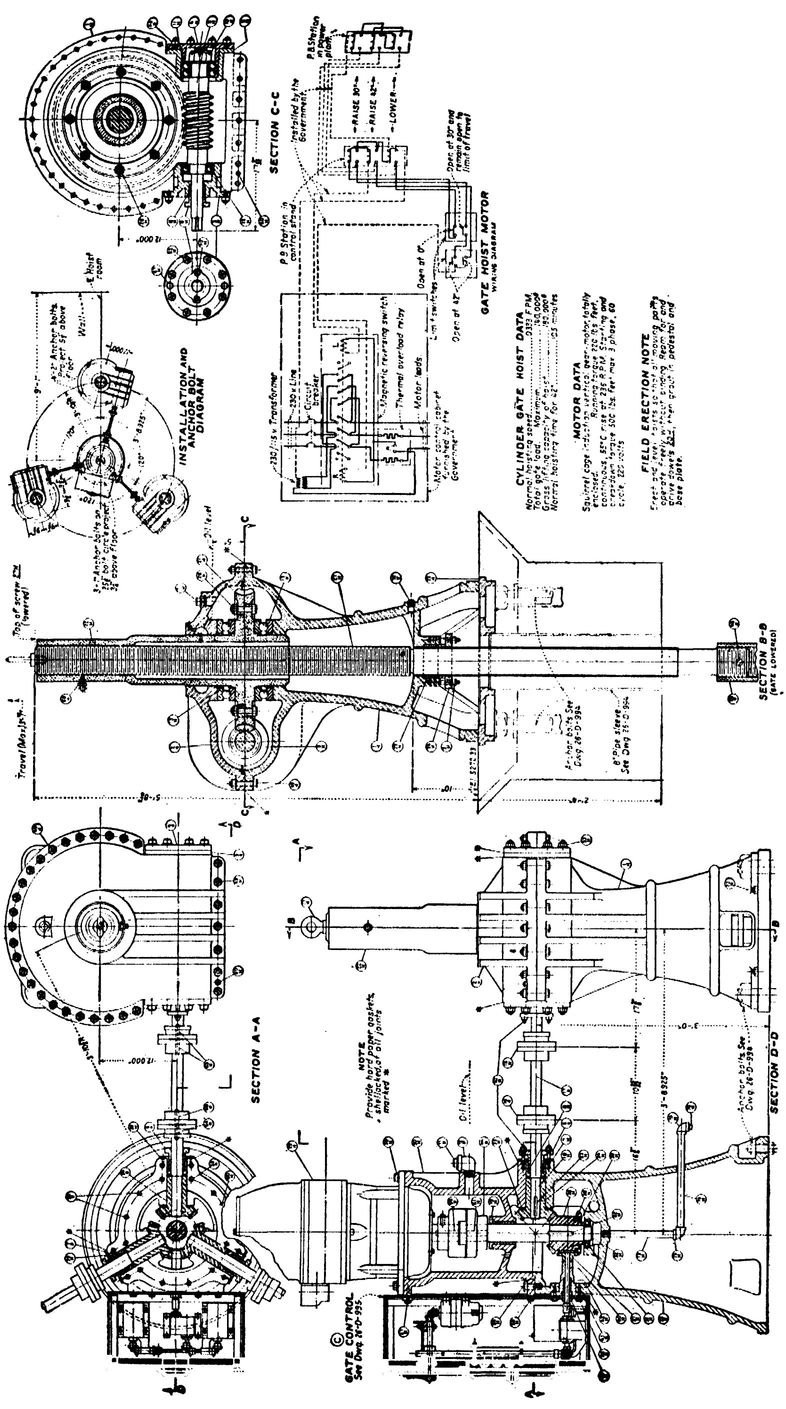


FIG. 45.—Ten-foot cylinder-gate hoist assembly. These views show construction and interrelation of parts of the mechanism in the upper control chamber in Fig. 43.

guides for such valves are not so desirable as nozzle orifices in the open air, so that a superabundance of air can be entrained by the jets whenever conditions demand it. Discharge guides when used should be made considerably larger than the jets that they are to enclose, so that where they are attached to their valve nozzles the annular cross-sectional area between their smallest diameters where they are bolted to the valves and the diameter of the valve orifice closely approaches that of the cross-sectional area of the jet.

Where possible, the valves should be allowed to protrude through the wall of their operating houses so that they can discharge into the open, and when this is done, the walls, if made of concrete, should preferably be formed with a liberal oversize octagonal opening to receive each valve, with heavy reinforcement provided around it, and a dovetail key slot formed to receive a supplemental fill-in of lean concrete around the valves after they have been installed. This will ensure that any later settlement of the wall will not pinch the encased valves and so prevent their free movement and operation.

Air inlets should be provided in the entrance transitions behind coaster gates or similar gates that must make emergency closures under high heads. The center lines of these ducts should be located one-half conduit diameter D downstream from the inlet or gate face of the bellmouth or transition. If closures under full spouting velocity are made but seldom, the air inlets may be made $0.08D$, which tests show will minimize vibration; but if frequent closures are required under such conditions, then the diameters of the air ducts should be $0.20D$ to prevent cavitation of the gate and the inlet transition structure. In this last case, it is recommended that an annular wind box be constructed around the entire circumference of the transition, with holes about 1 to 1¼ in. equally spaced to give uniform air distribution in the invert as well as the sides and top of the transition.

The desirability of conducting tests of carefully constructed scale models in the hydraulic laboratory cannot be overemphasized, for this will check the sufficiency of designs as nothing else but the actual operation of the prototype can do, and such tests will in many instances indicate and allow the inclusion of simple readjustments that will greatly improve the performance of the completed structures, while still permitting considerable savings in the final costs.

There are included under Reservoir Outlet Losses simple formulas that permit easy and rapid determination of various outlet designs, and these together with the needle-valve and tube-valve capacity curves make it possible to establish the sizes and number of outlets required to meet any individual set of conditions.

In those installations where branches from main supply manifolds lead off to needle valves, it will usually be found desirable to limit the maximum velocities through such wyes to 50 fps or less. This can be done easily by placing a tapered reducer behind the needle or tube valve so that the pipe behind the valve is made enough bigger than the entrance diameter of the valve as to maintain the velocity at 50 fps or less, as desired.

Expressed in terms of discharge coefficients for the inlets of needle valves, where the inlet diameter $D = 1.2$ times the outlet or discharge orifice diameter and the outlet is of the diverging round-lip type, the coefficient of discharge will be 0.542 for the inlet D and 0.780 for the outlet. Similarly, for needle valves with converging outlets and sharp-edged lips with an inlet diameter $D = 1.06$ times the discharge orifice diameter, the coefficient of discharge for inlet D is 0.571 and 0.640 for the outlet. For tube valves having an inlet $D1.075$ times the discharge orifice diameter, the coefficient for inlet D is 0.524 and 0.605 for the outlet. Free discharge-tube valves or conduit-tube valves such as those at Shasta Dam having inlets and outlets of equal size have a discharge coefficient of 0.76 for both inlet and outlet. Hollow-jet needle valves

have a coefficient of discharge for D of 0.7. Howell-Bunger valves have a coefficient of discharge for D of 0.9.

RESERVOIR OUTLET LOSSES[1]

These consist of (1) trash rack, (2) entrance, (3) transition, (4) bend or curve, (5) friction, (6) outlet, and in some cases losses due to gates or other changes in cross section of conduit.

1. Trash-rack losses.

Velocity through Rack, fps	Loss, ft
1.0	0.10
1.5	0.30
2.0	0.50

2. Entrance losses.
   a. Square corner opening (gate on face of dam) 0.5 of velocity head ($0.5h_v$), or balanced needle valve on upstream face of dam, 0.5 of velocity head ($0.5h_v$).
   b. Circular bellmouthed entrance, 0.1 of velocity head ($0.1h_v$).[2]
   c. Square bellmouthed transition, 0.2 of velocity head ($0.2h_v$).
   d. Gate in thin wall (orifice), as cylinder gate ports with unsuppressed contractions. 1.5 of velocity head ($1.5h_v$); (or $Q = 0.62AV$).
   Same with rounded corners, 1.0 of velocity head ($1.0h_v$); (or $Q = 0.71AV$).
   (Variation can be made between these values in special cases.)
3. Transition losses.
   a. Gradual contractions, 0.1 of increase in velocity head, $0.1(h_{v_2} - h_{v_1})$.
   b. Gradual expansions, 0.2 of decrease in velocity head, $0.2(h_{v_1} - h_{v_2})$.
   c. For abrupt changes, 0.5 of change in velocity head, $0.5(h_{v_2} - h_{v_1})$.
   d. For changes in cross section with no change in velocity, there is a small loss of head for which an allowance should be made.
4. Bend and curve losses.
   Loss of head due to a bend or curve is computed from the formula

$$H_B = 0.25\sqrt{\frac{\Delta}{90^\circ}}\left(\frac{v^2}{2g}\right)$$

   in which Δ is the angle of bend.
   Where the direction of flow changes abruptly through a right angle (as the vertical velocity in well at entrance of cylinder gates), the entire velocity head before the change is considered as lost.
5. Friction losses in conduits.
   Friction losses are generally computed from Kutter's formula with a value of $n = 0.014$ for concrete-lined tunnels and large conduits. For other types of conduits the friction loss may be taken from the following:
   a. Precast concrete pipe, $Q = 0.00546C_s d^{2.625}H^{0.5}$.
   b. Monolithic concrete conduits, $n = 0.014$.
   c. Steel pipe, $Q = \dfrac{0.78H^{0.53}D^{2.68}}{K_s^{0.53}}$

   or $H = K_s\dfrac{V^{1.9}}{D^{1.1}}$, where H is the loss of head in feet per 1,000 ft of pipe.

   (For values of K_s see Fred C. Scobey, *U.S. Dept. Agr. Bull.* 150, pp. 12, 89.)
   d. New cast-iron pipe, $Q = 130r^{0.63}s^{0.54}0.001^{-0.04}$ (Table 31, page 106, "H. & E. Tables").

[1] See also Appendix A.

[2] When using elliptical bellmouths (see pp. 340 and 342), $C = 0.97$ and 0.98 in $Q = C\sqrt{2gh}$, where h = total head in feet on center line of bellmouth.

Manning's formula $V = \frac{1.486r^{2/3}S^{1/2}}{n}$ has also been found very convenient for quickly determining s or the friction head by slide rule. (n in this formula has the same values as Kutter's n.)

6. Other losses.
   For losses due to gates in a conduit, if conduit changes shape, consider as transition (with gate open) and apply losses for a transition.

Balanced Needle-valve Discharge. Q (cfs) $= CA\sqrt{2gh}$. A = area of inlet or outlet in square feet using C as indicated on page 409. h = total head on center of inlet flange of valve including velocity head. Discharge of 4 to 96 in. diameter valves given by diagram (see Fig. 12, 13, 18, or 20).

ACKNOWLEDGMENTS

Sincere thanks are given to the U.S. Bureau of Reclamation by the author of this section for much of these data, and to D. J. MacCormack, J. K. Richardson, Byron H. Staats, Howard J. Pound, and many others for valuable assistance in preparation of the material.

ESTIMATING DATA

The tabulations and costs given in the following tables are, in each instance, accompanied by the date of purchase in the extreme right-hand column. It will be observed that prices vary greatly and are influenced not only by size and weight, but, even more markedly, by the date of purchase. The reader is cautioned to use these costs with discretion as many divergent factors, too numerous to mention, may otherwise cause him to arrive at overly optimistic conclusions with respect to his costs.

HYDRAULICALLY OPERATED HIGH-PRESSURE GATES

Size	Head, ft	Weight, lb	Pound price, cents	Date purchased
2'9" × 2'9"	43	15,570	13.8	1934
3'3" × 3'3"	76	24,500	13.2	1935
3'3" × 3'3"	94	30,000	13	1937
3.5' × 3.5'	62	31,500	8.5	1935
3.5' × 3.5'	101	36,500	11.5	1927
4' × 4'	100	45,000	11	1938
4' × 4'	138	50,500	10.6	1926
4.5' × 4.5'	129	37,500	15	1929
5' × 5'	67	71,000	8.9	1924
5' × 5'	85	65,000	9.7	1923
5' × 5'	110	74,000	11.9	1936
5' × 5'	83	49,700	10.8	1940
4' × 5'	300	59,000	9.6	1930
4' × 5'	55	50,000	10.3	1935
5' × 6'	55	75,000	10.3	1935
5' × 6'	73	71,500	8.4	1939
5' × 6'	100	87,100	8.5	1930
5' × 6'	135	83,750	10.6	1940
5'8" × 10'	182	133,400	7.9	1934
6' × 7.5'	570	84,375	8	1933
6' × 7.5'	185	76,600	9.5	

RING-FOLLOWER (R. F.), PARADOX (P.), AND RING-SEAL (R. S.) GATES

Type	Size, in.	Head, ft	Weight, lb.	Pound price, cents	Date purchased
R. F.	72	189	75,000	16.9	1938
R. S.	72	352	71,550	14.7	1939
P.	84	580	141,900	14.1	1935
R. F.	96	76	122,500	11.6	1938
R. F.	96	162	124,900	11.6	1936
P.	96	412	150,000	12.7	1934
R. F.	102	174	182,000	12.6	1939
P.	102	174	181,000	37.3	1937
P.	102	354	187,800	18.4	1935
R. F.	102	354	186,000	13.95	1936
R. S.	102	254	147,100	16.7	1938
R. S.	102	154	150,200	15.7	1938
R. S.	102	218	145,700	22	1938

NEEDLE VALVES

Type	Size, in.	Head, ft	Weight, lb	Control	Pound price, cents	Date purchased
b	24	50	4,500	Hydraulic	19	1940
a	24	82	11,500	Hydraulic	26.1	1924
c	30	107	8,500	Paradox	55.5	1937
a	36	214	28,000	Hydraulic	19.4	1921
c	36	98	14,000	Paradox	31	1936
c	36	76	13,750	Paradox	31.9	1936
a	42	101	35,300	Hydraulic	19.4	1928
b	42	381	28,500	Paradox	23.8	1930
c	42	97	19,000	Paradox	35	1937
a	48	214	21,000	Mechanical	30	1921
b	48	200	33,560	Paradox	18.7	1931
c	48	150	31,000	Paradox	24.6	1936
b	54	129	41,875	Paradox	21.5	1930
c	54	110	41,000	Paradox	23.5	1937
a	58	200	....	Hydraulic	....	1914
a	60	120	90,900	Hydraulic	11.5	1924
b	60	110	52,640	Hydraulic	15.2	1928
c	60	189	51,750	Paradox	17.4	1936
c	66	165	66,500	Paradox	19.8	1937
b	72	610	117,900	Paradox	15.4	1935
b	84	427	153,300	Paradox	14.8	1934
b	84	231	130,000	Paradox	10.3	1933
c	84	162	137,500	Paradox	11.5	1936 (Split body)

a. Balanced type. *b*. Internal differential. *c*. Interior differential.
Inlet diameters of majority of valves listed here are 20 per cent larger than their outlet diameters.

TUBE VALVES

A	28⅛	190	3,900	Manual	35.6	1939
B	36	481	9,500	Motor		
A	44	261	30,000	Motor		
A	52	137	25,500	Motor	24.5	1940
A	90	70	92,500	Motor	14.2	1940
B	102	122	....	Motor		
B	102	222	....	Motor		
B	102	322	....	Motor		

A. Free-discharge valve. *B*. Conduit or penstock valve.

HOLLOW-JET VALVES

No.	Valve size, in.	Designed head pressure, lb	No. of valves used	Name of dam	Status	Name of user
1	24	100	2	Jackson Gulch	1 built 1 advertised	Bureau of Reclamation
2	27	100	2	Deerfield	Installed, operating	Bureau of Reclamation
3	72	150	5	Anderson Ranch	Being fabricated	Bureau of Reclamation
4	72	150	2	Horsetooth	Being fabricated	Bureau of Reclamation
5	96	110	8	Friant	Being fabricated	Bureau of Reclamation
6	30	100	1	Granby	Advertised	Bureau of Reclamation
7	36	100	2	Anchor	Allocated	Bureau of Reclamation
8	48	100	2	Boysen	Allocated	Bureau of Reclamation
9	60	100	2	Enders	Allocated	Bureau of Reclamation
10	96	200	4	Bhakra	Allocated	Punjab, India
11	96	200	2	Hungry Horse	Allocated	Bureau of Reclamation

PENSTOCK BUTTERFLY VALVES

Type	Size, in.	Head, ft	Weight, lb	Control	Pound price, cents	Date purchased
	57	50	12,700	*C*	19	1940
	120	625	175,000	*D*	19.4	1933
	132	140	76,400	*C*	9.1	1932
	168	625	367,000	*D*	16.1	1933
	168	625	367,000	*D*	19.6	1935
	168	625	367,000	*D*	31.3	1937
	168	625	367,000	*D*	40.7	1939

C. Mechanical control. *D*. Hydraulic-rotor control.

Howell-Bunger Valves

No.	Valve size, in.	Head, ft	No. of valves used	Name of dam	Date shipped	Name of user
1	8	700	1	Nimo No. 2 Hydro Plant	Apr. 9, 1942	Ebasco International Corp., Columbia, South America
2	12	800	1	Los Angeles	Oct. 25, 1945	City of Los Angeles, Calif.
3	12	70	1	Norwalk	June 24, 1946	City of Norwalk, Conn.
4	30	150	1	Loveland	Sept. 8, 1944	California Water & Telephone Co., Monterey, Calif.
5	32	375	2	Copper Basin Outlet Works	Feb. 3, 1939	Metropolitan Water District of Southern California
6	36	87	1	Mill Creek	Feb. 14, 1941	U.S. Engineers, Walla Walla District, Wash.
7	42	102	1	Gene Wash	Apr. 6, 1938	Metropolitan Water District of Southern California
8	48	140	2	El Vado	1935	El Vada Dam, Chama, N.M.
9	54	146	1	Copper Basin	July 23, 1938	Metropolitan Water District of Southern California
10	54	130	1	Possum Kingdom	Oct. 13, 1939	Brazos River Conservation District. Possum Kingdom Dam, Tex.
11	54	138	1	Valle de Bravo	Nov. 18, 1946	Comision Federal de Electricidad, Valle de Bravo Dam, Mexico
12	60	140	2	Buchanan	Apr. 4, 1937	Lower Colorado River Authority, Buchanan Dam, Tex.
13	60	75	2	Nimrod	Aug. 25, 1941	U.S. Engineers, Little Rock District, Nimrod Dam, Ark.
14	60	290	1	Decew Falls Extension	Nov. 1, 1943	Hydro-Electric Power Commission Ontario, Canada
15	66	187	1	LaGrande	Sept. 18, 1943	City of Tacoma, Wash.
16	66	265	1	Alder	Dec. 24, 1943	City of Tacoma, Wash.
17	72	385	2	Ross	Oct. 16, 1944	City of Seattle, Wash.
18	78	127.5	1	Chatuge	June 3, 1943	Tennessee Valley Authority Chatuge Dam, N.C.
19	78	160.5	1	Nottley	June 10, 1943	Tennessee Valley Authority Notley Dam, Ga.
20	84	375	1	Fontana	Mar. 31, 1944	Tennessee Valley Authority Fontana Dam, N.C.
21	96	420	3	Mud Mountain	May 20, 1939	U.S. Engineers, Bonneville District, Mud Mountain Dam, Wash.
22	96	274	1	Colimilla	Nov. 18, 1936	Nueva Cia Electrica de Chapala, Colimilla Dam, Mexico
23	96	281	2	Watauga	Ordered Dec. 6, 1946	Tennessee Valley Authority Watauga Dam, Tenn.
24	96	75	3	Lador Falls	Ordered Jan. 28, 1947	British Columbia Power Commission, Campbell River Development, Lador Falls

SECTION 10

CANALS, FLUMES, COVERED CONDUITS, TUNNELS, AND PIPE LINES

By Julian Hinds

INTRODUCTION

1. Artificial channels for the conveyance of fluids fall into two primary divisions: those which merely guide the water as it flows down a sloping surface, and those which confine and guide the movement of water under pressure. These two classes are commonly referred to, respectively, as free-flow and pressure conduits. Free-flow conduits may be simple open channels or ditches, or they may be pipes or other enclosed structures, flowing partly full. In size, they may vary from enormous irrigation canals to a single furrow which guides the farmer's water or the small gutters and troughs found around the home or factory. Pressure conduits may be of wood, metal, glass, steel, concrete, or other suitable substance and although usually circular in cross section may be of any form. They vary in size from the 30 ft diameter steel pressure pipes at Boulder Canyon Dam to the minute tubing found on machines and scientific devices. Consideration is to be given herein only to those types of conduits commonly utilized in hydraulic engineering. Small hydraulic tubing, used in mechanical work, service pipes, plumbing fittings, and other miscellaneous water conduits will not be treated except as they may conform to the laws established for larger conduits. Also, no consideration will be given to the flow of fluids other than water.

FREE-FLOW CHANNELS

2. Effect of Slope and Frictional Resistance. All free-flow channels must be built on a slope, or at least the water surface must slope, to provide the fall required to produce and maintain motion. The velocity at which the water flows depends on the steepness of the slope, the size and shape of the channel, the roughness of its walls, and the viscosity and density of the water.

Ordinarily, variations in flow of water caused by variations in viscosity and density, which result from temperature changes are not important in comparison with other variables. Accordingly, these functions are usually ignored in hydraulic-engineering problems, their effects being included in experimental coefficients.

Resistance to the flow of fluids is usually referred to as frictional resistance, although it has little relation to ordinary friction between solids. A solid body sliding down an inclined plane in a vacuum will accelerate indefinitely. The tangential reaction between the solid and the surface of the plane is constant and practically independent of speed. Any excess of gravity over this reaction remains available for acceleration, regardless of velocity. This is not true of a fluid moving in a solid channel. The tangential reaction between the fluid particles and the channel walls results primarily from the impact of moving particles, the force required to overcome shearing resistance between the water particles, or a combination of these effects. Sliding, if it occurs at all, is of minor importance. These resistances increase with speed and rapidly approach equality with the component of gravity parallel to the

slope. Consequently, for any set of conditions there is a maximum speed, which will not be exceeded, however long the channel may be.

In solving problems involving resistance to fluid motion, reliance is placed on experimentally derived coefficients for empirical formulas, a variety of which have been proposed.

3. Mean Velocity. The velocity used in flow formulas is equivalent to the quantity of flow divided by the area of the water prism and is called the *mean velocity*. The actual velocity varies throughout the water prism in some manner that depends on the conditions of flow.[1] In certain problems, this variation must be taken into account, but its effect is usually hidden in empirical coefficients. If the velocity is very low and if other conditions are favorable, the flow may be streamlined, *i.e.*, each individual water particle may follow its own straight line down the channel.[2] Under conditions commonly encountered, the particles intermingle freely, turbulent flow being thus produced. The resistance to motion is fundamentally different for these two types of flow. Only turbulent flow will be considered in this chapter. Reference is made to Appendix A for the principles governing turbulent flow and for tables and diagrams useful in the design of both free-flow and pressure conduits.

4. Best Hydraulic Shape. The wetted perimeter offers resistance to flow and should be held to the minimum value consistent with the conditions or, conversely, the hydraulic radius for a given water cross section should be as large as practicable. The degree to which a conduit conforms to this criterion is a measure of its hydraulic efficiency.

The most efficient of all possible sections on this basis is a semicircle, open at the top and flowing full. The best closed section, flowing full, is a circle. The best polygonal section is one circumscribed about a semicircle for free flow, or about a circle for a closed conduit. For a specified number of sides, a regular polygon or half polygon is better than an irregular one, and the greater the number of sides, the greater the efficiency. Hydraulic efficiency is of importance chiefly as a means of reducing the size and consequently the cost of the waterway. It always should be considered but is often opposed by practical factors of greater importance.

For example, the cost of constructing and lining a semihexagon canal in earth section may exceed the cost of a slightly larger section on 1:1 or 1.5:1 slopes. Also, the slight hydraulic advantage of a circumscribed trapezoid over one with a wider bottom may be offset by the convenience of more working space. If the canal is in a deep cut, the cost of the freeboard part of the section is favored by a narrow bottom; but in combined cut and fill, a wide bottom may be cheapest, regardless of theoretical hydraulic efficiency.

Curved fillets at the intersection of the side slopes and the bottom improve the hydraulics of a trapezoidal canal and, with machine placing of lining, may also reduce cost.

It is notable that a circle flowing full is less efficient than any reasonably shaped open channel. For free flow in a closed circular conduit, the water area is smallest if the conduit is proportioned to run half full. Obviously, such an installation is not economical. Economy requires that both the size and the hydraulic efficiency be reduced to get the smallest conduit that will carry the required flow. Such a conduit will run at a depth equal to about 0.93 of the diameter (see Art. 39). Similar principles apply to other closed conduits under free-flow conditions.

5. Hydraulic Jump and Critical Depth Flow. Water flowing at or very near the critical depth (see Appendix A) is in indifferent equilibrium, and marked surface fluctuations may result from apparently minor channel irregularities. The relation of

[1] DODGE and THOMPSON, "Fluid Mechanics," McGraw-Hill Book Company, Inc., 1937, pp. 196*ff*.
[2] *Ibid.*, p. 171.

designed depth to critical depth should always be known, and where a safe margin above or below critical cannot be maintained by changing the shape of the channel, or otherwise, ample freeboard to care for possible disturbances should be provided.

Let Fig. 1 represent the energy curve for a flow of 800 cfs in a rectangular channel 10 ft wide on a slope that will give a normal depth of 5.40 ft ($s = 0.004$ for Manning's $n = 0.012$), the floor of the flume assumed to be at an elevation of 100 ft. Figure 1*b* is an enlargement of a portion of Fig. 1*a*. The elevation of the normal energy gradient will be depth plus velocity head plus the elevation of the floor

$$= 5.4 + 3.41 + 100 = 108.81$$

as indicated at *a* (Fig. 1*b*). Now assume an obstruction or imperfection equivalent to a 0.04-ft raise in the floor level. The new floor elevation is 100.04 ft, leaving a value of

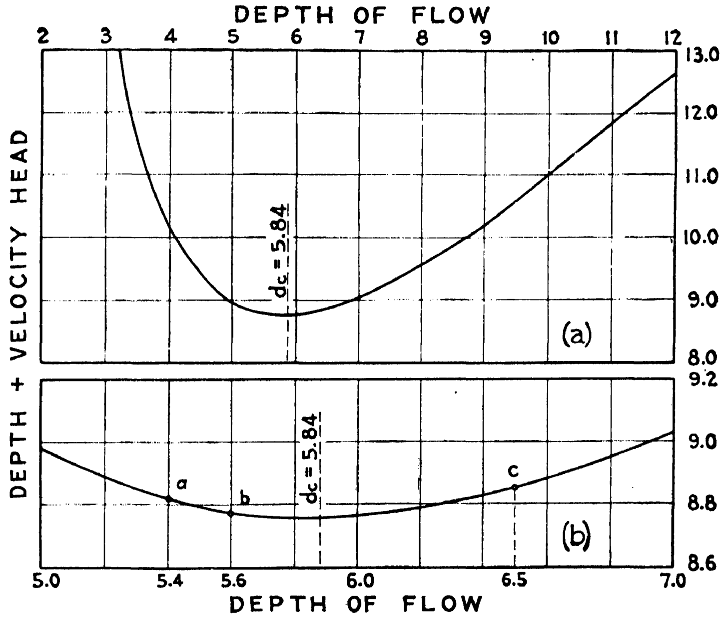


FIG. 1.—Critical depth flow.

8.77 for depth, plus velocity head. The depth will increase to 5.6 ft, as indicated at *b* (Fig. 1*b*), the result being a velocity head of 3.17, the reduced velocity yielding sufficient energy to surmount the obstruction.

If the obstruction should be equivalent to raising the floor 0.1 ft (1.85 per cent of the depth), the total energy available above the critical value is not sufficient to produce the required lift. The flow will then pass over the obstruction at critical depth with a minimum energy gradient of $5.84 + 2.92 + 100 + 0.1 = 108.86$. The water upstream from the obstruction must flow at a depth of 6.50 ft, as indicated at *c* (Fig. 1*b*). A hydraulic jump will occur some distance upstream followed by a backwater curve to give the required depth. Ample freeboard should be allowed or the condition removed by changing the depth and width of the channel until the depth plus velocity head for normal flow exceeds that for critical depth by at least 4 or 5 per cent of the depth of flow. Flow slightly above the critical depth is also unstable. Many channels, defective in this respect, have been subject to mysterious fluctuations.

6. Permissible Velocities. The cost of a canal or other conduit varies with its size. Therefore, the cost of the initial construction may be reduced by using the highest

possible velocity. However, if the velocity is made too high, the conduit may be damaged or destroyed by erosion. This must be avoided by limiting velocities according to the material of the bed and banks of the channel.

For clear water in smooth concrete or other hard-surfaced channels, the limiting velocity is beyond practical requirements, except perhaps in very steep chutes. Velocities above 40 fps for clear water in concrete channels have been found to do no harm. If the water carries abrasive materials, damage may occur with much lower velocities. Unless the abrasive material is particularly bad, velocities up to 10 or 12 ft should not prove injurious to wood or first-class concrete. Thin metal flumes may be damaged by coarse sand or gravel at 6 to 8 fps, and the galvanizing may be injured by even lower velocities. The abrasive materials may be in the water at its source or may ravel into the channel along its course. No definite relation has been established between the nature of abrasive material, material of channel bank, and permissible velocity; nor is it always possible to predict in advance the nature and amount of abrasive that will be encountered. Such materials should be excluded as far as practicable.

In unlined earthen channels, the limiting velocity involves many uncertain factors. Generally, a fine soil is more easily eroded than a coarse one, but the effect of grain size may be obscured by the presence or absence of a cementing or binding material. The tendency to erode is reduced by seasoning. Ground-water conditions exert an important influence. Seepage out of the channel, particularly if the water is turbid, tends to toughen the banks; infiltration reduces resistance to erosion.

Erosion can be avoided by designing for low velocities. If carried to an extreme, this results in large and costly canals, encourages the growth of aquatic plants, and increases seepage and evaporation.

If the water carries an appreciable amount of silt in suspension, too low a velocity will cause the canal to fill up until its usefulness is destroyed (to prevent this condition it must be cleaned at great expense). The erosive action of water on earth banks is decreased by silt in suspension. It is necessary to choose a velocity that will keep the silt in motion but that will not erode the banks of the canal. The margin of permissible velocities, thus determined, depends on the amount and nature of the silt in the water, the nature of the bank material, the size and shape of the canal, and perhaps on many other factors. The silt content of most turbid waters varies with the season, as does also the demand for water, and the resultant velocity of flow. Thus, a canal that will scour at one season may silt at another.

The determination of nonscouring, nonsilting velocities for earth canals has attracted the attention of many investigators. R. C. Kennedy,[1] one of the earliest writers, proposed the formula

$$V_o = Cd^{0.64} \tag{1}$$

in which V_o is a velocity that will neither silt nor scour, d the depth of the canal in feet, and C a coefficient whose value depends on the fineness of the soil particles, assumed the same for the silt in suspension as for the canal banks, as was generally the case in the canals for which the formula was devised. For the fine sandy silt of the Punjab, Kennedy suggested a value of 0.84 for C. For extremely fine soils in Egypt, Buckley found a value of 0.56, and for coarse silt, values as high as 1.0 have been suggested. This formula contains no allowance for the texture of the bank materials, the presence of cementatious or colloidal material, or the amount of silt in suspension.

Parker[2] suggests the inclusion of an allowance for the quantity of silt flowing, which seems reasonable, but no definite means of allowing for such a factor have been

[1] *Proc. Inst. Civil Eng.*, vol. 119, 1895.

[2] PARKER, PHILIP À. MORLEY, "The Control of Water," George Routledge & Sons, Ltd., 1925, p. 767.

established. Lacey[1] proposes that the shape of the channel is a primary factor in the control of silting and erosion. He also introduces a factor for the size of the silt grains.

Fortier and Scobey,[2] in 1926, express the view that the Kennedy formula gives excellent results for nonsilting velocities but point out that American practice indicates that higher velocities will not cause scour. They submit a table of maximum permissible velocities, reproduced herewith as Table 1, in which allowance is made for the composition and condition of the bank and the quality (but not the quantity) of silt in suspension. The effect of depth is ignored because of lack of data.

Lane,[3] in 1935, makes a thorough summary of existing formulas and discusses the problem in a comprehensive manner but derives no definite rules.

The final solution of the problem is not in sight. Ordinarily, a safe design will result if the maximum velocities are determined from Fortier and Scobey's list (Table 1) and the minimum velocity for silty water from Kennedy's formula. This rule must be tempered with judgment to allow for variables not included, and in important cases it is advisable to resort to research.

TABLE 1.—PERMISSIBLE CANAL VELOCITIES

Original material excavated for canal	Velocity, fps, after aging, of canals carrying		
	Clear water, no detritus	Water transporting colloidal silts	Water transporting noncolloidal silts, sands, gravels, or rock fragments
(1)	(2)	(3)	(4)
Fine sand (noncolloidal)	1.50	2.50	1.50
Sandy loam (noncolloidal)	1.75	2.50	2.00
Silt loam (noncolloidal)	2.00	3.00	2.00
Alluvial silts when noncolloidal	2.00	3.50	2.00
Ordinary firm loam	2.50	3.50	2.25
Volcanic ash	2.50	3.50	2.00
Fine gravel	2.50	5.00	3.75
Stiff clay (very colloidal)	3.75	5.00	3.00
Graded, loam to cobbles, when noncolloidal	3.75	5.00	5.00
Alluvial silts when colloidal	3.75	5.00	3.00
Graded, silt to cobbles, when colloidal	4.00	5.50	5.00
Coarse gravel (noncolloidal)	4.00	6.00	6.50
Cobbles and shingles	5.00	5.50	6.50
Shales and hardpans	6.00	6.00	5.00

Where the silt burden is important, it is better to make the slope a little too steep rather than a little too flat. A gradient that proves to be too steep can be controlled by checks.

The foregoing comments apply only to silt in suspension and not to coarser particles, usually referred to as the *bed load*, which are rolled along the bottom. The bed load may consist of particles of all sizes, from sand up to the largest stones that may be moved by the stream. Bed load may result from erosion of the channel banks, may enter the channel at the intake, or may be brought in elsewhere by wind, water, or

[1] Stable Channels in Alluvium, *Minutes of Proc. Inst. Civil Eng.*, vol. 225; also, *Engineering*, **129**, 179, Feb. 7, 1930.

[2] Permissible Canal Velocities, *Trans. A. S. C. E.*, **89**, 940, 1926.

[3] Stable Channels in Erodible Materials, *Proc. A. S. C. E.*, November, 1935, p. 1307.

weathering. Every precaution should be taken to exclude these coarser particles, and where this cannot be fully accomplished, means for removal should be provided.

CONVEYANCE LOSSES

7. General. There is an inevitable loss of water from all forms of conduits, possibly excepting a perfectly constructed metal pipe. Such losses are caused by leakage, seepage, absorption, and evaporation. The greatest losses occur in unlined earth canals, and the necessity of conserving water often leads to the use of lined conduits. The value of water lost is an important factor in all economic problems concerning the conveyance of water.

8. Losses from Concrete, Metal, and Wood Conduits. The leakage from well-constructed and well-maintained concrete, metal, and wood conduits is relatively small. Where such conduits occur in short lengths in systems composed chiefly of earthen channels, losses from them are negligible by comparison, hence such structures are treated as watertight. However, no conduit is completely tight, and in long lined systems the accumulation of small leakages may be appreciable. An example is the Colorado River Aqueduct, bringing water from the Colorado River to Los Angeles. The main line of this aqueduct is 242 miles in length, all concrete lined. The specifications require that all visible leaks be closed, yet it is expected than an appreciable loss of water will occur.

Data on losses from such conduits are scarce. Such leakage is usually expressed in terms of gallons per inch diameter of conduit per mile per day. Seven tests on 17 ft by 17 ft 6 in. concrete horseshoe conduits in the New York City water system showed leakages of 167 to 463 gal. per inch diameter, with an average of 283. An average leakage allowance of 300 to 400 gal. per inch diameter is liberal for any well-constructed concrete, steel, or timber free-flow conduit.

Leakage from new concrete conduits may exceed this allowance, but if visible leaks are repaired, the loss reduces rapidly as the concrete swells and as small openings fill with silt or algae.

9. Seepage from Earth Canals. The loss of water by seepage from an unlined canal is influenced by the nature and porosity of the soil; the depth, turbidity, and temperature of the water; the age and shape of the canal section; and the position of the ground-water level. How these factors and perhaps others affect the seepage is not fully understood. As with other problems involving soil texture, it is unlikely that the designer will ever be able to predict seepage losses with precision in advance of construction. However, the loss may be foretold with sufficient accuracy for practical purposes.

Seepage values suggested by Etcheverry and Harding[1] are shown in Table 2. These values depend only on the nature of the soil and the area in contact with the water prism and are independent of the volume of flow. The effect of depth of water on the rate of percolation is also ignored, although it may be appreciable. The data shown are based on average conditions. The higher values are for comparatively new canals. The seepage loss usually decreases noticeably with age, particularly if the water carries silt. Clay or other fine material is sometimes artificially added to the water to reduce seepage. Porous formations may be greatly improved by such means.

10. Losses from Lined Canals. Seepage losses may be reduced by lining. A carefully laid concrete lining 3 in. or more thick will reduce the loss per square foot per day to perhaps 0.04 cu ft. Thinner linings have proportional effects. For a cement-mortar lining 1 in. thick, carefully laid, a loss of 0.20 cu ft may be safely used. Oil linings and clay-puddle linings, if properly installed, have very beneficial effects.[2]

[1] Etcheverry and Harding, "Irrigation Practice and Engineering," McGraw-Hill Book Company, Inc., 2d ed., vol. II, p. 106, 1933.

[2] *Ibid.*, pp. 142*ff*.

TABLE 2.—CONVEYANCE LOSS IN CUBIC FEET PER SQUARE FOOT OF WETTED PERIMETER FOR CANALS NOT AFFECTED BY THE RISE OF GROUND WATER

Character of material	Cubic feet per square foot in 24 hr
Impervious clay loam	0.25–0.35
Medium clay loam underlaid with hardpan at depth of not over 2 to 3 ft below bed	0.35–0.50
Ordinary clay loam, silt soil, or lava-ash loam	0.50–0.75
Gravelly clay loam or sandy clay loam, cemented gravel, sand and clay	0.75–1 00
Sandy loam	1.00–1.50
Loose sandy soils	1.50–1.75
Gravelly sandy soils	2.00–2.50
Porous gravelly soils	2.50–3.00
Very gravelly soils	3.00–6.00

DESIGN OF CANALS

11. General. Earth canals are usually constructed trapezoidal in form. The side slopes are determined by the stability of the bank materials on the basis of experience. The dimensions of the channel and its setting on the ground are governed by costs and practical considerations. Hydraulic efficiency is not important in reasonably proportioned canals. The heights and widths of banks are determined by freeboard and stability requirements. Typical unlined canal sections are shown in Figs. 2, 3, and 4.

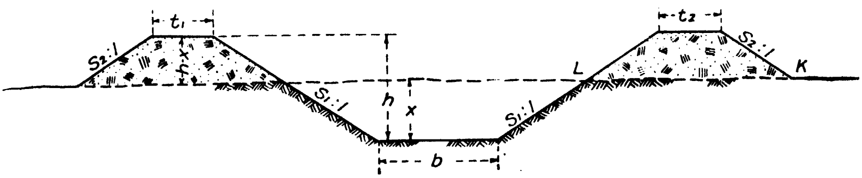


FIG. 2.—Typical canal section.

12. Bank Slopes. The side slopes of cuts and fills not exposed to the action of water must conform to the angle of repose of the materials, with due allowance for possible saturation by seepage. The steepest safe slopes are usually most economical. If the slopes within the waterway of an unlined canal are made too steep, the banks will erode or slough, failure being thus invited. Etcheverry and Harding[1] suggest slopes within the unlined waterway as follows:

For cuts in firm rock	¼:1
For cuts in fissured rock, more or less disintegrated rock, tough hardpan	½:1
For cuts in cemented gravel, stiff clay soils, ordinary hardpan	¾:1
For cuts in firm, gravelly, clay soil, or for side-hill cross section in average loam	1 :1
For cuts or fills in average loam or gravelly loam	1½:1
For cuts or fills in loose sandy loam	2 :1
For cuts or fills in very sandy soil	3 :1

For banks not within the waterway, the following slopes are recommended:

Rock and gravel fills	1¼:1
Fills of average loam, gravelly loam	1½:1
Sandy loam and sandy soil	2 :1

Lining protects the banks of the waterway from weathering and from the action of the water in the canal, but for slopes steeper than about ¾:1, the lining may have to act as a retaining wall and unless specially designed for that service may fail. Almost any coherent free-draining material can be maintained on a 1:1 slope if substantially lined. Lined rock cuts may stand on very steep slopes.

[1] *Ibid.*, p. 124.

13. Freeboard. A variation in the friction coefficient, accumulation of sand or silt, the growth of moss or other vegetation, centrifugal force on curves, wave action, increase in flow resulting from error at diversion, or the inflow of storm waters may raise the water level above that computed for a normal flow. A canal bank, therefore, must extend above the designed water level to provide a factor of safety. The amount of freeboard required is determined from experience. The lower limit, for an earth canal, is usually 1 ft for small canals, and 4 ft is a usual upper limit. Between these limits, the freeboard may be made about 1 ft plus 25 per cent of the depth. Where the canal is unusually wide, or where excessive wave action may occur, an additional allowance should be made. Allowance should also be made for the settlement of the bank.

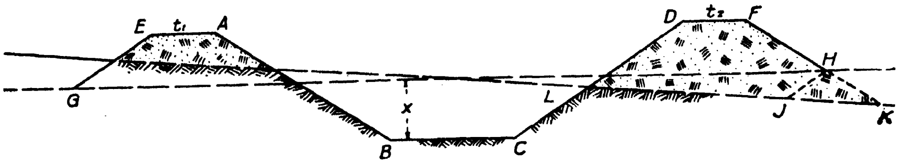


FIG. 3.—Typical canal section.

The top of lining, in lined canals, is not usually extended for the full height of the bank freeboard and may range from 6 in. to 2 ft above the water surface. The bank heights are sometimes reduced in lined canals, but this practice is questionable.

14. Top Width and Thickness of Banks. A canal bank must have sufficient strength to withstand the water pressure against it and sufficient thickness to prevent too free an escape of water. The top width of the bank is usually made about equal to the depth of the water in the canal with a minimum of about 4 ft. If a patrol road is required along the bank, the width should be not less than 12 ft. If the embankment is to be exposed to the water for a considerable height, it should be carefully compacted and its width may need to be increased. For first-class gravelly soil with just sufficient clay to ensure cohesion, the horizontal distance from L to K (Figs. 3 and 4) may be as little as four or five times the depth of water above L. For lighter soils and clays, this

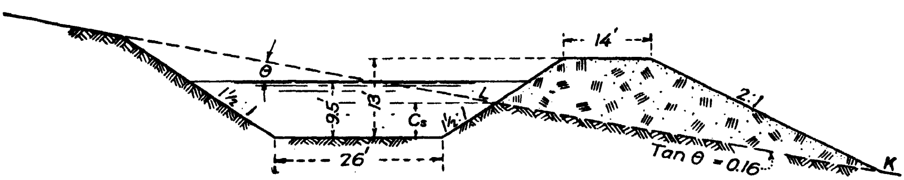


FIG. 4.—Typical canal section.

distance should be increased to a ratio of 8 or 10. The nature of the underlying soil must also be considered and the bank thickness made sufficient to avoid piping along the outer toe.

15. Spoil Banks and Berms. Deep cuts are sometimes required which yield more materials than needed for the construction of normal banks. If such materials are deposited adjacent to the canal, a level space, or berm, should be provided to protect the waterway from sloughing materials. For the same purpose, berms may also be required in the excavated slopes above the freeboard level. Spoil banks should be regular in form and roughly level. Waste materials may be used to construct an embankment along the uphill side of the canal to exclude cross-drainage water, or if not needed for this purpose, may be used to reinforce the downhill bank. A typical deep-cut canal section is shown in Fig. 5. Berms usually vary from 5 to 10 ft in width.

depending on the height of fills or cuts. The tops of banks should slope away from the canal, to exclude storm waters.

16. Shape and Size of Waterway. In an unlined canal, the area of the waterway is determined by the permissible velocity or by the available slope if the maximum permissible velocity is not to be attained. In a lined canal, the approximate area is usually determined by the available slope, although at times it is desirable to use other criteria. Steepening of the slope reduces the cost of the canal but may use up head needed for other purposes or, in case of an irrigation canal, may reduce the value of the works by eliminating a portion of the irrigable area. Where such factors can be evaluated, the canal velocity may be made such that the cost of the canal, plus the loss due to steepened slopes, is a minimum.

In small canals, the water depth is chosen arbitrarily or to give a desired shape to the waterway. The hydraulic efficiency being little affected by a reasonable range in shape, the ratio of depth to width may be chosen from economic or practical considerations.

In large canals, it may be necessary to limit the depth to avoid the expense of making high banks safe against water pressure or to minimize the danger of a bank

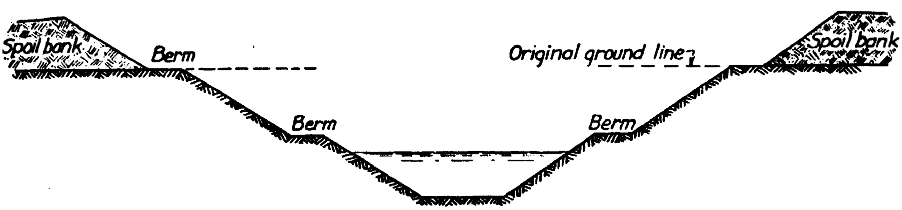


FIG. 5.—Typical deep-cut canal section.

failure. Depths in excess of 10 ft are usually avoided, but greater depths have been successfully employed. In rock cut or other firm material, where the waterway is largely in cut, the danger resulting from increased depth is not great.

In unlined canals where topographic conditions make the use of flat slopes difficult, wide shallow sections may be resorted to, to reduce velocities; although this device is not dependable if carried to an extreme. It is usually safer and cheaper in the long run to use reasonable depths with checks to control velocities.

If a canal is to be constructed across a nearly level terrain, without restriction as to the relation of the water level to ground surface, and if bank widths are not increased with canal width, the banks become lower and hence of smaller volume as the base is widened. If the excavation is limited to the materials required for the banks, the volume of earth work decreases as the canal is widened. If the width is made extreme, the cost per cubic yard of excavation and the cost for trimming become greater, and the cost of the whole may be increased. A bottom width in excess of six times the water depth is seldom required except for large canals where it is desired to limit the depth for safety. A value of b/d having been selected, the value of d is found by the equation

$$d = \sqrt{\frac{A}{\frac{b}{d} + S{:}S}} \tag{2}$$

where d is the depth, A the area of the water prism found by dividing the flow by the permissible velocity, and $S{:}S$ the side slope. The bottom width is usually chosen to the nearest foot, or 2 ft, and the depth recomputed to give the required area.

17. Hydraulic Computations. The actual computation of the shape of the waterway and its slope can best be illustrated by an example. Let it be required to design

an unlined waterway to be excavated in ordinary firm loam for a clear water flow of 1,000 cfs. The permissible mean velocity is 2.50 fps (see Table 1). The required side slopes are $1\frac{1}{2}:1$ (Art. 12). The value of n, for earth, straight and uniform, is 0.0225, and Manning's formula will be used.

A ratio of 4:1 for bottom width to depth will be tentatively assumed. The depth is found from Eq. (2). The computation proceeds thus:

$$A = 1{,}000 \div 2.5 = 400 \text{ sq ft}$$

$$\text{Trial } d = \sqrt{400 \div 5.5} = 8.53 \text{ ft}$$

$$\text{Trial } b = 4 \times 8.53 = 34.12 \text{ ft}$$

Adopt $$b = 34.00, \text{ the nearest full foot}$$

and $$d = 8.54 \text{ to give required area}$$

$$p = 64.8 \text{ ft}$$

$$r = 400 \div 64.8 = 6.17 \text{ ft}$$

Then, from the Manning formula,

$$2.50 = \frac{1.486}{0.0225} \times (6.17)^{2/3} s^{1/2}$$

$$s = 0.000127$$

If Kutter had been used instead of Manning, the slope would have been found from the Kutter equation.

It sometimes happens that it is desirable to limit both the slope and the velocity. Suppose that in the foregoing example topographic conditions require an hydraulic slope of $s = 0.00015$, all other conditions remaining as stated. If the canal shape is unaltered, the velocity will exceed the allowable limit of 2.50 fps, and scouring will result. The shape of the canal must be changed to reduce its hydraulic efficiency. Considering r, rather than s, as the unknown, Manning's equation becomes

$$2.50 = \frac{1.486}{0.0225} r^{2/3} \times (0.00015)^{1/2}$$

from which

$$r = 5.43$$

By trial, it is found that the required values of $A = 400$, $r = 5.43$, and side slopes $= 1.5:1$ are satisfied by making $b = 49.00$ and $d = 6.77$. This is an unusually wide section but may be used unless the slope can be controlled by some other more desirable means, such as relocation or the introduction of checks.

A more usual condition, particularly in lined canals, is to have only the slope and flow prescribed, the velocity and canal dimensions being optional. Assume that a flow of 1,000 cfs is to be carried in a lined canal, Manning's $n = 0.014$, side slopes 1.5:1, on a slope of 0.0004. A few preliminary figures indicate a section about 7.0 ft deep with a 15-ft bottom width for a first trial. The computations follow.

$$d = 7.0, \quad b = 15.0, \quad A = 178.5, \quad p = 40.24,$$
$$r = 4.43, \quad r^{2/3} = 2.70, \quad s^{1/2} = 0.02$$

$$V = \frac{1.486}{0.014} \times 2.70 \times 0.02 = 5.73$$

$$Q = AV = 1{,}020 \text{ cfs}$$

Try $$d = 6.9, \quad b = 15.0, \quad A = 174.91, \quad p = 39.87,$$
$$r = 4.38, \quad r^{2/3} = 2.68, \quad s^{1/2} = 0.02$$

$$V = \frac{1.486}{0.014} \times 2.68 \times 0.02 = 5.70$$

$$Q = AV = 1{,}000 \text{ cfs}$$

Variations required to adapt the computations to Kutter's formula are evident.

18. Locating the Canal on the Ground. If there is no restriction as to depth of cutting or the relation of the water surface to the ground level, the canal prism may be set into the ground a distance such that the excavated materials will just suffice for the construction of the banks. If the ground surface is smooth or regular, this may be accomplished precisely; but surface irregularities usually make it necessary to resort to averages. Balancing the cut and fill at any particular station may be accomplished algebraically or by trial estimates. For a canal in level ground, as illustrated in Fig. 2, algebraic expressions for the area in cutting and the area of the two banks may be equated giving the relation

$$bx + x^2S_1 = [(t_1 + t_2)(h - x) + (S_1 + S_2)(h - x)^2](1 + k) \tag{3}$$

where k is the percentage of shrinkage between area of cutting and area of compacted fill and all other symbols have the significance indicated in the figure. This equation may be solved for x, the only unknown.

If the canal is on gently sloping ground but still requires an uphill bank as in Fig. 3, find the economic cut for level section, then set the canal so that the intersection of the top of the level cutting with the ground surface comes halfway between the points G and H. This involves a small error which may be readily allowed for after a little practice.

In Fig. 4, the ground slope is such that the upper bank has disappeared. Algebraic expressions for the cut and fill, in terms of the canal dimension tan θ and the unknown center cut, may be equated and solved, allowance being made for swell of the fill as in previous examples; or the solution may be made by trial.

The foregoing discussion applies to the determination of the economic cut for a straight canal at a single station. On curves, the effect of curvature on the quantities in cut and fill must be allowed for. Contours are usually too irregular to be followed minutely by a canal of appreciable size. Consequently, the depth of cut must vary, and economy requires that portions of the materials be transported along the canal from points of excess cutting to points of deficient fill. To accomplish this, the locator computes the excess or deficiency in excavated material in each station, or fractional station, and carries an algebraic summation of these quantities forward as he progresses. The summation may be kept in the form of a table, or it may be platted on a profile of the line as a differential mass diagram. The location is moved uphill or downhill as required to keep this summation close to zero. Materials may be moved either backward or forward, and the distance of transportation must be considered in computing the cost of the work. Excessive haulage should be avoided by wasting and borrowing.

The permissible length of haul depends on the equipment used for excavation. With teams and scrapers, or carry-all power scrapers, the speed and cost of transportation is readily computed. With a power shovel, after the materials are once loaded into trucks, a moderately long haul is not expensive; but a considerable saving is possible if the use of trucks can be eliminated. The dragline, which is standard for canal excavation, is not suited to axial transportation of material over long distances. The cost of rehandling or of moving by bulldozers or trucks may exceed the cost of wasting and borrowing. Profiles designed for dragline excavation should be balanced in relatively short lengths, say 100 to 200 ft if practicable.

For important canals, particularly on side-hill locations, it is frequently required as a condition of safety that the water prism be set wholely in the original ground, except in occasional depressions where the banks are constructed with special care. Such a requirement removes the possibility of balanced cut and fill. Smaller canals with frequent turnouts, where the risk of breakage is not too great, may be set in shallow

cutting to facilitate diversion. The resulting deficiency in excavated materials is made up by borrowing.

19. Economy of Lined Canals. Canals may be lined to prevent erosion, to maintain nonsilting velocities on flat slopes, to reduce seepage losses, to avoid waterlogging of adjacent lands, or to reduce the cost of construction by reducing the cross-sectional

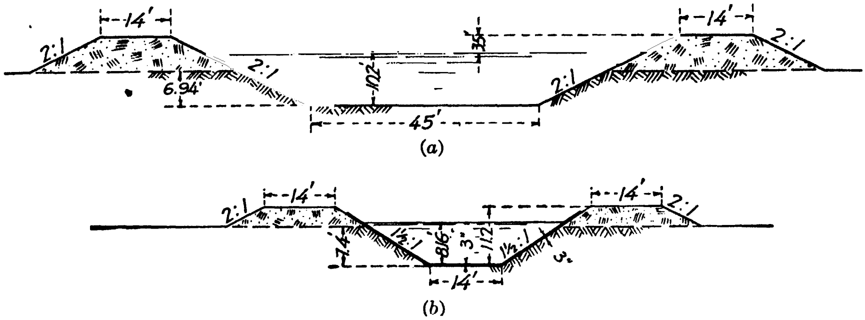


FIG. 6.—Comparison of (a) unlined and (b) lined canals.

area.[1] Usually, all these factors enter into lining problems. The interrelation of these factors will be illustrated by an example, as follows:

1. Required canal capacity, 1,000 cfs.
2. Loose, sandy soil, level, economic cutting permissible.
3. Permissible velocity, unlined, 1.50 fps (Table 1).
4. Permissible velocity, lined, not limited.
5. Maximum depth, approximately 10 ft.
6. Bank slopes, unlined, 2:1.
7. Bank slopes, lined, 1.5:1.
8. Concrete lining 3 in. thick, reinforced.
9. Top width of banks, 14 ft.
10. Shrinkage allowance, 10 per cent.
11. Seepage loss, unlined canal, 1.5 cu ft/sq ft/day (Table 2).
12. Seepage loss, lined canal, 0.05 cu ft/sq ft/day.
13. Manning's n, unlined canal, 0.0225.
14. Manning's n, lined canal, 0.013.
15. Available slope, 0.0002.
16. Cost of excavation, $0.30 per cubic yard, including placing in embankments.[1]
17. Cost of lining, $0.20 per square foot.[2]
18. Value of water, $2.00 per acre-foot.
19. Interest rate, 4 per cent per annum.
20. Variable operation over 8 months' period equivalent to about 6 months' continuous full flow.

For an unlined canal, the permissible velocity of 1.5 fps calls for a cross-sectional area of 667 sq ft, which is satisfied by a 45-ft bottom width and a depth of 10.2 ft, with $r = 7.36$ and $s = 0.000036$. This is a very flat slope, less than one-third of the available. The remainder must be consumed in falls, the cost of which will not be considered here. The resultant section, using a freeboard of 3.5 ft, is shown in Fig. 6*a*. The economic cut is 6.94 ft. The excavation is 15.2 cu yd/ft.

The seepage loss on a wetted perimeter of 90.6 ft is 136 cu ft per foot of canal per day. This is equivalent to 24,820 cu ft, or 0.57 acre-ft per foot of canal for the specified operating season, and at $2 per acre-foot is worth $1.14 per year, which at 4 per cent represents a capitalized value of $28.50 per foot of canal.

For the lined canal, the allowable depth of 10 ft calls for a bottom width of only about 6.4 ft, which is too narrow for efficient construction. If a base width of 14 ft is chosen, the velocity is 4.68 fps and the water depth is 8.16 ft. The section is shown in Fig. 6*b*. The total depth is 11.2 ft when a freeboard of $0.25d + 1$ ft is added. Allow-

[1] YOUNG, WALKER R., Low Cost Linings for Irrigation Canals, *Eng. News-Record*, Feb. 6, 1947, p. 64
[2] Cost as of about 1942, illustrative only.

ing for the thickness of lining, the total depth becomes 11.45 ft and the excavation bottom width is 14.15 ft. The economic cut is 7.4 ft, and the volume of excavated materials is 6.9 cu yd/ft. If the concrete lining is carried 2 ft above the water surface, the lined perimeter is 50.7 ft. The wetted perimeter is 43.4 ft. The leakage, at 0.05 cu ft/sq ft/day is 2.17 cu ft per foot of canal per day, 396 cu ft or 0.009 acre-ft per operating season. At $2, this equals $0.018 which, capitalized at 4 per cent, represents $0.45 per foot of canal.

The comparative costs (1947 prices) of the lined and unlined sections may be summarized as follows:

Unlined canal, per foot:	
Excavation 15.2 cu yd at $0.30	$ 4.56
Value of water lost	28.50
Total	$33.06
Lined canal, per foot:	
Excavation 6.9 cu yd at $0.30	$ 2 07
Concrete 50.7 sq ft at $0.20	10.14
Value of water lost	0.45
Total	$12.66

On the basis of construction alone, the lined canal costs nearly three times as much as the unlined canal but the value of the seepage lost reverses the ratio. This example was chosen purposely to be a decisive one. The economic balance is frequently close. Unlined canals in porous soils are seldom economically justified, and where the water supply is limited and water is valuable, lining is justified in all types of soils.

If the available fall is great, the lined canal may actually cost less to construct. This may also occur in deep rock cuts or in rough hillside work. Side-hill canals often require lining for safety, particularly if the ground is liable to slippage when saturated.

Where water values are low, irrigation laterals are not usually lined. Under a combination of high water values and limited supply, linings may extend to the smallest distributary, or the small lines may take the form of buried pipes. This condition reaches an extreme in the citrus areas of southern California.

20. Materials and Details of Paving. Concrete, cement mortar, gunite, and shotcrete are the most usual canal lining materials. The 3-in. reinforced-concrete lining of Fig. 6*b* would be classed as substantial for irrigation service under reasonably favorable climatic conditions. Linings 1½ in. thick have been used successfully in Oregon and other moderately cold climates. Such linings require well-drained banks. Where the drainage is inadequate, even the heaviest linings may be lifted by frost. Thicknesses of 3 or 4 in. are common in the colder parts of the United States. In frost-free areas, the maintenance of lining is simpler. The required thickness is also related to the size of the canal and the stability of the banks. A lining only 1½ or 2 in. thick would be considered thin for a canal as large as that of Fig. 6*b* but might be considered adequate for a small lateral.

The need for economy may lead to the use of extremely thin linings. Fair-sized ditches, with water depths of 3 or 4 ft, have been successfully lined with cement mortar ½ in. or even less in thickness applied as a plaster. Such linings require stable, well-drained banks and must be protected from mechanical damage. Thicknesses of 1 to 1½ in. are extensively used throughout California and southern Texas and have frequently been found successful in colder climates.

Linings of gunite or shotcrete (cement mortar applied by compressed air) are usually considered structurally superior to concrete or plaster linings and vary in thickness from ½ in. or less up to 1½ or 2 in. Greater thicknesses are occasionally used.

The thickness of lining is influenced by the purpose served by the canal. An open

canal on the Colorado River Aqueduct, bringing domestic water to southern California from the Colorado River, is lined with 6 to 8 in. of heavily reinforced concrete. This canal, which has a bottom width of 20 ft and a water depth of 10.2 ft, cannot readily be taken out of service for repairs. On a canal of similar size for a less exacting service, a thinner lining would suffice. If the canal banks must be built on steeper than stable slopes, the lining must be designed to hold it in place. In determining stability in arid areas, it must be recognized that soil moisture will be increased when the canal is put into operation. Canals lined on ½:1 slopes in apparently stable cuts have been known to collapse.

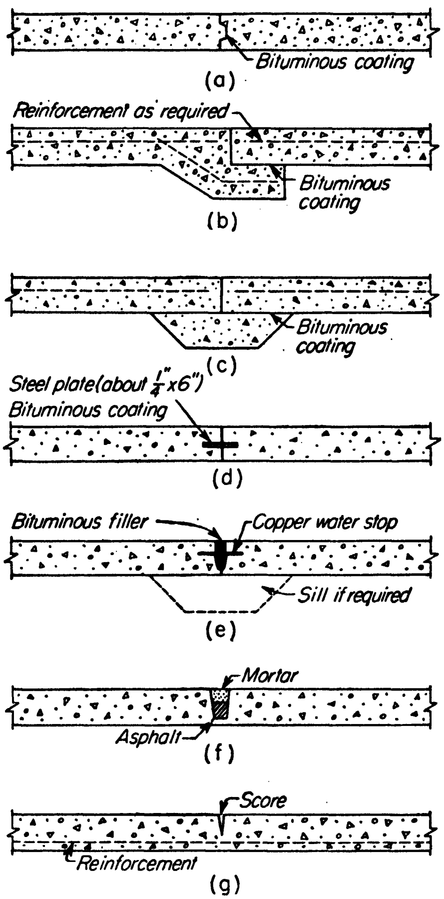


FIG. 7.—Typical canal-lining joints.

Concrete lining placed on bank slopes steeper than 1:1 requires forms. Such slopes are seldom economical except in rock cuts. Unformed lining can be placed on 1:1 slopes only with great care. Slopes of 1¼:1 work reasonably well and 1½:1 quite well. Plaster and gunite linings can be placed on any stable slope particularly if reinforced. Forms for lining should be metal faced or made of plywood or plastic sheets to ensure a minimum resistance to flow. Carefully constructed wood forms are frequently used. Unformed surfaces should be steel-trowel finished.

21. Contraction Joints in Concrete Lining. Temperature and moisture variations cause cracking of concrete lining. If the lining is not reinforced, the size and spacing of the cracks is governed by the thickness and tensile strength of the concrete and resistance to sliding along the bank. Theoretically, after one shrinkage crack has occurred, the lining will be dragged along the bank as it contracts for a distance such that the dragging force exceeds the tensile strength of the concrete. This may result in openings of sufficient width to permit appreciable leakage. Actually, many unreinforced canal linings have been successfully placed with no special provision for expansion and contraction. The swelling of the concrete when the canal is filled with water and the deposition of silt in the cracks reduce the leakage to a permissible value.

In important installations, the effect of shrinkage may be minimized by the use of special contraction joints. Typical joints are shown in Fig. 7. A successful joint must yield readily to shrinkage without permitting leakage. The tongue-and-groove joint shown at *a* is simple and is fairly effective where the lining is thick. Joints *b* and *c* are suitable for use with thin linings. The steel-plate water stop *d* has been widely used in the joints of closed conduits and is suitable for thick canal lining. A copper

water stop *e* may be inserted in a simple straight joint or in joints similar to *a*, *b*, or *c*. The asphaltic joint *f* has been successfully used. Many variations of these types are possible.

Because of the unknown sliding resistances in the joints and along the foundations, the spacing of contraction joints is not subject to exact computation. Examples may be found varying from twenty to one hundred times the thickness of the lining. A rough average is perhaps fifty times the thickness. If the lining is reinforced, greater spacing is permissible.

The type of joint may appreciably affect the cost of the lining. The tongue-and-groove joint type *a* and the bell-and-spigot joint type *b* require that the lining be placed in alternate sections. This is permissible with hand methods of placement but is not suited to modern paving machinery. The sills of type *c* can be placed ahead, and continuous construction is possible if a means of cutting the slab at the joint is provided. By providing special means for forming the joints, types *d*, *e*, and *f* permit continuous placing.

In the Pasco lateral system on the Columbia Basin Project in Washington, the U.S. Bureau of Reclamation is experimenting with 2- and 3-in. unreinforced-concrete linings with dummy joints similar to that shown at (*g*) in Fig. 7.[1] The joints are made ⅝ in. deep and ¼ in. wide and are spaced at 6-, 9-, and 12-ft intervals in separate reaches of canal. The joints are filled with a cold mastic type of asphalt prior to the application of a curing compound to the surface of the lining. Unreinforced gunite linings are being treated in the same way.

This type of dummy joint has long been used on reinforced linings.

22. Continuously Reinforced Linings. In a continuous lining, without contraction joints, cracking may be controlled by reinforcing steel. To accomplish this, the tensile strength of the steel, *at the elastic limit*, must exceed the ultimate tensile strength of the concrete at the time of first maximum shrinkage. If the first maximum shrinkage is accompanied by a temperature drop, the temperature stress in the steel must be allowed for.

The tensile strength of concrete is variable and uncertain but may be taken as approximately 10 per cent of the compressive strength. By using 2,500-psi concrete and hard-grade or rerolled rail steel with an elastic limit of 50,000 psi or more, about 0.5 per cent longitudinal reinforcement is required to fully control cracking. For lining reinforced in this manner, it is important that maximum as well as minimum strength of the concrete be controlled. If the tensile strength of the concrete is sufficient to stretch the steel beyond the elastic limit, the effectiveness of the reinforcement is impaired. Transverse reinforcement may be arbitrarily set at 0.2 or 0.25 per cent. The amount of steel required may be reduced by scoring the lining, as indicated in Fig. 7(*g*). If the scores are spaced so that full contraction between them is permissible, the steel need be sufficient only to break the uncut thickness.

The tensile strength of gunite is high, necessitating a high steel strength for full temperature reinforcement. This is offset by the use of cold-drawn wire of high unit strength. Reinforcement in plaster and gunite linings is usually nominal.

23. Linings of Miscellaneous Materials. Stone masonry lining is seldom used in the United States except for occasional paving around structures but is extensively used in other countries. Such linings may be laid up in lime or cement mortar and are 6 to 12 in. thick. If properly constructed, they are stable and reasonably watertight. The surface is sometimes hand-plastered to reduce resistance to flow.

Wood linings are used rarely where a temporary service is required or where permanent lining is to be deferred until settlement and earth movements resulting

[1] LIPP, MAURICE C., Experimental Canal Linings, *Western Construction News*, September, 1947, p. 82.

from new construction have ceased. Semicircular metal flumes, subsequently described, are sometimes installed as temporary ditch lining.

Canals through porous areas may be lined with clay spread in 3- to 6-in. layers and carefully worked or puddled into place. Such a lining, properly placed, may appreciably reduce seepage. Unless velocities are low, the clay is subject to erosion and may require protection by a layer of more stable material. Many leaky canals have been effectively sealed merely by sluicing clay into the flowing water. Canals carrying silty water seldom show a high seepage loss after a few seasons of service.

Asphalts and oils have been used to seal canals against leakage. Where suitable materials are available, an oil mix, similar to that used for road surfacing, may be spread on the bank and compacted. Such a lining will reduce seepage and increase resistance to erosion. It is cheaper than a concrete lining but is less permanent and less effective. An appreciable reduction in seepage may be effected merely by spraying a suitable oil over the canal surface. An oil of medium weight may be sprayed onto a dampened but not saturated surface, or a heavier oil or light asphalt may be applied in such manner as to form a thin continuous film on top of the ground. The latter type may require protection by a layer of soil. Such linings are inexpensive but are not permanent.

Walker R. Young, in the article previously mentioned, discusses work being done by the U.S. Bureau of Reclamation in an endeavor to develop low-cost lining. He states that "the high initial cost of standard types of lining is, at present, prohibitive for many projects. The need is great, therefore, for reducing the cost of canal lining or for discovering a satisfactory type of lining which at lower cost will still provide durability and serviceability."

He further states that "types of lining material already under consideration fall into three general groups, (1) hydraulic cement mixtures, (2) asphaltic materials, (3) earth materials. Hydraulic cement mixtures include concrete and mortar of portland cement and hydraulic lime with or without various additives. Structural variations include reinforced and unreinforced cast-in-place concrete and mortar, macadam and dry-tamped concrete, pneumatically applied mortar, and precast units. The use of hydraulic lime is expected to increase extensibility of the concrete, a characteristic highly desirable in canal linings. The addition of small amounts of asphalt emulsion or other plasticising agents should reduce the water content and therefore the shrinkage. The elimination of reinforcing steel, where design considerations will permit, will increase extensibility in addition to reducing unit costs. Asphaltic materials include asphaltic concrete (cast-in-place and precast), spray-applied asphaltic membranes, and prefabricated bituminous surfacing. The asphaltic membrane lining is essentially a stabilization of existing soil secured by the absorption of oils and cutbacks covered with a very thin membrane of a heavier asphalt. The earth materials include loosely placed, and compacted untreated earth, soil-cement and soil-resin mixtures, and bentonite and silt treated earth."

Young also goes into the economies resulting from the mechanized construction of canals and canal linings.

In the Pasco Lateral system of the Columbia Basin Project in the state of Washington, the U.S. Bureau of Reclamation is experimenting with asphaltic concrete linings. According to Maurice C. Lipp,[1] "The asphaltic concrete lining, mixed and laid hot, consists of a well-graded aggregate of ¾-in. maximum size, with an asphalt content of from 8 per cent to 10 per cent of 60–70 penetration asphalt cement. If the asphaltic mix is correctly designed and compacted, reducing the air voids to a minimum, it has been found that the resulting impermeability obviates the necessity of a surface seal. Another important factor favoring the elimination of seal treatment is the increased

[1] *Ibid.*

resistance to mud curling which tends to erode asphaltic concrete. Seal treatments which are sufficiently hard to resist this destructive effect may readily check and crack."

DESIGN OF FLUMES

24. General. The term *flume* is used to designate an artificial water channel of wood, metal, concrete, or masonry usually supported above the surface of the ground. Lined canals do not come within this classification. A flume is commonly used for crossing a topographical depression or difficult terrain.

The earliest flumes consisted of masonry channels supported by masonry piers and arches. The old Roman aqueducts are notable examples. Concrete flumes, with concrete arch supports, are still used to a limited extent, but present-day flumes are generally of lighter and more economical construction.

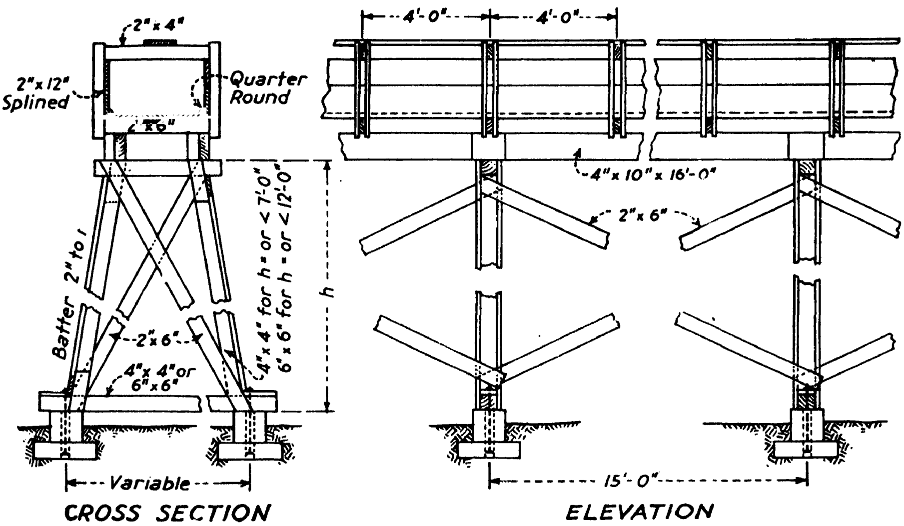


Fig. 8.—Small wooden flume.

25. Wooden Flumes. Because of the abundance and cheapness of timber, early American practice tended toward wooden flumes. In smaller sizes, such flumes consist of V-shaped or rectangular troughs, nailed together from plain boards, reinforced at 4- to 6-ft intervals with 2- by 4-in. collars, and supported on timber posts. In larger sizes, the collars become more important, the walls being formed of multiple boards held in place by nails or bolts. The side and bottom boards must have sufficient strength to carry the water load from collar to collar. The distance between bents is usually spanned by stringers, or girders, although in smaller sizes the flume itself may act as a girder. Typical wooden flumes are shown in Figs. 8 and 9.

The wall and floor boards usually should not be less than 1½ in. in thickness, as thinner boards tend to warp and crack. The boards may be tongue-and-groove, double-grooved with spiling, shiplapped, or plain-edged caulked and battened. Boards properly seasoned before use swell and tighten when wet. If the flume is to be used intermittently, as in irrigation service, a grooved joint is preferable, being less seriously affected by alternate wetting and drying. The collars usually consist of a complete rectangular frame with a tie across the top. The top tie may be replaced or supplemented by external braces. The crosstie requires an increased freeboard,

particularly if the flume is likely to carry appreciable quantities of moss, weeds, ice, or other floating materials.

Supporting trestles may consist of two simple posts for small flumes or of multiple-column frames for larger sizes. All framed joints should be connected by bolts, except for very light or temporary construction. The side and floor boards may be nailed into place, but nails are not otherwise permissible. A cross sill is usually provided at the bottom of the bent. This sill may rest directly on the ground but will have a longer life if supported on concrete or masonry piers.

The entire structure must be safe against overturning by wind action either loaded or empty. The outer posts may be battered to increase stability and should be anchored to the concrete supports. Unless the flume is relatively wide, the wind pressure on the lee wall may be taken as 50 per cent of normal, but full wind load

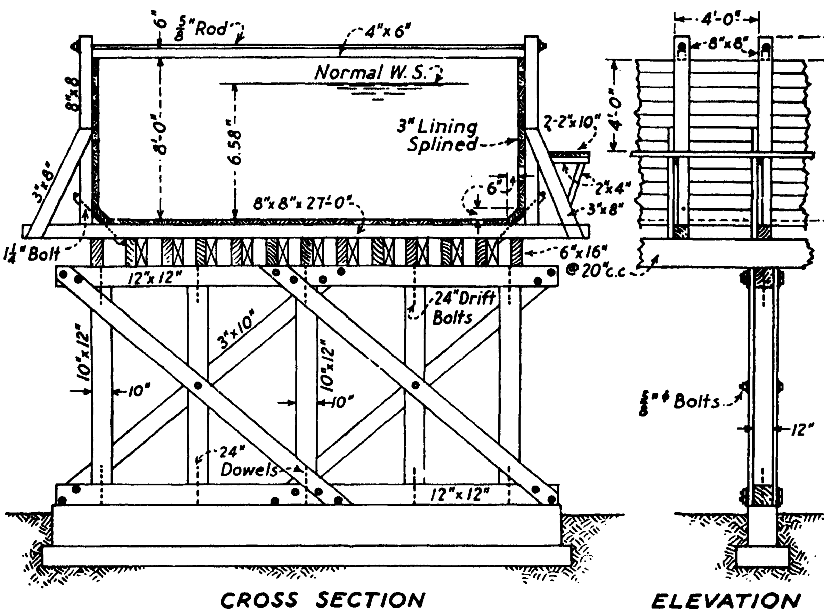


FIG. 9.—Large wooden flume.

should be assumed on each post. The allowance for wind pressure varies with conditions. An allowance of 20 psf is usual.

In some localities, earthquake forces may need to be considered. The magnitude of such forces depends on local seismographical conditions and the degree of safety desired. A frequent allowance is a horizontal force equal to 0.1 of the weight of the structure and water load, if any, applied at the center of the mass. The entire structure, full or empty, must be safe against overturning under this force.

Trestle posts and other structural members must be able to resist stresses resulting from earthquake or wind loads added to the usual loads. For the simultaneous action of wind, earthquake, and full water loads, it is permissible to allow a reasonable overstress, say 20 per cent.

Many wooden flumes constructed throughout the United States have given satisfactory service. They are not absolutely water-tight but if properly constructed and maintained, the water loss is not excessive. A first-quality seasoned fir or a yellow pine flume kept in continuous service will remain serviceable for 10 to 20 years. If creosoted, the life may be extended by 50 to 100 per cent. Cypress flumes may be

expected to outlast creosoted fir or pine. Redwood flumes have given satisfactory service for as long as 50 years. The life of the flume wall is shortened by intermittent service. The life of trestle bents may be prolonged by building the masonry footings well above the ground level. Imperfections should be repaired promptly, as leakage accelerates deterioration.

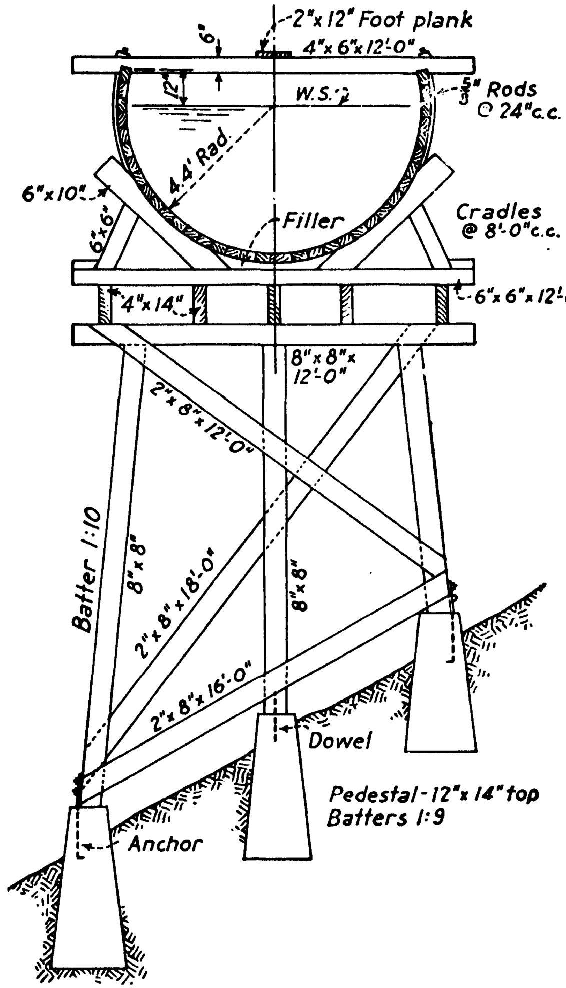


FIG. 10.—Wood-stave flume on trestle.

Wooden flumes are sometimes constructed semicircular in form of specially milled staves held together by semicircular steel bands, as illustrated in Fig. 10. The rods and crossties are spaced 2 to 4 ft apart, depending on the size of the flume. The rods must have sufficient strength to compress the staves closely together and to suspend the load. Malleable-iron washers spread the suspended load onto the top of the crosstie. The wood-stave flume may be tightened occasionally to overcome shrinkage.

26. Metal Flumes. Rectangular flumes are occasionally constructed of metal, steel plates of substantial thickness being used, supported by a steel framework, in much the same form as rectangular wooden flumes. Such construction is not usual.

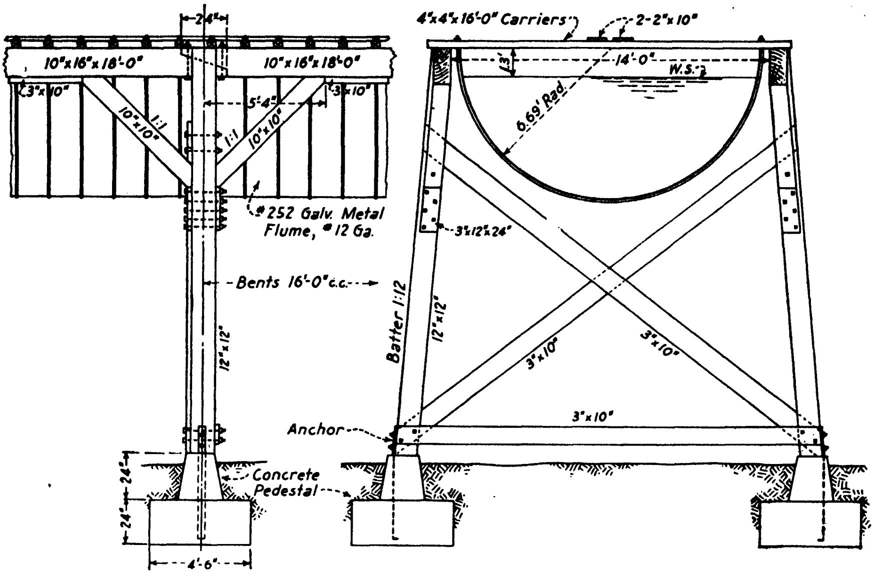


Fig. 11.—Typical metal flume.

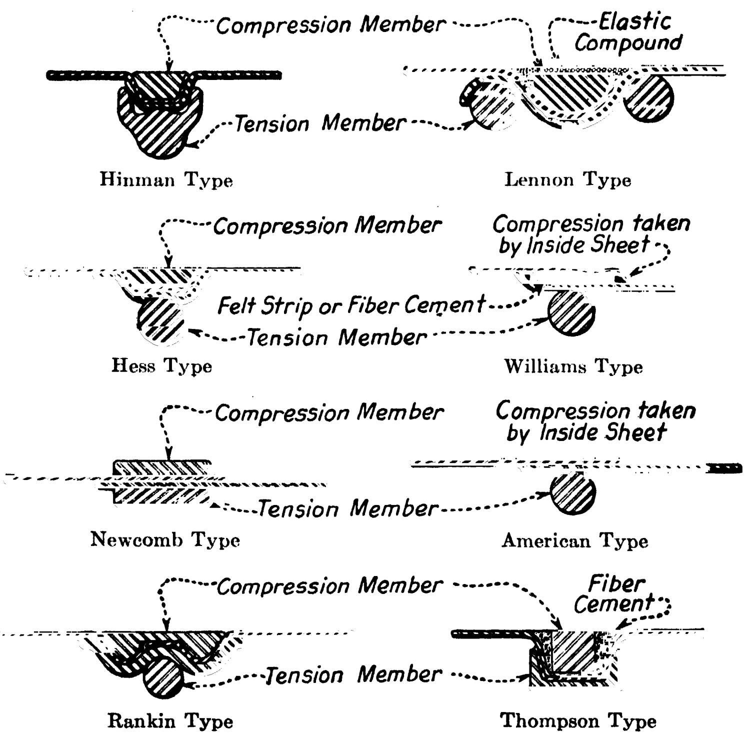


Fig. 12.—Metal flume joints.

However, a semicircular type of steel flume, erected on either wood or steel supports, is widely used. The flume barrel is composed of relatively thin steel sheets, rolled to semicircular form, the overlapping edges being pressed together between an outside rod, or hanger, and an inside compression member. The rod is supported by a crosstie, as illustrated in Fig. 11. The inside compression member reacts against the underside of the crosstie.

Many types of tension and compression members and of plate beads have been devised, some of which are shown in Fig. 12. The Hinman, Hess, and the Thompson types afford structural security and have been widely used. The Lennon and Rankin types are perhaps equally good but involve somewhat more metal. The Williams, Newcomb, and American types are simpler to manufacture and install, particularly on

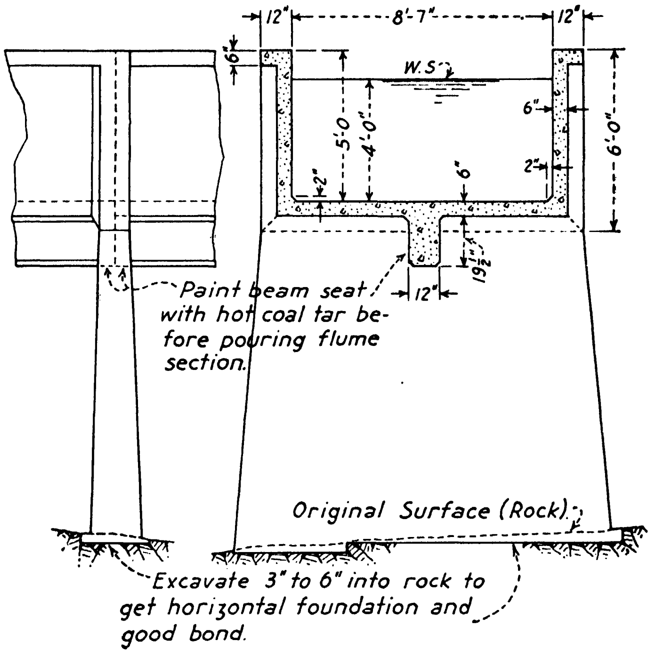


FIG. 13.—Typical concrete flume.

curves. A moderate curvature can be accommodated with these types without specially mitered sheets. The Williams and American types used the inside sheet for the compression member. These types have been successfully used in sizes up to about 60 in. in circumference.

The metal sheets and all metal parts coming in contact with them should be protected by galvanizing. The supports may be of wood or steel and are governed by the same laws of stability discussed under wooden flumes.

Metal flume sizes are designated by a number which is equal to the circumference of the semicircle in inches. Many such structures have been built, ranging in size from Nos. 24 to 360. The leakage is usually small, and a well-maintained flume should have a life of 15 to 30 years. When the galvanizing wears away, the interior should be protected with a suitable coal-tar paint or enamel.

27. Concrete and Gunite Flumes. Reinforced-concrete or gunite flumes if properly designed and installed are more permanent than timber or metal. Figure

13 shows a typical cross section and pier detail for a concrete flume. In this type, the lateral water pressure is resisted by the cantilever strength of the sides. Crossties may be used if desired. The load between piers is carried by the side walls, acting as girders, and the longitudinal center beam. Bell-and-spigot expansion joints are provided at the piers.

The installation in Fig. 14 is notable in that the component parts were precast to permit the replacement of an existing wooden flume with a minimum interruption in service.

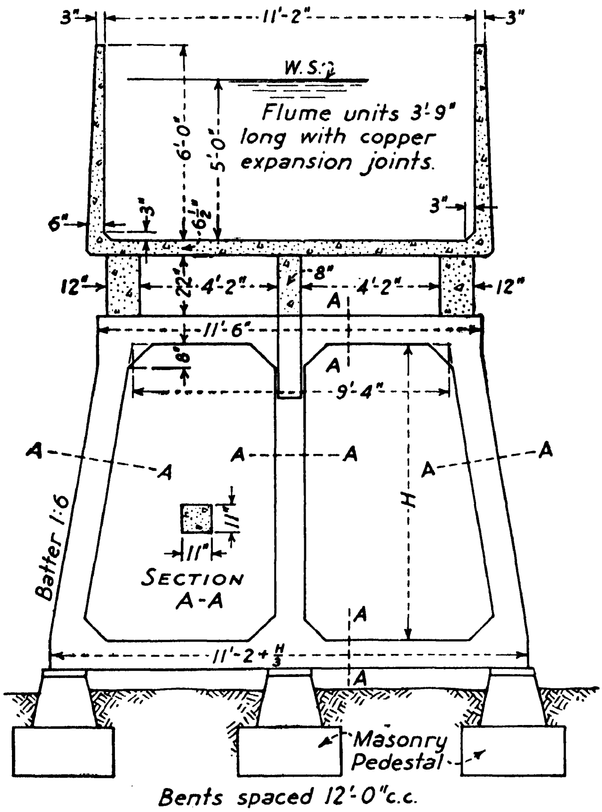


FIG. 14.—Precast concrete flume.

Gunite may be substituted for concrete in portions of either of these types of flumes. The gunite should be shot against smooth interior forms. The compressive strength of gunite is high, averaging 4,000 to 7,500 psi at 28 days. If carefully placed, a thin wall will afford all needed water-tightness, and gunite is particularly adapted to a light ribbed construction.

28. Bench Flumes. Frequently, where the ground is too rough or too steep to permit the use of a canal, a flume structure is erected on a bench, excavated into the hillside for that purpose. Typical installations are shown in Fig. 15. Such flumes may be constructed of any suitable material. If of reinforced concrete, the side walls are usually cantilevered up from the floor slab but may be supported by outside buttresses or tie bars. Gunite side walls are usually supported by posts or transverse ribs spaced 6 to 10 ft apart with tie bars across the top.

Rectangular wooden flumes are built on cross sills set directly on the foundation or supported on concrete pads. Semicircular wood or metal flumes are supported on cradles.

Bench flumes along an earth bluff or hillside represent a hazard if allowed to overflow, and demand ample spillway provision. Moss or debris lodging on the crossties

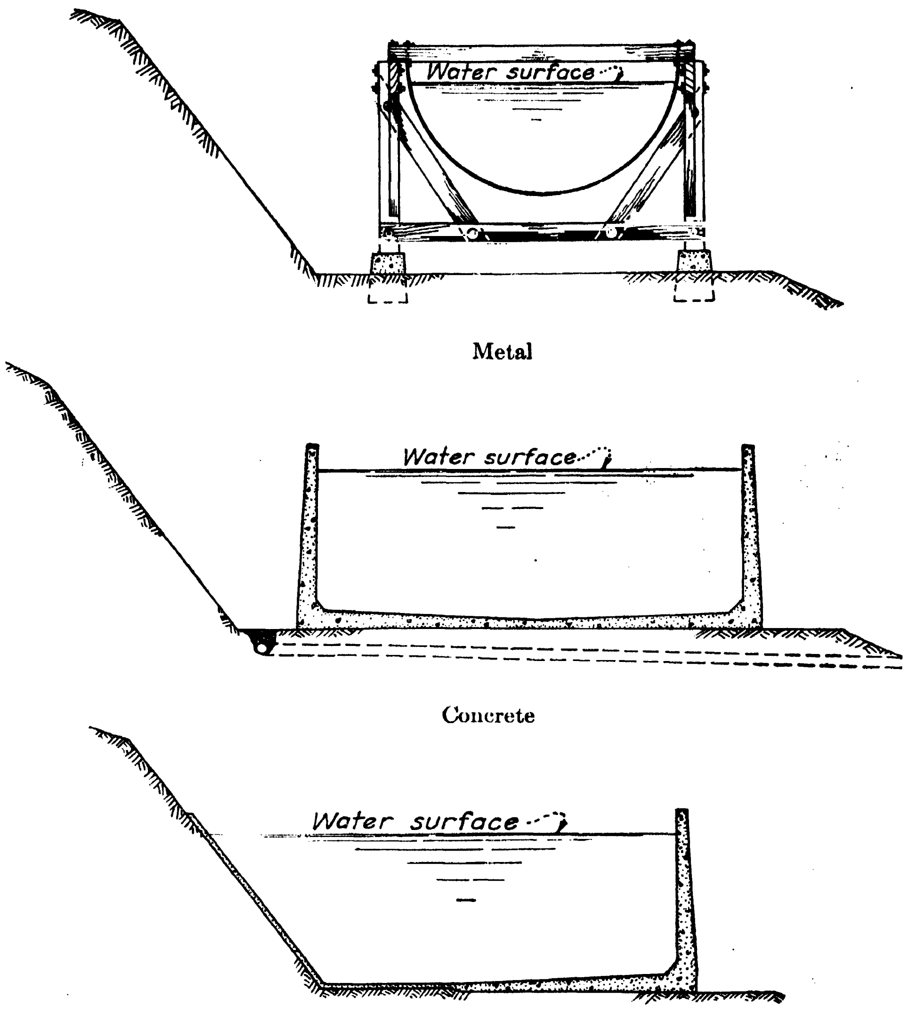


Fig. 15.—Typical bench flumes.

is a frequent source of danger. The uphill wall must be able to withstand outside earth pressure in case of a slide. Means must be provided for draining pockets on the upstream side.

29. Catenary Flumes. In a suspended circular flume, such as the metal flume of Fig. 11, the variable water pressure on the walls tends to distort the flume from its circular shape. The stiffness of the barrel resists such distortion, bending stresses

being thus set up. The resultant flexing of the flume may be reduced by giving the flume the shape of an hydrostatic catenary, the shape it would take if completely flexible. Flexing cannot be entirely removed by this means, as a given catenary is correct for only one depth of water in the flume. This type of flume is not commercially important.

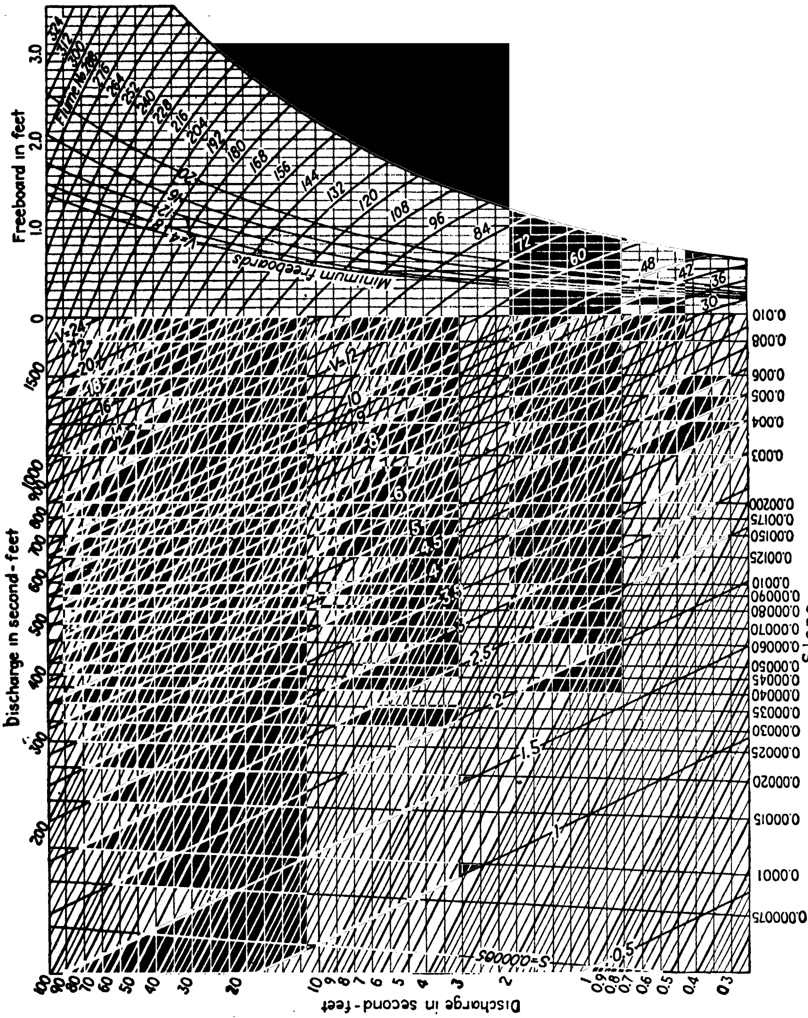


FIG. 16.—Discharge diagram for semicircular metal flumes, Kutter's n = 0.012.

30. Freeboard in Flumes. Freeboard in a flume is usually less than in a canal. A common rule is one-twelfth the diameter or width. A freeboard of $0.1d(0.9 + 0.1h_v)$ may be used for metal flumes,[1] where d is the depth of water and h_v is the velocity head. In very small flumes, a minimum of 2 or 3 in. is advisable. Special precaution is required where the foundation is subject to erosion or settlement as the result of possible spill.

[1] HINDS, JULIAN, Design, Construction and Use of Metal Flumes, *Eng. News-Record*, May 25, 1922, pp. 854*ff*.

31. Hydraulics of Flumes. The relation of flow to slope in flumes is fully covered by the principles established in Appendix A. Computations parallel the examples given for canals. Any suitable flow formula may be used, Kutter perhaps ranking first and Manning second in American practice.

Table 3 (Art. 39) will be found convenient for computing the hydraulic properties of circular flumes. For metal flumes the design may be made from Fig. 16, based on the Kutter formula, or from a similar diagram by Manning.

Velocities are usually higher in flumes than in adjacent conduits, and allowance must be made for losses and change in velocity head at entrance and exit. Particular attention must be paid to irregularities resulting from critical depth flow.

COVERED FREE-FLOW CONDUITS

32. General. It is frequently required that conduits operating partly full be covered. Such conduits may be of rectangular, circular, horseshoe, oval, or any other

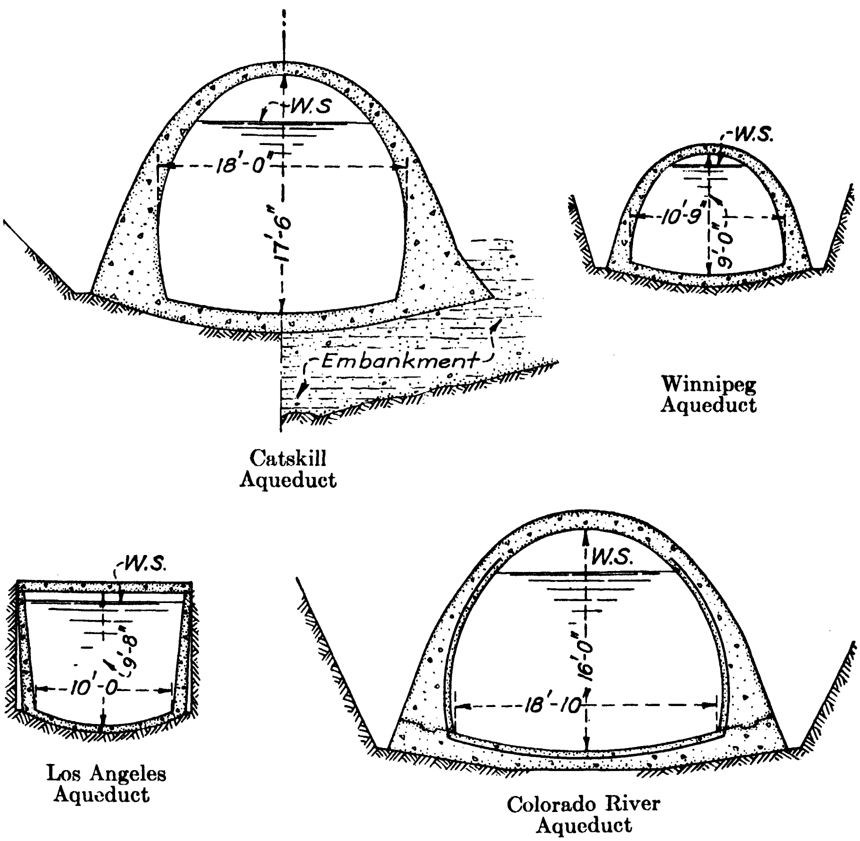


FIG. 17.—Typical aqueduct sections.

reasonable shape and are widely used as sewers and aqueducts. Typical sections of covered conduits are shown in Fig. 17.

33. Dimensions and Shapes. Structural efficiency, for sustaining external loads, rather than hydraulic efficiency controls the shape of a covered conduit. Rectangular

sections must be reinforced. Circular sections and other sections composed of arches may be reinforced or not as economy directs. The so-called *horseshoe section* is widely used, the arch wall, is unreinforced, being spread at the sides to keep the resultant within the middle third.

External loads depend on the depth and nature of the covering, details of trench, ground-water conditions, and the magnitude of superimposed loads. Approximate rules for design are given in Arts. 42, 43, and 44 for loads on buried pipes. Inadequate foundations, premature removal of forms, or improper backfilling may cause serious cracking. Backfill should be brought up evenly on the two sides of the structure by pushing the material into the trench. The dropping of heavy loads onto the arch should be avoided. Compaction, if required, should be carefully done. Transverse cracks are minimized by temperature control, proper curing, and frequently spaced transverse joints. Joints shown at *a*, *b*, *d*, and *e* (Fig. 7) may be used. Internal loads are usually assumed to be counteracted by external loads, if not, proper provision must be made.

34. Hydraulics of Covered Conduits. Computation procedures for flow in covered conduits are as described for canals, except that increased resistance as the water surface approaches or reaches the roof must be considered. The capacity of a square conduit drops 20 per cent when the flow reaches the roof. If the designed depth is only slightly less than full, a small obstruction may cause the flow to strike the roof and choke. Ample freeboard is essential. The design is unsafe unless the full conduit will carry the required discharge or a little more. With a curved roof, the reduction in capacity occurs gradually. Tables 3 and 4, Art. 39, are convenient for determining the function of circular and horseshoe conduits at part depth.

TUNNELS

35. General. Water lines occasionally take the form of tunnels, through high ground or mountains, over rugged terrain where the cost of a surface line is excessive, under congested city areas, and elsewhere as convenience and economy dictates. Tunnels may operate under pressure or flow partly full.

36. Dimensions and Shapes. A tunnel must be of sufficient size to carry the required flow on the slope available and in addition must conform to construction requirements. Unit costs vary with the size of the tunnel. The operations of men and equipment are hampered if the bore is too small. A minimum height of 6 or 7 ft is desirable even for hand excavation. Roughly, an 8- by 8-ft section is about the minimum for practicable machine excavation. Opinion is not crystallized as to the best minimum size. In good ground, the unit cost of excavation decreases as the diameter increases to a point that permits the use of full-sized shovel equipment, say up to 25 or 30 ft in diameter. If the ground is heavy, the cost per cubic yard for excavation may increase as the size increases above the optimum owing to excessive cost of support.

The cross section may be rectangular or circular or any combination of these forms. Straight sides and flat bottoms are usually considered easier to excavate. The circle is the best hydraulic shape and is also the most stable where the ground is insecure. Natural arching in a circular or curved section tends to prevent caving and to reduce the external loads where lining is required; also, the curvature adds strength to the lining. In rock, even if broken, the floor may be flat unless external water pressure is expected. In soft or swelling ground, the floor should be arched. In extremely heavy ground, a full circle may be essential. Typical tunnel sections are shown in Fig. 18.

37. Temporary Supports. Unstable ground must be held by temporary supports pending the placing and curing of the final lining. Timber or steel may be used for this purpose. In light ground, the spacing of the timber ribs or sets is chosen to per-

mit the use of a reasonable thickness of outside boards or lagging, and their size is chosen to fit the load to be supported. The timber size should be about one-twenty-fourth the tunnel diameter for light loading. For heavy ground, sizes up to one-twelfth or even greater may be required, and the spacing may be reduced until the timbers are set skin to skin. Each timber set may be independently complete, or the arches may be set on wall plates which are in turn supported on posts. The size and spacing of supports is determined by trial and experience as the work progresses. Wall plates, if used, should be removed between posts before concrete is placed.

Curved steel members may be used for the supporting ribs as illustrated in Fig. 19. Steel ribs cost more than timber but occupy less space, thus the amount of excavation

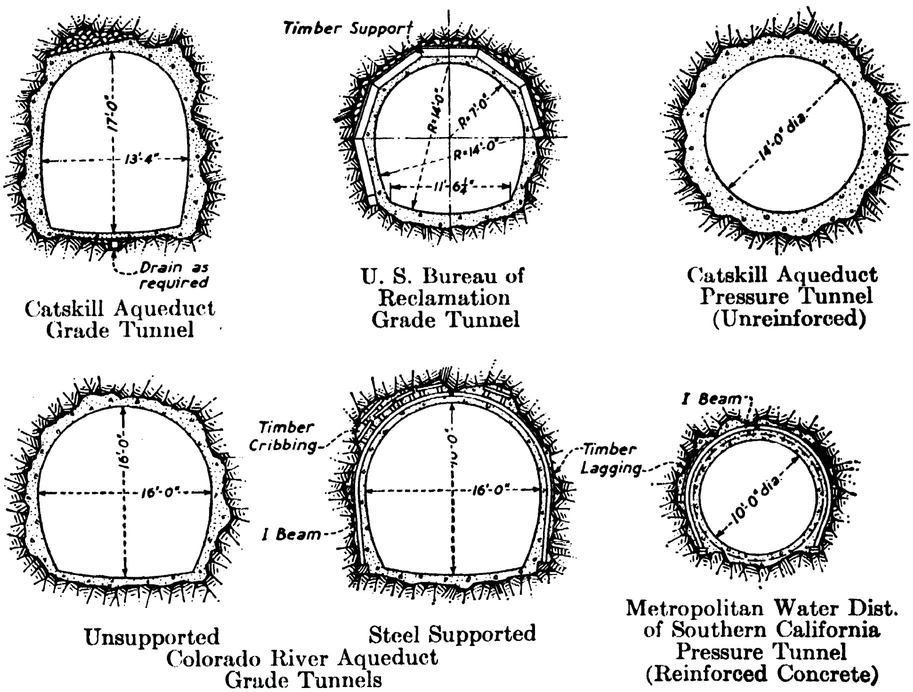


FIG. 18.—Typical tunnel sections.

and concrete is reduced. In dry sand or other running ground, steel ribs with specially corrugated metal plates may be used.

Both timber and steel are subject to rapid deterioration in wet tunnels, and neither contributes to the ultimate support except where steel is encased in concrete. The section must be sufficiently large to permit an adequate concrete lining inside the support.

Space outside the lagging must be filled with timber, broken rock, or other suitable material to stabilize the supports and to prevent the improper concentration of loads on the finished lining. In pressure tunnels, or where the ground is wet or moving, minimum blocking should be used and the space back of the lagging should be filled completely with concrete or grout.

Lightly fractured rock may be stabilized by a thin gunite coating, the need for temporary supports being thus eliminated. Such treatment is economical where adequate.

Table 3.—Area, Wetted Perimeter, and Hydraulic Radius of Partially Filled Circular Conduit Sections

$\frac{d}{D}$	$\frac{\text{Area}}{D^2}$	$\frac{\text{Wet. per.}}{D}$	$\frac{\text{Hyd. rad.}}{D}$	$\frac{d}{D}$	$\frac{\text{Area}}{D^2}$	$\frac{\text{Wet. per.}}{D}$	$\frac{\text{Hyd. rad.}}{D}$
0.01	0.0013	0.2003	0.0066	0.51	0.4027	1.5908	0.2531
0.02	0.0037	0.2838	0.0132	0.52	0.4127	1.6108	0.2561
0.03	0.0069	0.3482	0.0197	0.53	0.4227	1.6308	0.2591
0.04	0.0105	0.4027	0.0262	0.54	0.4327	1.6509	0.2620
0.05	0.0147	0.4510	0.0326	0.55	0.4426	1.6710	0.2649
0.06	0.0192	0.4949	0.0389	0.56	0.4526	1.6911	0.2676
0.07	0.0242	0.5355	0.0451	0.57	0.4625	1.7113	0.2703
0.08	0.0294	0.5735	0.0513	0.58	0.4723	1.7315	0.2728
0.09	0.0350	0.6094	0.0574	0.59	0.4822	1.7518	0.2753
0.10	0.0409	0.6435	0.0635	0.60	0.4920	1.7722	0.2776
0.11	0.0470	0.6761	0.0695	0.61	0.5018	1.7926	0.2797
0.12	0.0534	0.7075	0.0754	0.62	0.5115	1.8132	0.2818
0.13	0.0600	0.7377	0.0813	0.63	0.5212	1.8338	0.2839
0.14	0.0668	0.7670	0.0871	0.64	0.5308	1.8546	0.2860
0.15	0.0739	0.7954	0.0929	0.65	0.5404	1.8755	0.2881
0.16	0.0811	0.8230	0.0986	0.66	0.5499	1.8965	0.2899
0.17	0.0885	0.8500	0.1042	0.67	0.5594	1.9177	0.2917
0.18	0.0961	0.8763	0.1097	0.68	0.5687	1.9391	0.2935
0.19	0.1039	0.9020	0.1152	0.69	0.5780	1.9606	0.2950
0.20	0.1118	0.9273	0.1206	0.70	0.5872	1.9823	0.2962
0.21	0.1199	0.9521	0.1259	0.71	0.5964	2.0042	0.2973
0.22	0.1281	0.9764	0.1312	0.72	0.6054	2.0264	0.2984
0.23	0.1365	1.0003	0.1364	0.73	0.6143	2.0488	0.2995
0.24	0.1449	1.0239	0.1416	0.74	0.6231	2.0714	0.3006
0.25	0.1535	1.0472	0.1466	0.75	0.6318	2.0944	0.3017
0.26	0.1623	1.0701	0.1516	0.76	0.6404	2.1176	0.3025
0.27	0.1711	1.0928	0.1566	0.77	0.6489	2.1412	0.3032
0.28	0.1800	1.1152	0.1614	0.78	0.6573	2.1652	0.3037
0.29	0.1890	1.1373	0.1662	0.79	0.6655	2.1895	0.3040
0.30	0.1982	1.1593	0.1709	0.80	0.6736	2.2143	0.3042
0.31	0.2074	1.1810	0.1755	0.81	0.6815	2.2395	0.3044
0.32	0.2167	1.2025	0.1801	0.82	0.6893	2.2653	0.3043
0.33	0.2260	1.2239	0.1848	0.83	0.6969	2.2916	0.3041
0.34	0.2355	1.2451	0.1891	0.84	0.7043	2.3186	0.3038
0.35	0.2450	1.2661	0.1935	0.85	0.7115	2.3462	0.3033
0.36	0.2546	1.2870	0.1978	0.86	0.7186	2.3746	0.3026
0.37	0.2642	1.3078	0.2020	0.87	0.7254	2.4038	0.3017
0.38	0.2739	1.3284	0.2061	0.88	0.7320	2.4341	0.3008
0.39	0.2836	1.3490	0.2102	0.89	0.7384	2.4655	0.2996
0.40	0.2934	1.3694	0.2142	0.90	0.7445	2.4981	0.2980
0.41	0.3032	1.3898	0.2181	0.91	0.7504	2.5322	0.2963
0.42	0.3130	1.4101	0.2220	0.92	0.7560	2.5681	0.2944
0.43	0.3229	1.4303	0.2257	0.93	0.7642	2.6061	0.2922
0.44	0.3328	1.4505	0.2294	0.94	0.7662	2.6467	0.2896
0.45	0.3428	1.4706	0.2331	0.95	0.7707	2.6906	0.2864
0.46	0.3527	1.4907	0.2366	0.96	0.7749	2.7389	0.2830
0.47	0.3627	1.5108	0.2400	0.97	0.7785	2.7934	0.2787
0.48	0.3727	1.5308	0.2434	0.98	0.7816	2.8578	0.2735
0.49	0.3827	1.5508	0.2467	0.99	0.7841	2.9412	0.2665
0.50	0.3927	1.5708	0.2500	1.00	0.7854	3.1416	0.2500

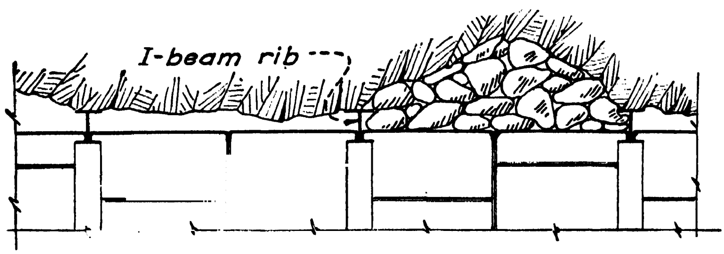
TABLE 4.—AREA, WETTED PERIMETER AND HYDRAULIC RADIUS OF PARTIALLY FILLED HORSESHOE CONDUIT SECTIONS

$\frac{d}{D}$	$\frac{\text{Area}}{D^2}$	$\frac{\text{Wet. per.}}{D}$	$\frac{\text{Hyd. rad.}}{D}$	$\frac{d}{D}$	$\frac{\text{Area}}{D^2}$	$\frac{\text{Wet. per.}}{D}$	$\frac{\text{Hyd. rad.}}{D}$
0.01	0.0019	0.2830	0.0066	0.51	0.4466	1.7162	0.2602
0.02	0.0053	0.4006	0.0132	0.52	0.4566	1.7362	0.2630
0.03	0.0097	0.4911	0.0198	0.53	0.4666	1.7562	0.2657
0.04	0.0150	0.5676	0.0264	0.54	0.4766	1.7763	0.2683
0.05	0.0209	0.6351	0.0329	0.55	0.4865	1.7964	0.2707
0.06	0.0275	0.6963	0.0394	0.56	0.4965	1.8165	0.2733
0.07	0.0346	0.7528	0.0459	0.57	0.5064	1.8367	0.2757
0.08	0.0421	0.8054	0.0524	0.58	0.5163	1.8569	0.2781
0.0886	0.0491	0.8482	0.0578	0.59	0.5261	1.8772	0.2804
0.09	0.0502	0.8513	0.0590	0.60	0.5359	1.8976	0.2824
0.10	0.0585	0.8732	0.0670				
0.11	0.0670	0.8950	0.0748	0.61	0.5457	1.9180	0.2844
0.12	0.0753	0.9166	0.0823	0.62	0.5555	1.9386	0.2864
0.13	0.0839	0.9382	0.0895	0.63	0.5651	1.9592	0.2884
0.14	0.0925	0.9597	0.0964	0.64	0.5748	1.9800	0.2902
0.15	0.1012	0.9811	0.1031	0.65	0.5843	2.0009	0.2920
0.16	0.1100	1.0024	0.1097	0.66	0.5938	2.0219	0.2937
0.17	0.1188	1.0236	0.1161	0.67	0.6033	2.0431	0.2953
0.18	0.1277	1.0448	0.1222	0.68	0.6126	2.0645	0.2967
0.19	0.1367	1.0658	0.1282	0.69	0.6219	2.0860	0.2981
0.20	0.1457	1.0868	0.1341	0.70	0.6312	2.1077	0.2994
0.21	0.1549	1.1078	0.1398	0.71	0.6403	2.1297	0.3006
0.22	0.1640	1.1286	0.1454	0.72	0.6493	2.1518	0.3018
0.23	0.1733	1.1494	0.1508	0.73	0.6582	2.1742	0.3028
0.24	0.1825	1.1702	0.1560	0.74	0.6671	2.1969	0.3036
0.25	0.1919	1.1909	0.1611	0.75	0.6758	2.2198	0.3044
0.26	0.2013	1.2115	0.1662	0.76	0.6844	2.2431	0.3050
0.27	0.2107	1.2321	0.1710	0.77	0.6929	2.2666	0.3055
0.28	0.2202	1.2526	0.1758	0.78	0.7012	2.2906	0.3060
0.29	0.2297	1.2731	0.1804	0.79	0.7094	2.3149	0.3064
0.30	0.2393	1.2935	0.1850	0.80	0.7175	2.3397	0.3067
0.31	0.2489	1.3139	0.1895	0.81	0.7254	2.3650	0.3067
0.32	0.2586	1.3342	0.1938	0.82	0.7332	2.3907	0.3066
0.33	0.2683	1.3546	0.1981	0.83	0.7408	2.4170	0.3064
0.34	0.2780	1.3748	0.2023	0.84	0.7482	2.4440	0.3061
0.35	0.2878	1.3951	0.2063	0.85	0.7554	2.4716	0.3056
0.36	0.2975	1.4153	0.2103	0.86	0.7625	2.5000	0.3050
0.37	0.3074	1.4355	0.2142	0.87	0.7693	2.5292	0.3042
0.38	0.3172	1.4556	0.2181	0.88	0.7759	2.5595	0.3032
0.39	0.3271	1.4758	0.2217	0.89	0.7823	2.5909	0.3020
0.40	0.3370	1.4959	0.2252	0.90	0.7884	2.6235	0.3005
0.41	0.3469	1.5160	0.2287	0.91	0.7943	2.6576	0.2988
0.42	0.3568	1.5360	0.2322	0.92	0.7999	2.6935	0.2969
0.43	0.3667	1.5561	0.2356	0.93	0.8052	2.7315	0.2947
0.44	0.3767	1.5761	0.2390	0.94	0.8101	2.7721	0.2922
0.45	0.3867	1.5962	0.2422	0.95	0.8146	2.8160	0.2893
0.46	0.3966	1.6162	0.2454	0.96	0.8188	2.8643	0.2858
0.47	0.4066	1.6362	0.2484	0.97	0.8224	2.9188	0.2816
0.48	0.4166	1.6562	0.2514	0.98	0.8256	2.9832	0.2766
0.49	0.4266	1.6762	0.2544	0.99	0.8280	3.0667	0.2696
0.50	0.4366	1.6962	0.2574	1.00	0.8293	3.2670	0.2538

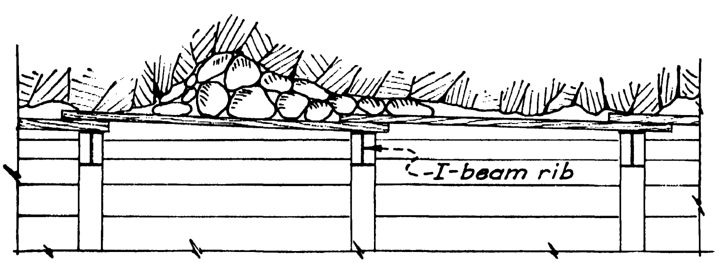
38. Tunnel Lining. Where the ground is stable without support, the sides and bottom of the tunnel may be lined to prevent seepage, to reduce frictional resistance, or both. The economic justification for such lining depends on the value of water and the cost of excavation and lining. The cost per cubic yard of excavation in tunnels is usually high and the unlined sides are rough, consequently lining is likely to pay.

Lining to prevent leakage may be of minimum thickness and may take the form of gunite shot into crevices and over porous areas. To improve flow, unevennesses in excavation must be filled. In computing the cost, the filling of overbreak must be amply allowed for.

The thickness of lining in caving grounds is determined by the external loads to be borne. A minimum thickness of 6 in. with an average thickness of not less than about



Steel Liner Plates



Timber Lagging

FIG. 19.—Steel-rib tunnel supports.

one-twelfth the finished diameter is frequently specified. A much greater thickness may be required in heavy ground. It is usually possible to estimate the required strength as the excavation progresses by observing the loads on the temporary supports. If the minimum supports are not overloaded, minimum lining is used. If the supports begin to show distress, the magnitude of the applied load is indicated by the strength of supports required. The permanent concrete lining should have an ample margin of safety over the necessary strength of the temporary support. External loads from fractured rock usually occur only on the roof and sides. Soft material may be squeezed up, thus loading the bottom. Clays and shales sometimes swell when wet and thus produce loads on the entire circumference, the magnitude of which can be determined only by observation as the tunnel is being driven. Linings through water-bearing areas, if tight, are subjected to external water pressure. Such pressure may be relieved by weep holes into the tunnel or by tile drains leading to the portals.

Curved portions of the lining are designed as arches and flat portions as slabs or beams. Where the pressures are heavy and likely to occur on the bottom as well as

on the roof and sides, a full circular lining is efficient. If the section is not circular, reinforcing steel may be required.

If the tunnel is to convey water under pressure, internal loads must be considered. In bad ground, or where the cover is light, the lining must be reinforced. If the rock is firm and the cover adequate, the reinforcement may be omitted if all voids between the lining and the rock are filled. A safe rule is to reinforce if the depth of cover is less than 1.5 times the internal pressure head for fairly good but broken rock. In badly broken ground or alluvium, reinforcement should always be provided. Where leakage is likely to cause settlement or slides, external drainage may be required. Particular precautions may be required to prevent the building up of pressures in fissures near portals.

39. Hydraulics of Tunnels. The computation of friction losses in tunnels involves no special problems. Flow-line tunnels require the same provisions for freeboard as

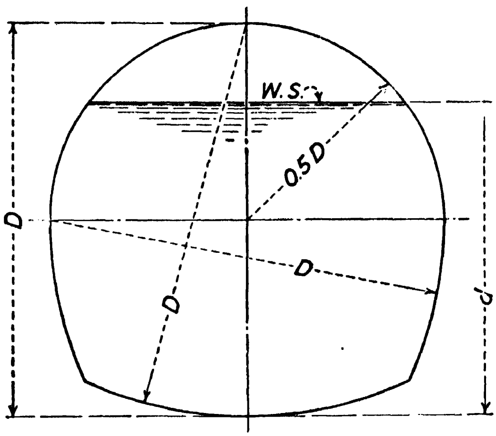


FIG. 20.—Standard horseshoe tunnel.

noted for covered conduits. The computation of areas and hydraulic radii is facilitated by special tables such as Tables 3 and 4. Table 3 gives factors for computing the functions of partly filled circular sections. Table 4 is for the more or less standard horseshoe section shown on Fig. 20. The maximum flow for a circular section occurs when the ratio of depth to diameter is 0.93, the Manning formula being used. For a depth of 0.80, the flow is the same as for the full circle. This latter depth is frequently used to determine the minimum freeboard. For the horseshoe section of Table 4, these depth ratios are, respectively, 0.94 and 0.82.

PIPE LINES

40. General. Pipes are used for the conveyance of water (or other fluids) under pressure, as in penstocks, inverted siphons, and city water lines. They also may be used as free-flow conduits running partly full, as in drains and sewers. Pipes for conveying water in appreciable quantities are made of steel, cast iron, concrete, wood, vitrified clay, and asbestos cement. Pipes of brass, copper, lead, glass, rubber, and other materials are used for special services.

41. Loads Due to Internal Pressures. Pipes must be capable of withstanding the bursting effort of the internal pressures. The total shell tension due to internal pressure is approximately

$$F = \tfrac{1}{2}pd \tag{4}$$

which gives a unit stress of

$$f_1 = \frac{0.5pd}{a} \tag{5}$$

where F = shell tension, lb/lin ft of pipe (one side).
f_1 = unit stress, psi.
p = internal pressure, psf.
d = diameter of the pipe, ft.
a = area of the pipe shell, or the tension-taking part of it, sq in. per foot of pipe (one side).

Any water hammer must be included in p. Reference should be made to Sec. 13 for methods to compute water hammer. Unless the pipe is vertical, the weight of the water and of the pipe shell introduce moments that vary with the manner of support.[1]

42. Vertical Loads Due to Backfill. Pipes are sometimes laid on the surface of the ground or on piers, bridge floors, etc. Vertical loads on such pipes consist of the

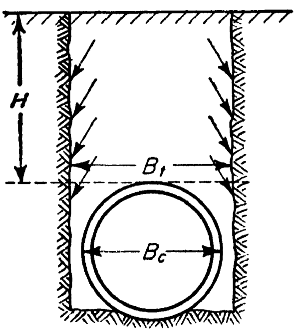


Fig. 21.—Pipe in trench.

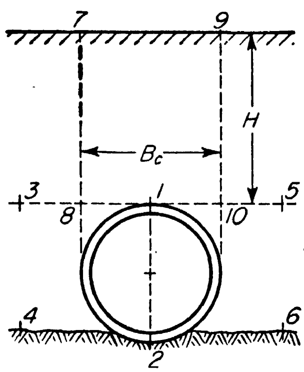


Fig. 22.—Pipe under fill.

weight of the pipe shell and the fluid in the pipe, and so far as ring stresses are concerned, generally need be considered only when the pipe is large and flexible, or at points of pier supports. This type of loading is discussed for steel pipe in Arts. 62 and 63.

Pipes used for the conveyance of water are usually buried in the ground. The most common installation is that illustrated in Fig. 21, where the pipe is laid in a narrow vertical sided trench and backfilled. Water pipes laid in city streets are usual examples.

In another common type of installation, the pipe is laid on the surface of the ground and covered with a fill, as illustrated in Fig. 22. Variations from these types include wide trenches, trenches with sloping sides, partial trenches, deep trenches with superimposed fills, etc.

The magnitude of the vertical loads in all these cases is affected by many factors, including rigidity of pipe, firmness of bedding, nature of foundation, and nature and compaction of the fill materials. An exhaustive study of such loadings has been under way at the Iowa State College since 1909 under the direction of Dean Anson Marston and his associates.[2] This work has been ably summarized by Prof. M. G. Spangler.[3]

[1] Paris, James M., Stress Coefficients for Large Horizontal Pipes, *Eng. News-Record*, **87**, 768, 1921.
[2] Marston, Anson, The Theory of External Loads on Closed Conduits in the Light of the Latest Experiments, *Iowa State College, Bull.* 96, 1930; and other bulletins referred to therein.
[3] Spangler, M. G., Underground Conduits—An Appraisal of Modern Research, *Proc., A. S. C. E.*, June, 1947, p. 855.

Pipe in Trenches. Unless compacted with unusual care, the fill alongside and above the trench pipe (Fig. 21) settles. The side fill, if loosely placed, may settle so much that it offers little or no support for the materials above the level of the top of the pipe. The weight is thus divided between the pipe and the shear or friction along the side walls of the trench. The shearing forces are indicated by inclined arrows. These forces may equal the shearing strength of the fill material, or the friction resistance between the fill and the trench wall, whichever is the smaller.

The actual division of load between the pipe and the trench walls is complex. If the pipe is flexible and so bedded that it readily yields, full shearing resistance may be developed for the full depth of the trench. If the pipe is rigid and firmly bedded, the wall forces may be somewhat less. However, the settlement of the fill above the top of the pipe will cause some wall shear, regardless of pipe rigidity.

With rigid pipe and ordinary bedding, and a trench width not greater than 1.5 times the outside diameter of the pipe, Marston's equation is

$$W_t = C_t w B_t^2 \tag{6}$$

where W_t = vertical external load on pipe, lb per ft of pipe.
C_t = experimental coefficient.
w = unit weight of fill material, lb/cu ft.
B_t = width of trench at top of pipe, ft.

For flexible pipe and thoroughly compacted side fills, Spangler suggests

$$W_t = C_t w B_c B_t \tag{7}$$

where B_c is the width of the conduit or pipe.

Values of C_t are computed from the equation

$$C_t = \frac{1 - \epsilon^{-2K\mu'\frac{H}{B_t}}}{2K\mu'} \tag{8}$$

where ϵ = base of natural logarithms.
H = height of fill above the top of the pipe, ft.
B_t = width of trench at top of pipe, ft.
μ' = coefficient of friction between fill and trench wall, which is equal to or less than μ, the coefficient of internal friction in fill.
K = Rankin's earth pressure factor.

Values of K are taken from the formula

$$K = \frac{\sqrt{\mu^2 + 1} - \mu}{\sqrt{\mu^2 + 1} + \mu} \tag{9}$$

Values of C_t for rigid types of pipes, well bedded on good foundations and back-filled with reasonable care with typical materials, may be taken from Fig. 23.

Pipe under Fills. Loading conditions for a pipe projecting above a prepared sub-grade and covered with fill are illustrated in Figs. 24 to 27. If such a pipe is rigid and firmly bedded, it will yield less than the fill alongside it, setting up shearing stresses along the planes 3-1 and 4-2 (Fig. 24) ignoring shearing stresses below 1-2. This causes the vertical load to exceed the weight of the material directly over the pipe. The maximum possible load is the weight of the material between planes 3-1 and 4-2 plus the maximum shearing strengths of the materials along the sides. This maximum is not always attained. In fact, if the pipe settles more than the fill alongside of it, the direction of the shear is reversed, causing the load on the pipe to be less than the weight of the fill directly above it.

Possible variations lead to four classifications of loading conditions which may be designated as complete and incomplete projection conditions, and incomplete and complete ditch conditions. The term ditch condition is used because the loading is similar to, but not necessarily identical with, that for pipes in narrow trenches (Fig. 21). The criteria are the deformation of a horizontal index plane through the top of the pipe and the existence and position of a plane of equal settlement.

The case of a relatively stiff well-bedded pipe covered with an uncompacted fill is illustrated in Fig. 24. The original position of the index plane is at 1-2. After settlement it sags at the sides, taking the final position 1′-2′.

The fill material immediately above the pipe is more heavily loaded and is hence more compressed than that at the sides. If a series of index planes are established between the top of the pipe and the surface of the fill, their deformation will decrease with ascending elevation. Under some conditions an elevation exists above which settlement is the same over the pipe and in the side fills, and so no load is transferred across the side planes. A horizontal plane through this elevation is called the plane of equal settlement, and its height above the top of the pipe is the height of equal settlement. If this plane is at or theoretically above the top of the fill, as illustrated in Fig. 24, the side shear is fully effective in loading the pipe, and the pipe is said to be in complete projection condition. An extreme example of this condition is a rigid pipe firmly bedded and covered with a loose and relatively shallow fill.

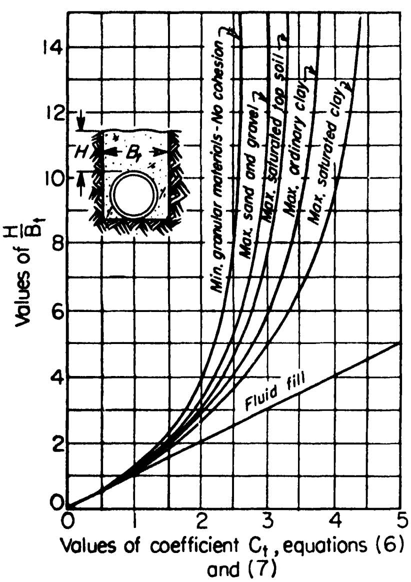


FIG. 23.—Pipe in trench. Values of C_t.

If the pipe is somewhat flexible or the side fill below the top of the pipe is well compacted, or both, the deformation (but not necessarily the displacement) of the index plane is reduced. The theoretical plane of equal settlement is lowered and may fall below the fill surface and become real, as indicated by line 5-6 in Fig. 25. There being no differential settlement above 5-6, there is no load transfer across 3-5 and 4-6, and the total load on the pipe is less than under the conditions of Fig. 24, but still more than the weight of the fill directly over the pipe. Hence, this is called incomplete projection condition. The limit of this condition is reached when 5-6 coincides with 1-2.

With further compaction of side fill, more pipe flexibility, or more yielding bedding, the deformation of the index plane may be reversed, creating a condition similar to that for a pipe in a trench or ditch. There is still a plane of equal settlement which rises above the top of the pipe with increased reversed distortion of the index plane. As long as it is within the fill, the incomplete ditch condition of Fig. 26 prevails and the load on the pipe is less than the weight of the fill directly above it, because of outward shear transfer across planes 1-5 and 2-6.

For an extremely flexible softly bedded pipe and firmly compacted side fills, this plane theoretically may pass above the top of the fill, becoming imaginary and producing the complete ditch condition shown in Fig. 27, which is mathematically similar to the trench condition of Fig. 21.

The fundamental equation for projecting conduits under fills, all conditions, is

$$W_f = C_f w B_c^2 \tag{10}$$

where W_f, C_f, w, and B_c have meanings similar to corresponding symbols in Eqs. (6) and (7).

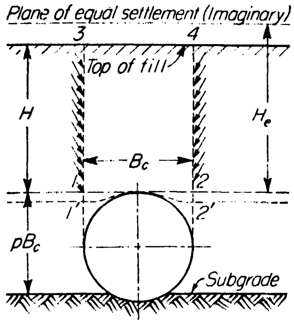


FIG. 24.—Complete projection condition.

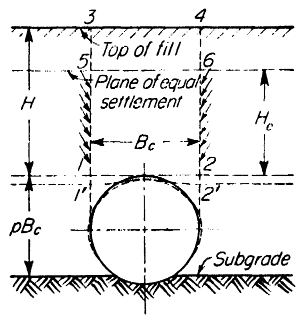


FIG. 25.—Incomplete projection condition.

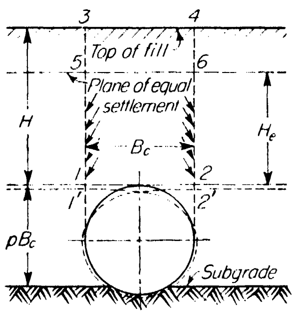


FIG. 26.—Incomplete ditch condition.

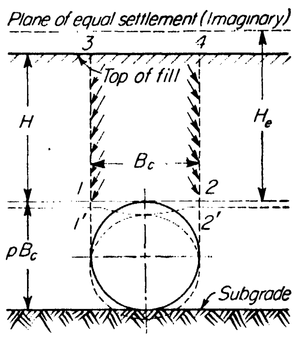


FIG. 27.—Complete ditch condition.

FIGS. 24–27.—Pipe under fill.

Values of C_f are computed from the equation

$$C_f = \frac{\epsilon^{\pm 2K\mu \frac{H_e}{B_c}} - 1}{\pm 2K\mu} + \frac{H - H_e}{B_c}\,\epsilon^{\pm 2K\mu \frac{H_e}{B_c}} \tag{11}$$

where H_e is the height of plane of equal settlement above the top of the pipe, *but not greater than* H, and other symbols are as previously described. Plus or minus signs are positive for projection conditions (Figs. 24 and 25) and negative for ditch conditions (Figs. 26 and 27). If the condition is not known, these signs agree with the sign of r_s as computed in Eq. (13).

Values of H_e are found from the equation

$$\epsilon^{\pm 2K\mu \frac{H_e}{B_c}} = \pm 2K\mu \frac{H_e}{B_c} \pm K\mu r_s p + 1 \tag{12}$$

where p is the ratio of the projecting portion of the conduit to its width, as indicated in Fig. 24, and where the plus or minus signs are evaluated as for Eq. (11).

The value of r_s is from the equation

$$r_s = \frac{(\Delta H_f + \Delta F_f) - (\Delta H_c + \Delta F_c)}{\Delta H_f} \tag{13}$$

where ΔH_f is the settlement of the side fill below the index plane, ΔF_f is the settlement of the foundation under the side fill, ΔH_c is vertical deflection of the conduit, and ΔF_c the foundation settlement under the pipe, all figured for a uniform height of fill, say above the index plane *before* adjusting for settlement.

The solution of these equations requires a knowledge of values of μ and of the settlement factors in Eq. (13). These involve elements of soil mechanics and conduit deflection that cannot be covered here. Because of the many variables involved, important installations call for field tests for soil settlement and sometimes for pipe deflections. Even for small pipes under low fills, the available data are limited. For values of μ varying from 0.3 to 1.0, $K\mu$ varies only from 0.17 to 0.1924, and back to 0.17.[1] As these two factors are used only as a product, a value of $K\mu = 0.19$ is a reasonable approximation.

For rigid pipes, Spangler[2] suggests a tentative value of r_s of +1.0 on unyielding foundation, and +0.5 to +0.8 on ordinary earth. For flexible pipe he found values ranging from 0.0 to +0.7, depending largely on the passive resistance of the side fill. These values are not particularly definite. The nature of the problem precludes definite generalization.

For cast-iron pipe up to 60 in. in diameter, for ordinary laying condition, Wiggin, Enger, and Schlick[3] adopt a compromise combined value of $r_sp = 0.75$.

Spangler[4] recommends a value of 0.70 for r_s until more comprehensive data are available.

The laborious solution of the foregoing equations may be avoided by the use of diagrams. A typical example is shown in Fig. 28. This diagram is for granular material with $K\mu = 0.1924$. The right-hand or lower curve is for complete projection condition. The left-hand curve is for complete ditch condition. The settlement ratio and the projection ratio are used as a product in this diagram. The line for $r_sp = 0$ is the border line between incomplete projection and incomplete ditch conditions, *i.e.*, where the load on the conduit is equal to the weight of the material directly over it.

Sloping Sided Trenches. It is generally assumed that the load on a pipe is not affected by the width of the trench above the top of the pipe; hence, for sloping sided trenches, B_t [Eq. (6)] is taken as the width of the trench at the level of the top of the pipe, the treatment being otherwise the same as for vertical sided trenches.

Wide Trenches. Schlick[5] found that as the width of the ditch increases, other conditions remaining constant, the load on the conduit increases in accordance with the trench-conduit law (pipe in trench, Fig. 21) until it equals that by the projecting-conduit load theory, and then remains constant for all greater widths. Hence, if the trench width (B_t) is more than about 1.5 times the outside conduit width (B_c), compute both for trench condition [Eq. (6) or (7)] and projection condition [Eq. (10)] and use the smaller value.

[1] *Ibid.*, p. 866.
[2] *Ibid.*, p. 868.
[3] WIGGIN, THOMAS H., M. L. ENGER, and W. J. SCHLICK, A Proposed New Method for Determining Barrel Thickness of Cast-iron Pipe, *Jour. A.W.W.A.*, May, 1939, p. 876.
[4] *Iowa State College Bull.* 112, p. 49.
[5] SCHLICK, W. J., *Iowa State College Bull.* 108, p. 6.

43. Loads Due to Surcharge and Concentrations. Pressures produced by loads on top of a fill are transmitted to any embedded structure. The effect of a concentrated load decreases in intensity with depth. Truck, airplane, and railway wheel loads are typical concentrations encountered in pipe design. Such loads are assumed to be distributed approximately in accordance with Boussenesq's equation for the distribution of stress within a semiinfinite elastic solid. Adapted to present purposes, *i.e.*, loads and pressures vertical and fill surface horizontal, this equation is

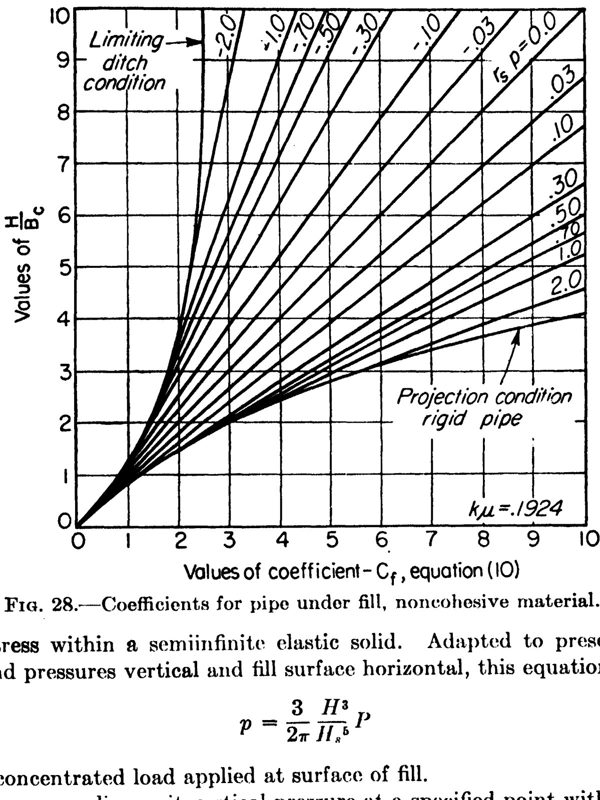


FIG. 28.—Coefficients for pipe under fill, noncohesive material.

$$p = \frac{3}{2\pi} \frac{H^3}{H_s^5} P \tag{14}$$

where P = concentrated load applied at surface of fill.

p = corresponding unit vertical pressure at a specified point within fill.

H = depth of such point below surface.

H_s = its slant distance from point of application of P.

The slant height, H_s, may be decomposed into its vertical and horizontal components and, if desired, the horizontal component may again be resolved into axial and transverse components, giving rise to the following variations of Eq. (14):

$$p = \frac{3}{2\pi} \frac{H^3}{(H^2 + r^2)^{5/2}} P \tag{15}$$

or

$$p = \frac{3}{2\pi} \frac{H^3}{(H^2 + x^2 + z^2)^{5/2}} P \tag{16}$$

where r is the horizontal component of H_s, x is the axial component, and z the transverse component, of r. Notwithstanding many dissimilarities with Boussenesq's conditions, experiment has shown these equations to be reasonably accurate for buried conduits.[1]

Table 5 will be found an aid in the solution of Eq. (16). A table for the solution

[1] SPANGLER, *op. cit.*, p. 869.

TABLE 5.—VALUES OF ϕ, EQ. (18), FOR A SINGLE CONCENTRATION

x or z	z or x	*H*								
		2	3	4	5	6	8	10	12	14
0	0	.1194	.0530	.0298	.0191	.0133	.0075	.0048	.0033	.0024
	1	.0683	.0408	.0256	.0173	.0124	.0072	.0047	.0033	.0024
	2	.0211	.0212	.0171	.0132	.0102	.0064	.0043	.0031	.0023
	3	.0063	.0094	.0098	.0089	.0076	.0054	.0038	.0028	.0022
	4	.0021	.0041	.0053	.0055	.0053	.0043	.0033	.0025	.0020
	5	.0008	.0019	.0028	.0034	.0035	.0033	.0027	.0022	.0018
	6	.0004	.0009	.0016	.0021	.0023	.0024	.0022	.0019	.0016
	7	.0002	.0005	.0009	.0013	.0015	.0018	.0018	.0016	.0014
	8	.0001	.0003	.0005	.0008	.0010	.0013	.0014	.0013	.0012
	9	.0001	.0002	.0003	.0005	.0007	.0010	.0011	.0011	.0010
	10	.0000	.0001	.0002	.0003	.0005	.0007	.0008	.0009	.0009
1	1	.0433	.0321	.0222	.0158	.0116	.0069	.0045	.0032	.0024
	2	.0157	.0176	.0151	.0121	.0096	.0062	.0042	.0030	.0023
	3	.0052	.0082	.0089	.0082	.0072	.0052	.0038	.0028	.0022
	4	.0019	.0037	.0049	.0052	.0050	.0041	.0032	.0025	.0020
	5	.0008	.0018	.0027	.0032	.0034	.0032	.0027	.0022	.0018
	6	.0004	.0009	.0015	.0020	.0023	.0024	.0022	.0019	.0016
	7	.0002	.0005	.0009	.0012	.0015	.0018	.0017	.0016	.0014
	8	.0001	.0003	.0005	.0008	.0010	.0013	.0014	.0013	.0012
	9	.0001	.0002	.0003	.0005	.0007	.0009	.0011	.0011	.0010
	10	.0000	.0001	.0002	.0003	.0005	.0007	.0008	.0009	.0009
2	2	.0077	.0108	.0109	.0095	.0080	.0056	.0039	.0029	.0022
	3	.0032	.0057	.0067	.0067	.0061	.0047	.0035	.0027	.0021
	4	.0014	.0028	.0039	.0044	.0044	.0038	.0030	.0024	.0019
	5	.0006	.0014	.0022	.0028	.0030	.0029	.0025	.0021	.0017
	6	.0003	.0008	.0013	.0018	.0020	.0022	.0021	.0018	.0015
	7	.0002	.0004	.0008	.0011	.0014	.0017	.0016	.0015	.0013
	8	.0001	.0002	.0005	.0007	.0009	.0012	.0013	.0013	.0012
	9	.0000	.0002	.0003	.0005	.0006	.0009	.0010	.0010	.0010
	10	.0000	.0001	.0002	.0003	.0004	.0007	.0008	.0009	.0008
3	3	.0017	.0034	.0045	.0049	.0048	.0040	.0032	.0025	.0020
	4	.0008	.0019	.0028	.0034	.0035	.0033	.0027	.0022	.0018
	5	.0004	.0011	.0017	.0022	.0025	.0026	.0023	.0020	.0016
	6	.0002	.0006	.0011	.0015	.0017	.0020	.0019	.0017	.0015
	7	.0001	.0004	.0006	.0010	.0012	.0015	.0015	.0014	.0013
	8	.0001	.0002	.0004	.0006	.0008	.0011	.0012	.0012	.0011
	9	.0000	.0001	.0003	.0004	.0006	.0008	.0010	.0010	.0010
	10	.0000	.0001	.0002	.0003	.0004	.0006	.0008	.0008	.0008
4	4	.0005	.0012	.0019	.0024	.0027	.0027	.0024	.0020	.0017
	5	.0003	.0007	.0012	.0017	.0020	.0022	.0020	.0018	.0015
	6	.0002	.0004	.0008	.0010	.0014	.0017	.0017	.0015	.0014
	7	.0001	.0003	.0005	.0008	.0010	.0013	.0014	.0013	.0012
	8	.0001	.0002	.0003	.0005	.0007	.0010	.0011	.0011	.0010
	9	.0000	.0001	.0002	.0004	.0005	.0007	.0009	.0009	.0009
	10	.0000	.0001	.0002	.0002	.0004	.0006	.0007	.0008	.0008
5	5	.0002	.0005	.0009	.0012	.0015	.0018	.0017	.0016	.0014
	6	.0001	.0003	.0006	.0009	.0011	.0014	.0015	.0014	.0012
	7	.0001	.0002	.0004	.0006	.0008	.0011	.0012	.0012	.0011
	8	.0000	.0001	.0003	.0004	.0006	.0008	.0010	.0010	.0010
	9	.0000	.0001	.0002	.0003	.0004	.0006	.0008	.0008	.0008
	10	.0000	.0001	.0001	.0002	.0003	.0005	.0006	.0007	.0007
6	6	.0001	.0002	.0004	.0006	.0009	.0011	.0012	.0012	.0011
	7	.0001	.0002	.0003	.0005	.0006	.0009	.0010	.0010	.0010
	8	.0000	.0001	.0002	.0003	.0005	.0007	.0008	.0009	.0009
	9	.0000	.0001	.0002	.0003	.0004	.0006	.0007	.0008	.0008
	10	.0000	.0000	.0001	.0002	.0003	.0004	.0006	.0006	.0006
7	7	.0000	.0001	.0002	.0004	.0005	.0007	.0009	.0009	.0009
	8	.0000	.0001	.0002	.0003	.0004	.0006	.0007	.0008	.0008
	9	.0000	.0001	.0001	.0002	.0003	.0005	.0006	.0007	.0007
	10	.0000	.0000	.0001	.0002	.0002	.0004	.0005	.0006	.0006
8	8	.0000	.0001	.0001	.0002	.0003	.0005	.0006	.0007	.0007
	9	.0000	.0000	.0001	.0002	.0002	.0004	.0005	.0006	.0006
	10	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0005	.0005
9	9	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0005	.0005
	10	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0004	.0005
10	10	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0004

of Eq. (14) will be found in the Proceedings of the American Society of Civil Engineers for May, 1933.[1]

The entries in Table 5 are values of ϕ in the equation

$$p = \phi P \tag{17}$$

where

$$\phi = \frac{3}{2\pi} \frac{H^3}{(H^2 + x^2 + z^2)^{5/2}} \tag{18}$$

As an example, find the unit pressure at a depth of 6 ft directly beneath a 10,000-lb wheel load. For $H = 6$ ft, $x = 0$, $z = 0$, $\phi = 0.0133$; hence the unit load is

$$10{,}000 \times 0.0133 = 133 \text{ psf.}$$

If the pressure at the same depth is desired at a point 4 ft axially and 3 ft transversely from the vertical through the load, the value of ϕ is 0.0035 and the unit pressure is $10{,}000 \times 0.0035 = 35$ psf.

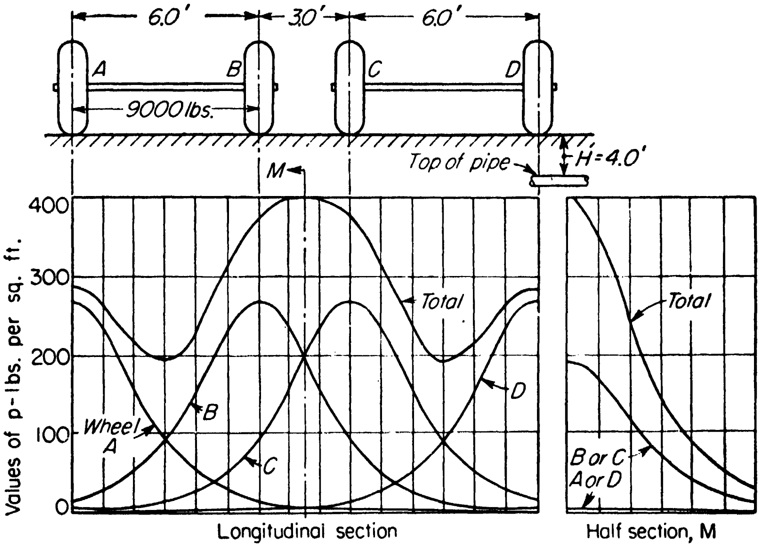


Fig. 29.—Combination of wheel loads.

Where two or more concentrations are involved, the effects are computed separately and added. Truck and trailer wheel combinations are common examples of multiple concentrations. They vary too widely to permit standardization except in special localities or for special purposes. As an example, the American Standards Association has proposed a tentative load pattern for cast-iron pipes in streets and highways.[2] The proposed loading consists of two passing trucks, each with a single rear axle, 6-ft tread, passing wheels 3-ft centers, each wheel carrying a load of 9,000 lb. The effects of the front wheels are neglected. The pressure distribution directly under the axles of such a load combination, at a depth of 4 ft below the surface, is shown in the longitudinal section of Fig. 29. Pressures from the individual wheels, as well as the total, are shown. The maximum pressure in this example is 400 psf, midway between the two center wheels.

[1] Progress Report, Committee on Earths and Earth Foundations, *Proc. A. S. C. E.*, May, 1933, p. 781.

[2] "Manual for the Computation of Strength and Thickness of Cast-iron Pipe," American Standards Association, December, 1939.

Forward and back from the line of the axles, the pressures become smaller. Along a transverse axis, halfway between the two center wheels, the pressure distribution is as shown in the half cross section.

The application of the American Standards Association loading on a 6-ft length of 5 ft diameter pipe, with 4 ft of cover, is illustrated in Fig. 30. The load plane is first divided into unit squares, as shown in the plan. Values of ϕ [Eq. (18)] for each wheel in each individual square are found by entering Table 5 with appropriate values of H, x, and z. Because of symmetry, only the loads in one corner of the area need be computed. The totals are found by multiplication, using the factors shown in plan (Fig. 30). If all wheel loads are identical, values of ϕ may be summated and then multiplied by the common wheel load. Computation on this basis, for the condition of Fig. 30, are shown in Table 6. If the concentrations vary, an appropriate alteration in procedure is required.

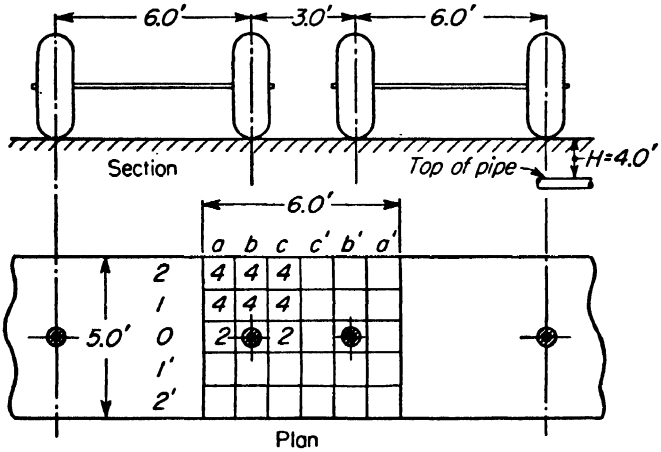


FIG. 30.—Example of multiple wheel loads.

The total load on the 5- by 6-ft area of Fig. 30 is 0.9554 times one wheel load, or 8,599 lb. The minimum load on a single square foot (square 2-a) is

$$0.0214 \times 9{,}000 = 193 \text{ lb},$$

and the maximum (square 0-c) is $0.0441 \times 9{,}000 = 397$ lb. The load per foot of pipe varies from 1,230 to 1,585 lb. The average is 1,433 lb. For other sized areas and other depth of cover the departure from average may be greater or smaller than here indicated. Whether the use of averages is permissible depends on the size and type of pipe and the installation conditions.

The pipe in Fig. 30 is assumed to run at right angles to the direction of travel. Figure 31 illustrates in plan only the procedure if the pipe runs parallel to the roadway. If the wheel loads are again placed in the centers of the squares to permit the use of Table 5 without interpolation, partial squares occur around the perimeter. Unit pressures for these partial squares may be taken as approximately equal to values for the corresponding whole squares. The multipliers indicated in Fig. 31 allow for the reduced areas. The quarter-square, 3-a, for example, occurs four times; hence, its multiplier is unity.

Distributed surcharges of considerable extent are usually assumed to continue downward indefinitely with undiminishing force and may be treated as additional fill of equivalent weight. In case of a long transverse load or a heavy concentration

on a moderately sized area, the loaded area may be subdivided into unit squares and the load on each square treated as a separate concentration, or, if desired, an algebraic solution may be worked out for particular cases.

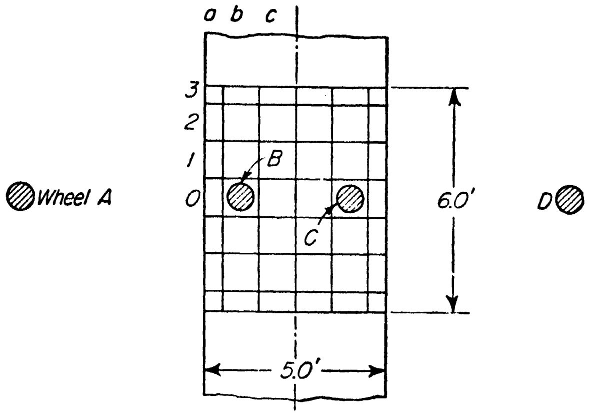


FIG. 31.—Pipe parallel to roadway.

44. Horizontal Backfill Pressures. Backfill pressures and reactions are vertical only at the top and bottom of the pipe; hence, it is necessary to consider horizontal components at other points. For rigid pipes under embankments, the side pressure, which is the active earth pressure, may be computed from the equation

$$p_h = Kp_v \tag{19}$$

TABLE 6.—LOAD FACTORS (ϕ) FOR AMERICAN STANDARDS ASSOCIATION TRUCK LOADS, H = 4 FT

Square	Wheel A		Wheel B		Wheel C		Wheel D		Total
	$x+z$	ϕ	$x+z$	ϕ	$x+z$	ϕ	$x+w$	ϕ	ϕ
2-a	2–5	.0022	1–2	.0151	2–4	.0039	2–10	.0002	.0214
2-b	2–6	.0013	0–2	.0171	2–3	.0067	2– 9	.0003	.0254
2-c	2–7	.0008	1–2	.0151	2–2	.0109	2– 8	.0005	.0273
1-a	1–5	.0027	1–1	.0222	1–4	.0049	1–10	.0002	.0300
1-b	1–6	.0015	0–1	.0256	1–3	.0089	1– 9	.0003	.0363
1-c	1–7	.0009	1–1	.0222	1–2	.0151	1– 8	.0005	.0387
Subtotal	...	.....	...	.....	...	.....	....	.....	.1791
Times 4	...	.....	...	.....	.....	.....	....	.....	.7164
0-a	0–5	.0028	0–1	.0256	0–4	.0053	0–10	.0002	.0339
0-b	0–6	.0016	0–0	.0298	0–3	.0098	0– 9	.0003	.0415
0-c	0–7	.0009	0–1	.0256	0–2	.0171	0– 8	.0005	.0441
Subtotal	...	.....	...	.....	...	.....	....	.....	.1195
Times 2	...	.....	...	.....	...	.....	....	.....	.2390
Total	...	.....	...	.....	...	.....	....	.....	.9554

Total load = .9554 × 9,000 = 8,599 lb on 6-ft length.
Average = 8,599 ÷ 6 = 1,433 lb per ft of pipe.

where p_h is the unit horizontal pressure, p_v is the unit vertical pressure corresponding to the total vertical load, including both fill and surcharge components, and K is the Rankin coefficient, Eq. (9). Ordinarily, K varies from 0.2 to 0.3 for the conditions of Fig. 22, or for wide trenches. In narrow trenches, unless the sides of the pipe are backfilled with special care, K may be zero.

Flexible conduits, such as thin steel pipes, brick conduits, and badly cracked concrete or clay pipes, suffer a shortening of the vertical diameter and a lengthening of the horizontal diameter of such magnitude as to develop *passive* resistance of the side materials, the maximum value of which equals the vertical pressure divided by K. Only the portion of this maximum required to establish equilibrium is actually developed. In poorly backfilled trenches this supporting pressure may be small or nonexistent.

45. Bedding and Load Distribution. Both the load on the pipe and the resistance of the pipe to rupture are influenced by the type of bedding and the care used in backfilling. As noted in Art. 42, if the pipe settles or yields, the vertical load on it is reduced. This is not usually considered an advantage. The yielding subgrade offers little lateral support and failure may occur for shallower fills than for a well-bedded pipe with little or no settlement. It is not sufficient, however, that the subgrade merely be unyielding. A pipe laid on a flat rock subgrade and covered with a loose fill may receive a heavy load and no side support. A rock trench should be overexcavated to allow a 6- to 12-in. bed of selected materials under the pipe.

In firm materials other than rock, the subgrade should be rounded to fit at least the lower quadrant of the pipe, and the backfill should be compacted at least up to the mid-depth, preferably higher. For trench pipe of appreciable flexural strength, or for thin pipe where the deflection required to establish adequate side support is permissible, compaction by puddling is adequate, particularly for granular materials. For deep trenches or friable pipe, the possibility of serious cracking may be reduced by carefully controlled hand or mechanical tamping. Careless tamping is not dependable.

In case of soft ground or rising water in the bottom of a trench, it may be advisable to bed the pipe on a 6- to 12-in. or thicker layer of crushed stone. Under highways, railways, or deep cuts, concrete bedding may be required, covering usually from 100 to 120 deg of the invert arc. In extreme cases, particularly for heavy concentrations and shallow cover, the concrete backfill may extend to or above the top of the pipe.

The fill alongside a pipe under an embankment should be thoroughly compacted, preferably to the level of the top of the pipe. Such compaction, if well done, reduces the load on the pipe and increases the load-carrying capacity.

For pipes and conduits of ordinary width, it is usual to assume vertical loads due to fills, concentrations, and other superloads, to be uniformly distributed across the pipe; and the vertical reaction over the full width of careful bedding. For wide conduits under shallow fills and other special conditions, it may be desirable to use the actual transverse distribution of concentrated loads. Averaging longitudinally, if the load is variable in that direction, depends on the stress distributing characteristics of the pipe.

Cast-iron, reinforced-concrete, and other substantial thick-walled conduits should be able to distribute type of loading shown over a 6-ft or greater length, but not more than the length of a joint of pipe. For less substantial construction, it may be preferable to design for the maximum per foot of pipe load.

Active side pressures on rigid projecting pipe, computed by Eq. (19), are trapezoidal. Unless the pipe is large or side stresses are unusually important, it will be adequate to assume them uniform and equal to the computed value at mid-depth of the pipe. Passive resistance to the horizontal deformation of flexible pipe may be

assumed parabolic, maximum at the mid-height of the pipe, and zero at points of contraflexure.

46. Support and Anchor Loads. Pipes supported above the ground on cradles or piers are subject to special stresses at the supports and to beam stresses between supports. A pipe barrel constitutes an efficient beam, and beam stresses need be considered only for relatively long spans.

Where a pipe is dead-ended, it is subject to an axial force due to the full internal pressure on the closed end of the pipe. An axial stress also occurs at bends. These forces may be resisted by tension in the pipe barrel or by anchors. In Fig. 32, let 1-2-3 represent the center line of a pipe with a bend at 2, and let 4-2 and 5-2 represent the product of water pressure and pipe area in the two ends. If the pipe is jointed and anchored, the force on the anchor is represented by 6-2. The bend may be horizontal, vertical, or inclined. If the pipe is dead-ended at 2, the force to be resisted by tension or by anchorage is 4-2.

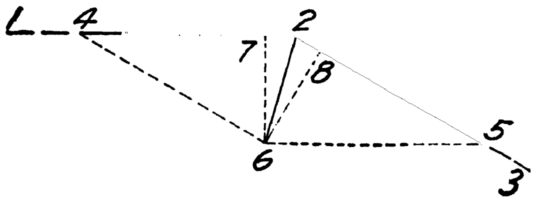


Fig. 32.—Forces acting on anchor.

In case of steel pipe without expansion joints, or continuously reinforced-concrete pipe, stresses due to temperature change may exceed those due to internal pressure and should be considered in the design of anchorages.

CONCRETE PIPE

47. Concrete Pipe. Concrete pipe may be precast or cast in place. Precast pipe is usually manufactured in short lengths at a central plant, transported to the job, laid in a trench, backfilled, and caulked at the joints. The sections may be cast in forms standing on end or spun centrifugally in horizontal outside molds. Cast-in-place pipe is constructed in its final position in the trench. Steel reinforcement provides against bursting pressures and external loads. Precast pipe for use in sewers and other nonpressure or very low pressure lines may be made without reinforcement.

Precast concrete pipe has been successfully made in diameters up to 12 ft 8 in.[1] Heads up to 500 ft are feasible, but except for small sizes, the practicable limit is perhaps between 300 and 400 ft. For heads above 80 to 100 ft, it is usual to insert a continuous welded steel cylinder in the pipe wall to ensure watertightness.

Cast-in-place pipe has been made up to 20 ft in diameter. Heads are usually limited to 100 to 150 ft because of the difficulty of securing dense concrete under adverse placing conditions. A steel cylinder insert may be used for watertightness.

48. Stresses in Concrete Pipes. The stress in the reinforcement due to the internal bursting pressure may be found from Eq. (5), a being taken as the area of reinforcing steel per foot of shell. This stress must be combined with the steel stresses resulting from external loads, the weight of the pipe shell, and the weight of the water in the pipe. The external loads are determined in accordance with Arts. 42, 43, and 44. The load on the top is usually assumed to be uniformly distributed. For a cast-in-place pipe, poured full width on a prepared foundation, the bottom reaction is also taken as uniform. For precast pipe, carefully bedded and compactly back-

[1] Whitman, N. D., Large Concrete Pressure Pipes, *Civil Eng.*, September, 1935, pp. 553*ff*.

filled, the reaction may be assumed uniformly distributed over the width of bedding. The assumption of a uniform distribution for side pressures is not likely to lead to appreciable error. Stress coefficients for rigid pipes, bedded over an 180-deg arc, also for 90-deg arc bedding, are shown in Table 7. Details of design must conform to the rules for combined direct stress and bending.[1] For small pipes, high heads, and liberal safety factors, bending stresses may be ignored.

TABLE 7.—THRUST AND MOMENT COEFFICIENTS FOR 1 LIN FT OF HORIZONTAL CONCRETE PIPE

Case	Loading	Direct stress, lb			Bending moment, ft-lb		
		Top	Side	Bottom	Top	Side	Bottom
1	Weight of pipe shell, 180-deg support	$-0.027Ws$	$+0.250Ws$	$+0.027Ws$	$+0.028Wsd$	$-0.031Wsd$	$+0.035Wsd$
	Weight of pipe shell, 90-deg support	$-0.053Ws$	$+0.250Ws$	$+0.053Ws$	$+0.033Wsd$	$-0.039Wsd$	$+0.051Wsd$
2	Weight of water in pipe, 180-deg support	$-0.186Ww$	$-0.068Ww$	$-0.451Ww$	$+0.028Wwd$	$-0.031Wwd$	$+0.035Wwd$
	Weight of water in pipe, 90-deg support	$-0.212Ww$	$-0.068Ww$	$-0.424Ww$	$+0.033Wwd$	$-0.039Wwd$	$+0.051Wwd$
3	Vertical external load, 180-deg support	0	$+0.500We$	0	$+0.063Wed$	$-0.063Wed$	$+0.063Wed$
	Vertical external load, 90-deg support	$-0.027We$	$+0.500We$	$+0.027We$	$+0.0685Wed$	$-0.0700Wed$	$+0.0783Wed$
4	Uniform horizontal external load	$+0.500H$	0	$+0.500H$	$-0.063Hd$	$+0.063Hd$	$-0.063Hd$
5	Triangular horizontal external load	$+0.313Ht$	0	$+0.687Ht$	$-0.052Htd$	$+0.063Htd$	$-0.073Htd$
6	Uniform internal pressure	$-0.500Pd$	$-0.500Pd$	$-0.500Pd$	0	0	0

For thrust (direct stress), + sign denotes compression.
For moment, + sign indicates tension on inside of pipe shell.
Reaction assumed vertical distributed uniformly.
d = average diameter of pipe, ft.
Ws = weight of pipe shell, lb/lin ft of pipe.
Ww = weight of water in 1 lin ft of pipe.
We = total vertical external load on pipe, lb/lin ft of pipe.
H = horizontal uniform external load on pipe, lb/lin ft of pipe.
Ht = horizontal triangular external load on pipe, lb/lin ft of pipe.
p = uniform hydrostatic internal pressure due to head to top of pipe, psf.

49. Allowable Unit Stresses. The reinforcing steel stretches under load, minute cracks being caused in the concrete which reduce watertightness. Consequently, it is customary to use low steel stresses with a decreasing scale as the pressure increases. From general experience, the following rule is suggested:

$$f' = 15{,}000 - 50H \tag{20}$$

[1] GUMENSKY D. B., Reinforced Concrete Members under Direct Tension and Bending, *Trans. A. S. C. E.*, **102**, 372, 1937.

where f' is the allowable stress in pounds per square inch, with a minimum limit of 8,000, and H is the internal pressure head in feet. If a steel cylinder is provided to ensure watertightness, the reinforcement stress need not be reduced for head. The steel in the cylinder may be considered as a part of the reinforcement against bursting pressure.

Allowable compressive stress in the concrete may follow prevailing rules for other reinforced structures. Concrete is usually designed for a 28-day strength of 4,000 psi for precast siphons and 3,000 psi for cast-in-place siphons.

50. Form of Reinforcement. The reinforcement may take the form of continuous wire or hot-rolled bars wound spirally or of standard bars welded or lapped into individual hoops. Welded hoops are usually tested individually to about 85 per cent of the elastic limit. For precast pipe, the circumferential and longitudinal steel is assembled into a cage which may be lifted and set onto the form. The longitudinal steel is nominal in amount. For cast-in-place pipe, the reinforcement is usually assembled in place in the trench and wired or welded into a continuous cage, although separate cage units may be shop fabricated as for precast pipe. If the cast-in-place pipe is *continuous* and exposed to appreciable temperature change, it should be reinforced against temperature cracks.[1]

For comparatively light external loads, the circumferential reinforcement may consist of a single circular cage in the center of the shell. For light internal pressures and moderate external loads, a single elliptical cage with the principal axis horizontal is economical. For heavier loadings, both inside and outside cages may be required; or inside, outside, and elliptical cages may be combined. When elliptical reinforcement is used, the top of the pipe must be marked as it is made.

51. Shell Thickness. The concrete shell must be of sufficient thickness to resist bending stresses. When such stresses are small, the thickness is made to equal to some arbitrary standard. For cast-in-place pipe, it is usual to increase the thickness as the head increases. A suggested rule is to make the thickness one-twelfth the inside diameter for heads up to 40 ft, with a 6 in. minimum, then add 1 in. for heads between 40 and 80 ft, 2 in. between 80 and 100 ft, 3 in. between 100 and 120 ft, 4 in. between 120 and 140 ft, and 5 in. above 140 ft.

The shell thickness of precast pipe is usually made one-twelfth the inside diameter for large pipes and one-twelfth the diameter plus 1 in. for pipes under 6 ft for all heads. It is not convenient, because of form expense, to vary the shell thickness for precast pipe. Added strength where required is secured by bedding the pipe in a lean concrete backfill under the bottom, supporting a 90- to 120-deg sector or even extending the concrete to the top of the pipe.

52. Centrifugal Concrete Pipe. Centrifugal concrete pipe is made by spinning horizontal external forms into which are placed the reinforcement, fixed in position, and the concrete mixture. The centrifugal force presses the concrete against the outer forms, a pipe of notable density, uniformity, and watertightness being produced. Such pipe has been manufactured in sizes from 8 to 84 in.

53. Prestressed Concrete Pipe. The necessity for reducing the allowable steel stress in accordance with Eq. (20) may be avoided by prestressing the reinforcement. Prestressing must be accomplished after the concrete, or at least part of it, has set. In large pipes or tanks this has been accomplished by threading hoops through grooves or holes left for them and then tightening with turnbuckles.

For commercial sizes of pipes, prestressing may be accomplished by winding rods or wires around previously prepared shells or cores. The shells may be cast in forms or spun, and may be plain concrete or contain a steel cylinder core. After curing, the

[1] See Art. 22, Continuously Reinforced Linings.

shell is revolved in a machine and the reinforcement is wound on helically under a predetermined tension which places the concrete under an initial compression.

The core, thus reinforced, is finally given an outside coating (¾ in. thick, more or less) of machine-applied mortar or gunite which completes the pipe.

For pipe without a steel cylinder, the design equations corresponding to a zero stress in the concrete shell and full working stress in the reinforcement under full internal loading are as follows:

$$a_s = \frac{F}{f_s} \tag{21}$$

$$f_{sp} = f_{sw}\left(\frac{a_c}{a_c + na_s}\right) \tag{22}$$

$$f_{cp} = \frac{a_{s_f}}{a_c}\, sp \tag{23}$$

where F is from Eq. (4), a_c is the area of concrete, a_s is the area of helical steel, f_{cp} is the initial unit compression in the concrete shell, f_{sw} is the allowable working stress in the steel, f_{sp} is the initial tension in the steel, and n is the modulus of elasticity ratio for steel and concrete. Values of F, a_c, and a_s are for a unit length of pipe.

Consider a 40-in. pipe, 4-in. shell, 240 psi internal water pressure, reinforced with prestressed continuous hot-rolled rods having an allowable working strength of 18,000 psi. The value of n will be assumed to be 12.[1]

From Eq. (4),

$$F = 240 \times 20 \times 12 = 57{,}600 \text{ lb/lin ft of pipe}$$

From Eq. (21),

$$a_s = \frac{57{,}600}{18{,}000} = 3.20 \text{ sq in./lin ft of pipe}$$

From Eq. (22), prestress in the steel is

$$f_{sp} = \frac{48}{48 + 12 \times 3.2} \times 18{,}000 = 10{,}000 \text{ psi}$$

From Eq. (23), the initial stress in the concrete is

$$f_{cp} = \frac{3.2}{48}\, 10{,}000 = 667 \text{ psi}$$

Prestressed to this extent, the concrete will just reach zero compression at full load. For a greater thickness of shell, the required initial steel stress is greater, and the initial concrete stress is less.

The foregoing figures are based on the use of structural grade steel. The use of high-strength wire will decrease the required steel area and increase initial and final stresses.

If it is desired to leave a residual compression in the shell, the amount of steel is found by adding $f_{cr}a_c$ to F in Eq. (21), and the required prestress by adding nf_{cr} to f_{sw} in Eq. (22), f_{cr} being the desired residual compression in the concrete.

Any prestress at all will reduce the porosity of the concrete, but there is no advantage in going below that given by Eq. (22).

Prestressing in noncylinder pipe is sometimes applied to the longitudinal steel. The longitudinal rods are held stretched between the end rings while the shell is being poured or spun until the concrete has set. When released, they compress the shell longitudinally and through Poisson's ratio expand it radially, thus partially prestressing the circumferential steel.

[1] Refer to any text on concrete design for values of n for the concrete to be used in specific cases.

Prestressed pipes also are on the market in which the inner concrete shell is poured or spun into a water-tight steel cylinder.[1] In such pipe the necessity for limiting the expansion of the concrete is absent. To do so would also limit the stress in the steel cylinder, thus reducing its usefulness as reinforcement. The object in prestressing in such case is to permit the efficient combination of low-strength structural grade cylinder steel with high-strength helical wire.

The design equations required to cause the steel in the cylinder and in the spirals to reach their separate working values simultaneously are as follows:

$$f_{yw}a_y + f_{sw}a_s = F \tag{24}$$

$$f_{sp} = \frac{a_c + na_y}{a_c + na_y + na_s}(f_{sw} - f_{yw}) \tag{25}$$

$$f_{yp} = \frac{na_s}{a_c + na_y} f_{sp} \tag{26}$$

$$nf_{cp} = f_{yp} \tag{27}$$

where a_y is the area of steel in the cylinder, f_{yw} is the allowable stress in the cylinder steel, f_{yp} is the initial unit compression in the cylinder steel, other symbols being as used in Eqs. (21) to (23).

The amount of steel in the cylinder can be varied only by selection of thickness of available standard plates, whereas the helical steel can be varied at will. Hence it is usual to start with an assumed plate thickness, checking later with other thicknesses if the first results are unsatisfactory.

Consider a 40-in. pipe, 2½-in.-thick core, within a $\frac{1}{16}$-in.-thick steel shell, with a safe working strength of 15,000 psi; spirally reinforced with prestressed spring steel wire, with a safe working strength of 50,000 psi; internal water pressure of 240 psi assumed to be applied at the inner surface of the steel cylinder, n taken as 12.

From the stated conditions,

$$a_c = 30 \text{ sq in.,} \qquad a_y = 0.75 \text{ sq in.}$$

From Eq. (4),

$$F = 240 \times 22.5 \times 12 = 64{,}800 \text{ lb}$$

From Eq. (24),

$$15{,}000 \times 0.75 + 50{,}000a_s = 64{,}800 \text{ lb}$$
$$a_s = 1.071 \text{ sq in.}$$

From Eq. (25),

$$f_{sp} = \frac{30 + 12 \times 0.75}{30 + 12 \times 0.75 + 12 \times 1.071}(50{,}000 - 15{,}000) = 26{,}700 \text{ psi}$$

From Eq. (26),

$$f_{yp} = \frac{12 \times 1.071}{30 + 12 \times 0.75} \times 26{,}700 = 8{,}800 \text{ psi}$$

From Eq. (27),

$$f_{cp} = \frac{8{,}800}{12} = 733 \text{ psi}$$

The theory of prestressing has been carefully checked by experiment on new pipes.[2] There appears to be no danger that the prestress eventually may be partially relieved by concrete flow, gradual yielding of the stressed steel, indentation of concrete or cylinder surfaces under the spiral wires, or other causes.

[1] LONGLEY, F. F., Prestressed Reinforced Concrete Pipe, *Water Works & Sewerage*, December, 1945, p. 367.

[2] ROSS, C. W., Tests of Prestressed Concrete Pipe Containing a Steel Cylinder, *Jour. Am. Concrete Inst.*, September, 1945, p. 37.

Prestressing of the kind described herein is not practical for heavy external loads requiring thick walls and double cage reinforcing.

54. Joints for Concrete Pipes. Precast pipe lengths vary from 4 to 16 ft, depending on pipe size and facilities for constructing and handling. Lengths up to 24 ft are being

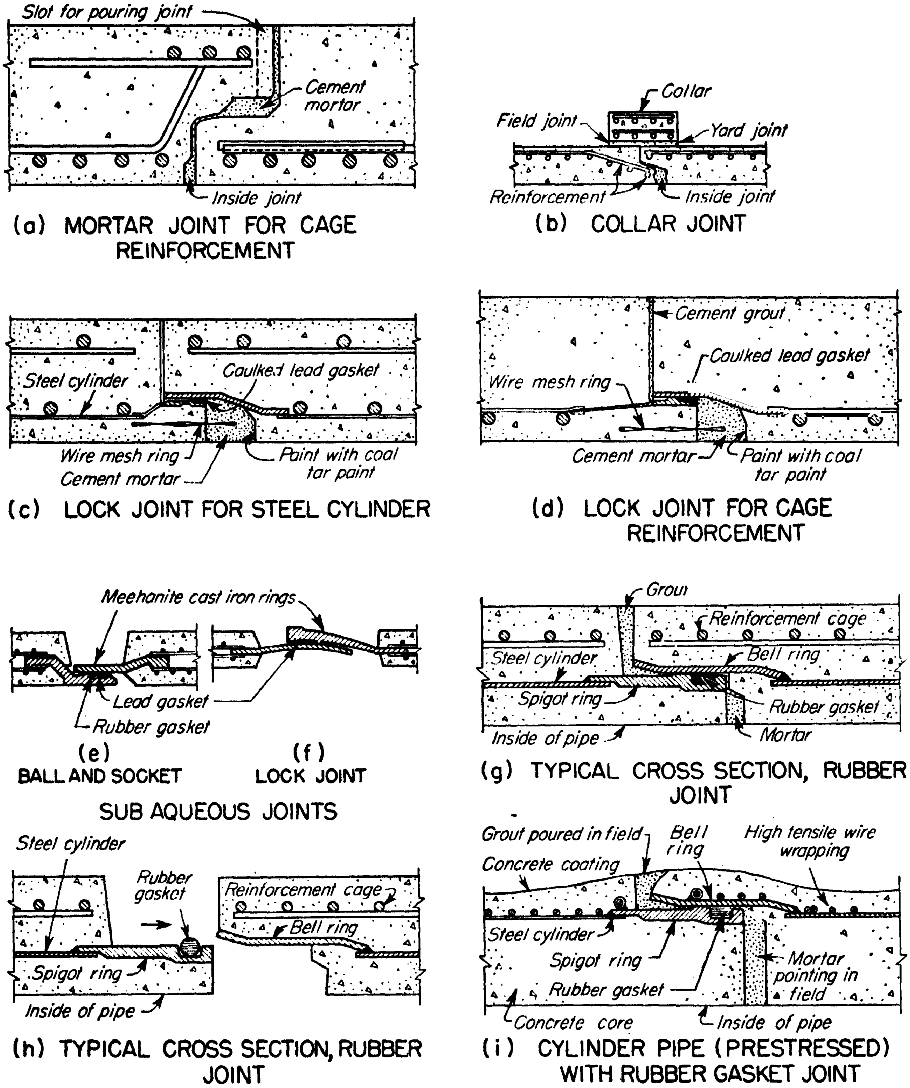


Fig. 33.—Joints in concrete pipe.

considered. For special work such as submarine lines, much longer lengths may be used. Joints generally take some kind of bell-and-spigot form and must be watertight even under slight movement caused by shrinkage or settlement. They should be self-centering to avoid offsets in the finished line. A few of the many successful joints are illustrated in Fig. 33.

A simple mortar caulked bell-and-spigot joint (not illustrated) may be used for

nominal heads in small drainage or irrigation pipe. The mortar joint shown at (*a*) in Fig. 33 was successfully used on the Colorado River aqueduct for nominal head in pipes up to 12 ft 8 in. in diameter. The cement mortar is poured after the completion of backfill through a port left at the top of the pipe. Generally this joint should be limited to low heads where slight leakage is not objectionable, as it can be made completely water-tight only with great care.

Joint (*b*) shown is widely used with spun pipe. If thoroughly caulked with dry cement mortar, it is satisfactory for relatively high heads. All the mortar joints are lacking in flexibility.

The lead-caulked joints shown at (*c*) and (*d*) have a long history of successful use. They require precision in manufacture but are relatively simple to install. They are enduring and remain water-tight under moderate longitudinal or flexural movement. The metal bell-and-spigot units are specially rolled shapes welded into rings and stretched beyond the elastic limit to precise diameters. Caulking and pointing are

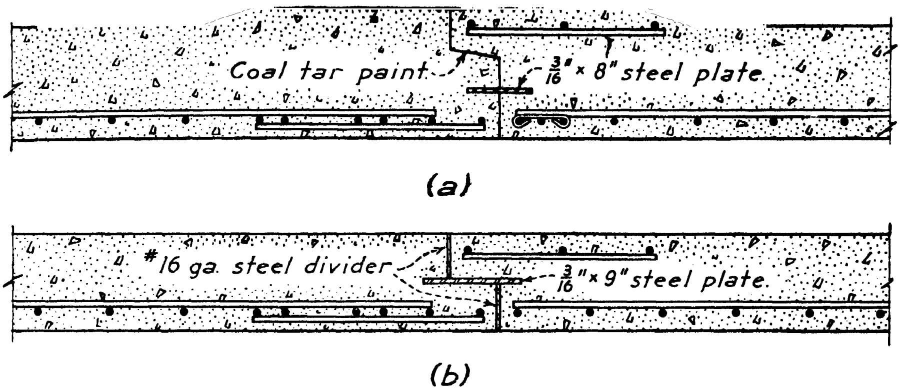


FIG. 34. Low-pressure joints in concrete pipe.

done after backfilling. The lead-caulked joints shown at (*e*) and (*f*) are two of many possible types for subaqueous work.

The rubber joint shown assembled at (*g*) and disassembled at (*h*) is rapidly replacing other types. Extreme ease of erection, flexibility, and watertightness are in its favor. Indications are that with complete protection from air and light the rubber in these joints will last indefinitely.

The joint illustrated at (*g*) and (*h*) is for the combination of a steel cylinder and cage reinforcement. It also can be adapted to cage reinforcement alone. An adaptation to prestressed cylinder pipe is shown at (*i*).

As in the case of lead joints, steel rings for rubber joints must be precisely manufactured and installed. The rubber band, slightly stretched over the spigot end, is compressed when the spigot is forced into the bell and completely fills the groove.

Cast-in-place pipe may be made continuous by running the longitudinal reinforcement through transverse construction joints. The reinforcement should be adequate in amount (approximately 0.5 of 1 per cent high-elastic-limit steel) and the joints should be keyed or provided with water stops similar to that shown at (*d*) in Fig. 7. If flexibility is desired to provide for possible settlement or earthquake movement, cast-in-place pipes may be jointed. By special adaptation, either the lead or the rubber joint of Fig. 33 may be used. Joints (*a*) and (*b*), Fig. 34, were used on the Colorado River aqueduct across seismically active alluvium areas. These joints were placed 20 ft apart. Type (*a*) required that the pipe be built in alternate sections. Type (*b*) was designed for continuous construction.

CAST-IRON PIPE

55. General. Cast iron is widely used for city water mains. Its first cost is greater than for pipes of other materials, but it has a long life and requires a minimum of maintenance and repair work.

Cast-iron water pipes have given satisfactory service for as long as 250 years.[1] Examples of installations made 50 to 60 years ago and still in satisfactory service are abundant. It cannot be concluded, however, that all cast-iron pipe installations are long-lived. Such pipes installed in highly corrosive soils or subjected to heavy electrolysis may fail in a few years.

The manufacture of cast-iron pipe is largely standardized. Sizes, weights, strengths, and other data for pipes for all usual purposes will be found in the manufacturers' catalogues. Standard specifications for pipe to meet both usual and unusual requirements may be acquired from the American Water Works Association, the New England Water Works Association, the American Society for Testing Materials, and other organizations and governmental agencies.

56. Classification, Lengths, and Weights. There are four types of cast-iron pipe according to method of manufacture, as follows:

Pit-cast pipe, manufactured by pouring molten metal into vertical sand molds.

Centrifugal pipe, spun in horizontal sand molds.

Centrifugal pipe, spun in horizontal metal molds.

Horizontally cast pipe, cast in horizontal green sand molds.

A discussion of these types and of their dimensions, weights, and strengths will be found in Sec. 20 of this book; also descriptions of joints and fittings.

57. Stresses in Cast-iron Pipe. The purchasers of cast-iron pipe for installation under ordinary or normal conditions usually rely on tables of safe bursting strength as published by pipe manufacturers, water works associations, or governmental agencies. Computations by the customer are required only for special conditions or for checking.

In simple cases, the thickness may be computed on the basis of bursting strength, with arbitrary allowances for foundry inaccuracies, corrosion, handling stresses, and ordinary backfill loads. Many formulas providing for these allowances have been proposed and used. A typical late example is the modified Fairchild formula:

$$t = \frac{d(p + p')}{2f_t} + \frac{0.28}{d^{0.15}} \tag{28}$$

where t = thickness, in.
d = internal diameter, in.
p = internal static pressures, psi.
p' = water hammer, psi.
f_t = allowable unit tension, psi.

For heavy external loads or any critical condition, a more adequate approach may be required. Such an approach is described in detail in "Manual for the Computation of Strength and Thickness of Cast-iron Pipe," approved and issued by the American Standards Association in December, 1939.

Because of the special elastic properties of cast iron, it is not possible to combine direct and flexural stresses in the simple manner used for other materials. It has been found experimentally that the internal bursting load and concentrated external load which combined will cause fracture may be computed by the following empirical formula:

[1] BABBITT and DOLAND, "Water Supply Engineering," McGraw-Hill Book Company, Inc., 1929, p. 367.

$$W_u' = \frac{W_u}{\sqrt{p_u}} \sqrt{p_u - p_u'} \tag{29}$$

where W_u = external load, in lb per ft of pipe, applied as illustrated in Fig. 35, which will just cause failure when acting alone.

p_u = internal pressure, in psi, which will just cause failure when acting alone.

p_u' and W_u' = any combination of internal pressures in psi and concentrated external load in lb per ft of pipe, applied as in Fig. 35, which will just cause failure when acting together.

The value of p_u in Eq. (29) is

$$p_u = \frac{2tf_u}{d} \tag{30}$$

where t = thickness of pipe, in.

f_u = ultimate tensile strength of pipe material in psi., as determined by full-length bursting tests.

d = internal diameter of pipe, in.

The value of W_u is computed from the equation

$$W_u = \frac{t^2R}{0.0795(d + t)} \tag{31}$$

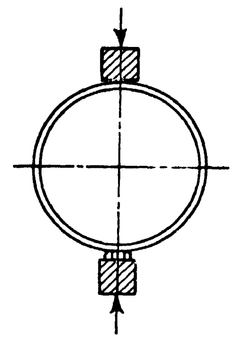


FIG. 35.—Standard test bearings, cast-iron pipe.

where R is the modulus of rupture (extreme fiber stress) in pounds per square inch, as determined by ring tests of the pipe material, loaded as illustrated in Fig. 35, the data being reduced in accordance with flexural theory for a thin ring of homogeneous materials.[1] Although the loading in Fig. 35 is usually referred to as three-edge bearing, it is treated as two-edge loading. The loads W_u' and p_u' are ultimate and must exceed actual loads by a safe margin.

Adjustment must be made for the difference in effectiveness between the concentrated load of Fig. 35 and actual trench loads. The relations of p_u' and W_u' to allowable trench loadings may be expressed thus:

$$p \lesseqgtr \frac{p_u'}{k_i} \tag{32}$$

$$W \lesseqgtr J\frac{W_u'}{k_i} \tag{33}$$

where p and W are permissible simultaneous values of internal pressure in pounds per square inch and external *trench loading* in pounds per foot of pipe, k_i is a factor of safety, and J is a trench load factor.

Values of J depend on the method of bedding. The American Standards Association[2] recognizes six bedding conditions for cast-iron pipe in trenches, as follows:

Field Condition	Explanation
A	Flat bottom trench, backfilling not tamped
B	Flat bottom trench, backfilling tamped
C	Pipe supported on blocks, backfilling not tamped
D	Pipe supported on blocks, backfilling tamped
E	Bottom of trench shaped to fit bottom of pipe for about 90 deg (unevennesses filled in by sand as required), backfilling not tamped
F	Same as *E* except that backfilling is tamped

[1] TIMOSHENKO, S. "Strength of Materials," D. Van Nostrand Company, Inc., 1930, p. 436.

[2] "Manual for the Computation of Strength and Thickness of Cast-iron Pipe," American Standards Association, December, 1939, part 1, p. 2.

Values of J derived from experiments at Iowa State College are shown in Table 8. These values are taken from Fig. 3, part 1, page 6, of the American Standards Association manual referred to. A value of 2.5 for k_i is recommended as a minimum.

The American Standards Association[1] recommends 11,000 psi ultimate tension and 31,000 psi modulus of rupture for pit-cast pipe. Corresponding recommendations for pipes produced by modern methods are 18,000 and 40,000 psi. Manufacturers recommend higher values for centrifugally spun pipe. Truck load or water hammer is allowed for where required, but it is not usual to allow for both simultaneously.

Wall thicknesses computed by the foregoing formulas are net. Specifications necessarily permit foundry tolerances, and to ensure that the minimum actual thickness shall not be less than the computed value, these tolerances must be added to values of t computed by formula. It is also usual to make an allowance for corrosion to ensure the maintenance of the required thickness for a long period of years.

Suggested additions for pit-cast pipe are 0.08 in. for corrosion, and for foundry tolerance 0.07 in. up to 8 in. in diameter, 0.08 from 10 to 24 in. in diameter, and 0.10 in. above 30 in. in diameter.

For low pressures and light external loading it is necessary, for practical reasons, to maintain certain minimum thicknesses. Above 12 in. in diameter this is accomplished by assuming a minimum internal pressure of 50 psi and an external cover of 5 ft. Suggested minimums for 12 in. and smaller pit-cast pipe are as follows:

Diameter, in.	Min Thickness, in.
3	0.37
4	0.40
6	0.43
8	0.46
10	0.50
12	0.54

TABLE 8.—VALUES OF c_i RATIOS TO THREE-EDGE BEARING

Field condition	Pipe diameter, in.															
	4	6	8	10	12	14	16	18	20	24	30	36	42	48	54	60
A	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15
B	1.29	1.32	1.34	1.36	1.38	1.41	1.43	1.45	1.47	1.52	1.58	1.64	1.69	1.72	1.75	1.77
C	0.22	0.31	0.40	0.50	0.60	0.67	0.73	0.78	0.81	0.87	0.93	0.96	0.98	0.99	0.99	1.00
D	0.82	0.83	0.84	0.86	0.88	0.91	0.95	0.98	1.01	1.07	1.14	1.19	1.23	1.25	1.28	1.31
E	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
F	1.75	1.78	1.80	1.83	1.85	1.88	1.90	1.93	1.95	2.00	2.08	2.14	2.20	2.25	2.29	2.31

C and *D* ratios are for an average block spacing of 6 ft.

STEEL PIPE

58. General. Steel is one of the most efficient pipe materials and is widely used for pipes varying from the smallest size up to 20 ft or more in diameter. In the smaller sizes, steel pipe may be machine made. Examples are ordinary water-service pipes and special pipes running up to 12 in. or more in diameter. Lengths of small pipe are joined in the field by threaded sleeves, welding, bolted flanges, or a variety of patented joints. Sections of larger pipe are joined by riveting, welding, or bolted flanges. For large sizes, the sections are made up from mill-sized sheets, rolled and

[1] *Ibid.*, part 2, p. 1.

trimmed to dimension, and fastened together by riveting, welding, or other means. Where shipping conditions permit, steel pipe is usually assembled in sections of convenient lengths in the shop. Large diameters are assembled in the shop to ensure correct dimensions and shipped knocked down.

59. Riveted Seams. Although riveted construction of steel pipe lines has been almost completely supplanted by welding, many old riveted lines are still in service. Comprehensive data on riveted joints will be found in an article by Peter Bier[1] of the U.S. Bureau of Reclamation, based on the standard specifications of the Pacific Coast Electric Association.[2] For light pressures, longitudinal seams may be simple lap joints with a single line of rivets. For higher pressures, single or double butt-strap joints with multiple lines of rivets are required. Single-lap circumferential joints are usually sufficient. The joints are made watertight by caulking, and rivets must be close enough to make the caulking effective.

Riveted joints cannot develop the full strength of the plate. Rivet heads and joint plates or offsets on the inside of the pipe materially increase flow resistance. Where permissible, this may be reduced by using single outside butt straps and countersunk rivets.

60. Welded Seams. With the rapid development of electrical welding processes, welded steel pipe is rapidly superseding other types. Typical welded joints are shown in Fig. 36. The simple butt joint shown at *a* is generally used in the shop, particularly for automatic machine welding. It may also be used for hand-welded field joints, although the lap joint *b* and the reinforced butt joint *c* are frequently preferred for field circumferential joints. These joints, when properly made, are as strong as the plate.

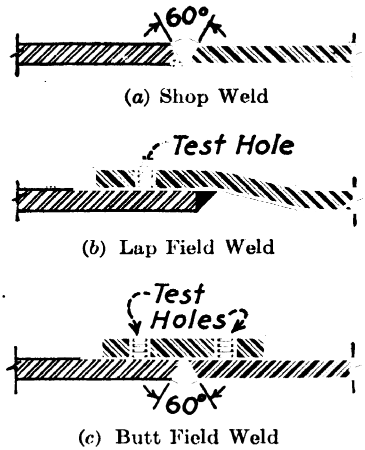


FIG. 36.—Typical pipe welds.

As the weld cools, internal stresses develop in the weld metal and the adjacent plate. These may be relieved for shop welds by annealing the finished section. Attempts at annealing field welds have not been successful.

The pipes are usually shop-welded into complete sections 30 ft more or less in length. These joints are bulk-headed and tested with water pressure exceeding the required working strength by a specified amount, then shipped to the field for assembly in the trench and welding together as at (*b*) or (*c*), Fig. 36. These field joints are immediately tested for leakage by forcing air or soapsuds into the test holes shown. After testing, the holes are closed with plugs or welded.

61. Cold Expanded Steel Pipe. A recent innovation is the cold expanded steel pipe. Shop-welded lengths are made up to a diameter slightly less than the finished diameter, the ends are stretched to the full size, and then the pipe is expanded by hydraulic pressure into a mandrel having the exact outside diameter desired for the finished pipe. The resulting pipe is true, straight, and smooth, and its elastic limit strength is greatly increased.

Pipe of this kind was used on the western portion of a line built to transport natural gas from western Texas to California. Special manganese plates

[1] BIER, PETER, Steel Penstock Design by a Graphical Method, *Eng. News-Record*, **99**, 629, 1927.
[2] Report on Penstock Design, Hydraulic Power Committee, National Electric Light Association, September, 1923.

from $\frac{9}{32}$ to $\frac{7}{16}$ in. were used. As the pipe leaves the welding machine, it is 30 ft long and has an outside diameter of $29\frac{1}{2}$ in. The ends are then expanded, and the pipe is placed in the mandrel and stretched by hydraulic pressure to a uniform 30 in. outside diameter. Two 30-ft expanded sections are welded together to make a 60-ft shipping section. Before shipping, all surfaces are sandblasted, primed, and coated with coal tar. The outer surface is wrapped with asbestos felt.

Cold working is claimed to increase the yield strength some 25 per cent. For this particular line, the yield strength is increased from a range of 42,000 to 47,000 psi to one of 52,000 to 65,000 psi.[1]

62. Plate Thickness. The tensile stress in the shell due to internal pressure is found from Eq. (5), a being taken as the net shell area (one side) in square inches per foot of pipe after deducting for rivet holes or joint efficiency. The value of f_1 must not exceed the allowable tensile strength of the steel. Plates for pipes are usually of a soft grade to facilitate riveting and welding, with an elastic limit of 27,000 to 30,000 psi. If water-hammer effects are amply allowed for, working values may be made half the elastic limit.

Steel pipe, being flexible, tends to deform when buried in a trench to the extent required to develop a more or less uniform external pressure (see Arts. 42 to 45). If such distortion is permissible, little concern need be felt for safety against moderate external loads. The amount of distortion can be reduced by careful bedding and thorough compaction of the backfill around the sides of the pipe. Distortion may also be reduced by subjecting the pipe to internal pressure during the backfilling operation and until initial settlement has taken place.

If distortion beyond the necessary elastic deflection is to be prevented, the pipe may be analyzed as an elastic shell, under loads computed as for other pipes, with due allowance for the greater flexibility.

An empty submerged pipe or one that may be emptied in such manner as to induce a vacuum may be subject to a uniform external collapsing force. The plate thickness required to resist buckling under uniform external pressure is approximately[2]

$$t = d\sqrt[3]{\frac{6}{E}p} \tag{34}$$

where t = shell thickness, in.
d = diameter of the pipe, ft.
p = uniform external pressure, psf.
E = modulus of elasticity, psi.

If E is 30,000,000, Eq. (34) becomes

$$t = 0.00585d\sqrt[3]{p} \tag{35}$$

Where the loads are light, it is customary to require an arbitrary minimum plate thickness, $\frac{1}{4}$ and $\frac{3}{8}$ in. being common values. For large pipes, the minimum may be made $\frac{1}{2}$ in., $\frac{5}{8}$ in., or more. Occasionally, plates $\frac{1}{16}$ to $\frac{1}{8}$ in. thicker than the theoretical are specified to allow a margin for corrosion.

The foregoing rules are for first-class substantial installations. Many water pipes of light-gage steel, both above ground and buried in trenches, have given years of satisfactory service. Where funds are limited and where corrosion effects are not severe, or where the service is temporary, such installations may be permissible.

63. Steel Pipe on Piers. Steel pipe above ground is usually supported at intervals from 20 to 60 ft. The supports may be concrete, steel, or wood. The permissible

[1] Expanding Pipe beyond Yield Point to Make It Rounder and Stronger, *Eng. News-Record*, Sept. 4, 1947, p. 86 (anonymous).

[2] TIMOSHENKO, *op. cit.*, p. 602, Eq. (197), $(1 - \mu^2)$ assumed = 1.0.

span is increased by ring girders at the support, of sufficient strength to hold the pipe rigidly to its circular form.[1] A typical ring-girder installation is shown in Fig. 37.

The girder in this example is formed from a 7½-in. 60-lb structural tee section. The flange is set into the pipe, forming a part of the barrel. Secondary stresses between the barrel and the girder are reduced by this means. This type of construction is patented. Any form of girder having sufficient strength to resist the distortion of the pipe may be used.

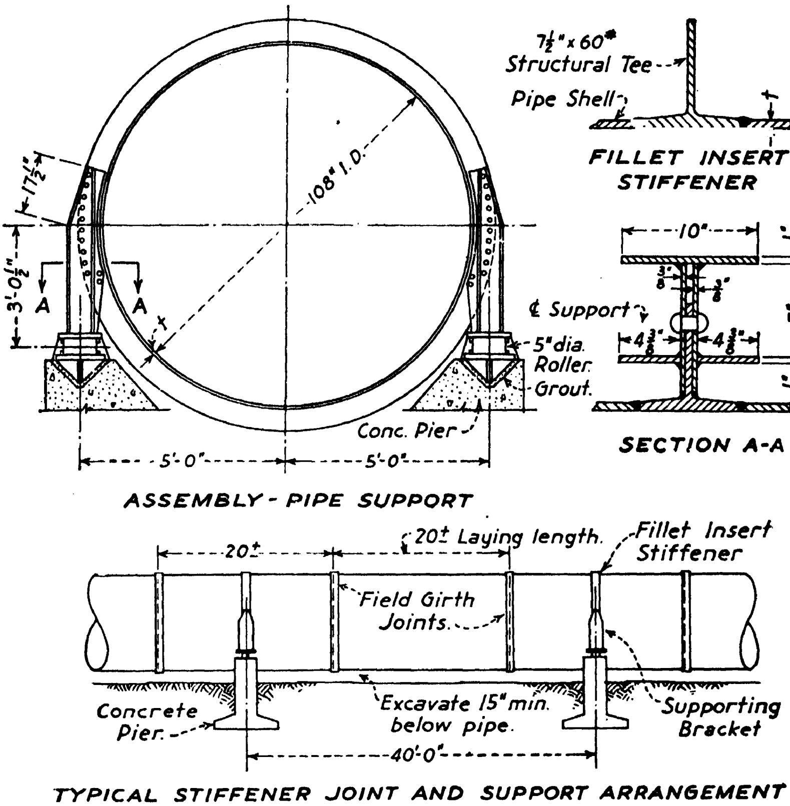


FIG. 37.—Steel pipe supported on ring girders.

64. Expansion Joints. Steel pipe buried in a trench or fill is usually welded or riveted continuously without joints. Movements due to temperature change are resisted by stresses in the steel. If the pipe is exposed, this is not always advisable, as the expanding pipe tends to move about, particularly at bends. To avoid this, expansion joints are provided. Typical expansion-joint details are shown in Fig. 38.

The device shown at *a* is simply a flexible flanged coupling of such width and material thickness that it may be stretched or compressed in the direction of the pipe axis, the necessary stress-relieving movement being thus allowed. Bends should be long-radius to avoid damage to the metal by flexing. The device shown at *b* operates in a similar manner.

In the joint shown at *c*, an outer sleeve 1 is attached to the pipe at 2. The other end of the pipe 3 slides within this sleeve. Leakage is prevented by packing 4 com-

[1] SCHORER HERMAN, Design of Large Pipe Line, *Trans. A. S. C. E.*, **98**, 101, 1933.

pressed by ring 5 into the space formed by the member 6 attached to the sleeve. It is desirable, although not essential, that the inside of the sleeve and the outside sliding surface of pipe 3 be armored with copper or bronze. This may be done by spraying with molten metal or by other available means. Joint *c* permits much greater movement than those shown at *a* and *b*.

Where expansion joints are provided, anchors are required to control the movement of the pipe. Exposed pipes should be painted a light color to reduce heat absorption.

65. Protective Coatings for Steel and Cast-iron Pipe. Some form of protective coating is desirable for both inner and outer surfaces of steel and cast-iron pipe. Asphaltic and coal-tar coatings are widely used. The molten material may be applied as a dip or by brush or spray. Coal-tar enamel is particularly efficient for surfaces in contact with running water. A satisfactory enamel must be sufficiently pliable not to crack at the lowest anticipated operating temperature and yet stiff enough not to

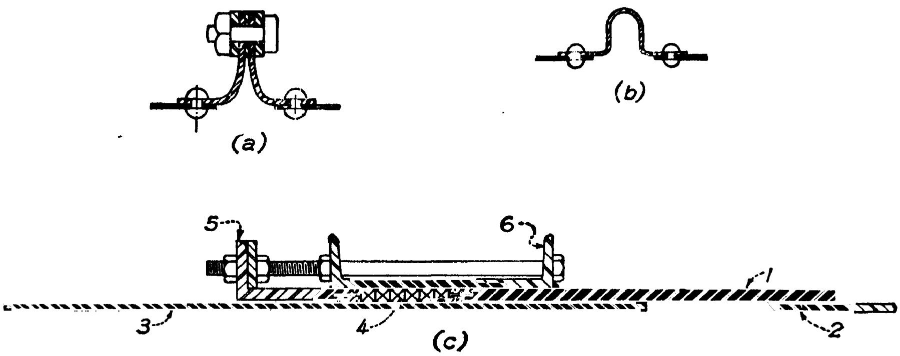


FIG. 38. Expansion joints in steel pipe.

run or sag at high temperatures. The enamel is applied hot, and the pipe should likewise be preheated if practicable. A thinner priming coat is generally used, and for best results, mill scale should be removed by sandblasting before coating. The inside enamel may be applied centrifugally, only the joint areas being left for hand daubing after erection. Under current practice, a good dip coating is about $\frac{1}{32}$ in. thick and first-class coal-tar enamel approximately $\frac{3}{32}$ in. These thicknesses are subject to change with the development and perfection of coating materials. The applied coating should be tested for imperfections by means of a light high-voltage electrical current, similar to the ignition voltage of an automobile, applied through a wire brush. Such a current will spark through imperfections.

To be effective, a bituminous coating on the outside of a pipe must itself be protected against soil stresses. A wrapping of bitumen-impregnated paper or burlap offers limited protection. A shell of concrete or gunite is more effective. A $\frac{3}{4}$-in. gunite shell (reinforced) has been successfully applied to steel pipes up to 11 ft in diameter both directly to the steel and over an enamel coating. Such coverings promise enduring protection against corrosion, but long-time tests are not available. A mixture of asphalt and aggregates, applied $\frac{1}{2}$ to 1 in. thick, may be used without added protection. In particularly corrosive soils, steel pipes may be completely encased in a thick concrete envelope.

The interior of pipes also may be protected with cement mortar. Pipes lined with 1 to 2 in. of mortar have been used extensively. More recently, thin linings $\frac{1}{4}$ to $\frac{1}{2}$ in. thick are being used with apparent success. In new pipe, these linings may be spun centrifugally before the pipe is laid. The joint spaces are gunited or hand-

plastered after erection. A patented machine[1] has been developed for placing thin mortar linings in pipes in place. The mortar is thrown against the pipe wall by radially spaced high-speed vanes rotating at 1,000 rpm. This is followed by slowly rotating trowels which smooth the surface.

WOOD-STAVE PIPE

66. General. Wood-stave pipe is widely used in the western United States, particularly for irrigation service. Diameters up to 17 ft have been installed. Its principal advantages are low first cost and high carrying capacity. The pipe barrel is made up of selected wooden staves, machine-shaped, held together by outside steel bands. The larger sizes are built in place, the staves breaking joint to form a continuous barrel. The curves are built in by bending the staves. Small sizes may be

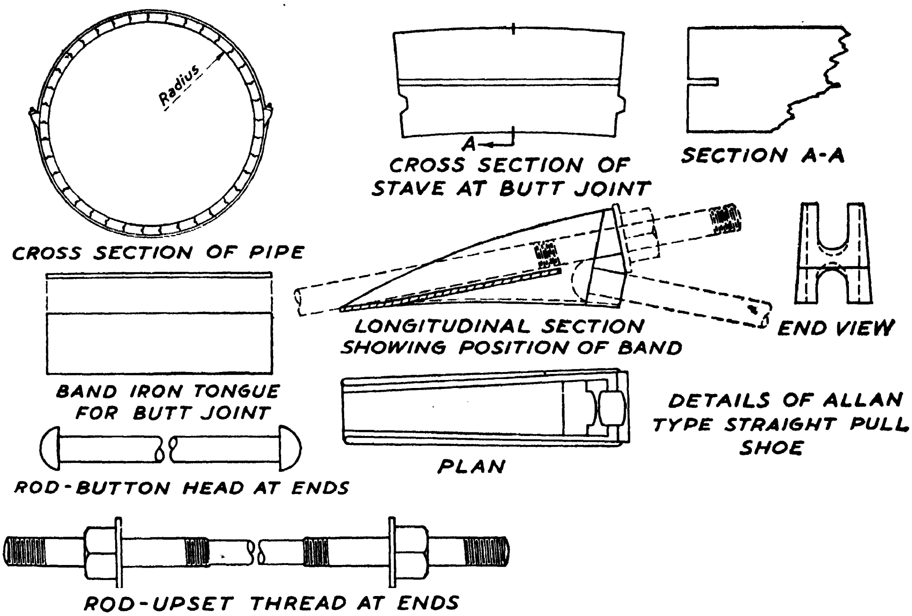


FIG. 39.—Details of continuous stave pipe with two-piece band.

wire wound in a factory, in convenient lengths, joined in the field by double spigot ends driven into wood-stave or metal sleeve couplings. Usually, machine-banded pipes are limited to 24 in. in diameter, although sizes up to 48 in. have been made. Stave and band details for continuous stave pipe are shown in Fig. 39.

67. Staves. The staves may be as wide as 8 in. and vary in thickness from 1½ to about 3 in. to meet the requirements of durability, strength, and rigidity. The faces of the stave are cylindrical. In continuous pipe, the ends are cut square, making butt joints staggered and hard driven in assembling. Steel tongues set in kerfs in the ends make the joints tight. The wood is generally redwood, fir, white or yellow pine, or spruce and should be clear, close grained, and free from defects.

68. Bands. The bands may be of wire or of round or flat rods sufficiently strong to take the tension from the internal pressure and from the compression in the staves from swelling of the wood. The pressure between the band and the staves must not cause progressive crushing of the staves, although some indentation is unavoidable.

[1] BANKS, W. G., and CLINTON INGLEE, A Relined Old Main Exceeds Its Original Capacity, *Water Works Eng.*, Aug. 19, 1936, pp. 1081*ff*.

For round bands, D. C. Henny[1] found that the design is safe if the required bearing pressure, when the contact width is equal to half the rod diameter, does not exceed the safe bearing strength of the wet wood. For flat bands, use the full width. He recommended safe bearings for wet redwood varying from about 750 psi for 3/8 in. to 600 psi for 7/8 in. round bands.

The bands must not only resist the internal water pressure but must produce a residual compression between staves to ensure watertightness; also, they must resist the swelling action of the staves. Etcheverry recommends[2] that band spacing be computed for edge bearing in the staves equal to 125 psi or 1.5 times the internal water pressure, whichever is the greater. The bands are usually provided with cold-rolled thread, the strength of the threaded portion being equal to that of the body of the rod. Shoes or washers furnished by the manufacturers ensure adequate bearing on the wood at the joints.

Band spacing determined from tensile strength of the bar is as follows:

$$L_s = \frac{af_s}{\frac{1}{24}pd + tf_w} \tag{36}$$

The spacing based on bearing pressure between the band and the wood is

$$L_b = \frac{(6d + t)e}{\frac{1}{24}pd + tf_w} \tag{37}$$

If these two values are not equal, the smaller controls. They may be made equal for round bands by choosing the band diameter from the equation

$$\phi = \frac{2}{\pi}(6d + t)\frac{f_b}{f_s} \tag{38}$$

Symbols in the three preceding equations have the following significances:

L_s = band spacing, in., based on tensile strength of band
L_b = band spacing, in., based on crushing of wood under band
a = area of one band, sq in.
e = safe bearing of band on stave, lb per linear inch of band (the safe bearing strength of the wood times the radius of round band or width of flat band)
d = internal diameter of pipe, ft
f_s = allowable stress in steel psi
f_b = residual compression between staves, psi, loaded pipe
f_w = ultimate edge bearing strength of wet staves against each other
p = water pressure, psi
t = stave thickness, in.
ϕ = diameter of round band, in.

The stretching of the bands under load tends to cause leakage because of reduced stave compression. Subject to this limitation, a stress of 15,000 psi may be considered safe.

Bands and fittings for machine-made pipe should be galvanized. As a further protection, the entire outer surface of the completed pipe is usually covered with tar or asphalt. Bands and fittings for continuous stave pipe may be dipped in a preservative bitumen compound or coated after installation.

69. Installation. The pipe may be installed in backfilled trench, laid on the surface, or supported above ground on cradles or piers. The spacing of cradles or piers is not standardized. A spacing of 6 or 8 ft is frequently found for pipes up to 6 ft in diameter. The beam strength of a wood-stave pipe is considerable but uncertain.

[1] ETCHEVERRY, B. A., "Irrigation Practice and Engineering," McGraw-Hill Book Company, Inc., 1915, vol. II, p. 282.
[2] *Ibid.*, p. 284.

70. Life of Wood Pipe.[1] The life of wood pipe depends on local conditions and is variable. If laid in a moisture-retaining soil and kept constantly full of water under a head of 50 ft or more, the useful life for selected pine, fir, or redwood should be 40 to 50 years. For lower internal pressures or less favorable soil conditions, the life is shorter, perhaps 10 to 15 years for pine and fir and 15 to 20 years for redwood. If the pipe is not constantly full or the pressure is low and the soil porous, the life may be as short as 4 or 5 years. Alternate wetting and drying are particularly injurious. Pipe on cradles or piers should last 25 or 30 years. Creosoting increases the life of pine or fir pipes materially, particularly under adverse moisture conditions.

VITRIFIED-CLAY PIPE

71. Pipes of vitrified clay are used for sewers and drains. When well burned, they are immune to attack by alkaline waters and sewage. They are not suitable for sustaining appreciable internal pressure and generally flow at part depth. If carefully bedded and backfilled, they are reasonably efficient for sustaining external loads. The pipe units are generally 2 to 4 ft long, but longer lengths up to possibly 8 ft are being offered by some manufacturers. Dimensions and strengths of standard pipe, according to the American Society for Testing Materials specifications, are shown in Table 9. These pipes are salt-glazed. Recent experiments indicate that unglazed pipe and ceramic glazed pipe are stronger and that they offer less resistance to flow than salt-glazed pipe. Such pipe can be purchased in some localities.

Joints are either bell-and-spigot or plain ends. Plain-end joints in drains may be left open to permit infiltration. Bell-and-spigot joints likewise may be left open or may be closed at the bottom and open at the top; or they may be made water-tight by caulking with cement or other suitable jointing material. Plain ends may be caulked

TABLE 9.—DIMENSIONS AND STRENGTH REQUIREMENTS OF CLAY SEWER PIPE

Internal diam., in.	Laying length, ft	Inside diam., ½ in. above base of socket[1]	Depth of socket, in.	Thickness of barrel, in.[2]	Avg crushing strength 3-edge bearing, lb per lin ft[3]
4	2	5¾	1¾	½	1,000
6	2, 2½	8¼	2¼	⅝	1,000
8	2, 2½, 3	10½	2½	¾	1,000
10	2, 2½, 3	12¾	2½	⅞	1,100
12	2, 2½, 3	15⅛	2¾	1	1,200
15	2, 2½, 3	18¾	2¾	1¼	1,370
18	2, 2½, 3	22½	3	1½	1,540
21	2, 2½, 3	25¾	3¼	1¾	1,810
24	2, 2½, 3	29⅜	3⅜	2	2,150
27	2½, 3	33⅛	3½	2¼	2,360
30	2½, 3	36⅝	3½	2½	2,580
33	2½, 3	40	4	2⅝	2,750
36	2½, 3	43¼	4	2¾	3,080

[1] Taper of socket 1:20.

[2] Thickness of socket not less than ¾ thickness of barrel.

[3] Pipe laid in trench will carry greater loads than under three-edge bearing test proportional to following minimum load factors:

Ordinary bedding	1.6
First-class bedding	2.0
Concrete cradle bedding	2.6

[1] *Ibid.*, p. 288.

into cement or steel collars. The possible use of rubber joints requiring little installation labor is being investigated.

CEMENT-ASBESTOS PIPE

72. A cement-asbestos pipe recently developed promises to be very useful, particularly in locations where concrete, cast iron, and steel are attacked by bad water or soil conditions. It is composed of cement and asbestos fiber, formed into a pipe under pressure. The resulting rocklike material has notable tensile strength and is claimed to be immune to all ordinary bad ground conditions. Because of plant cost and special technique involved, this pipe does not lend itself to construction on the job as does concrete pipe but is essentially a factory product. A familiar brand is Transite, produced by the Johns-Manville Company. In this type, double spigot ends are inserted in a collar of the same material, watertightness being secured by two rubber sealing rings. The manufacturers present much evidence to show that rubber protected from air and light is permanent. Similar products are offered by other manufacturers.

HYDRAULICS OF PIPE LINES

73. The theory of resistance to flow through pipe lines is covered in Appendix A. In American water-works practice friction in pipe lines is computed almost exclusively by the Hazen and Williams, Scobey (three variations), or Manning formula. Kutter is sometimes used. These formulas are given in Appendix A.

Losses, usually referred to as minor, occur at intakes and outlets, and at obstructions such as valves, branches, and changes in diameter. These losses are discussed in Appendix A.

EXAMPLES OF PIPE DESIGN

74. Steel Pipe. Let it be required to design an enameled-steel pipe line for the conditions of Fig. 40, flow 800 cfs, Hazen and Williams[1] $C_w = 130$, pipe buried with

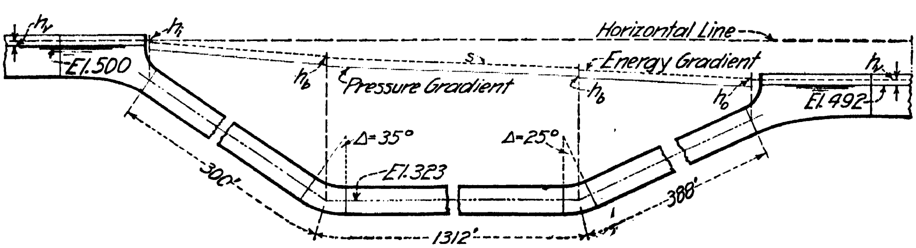


Fig. 40.—Inverted siphon used as basis for design example.

10 ft of fill on top of granular material weighing 110 lb/cu ft, channel velocities above inlet and below outlet 4.00 fps, first-class transition structures.

For convenience, the Hazen and Williams equation is converted to the form

$$Q = 0.432C_w d^{2.63} s^{0.54}$$

or, inserting the known value of Q and reducing,

$$d^{2.63} s^{0.54} = 14.25$$

The available fall of 8.00 ft, shown on Fig. 40, reduced by inlet, outlet, and bend losses and divided by the length of pipe gives the available slope s. The minor losses (inlet, outlet, and bend losses) not being known in advance, solution is made by a

[1] See Chap. 2, Sec. 2. Scobey's formula or any other accepted friction formula may be substituted if desired.

repetition of trials. The minor losses are first ignored, the corresponding value of s being $8.00 \div 2{,}000 = 0.004$. The required pipe diameter is 8.53 ft. The corresponding value of V is 14.00 and h_v is 3.05. The minor losses corresponding to this velocity are computed thus:

$$\begin{aligned}
& & \text{Ft} \\
h_i &= 0.1\Delta h_v = 0.1(3.05 - 0.25) &= 0.28 \\
h_0 &= 0.2\Delta h_v = 0.2(3.05 - 0.25) &= 0.56 \\
h_{b1} &= 0.25 h_v \sqrt{35/90} = 0.156 \times 3.05 &= 0.48 \\
h_{b2} &= 0.25 h_v \sqrt{25/90} = 0.132 \times 3.05 &= \underline{0.40} \\
& \text{Total} & 1.72
\end{aligned}$$

Subtracting this total from the available fall of 8.00 ft and dividing again by the length gives a new slope value of about 0.0031. A second trial shows this too small and a third gives $s = 0.00328$, $d = 8.88$, $V = 12.92$, and $h_v = 2.59$. The losses then are as follows:

$$\begin{aligned}
& & \text{Ft} \\
h_i &= 0.1(2.59 - 0.25) &= 0.23 \\
h_0 &= 0.2(2.59 - 0.25) &= 0.47 \\
h_{b1} &= 0.156 \times 2.59 &= 0.40 \\
h_{b2} &= 0.132 \times 2.59 &= \underline{0.34} \\
\text{Total minor losses} & &= 1.44 \\
\text{Friction loss} &= 2{,}000 \times 0.00328 &= \underline{6.56} \\
& \text{Total} & 8.00
\end{aligned}$$

For the purpose of illustration, the foregoing example is based on the direct use of the Hazen and Williams formula. In practice, the work is simplified by using curves or tables (see Appendix A).

The maximum head on the pipe occurs at the downstream end of the first bend. The friction head to the center of the bend is $300 \times 0.00328 = 0.98$. Adding h_v and h_i, the total drop is $0.98 + 2.34 + 0.23 = 3.55$ ft; hence the elevation of pressure gradient is $500 - 3.55 = 496.45$ and the head on the center of the pipe is $496.45 - 323.00 = 173.45$ ft. The bend loss is ignored in computing the pressure, as its point of full application is uncertain.

For an allowable stress of 13,500 psi and with $a = 12t$, Eq. (5) becomes

$$13{,}500 = \frac{0.5 \times 173.45 \times 62.5 \times 8.88}{12t}$$

from which

$$t = 0.297 \text{ in. (for tension)}$$

If the trench is 11 ft wide at the top of the pipe, H/B (Fig. 23), is 0.91 and the value of C_t [Eq. (6)] is 0.7, whence

$$W_t = 0.7 \times 110 \times 11^2 = 9{,}300 \text{ lb}$$

This is equivalent to a unit vertical pressure of 1,050 psf on the top of the pipe. Because the pipe is flexible, this pressure may be assumed uniform around the circumference and the thickness required to resist buckling [Eq. (34)] is

$$t = 0.00585 \times 8.88 \sqrt[3]{1{,}050} = 0.528 \text{ in.}$$

This thickness is greater than that required for tension and controls. A stock thickness of $\frac{1}{2}$, $\frac{17}{32}$, or $\frac{9}{16}$ in. may be chosen, remembering that Eq. (34) is only an approximate measure of the buckling strength of a buried steel pipe.

75. Concrete Pipe. Assume an alternative design for a precast concrete pipe, for the identical conditions of Fig. 40, using Scobey's formula $Q = 3.72C_s d^{2.625} H_f^{0.5}$ where H_f is fall per 1,000 ft of pipe (see Appendix A), with $C_s = 0.370$. Substituting given values, the equation becomes

$$800 = 3.72 \times 0.370 H_f^{0.5} d^{2.625}$$
$$581 = H_f^{0.5} d^{2.625}$$

The minor loss coefficients are the same as for steel pipe. These losses may first be ignored, as in the steel-pipe design, or as a first trial they may be made 1.44, the same as in the alternative steel pipe, assumed to have been computed first. This gives $H_f = 3.28$, $d = 9.01$, $V = 12.55$, $h_v = 2.45$. The corresponding minor losses are 1.36.

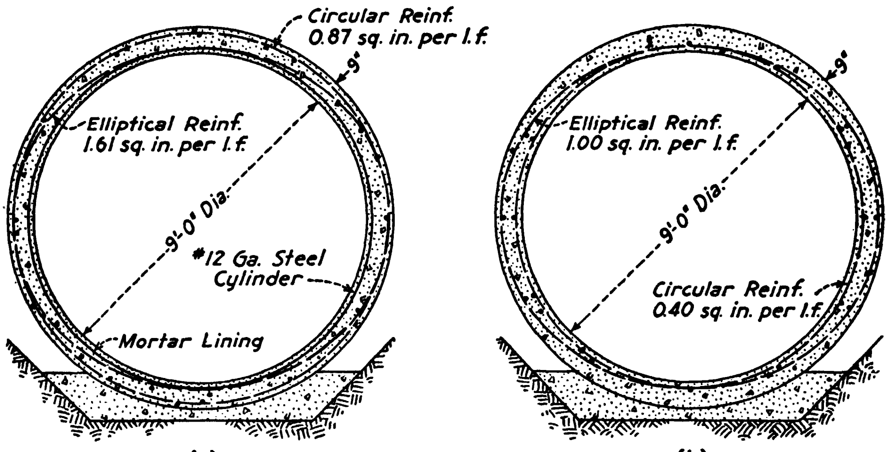


FIG. 41.—Pipe sections, design examples.

By solving again, $H_f = 3.32$, $d = 8.99$, $V = 12.60$, $h_v = 2.47$, with minor losses of 1.37, which is a satisfactory agreement. The diameter is made 9.00 ft.

On the assumption that the thickness of concrete shell is 9 in. and if the same clearances are allowed as for the steel pipe, the trench width for the concrete pipe is 12.5 ft. The vertical external pressure, acting on the outside diameter of the pipe, is 12,030 lb. Because of the rigidity of the pipe, only the active horizontal pressure on the sides of the pipe can be considered. The value of K in Eq. (9) is assumed to be

TABLE 10

Case	Direct stress			Bending moment		
	Top	Side	Bottom	Top	Side	Bottom
1. Weight of pipe shell	− 183	+ 861	+ 183	+1,109	−1,310	+1,713
2. Weight of water in pipe	− 843	− 270	− 1,686	+1,279	−1,512	+1,977
3. Vertical external load	− 325	+ 6,015	+ 325	+8,035	−8,211	+9,184
4. Uniform horizontal external load	+ 900	0	+ 900	−1,106	+1,106	−1,106
5. Triangular horizontal external load	+ 285	0	+ 625	− 461	+ 559	− 648
6. Uniform internal pressure	−51,477	−51,477	−51,477	0	0	0

0.15 because of the narrow clearance between the pipe and the sides of the trench. The uniform and the triangular horizontal loads on the pipe are computed to be

$$H = 1{,}800 \text{ lb}, \qquad H_t = 910 \text{ lb}$$

If the stress coefficients of Table 7 are used, the stresses are computed as in Table 10.

Because of the internal pressure, the lower portions of the pipe will require a steel cylinder. The thickness of the cylinder is chosen arbitrarily, and bar steel is added to take care of the moments and the remaining tension, as shown at *a* (Fig. 41). Tension due to internal water pressure, the last item in Table 10, reduces toward each end of the siphon, which permits a reduction of the steel bars to some minimum limit, say about 1.40 sq in. of steel per foot for this size of pipe. The steel cylinder may be omitted for heads under 80 ft or, with careful work, for heads under 100 ft. A minimum section is shown at *b* (Fig. 41).

ECONOMIC CONDUIT SIZES

76. Basic Principles.[1] If the slope is abundant and of little value, water conduits are made steep and small to save cost. However, elevating the water to provide slope

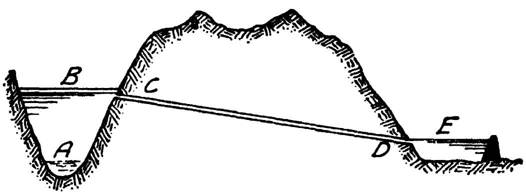


FIG. 42.—Illustrating economic analysis.

is frequently expensive, or head already in existence may have value for the production of power or for other purposes. In such cases, economic design requires a balancing of conduit cost against the cost or value of head.

A simple case is illustrated in Fig. 42, where a *stated discharge* is to be elevated from a natural stream at A by a dam B and conveyed through a tunnel CD to a fixed level in reservoir E. A graphical solution is illustrated in Fig. 43 where curve AB represents the cost of the tunnel, CD the cost of the dam, and EF the cost of both combined. The best slope is that corresponding to point G, the low point on the combined cost curve. This is an easy solution for a single conduit but is inadequate for more complicated problems.

In Fig. 42, the water is elevated by a dam. The same principle of analysis can be applied if it is elevated by pumping or other means, a cost-of-lift curve being substituted for the cost-of-dam curve (Fig. 43). The cost of lift must include the cost of equipment plus the *capitalized cost* of perpetual operation. If the elevation at the inlet is predetermined, the outlet elevation being subject to variation, the slope is determined by the *value of head* at the outlet, as for the production of power, or for furnishing a gravity supply to domestic or irrigation consumers. The value of fall for the production of power is equal to the *capitalized net return* from power sales less the cost of the power installation. For convenience, the term *value of a foot of head* is used to designate either the cost of its production at the inlet or its value at the outlet.

77. Cost-slope Tangent Method. Evidently the slopes for the two cost curves at H and J (Fig. 43) must be numerically equal, but opposite in direction, as otherwise the tangent, at G, would not be horizontal. Usually the region of uncertainty in height of dam is limited, and the curve CD is approximately straight within that

[1] For a more detailed discussion of economic slopes see Julian Hinds, Economic Water Conduit Size, *Eng. News-Record*, Jan. 28, 1937; Economic Sizes of Pressure Conduits, *Eng. News-Record*, Mar. 25, 1937.

region; hence, the economic slope may be located as follows: Take a trial height of dam corresponding to some arbitrarily chosen tunnel slope, as at *H*, and draw the tangent *HP* to the cost curve *CD*. Draw tangent *MN* with slope equal to slope of *HP* but reversed. If point of tangency *J* is on same vertical as *H*, then *J* marks the economic slope; otherwise, assume a new height of dam and repeat. Satisfactory adjustment is not difficult to make, as points of tangency are more or less indefinite.

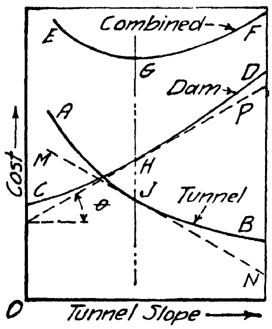


FIG. 43.—Illustrating economic analysis.

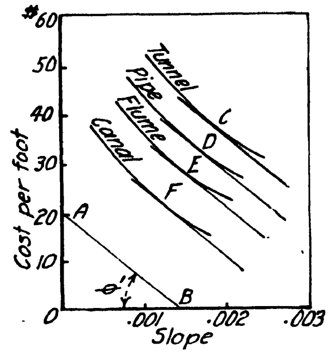


FIG. 44.—Illustrating economic analysis.

The significance of the slope-tangent method is shown by Fig. 46. Curve *AB* represents the cost-slope relationship for a foot of conduit for a specified flow. Slopes are measured on the horizontal *OM* and costs on the vertical *ON*. A trial slope *OC* gives a cost per foot of conduit equal to *CA*. If *CD* is drawn so that tan θ' is equal to the value of a foot of head, the distance *OD* will represent the cost (or value) of the fall in a foot of conduit on slope *C*. If *AE* is drawn parallel to *CD*, *OE* represents the combined cost. The lowest position of the point *E* occurs when *A* reaches *A'*; hence the economic slope is *OF*. In case of an irregular cost curve, as *GH*, the economic slope is marked by the lowest possible contact with the slope line, as at *K*.

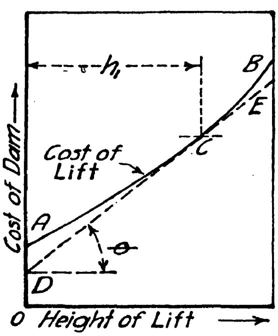


FIG. 45.—Illustrating economic analysis.

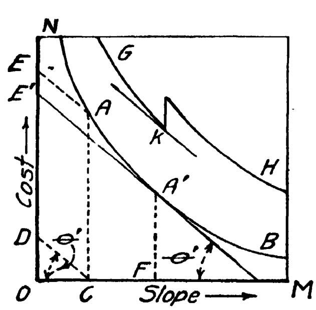


FIG. 46.—Illustrating economic analysis.

78. Application to Composite Conduits. The procedure for a conduit composed of many types is illustrated in Figs. 44 and 45. In Fig. 44, a cost-slope curve is plotted for each conduit type. If conditions vary, two or more curves may be required for a single type. These curves are on a linear-foot basis. The cost-of-lift curve is plotted separately, as in Fig. 45, and may represent the cost of a dam, capitalized cost of pumping, or the value of head at an outfall, as conditions require. A trial lift is

assumed as h_1 (Fig. 45), and the corresponding tangent ED is drawn, having an inclination θ to the horizontal. A line AB is then drawn on Fig. 44, tan θ' being made numerically equal to tan θ, each measured in the terms of its own diagram. Tangents parallel to AB are drawn, the economic slopes for the various conduits being marked at C, D, E, and F. These slopes are applied to the corresponding known conduit lengths, starting from some point of known elevation to give a computed height of lift to replace the assumed height h_1. If the slope of the line DE for the new height is essentially different from that for h_1, the solution should be repeated.

79. Application to High-head Pipes. In pipe lines under high head, the pressure introduces an additional variable. A simplified example of a high-head steel pipe is

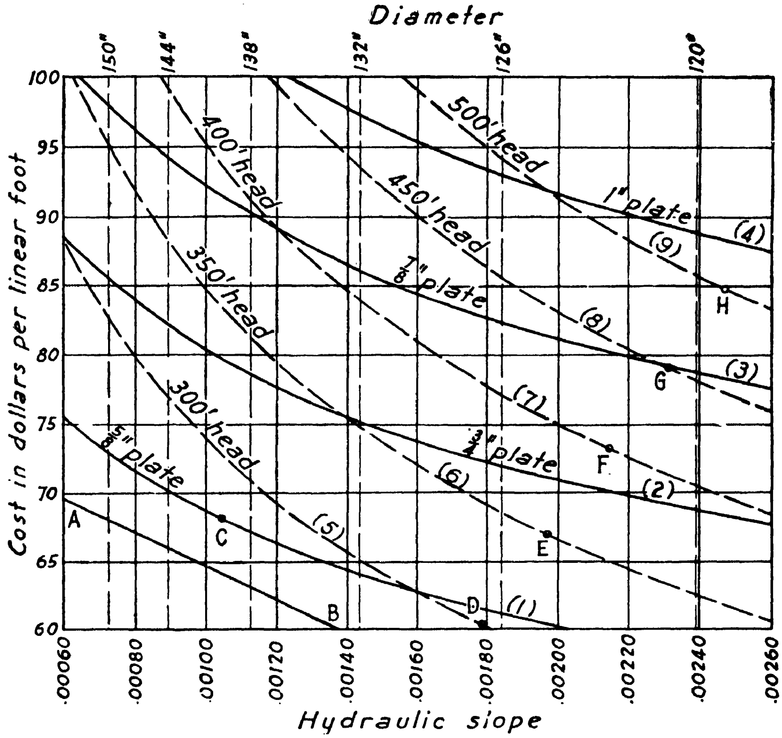


FIG. 47. Illustrating economic analysis.

illustrated in Fig. 47. Such pipes are limited by available plate thicknesses and usually by standard diameters. Cost-slope curves, based on constant plate thicknesses, take the form of curves 1, 2, 3, and 4. The ability of the pipes represented by any one of these curves to withstand head is variable. Another set of curves 5,6, 7, 8, and 9 may be drawn, representing the cost of pipe designed for constant head, without regard to standard plate thickness or pipe diameter.

Diameters, as well as slopes, are shown in Fig. 47 for convenience. The primary plotting, however, must be in slope coordinates which must be rectangular.

For low pressures, the economic slope tangent parallel to AB is applied to the lowest permissible plate thickness curve, the economic slope being marked as at C. For higher pressures, the tangent is applied to the constant-head lines as at D, E, F, G, and H. If these points fall on odd plate thicknesses or diameters, the nearest standard section of sufficient strength may be used.

Instead of plotting the constant-head lines, the permissible heads may be noted at the intersection of commercial plate thickness and standard diameters. The best practical size for any required head is the one of sufficient strength showing the lowest position of the sloping index line.

The best diameter decreases as the head increases. This is a well-known characteristic of deep inverted siphons. If the pipe is to be supported on a bridge, the cost of the bridge, or at least such part of the cost as is dependent on pipe size, must be included in the cost curve.

80. Controlling Elevations. Frequently, the controlling elevation is at neither the outlet nor the inlet but at some intermediate point, as at *F* (Fig. 48). Economic slopes upstream and downstream from the control are separately determined from the value of head at the inlet and outlet, respectively.

On the assumption that these slopes have been correctly determined for a tentatively chosen trial position of the control, it is then necessary to determine whether this choice is correct or whether the whole line should be raised or lowered from its trial position. The cost of the lift at the inlet can be reduced by lowering the entire line. The drop at *J* will be correspondingly reduced, a loss being thus introduced depending

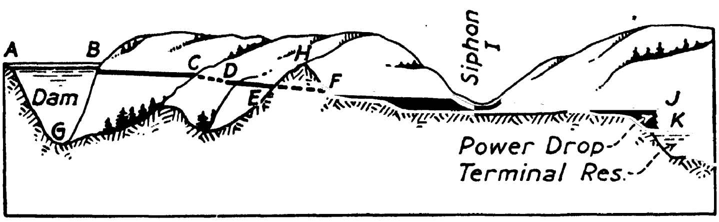


FIG. 48.—Illustrating economic analysis.

on value of head at the outlet. If the cost of head at the inlet exceeds the value of fall at the outlet, lowering the line will effect a net saving in these two items. However, any appreciable lowering will increase the length and cost of the tunnel *EF*, and the result may be an increase in the total. Raising the line will have an opposite effect. If the trial control is incorrectly chosen, the line must be raised or lowered until some definite change in topographic conditions is encountered. For the conditions illustrated, the tunnel would be shortened only slightly by raising the line but would be greatly increased if the line were lowered. This indicates the pocket at *F* as a possible control.

The control need not be at the end of a tunnel but may be any point marking a sharp division in construction conditions or costs. For a canal system, it may be a saddle, the boundary of a swampy region, a parcel of expensive right of way, or any of a number of similar obstacles. Raising or lowering the line may increase the cost of features other than the critical feature. Change of ground conditions, due to raising or lowering the line, may change costs throughout. All such factors must be taken into account. This seldom can be done mathematically because of irregularity of physical factors. The procedure is to select a trial control, determine economic slopes, and prepare an estimate to be compared with similar estimates for slightly higher and lower elevations and for other apparent controls.

Sometimes both inlet and outlet elevations are predetermined. If both the height of dam and water surface elevation at *E* (Fig. 42) are fixed, the tunnel should utilize all the available slope. The same is true of a long composite line, except that there exists the problem of distributing the slope between the various conduit types. To do this, plot cost-slope curves for all types, as in Fig. 44, then assume a trial value for tan θ' and locate points *C*, *D*, *E*, *F*, etc., which mark a coordinated set of economic slopes. Apply

these slopes to the known conduit lengths, and compute the required fall. If the required fall differs from the available fall, assume a new value for tan θ' and repeat the analysis, continuing until satisfactory agreement is secured. An intermediate control between fixed inlet and outlet levels may divide the line into two parts, for which the slopes are determined separately.

The principles herein outlined are of great assistance in the proportioning of long waterways, but they should not be applied with too great mathematical precision. Cost curves never can be precise. Also, the location of the point of tangency, as at G (Fig. 43) or C, D, E, or F (Fig. 44), is indefinite, and an appreciable variation from the exact value has little effect on the over-all economy.

SECTION 11

HYDROELECTRIC PLANTS

By J. C. Stevens and Calvin V. Davis

Sections 1 to 10 and 12 to 15 are all more or less common to the subject of hydroelectric developments. Space requirements do not permit more than a broad treatment of project types in this section, with the approach made largely from the viewpoint of project planning rather than that of detailed design.

POWER FROM FLOWING WATER

We may change the form of energy, but we can neither create nor destroy it. Water will work for us only to the extent that work has been performed on it. We cannot realize all the potential inherent work of the water because there are inevitable conversions of energy to nonuseful forms that we style *losses*.

In the hydrologic cycle, water is evaporated from the oceans and carried inland in the form of vapor by air currents. Cooling by adiabatic expansion of these air currents deflected upward by mountain ranges and by other means causes condensation of its vapor and precipitation as rain, snow, dew, on the land from whence it flows back to the ocean only to repeat the cycle. The work done on it by the energies of the sun, winds, and cooling forces places it on the uplands where work may be extracted from it in its descent to the oceans in direct correspondence to the work expended in putting it there.[1]

Energy and Work. Energy is the capacity to perform work. It is expressed in terms of the product of weight and length. The unit of energy is the product of a unit weight by a unit length, *i.e.*, the foot-pound, the gram-centimeter, the kilogram-meter.

Work is utilized energy and is measured in the same units as energy. The element of time is not involved.

Water in its descent to the oceans may be temporarily held in snow banks, glaciers, lakes and reservoirs, and in underground storage. It may be moving in sluggish streams, tumbling over falls, or flowing rapidly in rivers. Some of it is lost by evaporation, deep percolation, and transpiration of plants. Only the energy of water that is in motion can be utilized for work.

The energy of water exists in two forms; (1) potential energy, that due to position or elevation, and (2) kinetic energy, that due to its velocity of motion. These two forms are theoretically convertible one to the other. Energy may be measured with reference to any datum. The maximum potential energy of a pound of water is measured by its distance above sea level. The ocean has no potential energy because there is no lower level to which its water can fall. The energy of ocean waves can of course be realized.

The potential energy of a given volume of stored water with reference to any datum is the product of the weight of that volume and the distance of its center of gravity above that datum.

A rectangular tank of water of 100 sq ft surface and 20 ft deep whose water surface is 100 ft above sea level has a potential energy of $100 \times 20 \times 62.5 \times 90 = 1{,}125 \times 10^4$

[1] See Sec. 25 for a more detailed discussion of the hydrologic cycle.

ft-lb. This potential energy cannot perform work until it is set in motion. If a stream flows out of that tank and connects with a pipe supplying water to a perfect turbine, $1{,}125 \times 10^4$ ft-lb of work may be performed by the turbine as the tank empties—the potential energy has been converted to kinetic energy.

Power is utilized energy per unit of time, or the rate of performing work, and is expressed in horsepower, 550 ft-lb/sec, or kilowatts, 737 ft-lb/sec. The power from the tank of the preceding example will be at a decreasing rate because the head and flow diminish as the tank empties. Assume the outflow for the first second is 100 cu ft. The surface of the tank would be lowered 1.0 ft, and the center of gravity (head) of that 100 cu ft is 99.5 ft. The energy utilized in this first second, therefore, is 621,000 ft-lb, or 1,130 hp.

Now assume that a stream flows into the tank as fast as it is drawn off—a constant discharge of 100 cfs may then be passed through the turbine under a constant head of 100 ft, for the surface is not lowered and a constant output of 625,000 ft-lb/sec may be realized from our perfect turbine, equivalent to 1,136 hp, or 848 kw.

The potential energy of a stream of water at any cross section must be measured in terms of power, in which time is an indispensable element. It is the weight of water passing per second × the elevation of its water surface (not center of gravity) above the datum considered. The kinetic energy of a unit weight of the stream is measured by its velocity. It must also be measured in terms of power since velocity involves time. It is the weight per second times the velocity head, *i.e.*, the height the water would have to fall to produce that velocity.

If the water of the preceding example were drawn off at velocity of 10 fps, the surface elevation of the outlet channel would have to be $V^2/2g = 1.55$ ft lower than that in the tank in order to produce that velocity and the kinetic energy would be $6{,}250 \times 1.55 = 9{,}650$ ft-lb/sec. The total energy of a stream is the sum of its potential and kinetic energies. Thus the outlet stream has a total energy of 6,250 lb/sec × 98.45 = 615,350 potential plus, 9,650 kinetic, or a total of 625,000 ft-lb/sec of total energy. At the perfect turbine, all the potential energy has been converted to kinetic energy and the velocity head is 100 ft.

Of course the perfect turbine does not exist. Some of the potential energy is converted into heat by friction and turbulence so that the useful part is less than the theoretical potential.

Energy Line. The energy head is a convenient measure of the total energy of a stream of constant discharge at any particular section. It is the elevation of the water surface, potential energy, plus the velocity head, kinetic energy, of a unit weight of the stream. Although every unit of the stream has a different velocity, that usually considered is the velocity head corresponding to the mean velocity of the stream.[1] If the stream is flowing in a pipe, the energy head is the elevation of the pressure line, or the height to which water would stand in risers, plus the velocity head of the mean velocity in the pipe.

[1] Owing to the variable velocity distribution, an energy coefficient should be applied to the velocity head of the mean velocity in order to determine the true energy head. That is,

$$H_e = C_e \frac{V^2}{2g}$$

If the velocity distribution is known, the energy coefficient may be found from

$$C_e = \frac{\int v^3\, dA}{AV^3}$$

where v is mean velocity through the elementary area dA and V is the mean velocity for the stream $= Q/A$. This coefficient is always positive. In straight conduits of established regimen, it may vary from 1.02 to 1.10; at bends or where otherwise disturbed, it may reach a value as high as 2.0 or greater. Because of the lack of knowledge concerning velocity distribution, it has generally been ignored.

A line joining the energy heads at all points is the energy line. The energy lines would be horizontal if the energy converted to heat were included. Energy converted to heat however is considered lost; hence the energy line always slopes in the direction of flow and its fall in any length represents losses by friction, eddies, or impact in that length. Where sudden losses occur, the energy line drops more rapidly. Where only channel friction is involved, the slope of the energy line is the friction slope.

Figure 1 illustrates the principles of the foregoing example. The potential energy head of the tank full of water without inflow or outflow is that of the center of gravity of the tank of water Z. With inflow and outflow equal, however, the potential energy head is H. As the water passes into the canal, a drop of the water surface equal to the velocity head in the canal $V_1^2/2g$ must occur. At the entrance to the pipe line, an entrance loss h_1 is encountered as well as an additional drop for the higher velocity in the pipe. At any point in the pipe line, the pressure head h_p will be shown in a riser.

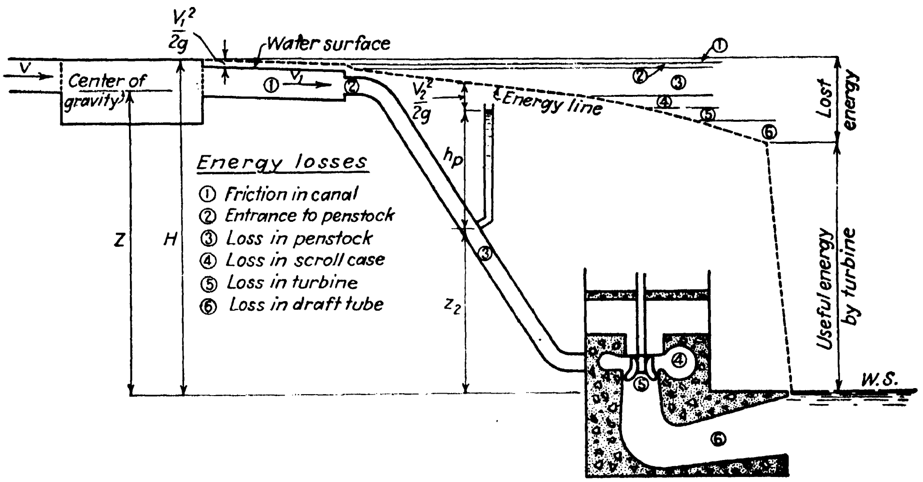


FIG 1.—Energy relations in a typical hydroelectric plant.

The energy head at any point is the pressure head plus the velocity head, and the line joining the energy heads is the energy line. The energy lost (converted to heat) is the sum of friction, entrance, bend, and other losses in all the conduits, including the turbine and draft tube. The useful energy is that of the power developed by the turbine. The sum of the useful energy and the lost energy must equal the original potential energy.

The Bernoulli theorem expresses the law of flow in conduits. For a constant discharge in a closed or open conduit, the theorem states that the energy head at any cross section must equal that at any other downstream section plus the intervening losses. Thus above any datum

$$Z_1 + \frac{V_1^2}{2g} = Z_2 + \frac{V_2^2}{2g} + h_c \quad (1)$$

In Fig. 2, Z is the elevation of a free water surface above datum whether it be in a piezometer tube or a quiescent or moving surface of a stream, V is the mean velocity, h_c the conduit losses between the two sections considered, and e the energy head above the chosen datum.

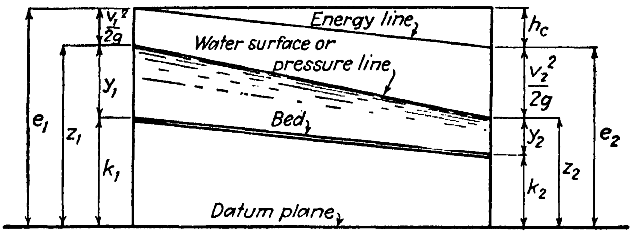
Obviously Z may be made up of a number of elements such as elevation of stream bed or pipe invert above datum k, pipe diameter D, depth in open channel y, or pressure

head above crown of pipe h. Frequently k and h are measured to the center line of the pipe, but if the pipe is large a distinction is necessary.

Head. *There are several heads involved in a hydroelectric plant which are defined as follows:*[1]

Gross head, simultaneous difference in elevation of the stream surfaces between points of diversion and return.

Operating head, simultaneous difference of elevations between the water surfaces of the forebay and tailrace with allowances for velocity heads.



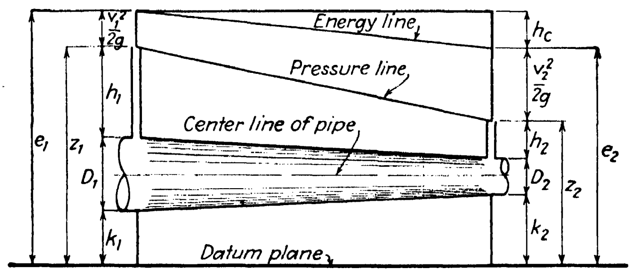


FIG. 2.—Energy relations in open and closed conduits.

Net or effective head, has different meanings for different types of development as follows:

1. For an open-flume turbine, the difference in elevation between (1) headwater in the flume at a section immediately in advance of the turbine plus velocity head, and (2) the tail water plus velocity head.

2. For an encased turbine, the difference between (1) elevation corresponding to the pressure head at entrance to the turbine casing plus velocity head in the penstock at the point of measurement, and (2) the elevation of the tail water plus velocity head at a section beyond the disturbances of exit from draft tube.

3. For an impulse wheel, including its setting, the difference between (1) elevation corresponding to the pressure head at entrance to the nozzle plus velocity head at that point, and (2) the elevation of the tail water as near the wheel as possible to be free from local disturbances. When considered as a machine, the effective head is measured from the lowest point of the pitch circle of the runner buckets (to which the jet is tangent) to the water surface corresponding to the pressure head at entrance to the nozzle plus velocity head.

[1] A.S.M.E. Power Test Code, series 1926.

Strictly speaking, the various heads above are the differences in energy heads. For the gross head, the velocities in the stream are generally disregarded, as are the velocity heads in the tailrace for the operating head. The net head, however, is important in determining efficiency tests of a turbine in its setting; hence it is important to use the difference in energy heads at entrance and exit of the setting.

The net head includes the losses in the casing, turbine, and draft tube, for they are charged to the efficiency of the wheel.

Formulas for the net head of the three cases above are as follows:

For one cased reaction wheel,

$$h = \left(Z_1 + \frac{V_1^2}{2g}\right) - \left(Z_2 + \frac{V_2^2}{2g}\right) = e_1 - e_2 \tag{2}$$

where Z_1 = elevation of pressure head at entrance of turbine casing.
V_1 = mean velocity at entrance of turbine casing.
Z_2 = elevation of tail water at draft-tube exit.
V_2 = mean velocity in the draft tube at its exit.
e_1 and e_2 = the respective energy heads.

For an open flume setting of a reaction wheel, the expression is the same as in (2) but the quantities have slightly different meanings: Z_1 is the elevation of water surface in the open flume just upstream of the turbine, V_1 the mean velocity in flume at that section, Z_2 the elevation of water surface in tailrace at draft-tube exit, and V_2 the mean velocity in the draft tube at its exit.

For the impulse wheel,

$$h = Z_1 + \frac{V_1^2}{2g} - Z_2 \tag{3}$$

in which Z_1 = elevation of pressure head at entrance to nozzle casing.
V_1 = velocity at the same point.
Z_2 = elevation of tailrace near the wheel.

The elevation of the nozzle above tailrace and the velocity head in the tailrace are lost as there must be clearance above the tail water for the wheel to revolve.

Efficiencies of elements composing a hydroelectric system are all measured as the ratio of energy output to input or of useful to total energy.

No element is perfect, its functioning involves lost energy (conversion to heat). The efficiency of a plant or system is the product of the efficiencies of its several elements, thus

$$E_s = E_c E_t E_g E_u E_l E_d \tag{4}$$

where E_s is the over-all system efficiency made up of the product of the several efficiencies of the conduits—canal, penstocks, tailrace, E_c; turbines, including scroll case and draft tube, E_t; generators, including exciters, E_g; step-up transformers, E_u; transmission lines, E_l; step-down transformers, E_d. Formula (4) expresses the over-all efficiency from the river intake to the distribution switches at the substation. To this could be added the efficiency of the distribution system, even to the customer's meters, his lights, water heaters, ranges, motors, etc.

For a constant discharge, the hydraulic efficiencies of the several elements can be expressed in terms of elevations or head above a given datum, and since that datum may be arbitrary such efficiencies will have different values depending upon the datum of reference. If all such efficiencies were referred to sea level, plants at low levels would have higher efficiencies than those at higher altitudes. Such a condition is, of course, intolerable. In effect, the efficiency of an element is the ratio of (1) total energy less losses to (2) total energy, but the datum of reference must be stated.

For purposes of illustration, the following analysis is presented. For the headworks of a system, there is a loss through the control gates in passing from stream to canal, and the efficiency becomes

$$\frac{Z_1 + \frac{V_1^2}{2g}}{Z_o} = \frac{e_1}{Z_o} \tag{5}$$

For the canal, the loss is mostly channel friction

$$\frac{Z_f + \frac{V_f^2}{2g}}{Z_1 + \frac{V_1^2}{2g}} = \frac{e_f}{e_1} \tag{6}$$

For the penstock, the loss is entrance and pipe friction

$$\frac{Z_t + \frac{V_t^2}{2g}}{Z_f + \frac{V_f^2}{2g}} = \frac{e_t}{e_f} \tag{7}$$

For the turbine, the loss is entrance, friction, impact and eddies in casing and draft tube

$$\frac{Z_d + \frac{V_d^2}{2g}}{Z_t + \frac{V_t^2}{2g}} = \frac{e_d}{e_t} \tag{8}$$

For the tailrace, the loss is eddying at draft-tube exits and channel friction

$$\frac{Z_r + \frac{V_r^2}{2g}}{Z_d + \frac{V_d^2}{2g}} = \frac{e_r}{e_d} \tag{9}$$

In the foregoing expressions, Z represents elevation of water surface and e of energy heads above datum, subscript $_1$ refers to the head of the canal below control gates, f to forebay, t to the entrance of the turbine, d to the draft-tube exit, and r to the river at its junction with the tailrace.

Z_o is the elevation of the normal river surface at the intake before water enters the canal (river velocities are neglected), and, as this may vary, the canal gates are manipulated to hold a given elevation in the canal intake for a given discharge. The efficiency of the headworks may therefore be variable even for a constant discharge. The elevation of the water surface at the head of the tailrace (exit of draft tube) Z_d may also be affected by the river stage, and this is reflected in variation in the efficiency of the turbine. The efficiencies of all the other elements will be sensibly constant for a constant discharge if canal, racks, etc., are kept clean and in good order.

Turbine efficiencies are specified for certain flows under certain heads and speeds and obviously must not vary arbitrarily with an arbitrary datum. The datum of reference is therefore considered to be moved to the water surface just downstream of the draft-tube exit where major turbulences have subsided.

The expression in Eq. (8) therefore must be modified by introducing a power head that represents the useful energy output of the wheel and is determined by tests; thus

$$\frac{h_p}{\left(Z_t + \frac{V_t^2}{2g}\right) - \left(Z_d + \frac{V_d^2}{2g}\right)} = \frac{h_p}{e_t - e_d} \tag{10}$$

becomes the correct expression for the efficiency of the turbine and setting under constant discharge, the datum of reference being the water surface of the tailrace at exit of the draft tubes.

With the preceding notation, the efficiency of the entire plant may be expressed independent of an arbitrary datum; thus

$$\frac{h_p}{Z_o - Z_r} \tag{11}$$

velocity heads in the river at the headworks and at its junction with the tailrace being neglected.

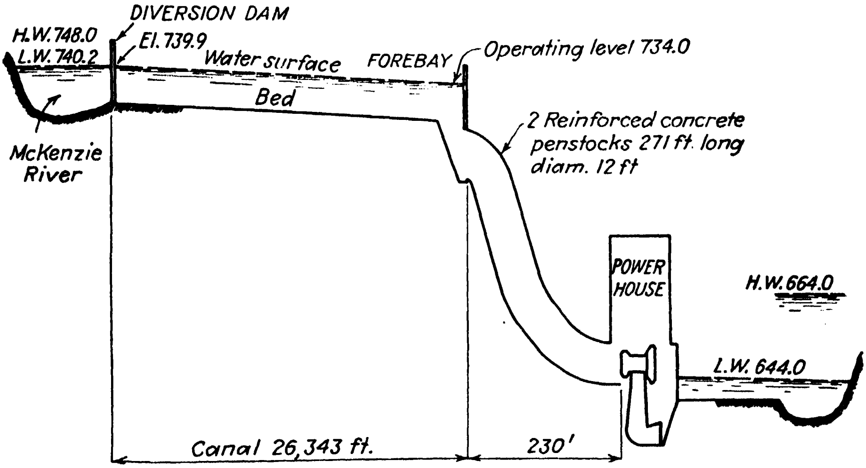


FIG. 3.—Leaburg plant, condensed profile.

The Leaburg hydroelectric plant on the McKenzie River near Eugene, Ore., is a typical plant having all the foregoing elements. Figure 3 is a condensed profile of the system. The full load discharge taken from the river is 2,200 cfs. The total drop in the river from the intake to the end of the tailrace at normal stages is 96.2 ft; hence the total potential energy is 24,100 theoretical hp.

The efficiency of each element of the Leaburg plant is given in Table 1 referred to the low water level at the junction of the tailrace with the river. If we had used sea-level elevations, by adding 644.0 ft to all heads, the indicated efficiencies would have been considerably greater. As used, the efficiency of the tailrace is nearly zero because the energy head at the end is practically zero when referred to the datum indicated, although there is yet a potential energy head of 644 ft above sea level.

Where dam and powerhouse are practically one, as at Hoover on the Colorado River (Fig. 10), the over-all plant efficiency is greater than at Leaburg because of the elimination of canal and tailrace.

The term *efficiency* is not often used for plant elements other than the generating equipment. It has been given herein merely to illustrate the relationship of each element to the whole in this regard and to show the effect of the datum of reference on indicated efficiencies. In practice, the lost head in each such element is found and deducted from the gross head to obtain the net power head.

The efficiency of a turbine depends upon the type, speed, head, and load. For moderate heads, the propeller type of turbine with adjustable blades has shown high efficiencies over a wide range of load and head. Most wheels show maximum efficiencies at about 80 per cent gate opening.

The efficiency of generators are generally greater the larger the unit, but they too depend upon the load carried. The efficiency of transformers increases rapidly with capacity and load within certain limits, whereas that of transmission lines increase with capacity but decrease with load.

The over-all efficiency of a plant is the product of the instantaneous efficiencies of its several pieces of equipment referred to the gross head on the water wheels. It obviously varies with capacity of units, head, load, and the number of units in service. Plant efficiencies are not always observed and frequently involve many complexities. In general, the plant efficiency is the ratio of the energy output of the generator to the water energy corresponding to the gross head (difference of forebay and tailrace levels) and that discharge through the wheels that results in the maximum efficiency, or it

TABLE 1.—EFFICIENCIES OF THE SEVERAL ELEMENTS OF THE LEABURG HYDROELECTRIC DEVELOPMENT

Reference datum is water surface at end of tailrace

	Low water		High water
Headworks:			
River WS Z_o, ft	96.2		104.0
Canal WS Z_1, ft	95.9		95.9
Velocity head $V_1^2/2g$, ft	0.2		0.2
Energy head e_1, ft	96.1		96.1
Efficiency of headworks e_1/Z_o, %	99.9		92.2
Canal:			
At head e_1, ft		96.1	
At forebay Z_f, ft		90.0	
Velocity head $V_f^2/2g$, ft		0.1	
Energy head e_f, ft		90.1	
Efficiency of canal e_f/e_1, %		93.7	
Penstocks:			
Energy head at entrance e_f, ft		90.1	
Pressure head at scroll case entrance Z_t, ft		86.0	
Velocity head $V_t^2/2g$, ft		2.5	
Energy head e_t, ft		88.5	
Efficiency of penstock e_t/e_f, %		98.2	
Tailrace:			
Elevation at end Z_r, ft	0		20.00
Velocity head $V_{r_2}/2g$, ft	0.2		0
Energy head e_r, ft	0.2		20.00
Elevation at head Z_d, ft	0.8		20.1
Velocity head at draft tube exit $V_d^2/2g$, ft	0.8		0.8
Energy head e_d, ft	1.6		20.9
Efficiency e_r/e_d, %	12.5		95.7
Turbines (for constant discharge of 1,100 cfs each):[1]			
Elevation of energy head:			
At entrance of turbine casing e_t, ft	88.5		88.5
At draft tube exit e_d, ft	1.6		20.9
Net head $e_t - e_d$, ft	86.9		67.6
Useful power head of turbine h'_p, ft	78.2		57.5
Efficiency $h_p/(e_t - e_d)$, %	90.1		85.0
For the entire plant:			
Power head of the turbine h_p, ft	78.2		57.5
Total head $Z_o - Z_r$, ft	96.2		84.0
Efficiency $h_p/(Z_o - Z_r)$, %	81.3		68.5

[1] From tests, bhp = 9,950 for 88-ft head and 1,100 cfs for turbine. For 86.8-ft head, bhp = 9,770 (proportional to head for constant discharge) which corresponds to an energy head for the turbine with 1,100 cfs of 78.2 ft. No tests were made from which the bhp under the lower head of 67.6 ft could be determined directly; it has been estimated.

may be for that discharge and load for which the indicated efficiency of the turbine is a maximum. In any case, it should be clearly defined.

Power Formulas. A cubic foot per second of water at 62.5 lb/cu ft falling 8.8 ft is equivalent to 1 hp and falling 11.8 ft is equivalent to 1 kw, therefore

$$\text{Theoretical hp} = \frac{Qh}{8.8} \tag{12}$$

$$\text{kw} = \frac{Qh}{11.8} \tag{13}$$

If E is the efficiency of the plant, the power that can be realized is given by

$$\text{hp} = \frac{Qh}{8.8} E \tag{14}$$

$$\text{kw} = \frac{Qh}{11.8} E \tag{15}$$

In the expression, E is the plant efficiency and h is the head on the water wheel defined by Eqs. (2), (3), or (4) as may be appropriate.

Useful energy is generally measured in terms of kilowatthours, occasionally in terms of horsepower-hours. Where the discharge and head are constant,

$$\text{hp-hr} = \frac{Qh}{8.8} Et \tag{16}$$

$$\text{kwhr} = \frac{Qh}{11.8} Et \tag{17}$$

where t is the time in hours for which the flow and head are constant or for which Q and h are average values. When the flow and head vary materially, the period considered is divided into smaller time intervals for which they are sensibly constant.

The horsepower-year and the kilowatt-year are terms sometimes used for power sales. On a 100 per cent load factor the relationships are

$$1 \text{ hp-year} = 0.746 \text{ kw-year} = 8{,}760 \text{ hp-hr} = 6{,}540 \text{ kwhr}$$

Power from any particular plant or system is limited by the capacity of the installed equipment. It may be limited also by the available water supply, head characteristics, and storage.

Firm power, or primary power, is that load, within the plant's capacity and characteristics, that may be supplied virtually at all times. It is fixed by the minimum stream flow, having due regard for the amount of regulating storage available and the load factor of the market supplied.

Surplus power, or secondary power, is all available power in excess of the firm power. It is limited by the generating capacity of the plant, by the head, and by the water available in excess of the firm water. A certain amount of surplus power may very closely approximate firm power in being available a large percentage of the time, whereas other amounts may be available for shorter periods.

Dump power is surplus power sold with no guarantee as to continuity of service, *i.e.*, it is delivered when, as, and if available.

Secondary power is another term for surplus power.

The capacity of a power plant is not easily defined. Name-plate capacity or rated capacity of a turbine is usually given in horsepower for a given head, discharge, and speed at which the best efficiency obtains. Obviously each of these quantities may vary within definite limits. The rated capacity of a-c generators is usually stated in

terms of definite speed, power factor, and temperature rise and is usually given in kilovolt-amperes. Each of these quantities may also vary within definite limits.

The A.I.E.E. definition of generating station capacity is "the maximum net power output that a generating station can produce without exceeding the operating limit of its component parts." The station or plant capacity can therefore be determined for a given station. It may be stated for a peak load over a given period as 15 min or 1 hr or for a continuous load. It would be higher for short periods than for continuous service if storage regulation exists but is limited by the temperature rise of generators. Until the station capacity has been fixed, the various factors having to do with capacity cannot acquire definite meanings. Where the capacity of a plant has not been fixed, it is customary to take name-plate capacity of generators as the plant capacity, which is often called *installed capacity.*

The average load of a plant or system during a given period of time is a hypothetical constant load over the same period that would produce the same energy output as the actual loading produced (A.I.E.E.).

The peak load is a maximum load consumed or produced by a unit or a group of units in a stated period of time. It may be the maximum instantaneous load or a maximum average load over a designated interval of time.

The maximum average load is generally used. In commercial transactions involving peak load, it is taken as the average load during a time interval of specified duration occurring within a given period of time, that time interval being selected during which the average power is greatest (A.I.E.E.).

The load factor is an index of the load characteristics. It is the ratio of the average load over a designated period to the peak load occurring in that period. It may apply to a generating or a consuming station and is usually determined from recording power meters. We may thus have a daily, weekly, monthly, or yearly load factor; it may apply to a single plant or to a system. Some plants of a system may be run continuously at a high load factor, whereas variations in load are taken by other plants of the system, either hydro or steam. Hydro plants designed to take such variations must have sufficient regulating storage to enable them to operate on a low load factor. They are often called *peak-load plants.* Operating on a 50 per cent load factor, there must be sufficient storage to enable such a plant, in effect, to utilize twice the inflow for half the time: on a 25 per cent load factor, the plant should be able to utilize four times the inflow for a quarter of the time, etc.—the lower the load factor, the greater the storage required.

As applied to the consumption of power, the load factor is the ratio of the average to maximum demand during any given period. It may apply to a single motor, an industrial plant, a city, or a consuming system. The maximum demand may be the highest consumer load during a 5-, 10-, or 15-min interval, the average of the two highest 5-min intervals, or otherwise as fixed by the management or by regulatory bodies. It is usually determined from a demand meter or taken from a graphic wattmeter. The period usually considered is a month for purposes of billing, although the sale rates of power are often based on the yearly load factor of the consumer.

The capacity factor is a measure of plant use. It is the ratio of the average load to the plant capacity. It may be computed for a day, month, year, or any other period of time. When the peak load just equals the plant capacity, the capacity factor and load factor are obviously the same. If the maximum demand is less than the plant capacity, the capacity factor may be either greater or less than the load factor, depending largely on the load factor itself.

The utilization factor is a measure of plant use as affected by water supply. It is the ratio of energy output to available energy within the capacity and characteristics of the plant. Where there is always sufficient water to run the plant at capacity, the

utilization factor is the same as the capacity factor. A shortage of water, however, will curtail the output and may either decrease or increase the utilization factor according to the plant load factor.

These several factors can be determined for any plant by analyzing past performance. They may also be forecast approximately by a complete analysis of stream flow and plant characteristics. To illustrate this subject matter, the power characteristics of the Bonneville development on Columbia River have been chosen. Two units of 43,200 kw and eight of 54,000 kw name-plate capacity each have been in service since Dec. 14, 1943. The plant capacity is 564,000 kw and the peaking capacity 570,000 kw. No more units are to be added.

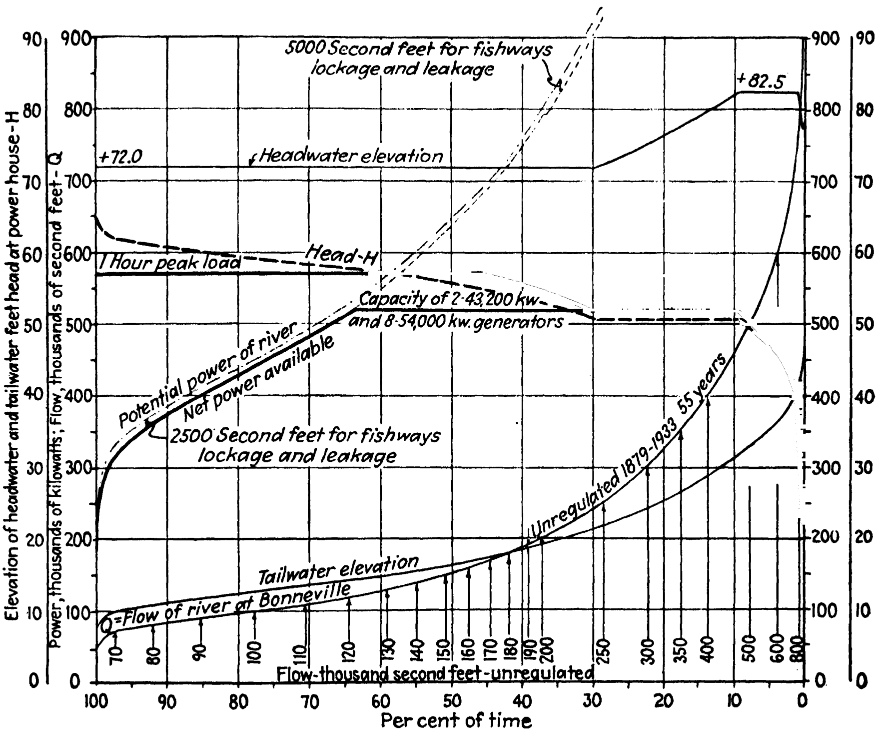


FIG. 4.—Power characteristics of the Bonneville plant.

Figure 4 shows power characteristics of this development, including the flow-duration curve for Columbia River for a 55-year period, the natural tail-water levels, the headwater levels as they are to be controlled by the crest gates on the dam, and the potential power output.

The power from this plant has three limitations: (1) by water supply, the minimum occurring during the winter months; (2) by the head which is diminished materially by the spring floods; and (3) by the capacity of the generators.

The combined rating of the generators is 576,000 kva, or 518,400 kw at 0.9 power factor based on 80°C temperature rise. The rated capacity can be exceeded for short periods of time, but this point is not covered by the specifications. The maximum plant capacity will be ultimately about 570,000 kw for a 1-hr peak.

The potential energy output during the average water year as indicated by the area

under the heavy line marked "net power available" is 4.15 billion kwhr, equivalent to an average load of 474,000 kw. The potential capacity factor of the plant, therefore, is $^{474}/_{570} = 83$ per cent for a 100 per cent load factor. For the same load factor, the output will be all there is available; hence the utilization factor is $^{474}/_{474} = 1.00$, obviously it is the same as the load factor.

There is no seasonal storage at Bonneville[1] but there is ample pondage to take care of daily regulation, about 200,000 acre-ft being available. This is sufficient to enable the plant to operate on about 55 per cent load factor.

Table 2 gives potential use data for the Bonneville plant. The last part of the table is of interest. Although the power available 100 per cent of the time is only 185,000 kw, it is permissible to consider firm power as somewhat greater. In this case, the power available 97 per cent of the time has been considered as firm power and the remainder as surplus power.

Table 2.—Potential Power Characteristics of the Bonneville Plant

Load factor, %	Output, billion kwhr	Average load, 1,000 kw	Capacity factor	Utilization factor
100	4.15	474	0.83	1.00
90	3.74	426	0.74	0.90
80	3.32	379	0.66	0.80
70	2.90	331	0.58	0.70
60	2.49	284	0.50	0.60
50	2.08	237	0.41	0.50

Load factor, %	Percentage of time power is available							
	97	95	90	85	80	70	60	55
	Thousands of kilowatts							
100	315	330	364	392	417	448	482	518
90	350	367	405	435	463	497	535	570
80	394	412	455	490	521	560		
70	450	471	520	560				
60	525	570						
55	570							

	Firm power	Surplus Power						
100	315	15	49	77	102	113	167	203
90	350	17	55	85	113	157	185	220
80	394	18	61	96	127	166	0	0
70	450	21	70	110	0	0	0	0
60	525	45	0	0	0	0	0	0
55	570	0	0	0	0	0	0	0

[1] The seasonal storage at Grand Coulee will also be available for all plants downstream, of which there are now two, Rock Island (Puget Sound Power & Light Co.) and Bonneville.

In determining the power available for any given percentage of time (from Fig. 4), it must be remembered that power is reduced at both extremities of the time scale. For example, the 70 per cent line indicates 480,000 kw on the left of the diagram, but this amount of power is not available for 6 per cent more of the time on account of high water; hence the power available 70 per cent of the time is in reality that indicated as available 76 per cent of the time, or 443,000 kw. Obviously the surplus power for any given time column is the difference between the firm power and the total power available for that percentage of time.

TYPES OF DEVELOPMENTS

Hydroelectric developments may be classified according to function, such as multiple-purpose or single-purpose and base-load or peak-load; according to hydraulic characteristics, such as run-of-river or storage; and according to the physical characteristics of the structures, such as indoor or outdoor or, more recently, above ground or underground. Multiple-purpose and single-purpose projects are described in Sec. 1, Reservoirs.

Where storage capacity is available to regulate the river and where flooding is infrequent, plants may be fitted to the base of the load curve. Plants having a combination of limited storage and head are frequently fitted to the peak of the load curve. The terms *base load* and *peak load* are not sharply definitive of a plant's characteristics, however, as the position which the generation occupies on the system load curve may vary widely with the conditions of flow not only at the plant under consideration but also at other plants in the system.

There follow descriptions of a wide variety of project types. A study of these will indicate the difficulty of categorically classifying each project.

In determining the proper type for a proposed development, the entire plan must be studied in the light of market conditions, including power demand, interconnection with other plants, future growth, available funds, and in light of physical conditions such as foundations for structures, storage facilities, climatic characteristics, accessibility. The final conclusion must be a composite of all these factors evaluated by sound engineering and business judgment.

A hydroelectric development includes in some form a water-diverting structure, conduit to carry water to the wheels, turbines and governors, generators, control and switching apparatus, housing for the equipment, transformers, transmission lines to distribution centers. In most cases, a forebay or a surge tank is provided in which head regulation is effected. Trash racks and gates are placed at the head of penstocks. Connected to the water-wheel cases are the draft tubes which utilize the head below the wheels, recovering the kinetic energy of the water without undue losses. The draft tube delivers the water to the tailrace, through which it is returned to the stream.

All these structures may be condensed into a short space as when the powerhouse is adjacent to the dam as at Hoover; or they may extend over many miles as in the Tiger Creek development on Mokelumne River, California, which returns water 21 miles below its diversion point.

In the West, hydroelectric developments are often combined with irrigation systems and may have superior, equal, or inferior rights to the water. The Exchequer Dam on Merced River stores 289,000 acre-ft for irrigation. A 33,500-hp hydroelectric plant at the dam generates power when, as, and if water is released for irrigation. The power system of the Salt River Valley Water Users' Assocation in the Phoenix area of Arizona is typical of the dual use of water for irrigation and power.

Most of the large modern developments by the Federal government are for multiple uses as Hoover, for flood control, irrigation, power, silt removal; Bonneville, for

navigation, power; Grand Coulee, for irrigation, flood control, power; Tennessee Valley plants, for navigation, power, flood control, silt removal; Fort Peck, for navigation, flood control, power.

The power from any plant or system may, of course, be used for pumping water for irrigation, but unless water for irrigation is involved as an inherent part of the plant's design, irrigation is not considered one of its multiple uses. A large part of the power at Grand Coulee will be used for pumping water from the storage space behind the dam for irrigation, but unless power were also available for commercial purposes, it would not be considered one of the multiple uses of the development.

Any development should utilize to the utmost such natural advantages of drop in the river, cutoffs across bends, auxiliary channels that may serve as head and tailraces, storage facilities, opportunities for spillway, land space available for operators' quarters and for switching apparatus, and, above all, secure foundations for structures.

Where natural falls are available, the development may consist of a simple diversion structure, forebay, tunnel, or pipe to the wheels that discharge directly into the river below the falls, as at Niagara and Snoqualmie.

Where there is no pronounced fall, two methods may be used to create head: (1) by a dam with adjacent powerhouse in the channel and (2) by a canal whose drop and length are less than that of the stream, ending in a forebay from which penstocks lead to the turbines. Any combination of these two methods may also be used where the dam and canal each develops substantial portions of the power head. Where the head is created solely by a dam, storage is also created and flowage rights must be secured. Where lands above the diversion cannot be flooded, the canalized river type must be adopted.

Multiple-use Developments. Where irrigation is involved, power must be developed before the water is used for irrigation. Power may thus utilize the head at high-diversion or storage dams, drops in canals, or drops from high canals to low-distributaries. In many cases, an exchange of irrigation water from one source to another will admit of power developments. In an irrigated section, power finds a ready market and the full utilization of water resources should prompt the parallel developments for both purposes. The water rights for irrigation in most of the western states are, in general, superior to those for power although power often has the superior right by reason of priority of use.

Where navigation is involved, power is considered a by-product of the development. The Federal government constitutionally exercises complete control over navigable waters, and navigation rights on such streams are superior to all other rights. However, there are all degrees of navigability. Many streams are navigable in theory but not in practice. The theoretical navigability[1] of Colorado River was an important factor in the constitutional right of the Federal government to construct and operate the Hoover project. That right has not been challenged.

The construction of dams and locks for navigation often makes possible substantial power developments as at Bonneville, Wilson, Wheeler, and Dnieprostroy (Russia). Where storage is created for navigation purposes, as at Fort Peck, power is, of course, available from the releases of water for navigation.

Dams to create storage solely for flood control rarely admit of power development. For example, the Miami Conservancy District dams create detention reservoirs whose sole purpose is to reduce the peaks of flood events. They are normally empty, and waters caught during floods are slowly released during and immediately following the flood period. Where capacity for flood control is included with storage for other

[1] **A steamboat actually ascended to above the Hoover Dam site in the early days, and regular boat schedules were maintained between Fort Yuma and the Gulf of California prior to construction of the Southern Pacific railroad.**

purposes, power developments are, of course, possible as at Lake Mead (Hoover Dam) where some 9,000,000 acre-ft capacity is reserved for flood control.

Storage for silt control has heretofore been combined with storage for other purposes. Lake Mead has 4,000,000 acre-ft reserved initially for silt. It is proposed to construct three reservoirs in the Colorado River basin for silt storage to preserve the integrity of Lake Mead in this regard. The streams are flashy and in their natural state are not power streams, but with the storage and heads created by the dams substantial power developments will be possible until the water-storage space is replaced by silt in the 100 or so years of their lives.

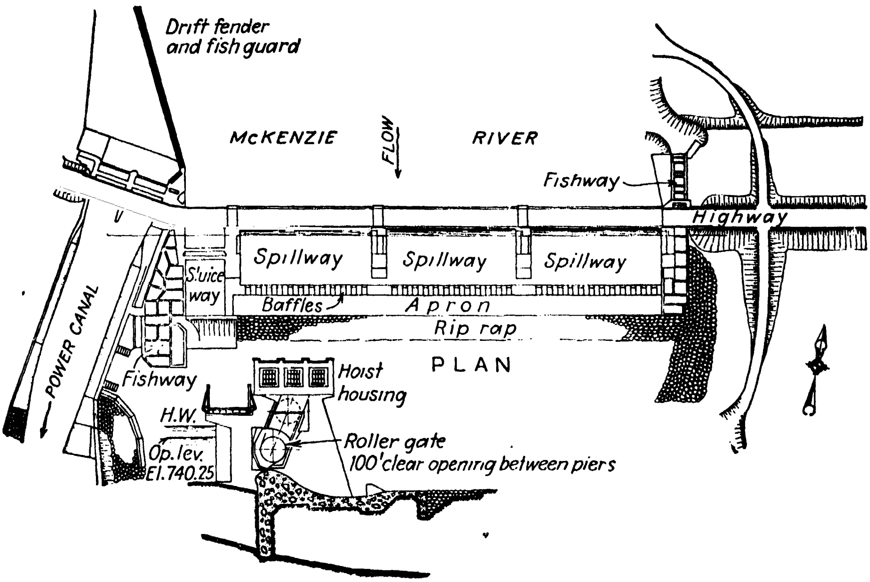


FIG. 5.—Leaburg diversion dam.

Multiple uses of water for domestic supply with power as a by-product are not uncommon, *e.g.*, the Owens River-Mono Lake supply for Los Angeles and the Hetch Hetchy project for San Francisco.

Typical Developments. The following plants have been selected from a long list of hydroelectric developments as being representative of their particular type.

Leaburg. This plant is chosen as an example of a small one having all the essential elements distinctly displayed—dam, canal, forebay, penstocks, powerhouse, and tailrace. It is a river-run plant (*i.e.*, without storage) owned by the city of Eugene, Ore. Development consists essentially of a diversion dam raising the water 20 ft into a 2,200 cfs capacity canal 5 miles long terminating in a forebay. Two 12 ft diameter concrete penstocks 238 ft long deliver water to two 10,000-hp turbines under a 90-ft head. Water is returned to McKenzie River through a tailrace ¼ mile long (see Fig. 3 for profile).

The dam (Fig. 5) is of the ogee overflow type with three 100-ft sections separated by piers. The crest is 8 ft above low water and is surmounted by three 105-ft roller gates controlling the 100-ft openings between piers for an additional height of 12½ ft. There is also a 30-ft Broome gate in a sluiceway with sill at river-bed level near the right end. Flood capacity with all gates open is 100,000 cfs. There is a fish ladder at

each end of the dam. The canal intake, with drift fender, is parallel to the river bank for a length of 180 ft and has an oscillating diaphragm of suspended chains intended to prevent fish from entering the canal. Diversion is through two 24-ft Broome gates. The partially lined earth canal section has a bed width of 30 ft, side slopes 1 on 1½, maximum depth 11.0 ft.

Waste from the forebay is effected through seven siphon spillways, five with nominal capacities of 500 cfs each, one for 250 cfs, and one for 125 cfs. Each penstock entrance is rectangular, 14 by 18 ft, and closed by a Broome gate. The rectangular section gradually changes to circular 12 ft diameter in 37 ft.

The scroll case is formed in concrete, heavily reinforced, and the penstock is reduced from 12 to 10 ft diameter at the entrance of the scroll case in a length of 18 ft. Sets of four piezometers are installed at the beginning and end of this reducer, forming

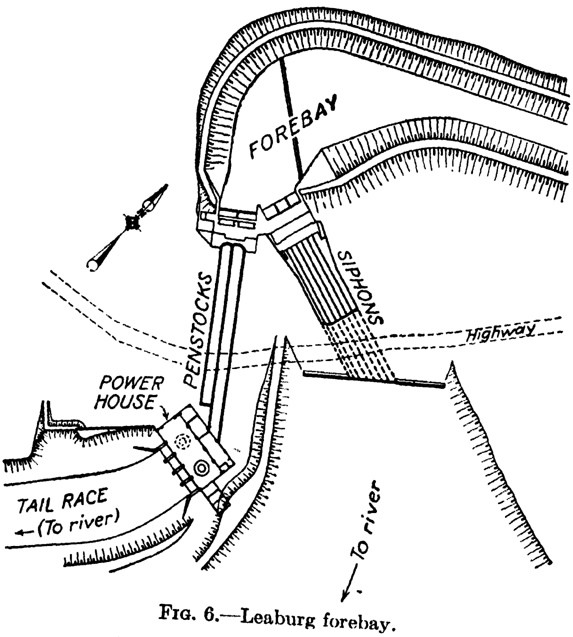


Fig. 6.—Leaburg forebay.

a Venturi section that was calibrated for flow. The draft tube is of the elbow type divided by a splitter pier into two sections beyond the elbow. The diameter at the entrance is 9 ft, and the exit is of two 7½- by 10-ft openings.

The substructure of the powerhouse is a concrete block 82 by 33 ft, in which the draft tubes and scroll cases are molded.

The main superstructure is 80.75 by 31.0 ft inside plan and the lowest point of the draft tube is 38.0 ft below the generator floor. An 82- by 15-ft electric bay is integral with the main building.

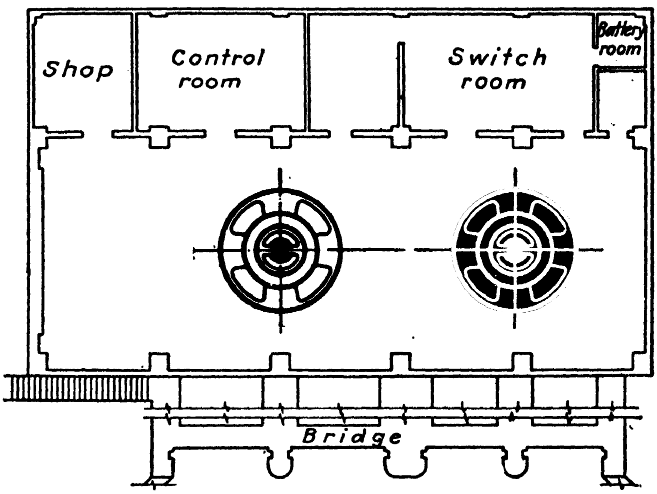
The powerhouse equipment consists of two reaction turbines rated at 9,750 hp at 89 ft head, 225 rpm; a governor; a generator rated at 9,375 kva at 0.80 power factor, 11,500 volts, 394 amp, 225 rpm, with direct-connected exciter; a 30-ton crane; air-compressor system; storage batteries; switchboard; annunciator system; and miscellaneous accessories.

The switching station is outside but adjacent to the powerhouse and contains six 2,500-kva water-cooled transformers 11,000/66,000 volts, circuit breakers, lightning

arresters, and miscellaneous switching equipment. By means of a small truck on a standard-gage track, any transformer may be taken into the powerhouse for untanking by the crane.

Figures 5 to 9 illustrate the general features of the Leaburg plant.

Hoover. This plant, a feature of the Boulder Canyon Project, is located on the Colorado River in Black Canyon, between Arizona and Nevada. It is chosen to illustrate a hydroelectric plant of large capacity. The head is created entirely by the dam, and the powerhouse is located immediately on its downstream side. The dam, powerhouse, and their appurtenances are owned, operated, and maintained by the Bureau of Reclamation. The generating equipment to the outgoing high-tension connections is furnished and owned by the Government. The generating, transmitting, and switching facilities are operated and maintained by the city of Los Angeles and the Southern California Edison Company as agencies under the supervision of a



GENERAL PLAN - MAIN FLOOR

FIG. 7.—Leaburg powerhouse.

Director of Power who is appointed by the Secretary of the Interior. The Boulder Canyon Project is a multiple-use development for flood control, improvement of navigation and regulation of the Colorado River, storage of water for reclamation of public lands, and generation of electrical energy.

The dam has a maximum height of 726.4 ft and a crest length of 1,244 ft. It may raise the water in the river a maximum depth of 583 ft. It is a concrete arch-gravity type, the thickness being 660 ft at the base and 45 ft at the top.

There are two lateral-flow spillways, one at each end of the dam, giving an aggregate length of 800 ft of spillway crest. Each crest is divided by piers into four 100-ft lengths closed by 16-ft drum gates capable of holding the water 24 ft above the spillway crest. Water passing the spillways is conveyed to the river below the powerhouse through two 50-ft concrete-lined tunnels in the canyon walls. The combined capacity of both spillways is 400,000 cfs. The storage capacity of Lake Mead is over 32 million acre-ft to the top of the gates in their highest position, El. 1,229. Active storage capacity, water surface at El. 1,229, is over 29 million acre-ft.

The four main steel header penstocks, each 30 ft in diameter, take water from intake towers (see Fig. 10) on the upstream side of the dam and are accessible in tunnels excavated in the canyon walls. From these header penstocks, fifteen 13-ft

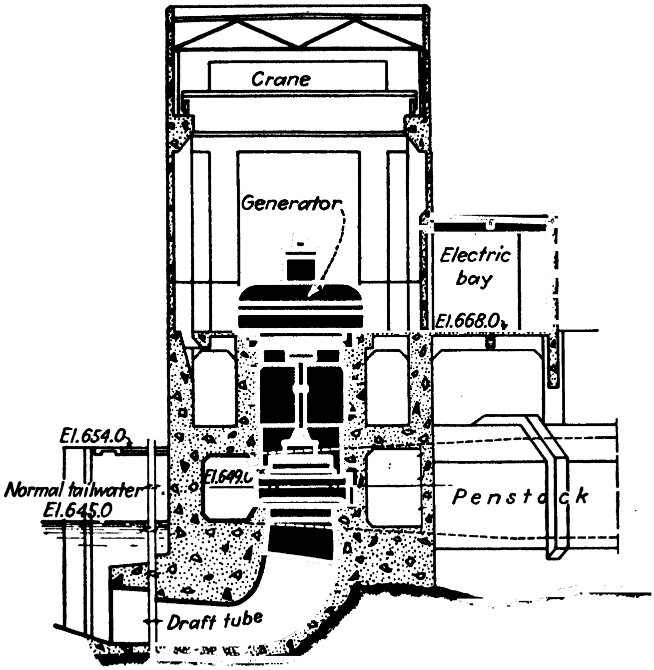


FIG. 8.—Leaburg generating unit.

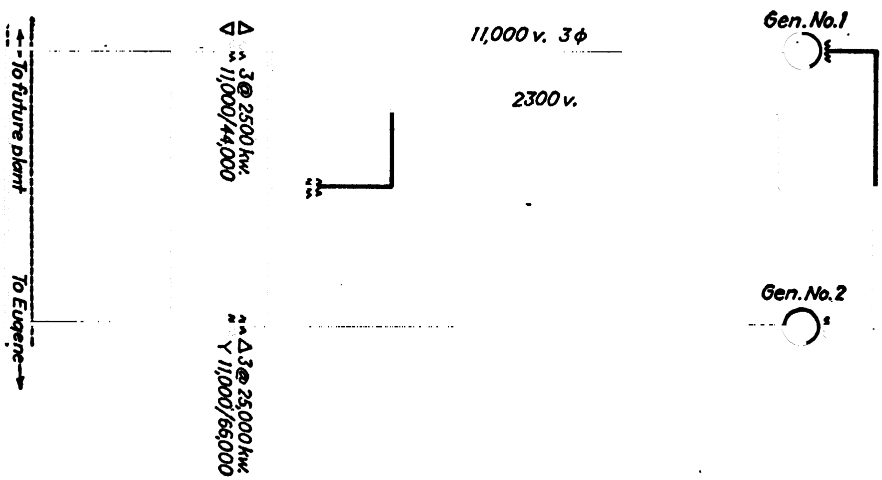


FIG. 9.—Leaburg wiring diagram.

steel branch pipes lead to the fifteen 115,000-hp turbines and another 13-ft pipe is divided into two 9-ft pipes for one 70,000-hp turbine and one 55,000-hp turbine. From the two inside header penstocks, there are taken twelve 8½-ft steel pipes for twelve 84-in. needle valves in the canyon-wall outlet works. Near the end of each of the two outer penstocks is a plug containing six 72-in. needle valves for the further control of releases.

Provision is made for fifteen 115,000-hp, one 70,000-hp, and one 55,000-hp reaction turbines, of which seven of the large units have been installed in the Nevada wing. In the Arizona wing, there are five of the large units and the 55,000-hp unit installed, and two of the large units and the 70,000-hp unit under contract (1951). The head varies between 420 and 590, three of the large units may have two speeds each of 150 and 180 rpm, respectively, by an interchange of runners. The turbine runner is a single casting 14.25 ft in diameter. Guaranteed efficiencies are as follows: full load, 89 per cent; three-fourths load, 91 per cent; one-half load, 83 per cent. The runners are set 10 ft below normal tailrace level, but the water in the draft tube can be displaced 5 ft below the runners by air pressure. There is a hydraulically operated

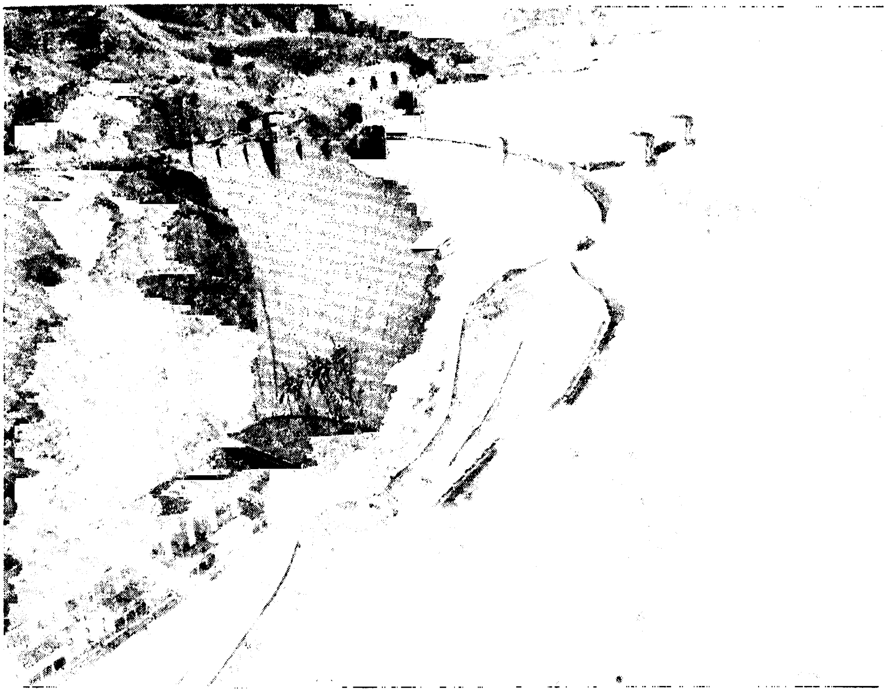


FIG. 10.—Hoover Dam as seen from the Arizona rim of Black Canyon.

butterfly valve in each penstock just upstream of the turbine. The scroll case is made up of steel castings or plate steel buried in concrete.

The generators for the large units are rated 82,500 kva, unity power factor, 16,500 volts, 60 cycles at 180 rpm. Three of these large units originally operated at 150 rpm, 13,800 volts, 50 cycles, but have been changed over to 180-rpm, 16,500-volt, 60-cycle operation. One of the smaller generators is rated 62,500 kva, 80 per cent power factor, 13,800 volts, 60 cycles at 257 rpm, and the other is rated 40,000 kva, unity power factor, 13,800 volts, 60 cycles at 257 rpm. The ultimate plant generating capacity is 1,340,000 kva or 1,327,500 kw, not including station-service generators. All of the units are operating or under contract (1951) except the 82,500-kva generator for unit N8 (Fig. 11). The total weight of a large generator is 2 million lb and the WR^2 is 105 million lb at a radius of 1 ft.

Excitation is obtained by main and pilot exciters on the main shaft. There are also two 3,500-hp station-service generating units in the central wing. Excitation is at 250 volts.

The auxiliary equipment includes a compressed-air system, a fire-protection water system, service and cooling water systems, oil supply and draining systems, governor oil systems, oil lubricating systems, and penstock water system for pressure regulators and water-jet eductors. There is also a pumping system for supplying domestic water to Boulder City under a pressure of 600 psi.

The transforming equipment is different for each power contractor. Figure 13 gives the essential details.

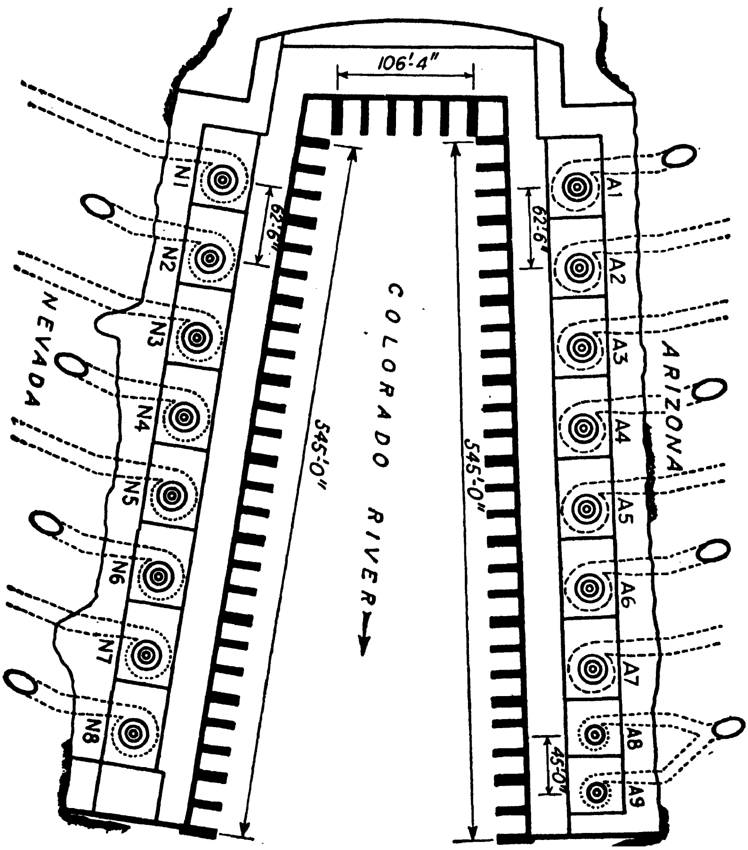


FIG. 11.—Hoover powerhouse.

The Boulder-to-Los Angeles transmission line of the city of Los Angeles consists of two three-phase 60-cycle circuits with a line-to-line voltage of 287.5 kv at the plant and 275 kv at the receiving end. The conductor is segmental hollow copper tube of 1.4 in. diameter, 512,000 cir mils, weighing 1.57 lb/ft and having an ultimate tensile strength of 21,600 lb. Splices are made by a bronze sleeve with double-wedge grip. The total weight of conductor in the 266 miles is 13½ million lb.

Figures 10 to 13 show the general features of this development.

There will be available about 4⅓ billion kwhr per year based on a plant efficiency of 83 per cent after allowing 10 per cent for shortages. This output is subject to an estimated diminution of 8¾ million kwhr each year due to consumption of water by upstream developments. The total installed capacity will be 1,340,000 kva. Rotary

condensers will be used at the receiving end, which with the line capacity will raise the power factor at the plant to unity.

Safe Harbor. This plant is chosen as representative of the low-head type where the head is created solely by the dam, of which the powerhouse is a part. It is on

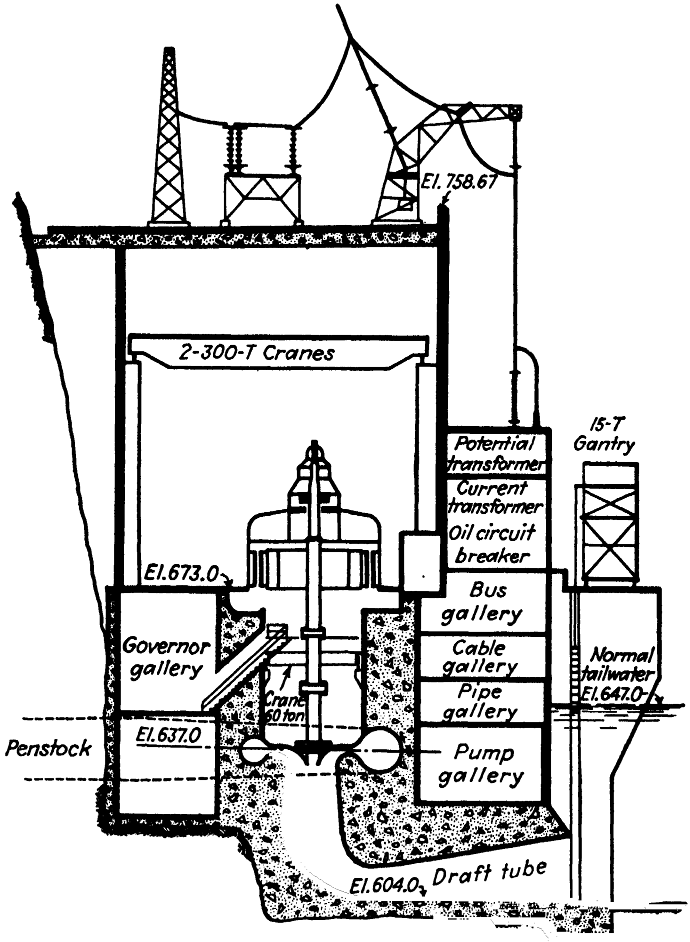


FIG. 12.—Hoover generating unit.

Susquehanna River 17 miles above the Pennsylvania-Maryland line. There are four major plants on this river in downstream order as follows:

Name	Operating company	Ultimate capacity, hp	Year present installations completed
York Haven	Metropolitan Edison Co.	29,000	1904
Safe Harbor	Safe Harbor W. P. Corp.	510,000	1940
Holtwood	Penn. Water & Power Co.	158,000	1924
Conowingo	Susquehanna Electric Co.	594,000	1928

The Safe Harbor plant has about 69,000 acre-ft of storage available but seldom is more than 15,000 used on account of the sacrifice of head. Freshets are anticipated when possible by drawing the pond down 5 to 7 ft, unwatering corresponding storage of 32,000 and 41,000 acre-ft, respectively. The mean annual flow of the Susquehanna River at this point is 36,100 cfs; the median flow, 21,600 cfs; maximum recorded flood, 787,000 cfs, Mar. 19, 1936.

The dam and powerhouse have a total length of 4,869 ft, of which 920 ft is occupied by the powerhouse.

The west spillway, occupying 1,356 ft, is an overflow section with crest 24 ft above normal tail water, and 32 ft more is controlled by twenty-four 48-ft Stoney crest gates. The east spillway occupies 454 ft of similar design with four similar crest gates and four double-leaf remote-controlled gates for close regulation of the pond

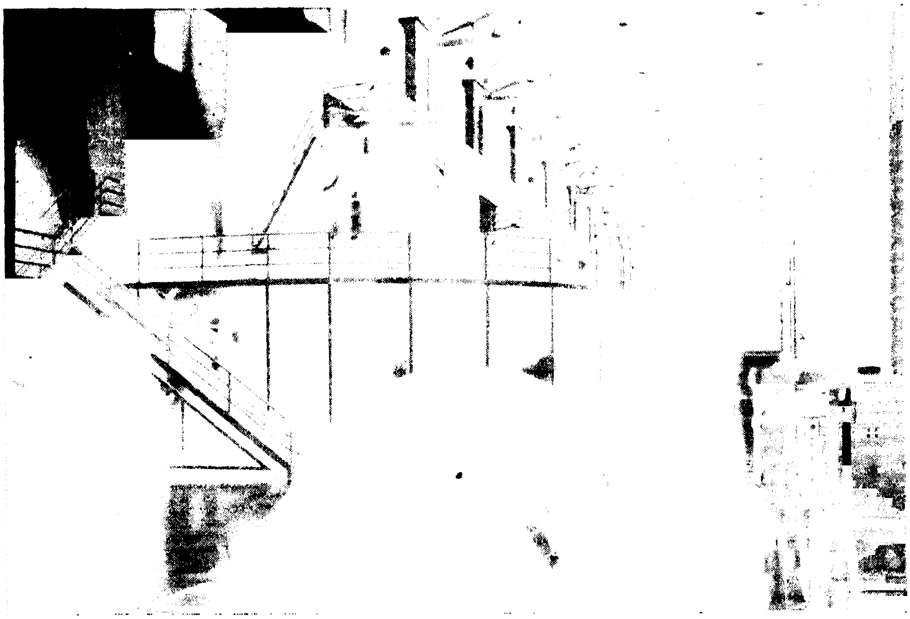


FIG. 13.—Generator room, Hoover power plant.

level. Two 150-ton and one 200-ton gantry cranes handle the gates. The total spillway capacity is 1,100,000 cfs. The remaining 2,139 ft is a nonoverflow dam with crest 60 ft above normal tail-water level.

The powerhouse proper is 638 ft long and 188 ft wide from the upstream face of the curtain wall to the draft-tube outlet. The penstocks, scroll case, and draft tube are molded in the concrete of the substructure. The superstructure, including the electrical bay, contains the generating, switching, and transforming equipment.

The penstocks may be closed by roller and sliding gates and the draft tube by stop logs to unwater the turbines, for which pumps are installed. The draft tube is of the elbow type and is divided into two symmetrical portions by a splitter pier below the elbow. The area at the entrance of the draft tube is about 300 sq ft and at the exit 990 sq ft.

The water reaches the intakes through a forebay made by a skimmer wall extending upstream from the dam parallel with the direction of flow, closing to the left bank of the river by a rock fill dike or ramp. The water flows under the skimmer wall which prevents surface ice and other floating debris from reaching the turbines.

Seven units have been installed, filling the original powerhouse building. In addition, five skeleton emplacements have been provided by constructing intakes with gates and stop-logged outlets for them so that the powerhouse can be extended without resorting to cofferdams.

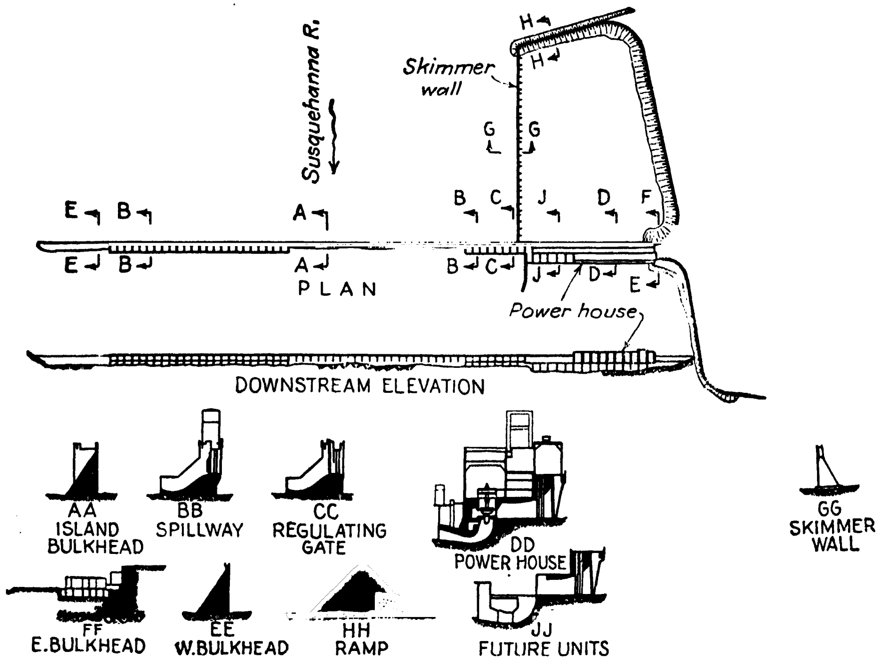


FIG. 14.—Safe Harbor plant.

The river below the powerhouse was deepened to form the tailrace, which has been completed for the ultimate development of 12 units.

The Manor and Conestoga substations are located on the east bank of the river near the powerhouse and are joined to it by tunnels in which control and power cables

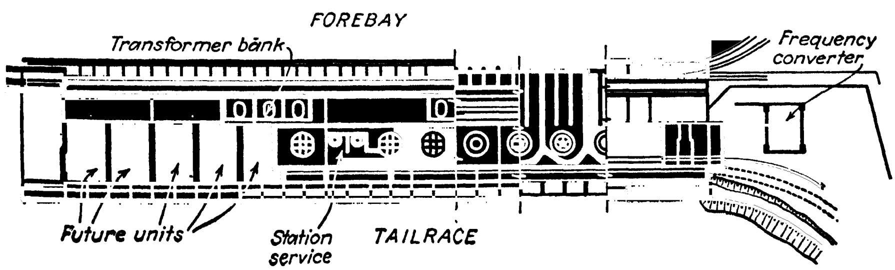


FIG. 15.—Safe Harbor powerhouse.

are carried. There is also included in the plant layout an operator's village consisting of 4 community and 21 other buildings of English architectural types.

The seven main propeller-type turbines are automatic adjustable-blade Kaplans rated at 42,500 hp, each nominally using 8,000 cfs under a head of 55 ft. Five turbines operate at a speed of 109.1 rpm, specific speed 150. Two units operate at a speed of

100 rpm, specific speed 137. There are five blades to each wheel running in a throat 18.33 ft in diameter. The mean velocity of the water at rated discharge is 5.67 fps at the entrance to the penstocks, about 40 fps through the runner, and 8.03 at the draft-tube exit.

The two service turbines are reaction wheels rated at 3,100 hp each under 55-ft head using 618 cfs at 180 rpm.

Five main generators are three-phase, 60 cycle, 13,800 volts star-connected, normally rated at 28,000 kw, 90 per cent power factor at 60C rise, with a maximum rating of 36,000 kva, 80 per cent power factor at 80C rise. They are equipped with Kingsbury thrust bearings below the rotor.

Two main generators are single-phase, 25 cycle, 13,300 volts, rated at 35,000 kva, 80 per cent power factor at 60C rise and 37,500 kva, 80 per cent power factor at 80C rise. They are equipped with G.E. spring thrust bearings below the rotor and an extra guide bearing above the rotor. Each unit has direct-connected main and pilot exciters, and there is also one 185-kw motor-driven spare exciter.

The two service generators are three-phase, 60 cycle, 480 volts, star-connected, open ventilated rated at 2,500 kva, 70 per cent power factor, 60C rise, with direct-connected exciters. A closed system of ventilation with surface air coolers is used for all main units. Ultimately the main units will comprise ten 60-cycle and two 25-cycle generators.

There are two banks of three single-phase transformers each rated at 84,000 kva self-cooled, or 126,000 kva with forced air cooling, 55C rise, 60 cycle, 13,800 delta/230,000 Y volts; and two three-phase transformers rated at 30,000 kva self-cooled, or 45,000 kva with forced air cooling, 13,800 delta/66,000 Y volts tap changing under load ± 10 per cent.

There is also a frequency changer of which the generator is single-phase, 25 cycle, 31,250 kva, at 80 per cent power factor, 13,300 volts, and the motor is three-phase, 60 cycle, 29,700 kva continuous, at 90 per cent power factor, 13,800 volts. Each machine has direct-connected main and pilot exciters. The generator is connected to the Conestoga substation and the motor to the main 60-cycle bus of the powerhouse.

The additional mechanical equipment of the plant consists essentially of the following:

Main governors, actuated with motor-driven flyballs, 10-in. valve, oil pressure 200 psi; twin pumps for each system which serves two units.

One centrifugal oil purifier each for transformers, governors, and lubricating oil.

Fire protection. CO_2 gas system of seventy 50-lb containers with manual or thermostatic control for generators, switching, and oil purification. The transformers are protected by a water system.

Water purification, 50,000-gal rapid sand filters, 100 gpm.

Heating system. Steam through aerofins and fans from two 104-bhp boilers, working pressure 125 psi.

Compressed air. Low-pressure system 100 psi from three 733-cfm compressors. High-pressure system 275 psi from two 80-cfm compressors.

Ice protection. Steam to each turbine stay vane for frazil ice. Gates and skimmer walls provided with air bubbler system.

Vacuum-cleaning system. Central-unit type.

Sewage systems. 30,000 gpd.

Two cranes 250-ton with two travelers of two hooks each. One transverse crane for transformer transfer 250 ton, one traveler single hook.

Miscellaneous electrical equipment includes:

House transformer; bank of three single-phase, 833 kva, self-cooled, 60-cycle,

13,800/460 volts delta-delta; 250 kva, three-phase induction regulator 10 per cent buck or boost.

60-cycle 13,800-volt switching equipment; double bus with oil circuit breakers for each piece of equipment. Circuit breakers rated at 2,000, 3,000, and 4,000 amp, 15,000 volt, 1,500,000 kva interrupting capacity.

220,000-volt switching equipment; two 265-kv vertical-break horn-gap disconnects on line to transformer banks and one horizontal break disconnect on each lightning arrester.

66,000-volt switching equipment; two 115-kv vertical-break motor operated horn-gap disconnects on transformer banks.

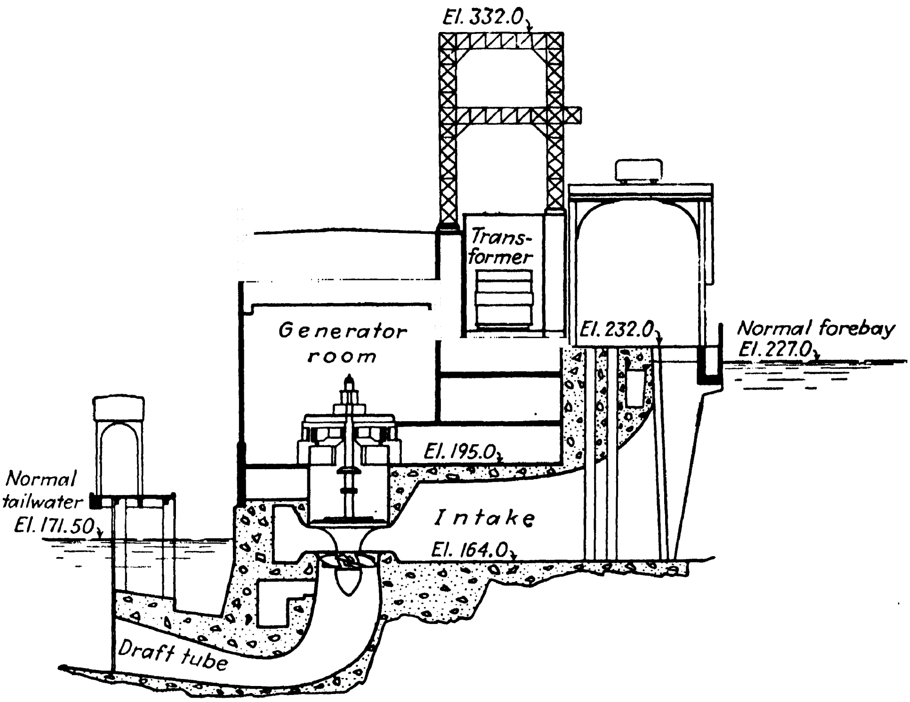


FIG 16.—Safe Harbor generating unit.

460-volt auxiliary system; double bus with bus section and bus-tie oil circuit breakers; duplicate feeders to safety switch groups throughout station.

Storage battery; two 120-cell batteries for control and emergency lighting, and one 24-cell battery for annunciator and signal system.

Annunciator and signal system; one 148-drop panel (ultimately 228) visual to main 60-cycle equipment; one 67-drop panel (ultimately 151) visual to fire-protection equipment; one 133-drop panel (ultimately 229) visual to 25-cycle equipment.

In the Conestoga substation, there are seven single-phase 132-kv step-up transformers each rated at 20,000 kva, 12,000/132,000 volts. The 25-cycle switching equipment includes a three-section low-tension bus with selector oil circuit breakers for the 25-cycle generator and frequency changer and single oil circuit breakers for the outgoing transformer circuits; high-tension air-break switches on each transformer and outgoing circuit and a high-tension transfer bus; a railroad trolley step-down substation is also located at Conestoga.

In Manor Substation, on the bluff just east of the powerhouse, there is provided space for large 220- and 66-kv substations. The 66-kv station is equipped with duplicate buses and seven oil circuit-breaker positions (initial installation of basic 1½-breaker scheme). The 220-kv station as at present constructed contains line and tie air-break switch facilities for controlling the two 220-kv circuits leaving Safe Harbor. The substations are normally operated from the powerhouse control room. The Conestoga substation is equipped for independent control at that station.

The air-bubbler system is for the protection of the spillway gates and the skimmer wall of the forebay against excessive stresses from ice thrust. By allowing air to bubble from the end of pipes submerged below the surface, ice formation is prevented. The air is supplied from large tanks at a pressure of about 100 psi. It is discharged

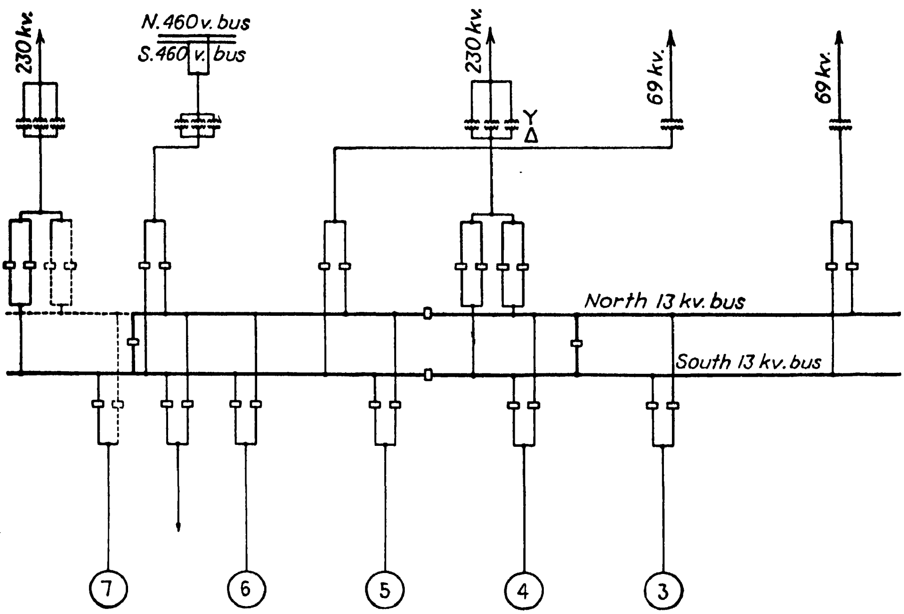


Fig. 17.—Safe Harbor wiring diagram.

from a ½-in. flexible bronze pipe in front of each gate, whose end is submerged 10 ft and about 2 ft upstream from the face of the gate. The air is controlled through a ⅟₁₆-in. orifice at the junction of the bronze pipe with the header. Two such tubes spaced 10 ft each side of the center of the 48-ft crest gate effectively prevent freezing. Each air tube is effective over a radius of about 10 ft on the water surface surrounding the tube.

Figures 14 to 17 show the general features of the Safe Harbor plant.

Norris. In this plant, the head is created solely by a dam with the powerhouse on the downstream side. It is a multiple-use project for navigation, flood control, and power and one of a series of developments in the Tennessee River drainage basin by the Tennessee Valley Authority, a Federal agency.

It is on Clinch River, a tributary of Tennessee River, 20 miles northeast of Knoxville. Clinch River has a minimum flow of 200, a maximum of 115,000, and a mean of 4,600 cfs. The dam creates a storage reservoir of 2,047,000 acre-ft capacity at normal level, El. 1,020, with a controlled flood surcharge of 520,000 acre-ft at El. 1,034 and an uncontrolled surcharge to El. 1,052 of 800,000 acre-ft, making a total

storage for flood control above El. 1,020 of 1,320,000 acre-ft. The normal drawdown of 65 ft (El. 1,020–955) gives a normal usable capacity of 1,500,000 acre-ft.

The dam is a concrete gravity type 265 ft above the stream bed with a top length of 1,860 ft, near the center of which is a 332-ft overflow spillway section, net overflow length 300 ft, with two piers controlled by three 100- by 14-ft drum gates on the spillway. In this section are eight outlets protected by trash racks and controlled by 5.7- by 10-ft slide gates for release of water for navigation or other purposes.

The powerhouse is at the left side of the spillway at the toe of the dam 209 by 69 ft in plan with the total height from bed rock to parapet of 153 ft. The two penstocks are steel pipes 20 ft in diameter within the concrete of the dam with entrances enlarged to rectangular openings and controlled by Broome gates 16.5 by 28.5 ft with 24-in. air

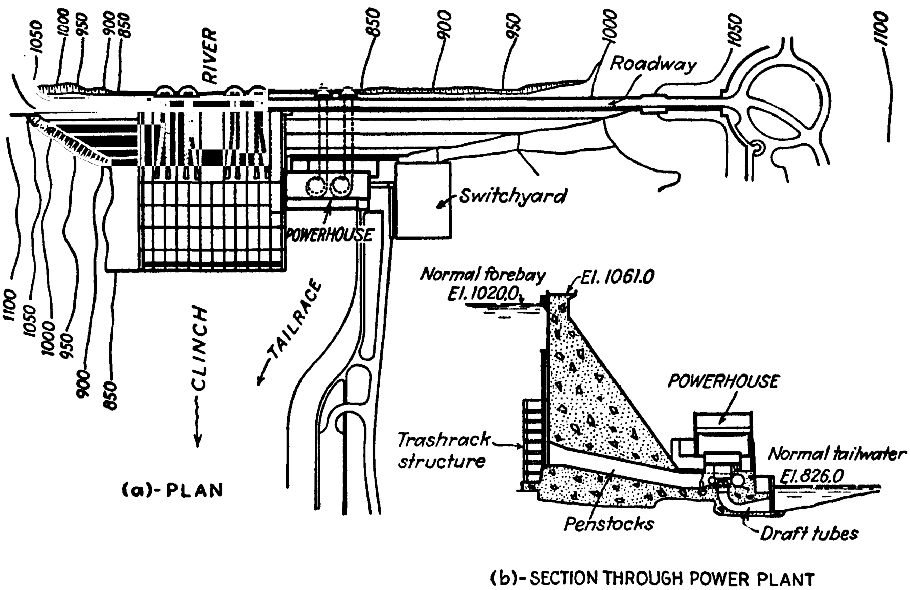


FIG. 18.—Norris plant.

vents below them. A heavy trash-rack structure protects the penstocks, the center lines of whose entrances are 153 ft below normal reservoir level.

The scroll case is of steel enclosed in concrete, and the elbow draft tube is formed in the concrete of the substructure of the powerhouse. The entrance area is 150 sq ft, and the exit area is 660 sq ft (see Fig. 18). The tailrace is excavated in the rock of the left river bank to a depth of 31 ft below normal tail water at the draft-tube exits and slopes gradually upward to the natural river bed. The tail-water level varies 6 ft between low and normal levels and 34 ft between normal and the maximum recorded, a total range of 40 ft.

The navigation benefits consist in releasing stored water to increase navigable depths in the Tennessee and Mississippi rivers, about 1,500,000 acre-ft being available for this purpose. It is estimated that the downstream flow can be increased to 6,000 cfs which would result in depth increases in Tennessee River of 1.7 ft above Chattanooga and 1.5 ft below Pickwick.

Two generating units have been installed—no additions are contemplated. The turbines are the reaction type rated for best efficiency of 91 per cent at 60,000 hp under 180-ft head and 55,000 hp under 165-ft head. Speed is 112.5 rpm, specific speed 48.8. Discharge at full gate is 4,100 cfs under 165-ft head. The runner is 161 in. in diameter.

The governors are of the oil-pressure actuator type with motor-driven flyballs and two servomotors.

The generators are rated at 56,000 kva, or 50,000 kw at 90 per cent power factor. They are three-phase 60-cycle 13,800-volt machines with a speed of 112.5 rpm and guaranteed efficiency of 97.3 per cent at full load and 90 per cent power factor. Excitation is by direct-connected pilot and main exciters of 275 kw, 250 volts.

There are two banks of three single-phase 18,667-kva transformers, 13,200 delta 154,000/100,000 Y self-cooled. For station service, there are two three-phase 750-kva 13,800/460-volt transformers.

Figures 18 and 19 show general features of the plant.

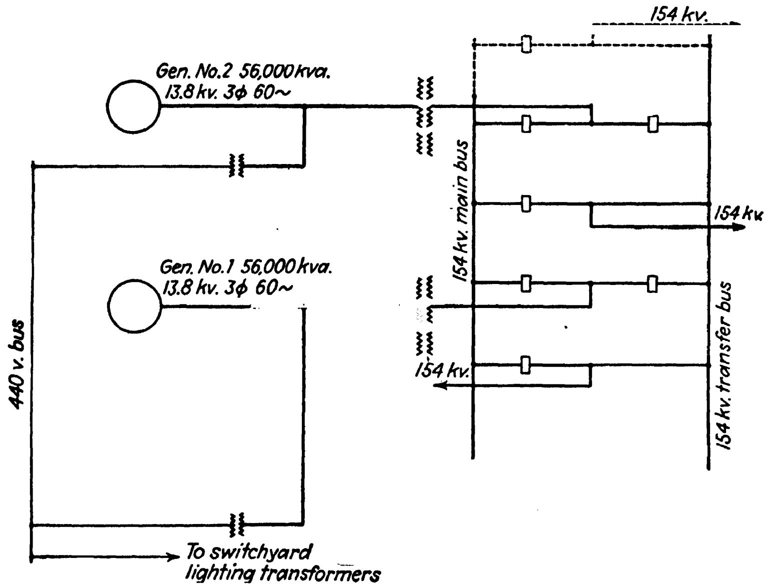


FIG. 19.—Norris wiring diagram.

Oak Grove. This plant is chosen as a distinct type on account of the high-head reaction turbine with long closed conduit and surge tank in place of forebay at the head of the penstocks. It is on the Oak Grove Fork of Clackamas River, Oregon, and is one in the system of hydro and steam plants owned by the Portland (Ore.) General Electric Co.

The diversion dam is a concrete-arch overflow type 68 ft high, 134 ft crest length, with 2-ft flashboards providing usable storage of 200 acre-ft—sufficient to run one unit 6 hr.

The closed conduit is 35,000 ft long from diversion dam to the surge tank, 34,200 ft of which is steel pipe 9 ft inside diameter. There are two tunnels near the intake aggregating 1,300 ft in length, concrete lined, 12.33 ft in diameter with a reinforced-concrete section between them 200 ft long of the same diameter. A 250-ft tunnel 14 ft in diameter joins the pipe to the surge tank. These tunnels have a capacity of 1,200 cfs, whereas the single pipe has a normal capacity of 400 cfs on a hydraulic gradient of 0.002. The maximum head on the supply pipe is 400 ft at the Cripple Creek crossing near the surge tank. Portions having a thickness of 3/8 in. and less are stiffened with 5- by 3-in. angles spaced 7.75 ft. There is a butterfly valve at each end of the 9-ft pipe. There are no expansion joints in the pipe line.

The surge tank is a differential type excavated for the most part in Cripple Creek

Knoll. The riser from the tunnel is 14 ft in diameter for 110 ft above the tunnel, enlarging to 31 ft in diameter for an additional height of 87 ft of which 30 ft is aboveground, with a 7-ft riser inside. A spillway carries overflow from the tank to Cripple Creek. Beyond the surge tank, the 14-ft tunnel continues at a lower level for 170 ft

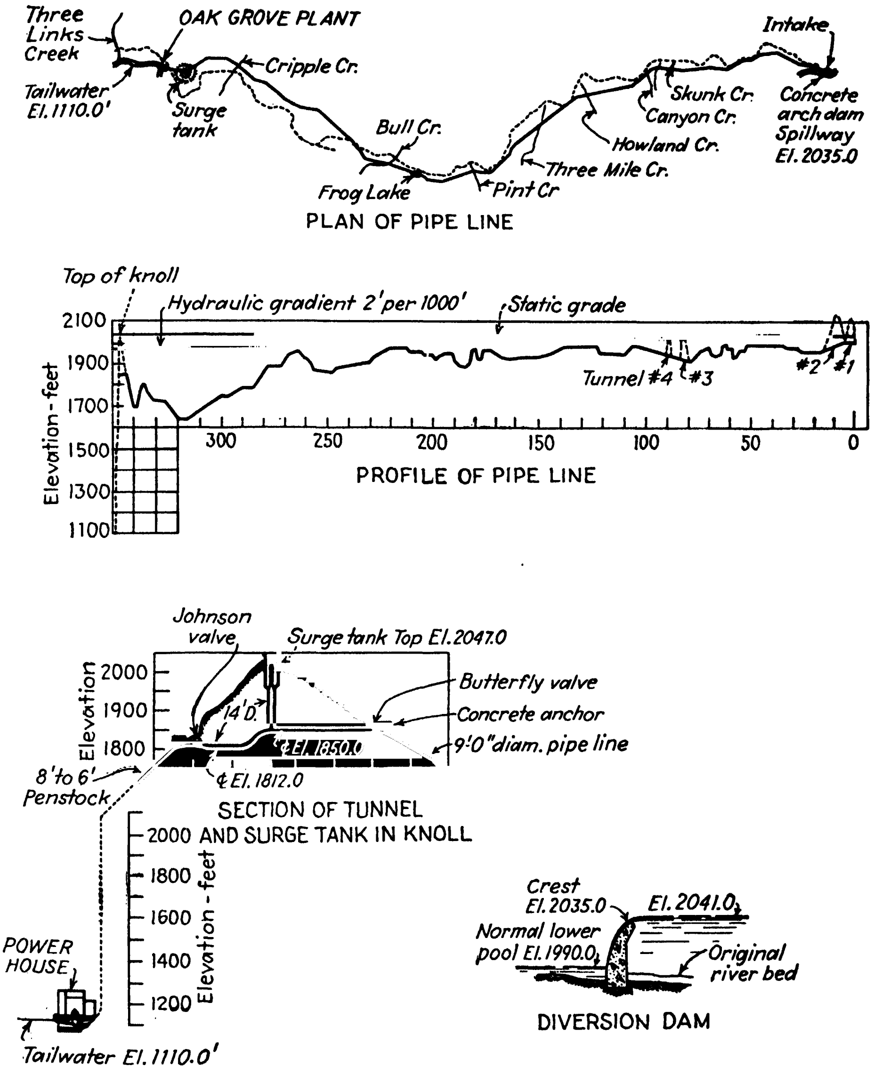


FIG. 20.—Oak Grove development.

to a transition dividing it into three 8-ft pipes. The surge tank is designed to be dead beat on on instantaneous change from 600 to 1,200 cfs.

There are two penstocks each 1,200 ft long, varying from 8 ft in diameter at the triple manifold where the static head is 240 ft to 6 ft at the turbine where the head is 930 ft. Each penstock is encased in a reinforced-concrete envelope 6 in. thick. They were built without expansion joints. At the head of the penstock for Unit 1 is a

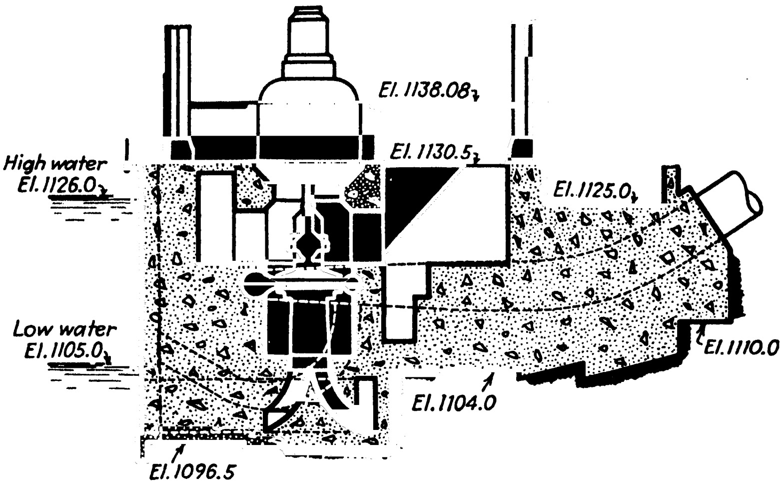


FIG. 21.—Oak Grove generating unit.

66-in. Johnson balanced valve and for Unit 2 a butterfly valve. Both have emergency controls at the powerhouse, or they will close automatically if the velocity through them becomes twice normal. At the end of each penstock is a 72-in. butterfly valve controlled by a motor and also by a reversible impulse wheel. It has an 8-in. needle by-pass. The casting for each butterfly valve also includes a Y branch containing a 40-in. relief valve with a capacity of 85 per cent of the flow of the penstocks. This relief valve is connected to the servo-motors for the turbine gates and opens as the turbine gates close so that the flow in the penstock need not vary with varying load.

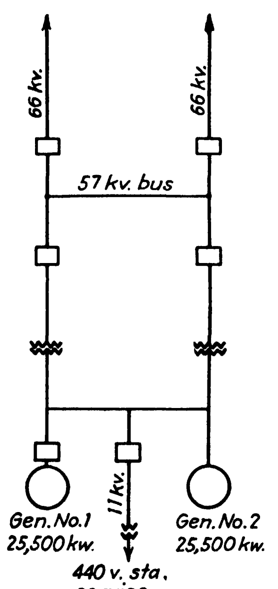


FIG. 22.—Oak Grove wiring diagram.

The draft tube is the Moody spreading type having an area of 26.5 sq ft at the entrance and 107 sq ft at the exit. The portion immediately below the turbine casing is of cast iron in two sections provided with wheels. These sections can be rolled out on tracks into a recess in the substructure to gain access to the turbine runner.

The turbines are each rated at 40,000 hp 514 rpm 400 cfs under a net head of 850 ft. The specific speed is 22.4. The casing is cast steel, and the runner is bronze with rubber seal rings. The governor is set for 5 sec for a full-stroke closing of the turbine gates upon sudden rejection of the total load, during which the speed will increase to 134 per cent of normal in 2½ sec, while the relief valve takes the rejected water with only 4 per cent rise in penstock pressure. The scroll case is of steel encased in concrete.

Generator 1 is rated at 30,000 kva, 11,000 volts, three phase, 60 cycle with 250 volt direct-connected exciter. Generator 2 is similar except that it has a pilot exciter. The transformers are rated at 10,000 kva, single phase, 11/66–100 kv. One bank has two such transformers connected open delta, the other, three transformers connected

delta-delta supplying two 66,000-volt transmission lines on which the voltage can be increased to 100,000 volts by adding the third transformer to the first bank and Y connecting the secondaries.

The manufacturers' guarantees were as follows:

Load	50 %	75 %	100 %
Turbine	86.5 %	90.4 %	85.7 %
Generator	96 0	97.0	97.6
Combined	83.1	87.6	83.6

The results of a test on Unit No. 1 made in May, 1925, gave the following;

For maximum load	
Discharge	447 cfs
Effective head	870 ft
Turbine output	37,500 hp
Turbine efficiency	84.4 %
At high efficiency	
Discharge	353 cfs
Effective head	888 ft
Turbine output	32,500 hp
Turbine efficiency	91.0 %

Figures 20 to 22 show the general features of this development.

Coordinated Hydroelectric Systems. In addition to the typical individual plants described in the foregoing, there is submitted below a brief description of three systems of plants on a single watershed that are all operated as a coordinated group.

There are obviously great advantages in the operation of a series of plants on the same stream system, particularly where storage is provided.

Water released from an upstream plant can seldom be used by succeeding plants on the stream to any advantage unless storage for reregulation is available at each plant, because the period between peak loads and the time interval required for released water to reach the lower plants can seldom be synchronized. In rare instances, water used for one peak can be used at a lower plant for the next peak during certain stages of the stream, but without storage much of the water is wasted. With sufficient storage capacity at each forebay, however, most of the water supply and all the fall can be used most advantageously.

Mokelumne River System. The hydroelectric plants on Mokelumne River in California constitute a complete development of a river, primarily for power but also with full regard to irrigation and domestic requirements below them. In addition to the five developments, actual and under construction, of the Pacific Gas and Electric Co., the East Bay Utility District has created, on the lower river, the Pardee reservoir and power plant of 20,000 hp capacity for domestic supply and power (see Fig. 23).

The complete Mokelumne development is planned to operate together with steam-electric plants to supply a market having about a 64 per cent load factor. The steam capacity is sufficient so that in years of good water supply the hydro can operate on base load at high capacity factor while steam carries the top of the load. In years with deficient water supply, the situation may be reversed, the steam operating at high capacity factor while the hydro operates at the top of the load in order to conserve stored water.

The first development on Mokelumne River was in 1857 for mining. In 1902, the old Electra plant was built—one of the pioneer hydroelectric developments, 30,000 hp, and among the first high-tension transmission lines, 60 kv, in California.

During these early years, storage on the head waters aggregated about 19,000 acre-ft in a number of small reservoirs. In 1928, a comprehensive plan for the maximum practicable development of the entire drainage basin was prepared, culminating in the five plants listed in Table 3. All except the Bear River plant have been completed to date (1951).

Additional storage for all except the Bear River plant is provided in Salt Springs Reservoir, of 142,000 acre-ft capacity, which was created by construction of the Salt Springs Dam, a rock fill 328 ft high, 1,300-ft crest length. Water tightness is secured by an upstream facing of heavily reinforced concrete laid in slabs 60 by 60 ft. The

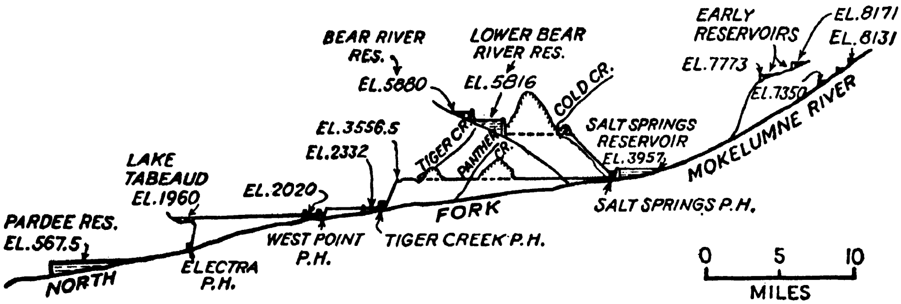


FIG. 23.—Mokelumne River system.

lateral-flow spillway has a capacity of 53,000 cfs over a 478-ft crest. In the unwatering tunnel have been sealed two 10-ft steel pipes each with a 129-in. hydraulically operated butterfly valve near the upper end and a 78-in. butterfly valve in the nozzled end.

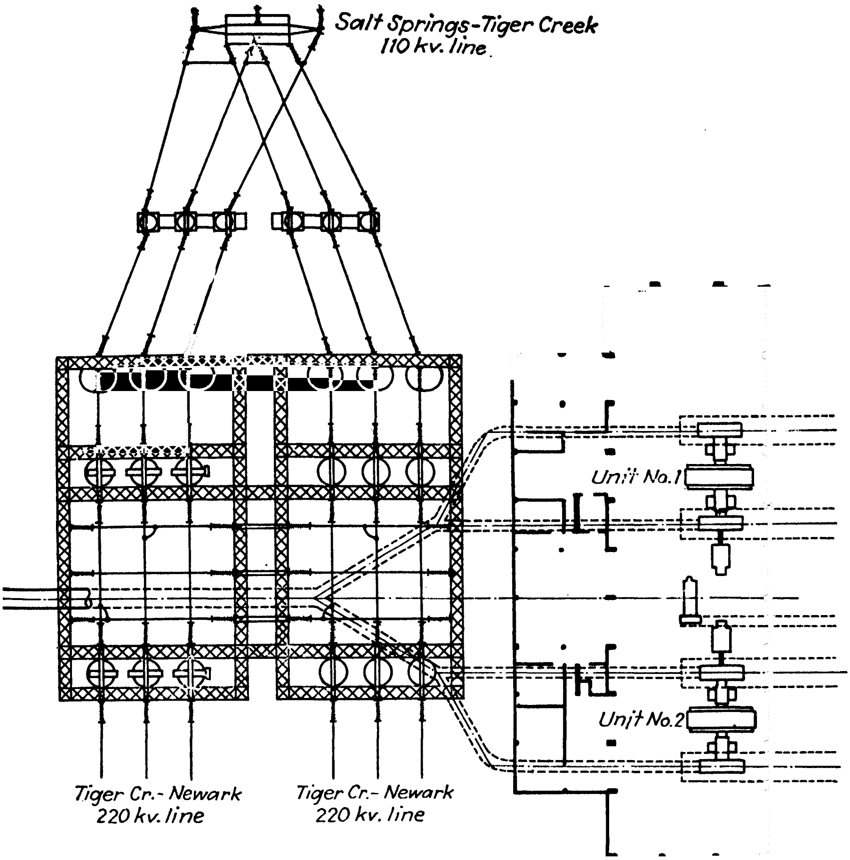
TABLE 3.—DATA ON MOKELUMNE RIVER DEVELOPMENTS

Plant	Maximum static head	Conduit capacity, cfs		Installed kva	Normal output	
		Average	Maximum		Million kwhr	Peak kw
Salt Springs	254	550	550	11,000	34.3	12,300
Bear River	2,104	130	200	33,000	125.6	29,000
Tiger Creek	1,218	550	625	60,000	321.1	58,000
West Point	312	575	675	16,000	89.2	15,000
Electra	1,268	575	975	99,000	407.0	95,000
Total	......	...	...	219,000	977.2	209,300

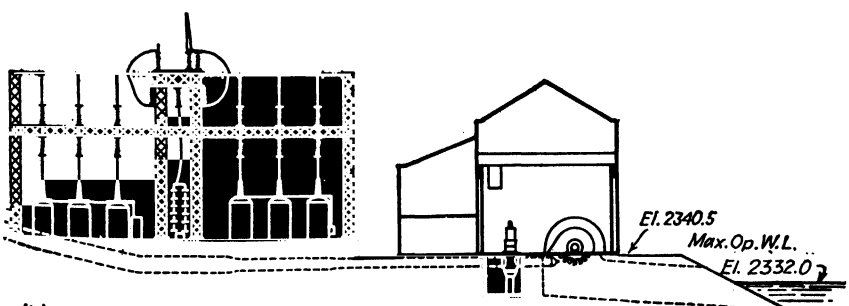
A 90-in. penstock takes off from one of the pipes for the Salt Springs power plant. The other, a 72-in. pipe, ends in a 54-in. needle valve discharging into the tailrace, by-passing the powerhouse, for emergency supply to the Tiger Creek conduit which heads in the tailrace.

Bear River has a fall of 2,500 ft in its last 6 miles. A dam 230 ft high will create usable storage of 48,000 acre-ft, from which a 2.6-mile pressure tunnel and steel penstock will deliver water to an impulse wheel in the Salt Springs powerhouse under a static head of 2,104 ft.

The Salt Springs powerhouse contains one reaction turbine using a constant dis-



(a)-PLAN OF TIGER CREEK POWER HOUSE AND SWITCHBOARD



(b)-SECTION PARALLEL TO PENSTOCK

FIG. 24.—Tiger Creek plant.

charge of 550 cfs under head varying from 175 to 254 ft. For heads less than 175 ft, some water is by-passed. This turbine is rated at 13,500 hp, 300 rpm. The generator is rated at 11,000 kva, 11,000 volts three phase, 60 cycle, 0.85 power factor. This unit has a long shaft necessitated by the desire to put the future Bear River impulse wheel and the Francis turbine on the same floor. The impulse wheel is rated at 40,000 hp, 300 rpm using 180 cfs under a net head of 1,970 ft. The generator will be rated at 33,000 kva, 11,000 volts, three phase, 60 cycle, 0.8 power factor. Water passing the wheels goes through the tailrace into the Tiger Creek conduit. Both units will not be run at full capacity simultaneously.

The Tiger Creek conduit to the Tiger Creek regulating reservoir is 17.8 miles long. The reinforced-concrete open flume is 13.3 miles long, 7 ft deep by 14.25 ft wide, on a slope of 0.0008, with a capacity of 550 cfs. There are 2.7 miles of tunnels of which 2.5 miles are unlined and two steel sag pipes (inverted siphons) 93 in. in diameter 0.08 and 0.14 miles long, respectively. Beyond the regulating reservoir, the conduit has a capacity of 625 cfs for 2.5 miles and discharges into a forebay at the head of the penstock. The regulating reservoir is formed by a dam 100 ft high in Tiger Creek giving a usable capacity of 158 acre-ft. The forebay has a capacity of 40 acre-ft. With this storage, a constant inflow of 550 cfs will permit a maximum draft of 625 cfs for 18 hr. Automatic gates release a constant discharge from the regulating reservoir which may be set by remote control from the Tiger Creek powerhouse. Storage in the forebay will take care of the plant load during any change in flow from the regulating reservoir.

One penstock, 4,750 ft long (slope length), leads to the Tiger Creek powerhouse. Its diameter decreases from 102 in. at the head to 72 in. at the Y outside the powerhouse where it branches into two 52-in. pipes, and each of these in turn subdivides into two 36-in. pipes for the four impulse wheels. The static head on the plant is 1,218 ft (see Fig. 24*a*).

The generating equipment of the Tiger Creek powerhouse consists of two units, each being driven by two wheels of the impulse type, each pair being rated at 36,000 hp under a net head of 1,190 ft at 225 rpm. There is one straight flow nozzle for each wheel, 10.5 in. in diameter. For a change in load, the governor first moves a jet deflector, then slowly adjusts the needle valve to the new load without undue disturbances in the penstock.

The generators are each rated at 30,000 kva, 11,000 volts, 60 cycle, 0.85 power factor, with direct-connected main and pilot exciters, driven by the two wheels at 225 rpm. The generators are connected directly to two banks of transformers, each set forming a single unit; *i.e.*, there are no circuit breakers between generator and transformer.

Two miles below the tailrace is a concrete arch dam 100 ft high in Tiger Creek forming an after bay of 1,015 acre-ft usable capacity in the top 17 ft to regulate the flow for the West Point plant next below on the stream. The conduit for the West Point plant heads in this after bay and is 14,300 ft long with a capacity of 575 cfs. There is no forebay, and the plant will run at a constant load. There is one reaction turbine driving a 15,000-kva 11,000-volt generator under a nearly constant head of 292 ft. The West Point plant is entirely automatic and controlled from the Tiger Creek plant. Power is transformed to 60,000 volts only and is used to supply the demands now served by the present Electra plant.

The old Electra plant was put into service in May, 1902, with five 2,000-kw units. In 1905, there were added two 5,000-kw units, and these seven units were in practically continuous operation since their installation until Nov. 30, 1949, when the old Electra plant was abandoned.

The new Electra plant operates under a static head of 1,268 ft. The forebay

has a capacity of 1,158 acre-ft. The powerhouse is essentially similar to Tiger Creek except that there are three units each rated at 37,500 hp, 225 rpm.

The four plants—Salt Springs, Tiger Creek, West Point, and Electra—utilizes the entire fall of Mokelumne River from the surface of Salt Springs Reservoir to the tailrace of the Electra plant, a drop of 3,262 ft. There only remain 120 ft of fall

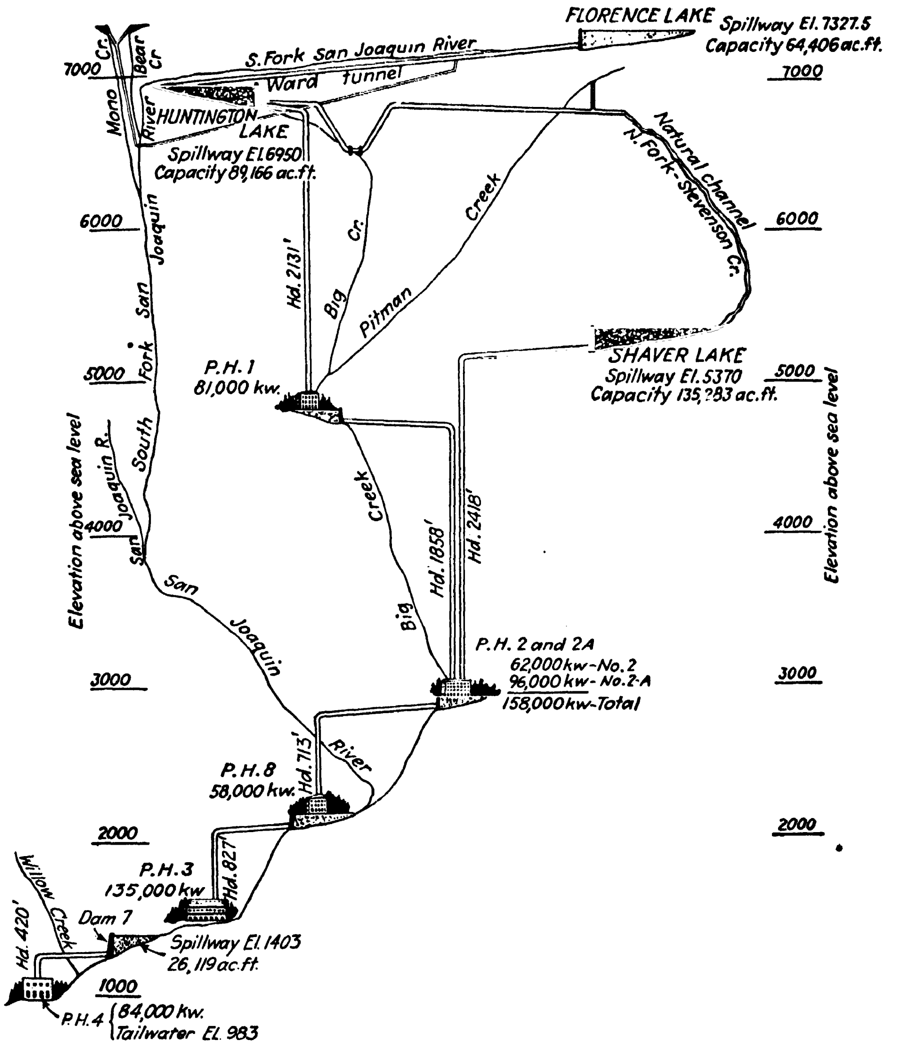


FIG. 25.—Big Creek-San Joaquin system.

between the Electra plant and Pardee Reservoir. When Bear River is finished, the additional storage will supplement the storage at the lower plants after which the Mokelumne River system will have been completed.

The energy from these plants, except West Point, will be conveyed to market centers over 220,000-volt transmission lines.

Big Creek-San Joaquin River System. This group of six plants is owned by the Southern California Edison Co. Figure 25 is an idealized profile of the system.

Storage is provided in Florence and Huntington lakes, connected by Ward Tunnel, into which the Mono and Bear Creek feeders flow, some 650 cfs being added to the system.

From Huntington Lake, one conduit delivers water into the head of Stevenson Creek, where it is stored in Shaver Lake, another conduit supplies water for Powerhouse 1 on Big Creek under a head of 2,131 ft. After passing the wheels, the water is gathered in an after bay that heads the penstocks to Powerhouse 2, operating under a head of 1,858 ft. Alongside 2 is Powerhouse 2A, which is supplied by Shaver Lake under a head of 2,418 ft—one of the highest head plants in this country. After leaving 2 and 2A, water is passed successively through Powerhouses 8, 3, and 4.

This system involves a total fall from Huntington Lake to the tailrace of Powerhouse 4 of about 6,000 ft. The normal annual output is 2.8 billion kwhr.

The aggregate storage provided is 315,000 acre-ft, and the aggregate generating capacity of the six plants is 516,000 kw. Plants 1, 2, and 2A utilize double overhung

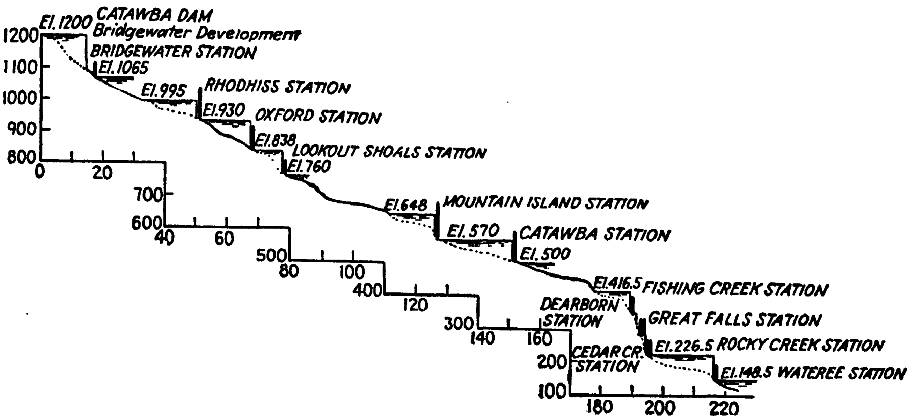


FIG. 26.—Catawba River system.

impulse wheels with jet deflectors controlled by the governors on unit No. 1 of plant 2A, other impulse wheels have relief valves. Plants 8, 3, and 4 contain reaction turbines, 3 being a close second to the Oak Grove Plant among high-head reaction-wheel installations. The units of plants 8 and 3 are equipped with relief valves, those of plant 4 are without relief valves.

Catawba River System. Another hydroelectric system in which practically the entire fall in a river length of 215 miles is utilized is that of the Duke Power Co. on Catawba River in North and South Carolina (Fig. 26). The company's entire system comprises a total generating capacity of 900,000 kw of which 445,000 is hydro. The combined usable storage capacity of the eight reservoirs is 800,000 acre-ft. The total fall is 1,050 ft, of which 785 ft is utilized for power.

The headwaters are heavily timbered, but the underlying rocks are granite and gneiss; hence the ground storage is limited and the streams are subject to extremes of high and low flows. Artificial storage is indispensable.

The plants are operated so as to derive the maximum amount of power from the available supply. Weather records are kept at each plant and reported daily to a central despatching office in Charlotte. During storms, hourly reports are received and plant loads distributed to use the resulting runoff to the greatest advantage.

Tennessee River System. The entire fall of Tennessee River from Knoxville, Tenn., to Kentucky Dam near the mouth, a distance of 625 miles and a fall of approximately 500 feet, has been utilized at nine reservoirs on the main stream, each of which

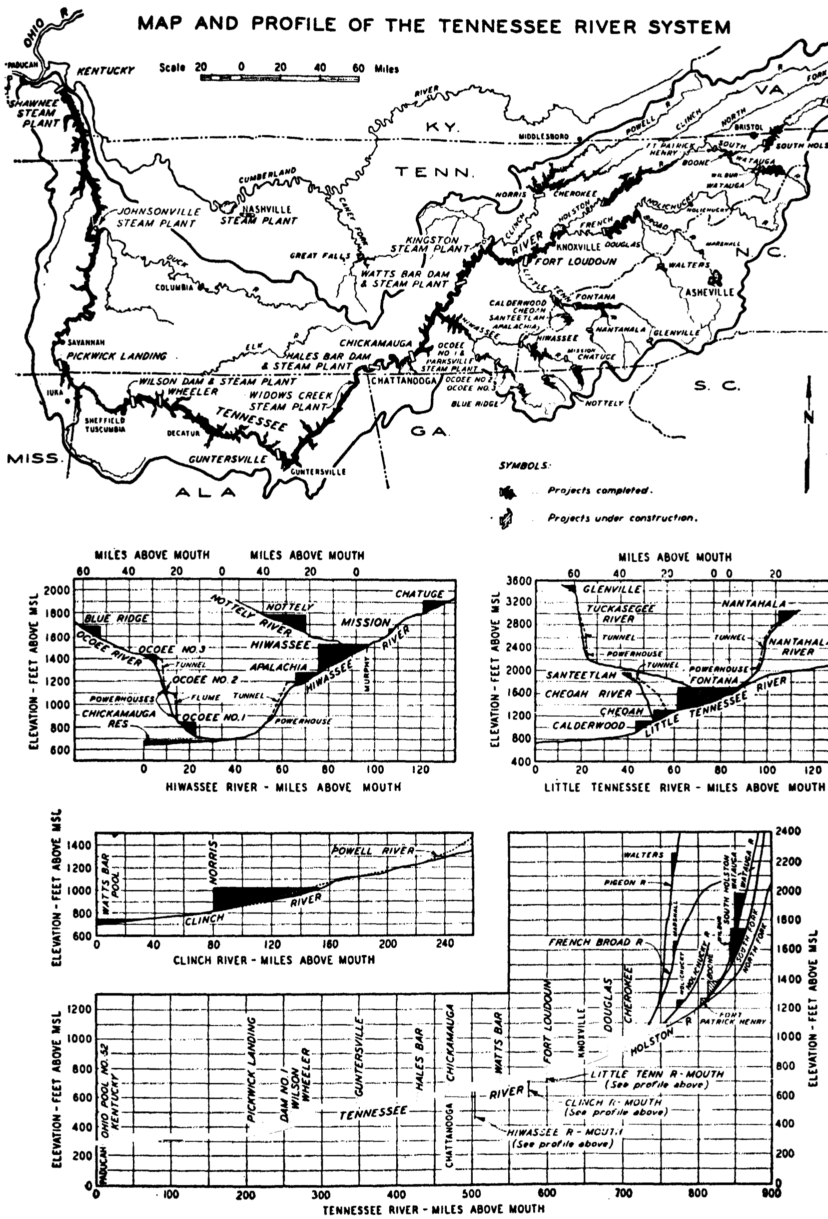


FIG. 27.—Tennessee River system.

is provided with a navigation lock and a powerhouse and seven of which have reserve storage space for floor control. This work is being done by the Tennessee Valley Authority. The flow in the main stream is stabilized by nine multipurpose reservoirs on tributaries entering the Tennessee River above Chattanooga, Tenn., and additional use of the water and available fall is made by seven power reservoirs on tribu-

taries. In addition, five power reservoirs, three of which have storage pools, owned by the Aluminum Company of America and its subsidiaries are operated in conjunction with the T.V.A. system under a mutual agreement.

The multipurpose hydroelectric system is being constructed with funds appropriated by Congress, which have been supplemented by funds derived from operation of the T.V.A. power system.

Figure 27 is a condensed map and profile of the T.V.A. system. It shows the completed reservoirs in the system, the two additional dams under construction, and several steam-electric plants completed and under construction, as well as a few privately owned power reservoirs operated independently of the T.V.A. system.

Types of Plants

In the preceding discussion, the various types of development were classified broadly according to function, *i.e.*, run-of-river, storage, high-head, low-head, multiple-purpose, single-purpose, etc. These functions are expressed largely in relation

FIG. 28.—Lower Salmon River plant, Snake River, Idaho.

to the over-all plan of development for the river system. In the following, the discussion of types continues within the more narrow limits of differences in structural characteristics.

Powerhouse structures may be classified as indoor, outdoor, semi-outdoor, and underground. All the plants described in the previous section are of the conventional indoor type. The features of this type are well known and require no additional elaboration here.

The outdoor type provides a metal housing for the generators and thus eliminates the conventional superstructure which houses all electrical and hydraulic equipment in the indoor type. In the early stages of development, it was thought that the outdoor type would not be suitable for severe climates. This notion has been largely dispelled by the experiences of the Idaho Power Company, and others, with several plants of the outdoor type which have been constructed in severe climates.

Typical of the most advanced practice in outdoor plant design is the Idaho Power Company's Lower Salmon Station on the Snake River, Idaho, shown by Figs. 28 and 29. This plant is composed of four units, each of 15,000 kw, operating under an average head of 56.5 ft. Three are driven by fixed-blade propeller turbines and one by a Kaplan turbine. Each generator is protected by a separate metal housing above the deck level. Operating and service bays are located below the deck. The transformers are located on a deck downstream from the gantry crane. Generators and turbines are serviced by a 125-ton gantry crane. Temperatures at the Lower Salmon site vary from 15F below zero to 115F.

Representative of modern semi-outdoor design is the Kentucky station (T.V.A.), shown by Fig. 30. This type combines some of the features of both the outdoor and

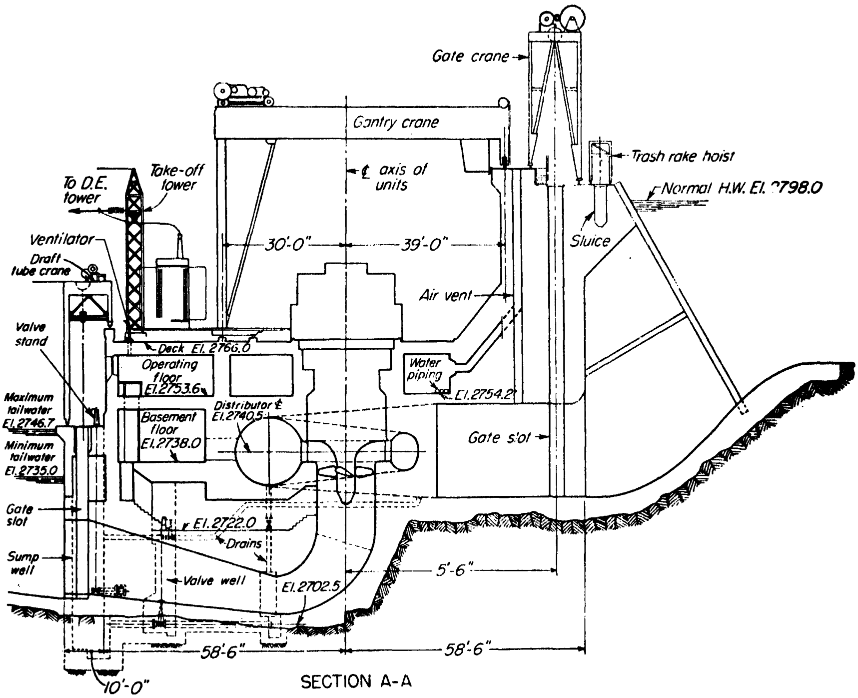


FIG. 29.—Lower Salmon River plant, Snake River, Idaho.

the indoor types. The generator room at Kentucky is 30 ft high, which is ample to clear the Kaplan head on the unit. The adjacent electrical bay is two floors high, and the downstream and end walls provide protection above tailwater 28 ft above the generator-room floor. A gantry crane hoists equipment through hatches in the roof and the main access road comes in at roof level. The Kentucky station houses four 32,000-kw units with provision for a fifth. The generators are driven by Kaplan adjustable-blade propellers which operate under an average head of 47 ft. The head varies from a maximum of 73 ft to a minimum, during extreme flood conditions of 6 ft. The T.V.A. has constructed five stations of this semi-outdoor type.[1]

The choice between the indoor, outdoor, or semi-outdoor types will be influenced by the relative economy of these types and by owner preference, the latter usually being the determining factor.

[1] PALO and MARKS, The Design of Hydroelectric Stations, *Trans. A.S.C.E.*, **3**, p. 1175.

The underground powerhouse represents one of the more important advances in hydroelectric design during the past decade. Economy and security are the two principal reasons for consideration of an underground station. Building costs have now (1950) reached a point where the cost of powerhouse superstructures will fall between $1 and $2 per cubic foot, exclusive of the crane and of electric and hydraulic equipment. Ordinarily, 1 cu yd of rock excavation in large tunnels would average only about half the cost of a cubic yard of powerhouse superstructure. It is frequently more economical, therefore, to excavate a powerhouse chamber underground than to build a superstructure above the ground.

This is not the only economy however. Consider, for example, the layout and section of the Guayabo station (Figs. 31 and 32) now (1950) under construction on

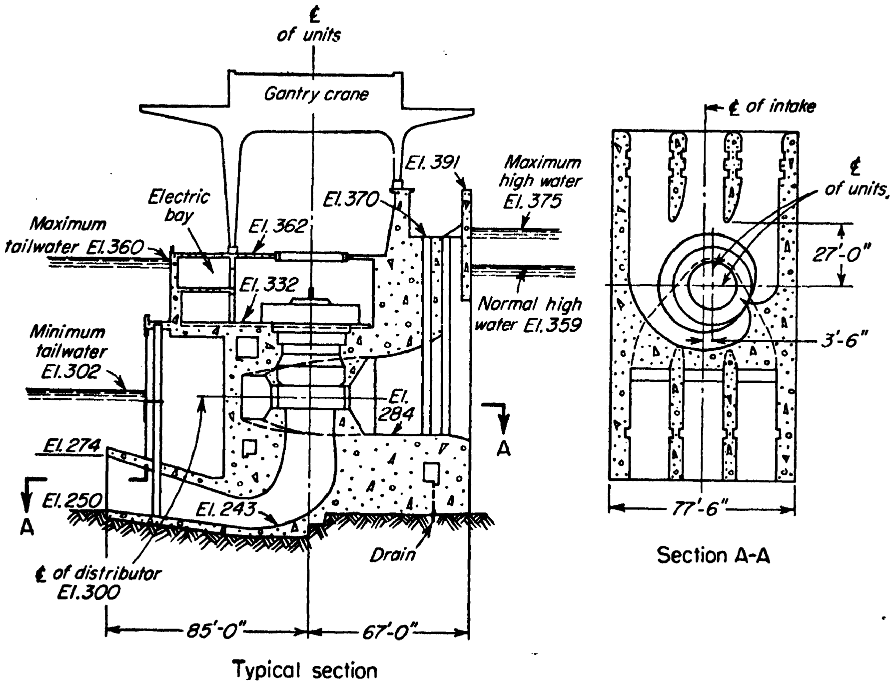


FIG. 30.—Semi-outdoor plant, Kentucky Dam, T.V.A.

the Rio Lempa River in El Salvador, Central America. It will be noted that the underground power station at this site will occupy a chamber excavated in rock directly below the intakes that form part of the dam on the north bank. The intake structure contains trash screens and sector intake gates which shut off or admit water as required, to steel-lined vertical-shaft penstocks leading to the turbines.

This arrangement minimizes the conduit losses and increases the net head on the turbines. The pressure-lined conduit is of minimum length thus effecting a second substantial saving as compared with the long pressure conduit which would be required to develop the head if the outdoor type of station had been located beyond the bend of the river, as shown by Fig. 31. Replacing the long pressure conduit is an unlined tail tunnel connecting the draft tube with the discharge channel beyond the bend in the river.

A third substantial saving was effected by the elimination of the surge tank. A surge chamber must be excavated in the tail tunnel but the cost of this chamber is small as compared with the cost of a tank.

The discharge channel at the outlet of the tail tunnel was located below the rapids which, during flood periods, may cause the tailwater at the dam to rise as much as 70 ft. Locating the outlet at this point also creates an increase in head under ordinary operating conditions as compared with the head available at the dam.

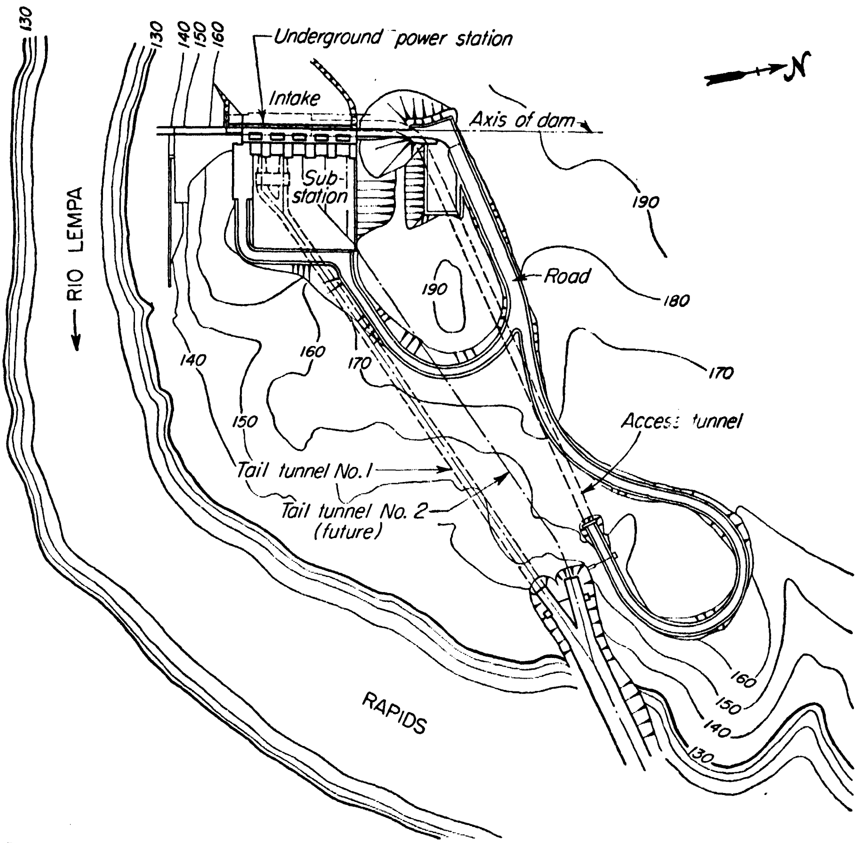


FIG. 31.—Plan of underground Guayabo power station, Rio Lempa project, El Salvador, Central America.

The most unusual features of the Guayabo station are the horizontal units and horizontal draft tube, as shown by Fig. 32. The advantages of this arrangement are:

1. The power-station chamber will be only slightly over one-half as high (and of about the same width) as required for vertical units, thus greatly reducing the excavation.
2. The elbows in the penstocks ahead of the turbines, required in vertical units, are eliminated and the penstocks enter the scroll cases tangentially without elbow bends.
3. The setting permits the use of a straight conical draft tube of simpler construction than an elbow tube.
4. The absence of penstock elbows and of the elbow draft tubes will conduce toward higher efficiency and smoother operation. The length of a horizontal conical draft tube is not a matter of equal concern as compared with vertical units, because it is in the direction of, and in lieu of, an equivalent length of the required tail tunnel.
5. The time of construction will be shortened materially. Instead of some 16 or more pours of concrete from foundation level to generator floor, the generator can be founded nearly on bedrock only after two pours of concrete.

6. By using an intermediate section of shaft, it can be so arranged that the removable parts of the turbines can be taken out without disturbing the generators.
7. The danger of cavitation and pitting will be almost eliminated.[1]

The only apparent disadvantage to this arrangement is that turbine movable parts will need to be moved horizontally for assembly or for dismantling without

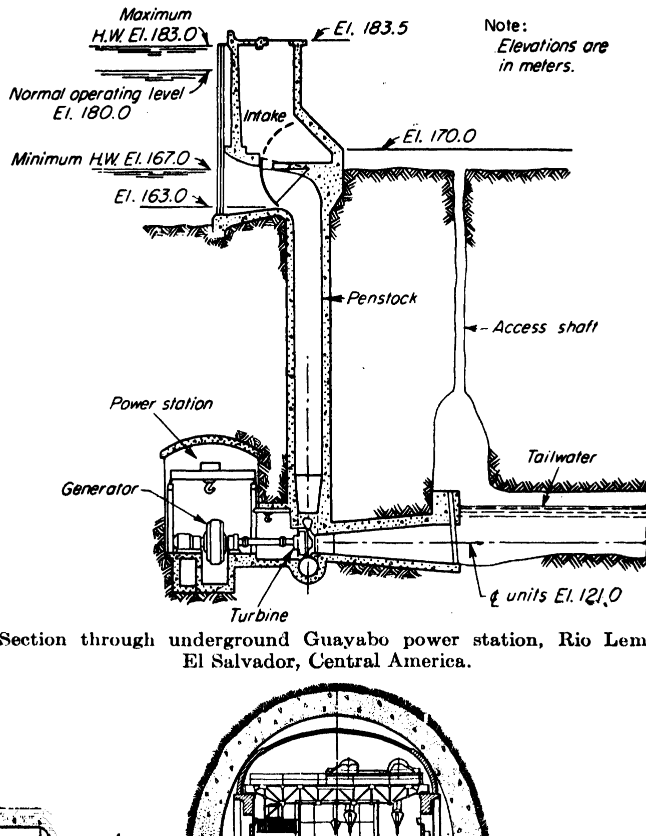


FIG. 32.—Section through underground Guayabo power station, Rio Lempa project, El Salvador, Central America.

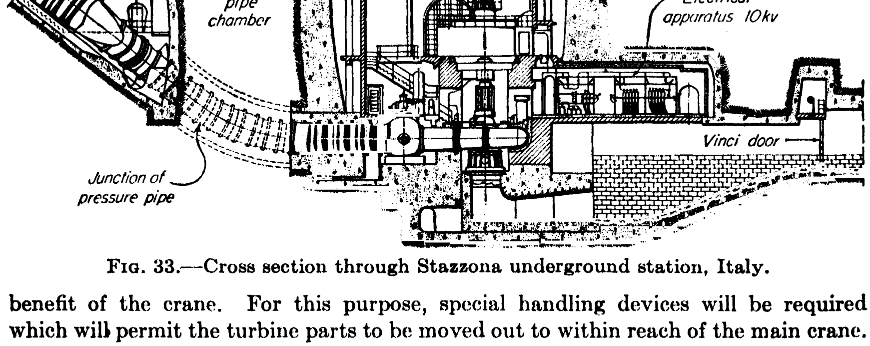


FIG. 33.—Cross section through Stazzona underground station, Italy.

benefit of the crane. For this purpose, special handling devices will be required which will permit the turbine parts to be moved out to within reach of the main crane.

[1] SORENSEN, K. E., Power Station of Radical Design Proposed for El Salvador, *Eng. News-Record*, June 15, 1950, p. 44.

The Guayabo station will be equipped with two horizontal-shaft hydraulic turbines, fed through the vertical penstock described above, driving the two horizontal generators. The plan provides extra floor space for erection and dismantling, switchgear, and operating room. The turbines will be rated each at 21,000 metric hp at a

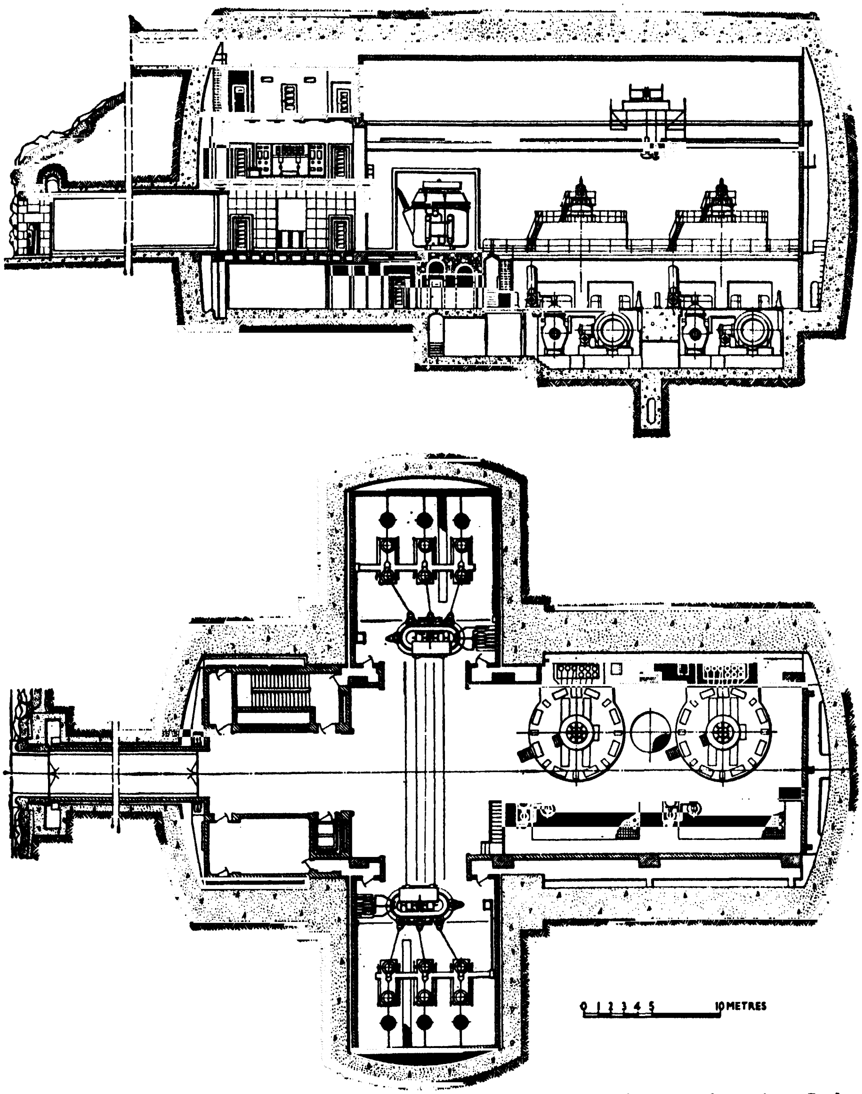


Fig. 34.—Longitudinal section and sectional plan, Stazzona underground station, Italy.

head of 50 m. The generators will be rated at 16,667 kva at 90 per cent power factor. The plan provides for an ultimate installation of five units.

The Stazzona station, another interesting underground plant, serving the city of Milan, Italy, is shown by Figs. 33 and 34. Water for this station comes from the Adda River. The plant has an installed capacity of 50,000 kw. The turbines operate

under a head of 91 m. The power station was erected in a central chamber 50 m long, 31 m high, and 18 m wide. Access is gained by a tunnel 150 m long, large enough to allow the transport of the largest parts of the machinery.

The station is erected inside an inner structure which is independent of the outer tunnel. The runways for the overhead traveling cranes are carried by reinforced-concrete columns and these, with all the remaining structures, are connected by means of a hinged joint to the main external chamber. This joint allows for a slight relative movement between the two structures.

Over the crane runway is a roof formed of hollow, prefabricated units, which completely isolates the roof of the main central power station from the roof of the main chamber. To keep the space between these two main structures dry, part of the cooling air exhausted from the generators is passed through this space before being discharged into the open.[1]

THE FOREBAY

A forebay is essentially a storage reservoir at the head of the penstocks. Where plants are located at the base of the dam, as at Hoover (Figs. 10 to 12) and at Norris (Fig. 18), the reservoir constitutes the forebay. Where plants are at the end of a

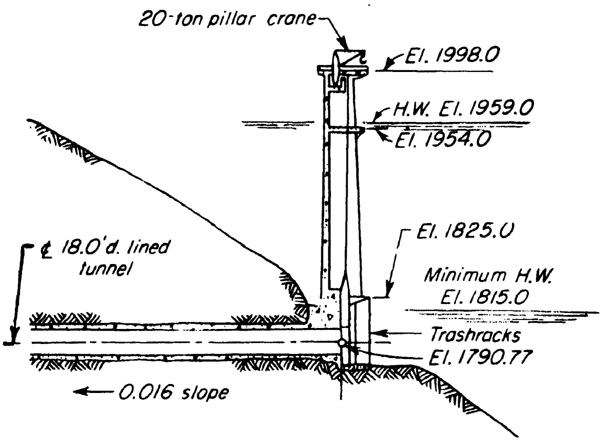


FIG. 35.—Intake tower, Watauga project.

canal, a forebay may be provided by enlarging the canal as at Leaburg (Fig. 8) or by building a dam to create a reservoir as at the Mokelumne plants. The purpose of a forebay is to store water rejected when the load on the plant is reduced and to supply water for initial increments of an increasing load while the water in the canal or pipe line is being accelerated.

Intakes. Intake structures vary widely, depending upon the type of plant. In sharp contrast are the intake tower for the Watauga project (T.V.A.) (Fig. 35), the submerged intake chamber for the South Holston project (T.V.A.) (Fig. 36), the intake structure at Fontana Dam (T.V.A.) (Fig. 37), and the integral intake and power-house structures for relatively low-head projects, such as shown by Figs. 14, 30, and 39.

The Watauga intake (Fig. 35) is a simple U-shaped concrete tower provided with trash racks and guides for the 18.8-ft-wide by 24.9-ft-high structural-steel intake gate with roller trains. A 20-ton capacity pillar service crane is supported on top of this tower.

[1] **Underground Power Stations,** *Water Power (London)*, November-December, 1950.

The South Holston intake (Fig. 36) is a concrete chamber which houses a 15 ft diameter butterfly valve. This valve controls the flow into a tunnel approximately 1,200 ft long, leading to a power plant containing one 35,000-kw unit acting under a rated net head of 180 ft. The intake structure is provided with stop logs and trash racks. Access to the chamber is from a tunnel leading to the left bank. This intake chamber is completely submerged under all operating conditions. The roof of the chamber would be cut away, following the drawdown of the reservoir, should it ever become necessary to remove the butterfly valve. It should be noted that it is extremely rare to use a butterfly valve as a head gate. So far as is known, the South Holston intake is unique in this respect.

The Fontana intake (Fig. 37)[1] offers an example of modern design in connection with high head plant. The power facilities at Fontana consist of the intake shown by

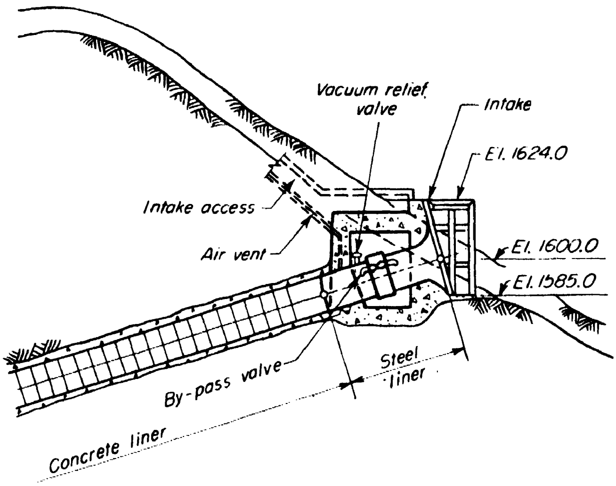


FIG. 36.—Intake chamber, South Holston project.

Fig. 37, three 14 ft diameter steel-lined intake conduits leading to the units in the power house at the foot of the dam. The power house contains two 66,600-kw units with provision for a third. These units operate under a rated head of 330 ft. Each discharges 2,600 cfs at rated head and capacity.

Each of the three trash-rack structures is supported on a concrete cantilever slab which projects 22 ft 2 in. from the face of the dam. In plan, the trash-rack structure is of semicircular shape with a 13 ft 0 in. outside radius. The circumference of the semicircle is subdivided into four straight panels, framed by concrete columns with concrete beam bracing. The column and beam structure has a total height of 97 ft, of which the lower 48 ft has the racks in the panel openings and the upper 49 ft is an extension for withdrawal and maintenance of the racks with the top 14 ft above minimum drawdown.

In the corner columns of the four panels, 7-in. guide channels are embedded. Four rack sections fit into each panel, each 12 ft 1½ in. high by 8 ft 4 in. wide. The vertical rack bars are 4 by ⅝ in. and the horizontal supporting members are 6 by ¾ in., spaced to give openings 6 in. wide by 2 ft 4 in. high. The concrete structure is designed for a differential head of 10 ft and the steel rack sections based on a differential head of 5 ft. The maximum flow velocity through the net rack area is 2.4 fps.

Figure 37 also shows the general arrangement of the intake gate. Each intake

[1] The Fontana Project, *Tech. Report* 12, Tennessee Valley Authority.

gate is of the tractor type and is designed to operate efficiently under a considerable head. Each gate is approximately 27 ft 10 in. high by 16 ft 6 in. wide and closes an opening 20 ft 10 in. high by 11 ft 6 in. center to center of the gate seals. The guides in which the intake gates operate are made up of channels for guide slots and formed plates to ensure a continuous surface for the gate tracks.

The intake conduit starts as an unlined concrete passage forming a transition from the rectangular gate section to the steel-lined circular section below. For the upper 198 ft of the conduit, the plate thickness of the liner is ¾ in. throughout. In this zone, the stresses caused by the water load were computed by assuming the conduit enclosure to be a hollow cylinder with large outside diameter. The resulting

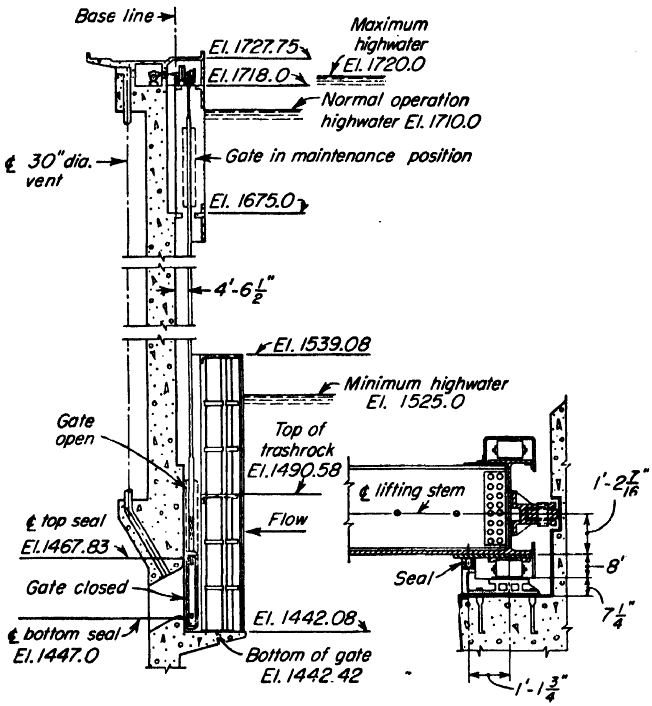


FIG. 37.—Intake, Fontana project.

stresses in this cylinder were combined with the computed stresses in the dam structure. Tensions of 75 psi or less were assumed to be carried by the concrete. All tension stresses above this limit were resisted by combined action of the liner and a horizontal band of reinforcing steel above the liner. For the lower 184 ft of the liner, where the full water load is carried by the steel plates, the plate thickness increases up to 1½ in.

Integral intakes constructed monolithically with the powerhouse substructures are shown by Figs. 28, 29, 30, and 39. In contrast to the usual practice of constructing each bay of such plants as separate blocks, the substructure and intake of the Petenwell station (Fig. 39) represent a single monolith over 140 ft long. No contraction joints were provided. Shrinkage cracks were held to a minimum by reducing the size of pours and building alternate units ahead of others.

The foundation of the Petenwell station consists of medium sand. A continuous

slab reinforced with 0.5 per cent of reinforcing steel supports the structure. No piling was used. The intake bulkhead wall separating the generator room from the headwater also contains 0.5 per cent of steel in a horizontal direction. The watertightness of the wall has been satisfactory.

Vertical-lift wheeled gates were provided for the Petenwell intake. The gates are of all-welded construction and the wheels are equipped with roller bearings to reduce hoisting effort and to eliminate by-pass or filler valves. The gantry crane is

DESIGN DATA FOR TRASH RACKS

SPACING OF VERTICAL BARS

Type of turbine and location of racks	*Space between bars*
Impulse turbines	*$\frac{1}{5}$ of nozzle diameter*
Racks in front of long pipe lines	*$\frac{3}{4}$ in.* *
Medium size Francis and Kaplan turbines	*2 to 3 in.*
Large size Kaplan turbines	*3 to 6 in.*

** For high head installation with pipe line, a mesh rack is recommended instead of bars.*

FORMULA FOR HEAD LOSS IN TRASH RACK

$$H_R = \beta \sin \alpha \frac{1.33 d V_0^2}{2ag}$$

where H_R = friction loss, feet

β = flow coefficient (see table)

d = maximum width of bar, inches

a = space between bars, inches

V_0 = water velocity upstream from rack, feet per second

α = angle between rack and direction of flow, degrees

g = 32.2, acceleration due to gravity, feet per second^2

** β, flow coefficient, for bar shapes shown with a ratio $\frac{l}{d} = 5$, where l = length (in direction of water flow) and d = with (horizontally across direction of flow.)*

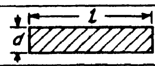
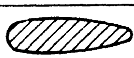
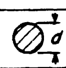
* β	*Bar shapes (cross section)*
2.42	
1.83	
1.67	
1.03	
0.92	
0.76	
1.79	

Fig. 38.—Design data for trash racks.

capable of lifting the gate against full unbalanced pressure to permit filling the spiral case. The head gates are heavy enough to close with flow through the units, which would be required if the turbine wicket gates became inoperative. In case of emergency, an additional gate slot is provided immediately upstream of the service slot to permit lowering of an extra gate in lieu of stop logs.

The trash racks, of all-welded construction, are built in sections which are self-supporting and can be removed entirely. The trash racks are designed to resist full headwater pressure, in case they become completely blocked by anchor ice or trash. The stress in the steel for this condition is 27,000 psi.

Equipment. The forebay has certain equipment peculiar to it, consisting essentially of the following:

1. Penstock closing gates, ventilators, hoists, stop logs
2. Trash racks with rakes and facilities for disposing of the debris
3. Ice-removal system
4. Spillway of the overflow, siphon, or automatic-gate type
5. Telephone system
6. Water-level gages

Penstock Gates. Several installations of various types have been described in the preceding paragraphs. In some cases, such gates are designed to be closed quickly in an emergency but cannot be opened under pressure. The sliding gate meets this requirement for small sizes, whereas for large sizes the Stoney gate is often used as it will close under its own weight. In other cases, the gates are not to be closed or

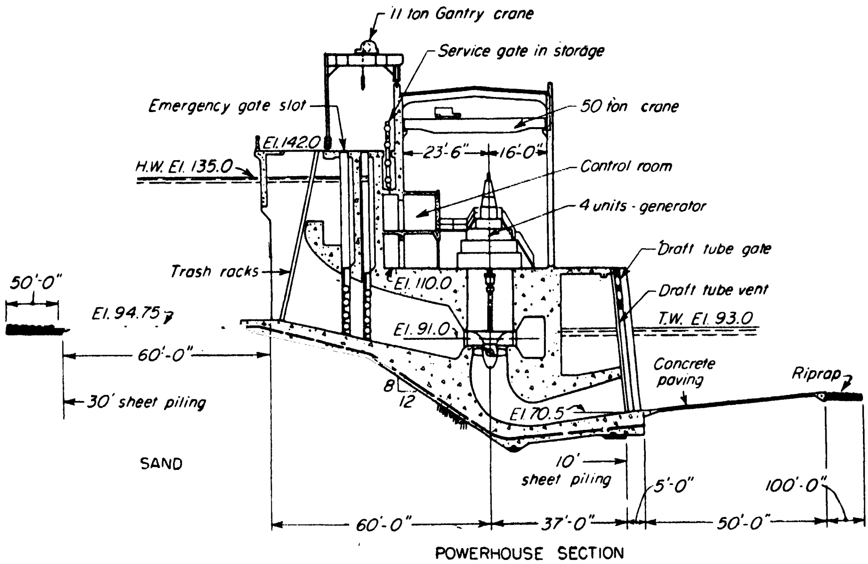


FIG. 39.—Petenwell power station.

opened except under balanced pressure. The turbine gates, or a butterfly or other type of gate in the penstocks near the turbine, are closed and the flow stopped before the forebay penstock gates are closed. Before opening, the penstocks are filled through a by-pass. Such requirements are met by the sliding gate. In still other cases, no lower gate is provided and the forebay gates are designed to close and open under full pressure. This requirement may be met by the Broome gate, the gate either with fixed wheels as the Bonneville crest gates or with a roller train as in the Stoney type. The radial gate (Taintor) operates under balanced pressure and is frequently used for penstock closures. The function of these gates determines their design.

A quick closure of the forebay gates in case of turbine failure might collapse the steel penstocks unless amply vented. At Oak Grove, a 66-in. Johnson balanced valve is in No. 1 penstock at the head, No. 2 contains a butterfly valve. Each penstock is also provided with a butterfly valve just ahead of the turbine. The Oak Grove penstocks are protected against collapse by a 6-in. envelope of concrete.

At Hoover, the penstocks may be closed by cylinder gates in the intake towers that will open or close under pressure.

A water-level fluctuation of 300 ft in Lake Mead had to be provided for in the intake towers; hence two cylinder gates were installed with sills at El. 894 and 1045, respectively. The maximum lake level is at El. 1,229. There are also butterfly valves in each penstock just above the turbine scroll case.

At Leaburg and Norris, the penstocks may be closed by Broome gates that will open and close under pressure. There are no other gates in the penstocks besides those of the turbine. The size of opening often governs the choice of gate and, if too large, it is divided by piers. The space in the penstock just downstream of the forebay gates should be amply vented to the atmosphere by standpipes reaching to above maximum forebay level.

Trash racks are necessary at the entrance to all penstocks to keep out debris larger than can pass through the turbine gates without injuring them. On impulse wheels, careful screening is necessary to prevent debris from choking the nozzle. Racks consist of flat steel bars set on edge, usually made up in panels for ease of handling. The bars are often joined by through bolts with pipe nipples for spacers. Oftentimes the bars are welded to the edge of crossbars, more space being thus left for rake teeth to pass between the bars. The rack panels are supported by beams spanning the bays and are usually inclined for ease of removing debris.

The maximum spacing of bars is generally specified by the water-wheel manufacturer. At Leaburg, there were spaced 3 in. except near the bed of the forebay where 1½-in. spacing was used to keep out the larger sizes of bed-load gravel.

The area of racks should be great enough to keep the velocity through them to 2 fps or less. This area should not include piers and supporting beams but does include the rack panels.

Figure 38 furnishes an acceptable basis for the preliminary design of racks through a comparison of head losses for various shapes and spacings of racks.[1]

The loading should be computed for debris and ice causing a partial stoppage of flow. The resulting head on the racks may be anywhere from 10 to 90 per cent of the normal head, depending largely on ice conditions.

Racks in intakes from large reservoirs are usually placed nearly vertical, and no provision is made for cleaning except as they become accessible with a lowering reservoir. At Norris (Fig. 18), they are placed in the upstream face of the dam. At Hoover (Fig. 10), the rack panels completely surround the intake tower, which is 342 ft above foundations, and practically cover its entire height.

Except in the major plants, cleaning the racks by hand rakes is generally followed, Mechanical rakes are available if their use is deemed necessary. At Hoover, the penstock racks will be manually cleaned as the reservoir level falls.

Disposal of the rakings is a feature often inadequately considered. At Leaburg, the debris is pushed into a chute that can be flushed by opening a forebay gate which sluices the trash into the river via the wasteway.

Ice Control. Where winters are severe, provisions must be made for handling ice in the forebay. Three kinds of ice are recognized: (1) Surface ice is the common crystalline type that forms on the surface of lakes and smooth rivers whenever the water reaches a temperature of 32F. (2) Frazil ice forms within the body of turbulent water whenever it has been cooled to 32F. It is often mistaken for snow. On riffles and turbulent streams, it forms in immense masses until the stream becomes a viscous mass causing the surface to rise, oftentimes to flow smoothly, whereupon surface ice, enclosing masses of frazil ice, is formed. (3) Anchor ice forms on the bed of streams and canals in clear water on cold clear nights where conditions are favorable for direct radiation of the earth heat into space. With a slight rise of temperature, *e.g.*, from the sun shining on the water, this ice may be loosened in large blocks and rise to float

[1] HAMM, H. W., Design Data for Trash Racks, *Elec. World*, July 30, 1949, p. 94.

downstream, carrying large quantities of gravel and other stream-bed material that clings to the underside of the block.

Except under rare cases of supercooling, when it lies undisturbed, water will change to ice whenever its temperature is lowered to 32F. A mixture of water and ice cannot fall below 32F until the water has been converted to ice. In such a mixture, the slightest tendency toward lowering the temperature below 32F increases the portion of ice, because heat is being transferred from the mixture to colder media; conversely, the slightest tendency toward a rising temperature will reduce the portion of ice, because heat is being absorbed from warmer media.

In the condition of losing heat, ice will adhere to any body in contact with the ice-water mixture that is absorbing heat, *e.g.*, racks and gates. In the condition of absorbing heat, ice will not so adhere. If then the temperatures of racks and gates of a forebay can be raised ever so slightly (a hundredth of a degree) above that of the mixture, ice will not adhere to them.

Except in ice jams, surface ice seldom gives much trouble, but frazil and anchor ice may accumulate to such an extent that plants sometimes have had to shut down on their account.

In some instances, the upper portion of the racks is housed in chambers that can be heated. In others, their temperature is raised by passing a low-voltage electrical current through them, for which the rack bars are specially insulated. In still other cases, an air-bubbler system is used to prevent formation of ice (see Safe Harbor plant). Safe Harbor also has a skimming wall, between the forebay and the river, underneath which water flows to the forebay, ice in the river being thus prevented from entering the turbines.

Wasteways. If the forebay is supplied by a canal or if it cannot absorb the water from rejected power loads, wasteways from the forebay must be provided. The long spillway crest is very effective but seldom is there space to accommodate one long enough to keep level fluctuations within prescribed limits. Where space is limited, some form of automatic wasteway is required—one that can discharge all surplus waters even to the full capacity of the plant without raising the forebay level an appreciable amount.

Siphons[1] make excellent wasteways from the forebays of hydroelectric plants. By designing them to inhale air as the forebay level lowers, thus reducing the flow, they can be made to discharge continuously any value between 3 and 100 per cent of maximum capacity. There are no moving parts, and the action is entirely automatic and very dependable. The forebay of the Leaburg plant is equipped with seven regulating siphons having a nominal combined capacity of 2,850 cfs. The forebay of the Waterville plant, also belonging to the City of Eugene, has two siphons of 350 and 700 cfs capacity, respectively, that keep the water level of the forebay, which is at the end of a 5-mile canal, within 0.3 ft of normal regardless of load variations.

The inlets must be kept free of ice, or the primers will fail to function. They should also be provided with racks to keep trash out of the primers.

Water-level Gages. In forebays, it is important to have water-level indicators, and frequently graphic recorders, on the switchboard by which the levels in the forebay may be known at all times. A precaution frequently adopted is to have a duplex indicator showing the levels on both sides of the racks. Alarm signals are frequently included to announce when the head loss through them become excessive.

In some plants, differential indicators or recorders register the difference in elevation of water in the forebay and tailrace, thus giving the total static head on the water wheels. They may be located in the generator room or mounted on the switchboard.

Where the flow is determined at a rated gaging station either in the canal, river, or

[1] Stevens, J. C., Siphons as Water Level Regulators, *Proc. A. S. C. E.*, October, 1938, p. 1627.

tailrace, it is often desirable to have the water levels at such stations registered in the powerhouse. Long-distance recorders and indicators for such service are available.

Flow registration through the turbines is also desirable wherever it can be accomplished. The performance tests of generating units are usually limited to one unit in a plant and referred to one head under a given range of loading. Such data are obtained to check guarantees, seldom for plant operation. If continuous records of turbine flow are available, much more effective and efficient plant operation may be secured.

A reducer in the penstock can frequently be used as a Venturi meter and calibrated at the time acceptance tests of units are made. At Leaburg, such a reducer 20 ft in length just ahead of the entrance to the scroll case was utilized by placing four piezometer taps, each in the 12- and 10.5-ft sections. The average diameter was determined at each set of piezometer taps. Each set of taps was connected to a manifold to each of which was attached a glass tube with a scale between them. The tops of the glass tubes were attached to a common manifold connected to the compressed-air system. By cocks in each line to the penstock and in the air line, the water columns in the glass tubes could be kept within the limits of the scale. This meter was calibrated at the time turbine tests were made, by the salt velocity method, giving the formula $Q = 963\sqrt{h}$, where h is the drop in head between the piezometer sections in feet of water.

Another method known as the Winter-Kennedy[1] principle for obtaining permanent plant flow records is coming into favor. The basic principle involves the conservation of angular momentum in the scroll case. Piezometer taps are placed in opposite walls of the scroll case at a selected section. The difference in pressure in these taps is an index of the discharge which is given by the general formula $Q = k\sqrt{P}$, where P is the difference in pressure. The calibration is made and the coefficients determined at the time turbine tests are made. Thereafter, plant performances and efficiencies can be kept as a routine matter.

Surge Tanks. For long penstocks where an open forebay is not available, a surge tank as a substitute forebay is employed. Its purpose is to relieve the line of excessive pressures due to manipulation of turbine gates as load on the generating unit varies and to supply initial water for an increasing load while the water in the pipe line is being accelerated. It is placed as near the turbines as practicable. The design of surge tanks is covered by Secs. 13 and 14.

PENSTOCKS

The pipe connecting the forebay and the scroll case of the turbine is generally termed the *penstock*. The principles of design are the same as for any other pipe except that an additional allowance must be made for water hammer due to governor control and gate manipulation.

Penstocks are made of steel, wood, or reinforced concrete. They may be placed aboveground on cradles, buried, half-buried, or placed on cradles within a tunnel. Occasionally a pressure tunnel is used for a considerable portion of the penstock length as at the Chelan Plant[2] of the Washington Water Power Co.

If the distance between forebay and powerhouse is short, a separate penstock for each turbine is preferable, as at Leaburg. If the head is high, separate penstocks for each unit are to be preferred, as at Oak Grove. For moderate heads and long lines, a single penstock may serve several turbines by a system of multiple bifurcations at the lower end. Using a header penstock with branches to each unit may be necessary,

[1] WINTER, IREAL A., Improved Type of Flow Meter for Hydraulic Turbines, *Trans. A. S. C. E.*, **99**, 847. The patents are owned by the Simplex Valve and Metering Co.

[2] *Elec. West*, Sept. 1, 1928.

as at Boulder, but should be avoided where possible on account of the loss of head at the junction.[1]

Water Hammer. When the velocity of water in a closed conduit is altered, a series of pressure waves is started which travel the length of the pipe, being gradually damped out by hydraulic friction. This effect is very noticeable in penstocks where load changes require corresponding changes in turbine gate openings. If a load is rejected, the gates close, velocity is checked, an initial positive increase in pressure being produced which changes to negative as the positive wave travels to the head of the pipe. Thereafter, at any given cross section the pressure values swing alternately above and below the normal for steady flow.

Conversely, if load increases, turbine gates open, velocity accelerates, and a negative pressure wave travels up the pipe followed by positive pressures as the column of water in the pipe accelerates. For checked velocity, the swings of pressure above and below normal are about equal, but for accelerated velocity the swing above normal may be considerably less than that below.

In penstock design, it is necessary to determine the excess pressure above normal that should be allowed for water hammer and also to provide against an intake of air on the passing of a negative pressure wave. The amount of pressure rise depends upon the elastic properties of the pipe line, whether it is of a single or of varying diameters and wall thicknesses, and upon the rate of gate closure.

The pipe period is the time required for a pressure wave to traverse its length and return. It is the same as the time for a sound wave to travel through the water in that particular pipe. This period is $2L/c$, where L is the total length of pipe and c the celerity of wave propagation. It has been found that these waves are reflected at sections where the conduit changes diameters or thickness. A resonant effect may build up pressures considerably greater than would occur in a simple pipe. If the time of gate movement is less than the pipe period, such resonant effects and afterwaves may become very important. In the analysis of such compound pipes, it is customary to substitute an equivalent pipe of uniform diameter. Space requirements prohibit development in this section of the theory of water hammer. Reference should be made to recent texts on advanced hydraulics for a full treatment of this subject.

Experience has shown that with properly designed penstocks, failures result not from governor action but from accidental conditions. The penstocks of the Moccasin plant (City of San Francisco) were ripped open because someone inadvertently closed the penstock gates at the powerhouse. The design of penstocks should include safety measures to reduce the possibility of such accidents to a minimum.

As a rule of thumb for short lines and low heads, it has been common to add 50 per cent of the normal operating head in a penstock to take care of water hammer. This is not an unreasonable allowance but coupled with it should be included relief, by-pass, and air valves, if necessary, proper governing, and such other precautions as will ensure that this surcharge will not be exceeded. In long lines under high heads, no such percentage increase is permissible, but the maximum water hammer for the most critical operating conditions must be painstakingly worked out. A detailed discussion of water hammer will be found in Sec. 13.

Reinforced-concrete pipe is used extensively for penstocks. At Leaburg they are 12 ft inside diameter under a maximum static head of 90 ft to which was added 50 per cent for water hammer. On the Metropolitan Aqueduct of Southern California,[2] concrete pipes have been built monolithic, 12 ft 4 in. inside diameter for heads of 160 ft or more on very steep grades.

If the heads, including water hammer with safety margins, have been fixed liber-

[1] STEVENS, J. C., Theoretical Energy Losses in Intersecting Pipes, *Eng. News-Record*, July 22, 1926.
[2] *Eng. News-Record*, **117**, 531, 1936.

ally, the working stresses in reinforcement can be taken at 60 per cent of the commercial elastic limit. A good quality of concrete with ample bonding must be provided to develop the full stress in the reinforcement.

Concrete penstocks may usually be buried and expansion joints omitted. In cold climates, exposed concrete pipe may be adversely affected by freezing of the water that penetrates the pipe walls under pressure. No concrete is absolutely waterproof.

Steel pipe is used for penstocks more than any other material. Great advance in the technique of welding has almost supplanted the use of rivets for joining the plates of penstocks. On important work, it is sometimes required that a definite portion of each longitudinal or girth weld be photographed by X-ray. These radiographs are then minutely examined for blowholes, slag pockets, cracks, poor fusion, and other

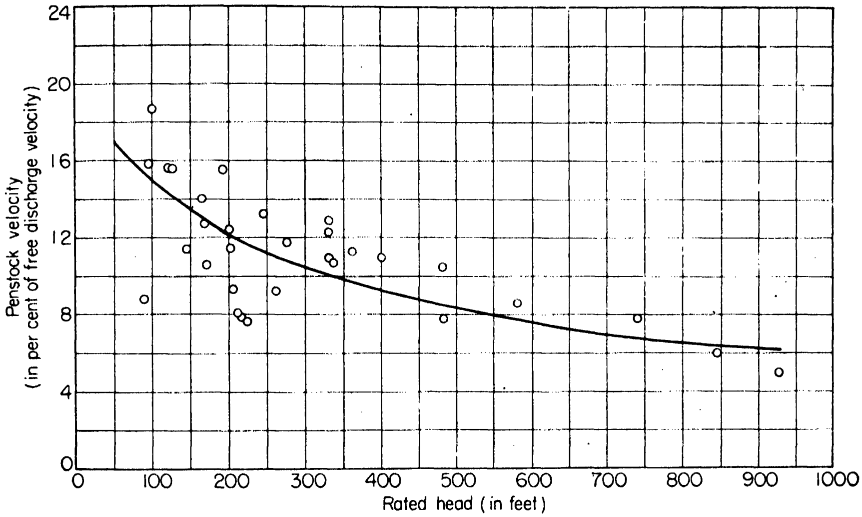


FIG. 40.—Penstock velocities at full turbine discharge. Existing plants.

defects. Joints that have been radiographed and had such defects corrected may be given an efficiency of 95 per cent, whereas 85 per cent should be adopted for joints not so inspected. This is somewhat higher than for riveted joints. Pipe of large diameter, especially if of minimum thickness, should be reinforced by regularly spaced stiffener angles. The Oak Grove 9-ft pipe has 5- by 3-in. stiffener angles spaced $7\frac{3}{4}$ ft wherever the plate thickness is $\frac{3}{8}$ in. (minimum thickness). The great advantage of welded pipe is its interior smoothness. With portable welding equipment, field repairs are made with equal or greater facility than for riveted pipe.

For riveted steel pipe, the joint efficiency varies with the type of joint about as follows:

Type of joint	Plate thickness, in.	Average efficiency, %
Single-riveted lap joint	$\frac{1}{4}$–$\frac{5}{8}$	60
Double-riveted lap joint	$\frac{1}{4}$–$\frac{3}{4}$	70
Triple-riveted lap joint	$\frac{1}{4}$–1	74
Double-riveted double butt joint	$\frac{3}{8}$–1	82

Economic Diameter. Experience has shown that the economical velocity for steel penstocks will fall somewhere between 10 and 20 fps. For preliminary investigations, there are several empirical formulas which may be used as a first trial. Two of these are:

$$V = 0.125\sqrt{2gH} \tag{18}$$

$$D = \left(\frac{P}{H}\right)^{0.466} \tag{19}$$

in which V = penstock velocity.
H = rated head.
P = rated horsepower.

These formulas, applied to a number of steel penstocks constructed by the Bureau of Reclamation, give the results shown by Table 4. The curve in Fig. 40 represents the average of penstock velocities, expressed as a percentage of spouting velocities for a number of large plants. This curve offers another approach to a preliminary study. In any final design, however, a study should be made to determine the value of power lost plus all annual charges for a range of penstock sizes. It is usual practice to keep the sum of these two values at a minimum.

TABLE 4.—COMPARISON OF ECONOMICAL VELOCITIES AND DIAMETERS OF SEVERAL PENSTOCKS WITH ACTUAL VELOCITIES AND DIAMETERS[1]

Power plant	Turbines		Actual penstock values			$V = 0.125\sqrt{2gh}$		$D = \left(\frac{P}{H}\right)^{0.466}$	
	Rated head	Rated hp	Q	V	D	V	D	V	D
Hoover	480	115,000	2,450	18.5	13.0	22.0	11.9	18.9	12.8
Grand Coulee	330	150,000	4,500	17.7	18.0	18.2	17.7	19.1	17.3
Shasta	330	103,000	2,800	15.8	15.0	18.2	14.0	16.9	14.5
Green Mountain	203	15,000	750	13.2	8.5	14.2	8.2	17.4	7.4
Seminoe	171	15,000	890	11.3	10.0	11.6	9.9	19.6	8.1
Marys Lake	205	11,300	550	10.9	8.0	14.3	7.0	16.6	6.5
Estes	482	21,000	449	13.5	6.5	22.0	5.1	17.0	5.8
Anderson Ranch	245	18,500	756	17.1	7.5	15.7	7.8	17.1	7.5
Heart Mountain	265	8,300	312	12.0	5.75	16.3	4.9	15.9	5.0
Kortes	200	18,500	912	14.3	9.0	14.2	9.0	17.3	8.2
Davis	120	62,200	5,220	13.7	22.0	11.0	24.6	19.6	18.4
Boysen	96	10,500	1,086	12.5	10.5	9.8	11.9	17.5	8.9
Hungry Horse	400	105,000	2,542	17.8	13.5	20.0	12.7	18.0	13.4
Canyon Ferry	125	23,500	1,930	13.5	13.5	11.2	14.8	18.6	11.5

[1] BIER, P. J., Welded Steel Penstocks, Design and Construction, *Engineering Monograph* 3, U.S. Bureau of Reclamation.

THE SUBSTRUCTURE

This portion constitutes the foundation of the building and contains the draft tubes, scroll cases, turbine settings, end reducers of the penstocks, penstock valves, and by-pass valves, if any, and in low-head plants, the penstocks and racks. The substructure is generally a concrete block with the passages and waterways molded therein, extending from the foundations to the generator floor. The scroll case may be formed in concrete as in Leaburg (Fig. 7) Safe Harbor (Fig. 15) or Petenwell (Fig. 39) or it may be of steel partly or wholly embedded in concrete, as at Hoover (Figs. 10 to 12) or in Apalachia (Figs. 42 to 44).

Over-all station economy must be the objective of the designer in selecting the size and spacing of the units, the elevation of the runner with respect to tail water, and the specific speed. In order to achieve minimum station cost, the turbine setting must be tailored to the particular site and cannot be established by statistical or empirical methods. This is especially true of the integral type of intake, such as shown by Figs. 30 and 39. This is illustrated by Fig. 41[1] which shows a somewhat exaggerated comparison of alternate deep and shallow settings for an integral intake and powerhouse substructure. In other words, the whole structure acts as a dam and both the over-all proportions and the stability are affected materially by the choice of setting. For the same head, the same power, and the same margin of safety against cavitation, a relatively deep setting, with respect to tail water, means both deeper excavation and more expensive structures. This additional cost, however, is offset in part by a higher allowable operating speed which results in smaller physical

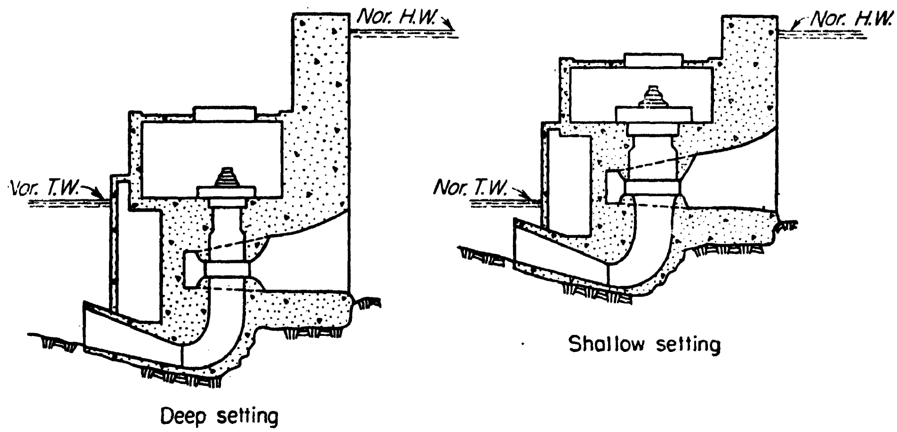


FIG. 41.—Comparison of deep and shallow settings.

dimensions and lower costs for the turbines and generators, and relatively less WR^2 will be required for a given degree of speed regulation.

The shallow setting, on the other hand, minimizes excavation and the structures. A slower operating speed, however, results in larger physical dimensions and costs for the generating units. The choice, therefore, will lie somewhere between the deep setting shown on the left of Fig. 41 and the shallow setting shown on the right. Where the rock is at low depth, the natural selection would be the smaller diameter, high-speed wheel, and conversely where the rock occurs at higher levels. For most sites, comparative estimates of several settings will be required to achieve the minimum.

In laying out the substructure, room must be provided for withdrawing the runner. It is also necessary to provide an inspection gallery surrounding the turbine and extending to the underside of the generator. This gallery is often referred to as the basement. The generator is conveniently supported on a heavy concrete barrel surrounding the turbine and extending upward to the generator floor to transfer the weight of the unit to the foundations. This barrel has openings in it for access to the turbine, ventilation, etc.

The Apalachia station (T.V.A.) (Figs. 42 to 44) offers an interesting example of a powerhouse substructure for a high-head installation. A concrete gravity dam, 150 ft high, a tunnel nearly 8 miles long, and the powerhouse containing two 37,500-kw units, operating under a 390-ft rated head are the principal elements of the project.

[1] RICH, G. R., Basic Hydraulics of Water Storage Projects, *Civil Eng.*, 1944, p. 351.

One 144-in. butterfly valve is located in each penstock about 300 ft upstream from the powerhouse, as shown by Fig. 42. The powerhouse substructure is 147 ft long and 87 ft wide, consisting of two unit blocks, each 44 ft long, and a service bay, 59 ft long. The superstructure encloses the generator room; a crane support, 48 ft in outside width and 143 ft long; and a 4-story equipment bay.

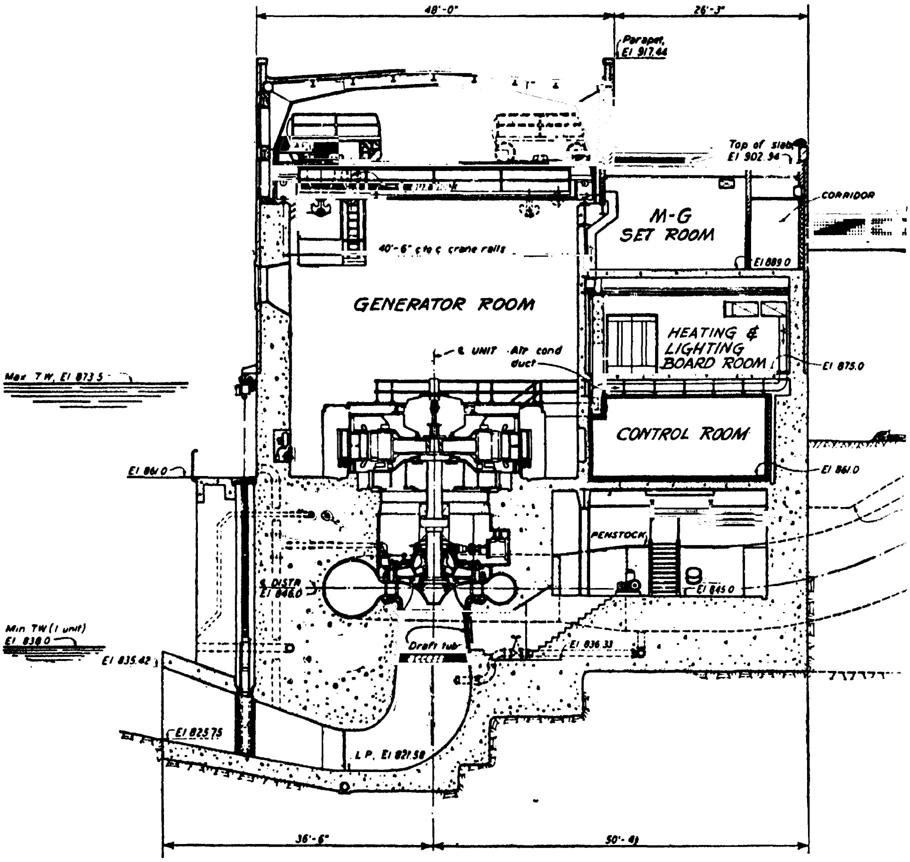


FIG. 42.—Cross section,

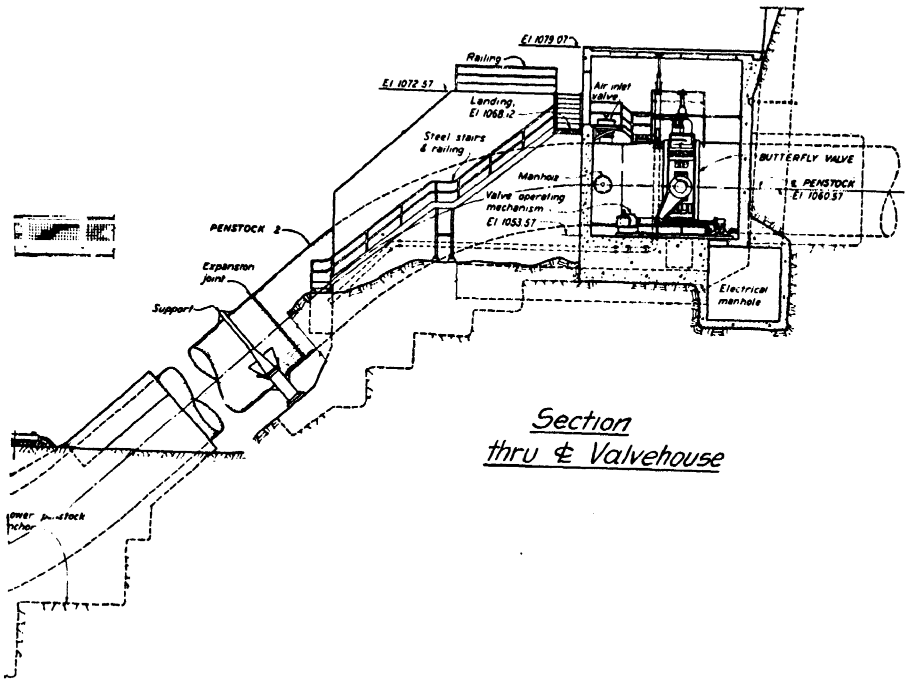
The substructure and outside walls of the superstructure are of concrete to levels above maximum tail-water elevation. The remainder of the superstructure consists of structural-steel framework with concrete floors and tile or concrete walls. The layout and design of the powerhouse were substantially influenced by the provision for tail water at El. 822.[1]

The draft tube is of the plain elbow type. It has two outlet openings separated by a 4-ft-thick center pier. At the bottom of the runner, the draft tube is 8 ft 7 in. in diameter; at the discharge end it has a total clear width of 21 ft 9 in. and a height of 9 ft 8 in. The velocity head at the draft tube exit at full gate discharge is 1.02 ft.

Cork-tar mastic is placed over the scroll case in the unit block and also over the upper half of the penstock where it goes through the east wall of the powerhouse. Between the unit block and the east wall, only the lower part of the penstock is

[1] The Apalachia, Ocoee No. 3, Nottley and Chatuge Projects, *Tech. Report* 5, Vol. 2, Tennessee Valley Authority.

embedded in concrete. These arrangements allow the penstock and the scroll case to expand and contract freely without causing undue stress in the concrete. Where the station is located at the toe of a dam, it is usual practice to provide a contraction joint between the dam and the powerhouse substructure. A flexible joint should be placed in the penstock where it crosses this contraction joint.



powerhouse, Apalachia project.

DRAFT TUBES

The draft tube is usually a very essential part of a turbine installation. It supplements the action of the runner by utilizing most of the energy remaining in the water at the discharge from the runner.

The draft tube is designed as a diverging discharge passage connecting the runner with the tailrace. It is shaped to decelerate the flow with a minimum of losses so that the kinetic energy remaining in the flow at discharge from the runner may be efficiently regained by conversion into the suction head, thereby increasing the total pressure difference on the runner. This regain of kinetic energy is usually the primary function of a draft tube.

The draft tube frequently serves a second purpose, that of regaining static suction head in cases where the runner is located above tail-water level. When the vertical distance of the runner above tail water is well within an atmospheric head and when

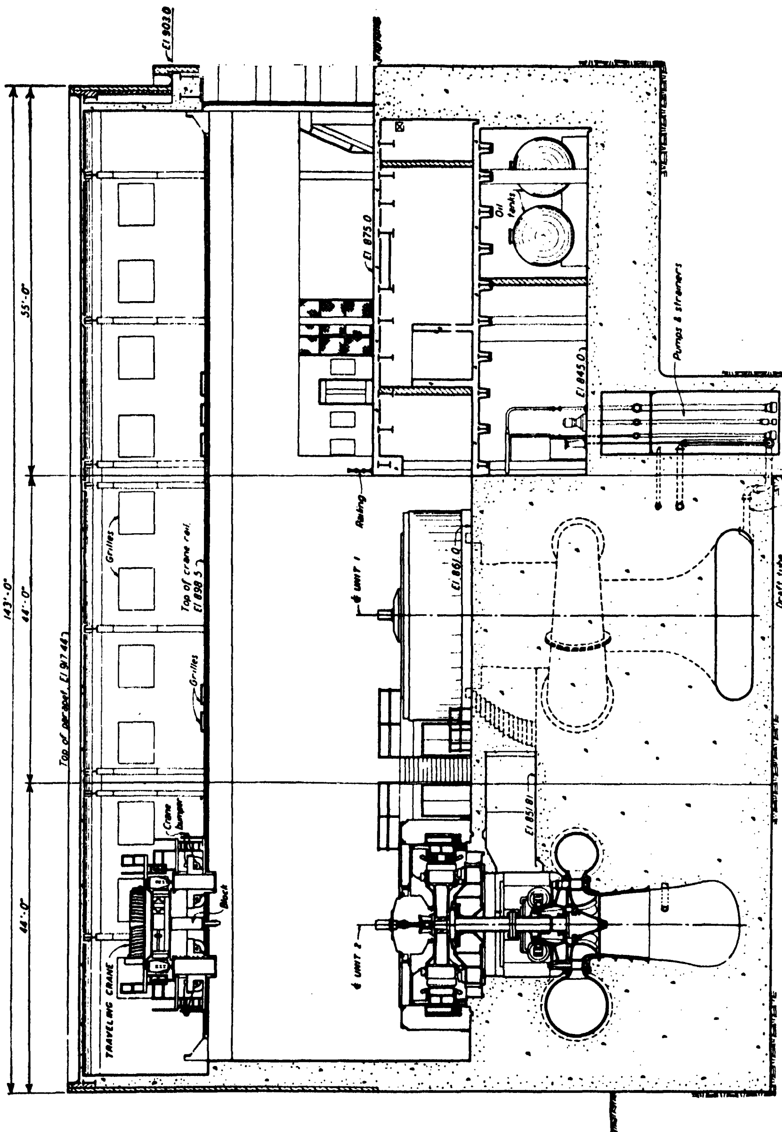


FIG. 43.—Longitudinal section, powerhouse, Apalachia project.

the outlet of the draft tube is sufficiently submerged to assure a water seal, the negative static draft head on the runner is added to the positive head from headwater to make up the total static head on the turbine. Although an atmospheric head is nearly 34 ft of water at seal level, the static draft head should never exceed 15 ft, and even this should not be reached except during low-water conditions of short duration with units of low specific speed and relatively high heads.

Draft tubes are not used with impulse wheels, and the head from the nozzle to tail water is necessarily lost.

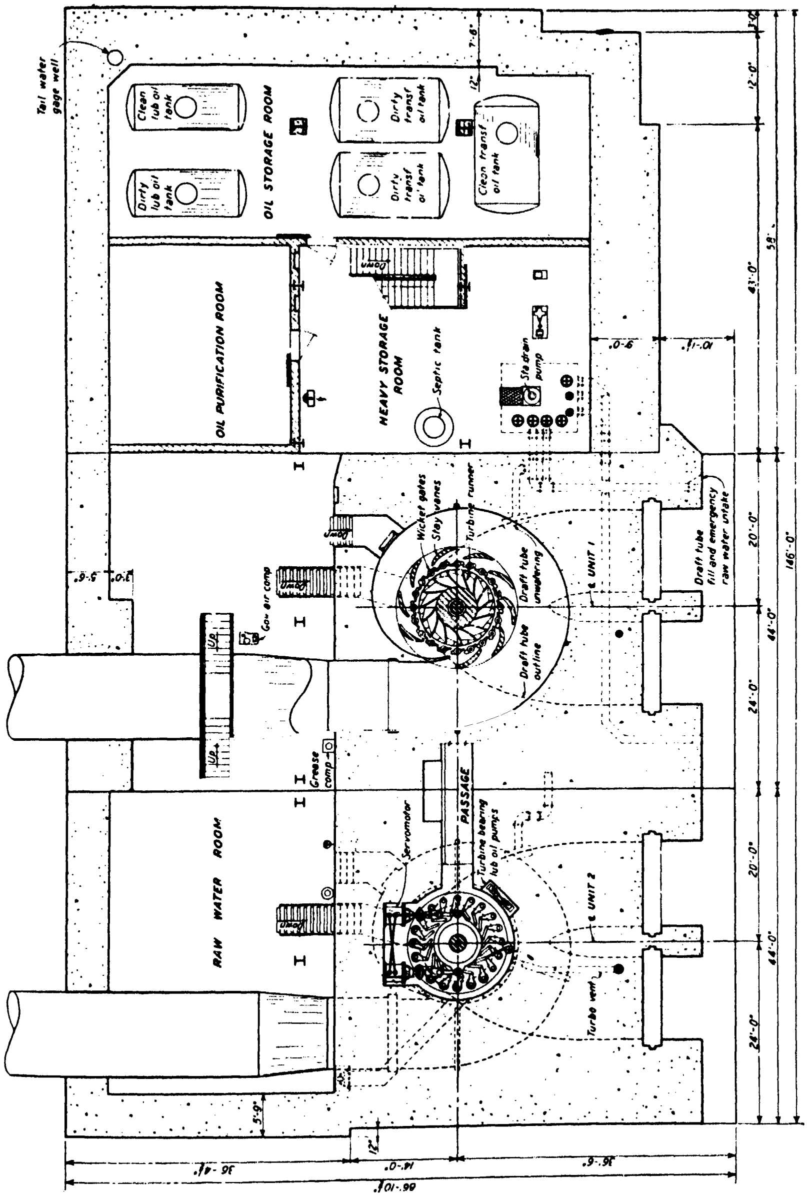


FIG. 44.—Sectional plan, powerhouse, Apalachia project.

Energy Relations. Using the tail-water level as datum, we can write the Bernoulli equation for the draft tube thus:

$$Z_1 + 2.3p_1 + \frac{V_1^2}{2g} = \frac{V_3^2}{2g} + h_f + h_i \qquad (20)$$

where Z_1 = elevation of turbine exit above tail water equivalent to the draft head.
p_1 = gage pressure (psi) at turbine exit.

V_1 = velocity at turbine exit.
V_2 = velocity at draft-tube exit.
V_3 = velocity in tailrace beyond disturbance from draft-tube exit.
h_f = friction loss in draft tube.
h_i = eddy loss at draft-tube exit.

For the vertical-tube type, the exit velocity head is lost, *i.e.*, $h_i = v_2^2/2g$. For the elbow or symmetrical type, some of the exit velocity is preserved in the direction of flow in the tailrace; hence it may be considered as a sudden enlargement in a conduit for which the loss is

$$h_i = \frac{(V_2 - V_3)^2}{2g} \tag{21}$$

The right-hand member of Eq. (20) is a relatively small quantity usually not over 2 ft. The exit velocity from the runner will depend on the specific speed of the turbine and may be anywhere between 20 and 40 fps; hence the greater V_1, the less the draft head, for the absolute pressure at the inlet to the draft tube cannot be less than the vapor pressure of water and should be substantially more.

The most efficient draft tube is a vertical tapered pipe expanding at the rate of about 8 deg central angle and approximately 4 to 5 inlet diameters in length. Rarely, however, is there space for such a tube and the elbow-type draft tube is generally used. In the past, the Moody-type spreading tube and the White hydraucone have been used, but with the advances in design, improved efficiency, and greater simplicity of construction of the elbow type, this latter type has virtually eliminated their use.[1]

The design of the draft tube is usually supplied by the turbine manufacturer and is an integral part of the turbine in determining turbine performance. Cavitation and pitting on the underside of runner blades may result with low-head high-specific-speed units if the draft head is too great or the rate of expansion too rapid. The draft heads on several installations are given in Table 5.

TABLE 5.—DRAFT HEADS FOR TURBINES

Plant	Type of wheel	Head, ft	Specific speed, rpm	Draft head, ft
Leaburg	Reaction	89	81.5	6.5
Hoover	Reaction	525	24.3	12.0
Safe Harbor	Kaplan	55	150	1.0
Norris	Reaction	165	48.8	11.5
Oak Grove	Reaction	850	22.4	10.0
Bonneville	Kaplan	45	158	− 7.0
Bonneville	Kaplan	64	102	− 2.4

Note that with the preceding Kaplan installations the runner is placed very near the low water level. This is done in order to minimize cavitation effects. For all the plants cited, the draft head is negative for high tail-water levels, *i.e.*, there is some back pressure on the draft outlet which, however, may be overcome by the dynamic pressure of the water at the draft-tube entrance. The earlier practice of utilizing a high draft head has sometimes resulted disastrously—in one important Western plant

[1] MOCKMORE, C. A., Flow Characteristics in Elbow Draft Tubes, *Trans. A.S.C.E.*, **103** (1938) p. 402.

TABLE 6.—NEGATIVE PRESSURES AT ENTRANCE OF DRAFT TUBES

Plant	Type of draft tube	Discharge, cfs Q	Head, ft	Draft head Z_1, ft	Mean velocity at draft tube, fps: Entrance V_1	Mean velocity at draft tube, fps: Exit V_2	Gage pressure, psi
Leaburg	Elbow	1,014	89.0	6.5	16.0	6.8	−4.0
Safe Harbor	Elbow	8,000	55.0	1.0	26.7	8.1	−4.4
Oak Grove	Moody	400	850.0	10.0	15.1	3.7	−5.3
Norris	Elbow	4,100	165.0	11.5	27.3	6.2	−9.3
Bonneville	Elbow	12,000	45.0	− 7.0	29.0	5.1	−3.6
Bonneville	Elbow	10,000	64.0	− 2.4	24.2	4.3	−1.9

the turbine runners were pitted so badly by cavitation that they dropped off into the draft tube before it was realized what was happening.

Negative pressures at the head of the draft tube may be computed from Eq. (20) for cases where the other factors are known. Table 6 gives the negative pressures for the plants listed in Table 5.

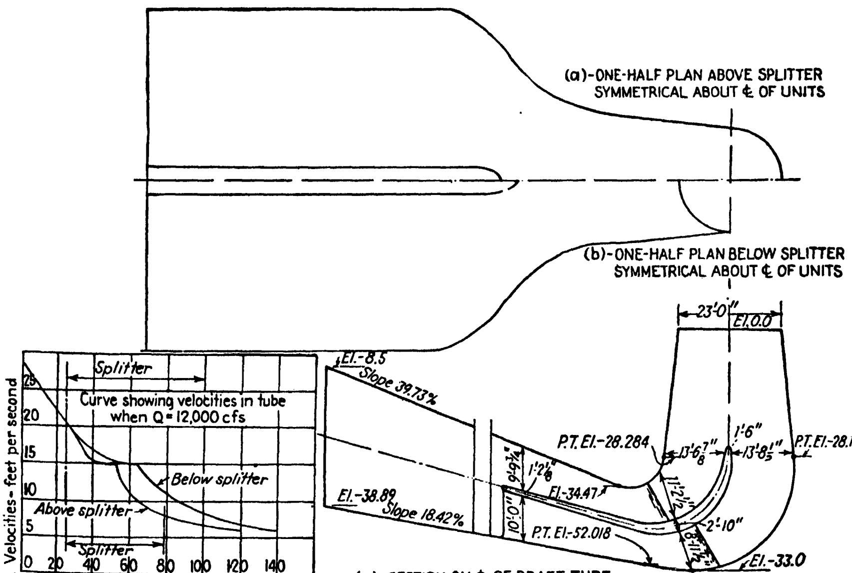


FIG. 45.—Bonneville draft tube.

The negative pressures indicated in the last column above are based on the kinetic energy of the mean velocity. Actually, the kinetic energy is greater than that indicated by reason of the velocity distribution in the draft tube.

It will be observed that for Bonneville, under a head of 45 ft, nearly 30 per cent of the energy remains in the water after it leaves the runner, which emphasizes the necessity of careful draft-tube design in order to realize as much of this as possible.

Figure 45 shows the essential dimensions of this draft tube. Note that it is divided both vertically and horizontally into four compartments. This is done to give better distribution of velocities in the tube, velocity-head recovery being made more complete.

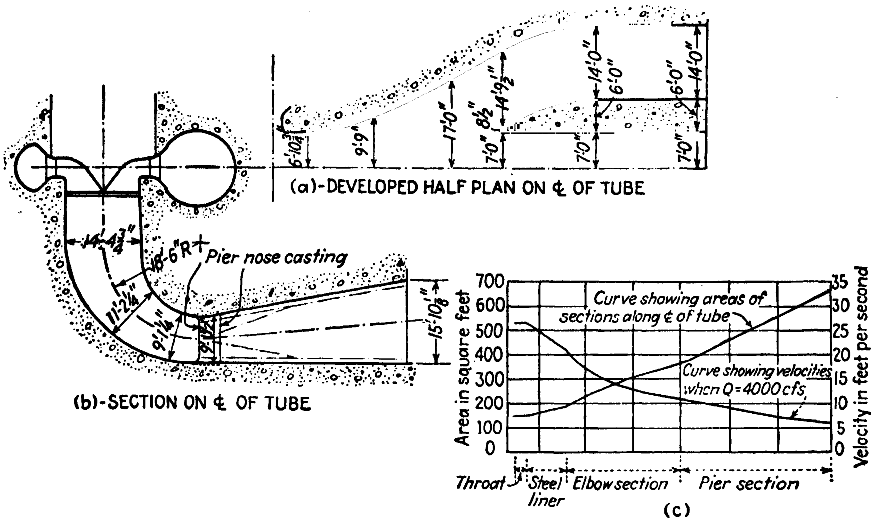


FIG. 46.—Norris draft tube.

Figure 46 illustrates the draft tube for the Norris plant, which is divided by two vertical splitter piers into three exit compartments for better recovery of the kinetic energy.

THE SUPERSTRUCTURE

The superstructure constitutes the main building, including everything above the main generator floor. The height of the building is generally determined by the required clearance for the traveling crane in handling the largest pieces of equipment, the turbine runner, and the shaft. An erection bay, usually at one end of the main floor, must be provided for the dismantling of the units. The shop may be adjacent to this service bay or elsewhere and is preferably served by the crane. An electrical bay is usually provided to house the switchboard and switching apparatus.

The building should be designed for wind, snow, and equipment loading as any other building of its class. In addition, the crane loads are generally carried by the walls. In some cases, maximum water level may be well above the generator floor; windows and doors must then be protected and the walls and foundations designed for pressure and uplift.

The framework which supports the roof and crane may be constructed of structural steel or reinforced concrete. Rigid frame bents are frequently used. A common type of structure consists of reinforced-concrete walls designed to support the crane load and the loads transmitted from the roof above. The roof and upper part of the superstructure above the crane may be either a steel truss supported on short columns, on the top of the concrete walls, or rigid frame bents.

Contraction joints are usually provided through each bay. These contraction joints should be continuous and extend in a vertical plane through the roof and superstructure walls down through the foundation.

POWERHOUSE EQUIPMENT[1]

Space requirements do not permit a complete discussion of powerhouse equipment. The principal hydraulic equipment of hydroelectric plants which justify full treatment in this book are covered by Secs. 12 to 15. This discussion will be limited, therefore, to the broad trends between, say, 1940 and the publication of this edition.

Turbines. In general, three types of turbines are in use at the present time: the propeller type, or axial-flow turbine; the Francis or reaction turbine; and the Pelton or impulse type.

The Francis or reaction turbine has been in use for almost a century and has gradually been improved until, at the present, there are several installations operating at above 94 per cent efficiency. The reaction turbine obtains rotative force by deflecting the flow of the water in closed passages and utilizing the consequent reaction. In general, this type of machine is used where the head available is between 75 and 900 ft. The highest head on which a Francis-type turbine has been installed was put in operation in 1942 at the Nantahala hydroelectric development of the Aluminum Company of America, in North Carolina. This turbine operates under a static head of 1,008 ft and has shown an efficiency of more than 93 per cent. Although considerable maintenance is required on this unit, the high efficiency obtained more than pays for the increased maintenance costs.

The propeller type, or axial-flow, turbine is essentially the same in shape as the ship's propeller, and the rotative force is obtained by forcing the water to flow past the propeller blades in a closed passage. The simplest type has blades which are fixed in position. The Kaplan type, or automatic adjustable-blade, turbine is designed to permit rotation of the blades on their hubs so that the angle that the blades make with direction of flow of the water can be changed automatically to provide for the maximum efficiency under varying conditions of load and head.

Propeller-type turbines are used almost exclusively at the present time for installations where the head is less than 75 ft. The Kaplan type is becoming increasingly popular because of its high efficiency over a large portion of the operating range, and because the cost of this type of machine is being reduced as the manufacturers become more familiar with its construction. The first Kaplan turbine in the United States was installed at the Lake Walk plant in Texas in 1927.[2]

Since fixed-blade propeller turbines are generally less expensive than the Kaplan adjustable-blade wheels, several plants have been designed recently using a combination of both Kaplan and fixed-blade machines. This was done on the Santee-Cooper project,[2] where two Kaplan units and two fixed-blade units were installed. In operating a plant of this kind, the operators must be careful always to have the fixed-blade wheels operating at the point of maximum efficiency, and to take the variations in load on the adjustable-blade units, which have high efficiencies under a large range of loads.

The possibility of keeping Kaplan wheels operating at high efficiencies at relative low loads permits having a number of units in operation at all tines, thus making available considerable "spinning reserve." On systems having frequent sizable load fluctuations, this factor has led to the installation of all Kaplan machines, rather than having part fixed-blade units. The fact that the adjustments of the blades are made automatically by the governor as the load varies removes any chance of operator error or inattention and, therefore, increases the over-all kilowatthour output of a plant.

The Pelton or impulse type of water wheel derives its rotative force from the

[1] This material on Powerhouse Equipment was contributed by E. Montford Fucik and Chester E. Bauman, respectively, Vice-President and Chief Electrical Engineer, Harza Engineering Co. It is reproduced through the courtesy of *Electric Light and Power*, January–February, 1948.

[2] Harza Engineering Co.

impulse of a free jet which impinges against buckets mounted on the rim of the water wheel. This type of unit is used on high head installations, above 900 ft. Refinements in the shape of the nozzles and buckets have raised the efficiencies obtainable with this machine to very close to 90 per cent.

The higher the speed at which a Francis or propeller-type water wheel is designed to operate under given head and flow conditions, the greater the probability of cavitation and consequent pitting of the blades and other steel parts. On the other hand, a higher speed results in a smaller and, therefore, cheaper generator. Recent experience has shown that if the blade areas which are most subject to cavitation, such as the bottom side of the blades at the training edge, are covered with stainless steel, the amount of pitting is drastically reduced.

At the present time, many turbines of the propeller type are provided at the time of manufacture with a thin (1/8 to 1/4 in.) layer of stainless steel in the areas most vulnerable to cavitation. This stainless steel is welded in place in areas left lower than the rest of the blade, and ground to a smooth contour. This procedure permits a higher speed to be used and results in substantial economies.

Several water wheels have recently been built entirely of stainless steel. This has been done where both cavitation and corrosion or erosion were expected to be unusually severe. Increasing use of stainless steel in the construction of water wheels can be expected, as its advantages become more fully realized and as the price of stainless steel becomes lower in respect to ordinary steel.

Generators. The largest hydroelectric generating units have steadily increased in size from less than 5,000 kva in 1900 to the 108,000 kva Grand Coulee units. With the advent of the larger machines, it was necessary to ship the generators in sections and finish the fabrication in the field. This created a manufacturing problem which has been solved very satisfactorily in recent years, so that the amount of fabrication in the field has been reduced to a minimum for installations in this country. However, for foreign projects, due to shipping limitations, it is still necessary to do most of the assembly in the field.

There is little chance for standardization in hydroelectric units. The hydraulic considerations determine the capacity and speed of the turbine and generator, and once this is decided, the generator kilowatt output can be increased only by operating at a temperature rise above normal, or at a higher power factor.

Class B insulation is now standard for all generators above 5,000 volts and 6,250 kva. However, there is a definite trend to specify Class B insulation for all hydro generators in spite of the fact that Class A is standard for all sizes below 6,250 kva, regardless of voltage. The 80C temperature rise above 40C ambient allowed by Class B insulation is seldom used for rating a machine. Normal rating is usually based on 60C rise, which allows for approximately 115 per cent of continuous capacity operation without injury to the machine.

Water-wheel generators are necessarily large due to the comparatively slow speed and the requirement of a large WR^2 for stability. However, the trend is toward lower short-circuit ratios and WR^2 as the result of faster operating system relays and breakers. One-cycle relays and three- or five-cycle breakers have eliminated in many cases the necessity of providing machines with high inertia to maintain system stability. This reduction is limited by hydraulic considerations to a value which will give stable operation of the combined turbine and generator without reference to the rest of the system.

The trend of generator voltages has been to the 15-kv classification, not only as the most popular, but also as a maximum. This seeming popularity is due mainly to the preponderance of large generators being installed. It is still more economical in many cases to use the lower voltages for small generators, due to reduction in cost of

generators and switchgear. It must be kept in mind that below 5,000 volts some of the saving is canceled if Class B insulation is desired. Increased knowledge of methods for carrying heavy currents has helped simplify the use of lower voltages. However, if low-voltage switching is employed, care must be taken that the maximum current rating available in air breakers is not exceeded. If it becomes necessary to use compressed air breakers, the saving may be canceled.

The power-factor rating for hydro generators is quite often different from steam units, due to the difference in location of plants. Steam units are usually located near the load center, while hydros are often located at some remote point and connected to the load by means of a transmission line. In order to avoid loading transmission facilities with reactive power, part of the steam units may supply reactive kva and act as spinning reserve, while the hydro supplies kilowatts. This would mean that the hydro generator could operate near unit power factor and a saving could be effected by purchasing the generator accordingly. In some cases, 95 per cent power factor machines have been used.

The application of amortisseur windings is a consideration for each project and must be decided on the merits of the particular application. The addition of this winding reduces the ratio of quadrature to direct-axis subtransient reactance from approximately 2 to 1.35, or still lower to about 1.25 if the windings are made continuous by connecting between poles. The increased cost can usually be justified for the nonconnected winding, but it is difficult to justify the extra cost of having them connected.

At one time, all hydro generators were of the open construction and self-cooled with air forced through the machine by the rotor. This system has been improved until today it is considered standard practice for large hydro generator stators to be completely enclosed and cooled by means of recirculating air passing over surface-type coolers placed symmetrically around the inner circumference of the enclosure. Quite often water for these coolers is obtained directly from the headwater of the dam, eliminating the need for pumps. The enclosed machine is naturally more expensive, but this is justified on the basis of keeping the machine clean and the possibility of better control of the machine temperature, which is conducive to longer life. It is important that space heaters be supplied to provide heat while the machine is shut down, thus minimizing the expansion and contraction and the possibility of condensation.

The conventional type of generator has two guide bearings, one below the rotor and a combined guide and thrust bearing above the rotor. The "umbrella type," with only a combination guide and thrust bearing below the rotor, is rapidly gaining in popularity. The rotor is bolted to the top of the shaft and can be removed without disturbing the bearings. This reduces the maximum weight of the largest piece and usually results in reduction of the crane capacity. The elimination of one bearing and its bracket also reduces the cost of the unit. In many cases the umbrella type will allow reduction in height of the powerhouse, due to lower height above floor line. On units of small diameter, the conventional type is recommended because of insufficient room under the rotor for access to the thrust bearing.

There are only three types of thrust bearings built in this country today; the Kingsbury, the spring type and the spherical type.

As shown in Fig. 47*A*, the lower or stationary part of the Kingsbury bearing consists of several independent segments, each supported slightly off center by an adjusting screw. During rotation, the slight tilt of each segment facilitates the drawing of oil between the bearing surfaces. In the adjustable hydroelectric thrust bearings, Fig. 47*A*, the shoes rest on the hardened end of jack screws which are individually adjusted to divide the load equally among the shoes. In equalizing Kingsbury bear-

FIG 47A —Kingsbury thrust bearing

FIG 47B — Kingsbury thrust bearing.

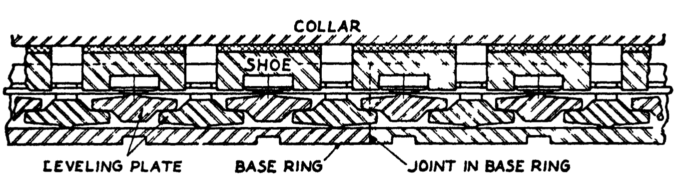


FIG 47C —Developed section of Kingsbury bearing, showing how the leveling plates of six-shoe thrust bearings distribute the load equally among the shoes

ings, the shoes rest on a series of interlocking leveling plates which by slight tilting equalize the load between the shoes and assure uniform thickness of oil films. Figure 47*B* shows such a hydroelectric bearing with shaft and runner omitted. The separate diagram marked Fig. 47*C* shows the principle of leveling plate equalization.

FIG. 48.—Spring-type thrust bearing.

The lower part of the spring-type bearing, shown in Fig. 48, is composed of more than one piece, but they are all doweled together to act as a one-piece plate supported on a large number of precompressed helical springs.

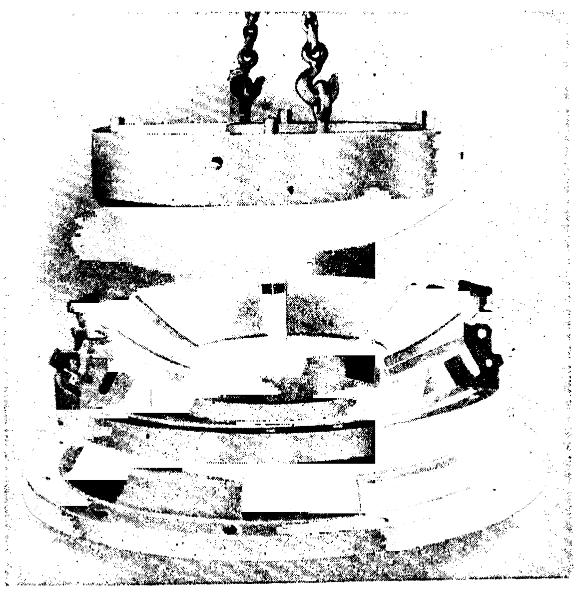


FIG. 49.—Spherical thrust bearing.

Kingsbury spherical thrust bearings (Fig. 49) are coming into wider use during recent years. This bearing combines in itself the functions of thrust and adjacent steady bearing. It eliminates entirely the side play which is more or less unavoidable with large cylindrical steady bearings. The shoes are not equalized, but are accurately mounted in the one-piece base ring.

All of these types of bearings run submerged in oil baths, which are cooled as required either by circulating water through a copper coil surrounding the bearing or, occasionally, by circulating oil through an outside cooler.

If trouble is experienced with a bearing, it usually occurs when the machine is started for the first time and then during the first few revolutions before the proper oil film is formed between the two bearing surfaces. Some trouble has also been experienced where a machine was rotated very slowly for an extended time. This low speed is not conducive to a good oil film.

Excitation. The latest practice now places both the main and the pilot exciters above and direct-connected to the generator by means of a smaller shaft fastened to the main shaft. Quick-response or high-speed excitation is recommended for better voltage regulation and stability. We can look for some improvement in this field in the form of devices for anticipating voltage changes and for producing higher internal generator voltages during fault conditions.

Governors. Reference should be made to Sec. 15 for a complete treatment of the subject of speed regulation.

Hydro units naturally have a larger WR^2 than steam units of comparable capacity and the water passing through the turbine has a high inertia, all of which makes a very different governing problem. When a Kaplan or adjustable-blade turbine is used, oil pressure, controlled by the governor through an oil head located on top of the pilot exciter and through a hollow shaft to the servo mechanism, operates the blades. Although governors have been driven mechanically by the main shaft, this method is considered obsolete, and governors driven electrically by means of a potential transformer or a permanent-magnet generator located on top of the generator shaft are now almost universally used. Since the permanent-magnet generator is attached directly to the device it is controlling and is not affected by voltage dips or surges due to external disturbances, this method provides more satisfactory operation.

Modern hydro generators are equipped with governor-operated air brakes. The ring header that supplies the air for brakes underneath the rotor also serves as an oil-pressure line for jacking up the rotor by means of a portable hand hydraulic pump. The brakes then serve the dual purpose of brake and jack.

Control. The tendency in small and medium-sized hydro plants is toward automatic and supervisory control. Modern governors can easily be equipped for automatic starting by means of a switch, operated either locally or by remote control. Automatic shutdown is normally supplied as standard equipment. For many machines, particularly those important to the system, it is considered good practice to equip the governor with a "speed-no-load" solenoid so that the many troubles that otherwise would cause the machine to be shutdown energize the speed-no-load solenoid instead. Thus the machine is brought quickly down to normal speed with the field still on and ready to be resynchronized and put back on the line, if the trouble does not warrant a complete shutdown. Of course, troubles such as differential faults energize the shutdown solenoid and the machine is completely shutdown, including tripping of the field breaker.

Control or limitation of runaway speed is considered essential for some installations. Machines are normally designed for speeds 100 per cent above normal. However, with an adjustable-blade Kaplan turbine, the runaway speed may be far in excess of this. For the Rio Negro[1] project in Uruguay, the normal speed is 125 rpm and, with maximum head and with the Kaplan blades in nearly flat position, the runaway speed would be 365 rpm. This is too far in excess of normal speed for economical design, so a stop nut controller was used. This device consists of a recorder controller which measures the headwater and tail-water levels and thereby obtains the

[1] Harza Engineering Co.

net head. Depending on the net head, the controller then adjusts a motor-operated stop nut which in turn limits the position of the wicket gates so that the maximum speed cannot exceed 325 rpm.

When a hydroelectric plant forms an important link in a large interconnected system, more elaborate control may be necessary. On projects of this kind, there have been provided both load control and telemetering over carrier current to a central load dispatcher, so that he virtually controlled the whole system. The telemetering is rather simple, consisting of generator potential transformers and current transformers feeding a base-rate sender which is used to key a carrier-current telemeter transmitter. The load controlling by the dispatcher is accomplished by using a carrier current set capable of sending three frequencies to the hydro plant, which are received by a "load control receiver." One frequency actuates the "raise" contact, the second actuates the "lower" contact, and the third frequency is for monitor purposes to prevent operation of the "raise" or "lower" due to any disturbance. The "load control receiver" operates the "master receiving contactor," a reversing contactor which in turn operates the "governor motor contactor," also a reversing contactor which controls a reversible motor on the governor of the generator for raising or lowering the kilowatt output.

Switchgear. For switchgear of the generator voltage class and below, the trend is definitely toward air breakers enclosed with the rest of the switchgear in streamlined metal housings. For stations where one or two generators feed a transformer bank, one of the latest developments is unit-type switchgear attached to its own generator by a throat. By placing all instruments and control on this switchgear, the wiring and connections can all be completed at the factory and the assembly in the field is quite simple, since all connections are made through the throat, eliminating the need for conduit between generator and switchgear. Where three or more generators are used and centralized control by an operator is desired, this scheme offers little, if any, advantage.

There is also a tendency to use metal-clad switchgear outdoor in the 15-kv class and below. It requires less space and is usually more economical than building steel structures and using outdoor breakers and other switchgear.

A number of 34.5-kv indoor compressed-air breakers have been applied, but their use is still limited. They are particularly desirable in countries where oil must be imported; also in locations where it is necessary or desirable to build an indoor station using this voltage.

The use of a ground-fault bus in metal-clad switchgear is becoming very popular and affords a very effective bus-protection scheme. Switchgear that is normally furnished with isolated-phase construction can easily be insulated from ground and then connected through a current transformer to the station ground. This construction makes it impossible to have a fault in the switchgear that does not involve a ground which will operate a ground relay in the secondary of the current transformer. This of course assumes that this part of the system is grounded.

Along with air breakers has come the application of air-cooled transformers for auxiliary service, making possible a complete metal-enclosed structure without the hazard of oil fires.

Transformers. The economical use of materials forced on us by the war led to changes in both the loading and the design of power transformers. There is a trend now to load not only the transformers but other equipment, including generators, according to temperature rather than by name-plate rating. A so-called "hot-spot" temperature relay, placed in the top oil of the transformer and also heated by load current from a current transformer, is designed to match the heating characteristics of the power transformer. This relay is equipped with two contacts: the first designed

to give a warning, and the second to disconnect the transformer before it is damaged. Forced-oil, forced-air, or water-cooled transformers, built with approximately 60 per cent of the material required for self-cooled transformers, were used considerably during the war and will continue to have many applications, particularly where transformers are not normally fully loaded, yet peak capacity is required.

These transformers are built to meet the standard top-oil temperature rise, based on the assumption that the hot spot is actually about 10 deg above this. This standard was based on a transformer without forced-oil circulation, so if the transformer is loaded according to temperature the forced-oil, forced-air (or water) transformer will safely carry more than its name-plate rating due to elimination, or at least material reduction, of the difference between hot-spot and top-oil temperature. The weight is also considerably less, which may be worth considering if the transformers are mounted on the draft-tube deck. Naturally, the losses at full load are higher than for the self-cooled type, and this must be balanced against the lower first cost. At least one of the oil pumps must be running to carry any load; in fact, it may not even carry excitation losses if both pumps are shut down. There is also the consideration of more maintenance with pumps and fans.

REQUIREMENTS FOR MIGRATORY FISH

It is becoming increasingly important to provide facilities for passing migratory fish. They come in from the sea and swim to spawning grounds in the upper reaches of our rivers. It is important that the fish shall not be stopped from their migration by barriers which may either prevent them from getting upstream or undoubtedly delay them in reaching the spawning grounds.

Migratory salmon will, as a last resort, leap over many obstacles provided they have a sufficient length of clear run in water of adequate depth. It is known that salmon have leaped as high as 11 ft in getting over obstacles. Generally, however, 6 ft is taken as the maximum which the fish can jump. As a result of recent legislation and the requirements of conservation agencies, the investment in fish-passing facilities is now becoming a major item in the construction of many hydroelectric projects. As a matter of fact, the investment in such facilities has equaled the cost of the turbines and generators in several large projects.

Basically, there are three essentials for fish-passing facilities. The fish should be able to (1) swim through these facilities without exerting any unusual effort, (2) pass

TABLE 7

	Bonneville	McNary
Ladders:		
Number	3	2
Dimensions	One 37 ft wide, two 40 ft wide; all approx 1,310 ft long	30 ft wide; approx 2,100 ft long
Gradient	1 vertical on 16 horizontal	1 vertical on 20 horizontal
Velocity	Approx 1 fps	Approx 2 fps
Discharge	160 cfs (avg)	180 cfs (avg)
Resting pools	40 ft wide, 16 ft long	30 ft wide, 20 ft long
Depth of pools	6 ft	6 ft
Overflow depths	1–1.5 ft	1–1.5 ft
Locks:		
Number	6	1
Dimensions	20 ft wide, 30 ft long	20 ft wide, 30 ft long
Height of lift	50 ft to normal pool (avg)	86 ft to normal pool (avg)

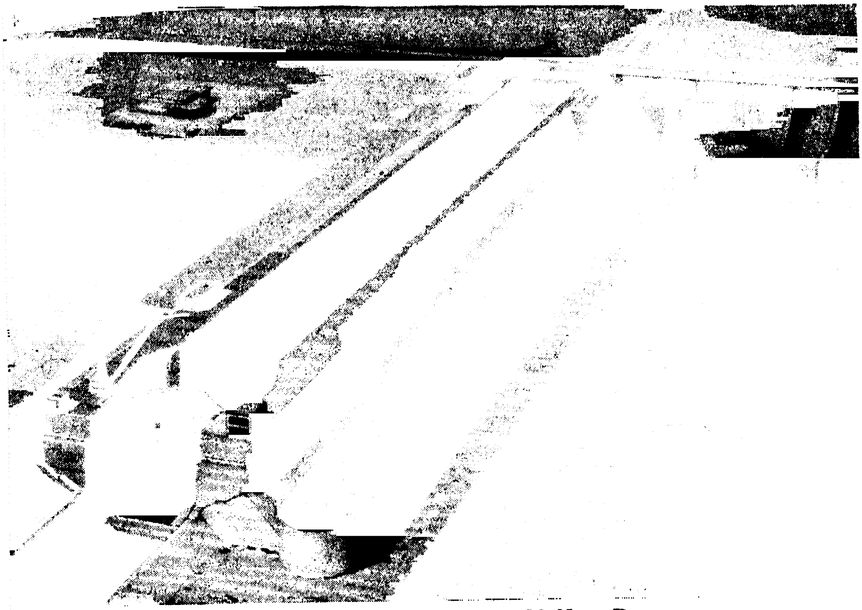


Fig. 50.—Fish-passing facilities, McNary Dam.

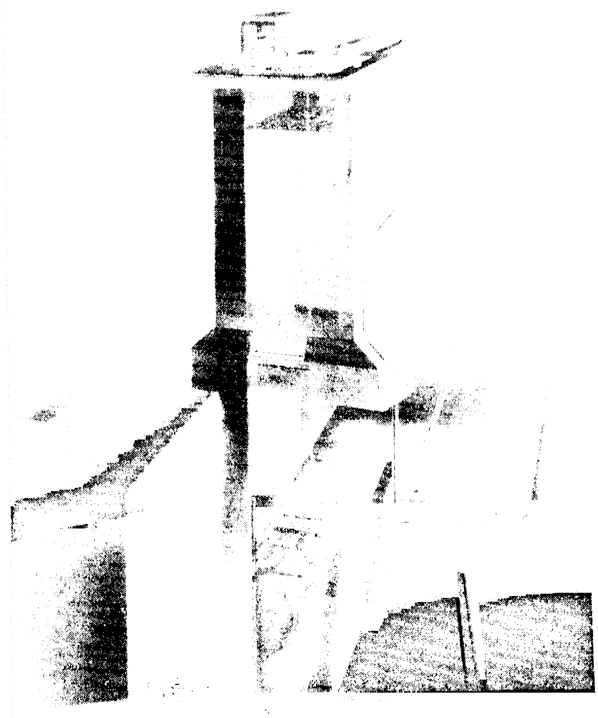


Fig. 51.—Fish lock, McNary Dam.

through these facilities without any unusual risk of injury, (3) easily and quickly find the entrances to the ladders or locks or whatever means may be provided for getting them over the barrier.

Typical of modern practice are the fish ladders and fish locks provided at the Bonneville and McNary hydroelectric projects, both on the Columbia River. A summary is given in Table 7.

During periods of low flows, the submerged rock ledge on the Washington side of McNary Dam would block Washington-shore facilities to upstream migrants. Therefore, one longitudinal and three lateral approach channels are cut through this ledge to provide effective passage of the migrants from a natural deep hole in the center of the channel below the spillway dam. Three entrance bays will be provided in the Washington-shore fish ladder, as shown by Fig. 50. Each of these bays has a telescoping leaf gate or weir which provides attractive entrance velocities under varying stages. The dimensions of the McNary ladder are shown by Table 7.

The McNary fish lock (Fig. 51) will have the same principal features, as to size and plan of operation, as those now installed in the Bonneville Dam, as shown by Table 7. This lock has four automatically adjustable weirs in the ladder below the lock entrance. An entrance pool downstream from the lock entrance opens into the lower end of the fish ladder, and a flume discharges the fish into the higher end of the fish ladder between the regulating weir and the counting station. The normal operating period for a complete cycle of the locking operation is about 15 min. The height through which the fish will be raised is 92 ft as compared with 60 ft at Bonneville Dam. The lock and fish ladder will not operate simultaneously.

In the case of anadromous fish it is just as important to pass fingerlings and other migrants downstream as it is to pass the adult fish upstream. Some experiments have indicated the mortality of downstream migrants that go through the water wheels. Such experiments are very difficult to conduct and the results vary over a wide range. They do, however, indicate an average mortality of about 15 per cent. The state and Federal agencies having these problems in hand are urging that facilities be provided to pass these migrants around the hydroelectric plant instead of permitting them to go through the turbines.

The method now in vogue is to screen the inlet passages to penstocks and other intakes with traveling screens having a mesh with net square openings of $\frac{1}{8}$ to $\frac{1}{4}$ in. These screens are provided with water jets under 40 to 60 psi pressure from the downstream side that automatically keep the screens clean. In the piers and walls just upstream from the screens are a series of horizontal openings 6 to 10 in. in diameter that join a vertical header. The headers join pipes or openings in the concrete that by-pass the entire plant by discharging into a bay with adjustable overflow gates or other means by which the fish may reach the tail water of the plant without an excessive drop.

Neither at the Bonneville nor McNary plants is the water entering the penstocks screened for fish. At Bonneville, however, there are two openings that supply water to the fish ladders, each of which is closed with five 10-ft traveling screens, automatically flushed, that screen water to a depth of 40 ft. Somewhat similar screens are being installed at McNary for similar purposes. There are, however, a number of smaller plants in the Pacific Northwest in which the intakes to the turbines have fine-mesh automatic traveling screens to prevent fish from going through the turbines.

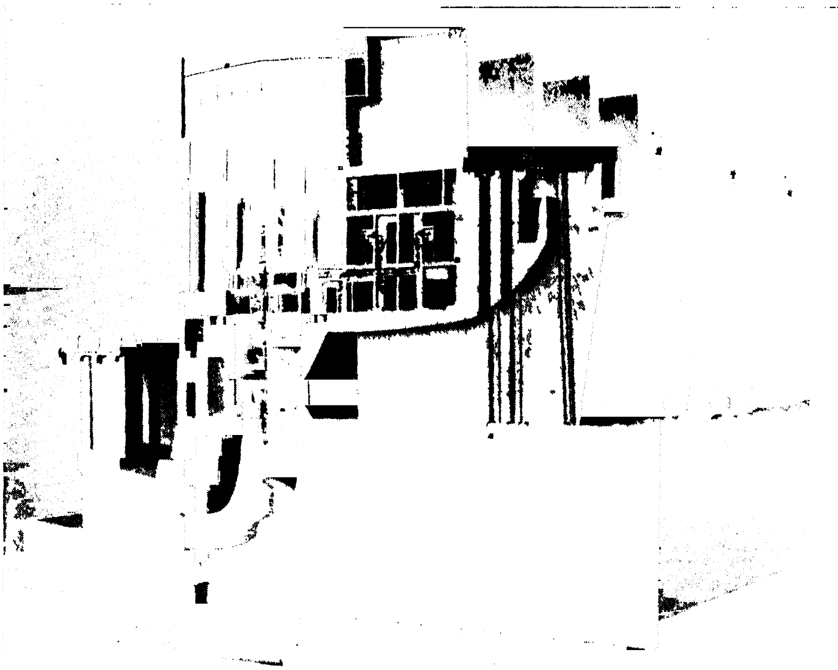


FIG 1—A modern hydroelectric powerhouse Scale model of Safe Harbor Plant of the Safe Harbor Water Power Corp containing six Kaplan-type turbines each of 42 000 hp under 55-ft head, built by S. Morgan Smith Co and Baldwin-Southwark Corp, I P. Morris Division.

SECTION 12

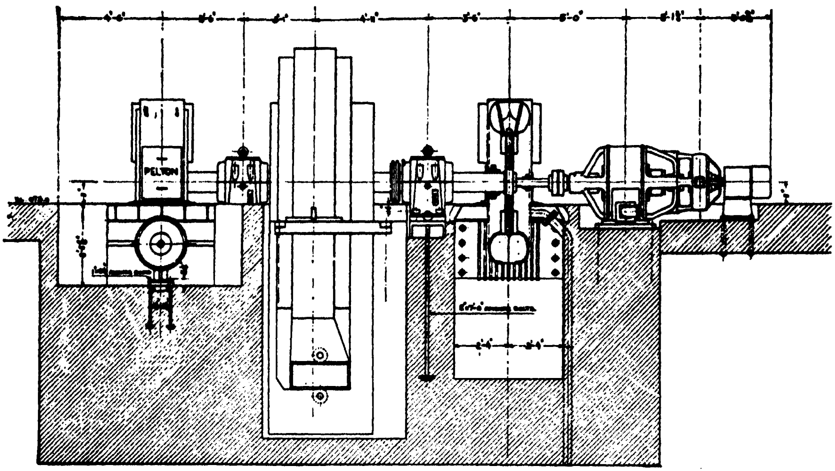
HYDRAULIC MACHINERY

BY LEWIS F. MOODY

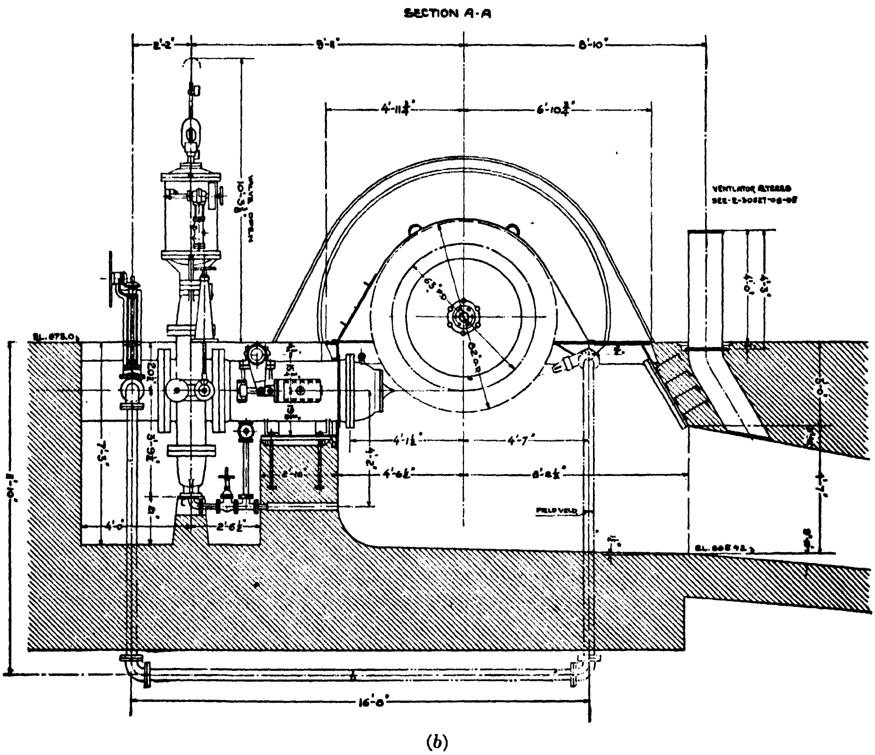
HYDRAULIC TURBINES

General Classification. Hydraulic power generating machines, or hydraulic prime movers, comprise most broadly the following gravity water wheels, such as overshot, breast, and undershot wheels utilizing the elevation head of water, which are now practically obsolete; positive-displacement motors, utilizing direct pressure without material effects of changes in the fluid velocity, such as a reciprocating engine or rotary displacement machine (for example, a reversed gear pump driven by water pressure), a type having little application; and turbines Turbines are the only prime movers of importance in modern hydraulic power development and are the only class that will be considered here Turbines involve in their action a continuous transformation of the potential energy of the fluid into kinetic energy (and in reaction turbines a subsequent reconversion) and a conversion of kinetic energy, or of both kinetic and potential energy, into useful work, as will be explained

Hydraulic turbines are classified, with respect to their hydraulic action, in two main divisions *impulse turbines* and *reaction turbines* These terms should be regarded as names, having no hydraulic significance in differentiating between their actions but being firmly intrenched in general usage



(a)



(b)

FIG. 2(*a and b*).—Pelton wheel. Station service unit for Hoover Dam power plant. "Double overhung" arrangement. 3,500 hp, 490 ft head, 300 rpm. Pelton Water Wheel Co. (*a*) Side elevation. (*b*) End elevation.

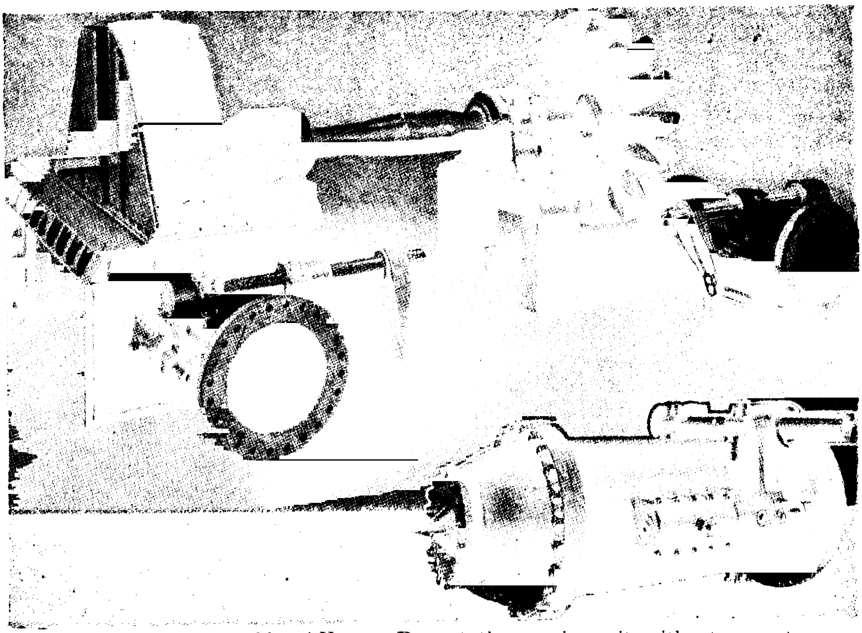


FIG. 2*c*.—Shop assembly of Hoover Dam station service unit, without generator.

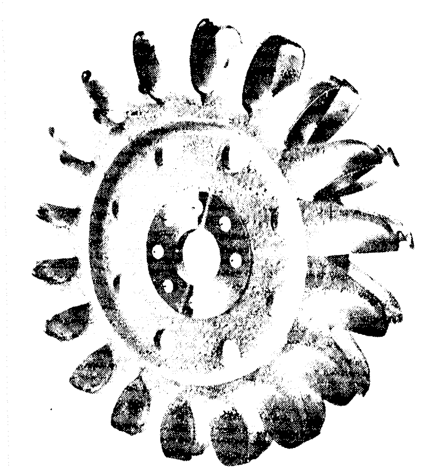


FIG. 2*d*.—Runner for Hoover Dam station service unit. Integrally cast buckets.

FIG. 3.—Pelton wheel runner for 2,900-hp single-overhung unit for Andes Copper Co. 2,150 ft head, 900 rpm. Detachable buckets. (*Baldwin-Southwark Corp., I. P. Morris Division, Associates of Pelton Water Wheel Co.*)

Impulse turbines are represented in modern practice by a single type, the Pelton water wheel, illustrated in Figs. 2 and 3. In an impulse turbine, all the available head is converted into kinetic energy or velocity head in a contracting nozzle (Fig. 4) by which the water is formed into a free jet before acting upon the runner or revolving element, and in its subsequent flow and throughout its action in the runner the water flows with free surfaces in buckets which it only partly fills, its flow thus being similar

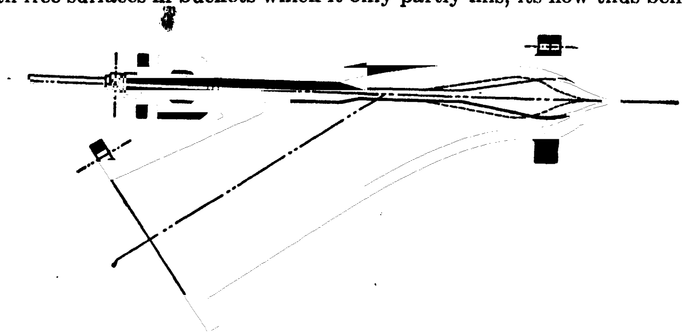


FIG. 4a.—Doble-type nozzle for Pelton water wheel. (*Pelton Water Wheel Co.*)

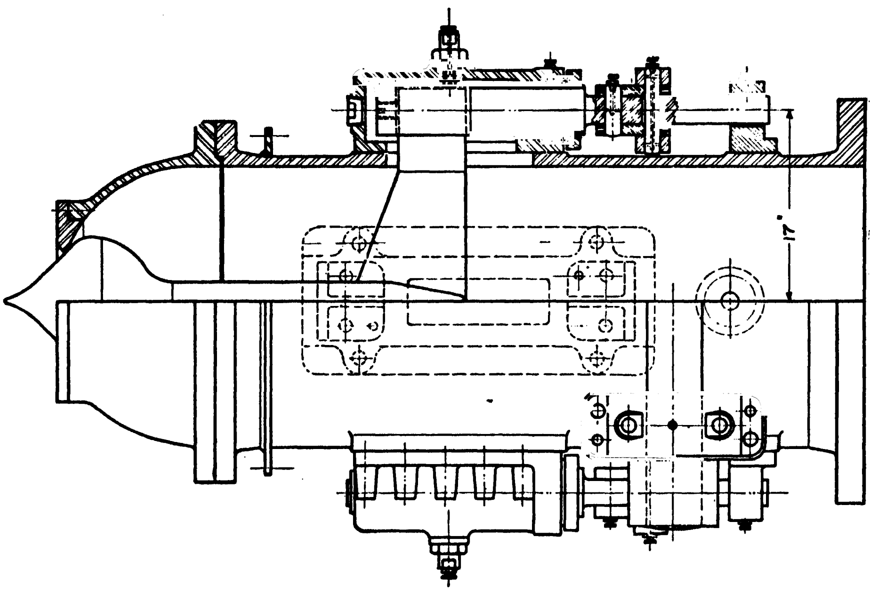


FIG. 4b.—Straight-flow nozzle for Pelton water wheel. (*Pelton Water Wheel Co.*)

to that in an open channel. During this action, the water is in contact with the air, and although the Pelton wheel is provided with a housing to prevent splashing and to guide the discharge, this housing contains air at substantially atmospheric pressure, through which the water discharged from the buckets falls freely through the discharge passage into the tail water.

In reaction turbines, the entire flow from headwater to tail water takes place in a closed conduit system, being laterally constrained on all sides and not open to the air at any point. At the entrance to the runner, only a part of the available head is

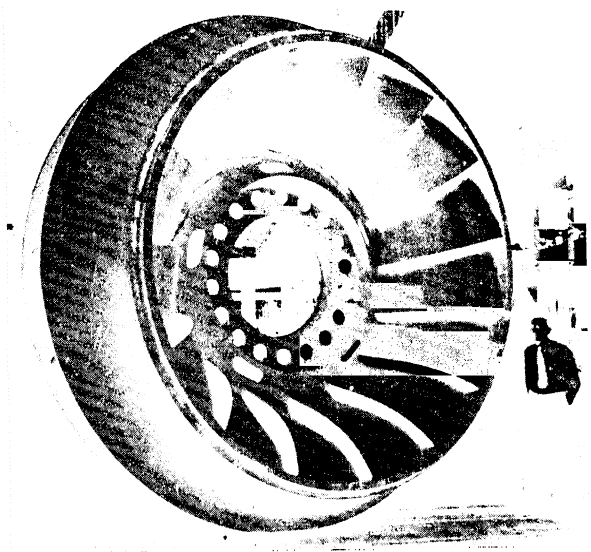


FIG. 5.—Francis turbine Runner. 115,000-hp turbine for Hoover Dam power plant. (*Baldwin-Southward Corp., I. P. Morris Division.*)

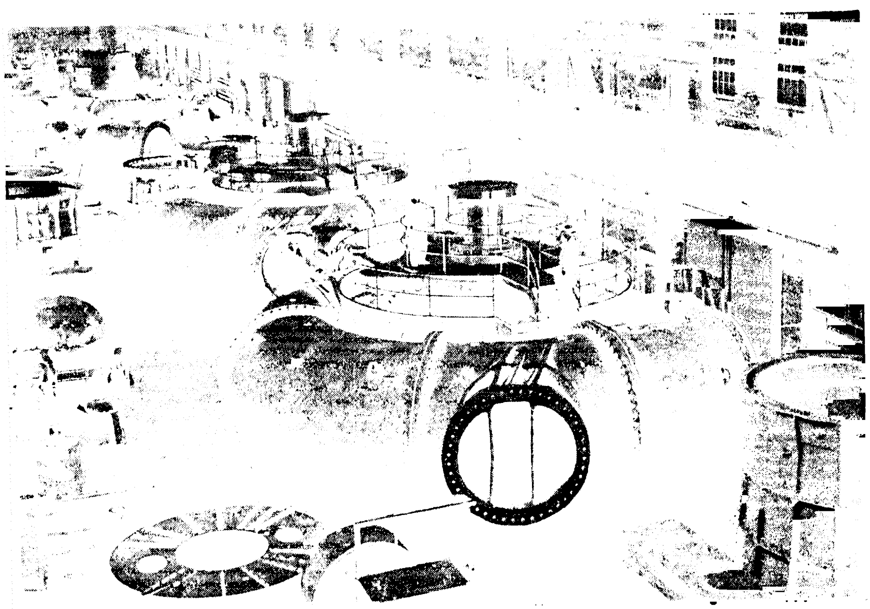


FIG. 6.—Two 115,000-hp turbines for Hoover Dam plant, U.S. Reclamation Service, erected in shop. (*Allis-Chalmers Mfg. Co.*)

converted into kinetic energy, a substantial part remaining in pressure head which varies throughout the passage through the turbine. The flow thus resembles that in a closed pipe system. In all modern forms of both impulse and reaction turbines, the water in entering the runner exerts upon it a force in the direction of flow. termed

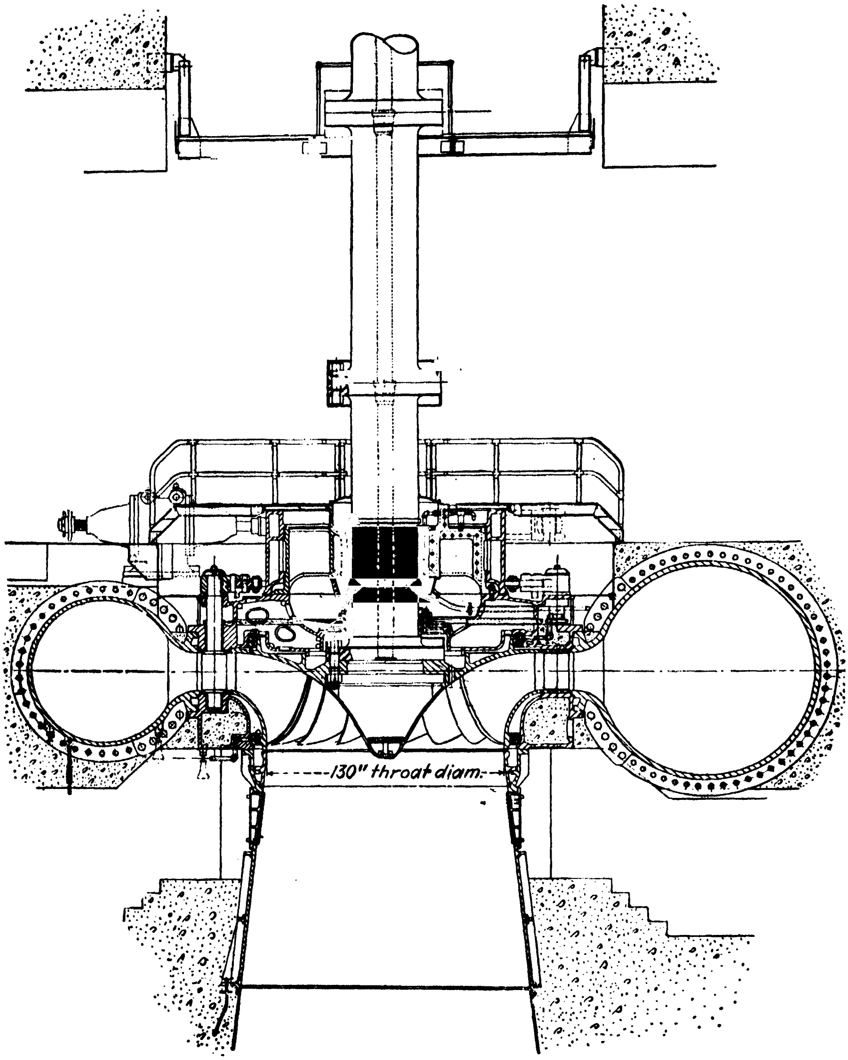


FIG. 7.—Sectional elevation of 115,000-hp turbines for Hoover Dam power plant. 510 ft head, 180 rpm. (*I. P. Morris Division, Baldwin-Southwark Corp.*)

in hydraulics an **impulse**; and at discharge from the runner, it exerts a reaction opposite to its direction of flow.

Reaction turbines are represented in modern practice exclusively by two types, the Francis turbine, illustrated in Figs. 5 to 11, and the propeller turbine, illustrated in Figs. 12 to 18. Reaction turbines may be designed to operate properly with inward, outward,

or axial flow through the runner (these directions referring to the velocity components in a meridian plane, a plane containing the runner axis), and the general theory to be outlined later can be applied to any of these cases. Historically, the Fourneyron turbine had outward radial flow through the runner, the Jonval turbine employed axial flow through an annular vane passage, but these have given way to the Francis turbine. In this turbine, the flow passes inwardly through a circular series of guide vanes, or wicket gates (Fig. 10), pivotally adjustable for regulation and having contracting passages between them in which a part of the head is converted into velocity head. The streams issue from the guide vanes in a diagonal direction having both

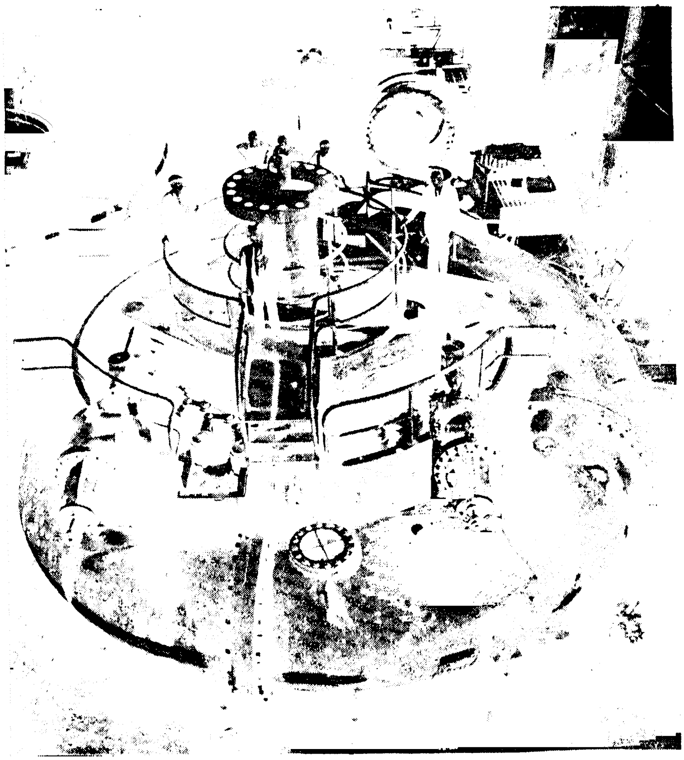


FIG. 8.—115,000-hp turbine for Hoover Dam power plant, erected in shop. Cast steel casing. (*I. P. Morris Division, Baldwin-Southwark Corp.*)

radial inward and tangential velocity components. The streams then merge within a transition space represented by the clearance between the guide vanes and runner vanes and form a continuous ring of revolving and inwardly progressing water. The water then enters the runner passages in which the radial component of motion is gradually turned between inner and outer walls of the runner either partially or completely into the axial direction, while the tangential or whirl components are gradually deflected by the vanes until at discharge from the runner a small whirl component remains. The flow then passes through a draft tube which by means of progressively increasing cross-sectional areas gradually reduces the velocity, a large part of the residual kinetic energy remaining in the runner discharge being reconverted into effective head, which is utilized through a reduction in the static pressure against which the runner discharges.

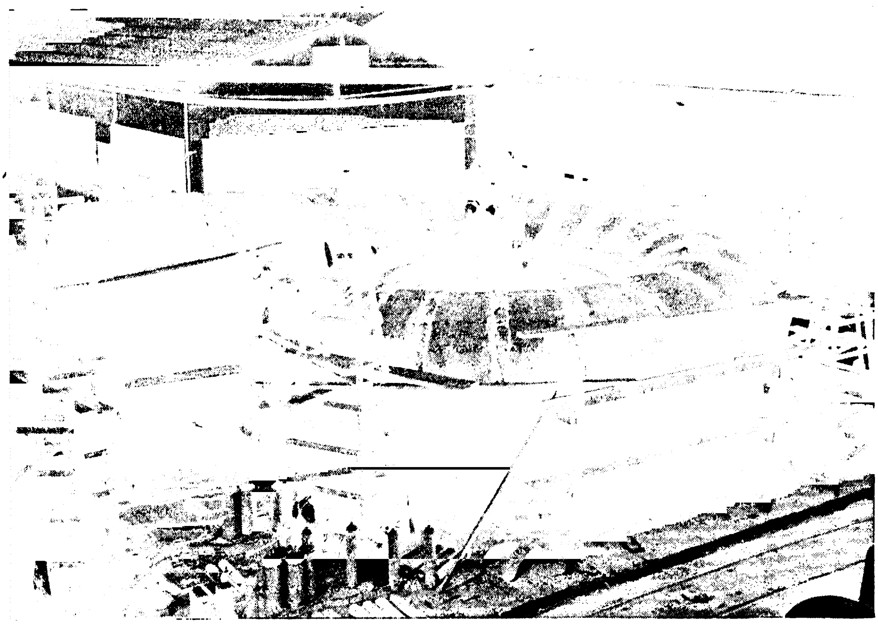


FIG. 9.—Shop erection of plate steel spiral casing for a Francis turbine. 80,000-hp turbine for Hiwassee Development, Tennessee Valley Authority. Casing inlet diameter 18 ft. (*Newport News Shipbuilding and Dry Dock Co.*)

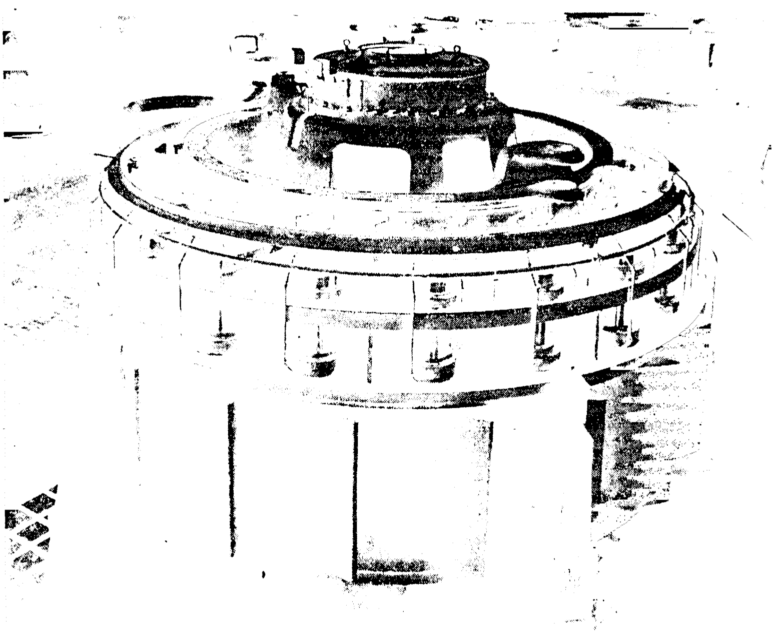


FIG. 10.—Shop assembly of the wicket gates and operating mechanism for a Francis turbine. 18,000-hp turbine for Loup River Project, Columbus Development. 112 ft head, 150 rpm. (*I. P. Morris Division, Baldwin-Southwark Corp.*)

In the propeller turbine, the water is confined within a closed conduit system and has no exposed free surface and is therefore classed as a reaction turbine. The flow through the runner is axial with respect to the meridian components, but unlike the axial-flow Jonval turbine, the hub is relatively small and the major portion of the area

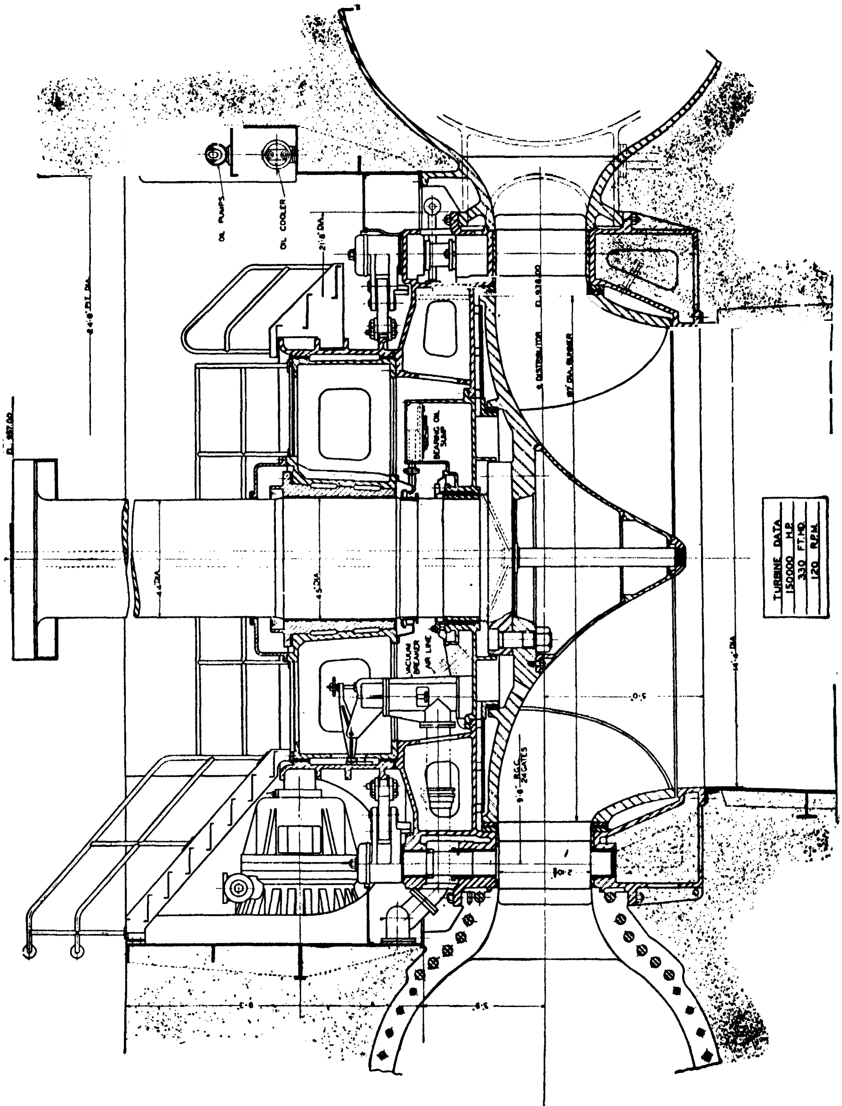


FIG. 11.—Sectional elevation of 150,000-hp turbines for Grand Coulee Plant, U.S. Reclamation Service. The most powerful hydraulic turbines in the world. 330 ft head, 120 rpm. *(Newport News Shipbuilding and Dry Dock Co.)*

within the throat section of the water passage is occupied by the flow; the runner vanes or blades, few in number, are nearly flat with little curvature and limited area, are pitched in a direction more nearly tangential than axial, and are unshrouded or have free outer ends like a marine propeller, revolving within the circular stationary

wall with as small running clearance as is practical. The relative velocity between the blades and water is high, and there is comparatively little change in this relative velocity as the water passes through the runner. In this last feature, the propeller turbine approaches the action in an impulse turbine in which the relative velocity remains practically constant in its passage through the runner buckets.

An important subdivision of the propeller turbine is the Kaplan turbine in which the individual runner blades are pivotally mounted in the hub so that their inclination may be adjusted during operation, by the governor, simultaneously with the adjust-

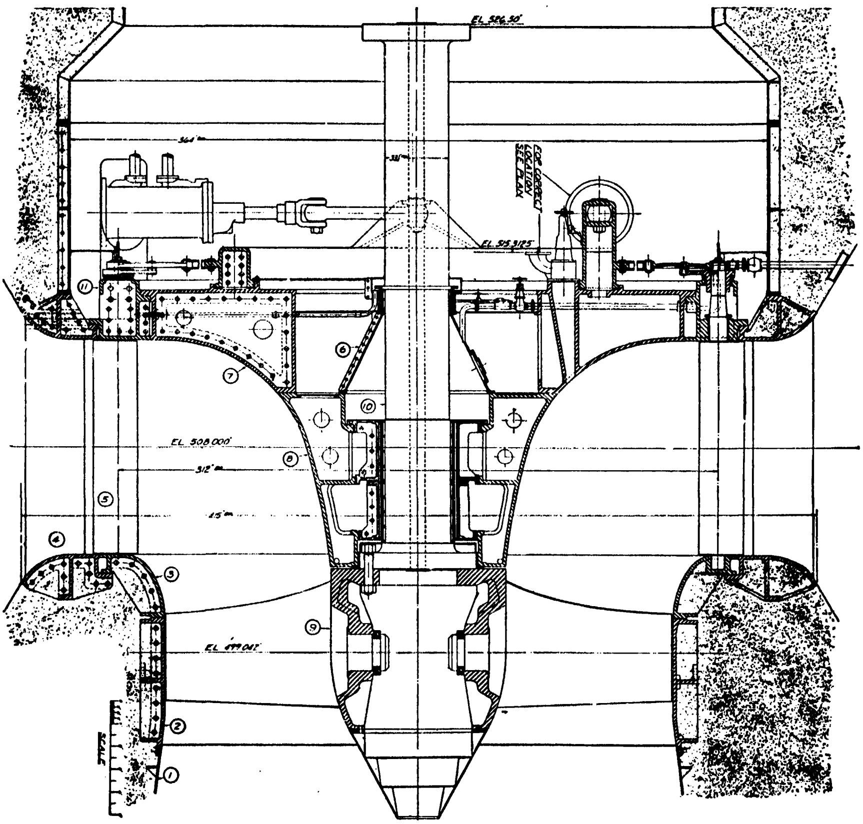


FIG. 12.—45,000-hp propeller turbine for Wheeler power plant, Tennessee Valley Authority. Fixed-blade runner type, 48 ft head, 85.7 rpm. (*I. P. Morris Division, Baldwin-Southwark Corp.*)

ment of the guide vanes, to meet changing power demands and changes in head. This adjustment is accomplished by means of levers, links, and a crosshead within the enlarged runner hub as shown in Fig. 15, the mechanism being operated by an adjusting rod through the turbine shaft and actuated by a hydraulic piston within a cylinder formed by an enlargement of the shaft. The piston is moved by oil pressure admitted through a concentric pipe within the shaft bore passing down from the upper end of the shaft and controlled by a valve which adjusts the runner blades to correspond to the positions of the turbine guide vanes or wicket gates. A special type of Kaplan turbine has been introduced by Terry (Fig. 17), in which the runner-blade

adjustment is accomplished by a mechanism within the hub responsive to the pressure on the blades and the relative direction of flow and by means of which the runner blades automatically adjust themselves to suit the wicket-gate position.

Reaction turbines have been applied up to heads over 1,000 ft.[1] Beyond this, the impulse turbine of the Pelton type is employed. Pelton wheels have been used up to heads of more than 1 mile.[2] Propeller turbines have been employed for heads up to slightly over 100 ft.[3]

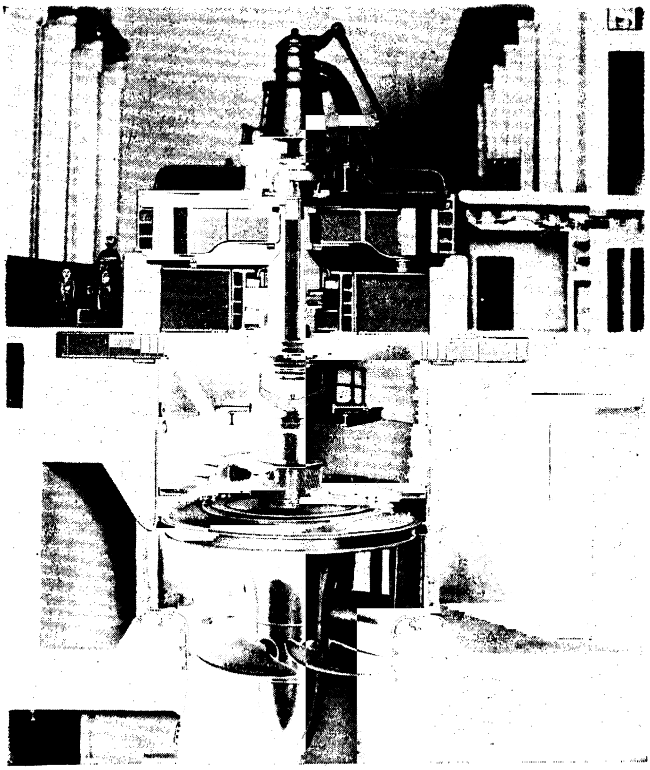


FIG. 13.—Model of Safe Harbor Plant, Safe Harbor Water Power Corp., with part of concrete casing removed to show internal construction of turbine. One of the 42,000-hp Kaplan-type turbines, three built by I. P. Morris Division, Baldwin-Southwark Corp. and three by S. Morgan Smith Co.

Except for small installations under low heads, reaction turbines are equipped with spiral or volute casings, these being of circular section and constructed of cast steel (Fig. 6) for the highest heads and of plate steel (Fig. 9) for moderate heads. In low-head plants (less than about 100 ft), the spiral casings are usually of partly rectangular section, molded in the concrete substructure of the powerhouse (Fig. 1). For small installations under the lowest heads (about 30 ft or less), open-flume settings

[1] Piottino plant, Switzerland. Two Francis turbines of 28,800 hp each under 1,060-ft head. Built by Ateliers des Charmilles, Geneva.

[2] Chandoline plant, Switzerland. Three Pelton turbines of 50,000 hp each under 5,740-ft head. Built by Ateliers des Charmilles.

[3] Marne plant, Italy. Kaplan turbine of 7,320 hp under 105-ft head. Also the Shannon plant, Ireland, under approximately the same head.

may be used, the runner and guide vanes being placed in an open rectangular or partly spiral flume into which the free surface of headwater extends. After the water enters

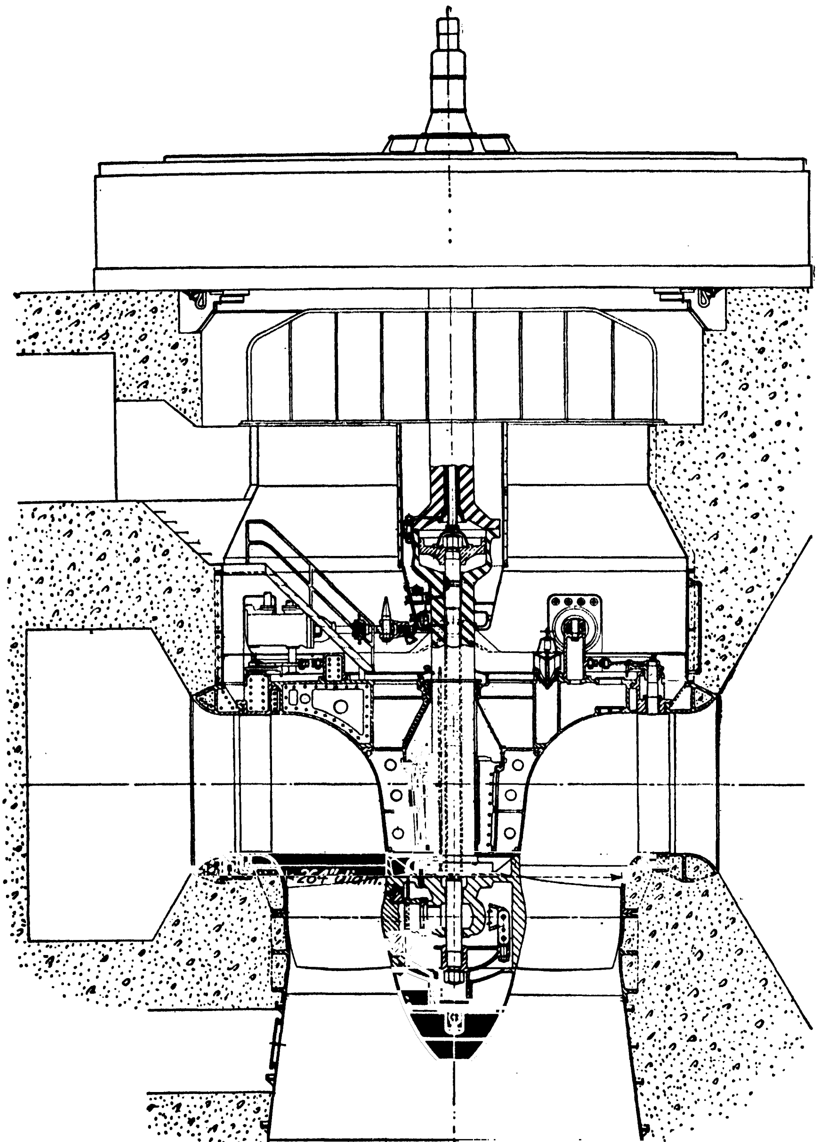


FIG. 14.—Sectional elevation of 36,000-hp Kaplan-type turbine for Chickamauga Development, Tennessee Valley Authority. 36 ft head, 75 rpm, runner throat diameter 22 ft. (*I. P. Morris Division, Baldwin-Southwark Corp.*)

the guide-vane channels, it no longer exposes a free surface at any point until it is discharged at the end of the draft tube into the tailrace.

The foregoing statements regarding the absence of free surfaces exposed to the air in reaction turbines should be qualified to this extent: many high-speed Francis and propeller turbines are equipped with automatically controlled valves admitting air to the innermost portion of the runner passages during part-gate or light-load operation. This permits a portion of the flow to approach open-channel or free-jet charac-

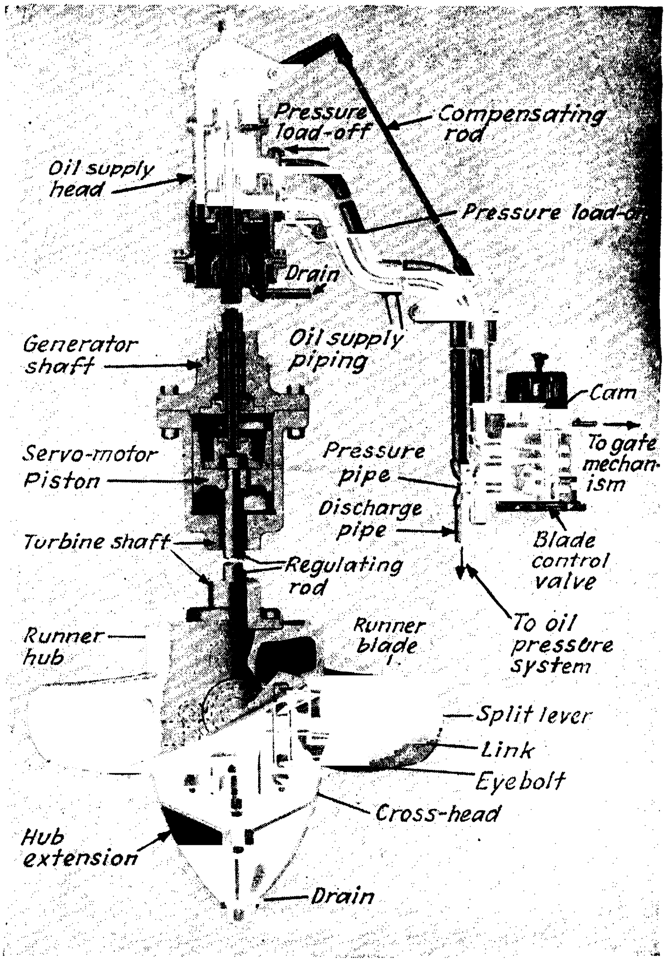


FIG. 15.—Diagrammatic sketch showing arrangement of the blade-operating mechanism of a Kaplan-type turbine. (*S. Morgan Smith Co.*)

teristics, but the pressure of the air within the passages is not atmospheric and the action takes place within completely enclosed spaces. This admission of limited quantities of air improves the efficiency of Francis turbines under small loads, and the air is shut off progressively and automatically as the guide vanes are opened. In fixed-blade propeller turbines, the air admission is frequently advantageous nearly up to normal gate opening.

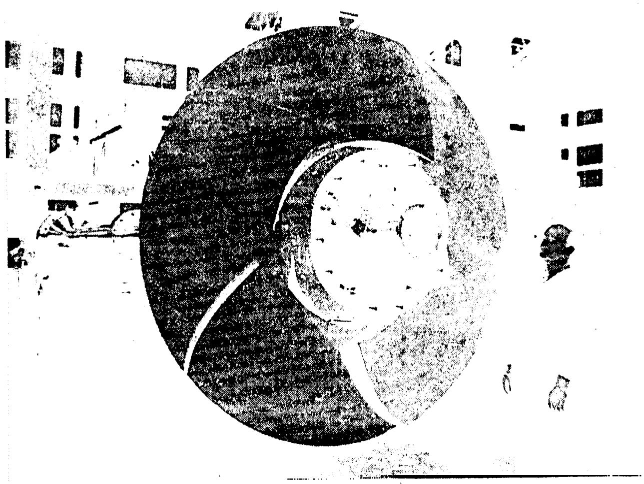


FIG. 16a.—Kaplan-type runner for Greenwood County, South Carolina, Buzzards Roost development, showing interior of hub. 7,400 hp, 60 ft head, 240 rpm. Runner blades in closed position. (*I. P. Morris Division, Baldwin-Southwark Corp.*)

(*b*)

(*c*)

FIG. 16(*b* and *c*).—Kaplan turbine for Buzzards Roost plant, showing runner, shaft, and blade-operating cylinder. *b*. Blades closed. *c*. Blades open.

Pelton wheels are regulated by needle nozzles, two forms of which are shown in Figs. 4a and b. Figure 4a shows the conventional form, originated by Abner Doble, and Fig. 4b shows one of the recent modifications, the straight-flow nozzle introduced by the Pelton Water Wheel Co. to avoid the disturbance of the jet by curvatures of the approach piping. Since for quick regulation the flow in the penstock or pipe line

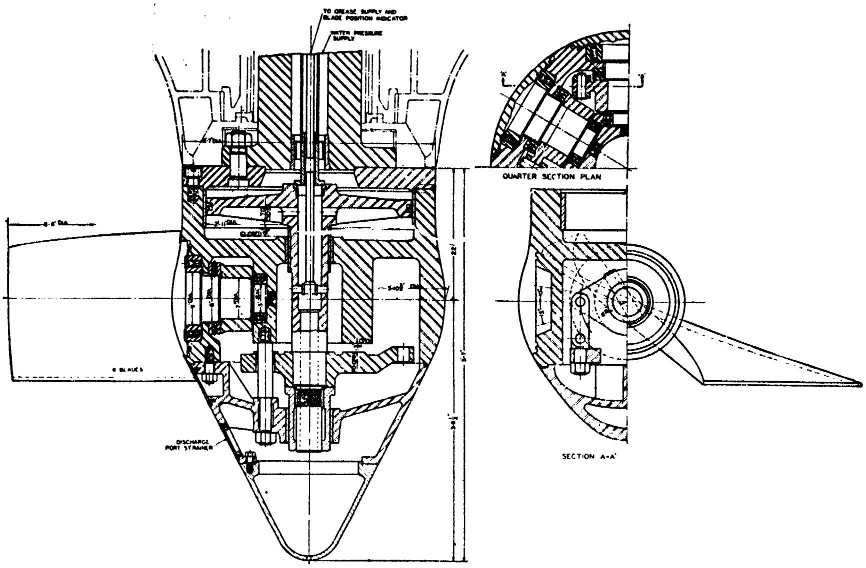


FIG. 17.—Runner section of Terry turbine for Austin Dam plant, showing hydraulic balancing cylinder and mechanism for automatic blade adjustment. (*Newport News Shipbuilding and Dry Dock Co.*)

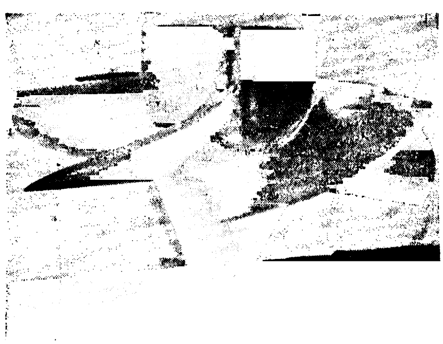


FIG. 18.—10,000-hp Terry-type runner for Austin Dam plant, Texas, of Lower Colorado River Authority. 61 ft head, 200 rpm.

cannot be suddenly altered because of its great inertia, the needle nozzle is supplemented by either a jet deflector or an auxiliary nozzle. The governor acts immediately to by-pass a part of the jet by means of either the deflector or auxiliary nozzle so that a part of the flow is removed from acting on the runner, after which a delayed action controlled by a dashpot permits the needle to be slowly closed and the deflector or auxiliary nozzle simultaneously withdrawn from action.

In reaction turbines having long penstocks, as in high-head installations, relief valves are applied (Fig. 19) temporarily to by-pass part of the flow from the turbine casing to the tailrace during quick closure of the wicket gates or guide vanes, in order to avoid too rapid deceleration of the penstock water column with consequent undue

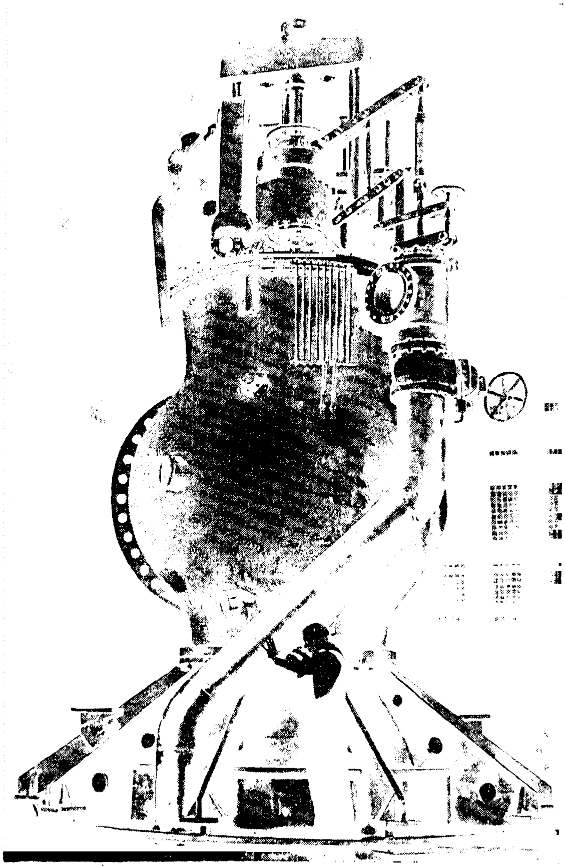


FIG. 19.—Relief valve or pressure regulator erected in shop, for 115,000-hp turbine at Hoover Dam. (*Allis-Chalmers Mfg. Co.*)

rise in pressure due to inertia effects or water hammer. Dashpot control permits the subsequent slow closure of the relief valve to minimize loss of water.

EFFECTIVE HEAD, AVAILABLE POWER, EFFICIENCY

Figure 20 illustrates diagrammatically a Francis turbine supplied by a penstock and discharging through its draft tube into the tailrace. The turbine proper is, according to standard practice,[1] taken to begin at the intake flange of the turbine casing and to end at a section of the tailrace just beyond the physical end of the draft tube. The turbine thus comprises the casing, guide vanes, runner, and draft tube.

[1] See Testing Code approved by Machinery Builders' Society, Oct. 11, 1917; Test Code for Hydraulic Prime Movers, American Society of Mechanical Engineers, approved, June, 1938.

The interdependence of the actions of the draft tube and the primary elements of the turbine requires that the draft tube be considered an integral portion of the turbine.

The effective head under which the turbine operates corresponds to "the difference between the total energy contained in the water immediately before its entrance into the turbine, and its total energy immediately after discharge from the draft tube."[1]

If we remember that an amount of water weighing W pounds falling through a vertical distance H is capable of delivering WH foot-pounds of work, an amount of head may be considered either as an elevation H above some selected datum of reference or as specific energy, *i.e.*, an amount of energy in foot-pounds per pound of water flowing, or WH/W. If a gage glass is connected to a piezometer orifice in the penstock adjacent to the inlet of the turbine casing, the potential energy in the water entering the turbine will be represented by the height to which the water rises in the gage glass. To this must be added the velocity head corresponding to the kinetic energy in the entering flow. This corrected elevation may be thought of as a virtual elevation equivalent to that of still water in a large forebay feeding the turbine

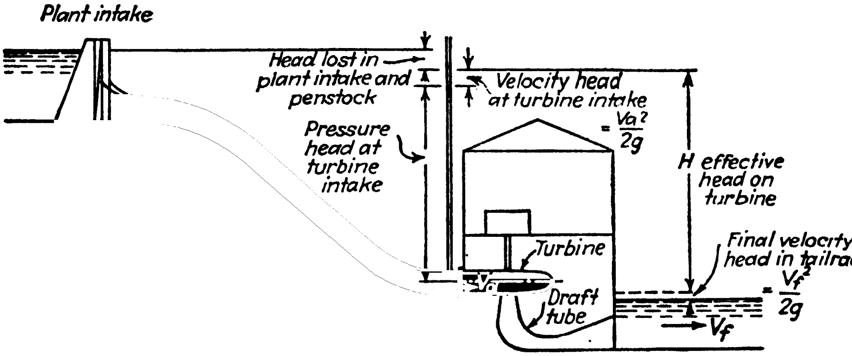


FIG. 20.

directly. In the same way, on the tailrace side the tail-water elevation must be corrected by adding the velocity head at the point of measurement to give an equivalent still-water elevation representing the total energy remaining in the water at discharge from the turbine. The difference between these two equivalent elevations represents the effective head H; or in the words of the A.S.M.E. Test Code:

"The effective head on the turbine shall be taken as the difference between the elevation corresponding to the pressure head in the penstock at the entrance to the turbine casing and the elevation of tailwater, the above difference being corrected by adding the velocity head in the penstock at the section of measurement and subtracting the residual velocity head at the section of measurement in the tailrace."

If the specific weight of the water is w pounds per cubic foot and the quantity passing through the turbine is Q cubic feet per second, the power available due to wQ pounds of water falling H feet is wQH foot-pounds per second, and the available horsepower, or water horsepower, is

$$\text{whp} = \frac{wQH}{550} \tag{1}$$

Of course not all this power can be delivered through the main shaft of the turbine to the electrical generator or other driven machine, for a portion is lost in hydraulic

[1] Test Code for Hydraulic Prime Movers, *A.S.M.E.*, June, 1938.

resistances and eddies and a small portion in leakage around the runner and in mechanical disk friction on the runner crown and bearing friction on the shaft. By calling the turbine efficiency e, the delivered or brake horsepower is

$$\text{hp} = \frac{wQHe}{550} \tag{2}$$

FUNDAMENTAL PRINCIPLES OF TURBINE ACTION

Relative and Absolute Velocities. Figure 21a illustrates a simple inward-flow reaction turbine. Here the water first passes through the guide vanes which have gradually contracting passages between them from which the water issues with a considerable velocity in an oblique direction having both radial and tangential com-

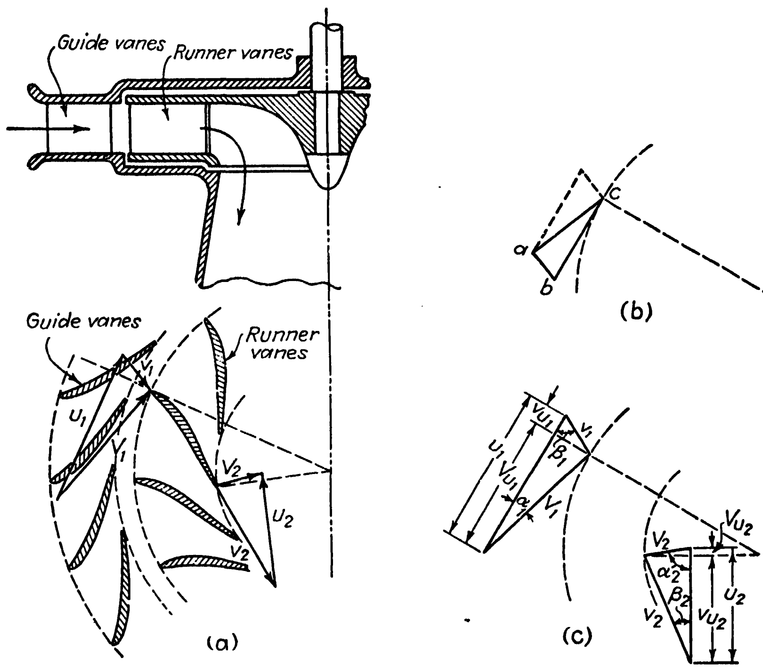


FIG. 21.

ponents with respect to the turbine axis. When the water reaches the entrance tips of the runner vanes, it has a velocity V_1. We must now consider the velocity both with respect to the stationary system and also with respect to the revolving runner; the first, denoted by V_1, represents the motion as it would appear to a stationary observer and is termed the *absolute velocity*. The second, denoted by v_1, represents the motion as it would appear to an observer riding around on the runner and is termed the *relative velocity*. V_1 and v_1 are related by forming a vector triangle with u_1, the linear velocity of the corresponding point on the runner, here the runner-vane entrance tips. All velocities will be expressed in feet per second.

This relation may be understood by considering the movement of a particle in a small time interval Δt. Suppose Fig. 21b represents a plane revolving with the runner. A particle starts at a and moves to b, ab being the relative motion, or the motion as it would appear to an observer moving with the runner. If we now change our point

of view to that of a stationary observer, the particle is seen to have started at a and moved to c, while the point b on the runner has moved from b to c. Thus bc is the movement of the runner and ac the absolute motion of the particle. The absolute motion ac is thus seen to be the resultant of ab and bc. The velocities are proportional to the movements in the given time, so that the absolute velocity V_1 is the resultant of the relative velocity v_1 and the linear velocity of the point of reference on the runner, u_1, the three forming a closed triangle. A similar relation applies at the exit from the runner, where v_2 is the exit velocity relatively to the runner, u_2 the linear velocity of the exit point on the runner, and V_2 their resultant, the absolute velocity of discharge from the runner. Since u_1 and u_2 are the circumferential velocities of two points on the same runner, they are proportional to their radial distances from the axis of rotation

$$u_1 = \frac{2\pi r_1 N}{60}, \qquad u_2 = \frac{2\pi r_2 N}{60} \tag{3}$$

and $u_1/u_2 = r_1/r_2$, N being the rotational speed, rpm, and r_1 and r_2 the radii, ft.

It is convenient to extend the system of notation to include the angles of the inflow and outflow triangles and the tangential components of the velocities, or the whirl components. These designations are shown in Fig. 21c. The direction of v_2, *i.e.*, the angle β_2, is the direction imparted by the runner vanes at discharge, so that β_2 is substantially the inclination of the discharge portion of the runner vane. Similarly, when the turbine is running at its proper speed, the direction of v_1 should closely conform to the direction of the runner vane at the entrance, to avoid "shock" or eddy formation.

Force and Torque Relations. Let us now consider the forces exerted on the runner by the impulse and reaction of the entering and leaving water and the moments of these forces upon the runner. At every point of the outer periphery of the runner, a cylindrical surface containing the entrance edges of the vanes, the flow enters with a velocity V_1, and every stream element comprising a flow of dq cubic feet per second exerts upon the runner a forward force or impulse $w(dq)V_1/g$, according to the momentum principle or that of the impulse of a jet. The tangential component of this force is everywhere $\frac{w(dq)V_1}{g}\cos\alpha_1$, or in our notation $\frac{w(dq)V_{u1}}{g}$, and its moment acting on the runner is $\frac{w(dq)V_{u1}}{g}r_1$. Hence the total moment of all the entering water is $wqV_{u1}r_1/g$, in which q is the total quantity of water passing through the runner per second. Similarly, at discharge from the runner, the total moment of the backward reaction of the leaving water all the way around the inner periphery is $wqV_{u2}r_2/g$. Hence the net turning moment exerted on the runner is $\frac{wq}{g}(r_1V_{u1} - r_2V_{u2})$; and this is balanced by the resisting torque, due to the useful load on the shaft, and mechanical friction. If $q = Q - Q_L$, in which Q_L is the leakage loss, or flow that by-passes the runner through the clearance spaces, and e_v is the volumetric efficiency $(Q - Q_L)/Q$, we can put $q = e_vQ$.

This moment when multiplied by $2\pi N/(60 \times 550)$ gives the horsepower transmitted from the water to the runner; and this power multiplied by the mechanical efficiency e_m, to allow for the losses in bearing friction and in stuffing-box friction on the shaft, is equal to the power delivered on the shaft to the generator or other driven machine, *i.e.*, the delivered or brake horsepower. Hence,

$$\frac{2\pi N w Q e_v e_m}{60 \times 550 g}(r_1V_{u1} - r_2V_{u2}) = \frac{wQHe}{550}$$

If we subdivide the efficiency into the product of three factors, the volumetric efficiency e_v, the mechanical efficiency e_m, and the hydraulic efficiency e_h, then $e = e_v e_m e_h$; and if we remember that $2\pi r_1 N/60 = u_1$ and $2\pi r_2 N/60 = u_2$, the preceding relation becomes the Euler formula,

$$u_1 V_{u1} - u_2 V_{u2} = gHe_h \tag{4}$$

This is a relation of major importance in turbine design and analysis.[1]

The foregoing moment equation may be applied to the case of the flow in a vane-free space, such as the flow in the casing, in the transition space between the guide vanes and runner, and in the draft tube.

Consider the equilibrium of the ring of water occupying the doubly hatched annular space shown (Fig. 22) in a conduit bounded by surfaces of revolution. The space is free of vanes.

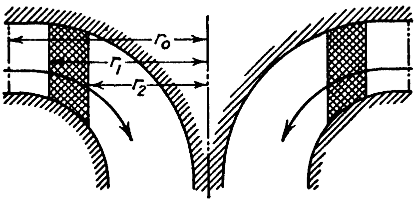


FIG. 22.

The moment exerted by the water entering and leaving the annular space must be equated to zero, for there are no vanes to resist it or to abstract power. Since steady flow is being considered, constant with respect to time, any moment exerted on the free mass of water within the annular space would accelerate it indefinitely and destroy the balance of forces essential to steady flow. Hence in this case,

$$\frac{wq}{g}(r_1 V_{u1} - r_2 V_{u2}) = 0$$

and

$$r_1 V_{u1} = r_2 V_{u2}$$

If the flow enters at some radius r_0 with tangential velocity V_{u0}, then V_u at any point is given by

$$rV_u = r_0 V_{u0} = \text{(a constant)} \tag{5}$$

i.e., the whirl component of the velocity varies inversely as the radius. This is the vortex law, or the principle of constancy of moment of momentum.

Although the principles just derived have been demonstrated for a purely radial inward-flow runner, they are equally applicable to the usual Francis runner (Fig. 23) in which the flow lines in the meridian section curve from a radial toward the axial direction. Here the inflow and outflow triangles for a given flow line cannot be drawn in one plane or surface; but the outflow triangle must be constructed, as in the case shown, on the development of a conical surface tangent to the flow line at the runner exit point 2. The preceding relations, however, apply unchanged.

Head or Energy Relations. It is also useful to trace the head or specific energy relations governing the flow by applying the Bernoulli theorem. With reference to Fig. 24, imagine a piezometer connected to the space at the entrance to the runner, at point 1, recording the pressure head h_{p1}. By taking the surface of tail water as datum, the elevation of point 1 is z_1. The velocity at this point is V_1 and the velocity head $V_1^2/2g$. Hence the total dynamic head at 1, or the specific energy of the water flowing into the runner at this point, is $h_{p1} + z_1 + \frac{V_1^2}{2g}$. Of this energy or head, a

[1] Except in small turbines and those of the low-speed type, the bearing friction and gland friction are of small proportions amounting usually to only a fraction of a per cent, and the leakage loss seldom exceeds 1 or 2 per cent, so that in the region of specific speeds greater than about 30 the hydraulic efficiency e_h will exceed the turbine efficiency e by an amount seldom greater than 1 or 2 per cent.

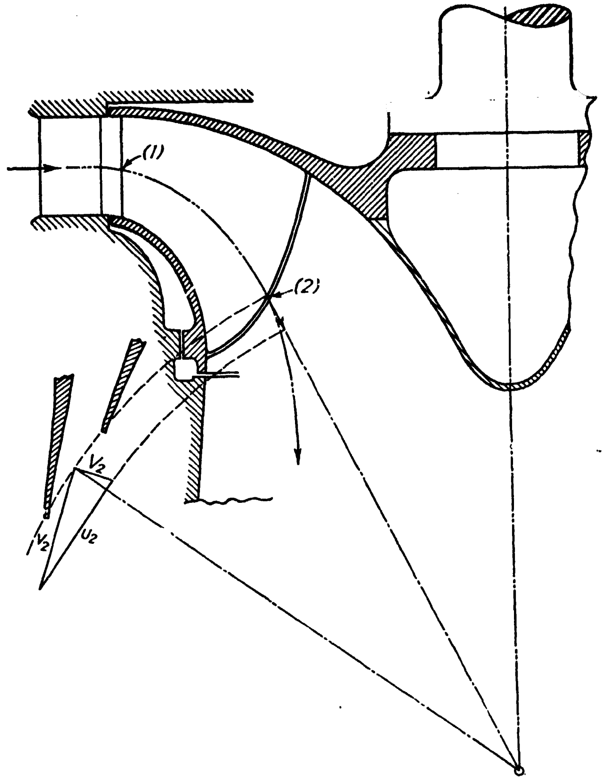


FIG. 23.

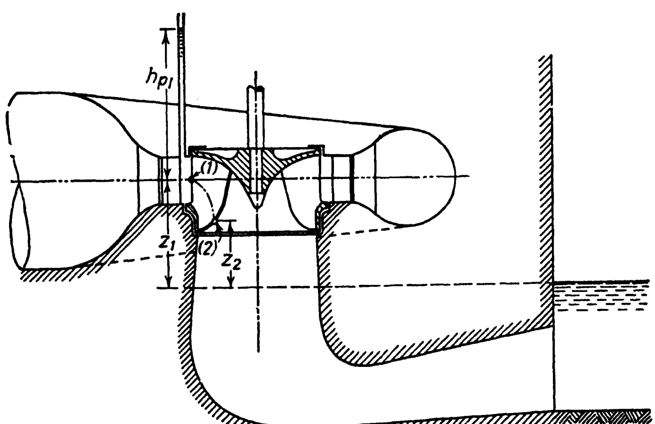


FIG. 24.

certain portion h_L is lost in surface friction and eddies in the runner, another portion is transmitted to the runner to drive it against its useful load, and the remainder is the total dynamic head at point 2. The amount of head transmitted to the runner is the total foot-pounds per pound of water flowing, or

$$\frac{wqHe_h}{wq} = He_h$$

so that the Bernoulli formula takes the form

$$h_{p1} + z_1 + \frac{V_1^2}{2g} - h_L - He_h = h_{p2} + z_2 + \frac{V_2^2}{2g} \tag{6}$$

This is the Bernoulli formula expressing the energy balance with respect to the stationary system.

We can also imagine ourselves to be moving with the runner and can write the Bernoulli formula as it applies to the relative movement of the water in the runner.

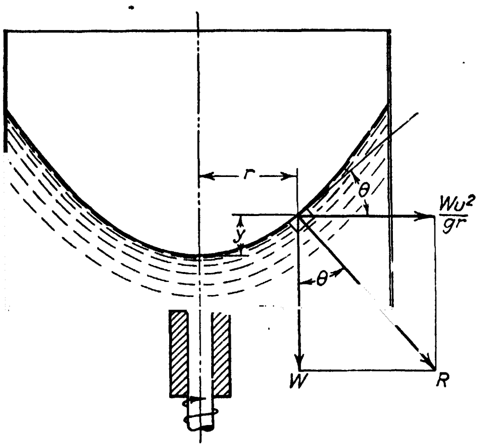


FIG. 25a.

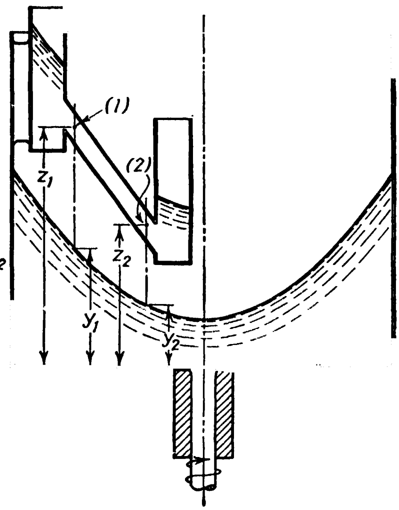


FIG. 25b.

Let us first briefly review the principles of hydrostatics governing liquid at rest relatively to a rotating system turning uniformly about a vertical axis. In the open tank shown (Fig. 25*a*) rotating n revolutions per second, containing water with a free surface, a particle in the surface is subject to a downward force W, its weight, and a centrifugal force Wu^2/gr, u being the linear circumferential velocity of a point in the rotating system at radius r. Since the water is in equilibrium, at rest with respect to the tank, the resultant of the forces acting on the particle must be normal to the free surface, there being no force to resist motion along the surface. Hence the angle of inclination θ of the surface is such that

$$\tan\theta = \frac{Wu^2}{Wgr} = \frac{u^2}{gr} = \frac{dy}{dr}$$

as will be clear from the sketch. Putting $u = 2\pi rn$, $\frac{dy}{dr} = \frac{4\pi^2n^2r}{g}$, and $dy = \frac{4\pi^2n^2}{g}rdr$ is the differential equation of the surface curve. Integrating this, between $y = 0$ and $y = y$,

$$y = \frac{4\pi^2n^2r^2}{2g} = \frac{u^2}{2g} \tag{7}$$

which means that the surface is a paraboloid of revolution and the elevation of any point in the surface above that at the axis is equal to the velocity head corresponding to the linear velocity of the system at the point considered.

Now consider a pipe system such as shown (Fig. 25*b*), carried by the revolving tank and with water flowing with steady flow from one small tank into another through a pipe, revolving with the system. Let us write the Bernoulli formula between sections 1 and 2 of the pipe in terms of the relative velocities v_1 and v_2 of the flow in the pipe. Since a particle in the free surface of water at rest with respect to the large tank has no tendency to move relatively to the rotating system, it is the elevation of water above this surface which measures the head available to produce flow. Hence it is only necessary in applying the Bernoulli formula relatively to the system to replace our usual horizontal datum plane by a paraboloidal datum surface coincident with or parallel to an actual or imaginary water surface corresponding to the speed of rotation. That is, instead of the elevation heads z_1 and z_2, which would apply if the system were stationary, we must use

$$z_1 - y_1 = z_1 - \frac{u_1^2}{2g} \qquad \text{and} \qquad z_2 - y_2 = z_2 - \frac{u_2^2}{2g}$$

Our Bernoulli formula then becomes

$$h_{p1} + \left(z_1 - \frac{u_1^2}{2g}\right) + \frac{v_1^2}{2g} - h_L = h_{p2} + \left(z_2 - \frac{u_2^2}{2g}\right) + \frac{v_2^2}{2g}$$

This form of the Bernoulli formula for a rotating system applies directly to the turbine runner passages. By treating sections 1 and 2 as points on the revolving runner, it is clear that the pressure heads h_{p1} and h_{p2} merely measure intensities of the static pressure and are the same for points just outside the runner or just within it, or the same whether the observer is stationary or imagined to be moving with the runner. There is no need to introduce between points 1 and 2 on the runner any term representing the extraction of energy or the performance of work beyond the friction and eddy losses h_L within the passages, because between these points no relatively moving element is interposed. Hence we can write for the runner,

$$h_{p1} + z_1 - \frac{u_1^2}{2g} + \frac{v_1^2}{2g} - h_L = h_{p2} + z_2 - \frac{u_2^2}{2g} + \frac{v_2^2}{2g} \tag{8}$$

If we subtract this from the Bernoulli formula previously written with reference to the stationary system

$$h_{p1} + z_1 + \frac{V_1^2}{2g} - h_L - He_h = h_{p2} + z_2 + \frac{V_2^2}{2g}$$

we obtain

$$\frac{V_1^2 - v_1^2 + u_1^2}{2g} - \frac{V_2^2 - v_2^2 + u_2^2}{2g} = He_h \tag{9}$$

This is a useful relation connecting the absolute and relative velocities of the water, the velocities of the runner, the effective head on the turbine and its hydraulic efficiency.

If we now insert the trigonometric properties of the vector velocity triangles

$$v_1^2 = V_1^2 + u_1^2 - 2u_1V_1 \cos \alpha_1 = V_1^2 + u_1^2 - 2u_1V_{u1}$$

and

$$v_2^2 = V_2^2 + u_2^2 - 2u_2V_2 \cos \alpha_2 = V_2^2 + u_2^2 - 2u_2V_{u2}$$

we obtain

$$\frac{2u_1V_{u1} - 2u_2V_{u2}}{2g} = He_h$$

or

$$u_1V_{u1} - u_2V_{u2} = gHe_h$$

the same relation as previously derived from the balance of moments.

The foregoing principles are the fundamentals of turbine theory and should make clear the force relations and energy transformations involved and serve to explain the action. Although they are important in turbine design, the application of turbines to the conditions in a power installation and the problems of the user of turbines can be best handled by means of another series of principles based on dimensional relations and the laws of dynamic similarity, which will now be outlined.

PRINCIPLES OF DYNAMIC SIMILARITY FOR TURBINES

Consider a series of homologous turbines, or turbines that are geometrically similar, each a photographic reproduction of the others, or all made from the same drawings but to different scales. Then imagine these to be operated under various effective heads. Any one of these turbines may of course be operated at various speeds; but there is one best speed at which the maximum efficiency will be developed, and if the design is correct this will be the designed speed or normal speed.

We have seen that at the proper speed the direction of the runner vanes at the entrance must agree with the relative direction β_1 of the entering flow. At any other speed, the flow will part from one side or the other of the vane, leaving an eddy-filled space which consumes energy and converts velocity head into heat, which is hydraulically unutilizable. Hence only one fixed shape of inflow triangle is permissible to avoid this so-called *shock* loss.

In the same way, at discharge from the runner there is one relation between the velocities that is most favorable, and any other will be inconsistent with the highest efficiency. The important element here is the residual velocity head remaining in the water leaving the runner $V_2^2/2g$. A considerable portion of this, usually an amount of the order of some 80 per cent, can be reconverted into effective pressure head by deceleration in the gradually enlarging draft tube, but it is inherent in the deceleration process that an appreciable part of this energy must be lost. Consequently, it is important that V_2 be held near a minimum value. The smallest value of V_2 would be secured by discharging the water perpendicularly to the surface of revolution generated by the discharge edges of the runner vanes, for the entire discharge must pass through this area and can do so at minimum velocity when the velocity is normal to the area. However, there are also internal losses in the runner, and the conduit friction or resistance of the vane surfaces must also be minimized. This frictional loss is closely proportional to the relative velocity head $v_2^2/2g$. The optimum condition requires the sum of these two losses to be a minimum and results in a small angle $(90^\circ - \alpha_2)$ of the absolute velocity of discharge V_2, this angle in most cases having values of 5 to 15 deg, to give an idea of its order of magnitude.[1]

It is thus seen that for correct operation the shapes of the two vector velocity triangles are fixed and the angles must be kept unaltered. The values of the velocities may change but all in the same proportion. The scale of the triangles may be altered, but all velocities must retain the same ratios to each other. Thus if the velocity V_1 is doubled, u_1 must also be made twice as great and the speed of the turbine N must be doubled also, and all the remaining velocities will be increased in like ratio.

If a turbine of given geometric form is built in any size and operated under any head, we will suppose that the speed N has been adjusted to conform to a fixed shape of each velocity triangle. It would be reasonable to assume that the efficiency would then remain unaltered and independent of turbine size or head; but this conclusion is subject to a relatively small correction from the fact that the frictional losses may not vary in exact proportion to the velocity-head and eddy losses, as is well known in the case of pipe flow. As is known, to preserve strict dynamic similarity for conduit flow so that the flow patterns will be identical, the viscous forces must remain proportional

[1] See *Trans. A.S.M.E.*, **43**, 1113, 1921.

to the inertia forces, which is possible only when the Reynolds number remains constant. This is considered by the author to be of little importance in turbine calculations, for turbine conduits are large, even in the usual sizes of laboratory models, the velocities are high, and the surfaces exposed to the flow are relatively rough, so that the Reynolds numbers fall far beyond the region of the Reynolds critical velocity and the turbine passages fall into the class of rough conduits, as distinguished by von Karman and Prandtl,[1] in which the flow is completely turbulent and viscous forces are negligible. On the basis of this reasoning, variations of head on a given turbine should have negligible effect on the efficiency, a conclusion that seems to be in agreement with such data as are available for turbines of usual sizes. There is a factor, however, that does effect the efficiency, *viz.*, the lack of complete geometrical similarity. The similarity does not extend to the surface texture of the conduit walls, where there is more nearly a constancy of the degree of absolute roughness than of relative roughness necessary to complete homogeneity.

Consequently, the losses in hydraulic wall friction will not remain proportionately constant with change in size of turbine. However, the change in efficiency due to this cause is comparatively minor in amount, and its effect on the validity of the relations about to be developed has been found to be negligible. In a later section, methods of correcting efficiencies for the effect of turbine size will be given.

If we assume that the efficiencies of homologous turbines of differing sizes and operating under different heads will remain substantially constant, or that the turbines are truly homologous even as to surface roughness, and postulate that each turbine is to be operated at its correct speed for its size and head so that the velocity triangles will also be homologous, then all velocities will have a constant ratio to any one velocity selected as a reference value. Hence the relation between relative and absolute velocities derived above

$$\frac{V_1^2 - v_1^2 + u_1^2}{2g} - \frac{V_2^2 - v_2^2 + u_2^2}{2g} = He_h$$

can be expressed as constant $\times \dfrac{V_1^2}{2g} = He_h$; so that if e_h is substantially constant, it is seen that for correct similarity of operating conditions the velocity head corresponding to any velocity must be kept in constant ratio to the effective head on the turbine. The flow through the turbine will then follow the same laws as the flow through a nozzle, for example. These conclusions give us the following rules of similarity for turbines.

Any velocity head must vary as the effective head on the turbine $\dfrac{V^2}{2g} \propto H$.

Any velocity must vary as $\sqrt{H}$; $V \propto H^{1/2}$ (considering g as always practically the same).

The discharge must vary as velocity $\times$ area, $Q \propto VA \propto H^{1/2}D^2$, where D denotes any representative dimension of the turbine, such as the runner-throat diameter. (Any conduit area A varies as the square of the linear dimensions.)

Power output varies as QH; $HP \propto H^{1/2}D^2H \propto D^2H^{3/2}$.

Since the velocity u of a point on the runner must vary in proportion to the other velocities, $u = \dfrac{\pi DN}{60}$ and $u \propto H^{1/2}$. Hence the turbine speed, rpm, varies as u/D and $N \propto H^{1/2}/D$.

[1] PRANDTL, L., Neuere Ergebnisse der Turbulenzforschung, *Z. Ver. deut. Ing.*, **77**, No. 5, 105–114, 1933.
KARMAN, TH. VON, Some Aspects of the Turbulence Problem, *Mech. Eng.*, July, 1935, pp. 407–412.
LINDQUIST, ERIK, On Velocity Formulas for Open Channels and Pipes, *Trans. World Power Conf. Section Meeting, Scandinavia*, **1**, 177–234, 1933.
BAKHMETEFF, BORIS A., "The Mechanics of Turbulent Flow," Princeton University Press, 1936.

These relations are convenient for use in such problems as the application of model tests to determine the expected performance of a large unit of the same design or to transfer the results obtained in one turbine to another of homologous form. For example, if a model of throat diameter D develops a horsepower HP under a head H when running at a speed N, we can write

$$HP \propto D^2H^{3/2} \qquad \text{or} \qquad HP = HP_1D^2H^{3/2} \tag{10}$$

in which HP_1 is the power that would be developed by a homologous turbine of 1 ft throat diameter under 1 ft head as can be seen by putting $D = 1$ and $H = 1$ in the last relation. Knowing HP, D, and H, we can compute HP_1 from the model tests. Then a homologous turbine of D' throat diameter operating at the proper corresponding speed under a head H' would develop a power of $HP' = HP_1D'^2H'^{3/2}$. The proper speed can be found by the use of the relation $u \propto H^{1/2}$, or $u =$ (const.) $H^{1/2}$, and using the customary notation the constant is called $\phi\sqrt{2g}$, so that

$$u = (\phi\sqrt{2g})H^{1/2} = \phi\sqrt{2gH} \tag{11}$$

i.e., ϕ is the ratio of the circumferential velocity of a point on the runner to the theoretic spouting velocity of the water, $\sqrt{2gH}$. Then ϕ may be computed from the model test and must remain the same for the large unit, *i.e.*,

$$\phi = \frac{u}{\sqrt{2gH}} = \frac{\pi DN}{60\sqrt{2g}\,H^{1/2}} = \frac{\pi D'N'}{60\sqrt{2g}\,H'^{1/2}}$$

from which N' is found.

Specific Speed. Modern practice in turbine design rests on the firm basis of actual results secured on laboratory models and large units, and each experienced manufacturer has available such test records for an extensive series of designs of progressively varying characteristics, each giving satisfactory performance. No matter how elaborate the method of design based on theory, no important installation would be carried out without confirming the calculations by tests on a model or closely similar field installation. The complexity of the flow conditions and the manifold factors involved make this procedure the sole source of assurance as to successful performance of the installation and at the same time make it the most economical and timesaving method of design. Modified and radically new designs are continually being produced, however, and a sound theoretical basis of design cannot be dispensed with in favor of exclusively empirical methods.

The usual problem that first arises in planning a new hydroelectric project is the selection of the proper type of turbine and its proper speed to suit the given conditions of power capacity and effective head. This step has been essentially simplified by a relation of basic importance, which has systematized the entire field of turbine design and performance and in modified form also covers the field of pumps, and is applicable to hydraulic machinery in general. This is the specific-speed principle.

This may be viewed as a combination in a single expression of the foregoing principles of similarity and amounts to a definition of the rotary speed of the turbine N in dimensional form, the dimensions used being not the fundamental dimensions of length, mass, and time but those entering into the usual turbine problem, *viz.*, head and power output.

We have just found that $HP \propto D^2H^{3/2}$ when $N \propto H^{1/2}/D$. From the first expression, $D \propto \sqrt{HP}/H^{3/4}$, and inserting this in the second, $N \propto \frac{H^{1/2}H^{3/4}}{\sqrt{HP}} \propto \frac{H^{5/4}}{\sqrt{HP}}$. By putting this in the form of an equation instead of a proportionality,

$$N = (\text{a constant})\,\frac{H^{5/4}}{\sqrt{HP}}$$

To determine the constant, it is necessary only to put the head and *HP* each equal to unity, from which it is seen that the constant is the speed (in revolutions per minute) of a homologous turbine of such size that it would develop 1 hp under 1 ft head. This is called the *specific speed* and is denoted N_s, so that the fundamental relation is simply

$$N = N_s \frac{H^{5/4}}{\sqrt{HP}} \tag{12}$$

N_s is a constant for all geometrically similar turbines of any size under any head, each being operated at its proper speed corresponding to the head and size.

If the type or design is changed, we have, of course, another series of homologous turbines with a new value of the constant. N_s is therefore a quantity that sets forth in a single numerical value the inherent ability of a given form of turbine to develop speed for a given power and head; or if the expression is squared and solved for *HP*, it is seen that N_s^2 represents the power capability of the design at a given head and speed. N_s has been described by Arnold Pfau as expressing the inherent speediness of a turbine.

Specific speed may be briefly defined as the speed of a homologous 1-hp turbine under 1 ft head. It is of course unnecessary actually to build a 1-hp model to find the value of N_s, for it can be readily computed from the tests of any size of model or installed unit by inserting the actual head, horsepower, and speed from the test in

$$N_s = N \frac{\sqrt{HP}}{H^{5/4}}^{*} \tag{13}$$

Then having found N_s, this value remains constant for any other size or head, for the same design, and we can find the corresponding speed of another homologous turbine from the preceding expression $N = N_s \dfrac{H^{5/4}}{\sqrt{HP}}$, in which *HP* and *H* will be those for the new installation. The chart of Fig. 26*a* provides a graphical solution of this formula.

In the metric system, N_s is the speed of a homologous turbine of a size to develop 1 metric *HP* under 1 m head; the metric N_s is equal to 4.45 times the N_s in the foot-pound system.

In a multiple-runner turbine, the specific speed is expressed as that of a single runner; and in a multiple-jet Pelton wheel, the specific speed is based on the horsepower of a single jet.

The entire field of turbines can be classified on the basis of the specific-speed values. Indeed this classification can be extended to include other prime movers such as gravity water wheels and steam turbines and can also be extended to cover in a single comprehensive series, pumps, blowers, fans, and marine propellers.[1] We shall here limit our consideration to hydraulic turbines of impulse, reaction, and propeller types.

As might be expected, there are some forms and proportions in turbine design that are particularly favorable to high efficiency with a corresponding range of specific speed values. Specific speeds that are abnormally high or low as compared with this normal range naturally entail impaired efficiencies.

Since for a given power capacity and head the actual speed is directly proportional to the specific speed, as shown by the specific-speed formula, and since high rpm

* The relation is convenient for calculation on the Mannheim slide rule by putting $H^{5/4} = H\sqrt{\sqrt{H}}$.

[1] MOODY, L. F., Die spezifische Umlaufzahl von Wasserturbinen, *Z. für das gesamte Turbinenwesen*, Sept. 10, 1909, p. 392.

MOODY, L. F., The Present Trend of Turbine Development, *Trans. A.S.M.E.*, **43**, 1113, 1921.

TAYLOR and MOODY, "The Hydraulic Turbine in Evolution," *Engineers and Engineering*, **39**, 241, July, 1922.

results in reduced diameters and weights of the driven machine, and to some extent of the turbine itself, and also in reduced size and cost of the powerhouse structure, there is from economic considerations a strong attraction toward higher and higher specific speeds, even at the expense of some sacrifice in efficiency. It should be understood, however, that in the great majority of hydroelectric developments the major

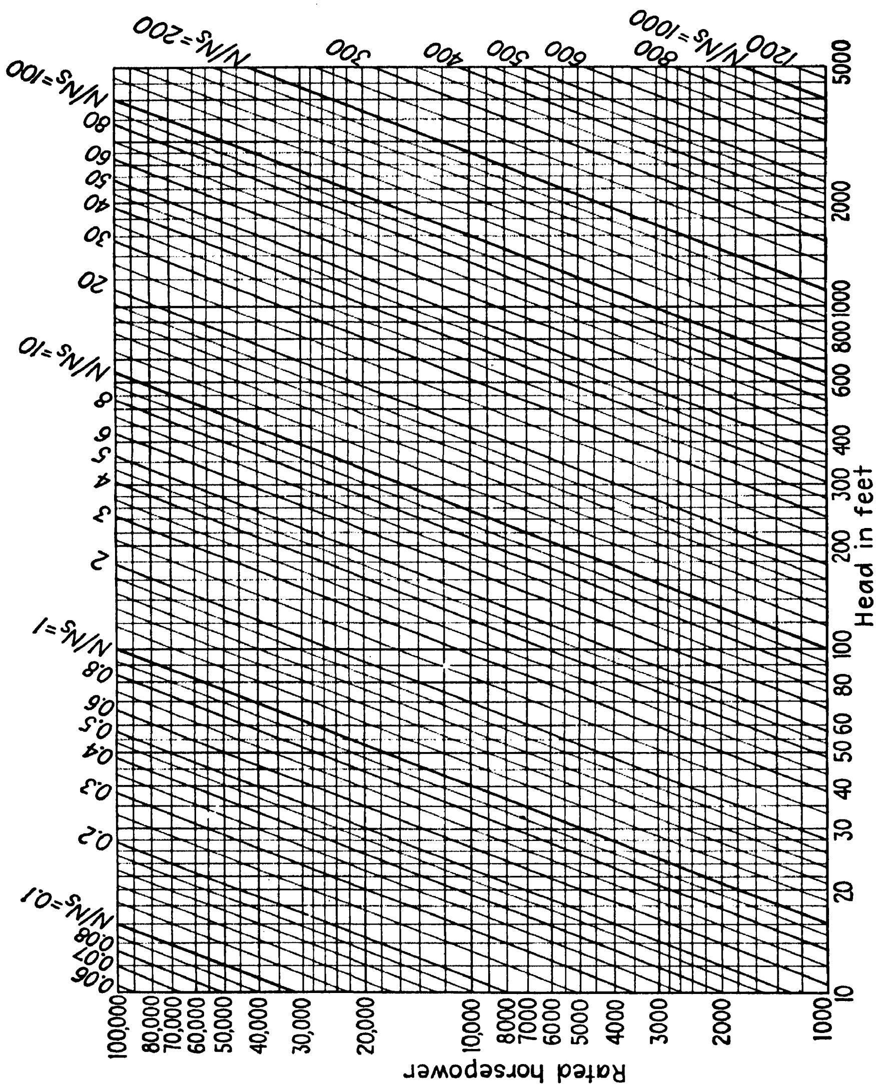


FIG. 26a.—Chart for the solution of the specific-speed formula for turbines.

portion of the cost is in the dams, canals, reservoirs, pipe lines, transmission lines, and similar fixed works and only a small portion in the powerhouse and machinery; so a saving in first cost of the latter elements must be justified in comparison with the sacrifice in earnings on the whole project entailed by a reduction in efficiency; and it will be appreciated that in most cases a considerable increase in specific speed will be beneficial only when the impairment in efficiency is slight. These considerations are

of special importance in low-head installations, where the actual speeds are low and the machinery large. The continual demand for the highest possible speeds has in recent years resulted in a great increase in the range of specific speeds available without material sacrifice in efficiency, and there is now a wide band of specific-speed values consistent with high attainable efficiency, as will be seen from the curve of highest well-established efficiencies plotted against specific speeds (Fig. 26*b*). This curve has been drawn as an envelope of efficiencies obtained in many installations in which records of high efficiency were established under approved test conditions.

In using this curve as a guide to the efficiency that may reasonably be expected in a new installation, it must be understood that this curve does not represent an average performance, as the great majority of points from existing plants fall below it. It actually passes through the points from a limited number of installations where, owing to particularly favorable designs and test conditions, maximum results were obtained; and as every new plant cannot be expected to equal the world's record, it would be reasonable not to count on the attainment of an efficiency within a margin

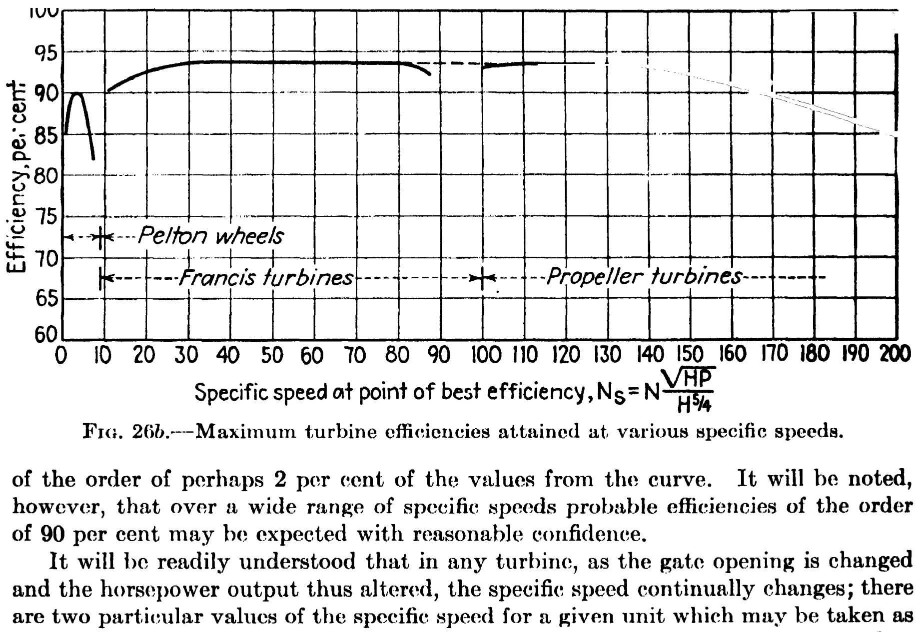


FIG. 26*b*.—Maximum turbine efficiencies attained at various specific speeds.

of the order of perhaps 2 per cent of the values from the curve. It will be noted, however, that over a wide range of specific speeds probable efficiencies of the order of 90 per cent may be expected with reasonable confidence.

It will be readily understood that in any turbine, as the gate opening is changed and the horsepower output thus altered, the specific speed continually changes; there are two particular values of the specific speed for a given unit which may be taken as representative. The usual value used in classifying turbines is the specific speed at the rated capacity or guaranteed horsepower under the normal head for which the unit is designed. This may be called the *rated specific speed.* Except in the case of high-speed Francis and fixed-blade propeller turbines, it is possible to have the point of highest efficiency occur at a power somewhat less than full rated load, and this power for maximum efficiency may be called the *normal load* and the corresponding specific speed the *normal value.* The curve of Fig. 26*b* is based on the normal specific speeds, or the specific speeds for the points of highest efficiency.

The curve of efficiency versus horsepower output for a given turbine operated at varying gate opening has different characteristic forms for various turbine types and specific speeds, as illustrated in Fig. 27. The Pelton wheel, having very low specific speeds, gives a flat-topped efficiency curve with high part-gate and over-gate efficiencies and only small impairment of efficiency over a wide range of nozzle openings

and outputs. The low-speed Francis turbine has similar characteristics.[1] Both types are therefore suitable for plants serving to regulate a power system and taking the load variations. High-speed Francis turbines show poor part-load efficiencies and little, if any, overload capacity beyond the point of maximum efficiency, *i.e.*, they have a sharply peaked efficiency curve and are therefore best suited to operation under block load or within a narrow range of outputs. This characteristic is further intensified in fixed-blade propeller turbines. In run-of-river plants, such as low-head developments with limited pondage, where under reduced demand the unused flow must be wasted over the spillway, the low part-gate efficiencies are of little consequence.

When a high degree of regulation is of importance in low-head plants, high part-gate and over-gate efficiencies may be secured in high-speed propeller turbines by making the runner blades automatically adjustable in the hubs in response to changes

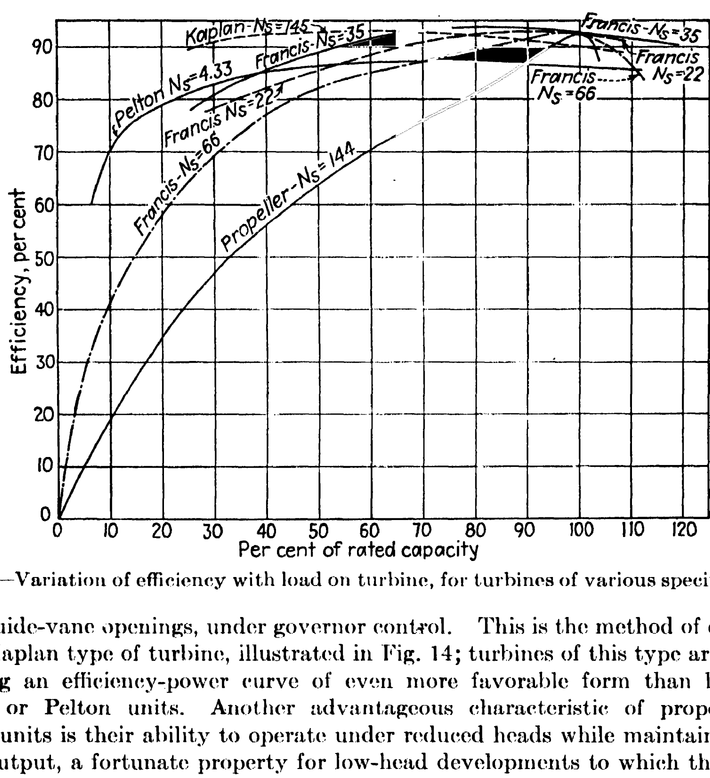


FIG. 27.—Variation of efficiency with load on turbine, for turbines of various specific speeds.

in the guide-vane openings, under governor control. This is the method of operation of the Kaplan type of turbine, illustrated in Fig. 14; turbines of this type are capable of giving an efficiency-power curve of even more favorable form than low-speed Francis or Pelton units. Another advantageous characteristic of propeller and Kaplan units is their ability to operate under reduced heads while maintaining good power output, a fortunate property for low-head developments to which these types are suited (owing to their high specific speeds), for such developments are usually subject to wide head variations.

It will be easily understood that in plants having high heads there is little need for using high specific speed, for even a low specific speed in combination with the high head will usually give as high actual rpm as is desired as may be seen from the specific-speed formula. On the other hand, with low heads and large unit capacities, a normal value of specific speed would result in extremely low rpms, very large diameters of turbine and generator, and large powerhouse structures, as previously mentioned. Consequently, low-speed turbine types meet the usual demands for high-head develop-

[1] The curves in Fig. 27 for Francis turbines are from units equipped with spreading draft tubes and show somewhat higher part-gate efficiencies than are usually obtained.

ments, and high-speed types are necessary for low-head plants. This is another fortunate consideration, for, as will be shown, high-speed turbines are unsuited for high-head application owing to the limitations imposed by another and most important factor, *viz.*, the cavitation limit; whereas low-speed types can be operated under high heads owing to their favorable cavitation characteristics. In selecting the specific speed suitable for a given head, it is the cavitation requirements that are the controlling consideration, and this subject will now be taken up.

CAVITATION[1]

Let us write the Bernoulli formula for the draft tube of the turbine (Fig. 28) between section 2, at the outlet of the runner to section f, the point of final discharge into the tailrace, and take the tail-water surface as datum level. Then, by calling the elevation of section 2 above tail water H_s (the static draft head) and expressing all

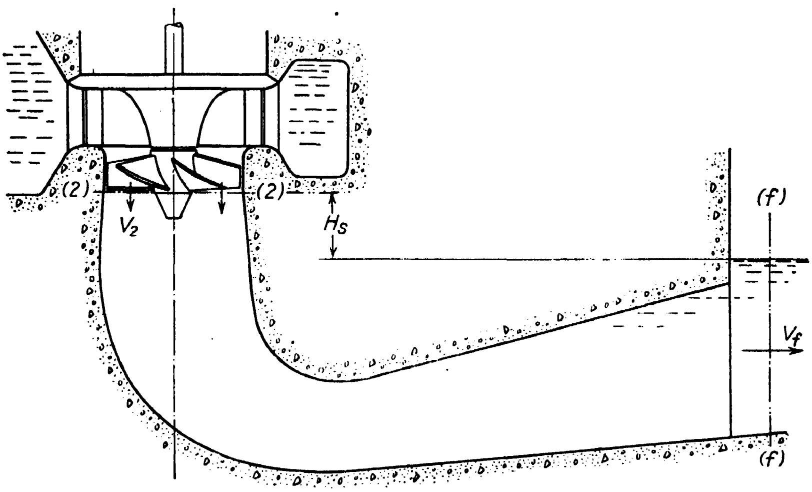


FIG. 28.

static pressure heads as absolute pressures, H_{p2} denoting the absolute pressure head at 2 and H_a the pressure head of the atmosphere,

$$H_{p2} + H_s + \frac{V_2^2}{2g} - H_L = H_a + O + \frac{V_f^2}{2g}$$

in which H_L is the loss of head in eddies and friction within the draft tube and V_f the final discharge velocity in the tailrace.

The total loss of energy is the internal loss plus the final residual velocity head rejected into the tailrace, $V_f^2/2g$. If the effective head on the turbine varies, and if the rotational speed of the runner is always adjusted to suit the head so that ϕ remains constant, V_2 will remain at constant angle of obliquity α_2 to the plane of section 2 and its axial component will remain in fixed ratio to V_2; and from the principle of continuity of flow, the velocity at any point in the tube and that at section f will likewise remain in fixed ratio to V_2. From the principles of similarity, the eddy losses within the tube and the final outflow loss $V_f^2/2g$ will then always be directly proportional to $V_2^2/2g$; if we make the reasonable assumption that the flow is completely turbulent, we can also take the frictional losses proportional to the velocity head at any point or

[1] ROGERS and MOODY, Inter-relation of Operation and Design of Hydraulic Turbines, *Engineers and Engineering*, 1925. THOMA, D., Experimental Research in the Field of Water Power, *Trans. First World Power Conf.*, II, 536, 1924.

to $V_2^2/2g$. The total loss can then be put equal to $K(V_2^2/2g)$, K being a constant coefficient for a given turbine.

Then the Bernoulli formula gives

$$H_{p2} = H_a - H_s - \frac{V_2^2}{2g} + H_L + \frac{V_f^2}{2g} = H_a - H_s - \frac{V_2^2}{2g} + K\frac{V_2^2}{2g}$$
$$= H_a - H_s - (1 - K)\frac{V_2^2}{2g}$$

Now the head representing the total kinetic energy of the flow entering the draft tube is $V_2^2/2g$; it is the purpose of the draft tube, by gradually decelerating the velocity, to reconvert as much as possible of this kinetic energy into effective pressure head in order to reduce the back pressure H_{p2} against which the runner discharges. The efficiency of the draft tube can therefore be expressed as

$$e_d = \frac{\dfrac{V_2^2}{2g} - K\dfrac{V_2^2}{2g}}{\dfrac{V_2^2}{2g}} = 1 - K$$

so that the equation for H_{p2} becomes, simply,

$$H_{p2} = H_a - H_s - e_d\frac{V_2^2}{2g} \tag{14}$$

H_{p2} represents the average pressure head at the runner discharge. But on the runner vanes there will be a higher pressure on the vane faces and a lower pressure on the back surfaces; it is this pressure difference that drives the runner around and develops mechanical power on the shaft. Pressures at local points in section 2 will therefore alternately exceed and fall below the average pressure H_{p2}; at some local point on the back of a runner vane the pressure will fall below H_{p2}; and at the local point of minimum pressure we shall call the absolute pressure head H_m. The pressure difference $H_{p2} - H_m$, the local pressure drop, is due to the flow through the turbine and under dynamically similar conditions is proportional to the effective head producing the flow, *i.e.*, the effective head on the turbine, H. Hence we can write $H_{p2} - H_m = K_cH$, where K_c is a constant coefficient for a given turbine, its value depending on the particular shape of the vanes. The absolute pressure head at the point of minimum pressure in the turbine is then

$$H_m = H_{p2} - K_cH = H_a - H_s - e_d\frac{V_2^2}{2g} - K_cH$$

If we consider that when hydraulic similarity is preserved, by keeping the runner speed in proper relation to the head, $V_2^2/2g$ is proportional to H, we can combine the last two terms and put

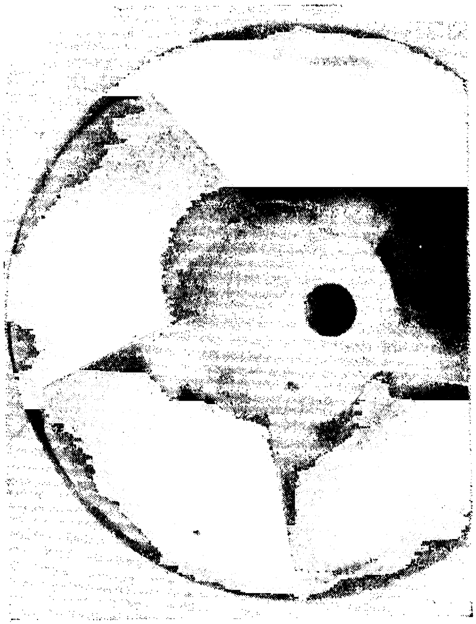
$$e_d\frac{V_2^2}{2g} + K_cH = \sigma H$$

in which σ is a constant coefficient for a given turbine (or for any geometrically similar turbine) operating at a given ϕ. Then we have

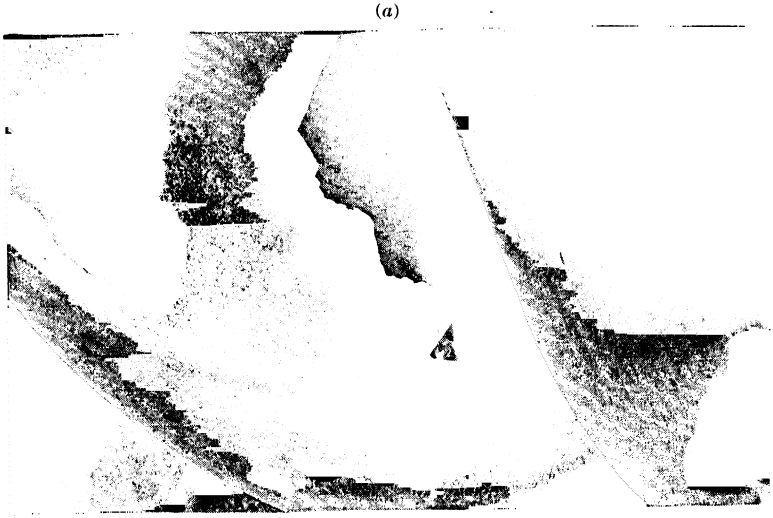
$$H_m = H_a - H_s - \sigma H \tag{15}$$

Now suppose that both headwater and tail water are progressively lowered by equal amounts. The effective head H will remain constant, but H_s will increase so that H_m will be progressively lowered. There is, however, a limit to the possible reduction of H_m. When H_m reaches the vapor pressure of the water, the point at which the water boils, the pressure can go no lower as long as water is present as a liquid. Beyond this amount of reduction, the water cannot exist as a liquid. When

H_m is reduced to this limiting value H_{vp}, the vapor pressure head, the water begins to boil and the passages become partly occupied by vapor pockets within the flowing stream. The formation of these vapor pockets, or vapor-filled cavities, in the stream is called *cavitation*.



(*a*)



(*b*)

FIG. 29. (*a*) Stroboscopic photograph showing cavitation in a Kaplan runner. The light-toned regions at the periphery and near the hub indicate vapor-filled cavities in the flow. (See Sharp, R. E. B., Cavitation of Hydraulic Turbine Runners, *Trans. A.S.M.E.*, Oct., 1940.) (*b*) Pitting caused by cavitation in a Francis runner.

When the pressure at some point in the turbine reaches the vapor pressure, critical conditions ensue and the turbine becomes subject to a virulent disease known as *pitting*, a consequence of the cavitation. Other unwelcome accompaniments of cavitation are mechanical vibration of the machine and surrounding structures, and when the extent of the cavitation increases, an impairment of power and efficiency results. This phenomenon is not limited to turbines, but may also occur in pumps and in stationary conduits at points of low pressure and high velocity, as for example in sluiceways through dams. The guarding against its occurrence is a vital consideration in the selection of type and specific speed of a turbine and in fixing the runner elevation above tail water.

The cavitation phenomenon can be explained as follows: Consider the flow through a turbine runner where the total draft head is excessive and where there is a failure of the vane contour to conform to the natural flow lines, due to the curvature being too sharp for the pressure and velocity conditions. At such a point in the runner, usually on the back of the vanes near the discharge orifice and near the outer periphery, where the relative velocity is high, the flowing stream parts from the vane surface and leaves a void filled with eddies; and when the absolute pressure is reduced to the vapor pressure, this void or cavity becomes filled with water vapor, air and other gases (Fig. 29*a*). As the flow continues downstream, the static pressure rises again and then exceeds the vapor pressure. Moreover the flow in large conduits at high velocities is never actually steady but is turbulent and subject to continual variations of velocity and pulsations of pressure. Hence, when a particle of the flowing liquid reaches a point where the local pressure just attains the vapor pressure limit, at one instant the particle will be under this pressure and vaporize, forming a cavity filled with vapor; at the next instant, the pressure will rise above the vapor pressure and the vapor will suddenly condense and return to liquid, producing a collapse of the cavity and an explosion or, more strictly, an *implosion*. This action is not confined to the larger cavities but extends into the pores of the metal. The water rushing in to fill the collapsed cavity will also enter the vapor-filled pores until instantaneously stopped by the bottom surface of the pore, where water-hammer action takes place. This is capable of producing pressure intensities on the areas of the same order as the tensile strength of the metal, and under the continual repetition of the shocks, the metal fails locally under fatigue and small particles are irregularly broken away, giving the surface a peculiar spongy appearance (Fig. 29*b*). The action, called *pitting*, is thus believed to be primarily mechanical, as just described; it was formerly thought to be mainly chemical, in the nature of rusting, or electrolytic; but it can be produced in wood, concrete, and even in glass, which points to a mechanical origin as outlined in the preceding theory. This being accepted as a logical explanation, it should be possible to prevent pitting by avoiding the occurrence anywhere in the turbine of a local pressure head so low as to approach the vapor pressure of the water. This method of avoiding cavitation and pitting has been found from experience to be effective, and experience has thus verified the conclusions of the foregoing theory.

In the formula for the minimum local pressure, we find that the critical point at which cavitation occurs is given by putting $H_m = H_{vp}$, the vapor pressure head, so that

$$H_{vp} = H_a - H_s - \sigma_c H \qquad \text{and} \qquad \sigma_c = \frac{H_a - H_{vp} - H_s}{H}$$

in which σ_c is the critical sigma, or the value of σ at which cavitation begins. We can call $H_a - H_{vp} = H_b$ the height of the barometric water column, or the height to which

water may be drawn up in a water barometer. Then

$$\sigma_c = \frac{H_b - H_s}{H} \tag{16}$$

This is the Thoma formula.

For plants at moderate elevations above sea level and for usual temperature ranges, we can in general use

$$H_b = H_a - H_{vp} = 34 \text{ ft} - 1.2 \text{ ft} = 32.8 \text{ ft}$$

It will be readily perceived that if a plant is installed with excessive static draft head H_s, *i.e.*, too high a setting of the runner above tail water, so that the actual plant

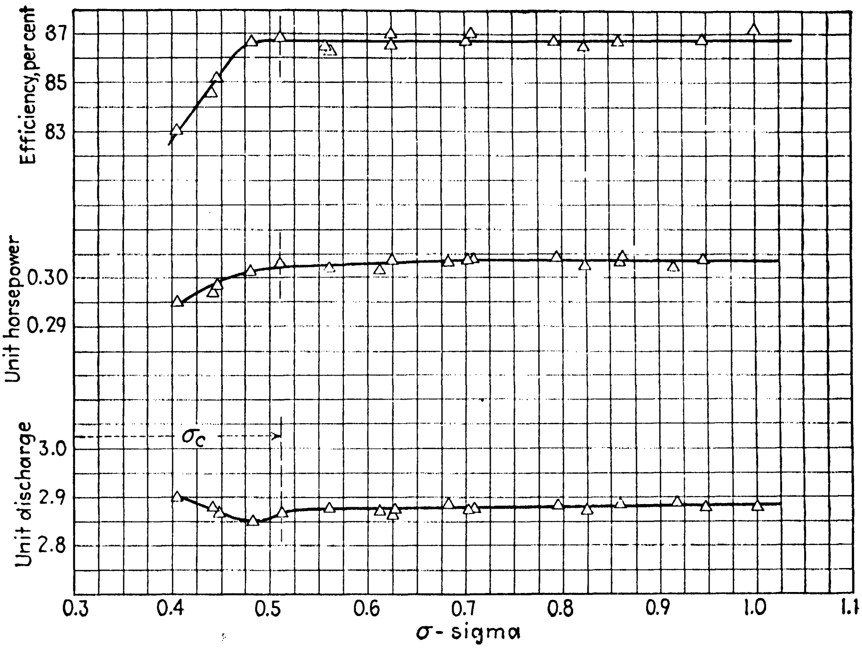


FIG. 30.—Test curves of efficiency, power, and discharge vs. cavitation coefficient sigma, showing evidence of cavitation at the critical value of sigma.

sigma is lower then the critical sigma σ_c at which cavitation begins, then portions of the water passages will be occupied by vapor and the passages will not remain completely filled with liquid. The stream areas and the lines of flow will be changed, and the principles of similarity will fail to apply, for we have no longer preserved complete geometrical similitude of the actual water stream.

This gives us a method of determining the numerical value of σ_c for a given turbine. A homologous model may be tested by progressively increasing H_s while keeping H, N, and all other factors constant. So long as no cavitation occurs, the conditions of similarity will continue to be satisfied and no material change in efficiency, power output, or discharge will occur. A drop in any or all of the quantities therefore indicates a departure from conditions of similarity, and the sigma at which such a drop begins is the critical sigma for the particular turbine. Figure 30 illustrates the method, the curves of efficiency, HP, and Q being plotted against σ. The drop in the

efficiency curve is usually the most sensitive and reliable index of cavitation. We must then be sure that in the actual installation the σ for the plant is not less than this σ_c but exceeds it, some margin being thus allowed against cavitation.

It is possible in making the cavitation tests to vary H instead of H_s, the turbine speed N being always kept such that ϕ remains constant. We can then plot the values of HP and Q reduced to unit head, HP_1 and Q_1, against σ as abscissa, as before. The first method is preferable when the laboratory arrangement permits. The number of laboratories equipped to make cavitation tests is limited, and another method must be used where laboratory determinations are lacking; indeed this second method is of importance to supplement the first, even when cavitation tests have been made. This second method is the use of field experience in operating plants to indicate whether pitting has actually occurred at the values of sigma existing during operation. Since the most serious result of cavitation is pitting, what we are most concerned with is the value of σ where pitting begins. Some turbines are known to operate without discoverable loss of power or efficiency but to develop distinct pitting; evidence of pitting is therefore a sensitive index of cavitation.

SELECTION OF TYPE AND SPEED OF TURBINE FOR A NEW PLANT

Since the critical sigma or degree of resistance to cavitation of a particular design of turbine is greatly affected by the design, so that two turbines of the same specific

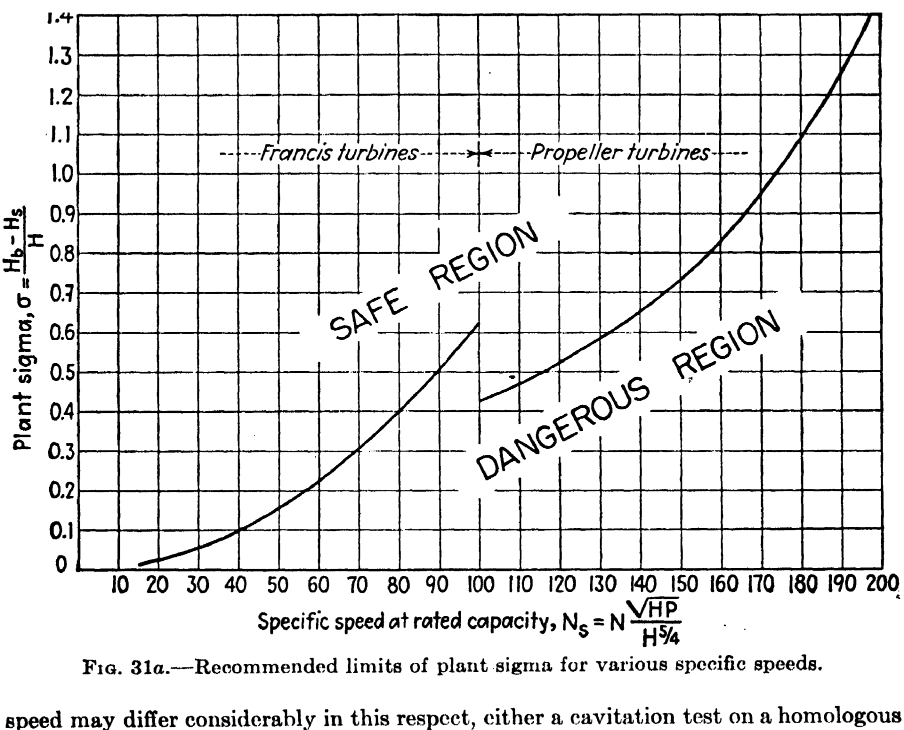


FIG. 31*a*.—Recommended limits of plant sigma for various specific speeds.

speed may differ considerably in this respect, either a cavitation test on a homologous model or, still better, actual field experience from a nonpitting homologous installation is needed if a new installation is to be carried out safely without allowing a considerable margin in the value of the plant sigma, *i.e.*, in the value of H_s to be used in the actual setting.

Such information is particularly needed with propeller turbines for which the critical sigma is greatly dependent on the particular design, especially on the proportional blade area of the runner. Ample blade area is necessary to keep sigma within reasonable limits. In the absence of these data, a fair degree of guidance for estimating the safe sigma may be obtained from curves based on pitting experience and available cavitation tests on representative turbines of various specific speeds. The value of the critical sigma in turbines of normal or conventional design is greatly

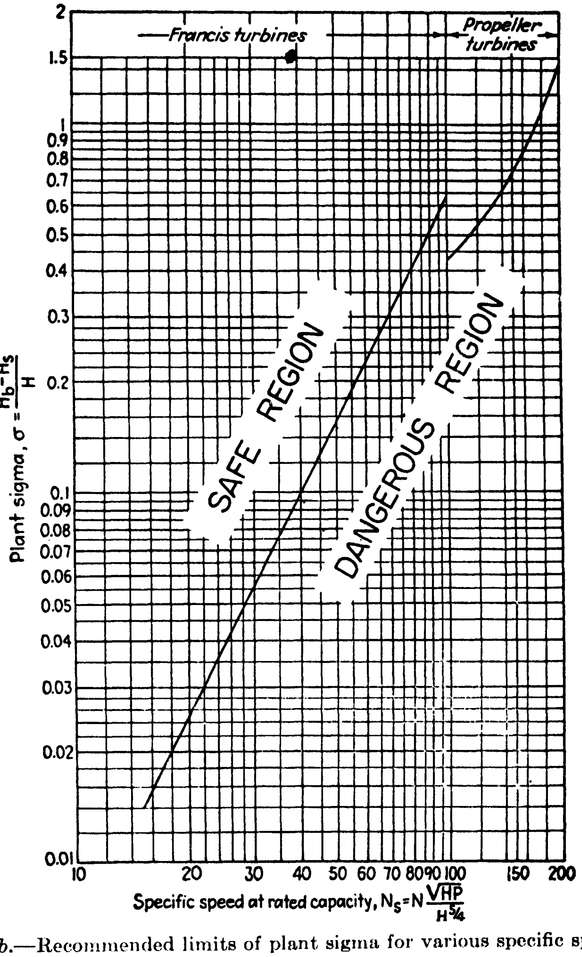


FIG. 31b.—Recommended limits of plant sigma for various specific speeds.

dependent on the specific speed so that recommended limiting values of plant sigma plotted as a function of N_s provide a useful curve, given in Fig. 31a, which serves as a valuable index to the selection of type and specific speed of turbine suitable for a given head. This curve is intended to show the lowest value of plant sigma that could be safely used for any given value of the N_s at rated capacity and in the absence of definite pitting experience or cavitation test on a given design. The actual plant sigma should exceed or at least equal this minimum value in every case. This curve has been drawn as an envelope defining the lower boundary of the region of plants that

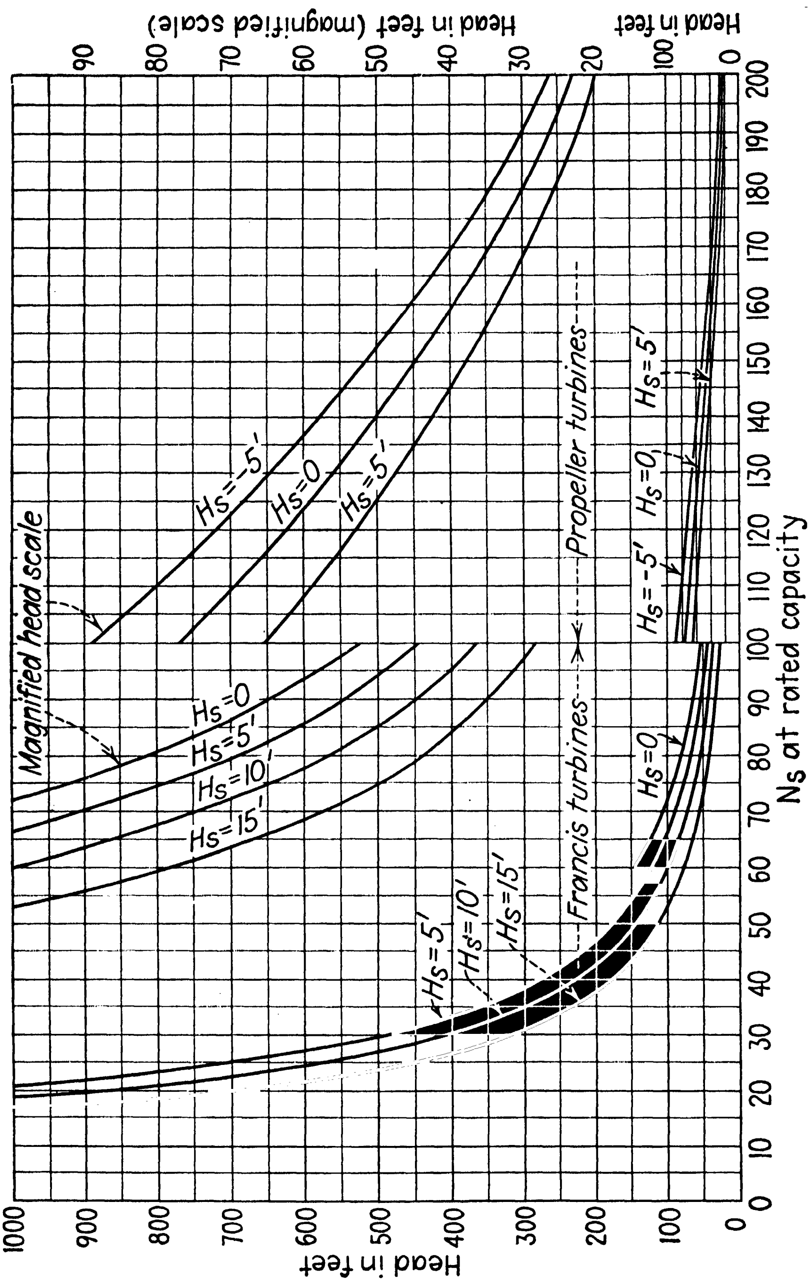


FIG. 32a.—Recommended limits of specific speed for various effective heads and different values of draft head.

have shown no serious pitting as well as points based on critical sigmas from cavitation tests (allowing a minimum margin), and although necessarily an approximation owing to the character of the data, it offers a fair index of the present limits of safe practice for turbines of normal design. The values for propeller turbines correspond to the fixed-blade type; for the Kaplan type (adjustable runner blades), it is recommended that plant sigmas be taken about 10 per cent greater than the values plotted. The same curve is plotted in logarithmic coordinates in Fig. 31*b*. For Francis turbines, the sigma vs. N_s curve agrees with the empirical approximation $\sigma = 0.625(N_s/100)^2$, in which σ is the recommended safe minimum plant sigma and N_s the specific speed at rated capacity. For propeller turbines, the plotted values agree within a fair degree of approximation with the empirical relation $\sigma = 0.28 + (1/7.5)(N_s/100)$.[3] With the N_s relation, the Thoma formula, and such a curve of cavitation factors, the suitable type and speed of turbine can be selected for any installation.

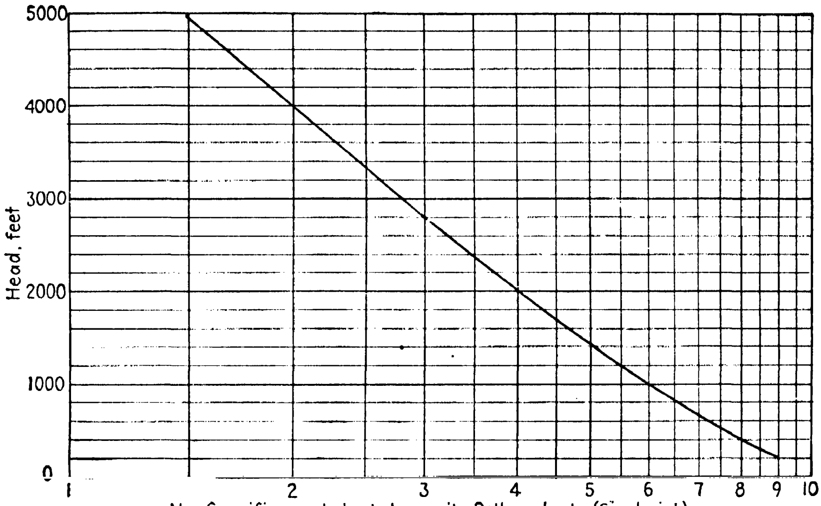


FIG. 32*b*.—Limiting heads for various specific speeds of Pelton wheels, based on usual practice.

Other factors, such as form of efficiency curve, may also influence the selection, but in no case may a cavitation factor be used that is unsafe; *i.e.*, the specific speed and draft head must be in proper relation to the effective head on the plant, if satisfactory performance and life of the vital parts are to be assured.

Since the designer of a hydroelectric plant usually wishes to avoid undue depth of excavation for the powerhouse foundation and also usually wishes to place the runner above high tail water for easy accessibility, a fairly large value of H_s, say from 5 to 15 ft, is usually desired. On the other hand, the engineer of the turbine manufacturer is faced with the vital necessity of limiting H_s in accordance with a safe value of sigma and at the same time providing as high a specific speed as is feasible, in the interest of economy in cost of turbine, generator, and powerhouse. The economy secured by increase of specific speed is so substantial and indeed necessary, in the case of low-head plants where the machinery is relatively large, that it becomes advantageous to adopt high specific-speed turbines, usually of the propeller type, and to secure the necessary plant sigma in many cases by disregarding the question of easy access to the runner and by setting the runner below the tail-water elevation, thus using a negative value of H_s.

An initial solution of the problem of selecting type, N_s and N, can be obtained from the chart (Fig. 32a). Here the definite limits of head vs. N_s are shown for several values of H_s, calculated from the values of plant sigma in Fig. 31.

Each point on the chart has been computed from the Thoma formula giving $H = \frac{H_b - H_s}{\sigma}$. The curves in the upper part of the chart duplicate those at the bottom but to a magnified scale of heads. For Francis turbines, H_s is to be measured to the throat or point of smallest diameter of the water passage, or to the lowest point of the runner vanes if higher than the throat; for propeller turbines, H_s is measured to the mid-height of the runner blades at the outer tips or runner periphery, since this basis was used in constructing the curve of plant sigmas (Fig. 31). The curves of the chart represent a surface enveloping points corresponding to approved practice and giving the highest heads to which various specific speeds have been satisfactorily applied with various draft heads. Naturally a conservative designer will not desire

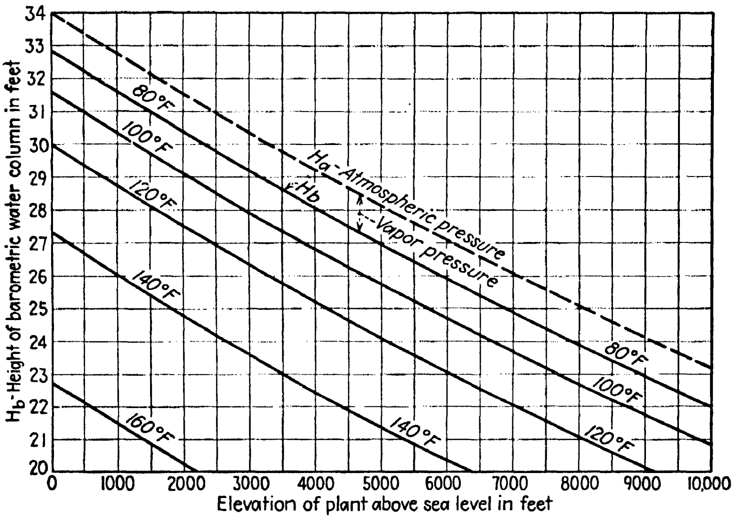


FIG. 33.—Height of barometric water column H_b corresponding to atmospheric pressure at various elevations above sea level and various water temperatures.

to adopt a value at the limit of existing practice, but will allow some margin, unless fortified by service records or cavitation tests on a turbine homologous to the particular design contemplated. In the case of horizontal-shaft units (now rarely used except for special purposes such as driving horizontal-shaft pulp grinders), H_s should be measured to the upper end of the outlet diameter and not to the shaft center line, since the point of minimum pressure will be at the upper side of the water passage.

The chart is based on a value of H_b of 32.8 ft, corresponding to plants located at or near sea level and with water temperatures not exceeding about 80F. For plants subject to higher water temperatures, the value of H_b must be reduced to allow for the increased vapor pressure, the correction being computed from steam tables and the correct H_b entered in the Thoma formula. For plants to be erected at higher altitudes as in mountainous regions, H_b must be reduced to allow for the corresponding decrease in barometric pressure H_a; this correction may be obtained from Fig. 33.

For Francis turbines, the lines of the chart correspond to the empirical equation for recommended plant sigmas, from which

$$H = \frac{H_b - H_s}{0.625\left(\frac{N_s}{100}\right)^2} \qquad \text{or} \qquad N_s = 100\sqrt{\frac{H_b - H_s}{0.625H}}.$$

Based on the empirical approximation for sigma previously mentioned, the corresponding relations for propeller turbines are

$$H = \frac{H_b - H_s}{0.28 + \frac{1}{7.5}\left(\frac{N_s}{100}\right)^3} \qquad \text{or} \qquad N_s = 195\left(\frac{H_b - H_s}{H} - 0.28\right)^{1/3}$$

but the values given in the chart are preferable and give a direct solution.

Since the application of propeller turbines and particularly of Kaplan turbines is being continually extended and cavitation research is being carried forward, it is to be expected that accumulating experience will progressively modify the practice in this field. The chart values merely represent what the author regards as conservative practice in the light of present knowledge. As already pointed out, individual designs of propeller turbines may vary considerably from any generalized rule for cavitation factors. The only safe practice in the field of propeller turbines is to carry out cavitation tests on a model of any proposed installation, covering speeds corresponding to the full range of heads under which the turbine must operate.

Of course the actual N used for a specific installation in a hydroelectric plant must be a synchronous speed for the generator. For 60-cycle frequency, which is now standard, $N = (7{,}200/n)$, in which n is the number of poles in the generator, which must be an even integer. For 25 cycles, still used in some regions in North America, $N = (3{,}000/n)$; for 50 cycles, used in Europe and South America, $N = (6{,}000/n)$.

To illustrate the use of these principles, let us apply them to a typical problem and select the type and speed of a 10,000-hp turbine for an effective head of 100 ft to suit a 60-cycle generator. It is desired to place the runner 10 ft above tail water and to provide for a possible water temperature of 80F. If the plant is near sea level, the required cavitation factor is

$$\sigma = \frac{H_b - H_s}{100} = \frac{32.8 - 10}{100} = 0.228$$

From the plant sigma vs. N_s chart, the permissible specific speed is 60. This result could have been read directly from the N_s vs. H chart. This corresponds to a Francis turbine of good attainable efficiency, as seen from the N_s vs. efficiency curve. The permissible speed of the unit is

$$N = N_s\frac{H^{5/4}}{\sqrt{HP}} = 60\frac{(100)^{5/4}}{\sqrt{10{,}000}} = 60\,\frac{316}{100} = 189.7 \text{ rpm}$$

The nearest synchronous speed that does not exceed this is

$$N = \frac{7{,}200}{n} = \frac{7{,}200}{38 \text{ poles}} = \underline{\underline{189.5}} \text{ rpm}$$

which is satisfactory. If the plant is to be located at 1,000 ft above sea level, then from the chart for barometric head, $H_b = 31.6$ ft and $\sigma = \frac{31.6 - 10}{100} = 0.216$. The permissible specific speed is now 59, from the plant sigma vs. N_s chart. Or, this value could be read from the H vs. N_s chart by using a virtual

$$H_s = 10 + 32.8 - 31.6 = 11.2 \text{ ft}$$

For this case, the permissible speed is $N = 59(^{316}/_{100}) = 186.5$ rpm. It will

now be necessary to go to the next lower synchronous speed and increase the number of poles to 40, the result being $N = \frac{7,200}{40} = 180$ rpm.

In the field of impulse turbines, no analysis of specific speed limitations has yet been made on the basis of cavitation theory, nor have cavitation tests been carried out in the laboratory, so far as the author is aware. Data based on field experience are complicated by considerations of the abrasion or grooving of nozzle tips, needles, and buckets of Pelton wheels under high heads, an action that is dependent on the particular metals employed and the amount of sand or silt carried by the water. In addition to this source of wear, and perhaps tending to initiate or to aggravate it, is the possibility of pitting due to cavitation, particularly at the needle tip, the discharge edge of the nozzle, the rear surface of the buckets near the cutting edge, and even in the bucket bowls. If sufficient field experience data or cavitation tests in the laboratory were available, it would be possible to investigate the safe limits of sigma for Pelton wheels on the basis of the Thoma formula. For the usual case of discharge against the atmosphere and the absence of a draft tube, H_s in the Thoma formula becomes zero and the formula becomes simply $\sigma = H_b/H$. Experience has shown that, as in other turbines, low specific speeds must be used for very high heads and the use of high specific speeds must be limited to low heads. This is consistent with a σ vs. N_s curve of a form similar to the curves for Francis and propeller turbines.

In Fig. 32*a*, the scale does not permit the extension of the curves into the region of Pelton wheels. This extension is shown in Fig. 32*b*, based on a curve showing generally used specific speeds at various heads, constructed by the S. Morgan Smith Co.[1] This may be used as a guide reasonably representing safe practice based on experience.

CHARACTERISTIC PROPORTIONS OF TURBINE RUNNERS

Figure 34 shows the typical forms of runner corresponding to various specific speeds. The three runners in Fig. 34 are plotted to the same scale, the dimensions being consistent with a constant *HP* output under a constant head, so that the figures show the great change in dimensions of a turbine of a given capacity involved in a change of specific speed. Since under a constant head the actual rpm is directly proportional to the specific speed, the values of N_s shown are a direct indication of the comparative speeds at which the various types would operate at a given power and head. Evidently, for example, a high-speed Francis turbine substituted for a propeller turbine would require a much heavier runner and larger turbine and would run only about one-half as fast, a generator of nearly twice the diameter of that needed for the propeller turbine being required. If a Pelton wheel were substituted for the propeller turbine, it would require an enormous increase in dimensions and would run at a speed of the order of about one-fiftieth of that of the propeller unit.

The wide range of specific speeds available is obtained by altering the design proportions. Thus for a very low specific-speed Francis turbine, a large entrance diameter D_1 is used (see Fig. 34) and a small throat diameter D, the breadth B of the distributor and guide vanes being small. The disk friction on the runner hub and shroud ring and the leakage loss through the runner clearances are high, and the maximum efficiency is somewhat less than for a more normal specific speed; but the efficiency remains high for part loads and overloads, and the efficiency curve is flat and of favorable form for operation under widely varying load demands.

Increase in specific speed requires the reduction of D_1 to suit a higher rpm without going to a peripheral velocity u_1 out of proportion to the water velocities corresponding to the head. To accommodate the required quantity of flow, B must be increased

[1] Impulse Turbines, *Bull.* 138, 1938.

to compensate for the reduced diameter and thus to maintain a proper entrance area. For a high-speed Francis turbine, D_1 becomes smaller than D, the vanes are large and are usually given a complex spoon formation, and the high relative velocity of the vanes through the water involves high surface friction. The restriction in D to suit the high rpm involves a high value of the kinetic energy discharged from the runner, and the performance becomes greatly dependent on the ability of the draft tube to regain as much as possible of this energy. Still higher specific speeds have been made possible by the development of the propeller turbine in which the shroud-ring friction is eliminated; the vanes become nearly flat blades inclined at a small angle to the tangential direction and have a high relative velocity through the water. These blades are few in number, usually four to six, and of reduced length in the direction of flow, so that they normally overlap each other but little or not at all, their surface friction thus being kept from reaching inordinate amounts.

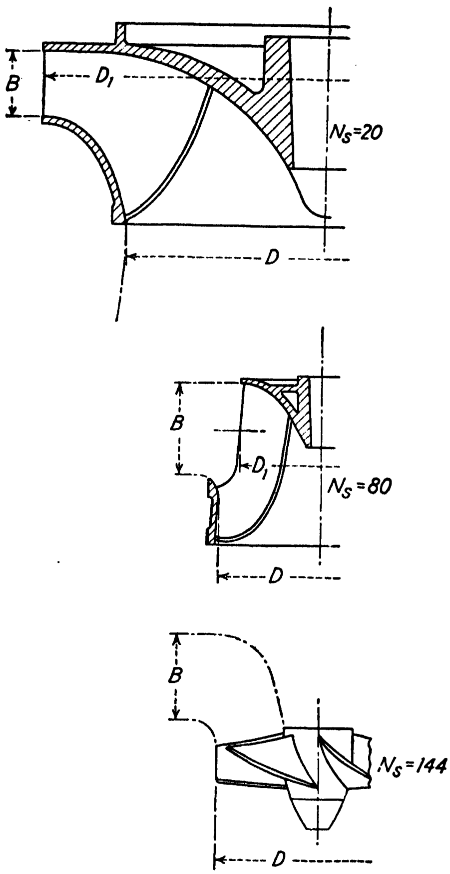


FIG. 34.

At the lower end of the specific-speed range, reduction below the speeds of low-speed Francis turbines is effected by going to partial admission; *i.e.*, instead of admitting water all the way around the runner periphery, it is admitted at only one or two points; and to reduce the friction and turbulence of the entering flow, this admission takes place through circular contracting nozzles equipped with adjustable needles for regulation, forming the flow into undisturbed jets with little loss of head. The most important and valuable feature of the Pelton wheel is the use of free jets, surrounded by air, and all the space in the runner not occupied by the flow is filled with air, and through the runner the flow has a free surface as in open channels. The pressure through the runner space, being atmospheric, can never be appreciably reduced at any point to any material extent below the atmospheric value, and the possibility of cavitation is greatly reduced. This adapts the Pelton wheel to use under higher heads which would be prohibitive for Francis turbines owing to cavitation. The runner vanes become open buckets of double-discharge type, as shown in the figures.

Although the proportions of turbines may be varied somewhat according to the choice of the designer, the general characteristics for any given specific speed are fairly determinate in ordinary practice, and the following curve sheet (Fig. 35) gives values for Francis turbines consistent with normal design and high efficiencies. The design calculations are carried out for the point of best efficiency, the velocity triangles being

drawn, the vane angles fixed to suit the relative velocity at the runner entrance, and the areas A_1 and A_2 of the discharge orifices of the guide vanes and runner vanes calculated so that at each point the velocity multiplied by the area and a coefficient of discharge is equal to Q, the discharge.

The coefficient of discharge of the guide vanes is nearly unity, and that of the runner outlet is usually of the order of, say, 0.85 to 0.90 but is dependent on the characteristics of the design. The most important magnitude controlling the discharge and therefore the power capacity is the outflow area of the runner, which is carefully checked by template and calipers during manufacture to ensure its conformity to the

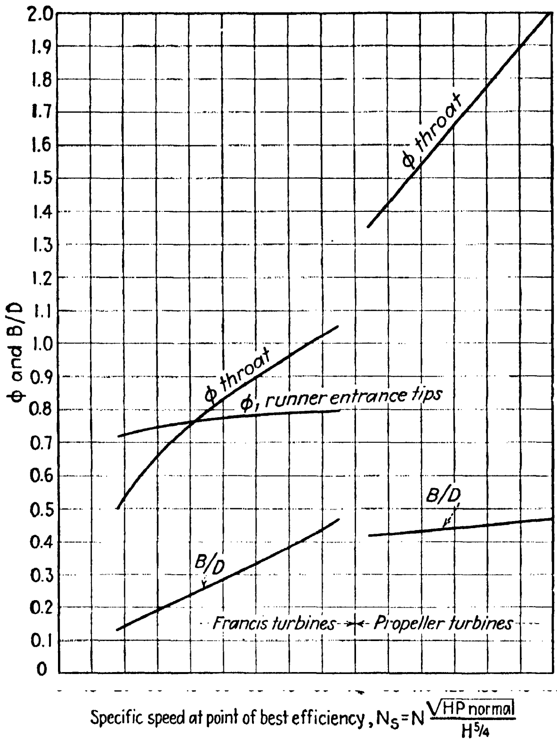


FIG. 35.—Design factors for selecting runner proportions of reaction turbines of various specific speeds.

design calculations and to the value corresponding to the test model. Space does not permit the covering of the extensive subject of turbine design, but it is hoped that the general principles and guides to proportioning will prove helpful to designers as well as users of turbines.

In the curves and tables, $\phi = \pi DN/60\sqrt{2gH}$; $\phi_1 = \pi D_1 N/60\sqrt{2gH}$; B, D, and D_1 being expressed in feet. From the curves of ϕ, ϕ_1, and B/D, these dimensions can be computed.

The percentage of rated load at which the point of best efficiency is taken will depend largely on the requirements of the installation but is limited by the attainable turbine characteristics; in the absence of definite requirements, average figures for usual conditions may be taken within the approximate limits given in the following table, which also gives the corresponding ratios of N_s at best efficiency to the N_s at

rated capacity. Since at given head and speed N_s varies directly as the square root of the horsepower,

$$\frac{N_s \text{ at best efficiency}}{N_s \text{ at rated capacity}} = \sqrt{\frac{HP \text{ at best efficiency}}{HP \text{ at rated capacity}}}$$

Type of turbine	Pelton	Francis						Propeller
N_s at rated capacity	2–6	15	25	35	50	75	90–100	100–180
Ratio $\frac{HP \text{ at best eff.}}{HP \text{ at rated cap.}}$	0.75–0.85	0.75[1]–0.85	0.80–0.875	0.84–0.90	0.90–0.95	0.98–1.00	1.00	1.00
Ratio $\frac{N_s \text{ at best eff.}}{N_s \text{ at rated cap.}}$	0.865–0.92	0.865[1]–0.92	0.895–0.935	0.915–0.95	0.95–0.975	0.99–1.00	1.00	1.00

[1] Upper figures, symmetrical draft tubes; lower figures, elbow draft tubes.

Kaplan turbines permit a wide degree of flexibility in this respect, and the values adopted will be governed largely by the plant conditions, with a usual range of the same order as in Pelton wheels. The turbine builders usually allow about 5 or 6 per cent margin in power beyond the guaranteed rated capacity to provide a range for governing and a margin for variations of actual from computed performance.

PELTON WHEELS

In considering the design proportions of Pelton wheels, space does not permit the inclusion of the complex subject of bucket design, and we shall limit ourselves to the determination of the leading dimensions of the wheel or runner and the jet. The most important dimensions are the pitch diameter D of the runner and the diameter d of the jet after it has reached its vena contracta, after which its diameter remains practically uniform until it enters the buckets. Both diameters are expressed here in feet. The diameter ratio (D/d) is an important factor and is the principal one in fixing the specific speed. We shall consider the most common type of Pelton wheel, that with a single jet, and shall consider a single runner. The specific speed is defined for this case. Naturally the effective specific speed of a unit having two single-nozzle runners or a single runner with two nozzles will be 1.41, or $\sqrt{2}$ times the specific speed of the single-nozzle runner, for the specific speed is proportional to the square root of the horsepower. The pitch diameter of the runner is defined as the diameter of a circle tangent to the center line of the nozzle and jet; and the characteristic ϕ is the ratio of the velocity of the runner at the pitch diameter to $\sqrt{2gH}$, H being the effective head.

Before proceeding with the discussion of the Pelton wheel, the effective head should be defined. The following definition is quoted from the A.S.M.E. Test Code for Hydraulic Prime Movers:

" . . . the effective head shall be taken as the elevation corresponding to the pressure head at entrance to the nozzle (or the average of the pressure heads for two or more nozzles) plus the velocity head at this point (or points), minus the elevation of the lowest point of the pitch circle of the runner buckets. The pitch circle is that circle which is tangent to the axis of the power jet."

This is the effective head of the machine and is the amount properly chargeable against it; if the tail-water level were brought up to the point where it would just clear the buckets, then, by disregarding the negligible depth of buckets beyond the pitch circle, it would be the available head at the powerhouse. Actually, however,

the normal tail-water level must be some distance below the wheel to avoid submerging the runner buckets when the tail-water level is subject to variations and to avoid submerging the bottom of the generator in a horizontal shaft unit. Hence there is a free drop between the runner and tail water which is entirely lost, unless the discharge passage is run filled with water and operated as a draft tube, the wheel housing made airtight, and the wheel run in a partial vacuum. Such draft-tube action is seldom used with Pelton wheels, for it requires float-controlled air admission to regulate the free water surface, stuffing boxes where the shaft enters the housing, and other complications, and in high-head plants the amount of head saved is inconsiderable. Thus if the elevation of the jet above tail water is 10 ft and the effective head 3,000 ft, the saving would be only ⅓ of 1 per cent. However for lower head installations, the provision of draft-tube action would be well worth considering in some cases. Experiments have shown no gain from it due to the reduction of windage loss of the runner when rotating in a partial vacuum. The conditions here considered will assume that atmospheric pressure exists within the wheel housing.

It should be pointed out that in comparing the installation efficiencies obtainable from Pelton and Francis turbines the Pelton efficiencies should be discounted to allow for this free fall, in order to obtain comparable values, for the Francis turbine is charged with the head down to tail water in computing its efficiency as already defined.

The water discharges from the nozzle under the effective head H, and the discharge is $Q = C_v \sqrt{2gH} \frac{\pi}{4} d^2$, C_v being the coefficient of velocity of the jet and d the diameter of the jet (vena contracta) and not that of the nozzle orifice. The horsepower is $HP = wQHe/550$, and the linear velocity of the runner is $\pi DN/60 = \phi \sqrt{2gH}$, so that $D = 60\phi \sqrt{2gH}/\pi N$. Expressing d in terms of (d/D), we have

$$d = \left(\frac{d}{D}\right) \frac{60\phi \sqrt{2gH}}{\pi N}. \tag{17}$$

We then have

$$HP = \frac{wHeC_v}{550} \sqrt{2gH} \frac{\pi}{4} \left(\frac{d}{D}\right)^2 \frac{(60)^2\phi^2 2gH}{\pi^2 N^2} \tag{18}$$

and collecting the constant terms

$$HP = \frac{(60)^2 w (2g)^{3/2}}{550 \times 4\pi} eC_v\phi^2 \left(\frac{d}{D}\right)^2 \frac{H^{5/2}}{N^2} \tag{19}$$

Hence for geometrically similar units operating at correct speeds, *i.e.*, at constant ϕ, $HP = (\text{const.})H^{5/2}/N^2$, or $N = (\text{const.})H^{5/4}/\sqrt{HP}$. This is merely the specific-speed relation already derived from general considerations, or $N = N_s\, H^{5/4}/\sqrt{HP}$. The specific speed is then

$$N_s = N \frac{\sqrt{HP}}{H^{5/4}} = 30 \sqrt{\frac{w}{550\pi}}\, (2g)^{3/4} \frac{\sqrt{eC_v}\phi}{\left(\frac{D}{d}\right)}$$

and putting $w = 62.4$ and $2g = 64.3$,

$$N_s = 129.3 \sqrt{eC_v} \frac{\phi}{\left(\frac{D}{d}\right)} \tag{20}$$

It will be noted that homologous wheels of any size under any head, when running at their proper corresponding speeds and disregarding slight changes in relative friction, will have the same efficiency, C_v, ϕ, and D/d.

Hence this analysis shows that the factor N_s as just derived is a quantity that remains constant for a given design regardless of size or head. This fact specifically verifies the applicability of the specific-speed principle to impulse wheels, a conclusion

that could have been foreseen since the original derivation was general. The last expression is, however, useful as showing the influence of the various factors on the value of N_s.

The value of C_v is subject to little variation in well-designed needle nozzles and can be taken as 0.97 to 0.99 as a good approximation. The following table gives typical values of ϕ for average practice:

N_s at best eff.	2	3	4	5	6	7
ϕ	0.47	0.46	0.45	0.44	0.433	0.425

With the aid of this table and the preceding formulas, the approximate leading dimensions D and d may be readily computed.

It will be noted that ϕ varies slowly, and since the efficiency does not vary greatly for moderate values of specific speed, it is seen that the specific speed is mainly dependent on the diameter ratio D/d. A convenient approximation can be obtained, for normal values of N_s, say 2 to 5, by using an average efficiency of 0.85 and an average ϕ of 0.46, which give approximately

$$N_s = \frac{54}{\left(\frac{D}{d}\right)}$$

A normal diameter ratio, favorable to highest efficiency, is D/d = about 18. This, it will be noted, corresponds to about $N_s = 3$. High efficiency is obtainable from D/d = about 27, corresponding to N_s = about 2; to (D/d) = about 12, corresponding to N_s = about 4.5. Higher specific speeds involve a sacrifice of efficiency to an increasing extent.

With normal and low specific speeds, the buckets can be cast separately and attached by lugs and bolts to the wheel hub (Fig. 3) or to a demountable rim, for easy replacement in the event of wear, which suits the wheels to high-head operation. With specific speeds higher than normal, the buckets are large and closely spaced and must be cast integrally with the hubs (Fig. 2*d*).

SCALE EFFECT

By scale effect is here meant the effect of size of turbine on its efficiency, in a series of homologous turbines.

In the earlier section on dynamic similarity, it was noted that if the water passages in a turbine are considered to fall in the class of *rough conduits*, as termed by Karman and Prandtl, with high Reynolds numbers, the flow may be considered to be completely turbulent and the viscous forces negligible, so that the surface friction losses in a given turbine may be expected to vary directly with the velocity heads and, therefore, with the effective head on the turbine. Hence a given turbine should have a substantially constant efficiency when operated at its proper speed under various heads. It was also noted however that so-called *homologous* turbines of different actual dimensions, geometrically similar in design and proportions, are not completely homologous with respect to the surface roughness. The variation in degree of relative roughness with change of size of turbine will cause a small, but appreciable, variation in the proportion of the effective head lost in hydraulic friction; therefore the efficiency will change somewhat with change of size.

Of course the hydraulic friction is not the entire source of loss in a turbine. There is also mechanical bearing friction, which, however, is very small in units of usual size.

although appreciable in a laboratory model; and there are eddy or velocity head losses due to the final kinetic energy loss at the exit from the draft tube and at points of sudden enlargement in the turbine, as at the outlet from the guide vanes and runner where the vane thicknesses cause small eddy spaces at their ends. Such enlargement and outlet losses will tend to bear a fixed ratio to the velocity heads and therefore to the head on the turbine and consequently represent an unvarying element of proportional loss, not causing any change in efficiency.

In undertaking the formulation of the effect of change of size on efficiency, a sense of proportion makes it clear that any elaborate formulation would be uncalled for. In efficiency tests of turbines, there is a wide range of possible error both in the field and laboratory, due for example to the difficulty in measuring the water quantity; and field measurements of efficiency are subject to considerable possible errors of the order of a per cent or more even in well-conducted tests. In comparing the efficiencies of an installed unit and model, or even of two installed units of different size, there is rarely exact or complete geometrical similarity. Moreover, the proposed formula will be applied to model tests in different laboratories, and each laboratory will have a slightly different scale of efficiencies. Since the complexity of the factors involved in the problem makes a completely rational solution impractical, and the solution must be in part empirical, the degree of approximation involved in the data which must be relied upon will justify only a simple formulation of the desired relation.

As a first simplification, we shall not attempt to separate consideration of the various sources of loss, but shall treat all the losses in bulk and assume them to vary in the same manner, although not to the same degree, as the hydraulic friction, which will be considered to be of the same nature as pipe friction. Calling h_L the total loss of head in the turbine, and putting the efficiency,

$$e = \frac{H - h_L}{H} = 1 - \frac{h_L}{H}$$

we can write

$$1 - e = \frac{h_L}{H};$$

and using the Darcy formula for pipe friction:

$$1 - e = \frac{f \frac{L}{d} \frac{V^2}{2g}}{H}$$

in which L is an equivalent length of water passage, d the effective diameter of an equivalent pipe, and f a variable coefficient which is a function of the Reynolds number and of the relative roughness of the surfaces. As noted above, we can reasonably discard the dependence on Reynolds number. As a simple functional relationship we can use the well-established exponential relation between f and d, which has been found practically satisfactory for pipes over a wide range of sizes, namely $f = \frac{\text{const.}}{d^n}$, giving the exponential formula $h_L = \text{const.} \frac{L}{d^{1+n}} \frac{V^2}{2g}$ for a given material or constant absolute roughness.

The value of $(1 + n)$ for pipes has been evaluated by many authorities, with values ranging from 1.25 to 1.33 and even to 1.4 (Forchheimer).[1]

[1] MOODY, L. F., The Propeller Type Turbine, *Trans. A. S. C. E.*, **89**, 625–647, 1926. FORCHHEIMER, P., "Hydraulik," 3d ed., B. G. Teubner, Leipzig, 1930. STRICKLER, A., Beitrage zur Frage der Geschwindigkeitsformel, etc., *Schweitzerische Bauzeitung*, June 7, 1924, p. 265.

STAUFFER, F., Einflüsse auf den Wirkungsgrad von Wasserturbinen, *Z. Ver. deut. Ing.*, **69**, 415–17, 1925. Discussion by L. F. Moody in *Mech. Eng.*, November, 1935, pp. 730–731.

Considering the effect of the nonfrictional losses, the value $(1 + n) = 1.25$ was proposed as a good probable approximation, or $n = \frac{1}{4}$. We then have

$$1 - e = \frac{\text{const.}}{d^n} \frac{L}{d} \frac{V^2}{2gH} \tag{21}$$

In two homologous turbines operating under dynamically similar conditions, distinguishing the values for one of them by the subscript $_1$,

$$\frac{1 - e_1}{1 - e} = \frac{\dfrac{\text{const.}}{d_1^n} \dfrac{L_1}{d_1} \dfrac{V_1^2}{2gH_1}}{\dfrac{\text{const.}}{d^n} \dfrac{L}{d} \dfrac{V^2}{2gH}}$$

From their geometrical similarity $L_1/d_1 = L/d$; and for dynamic similarity with the turbines operating at the proper corresponding speeds $V_1^2/2gH_1 = V^2/2gH$, so that

$$\frac{1 - e_1}{1 - e} = \left(\frac{d}{d_1}\right)^n \tag{22}$$

Here d and d_1 are the equivalent diameters representing the water passages. The ratio d/d_1 is merely a ratio of two corresponding linear dimensions and can be represented by any convenient characteristic lengths, such as the throat diameters.

From the comparison of field tests of large units with model tests in one of the manufacturer's laboratories[1] and by solving in each case for n, the average value of n was found to be $\frac{1}{4}$. The efficiency e of a model having been found, that of a large unit can be readily computed by this formula, which has been found to agree well with the efficiency data of a large number of tests. Certain laboratories, probably having a somewhat higher scale of results, find that it tends to give expected field efficiencies that are slightly too high. Recent data tend to indicate an exponent of $\frac{1}{5}$ rather than $\frac{1}{4}$, for general application. The formula is therefore

$$\frac{1 - e_1}{1 - e} = \left(\frac{D}{D_1}\right)^{\frac{1}{5}} \tag{23}$$

A leading French manufacturer applies this step-up ratio with exponent $\frac{1}{4}$ to only $\frac{3}{4}$ of the total loss, with satisfactory results. The formula as given applies only to Francis and propeller turbines; for Pelton wheels having very small friction losses and frequently having more disturbed jets in large units than in models, no increase of efficiency with size can be counted upon. In some cases, field efficiencies are found to be slightly less than the model efficiencies. The formula has been found to be satisfactory for centrifugal pumps in some cases, but in the field of pumps insufficient evidence has been collected to provide sufficient basis for its general adoption until it has been further tested.

Although the efficiency changes perceptibly with change in size of homologous turbines, little change in the proportional horsepower has been detected, in general; therefore the principles of similarity may be applied without correction to the power, the efficiency alone being corrected.

Another form of efficiency step-up formula, based on the Karman-Prandtl friction formula, instead of on the exponential formula, is[2]

$$\frac{1 - e_1}{1 - e} = \left(\frac{\log CD}{\log CD_1}\right) \tag{24}$$

Here common logarithms are to be used, with a value of C, averaged from comparative tests, of 800. The results are, however, almost identical with the previous

[1] I. P. Morris Laboratory, Baldwin Locomotive Works, Eddystone, Pa.
[2] Discussion by L. F. Moody, *Mech. Eng.*, November, 1935, pp. 730–731.

one-quarter-power formula (maximum difference only ¼ of 1 per cent), and except for a more rational basis it has nothing to recommend it over the previous form, which is an entirely satisfactory approximation.

The conclusion from the foregoing discussion is that the proportional loss of head in a turbine of given design, or the percentage of loss $(1 - e)$, varies very nearly in inverse ratio to the one-fifth power of the diameter.

COMBINED OPERATION OF SEVERAL TURBINE UNITS

We have so far discussed only the efficiency of a single turbine. When there are several units in a station, the question arises as to the best distribution of the station load among the units to give the best attainable station efficiency for any value of the station load. The notion was formerly held by some operators that with a number of similar units the load variation should be carried by one unit, while the other units are permitted to run at their points of best efficiency. This assumption will be shown to be in error, and a simple principle will be explained by which the best distribution can be readily prescribed for any load and between like or unlike units.

In stations having a small number of units and subject to considerable variation of load demand, the best solution of the problem of selection of units is frequently the combination of one unit having a flat efficiency curve, such as a Kaplan or moderate

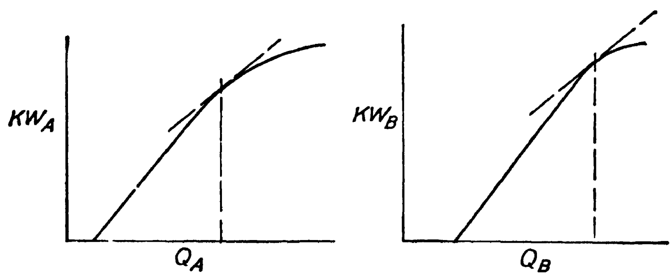


FIG. 36.

specific-speed Francis turbine, with several high specific-speed turbines, the first unit being used to take the greater portion of the load swings. The principle of this section is needed in working out a load schedule for such a plant.[1]

Consider a plant containing two unlike units, which we shall call units A and B. Curves are plotted giving the kilowatt output of the generator of each unit vs. the water quantity used at various gate openings, as shown in Fig. 36.

Suppose the units are running with discharges Q_A and Q_B and generator outputs KW_A and KW_B, respectively.

Let us find whether some other division of a given total quantity of water Q for the station will produce a greater total output KW, and let us determine the best division of load to give a maximum total output for a given total Q. Put the total discharge $Q = Q_A + Q_B$ with Q given and constant and Q_A and Q_B varying, and the total output $KW = KW_A + KW_B$. Using, say, Q_A as the independent variable, we should have $dKW/dQ_A = 0$, for maximum KW; or $(dKW_A/dQ_A) + (dKW_B/dQ_A) = 0$. Now to maintain Q constant, an increment of discharge dQ_A of unit A must be balanced by an equal decrement $-dQ_B$ of the discharge of unit B, or from the first equation above $dQ_A = -dQ_B$. Hence,

$$\frac{dKW_A}{dQ_A} - \frac{dKW_B}{dQ_B} = 0 \qquad \text{or} \qquad \frac{dKW_A}{dQ_A} = \frac{dKW_B}{dQ_B} \tag{25}$$

[1] ROGERS and MOODY, Inter-relation of Operation and Design of Hydraulic Turbines, *Engineers and Engineering*, **42**, 169, July, 1925. The principle was described in an unpublished note by A. Streiff, 1915.

Therefore for best economy the two units must be operated at points of equal slope on their output curves, as indicated in the figure. Since the same relation holds between any two units, as between A and a third unit C, all units in a station should be operated at any time at points of equal slope of their output vs Q curves.

If all the units in a station are exactly similar, having identical curves, they must consequently share the load equally. Of course, as soon as the load drops to the extent of the capacity of one unit, it should for best economy be shut down completely and the load redistributed among the remaining units. It is assumed here that the curves are convex upward, the usual form. If one or both have concave portions, the preceding solution may correspond to a minimum rather than maximum output, and such cases require further consideration, as discussed in the foregoing reference, the distinction between maximum and minimum being determined from the second derivative.

A method used by Prof. P. Danel, of Grenoble, will help to visualize this problem. Suppose curve sheet B is drawn on tracing paper, inverted and placed over curve A with corresponding axes parallel, as in Fig. 37. At point 1 where the curves intersect, unit A is delivering KW_A with discharge Q_A and unit B is delivering KW_B with a discharge Q_B, so that the total output KW is the vertical height of the combined diagram between the two base lines and the total discharge Q is the horizontal width of the combined diagram between the vertical axes. The same relation holds for any other intersection as at 2, and so long as the curves intersect this relation is satisfied.

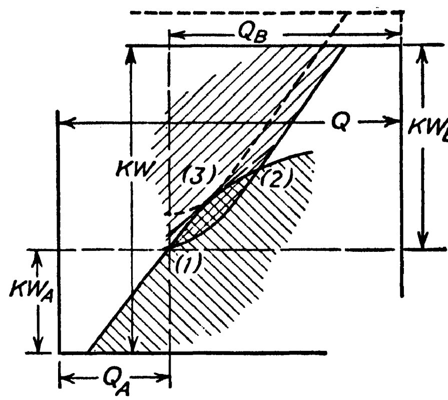


Fig. 37.

To obtain maximum output for a given Q, we can move curve B upward until the two curves are just tangent, as shown dotted. For maximum economy, the units must therefore be operated at point 3, the point of tangency, the gates being adjusted so that the units discharge the corresponding quantities. At this point 3, the curves have a common tangent, so that the units are operating at points of equal slope on their output-Q curves, as previously found. This method can readily be applied to special cases, such as curves with concave portions, and the solution is automatic.

PUMPS

General Classification. Pumps are machines that convert mechanical energy into hydraulic energy, a process which is the reverse of that of prime movers. Considered broadly, pumps comprise classes corresponding to those mentioned under prime movers: gravity lift, such as chain pumps, rarely used; displacement pumps, of the reciprocating type or piston pumps, or rotary as in lobed-wheel pumps and blowers or gear pumps, in which the fluid is moved without change in velocity in the spaces between intermeshing rotors or gear teeth from a point of low pressure to one of high pressure; and pumps that utilize the reversed reaction-turbine process, involving transformations between pressure head and velocity head and conversion of mechanical into hydraulic energy.

Only the last class will be considered here. There is no conventional name for this class as a whole, which includes centrifugal pumps (Figs. 38 to 47), in which the flow through the runner or impeller is radially outward, and axial-flow screw or propeller pumps (Figs. 58 and 59). Mixed-flow or diagonal-flow pumps, intermediate

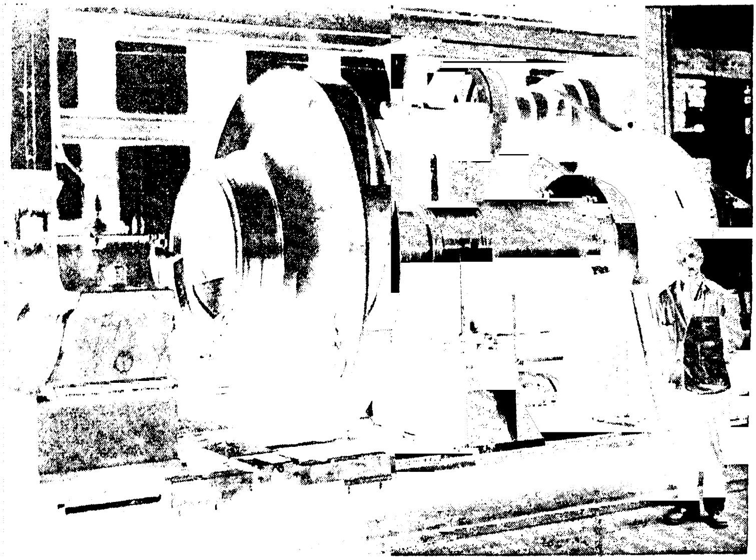


FIG. 38.—Impeller and shaft for 42-in. pumps for Eagle Mountain and Hayfield stations.

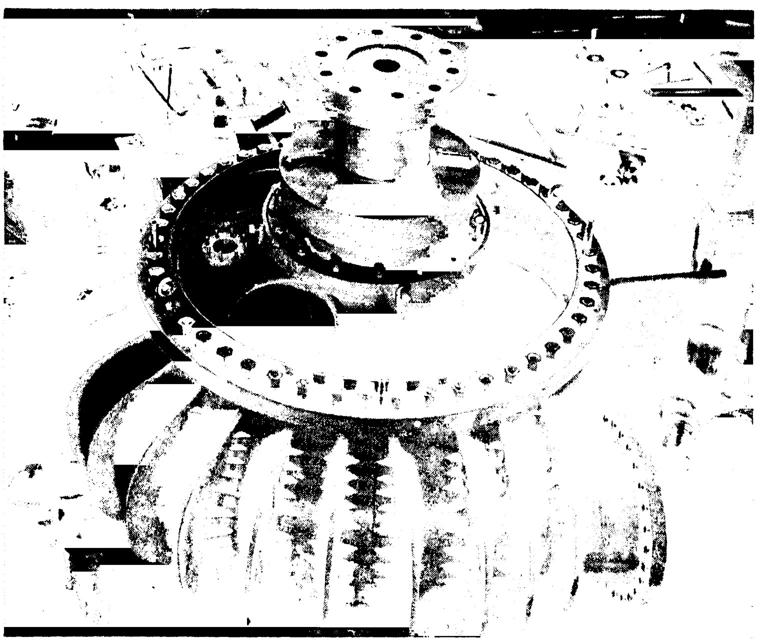


FIG. 39.—Shop assembly of one of six pumps for Eagle Mountain and Hayfield stations, Colorado River Aqueduct, Metropolitan Water District of Southern California. Average operating head: 440 ft at Eagle Mountain, 444 ft at Hayfield. Normal capacity 200 cfs, 12,500 hp per unit, 450 rpm. (*Worthington Pump and Machinery Corp.*)

between these types (Figs. 55 to 57), are included under centrifugal pumps as a subclassification.

Centrifugal pumps thus correspond to Francis turbines with reversed flow direction, and screw or propeller pumps correspond to propeller turbines reversed. It must

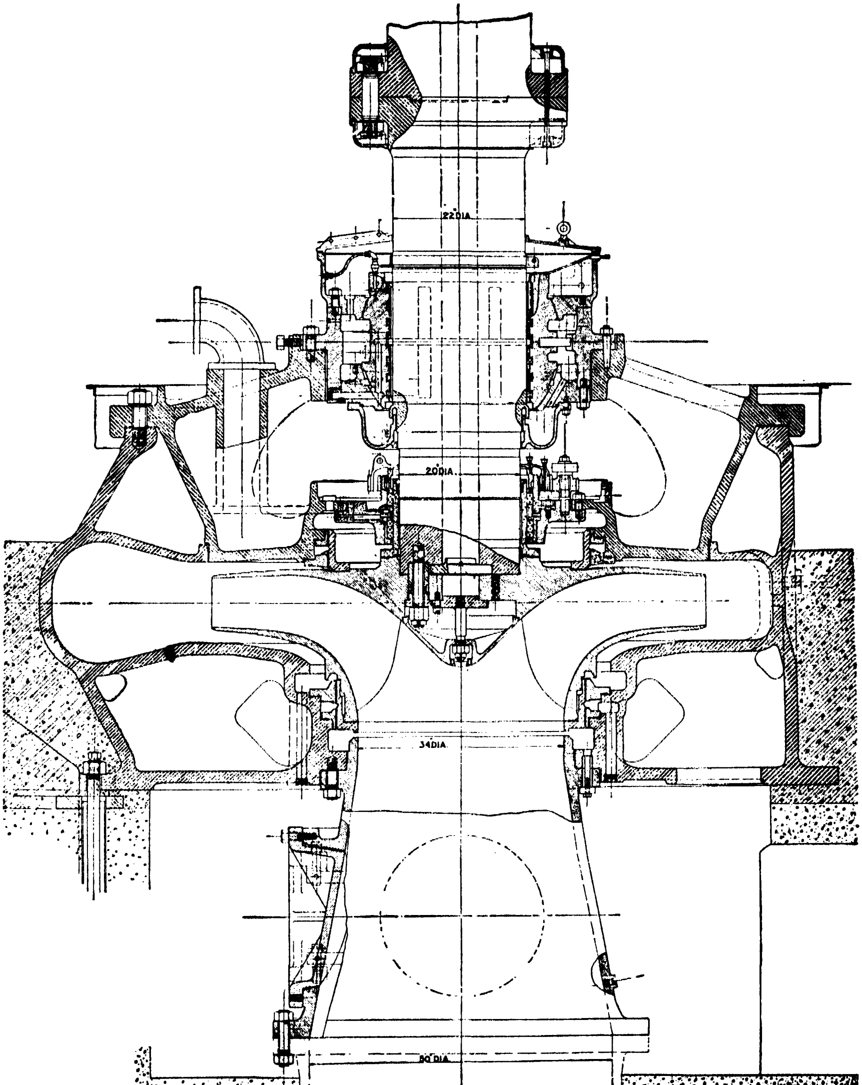


FIG. 40.—Sectional elevation of Eagle Mountain and Hayfield pumps, Colorado River Aqueduct. (*Worthington Pump and Machinery Corp.*)

not be mistakenly inferred that a turbine of normal design can be arbitrarily reversed in direction of rotation and employed as a pump; the changed conditions require fundamental changes in form so that the designs are essentially different. It is possible,

however, to design a reversible machine with special design characteristics so that the same machine will operate as either a pump or turbine (the reversible pump-turbine, a class of hydraulic machine distinct from both the pump and turbine).[1]

The form of centrifugal pump most frequently used is the simple combination of an axial inlet or suction pipe, a radial outward-flow impeller, and a volute or spiral casing. Usually no stationary guide vanes are used either at the inlet or outlet. The impeller transforms mechanical energy into both pressure head and velocity head in the fluid, and the volute casing, often supplemented by a diverging conical discharge pipe, acts as a diffuser which gradually decelerates the flow velocity and converts most of the velocity head into pressure head at the discharge flange of the pump. Some special forms of pump use guide vanes at the entrance (see, for example, the Moody spiral pump[2]); and in some forms of multistage pumps used for high heads and small quantities of fluid and thus involving low specific speeds, the volute casing is replaced by discharge guide vanes or diffusion vanes, which provide decelerating passages between them. Such an arrangement is sometimes rather confusingly called a *turbine pump*, and it evidently corresponds in its relation of elements to a reversed Francis turbine, although as previously noted, the forms of the elements must be quite different from those of a turbine. Diffusion vanes are sometimes used in single-stage pumps (Fig. 42) and particularly in axial-flow propeller pumps of high specific speed (Figs. 58 and 59).

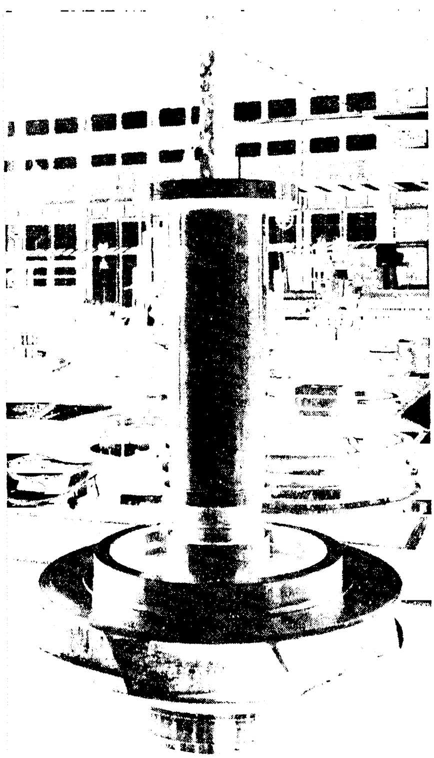


FIG. 41.—Impeller and shaft for one of pumps for Intake station, Colorado River Aqueduct, Metropolitan Water District of Southern California. (*Byron Jackson Co. and Pelton Water Wheel Co.*)

Pumps of small and moderate sizes are commonly arranged with horizontal shafts, and a wide range of sizes may be secured from the pump manufacturers as standardized or stock products with known performance characteristics as determined from shop tests. A common arrangement is with double-suction impellers, equivalent to two single-suction impellers placed back-to-back and made as a single casting; this arrangement gives balanced end thrust on the shaft (Figs. 45 to 47).

For low and moderate heads and large capacities, as in water supply, drainage, and irrigation systems and condenser pumps, high specific speeds are needed, and here mixed-flow and propeller pumps are used often of large size and in vertical-shaft single-suction units. For low-head pumps required to operate under a widely varying head, adjustable-blade propeller pumps have been used to a limited extent. These

[1] MOODY, L. F., U. S. Patents 1,530,569; 1,919,376; 2,010,555.
NAGLER, F., U. S. Patent 1,494,008.

[2] U. S. Patent 1,321,538.

are similar in construction to Kaplan turbines, but without the simultaneously adjustable guide vanes of the turbine.

At the other extreme requiring very low specific speeds, as in high-lift pumps such as mine drainage, deep-well pumps, and boiler-feed pumps, the method of placing a

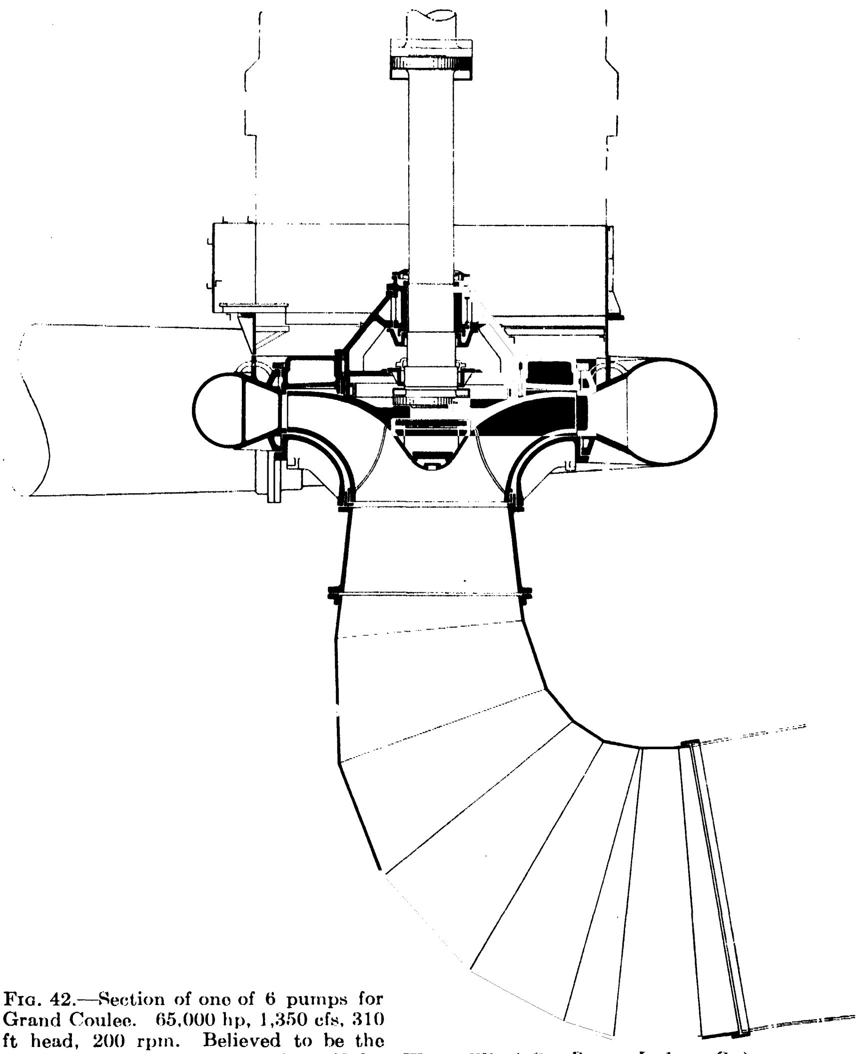


FIG. 42.—Section of one of 6 pumps for Grand Coulee. 65,000 hp, 1,350 cfs, 310 ft head, 200 rpm. Believed to be the most powerful pumps ever built. (*Pelton Water Wheel Co. Byron Jackson Co.*)

number of pumps in series is adopted, as many as 9 or 10 stages being frequently employed. The successive impellers are often placed in a single casing containing the proper water passages and forming a single multistage unit. Figure 63 shows a hot-oil pump of 19 stages for a head of 8,600 ft, believed to be the highest head pump ever built. The Byron Jackson Co. have also built a small mine pump having 54 stages in a single casing.

Head, Power, Efficiency. The head against which a pump operates, called the *total dynamic head*, and here designated by H, is defined by the A.S.M.E. Test Code for Centrifugal and Rotary Pumps (approved 1927) as follows:

"The net total head is the total dynamic head produced by the pump and is represented by the difference between the absolute heads at the discharge and suction nozzles, taking into account any difference between the elevations of these nozzles with respect to a fixed datum and any difference between the velocity heads at the points of measurement."

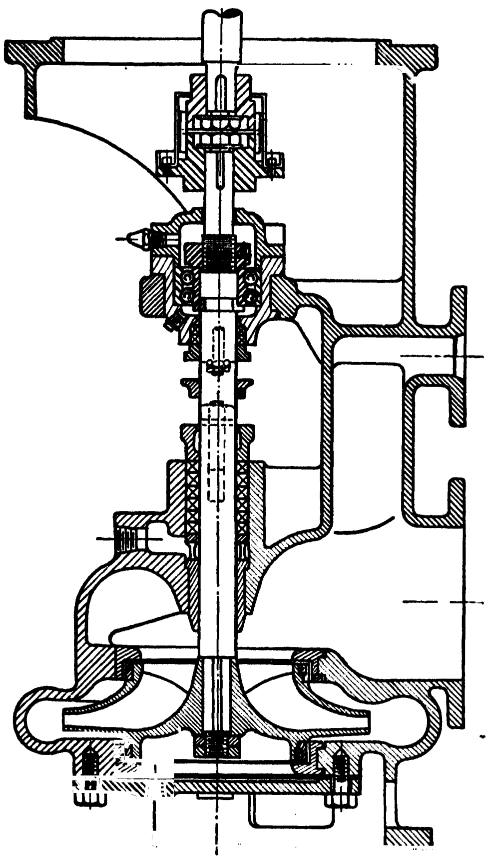


FIG. 43.—Section of top-suction condensate pump. (*Warren Steam Pump Co., Inc.*)

The definition is thus seen to be the same in substance as that of the effective head on a turbine previously given.

As formulated in the Test Code

$$H = \left(H_{pd} + \frac{V_d^2}{2g}\right) - \left(H_{ps} + \frac{V_s^2}{2g}\right)$$

in which H_{pd} is the absolute discharge pressure head in feet, H_{ps} the absolute suction pressure head in feet (both with respect to a fixed datum elevation), and V_d and V_s the discharge and suction velocities in feet per second.

When the suction pressure is measured close to the impeller and the flow direction is not controlled by fixed guide vanes, the centrifugal pressure due to rotating flow may cause serious error in the gage reading. This is particularly pertinent to vertical-shaft pumps in open sumps (Fig. 58) having short inlet passages. In such cases the total dynamic head should be taken as the reading of the discharge gage plus the velocity head at the gage section plus the elevation of the gage zero above the free-water surface in the sump.

The water horsepower is also defined exactly as for turbines, whp $= wQH/550$. The efficiency of a pump is the ratio of the water horsepower, which is the useful work delivered, to the energy supplied to the pump by the shaft of the driving motor, or the shaft horsepower, shp,

$$e = \frac{\text{whp}}{\text{shp}} = \frac{wQH}{550\,\text{shp}} \tag{26}$$

Fundamental Principles. The principles are the same as those explained above in the corresponding section for turbines. When applied to pumps, they take the following form. Referring to Fig. 21*a* the subscript $_1$ will now represent the conditions at entrance to the impeller, and $_2$ those at discharge; and that figure will represent the

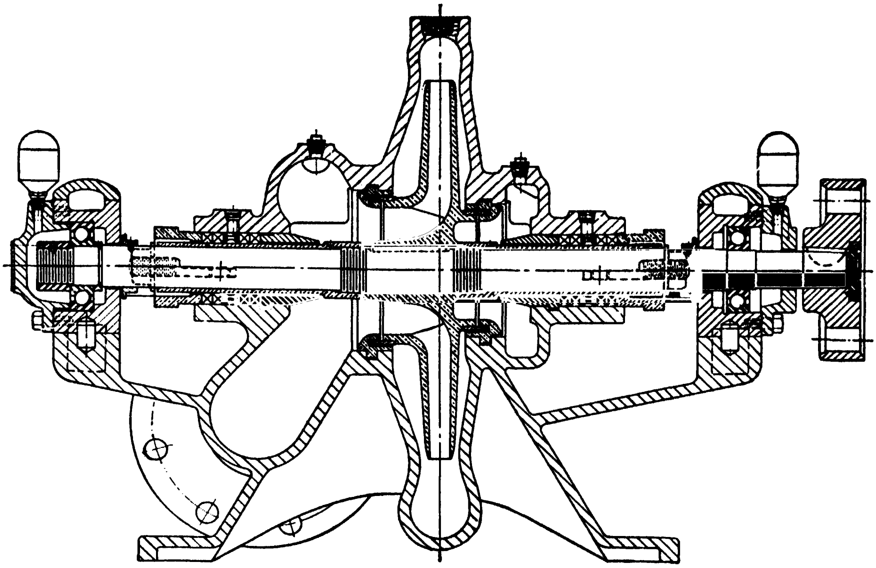


FIG. 44.—Single-suction, horizontal-shaft, low-specific-speed pump. (*De Laval Steam Turbine Co.*)

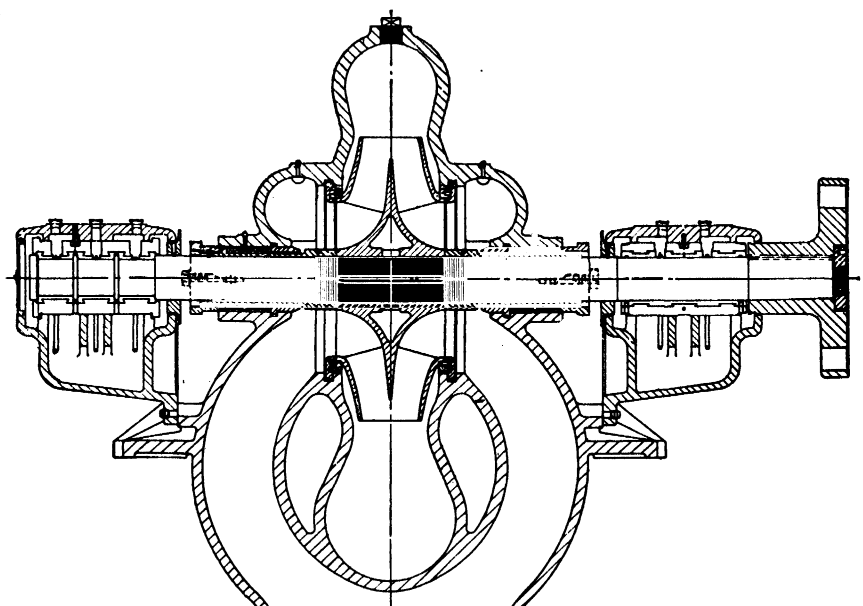


FIG. 45.—Double-suction, horizontal-shaft, split-casing pump. (*De Laval Steam Turbine Co.*)

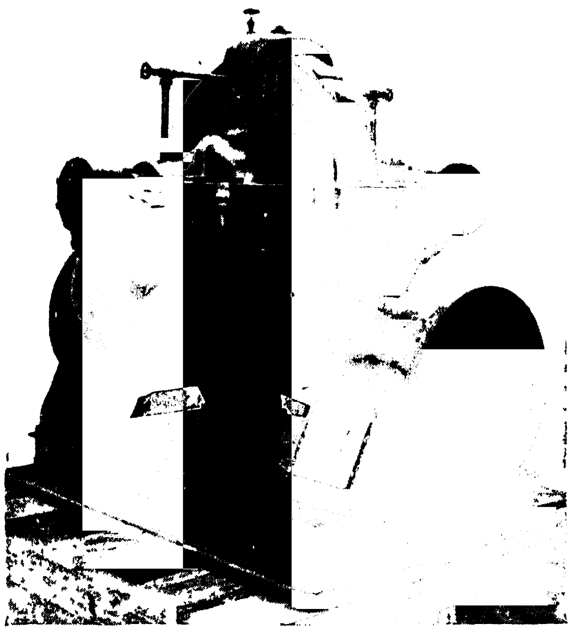


FIG. 46. Double-suction, horizontal-shaft, split-casing pump. (*Ingersoll-Rand Co.*)

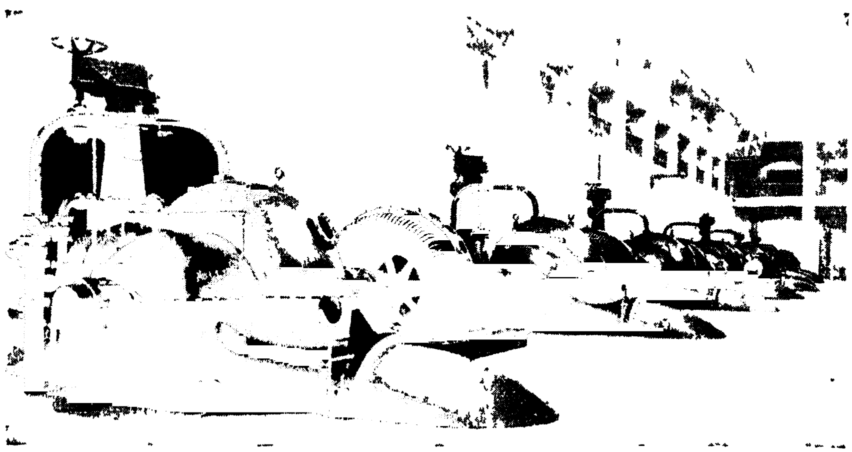


FIG. 47. A modern pumping station. West Side Treatment Works, Sanitary District of Chicago. Five 54- by 48-in. pumps, 2,000 hp each, 360 rpm; and two 42- by 36-in. pumps, 1,000 hp each, 400 rpm. (*Allis-Chalmers Mfg. Co.*)

pump action if the subscripts $_1$ and $_2$ are interchanged and all velocity directions reversed (Fig. 48).

If the same force and torque relations as developed for turbines are applied to pumps, the net turning moment exerted on the impeller is $\frac{wq}{g}(r_2V_{u2} - r_1V_{u1})$, where q is the quantity of water flowing through the impeller. Here Q, the discharge from

the pump, is equal to q minus the leakage loss Q_L through the clearances, and we can put $Q = e_v q$, e_v being the volumetric efficiency, $e_v = Q/(Q + Q_L)$. The turning moment on the impeller multiplied by $2\pi N/(60 \times 550)$ gives the shaft horsepower multiplied by the mechanical efficiency e_m to allow for bearing and stuffing box friction. That is,

$$\frac{2\pi N w Q}{60 \times 550 e_v g}(r_2 V_{u2} - r_1 V_{u1}) = e_m \text{ shp} = e_m \frac{wQH}{550e}$$

Introducing as before $\frac{2\pi N r_2}{60} = u_2$ and $\frac{2\pi N r_1}{60} = u_1$, this relation becomes

$$\frac{u_2 V_{u2} - u_1 V_{u1}}{e_v g} = \frac{e_m H}{e}$$

and if we put $e = e_v e_m e_h$, in which e_h is the hydraulic efficiency, we have the Euler formula for pumps:

$$u_2 V_{u2} - u_1 V_{u1} = \frac{gH}{e_h} \tag{27}$$

This corresponds to Eq. (4) for turbines. For the usual centrifugal pump having no guide vanes at the entrance to the impeller to give initial whirl (Fig. 40) or having guide vanes to prevent initial whirl, $V_{u1} = 0$ and

$$u_2 V_{u2} = \frac{gH}{e_h} \tag{28}$$

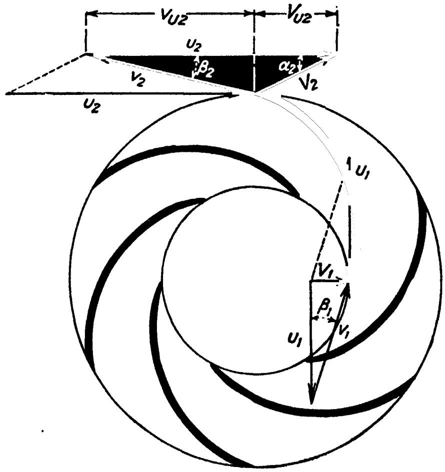


FIG. 48.—Relative and absolute velocities in a pump impeller.

The amount of head corresponding to the energy transmitted from the impeller to the water is $wqH/wqe_h = H/e_h$, and the Bernoulli formula applied to the pump, with reference to the stationary system [corresponding to Eq. (6) for turbines], is

$$h_{p1} + z_1 + \frac{V_1^2}{2g} - h_L + \frac{H}{e_h} = h_{p2} + z_2 + \frac{V_2^2}{2g} \tag{29}$$

Equation (8) applies unchanged to pumps, *viz.*,

$$h_{p1} + z_1 - \frac{u_1^2}{2g} + \frac{v_1^2}{2g} - h_L = h_{p2} + z_2 - \frac{u_2^2}{2g} + \frac{v_2^2}{2g} \tag{8}$$

and subtracting this from Eq. (29), we have

$$\frac{V_2^2 - v_2^2 + u_2^2}{2g} - \frac{V_1^2 - v_1^2 + u_1^2}{2g} = \frac{H}{e_h} \tag{30}$$

which corresponds to Eq. (9) for turbines.

Equations (27) and (28) although forming a sound basis for pump design cannot be applied without taking account of some important distinctions. V_{u2} in Eq. (28) is the actual tangential velocity of flow and cannot be taken from an apparent velocity triangle constructed from the vane angle. If the impeller contained an infinite number of vanes, the relative direction of discharge could be properly taken as the direction of the vanes, and the discharge velocity could be taken as uniform around the circum-

ference. Actually, with a finite number of vanes, the pressure intensity adjacent to the driving face of a vane is necessarily greater than that in the region of the rear surface of the preceding vane; and the velocity heads of the flow on opposite sides of an impeller channel therefore differ correspondingly but in reversed order, and the velocity is not uniform across the channel. Also although the flow adjacent to the vanes has rotational momentum effectively imparted to it, the portion of the flow intermediate between the vanes is less effectively acted upon and its flow lines near the discharge periphery of the impeller tend to approach the free path corresponding to a free vortex. Allowances for these considerations must be made by applying a coefficient (usually of the order of 0.7 to 0.8) to represent the ratio of actual momentum imparted to that realizable with an infinite number of vanes, *i.e.*, the apparent V_{u2} is multiplied by the coefficient K.

The following method for evaluating K, for radial and mixed-flow pumps with overlapping vanes, has been developed by the author with the cooperation of the Worthington Pump and Machinery Corp.

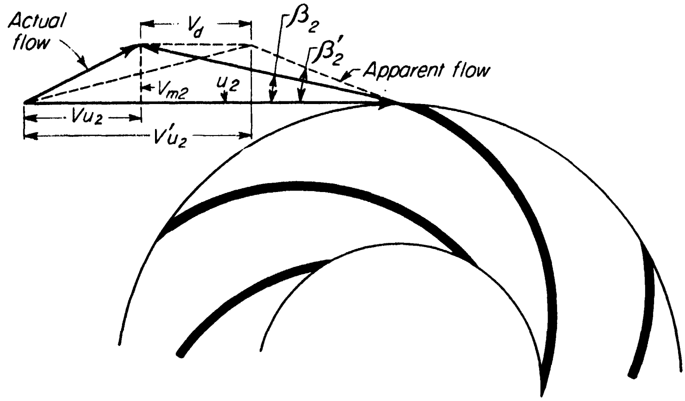


FIG. 49. Actual and apparent flow.

In Fig. 49 the dotted triangle represents the apparent flow based on the angle β_2' of the impeller vane. The solid triangle represents the actual flow based on the angle β_2 of the true average direction of discharge from the impeller. The meridian velocity component V_{m2}, or the radial component in a radial-flow impeller, remains unchanged, so that the actual flow triangle differs from the apparent triangle by a shift of the vertex parallel to u_2 through the distance V_d. The actual whirl component V_{u2} is calculated from the Euler formula

$$u_2V_{u2} - u_1V_{u1} = \frac{gH}{\epsilon_h}$$

V_d may be regarded as a differential velocity added vectorially to the apparent triangle to bring it into conformity with the actual triangle, changing the apparent whirl component V_{u2}' into the actual whirl component V_{u2} of the absolute flow. The ratio $V_{u2}/V_{u2}' = K$, the conversion coefficient. With an infinite number of vanes V_d is zero and $K = 1$.

The torque on the impeller

$$\frac{wq}{g}(r_2V_{u2} - r_1V_{u1})$$

may be equated to the average pressure difference between face and back of a vane

multiplied by A_{mer}, the vane area projected in a meridian plane, and by its mean radius r_m to its center of gravity, and by the number of vanes n. If the pressure difference corresponds to a differential pressure head Δh_p,

$$\frac{wq}{g}(r_2V_{u2} - r_1V_{u1}) = wA_{mer}nr_m\,\Delta h_p{}^*$$

Since V_d is a velocity, representing transverse flow in the circumferential direction across a vane passage from vane to vane, the corresponding velocity head $V_d^2/2g$ must correspond to a pressure head difference between the face of one vane and the back of the preceding vane, that is, Δh_p. We can therefore put $V_d^2/2g = (C/2)\,\Delta h_p$ in which C is a coefficient to be determined by test for any given type of impeller.

For normal operation with zero initial whirl

$$V_{u1} = O \qquad \text{and} \qquad \frac{V_d^2}{2g} = \frac{C}{2}\frac{q}{g}\frac{r_2V_{u2}}{A_{mer}nr_m}$$

and putting $q = A_{per}V_{m2}$ in which A_{per} is the peripheral area of the impeller normal to V_{m2}, equal to $2\pi r_2B$ − (vane thickness correction), we have

$$V_d = \sqrt{\frac{C}{n}\frac{A_{per}}{A_{mer}}\frac{r_2}{r_m}V_{m2}V_{u2}}$$

Here $A_{per}r_2$ and $A_{mer}r_m$ are the static moments of the two areas about the impeller axis.

Then since $K = \frac{V_{u2}}{V_{u2}'} = \frac{V_{u2}}{V_{u2} + V_d}$, we have

$$K = \frac{1}{1 + \sqrt{\frac{C}{n}\frac{A_{per}}{A_{mer}}\frac{r_2}{r_m}\frac{V_{m2}}{V_{u2}}}}$$

and
$$V_{u2}' = \frac{gH}{K\epsilon_h u_2}$$

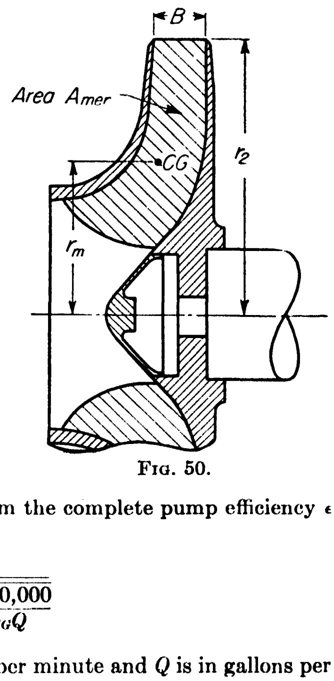


FIG. 50.

A common practice combines $(K\epsilon_h)$ in a single coefficient sometimes called the manometric coefficient which we denote K_m (see Fig. 64).

The hydraulic efficiency ϵ_h may be estimated from the complete pump efficiency ϵ by an empirical relation:

$$1 - \epsilon_h = \frac{1 - \epsilon}{1 + \frac{1}{2}\sqrt{\frac{10{,}000{,}000}{N_{SG}Q}}}$$

in which N_{SG} is the specific speed based on gallons per minute and Q is in gallons per minute. In a double-suction pump both N_{SG} and Q are to be taken for the complete pump. The coefficient C, from an analysis by Osborne[1] of test data of the Worthington Pump and Machinery Corp., may be taken as $C = 0.4$ as a satisfactory average.

No attempt will be made here to cover the detailed methods of pump design, for excellent designs are already developed for a wide range of specific speeds, and pumps of satisfactory performance are standard products of the leading pump manu-

* SPANNHAKE, Problems of Modern Pump and Turbine Design, *Trans. A.S.M.E.*, 1933, Eq. (8).

[1] OSBORNE, W. C. (Assistant in Mechanical Engineering, Princeton University), A Method of Calculating the Degree of Flow Deviation at the Discharge of Centrifugal Pump Impellers, A.S.M.E. annual meeting, Nov. 26–Dec. 1, 1950.

facturers. Attention will therefore be directed to the selection and application of pumps for various conditions of use.

Specific Speed. The principles of dynamic similarity explained in the case of turbines are equally valid when applied to pumps; and if a series of homologous pumps of various sizes are operated against various heads, we can say here as we did for turbines that $HP \propto D^2H^{3/2}$ if each pump is operated at its proper speed for its size and head so that $N \propto H^{1/2}/D$. This leads to the same specific speed relation as for turbines. However, this is not so convenient a relation in the pump field as another, which will be used instead. In specifying or designing a pump, the quantities in which we are most interested are not horsepower, head, and speed but discharge, head, and speed; *i.e.*, pumps are rated according to discharge capacity rather than power requirements. We shall therefore obtain a more convenient relation if we deal with Q instead of HP. By applying the same principles of similarity as before $Q \propto D^2H^{1/2}$ when $N \propto H^{1/2}/D$. Eliminating D, we have

$$N \propto \frac{H^{1/2}}{\sqrt{\dfrac{Q}{H^{1/2}}}} \propto \frac{H^{3/4}}{\sqrt{Q}}$$

If this is expressed as an equation instead of a proportionality,

$$N = (\text{a constant})\,\frac{H^{3/4}}{\sqrt{Q}}$$

By putting H equal to 1 ft and Q equal to 1 cfs, the constant is seen to be the speed, in revolutions per minute, of a homologous pump of such size that it would deliver 1 cfs against 1-ft head. This is taken as the specific speed of a pump and is commonly denoted N_s. When it is desired to apply both specific-speed functions in the same problem, this new specific speed, based on quantity, may be distinguished by using the notation N_{sq}. This specific speed is equally applicable to turbines, although not so convenient in most turbine problems as the specific speed based on HP. By expressing turbine specific speeds as N_{sq}, a comprehensive chart can be drawn showing both turbine and pump characteristics on a single diagram.[1] When not otherwise noted, N_s will be used in this discussion to mean specific speed based on quantity, when dealing with pumps.

As in the case of turbines, this new N_s (or N_{sq}) is a constant for all geometrically similar pumps of any size and head, each being operated at its proper speed corresponding to the head and size. The type of a pump can therefore be determined by inserting its values of Q, H, and N in

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} \tag{31}$$

and finding its N_s $(=N_{sq})$.

Some inconvenience, in centrifugal-pump calculations, results from lack of standardization of units of discharge. For small and moderate sizes of pumps, it is a trade custom to specify the discharge in gallons per minute and for large pumps to use either cubic feet per second or millions of gallons per day. In the interest of standardization and to avoid unwieldy numerical values, we shall base the specific speed on the primary units of cubic feet per second as defined above, unless otherwise noted. Specific speed is, however, frequently stated on the basis of quantity in gallons per minute or

$$N_s = N\,\frac{\sqrt{Q\ \text{gpm}}}{H^{3/4}} \tag{32}$$

[1] MOODY, L. F., Present Trend of Turbine Development, *Trans. A.S.M.E.*, **43**, 1132, 1921.

This value of N_s will be distinguished here by the symbol N_{sG}. To convert N_{sG} into N_s (or N_{sq}), the following conversion factors are used: 7.48 U.S. gal = 1 cu ft; 7.48 × 60 = 449 gpm = 1 cfs.

$$N_s = \frac{N\sqrt{\frac{Q\text{ gpm}}{449}}}{H^{3/4}} = \frac{N}{\sqrt{449}}\,\frac{\sqrt{Q}\text{ gpm}}{H^{3/4}} = \frac{N_{sG}}{21.2}$$

The specific speeds here employed will be the values for a single-suction, single-stage pump. Thus the quantity to be used in figuring the N_s for a double-suction pump is that for only one-half the double impeller, or one-half the total Q of the unit.

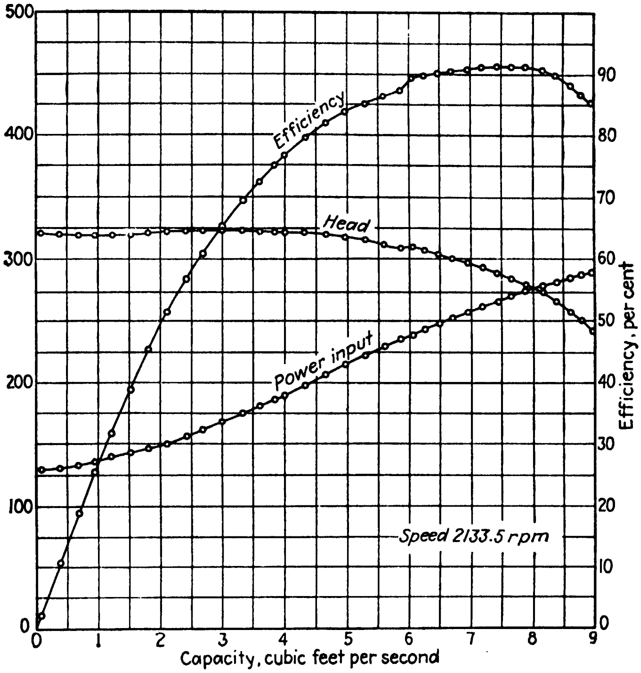


FIG. 51.—Characteristic curves of a Byron Jackson–Pelton model pump, tested at California Institute of Technology for Metropolitan Water District of Southern California.

For a multistage pump, the head to be used in figuring N_s is the total head of the unit divided by the number of stages.

We shall also use for pumps the specific speed for the point of best efficiency, which should be the point of normal operation. Figures 51 and 52 show the efficiency and head characteristic curves for pumps of moderate specific speed and high efficiency.

As in the case of turbines, there are forms and proportions of pumps favorable to high efficiency, corresponding to a particular range of specific speeds; and abnormally high or low specific speeds require abnormal proportions and are accompanied by impaired efficiencies. Figure 53 indicates the trend of highest recorded efficiencies, obtained in nearly every case from tests of very large pumps. Small pumps and those of the usual commercial sizes naturally cannot be expected to approach these values, since, just as in the case of turbines, efficiencies increase with an increase in dimensions. Sufficient comparative tests of homologous pumps are not available to establish firmly

the applicability to pumps of the turbine efficiency step-up formula

$$\frac{1-e}{1-e_1} = \left(\frac{D_1}{D}\right)^{1/5}$$

but the average exponent of four such comparisons for pumps gives a value of $\frac{1}{5}$, so that this formula may be applied tentatively. To show the effect of size, in terms of capacity, on the efficiencies of good commercial pumps, Fig. 54 is given, prepared by G. F. Wislicenus and revised to include recent test results of the Worthington Pump

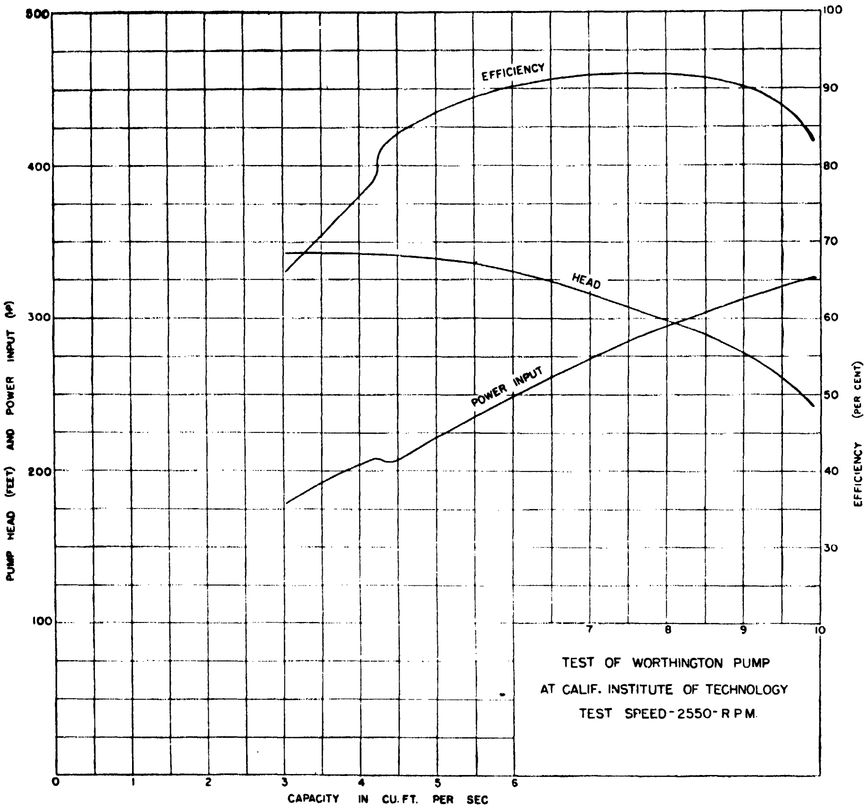


Fig. 52.—Characteristic curves of a Worthington model pump, tested at California Institute of Technology.

and Machinery Corp. The curve of Fig. 53 will serve to indicate the region of specific speeds most favorable to high efficiencies and the comparative effect of variations of N_s; probable efficiencies to be expected for any specific speed should of course be taken considerably below the plotted values which represent maximum and not average results. It will be noted that below a specific speed of 50 (cfs units) or 1,000 gpm units the efficiencies drop rapidly. This is mainly due to the absence of data for large pumps in this region. The efficiency of a radial-flow centrifugal pump of given unit capacity is practically unaffected by subdividing its inlet and changing from single-suction to double-suction design, so that for a double-suction pump of this type the specific speed in Fig. 53 should be based on the total unit capacity.

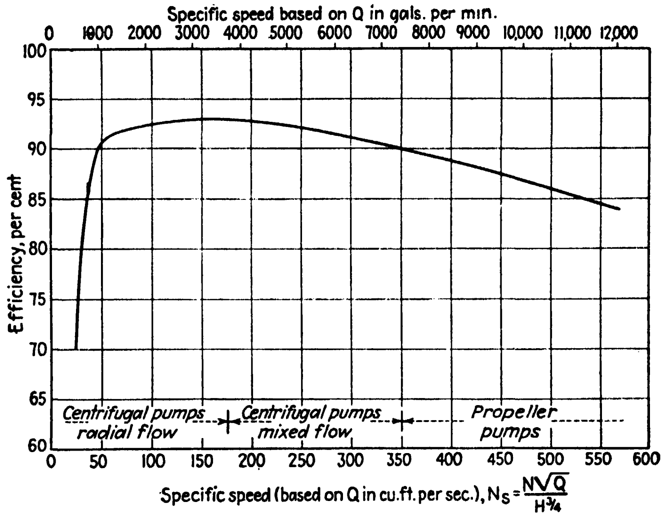


FIG. 53.—Maximum pump efficiencies secured at various specific speeds.

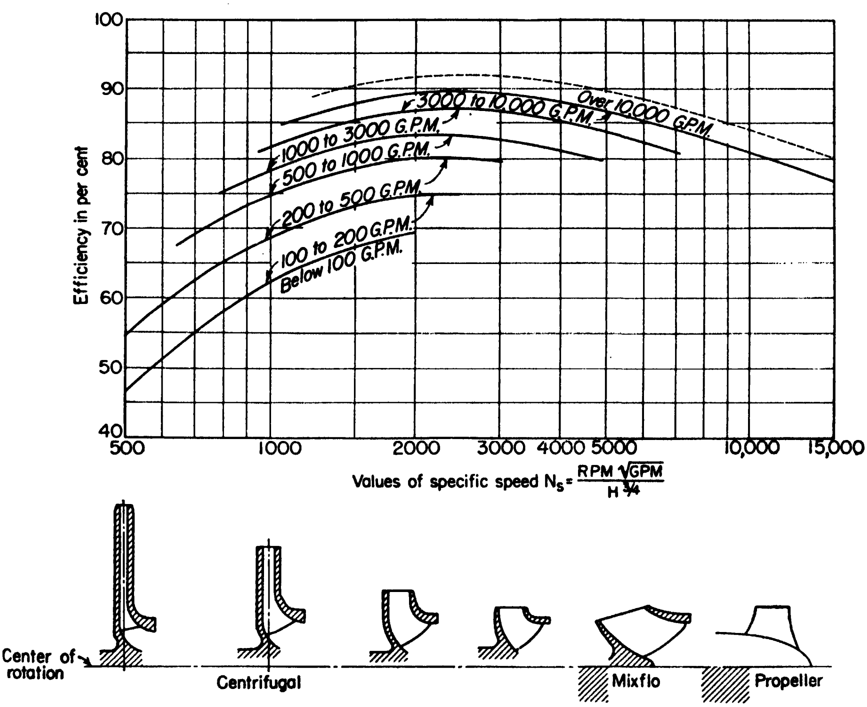


FIG. 54.—Effect of rated capacity on efficiency of good commercial pumps.

For very high heads and small quantities to be pumped, as in boiler-feed pumps or mine pumps, the specific speed would work out far below the values for normal design and good efficiency, even when extremely high-speed driving motors are used. Instead of attempting to develop the head in a single impeller, the head is subdivided in a number of stages, and several pumps are placed in series, either as separate units or as a series incorporated in a single unit and with all the impellers on one shaft and within a common casing. For example, if s stages are used and the specific speed of the entire unit is $N_s = \sqrt{Q}/H^{3/4}$, the specific speed of one stage corresponding to a single impeller will be increased to $N_{s1} = N \frac{\sqrt{Q}}{(H/s)^{3/4}} = s^{3/4}N_s$, which can be increased to a value that will fall within the range of good design and normal proportions. For

Fig. 55.—Vertical-shaft, mixed flow pump with volute casing. (*Worthington Pump and Machinery Corp.*)

instance, suppose the specific speed for the total H and Q of a pump taken as a unit works out at a value of 10, which would require abnormal design and low efficiency. By using a number of stages such as 9, the specific speed of an individual impeller would be increased to $10(9)^{3/4} = 52$, which is within the region of normal design and good efficiency (see Fig. 53).

On the other hand, the specific speed of a pump unit may be increased by placing a number of impellers on the same shaft arranged to pump in parallel. If the number of impellers in parallel is n, the specific speed of the unit will be $N_s = N \frac{\sqrt{nQ}}{H^{3/4}} = N_{s1}\sqrt{n}$, where N_{s1} is the specific speed of a single-suction impeller. Thus the specific speed of a double-suction pump, having a double impeller, will be $N_s = N \frac{\sqrt{nQ}}{H^{3/4}} = 1.41N_{s1}$. The value of the specific speed to be used in determining the form and characteristics of the impeller is that corresponding to one stage and single suction, which is the value used in the curves shown here.

Another reason for using a number of stages in a high-head pump is the desire not to exceed about 500 or 600 ft head per stage to avoid abrasion or wear of the parts exposed to high velocity; and frequently the head per stage is limited by cavitation requirements. The various design proportions of a pump may be expressed as functions of the specific speed. The impeller diameter, for example, can be fixed from $u_2 = 2\pi r_2 N/60 = \phi\sqrt{2gH}$; and Fig. 64 indicates reasonable values of ϕ as a function of N_s.

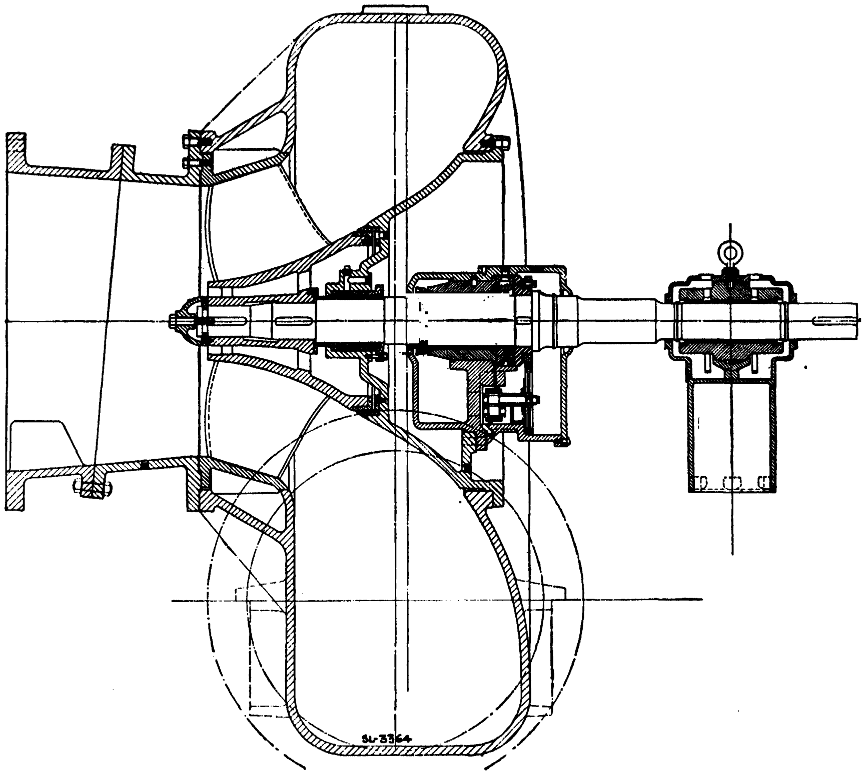


FIG. 56.—Sectional elevation of a horizontal-shaft, mixed-flow pump. (*Worthington Pump and Machinery Corp.*)

Cavitation. As already explained in the section for turbines, the same reasoning there set forth applies equally to pumps, including the Thoma formula $\sigma = \dfrac{H_b - H_s}{H}$, the criterion of cavitation; and it will be unnecessary to repeat its derivation. The charts in Figs. 65 and 66 correspond to those given for turbines. For values of H_b, refer to Fig. 33. The solid line in Fig. 65 gives recommended minimum values of sigma for pumps of normal design, and Fig. 66 is based on these values. Figure 66 and the full line of Fig. 65 agree with the empirical relation $\sigma = 0.1455(N_s/100)^{4/3}$ or $\sigma = 0.0535(N_{sg}/1000)^{4/3}$. Centrifugal pumps of generally similar design characteristics give fairly consistent sigma values; but points taken from various commercial types of pumps show a wide scatter, some pumps showing double the sigma value of others of the same specific speed. The full line on Fig. 65 merely gives a reasonable limit for normal designs. For installations close to this limit, cavitation tests or field

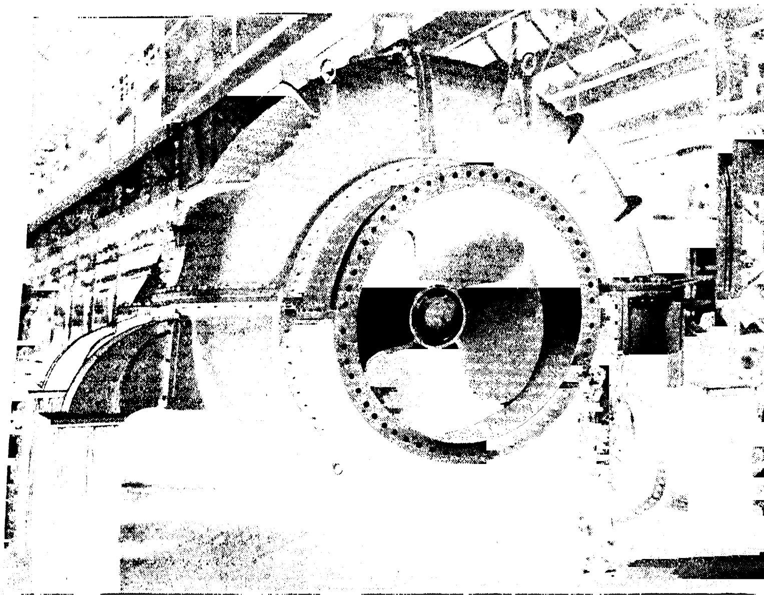


FIG. 57.—80-in. mixed-flow pump. (*Worthington Pump and Machinery Corp.*)

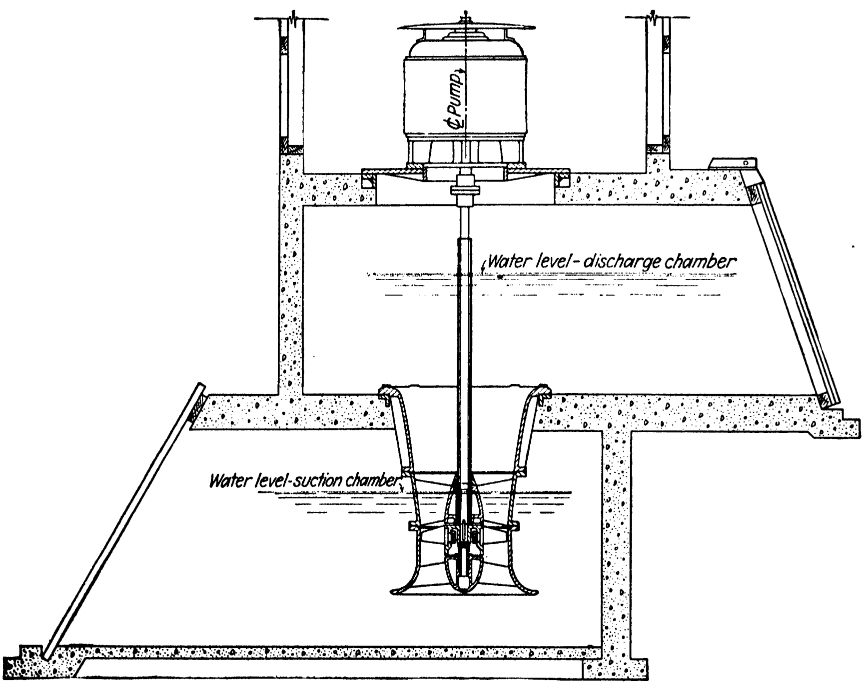


FIG. 58.—Axial-flow propeller pump, with diffusion vanes. (*Byron Jackson Co.*)

experience should be available for the particular design contemplated. This is emphasized in the field of propeller pumps where the variation of critical sigmas due to changes in design characteristics, particularly the proportional blade area, for a given specific speed, is great. The curve of sigmas for propeller pumps merely serves to show values that have been secured and to indicate the trend of this variable.

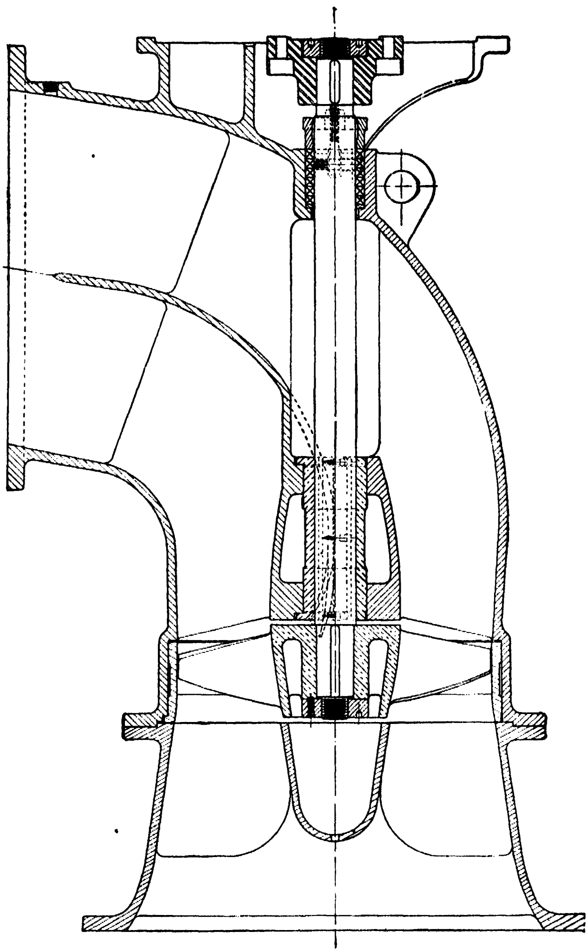


Fig. 59.—Axial-flow propeller pump, with diffusion vanes and discharge elbow. (*Warren Steam Pump Co., Inc.*)

The values of sigma used in plotting the charts are based on the suction lift H_s measured by pressure gage at the pump suction flange corrected for velocity head and expressed as the elevation of the shaft center line in horizontal-shaft pumps, or the casing center line in vertical-shaft centrifugal pumps, above the equivalent water surface corresponding to the suction pressure and velocity head. In vertical-shaft propeller pumps, the suction lift is measured up to the mid-height of the impeller.

With reference to Fig. 67, h_p is the height of water column in a gage glass connected to the pump suction pipe at a point z feet below the shaft center line. h_p

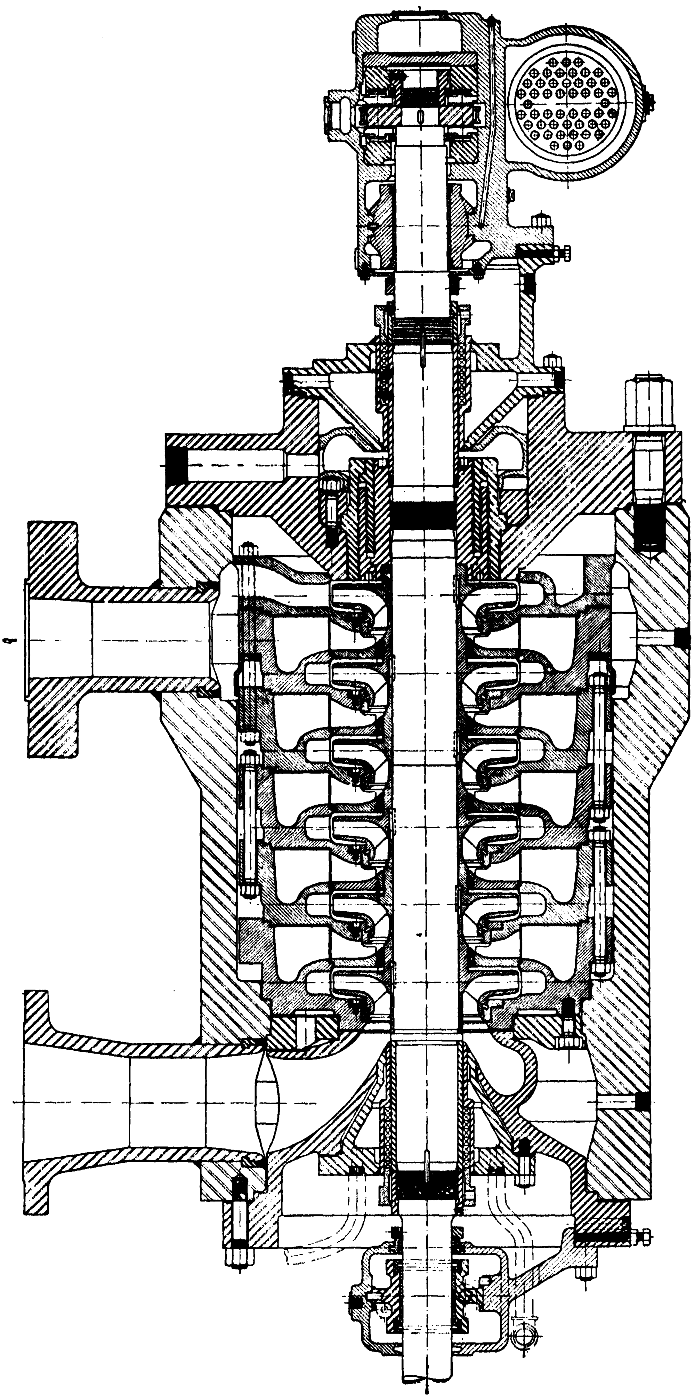


FIG. 60.—Six-stage boiler-feed pump. Forged-steel barrel-type casing. (*Worthington Pump and Machinery Corp.*)

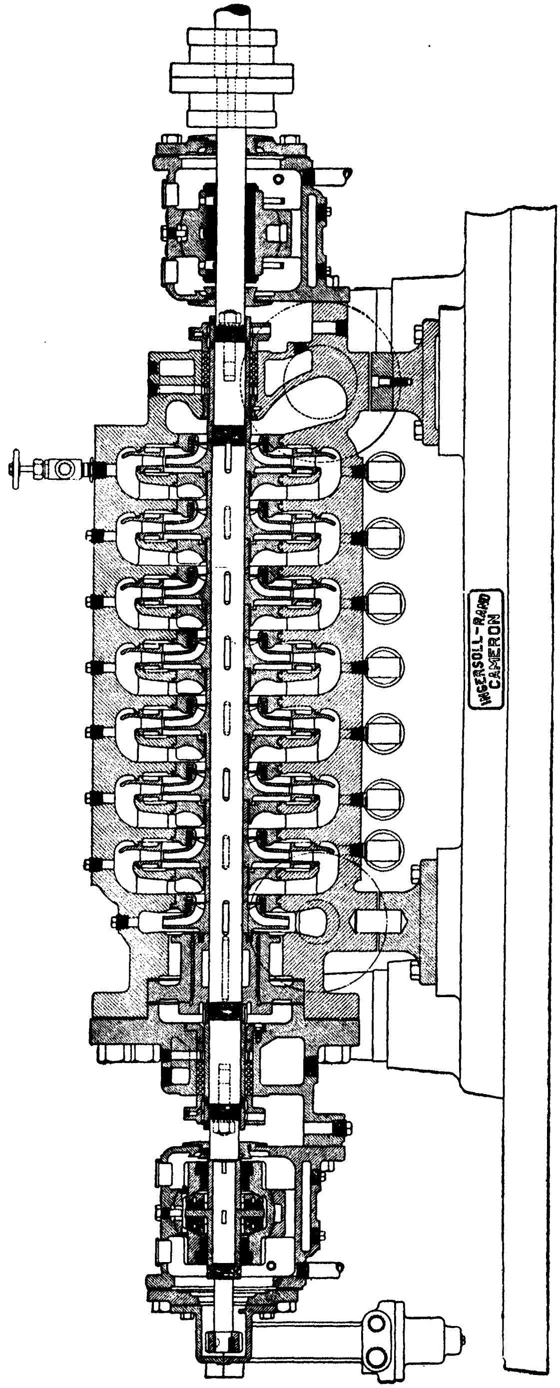


Fig. 61.—Eight-stage pump. Diffusion-vane type. Split casing. (*Ingersoll-Rand Co., Cameron Pump Division.*)

must be increased by the velocity head in the suction pipe at the section where the gage is connected The top of the water column so corrected is the elevation of the equivalent or virtual sump elevation shown by the dashed line The value of H_s referred to the pump axis is therefore $H_s = z - (h_p + v_1^2/2g)$ When the pump operates under positive suction pressure, this is termed *suction head*, which is merely

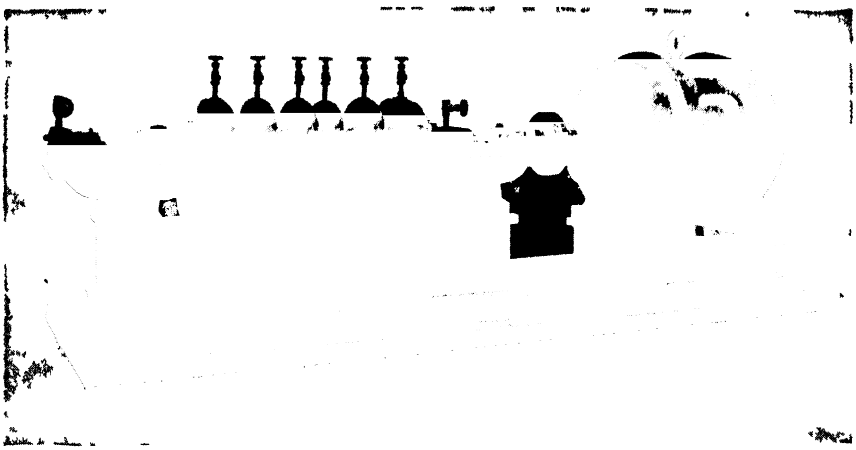


Fig 62 Six-stage pump Split-casing type (*Ingersoll-Rand Co, Cameron Pump Division*)

Fig 63 Nineteen-stage hot-oil pump Believed to be the highest head pump ever built 8,600 ft head, 875 gpm (*Byron Jackson Co*)

$-H_s$. In applying these sigmas to large horizontal-shaft installations, it is advisable to figure the suction lift in the large pump to the upper side of the impeller entrance diameter rather than to the shaft center line The values are also based on operation at the point of best efficiency When a pump is expected to operate at greater than normal discharges, as in cases where the operating head is subject to considerable variation, higher sigma values should be used.

The solid line of Fig 65 was constructed from a large number of test points as best

representing the trend of sigma with respect to specific speed. It is interesting to note that the empirical formula representing this curve agrees with a theoretical relation proposed by G. F. Wislicenus.[1]

Cavitation in a pump of normal design is usually initiated close to the inlet tips of the impeller vanes, where the pressure is low and where the disturbance due to the impingement of the flow on the vane tips causes a tendency toward separation. Consider a given impeller operating at constant capacity, speed, and constant suction head or suction lift. Then let us suppose that the pump is altered in form at points affecting the flow only after it has passed the inlet section and has proceeded far enough downstream to avoid any effect of the alteration on the flow distribution in the inlet section. Such an alteration may comprise a change of form of the pump casing or of a discharge elbow, for example, or the shape of diffusion vanes; or it may comprise an elongation or shortening of the impeller vanes at the discharge end, which may involve a change in

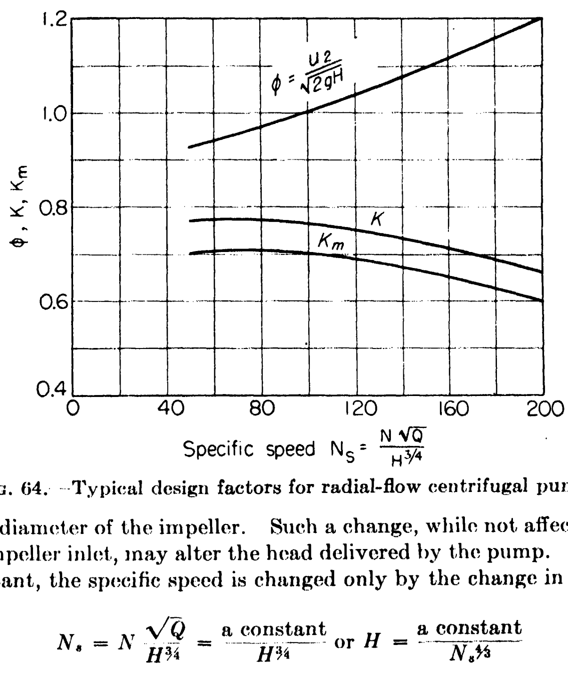


FIG. 64.—Typical design factors for radial-flow centrifugal pumps.

the discharge diameter of the impeller. Such a change, while not affecting the conditions at the impeller inlet, may alter the head delivered by the pump. Since Q and N are held constant, the specific speed is changed only by the change in H:

$$N_s = N\frac{\sqrt{Q}}{H^{3/4}} = \frac{\text{a constant}}{H^{3/4}} \text{ or } H = \frac{\text{a constant}}{N_s^{4/3}}$$

Expressing the Thoma cavitation formula $\sigma_c = \frac{H_b - H_s}{H}$ in the form $\sigma_c = \frac{H_{sv}}{H}$, in which H_{sv} is the excess of the absolute suction head above the vapor pressure, or the net positive suction head, we insert the above value for H, giving,

$$\sigma_c = \frac{H_{sv}}{\text{constant}} N_s^{4/3}$$

Since H_{sv} is being held constant, we have $\sigma_c = (\text{a constant})N_s^{4/3}$. Hence for variations in the form of the pump such as described, σ_c is proportional to $N_s^{4/3}$. This relation should determine the slope of the σ vs. N_s curve in pumps of normal design, at least for small changes in N_s, at any part of the field; and experience shows that it is substan-

[1] WISLICENUS, WATSON, and KARASSIK, Cavitation Characteristics of Centrifugal Pumps Described by Similarity Considerations, *Trans. A.S.M.E.*, 17, 1939.

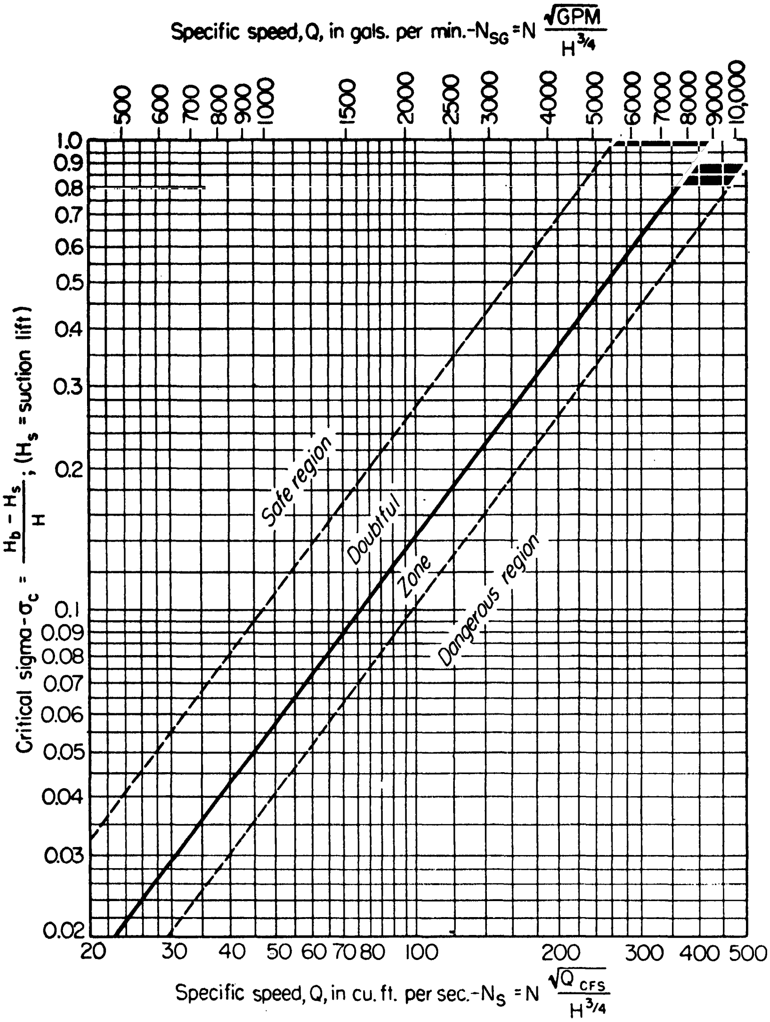


FIG. 65.—Limiting values of cavitation coefficient sigma at various specific speeds, recommended for safe practice. For single-stage, single-suction pumps with overhung impellers.

tially true for pumps of all types throughout the entire range of specific speeds developed.

Wislicenus expressed the constant in terms of the "suction specific speed,"

$$S = \frac{N \sqrt{Q_{\text{gpm}}}}{H_{sv}^{3/4}}$$

a specific speed based on H_{sv} instead of H. Then

$$\sigma_c = \frac{H_{sv}}{H} = \left(\frac{N \sqrt{Q_{\text{gpm}}}}{S}\right)^{4/3} \Big/ H = \left(\frac{N \sqrt{Q_{\text{gpm}}}}{SH^{3/4}}\right)^{4/3} = \left(\frac{N_{sg}}{S}\right)^{4/3}$$

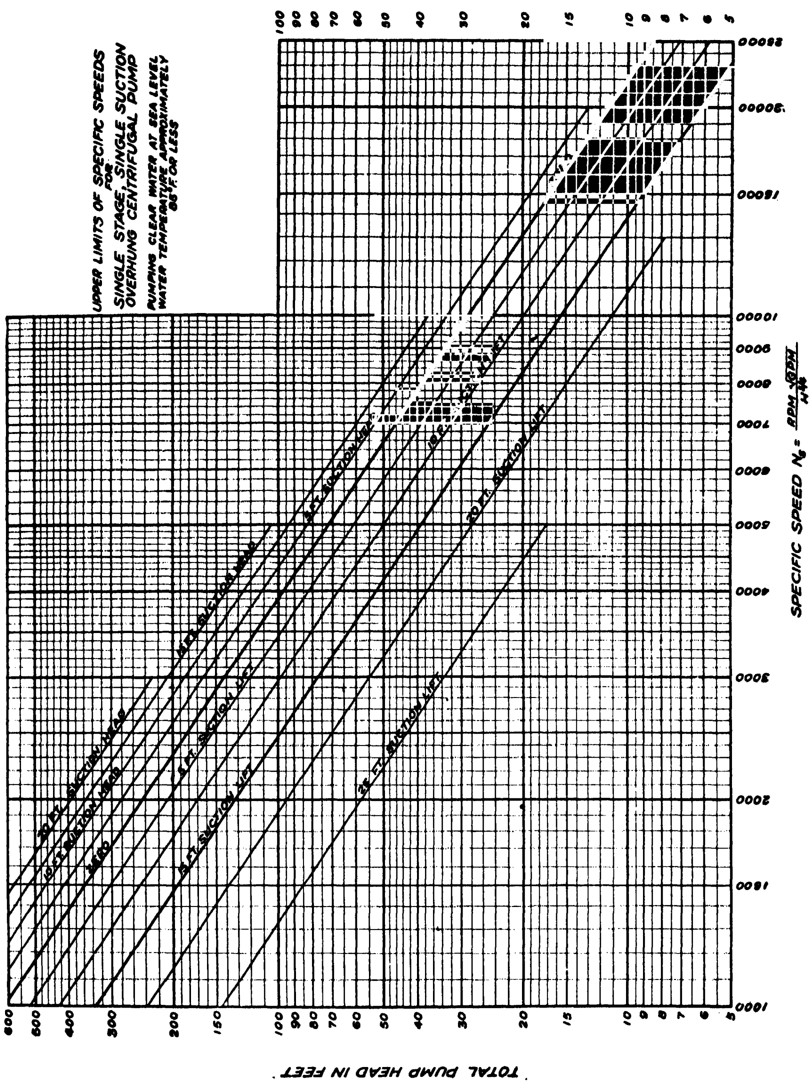


FIG. 66.—Chart of limiting head for various specific speeds, recommended for safe practice. For single-stage, single-suction pumps with overhung impellers. At sea level with water temperature 80F.

The solid line of Fig. 65 corresponds to a value of S of 8,990 or in round numbers

$$\sigma_c = \left(\frac{N_{sq}}{9{,}000}\right)^{4/3}$$

If this relation is combined with the specific-speed formula, we obtain the simple relation

$$N = \frac{8{,}990(H_b - H_s)^{3/4}}{\sqrt{Q_{gpm}}}$$

or in round numbers

$$N = \frac{9{,}000(H_b - H_s)^{3/4}}{\sqrt{Q_{gpm}}}$$

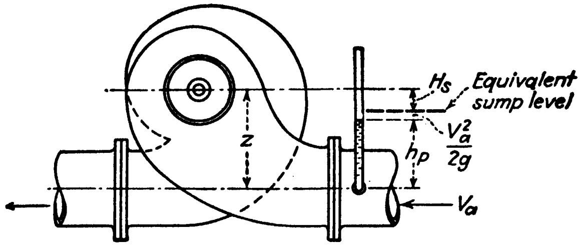


FIG. 67.

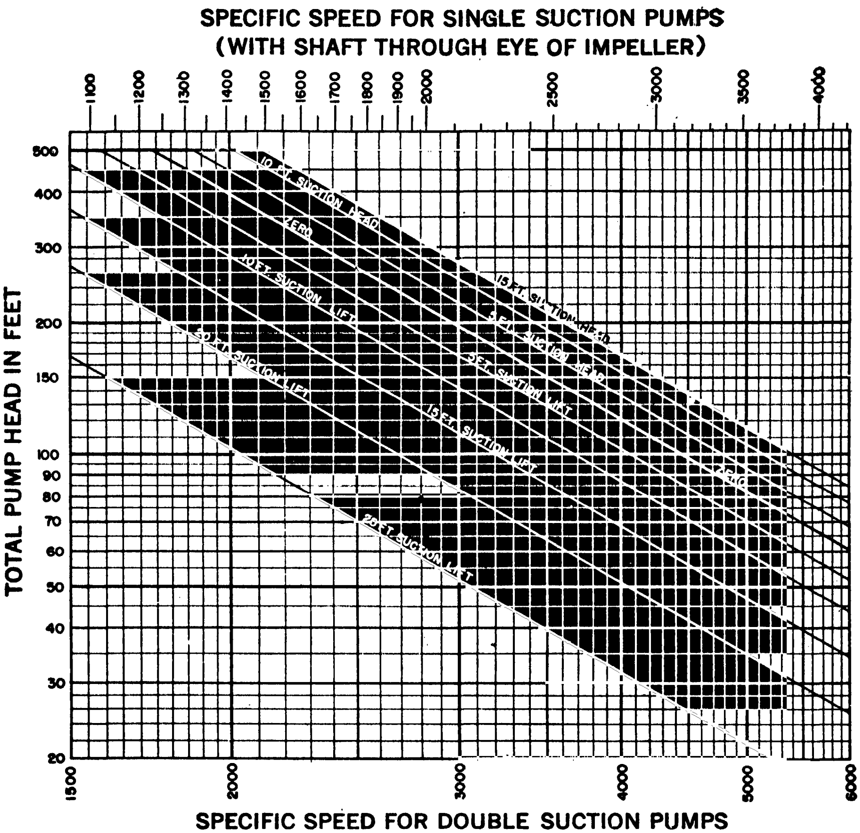


FIG. 68.—Chart of limiting head for various specific speeds, for double-suction pumps and single-suction pumps with shaft through eye of impeller. For sea level and water at 80F. Specific speeds shown correspond to the total pump capacity (both suctions in a double-suction pump).

in which Q_{gpm} is in gallons per minute. With Q expressed in cubic feet per second, this becomes

$$N = \frac{425(H_b - H_s)^{3/4}}{\sqrt{Q}}$$

This relation is independent of the head H and makes possible the construction of the

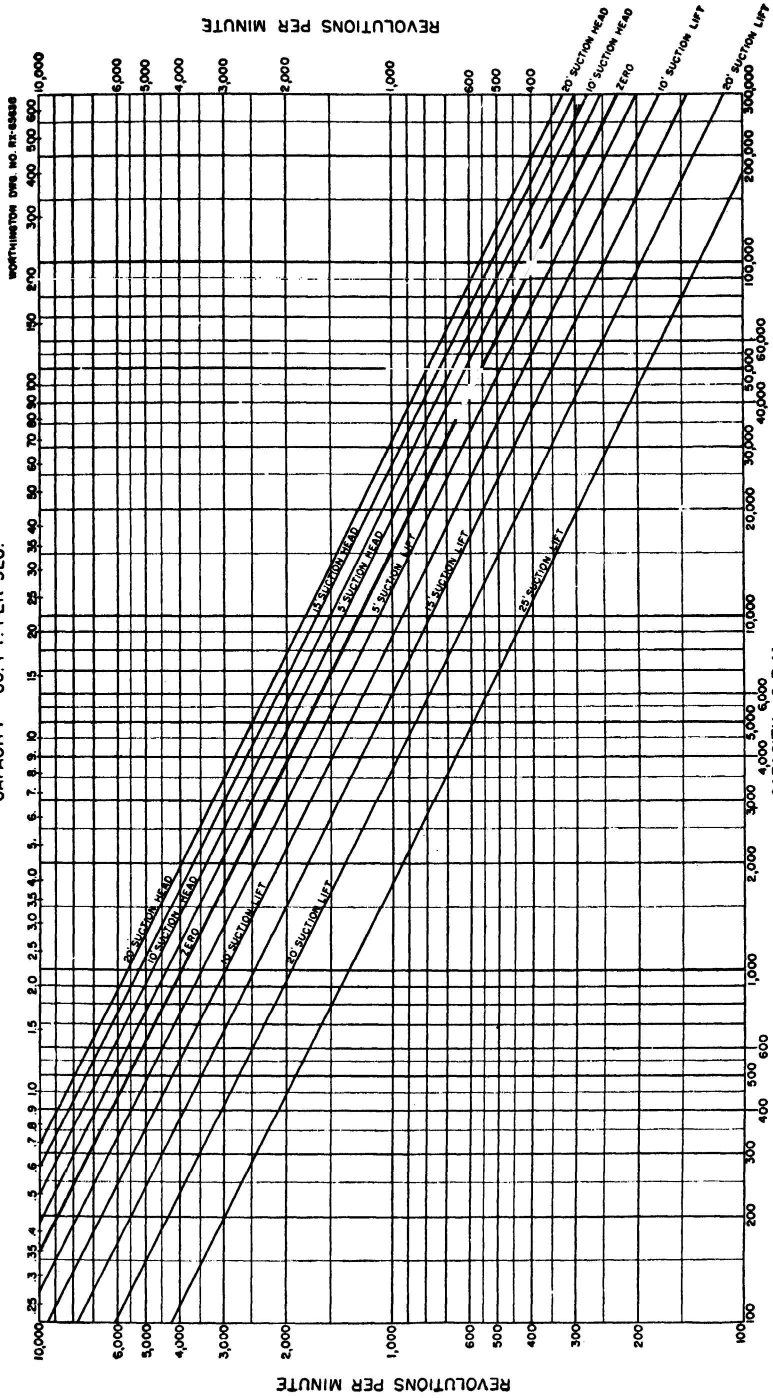


FIG. 69.—Chart of limiting speeds for various capacities and suction heads, for single-stage, single-suction pumps with overhung impellers. The speeds given are the highest safe values corresponding to the conditions of Figs. 65 and 66.

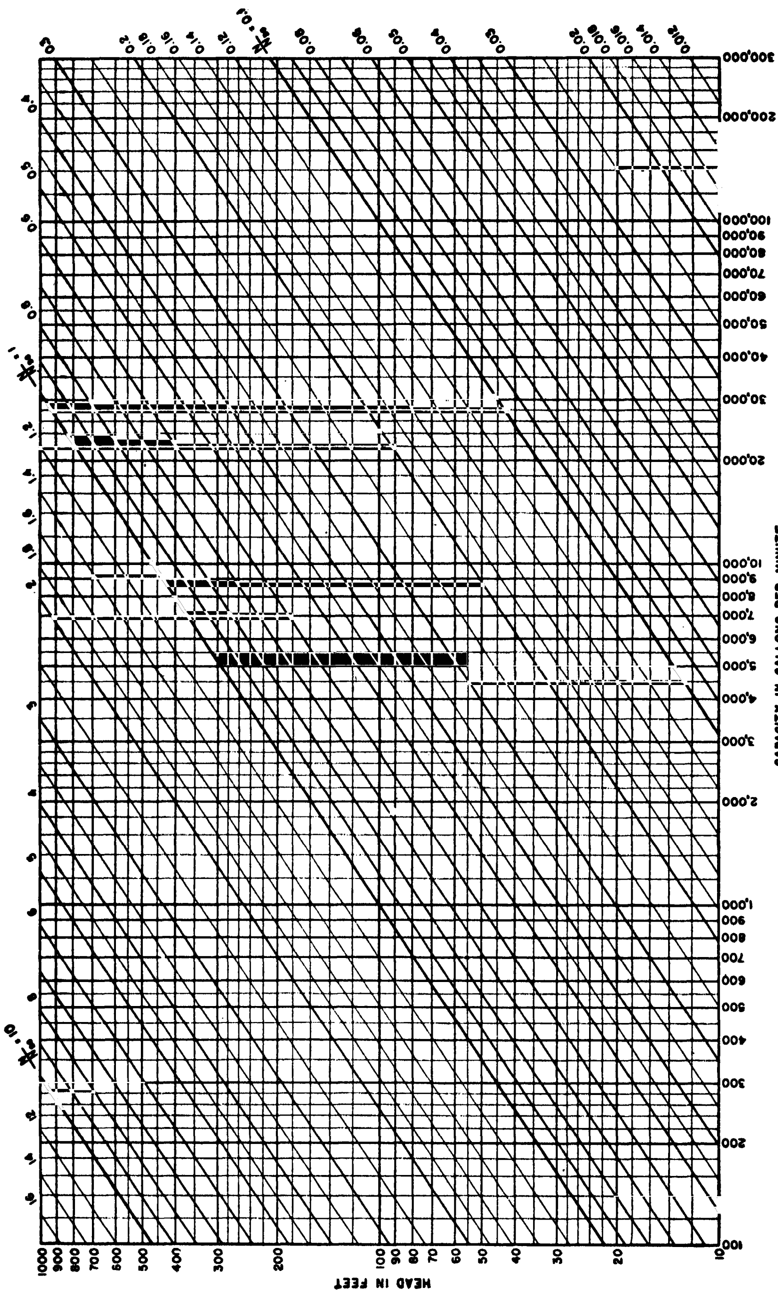


FIG. 70.—Chart for the solution of the specific-speed formula for pumps.

simple chart of Fig. 69. (Figure 10 of the last reference was based on the same principle.)

The charts and curves of Figs. 65, 66, and 69 are for single-suction pumps with overhung impellers and are applicable to pumps of the highest efficiency. For pumps in which the shafts extend through the suction space, such as double-suction and multistage pumps, the reduction of the effective eye or throat area due to the shaft, dependent on the size of shaft and consequently on the head, requires a downward revision of the capacity and specific speed. For such pumps of usual standard or commercial types, Fig. 68 gives the safe upper limit of head for given specific speeds or upper limit of specific speed for given heads. Since this chart represents approved commercial practice, not necessarily limited to pump proportions for the highest attainable efficiencies, the limits for the lowest specific speeds are slightly more liberal than the corresponding values for overhung pumps in Fig. 66.[1]

Figure 70 is a chart to facilitate the solution of the specific-speed formula.

LABORATORY AND MODEL TESTS OF PUMPS[2]

Shop Test on Full-sized Pump at Full Head. In order that a shop or laboratory test should correctly predict the results to be secured in the field installation, the setting as well as the pump proper should be duplicated. On the intake side the setting includes the passage between the impeller and a point in the supply piping or sump where the velocity head is so low that variations in passages beyond that point will have a negligible effect. On the discharge side it includes the passage between the pump casing and the point where the guaranteed head is to be measured. To represent true pump performance, the discharge head should be measured, both in the shop or laboratory and in the field installation, in a straight section of discharge pipe a sufficient distance beyond the pump discharge flange to permit the flow to reach substantially normal distribution across the section of measurement, and thus to provide a "smoothing section" to overcome the distortion of flow caused by the pump volute or by a discharge elbow.

If the shop or laboratory test is made at the same head and speed as the field installation, then, in the absence of a cavitation test, the suction head or lift must be the same if the atmospheric pressure and water temperature are the same. If the atmospheric pressure or water temperature is different, the suction head or lift must be such as to give the same cavitation factor $\sigma = \dfrac{H_b - H_s}{H}$, in which $H_b = H_a - H_{vp}$, the atmospheric pressure head minus the vapor pressure head of the water corresponding to its temperature. H_s is the net suction lift $= z - \left(h_p + \dfrac{Va^2}{2g}\right)$ as in Fig. 67; and H is the total pump head.

If, however, cavitation tests have been made on the pump and it has been estab-

[1] The cavitation limits given by the chart Fig. 68 for pumps with the shaft passing through the impeller eye are in accordance with the following semi-empirical formulas, in which $H_{sv} = H_b - H_s$:

For single-suction pumps, $$H = 26 \frac{H_{sv}\left(1 - 0.25\sqrt{\dfrac{H_{sv}}{32.8}}\right)^{2/3}}{\left(\dfrac{N_{sg}}{1{,}000}\right)^{1.7}}$$

For double-suction pumps, $$H = 47 \frac{H_{sv}\left(1 - 0.25\sqrt{\dfrac{H_{sv}}{32.8}}\right)^{2/3}}{\left(\dfrac{N_{sg}}{1{,}000}\right)^{1.7}}$$

[2] Based on data contained in "Standards of Hydraulic Institute," Centrifugal Pump Section, Copyright, 1948, by Hydraulic Institute, New York.

lished that the cavitation sigma of the installation is substantially in excess of the critical sigma throughout the range of operation, then if the sigma for the shop or laboratory tests is also kept well above the critical sigma, its exact value is of no moment.

Example: A four-stage boiler feed pump rated at a capacity of 400 gpm is to operate against a total head of 900 ft, handling water at 250F and running at 3,550 rpm. The suction gage pressure is to be 21 psi in a 4-in. suction pipe, at the elevation of the pump axis. The pump is to be tested in the shop under full head and at the same speed, handling water at 80F. What suction head or lift should be applied in the shop test to make it correctly represent the field conditions?

The first condition is that the specific speed must be the same in the shop and the field operation, which will be secured with the head, speed, and capacity the same in both cases. The second condition is that the cavitation factor sigma must be the same in shop and field operations, to make the suction conditions in the shop test representative of the field conditions. Calling

$$H_{sv} = H_b - H_s = H_a - H_{vp} - H_s$$

in which H_{sv} is the "net positive suction head," or the excess of the absolute suction head over the vapor pressure head, then $\sigma = H_{sv}/H$.

In this example, from steam tables, at 250F the specific volume of the water is 0.017 cu ft/lb, the specific weight is $w = \dfrac{1}{0.017} = 58.8$ lb/cu ft, and the vapor pressure is 30 psi absolute. The suction pressure head is $h_p = \dfrac{(21)(144)}{58.8} = 51.4$ ft, the pipe velocity V_a is 10.2 fps, and the velocity head is 1.6 ft, so that $H_s = z - \left(h_p + \dfrac{V_a{}^2}{2g}\right) = 0 - 51.4 - 1.6 = -53.0$ ft (or 53 ft positive suction head).

Hence $H_{sv} = H_a - H_{vp} - H_s = \dfrac{(14.7 - 30)}{58.8}(144) + 53.0 = 15.5$ ft. The head per stage is $H = 900/4 = 225$ ft, and $\sigma = \dfrac{H_{sv}}{H} = \dfrac{15.5}{225} = 0.069$.

To maintain the same σ in the shop test with the water temperature 80F and the vapor pressure 0.5 psi, $H_{sv} = \sigma H$ must remain the same and the suction lift must be

$$\begin{aligned} H_s &= H_a - H_{vp} - H_{sv} \\ &= \frac{(14.7 - 0.5)(144)}{62.3} - 15.5 = 32.8 - 15.5 = 17.3 \text{ ft} \end{aligned}$$

Shop Test on Full-sized Pump at Reduced Head. When laboratory or shop limitations such as available power preclude full-head tests, reduced-head tests are permissible and are in general closely representative of tests at full head. However, in tests at reduced power the relative loss in mechanical bearing friction and stuffing-box friction may be increased (an effect which may be appreciable in small pumps); and the hydraulic friction losses may be relatively increased when the Reynolds number for the water passages is reduced (an effect which may be appreciable in small pumps of low specific speed). It is recommended that when reduced head tests are used as acceptance tests, the guarantees specify the test head, and that the performance guarantees be based on the reduced head conditions.

In any such reduced head test, two conditions must be maintained: the operating speed must be such as to keep the specific speed the same as in the field installation, and the cavitation sigma must be the same. As noted, however, if it has been established by cavitation tests that sigma is substantially in excess of the critical sigma, then the second requirement is unnecessary.

In order to maintain hydraulic similarity with the field operation, the capacity Q_1 for the test and the test speed N_1 must be reduced below the installation capacity Q and speed N, respectively, in the ratio

$$\frac{Q_1}{Q} = \frac{N_1}{N} = \sqrt{\frac{H_1}{H}}$$

where H_1 and H are the pump heads for the test and installation, respectively.

Example: With the same field conditions as in the preceding example, the shop test on the same pump is to be made at a reduced head of 720 or 180 ft per stage with water at 80F. What capacity, speed, and suction lift or head should be used in the shop test?

From the above relations, the capacity to be used in the shop test is

$$Q_1 = Q\sqrt{\frac{H_1}{H}} = 400\sqrt{\frac{180}{225}} = 358 \text{ gpm}$$

The speed to be used in the shop test is

$$N_1 = N\sqrt{\frac{H_1}{H}} = 3{,}550\sqrt{0.8} = 3{,}170 \text{ rpm}$$

The specific speed will now be the same in shop and field:

$$N_s = N\frac{\sqrt{Q}}{H^{3/4}} = 3{,}550\frac{\sqrt{400}}{(225)^{3/4}} = 3{,}170\frac{\sqrt{358}}{(180)^{3/4}} = 1{,}220$$

To keep the cavitation factor the same in shop and field, we found in the preceding example $\sigma = 0.069$ and $H_b = H_a - H_{vp} = 32.8$ for the shop test. Then for the shop test

$$H_s = H_b - \sigma H_1 = 32.8 - (0.069)(180) = 20.4 \text{ ft.}$$

To reproduce the field conditions, the shop test must therefore be run with a suction lift of 20.4 ft.

Model Tests. In many cases involving installed units of large size, model tests are of great utility. Even when it might be feasible to test the large unit in the shop, a model may often be tested with greater accuracy and thoroughness; and by adopting a standardized size of model for various pumps, properly comparable performances can be secured. Model testing in advance of final design and installation of a large unit, as standard procedure, not only provides insurance of performance in advance but makes alterations possible in time for incorporation in the installed unit. It is not essential that the model test head be the same as that in the installation.

The model should have complete geometric similarity with the prototype, not only in the pump proper but in the intake and discharge conduits as specified for tests on full-sized pumps. The model should be run at such speed under the test head that the specific speed remains the same as that of the installed unit; and if cavitation tests are not available, the suction head or lift should be such as to give the same sigma value as in the installation.

If corresponding diameters of model and prototype are D_1 and D, respectively, then the model speed N_1 and capacity Q_1 under the test head H_1, must agree with the relations:

$$\frac{N_1}{N} = \frac{D}{D_1}\sqrt{\frac{H_1}{H}} \qquad \text{and} \qquad \frac{Q_1}{Q} = \left(\frac{D_1}{D}\right)^2\sqrt{\frac{H_1}{H}}$$

In testing a model of reduced size, the above conditions being observed, complete hydraulic similarity will not be secured unless the relative roughness of the impeller and pump casing surfaces is the same. With the same surface texture in model and prototype, the model efficiency will be lower than that of the larger unit; and greater relative clearances and shaft friction in the model will also reduce its efficiency.

In pumps of high or medium specific speed, and in models of reasonable size, with correct relative clearances and with impeller and diffuser surfaces of smoother finish than in the larger prototype, a close approximation to the prototype efficiency may be secured if the model bearing and stuffing-box friction is minimized by a special design. In such models, operated under a head which is not too low, the Reynolds number of the flow passages is high enough to cause completely turbulent flow and to make viscous forces negligible and the Reynolds number without effect. In such cases

little increase in prototype efficiency over model can be counted upon. When model tests are to serve as acceptance tests, it is recommended that the efficiency guarantees be dependent on the model performance rather than on inferred field performance computed by formula.

Not all pumps are well adapted for model testing. In installations in which free-water surface phenomena may affect the performance, complete hydraulic similarity prescribes constant Froude number, which requires a constant ratio of pumping head to the linear dimensions of the pump. To keep sigma the same, it is necessary to reduce the barometric pressure in the same ratio by enclosing the whole setting in a closed chamber. These conditions apply, for example, when a limited depth of free sump surface over the suction inlet may cause prerotation, or vortices drawing air into the pump. In pumps such as condensate pumps intended to operate under cavitating conditions, when vapor separation produces free surfaces within the pump, it is recommended that the pumps be tested in their full size at full head and speed.

Example: A single-stage pump to deliver 200 cfs against a head of 400 ft at 450 rpm and with a positive suction head, including velocity head, of 10 ft has an impeller diameter of 6.8 ft. The pump being too large for a shop or laboratory test, a model with 18 in. impeller is to be tested at a reduced head of 320 ft. At what speed, capacity, and suction head should the test be run?

Applying the above relations:

$$N_1 = N \frac{D}{D_1} \sqrt{\frac{H_1}{H}} = 450 \frac{6.8 \text{ ft}}{1.5 \text{ ft}} \sqrt{\frac{320}{400}} = 1{,}825 \text{ rpm}$$

$$Q_1 = Q \left(\frac{D_1}{D}\right)^2 \sqrt{\frac{H_1}{H}} = 200 \left(\frac{1.5}{6.8}\right)^2 \sqrt{\frac{320}{400}} = 8.73 \text{ cfs, or } 3{,}920 \text{ gpm}$$

To check these results, the specific speed of the prototype is

$$N_s = N \frac{\sqrt{Q}}{H^{3/4}} = 450 \frac{\sqrt{200}}{400^{3/4}} = 71.2 \text{ in the cfs system}$$

and that of the model is

$$N_{s1} = 1{,}825 \frac{\sqrt{8.73}}{320^{3/4}} = 71.2 \text{ (or 1,510 in the gpm system)}$$

The cavitation factor for the field installation, assuming a water temperature of 80F as a maximum probable value, and $H_b = 32.8$ ft as in the first example, will be

$$\sigma = \frac{H_b - H_s}{H} = \frac{32.8 + 10}{400} = 0.107,$$

which should be the same in the test. With the water temperature approximately the same,

$$\sigma = \frac{H_b - H_{s1}}{H_1}$$

and

$$H_{s1} = H_b - \sigma H = 32.8 - (0.107)(320) = -1.45 \text{ ft}$$

Hence the model should be tested with a positive suction head of 1.45 ft, to reproduce the field conditions.

Test of Full-sized Pump or Model at Increased Head. Under some conditions it may be desirable to carry out shop or laboratory tests at a higher head than the installation head. This may be due, for example, to the limitations of test motors or electrical frequency. A more usual cause may be the inability to provide sufficient suction lift in the shop. The required sigma can then be obtained by an increase in the pumping head instead of an increase in suction lift.

Cavitation Tests. The critical value of sigma at which cavitation begins can be found by running the pump at constant specific speed and varying sigma by progressively altering the relation between suction lift and pumping head. The efficiency and the head corrected for constant speed can then be plotted against sigma. The

point where the curves break away from the horizontal, as sigma is reduced, indicates the critical sigma. For pumps of abnormal design and those which are to operate close to or beyond the recommended limits of Figs. 65 to 69, cavitation tests are of great importance, sometimes of greater importance than accurate efficiency tests. Three typical arrangements are shown for determining the cavitation characteristics of pumps.

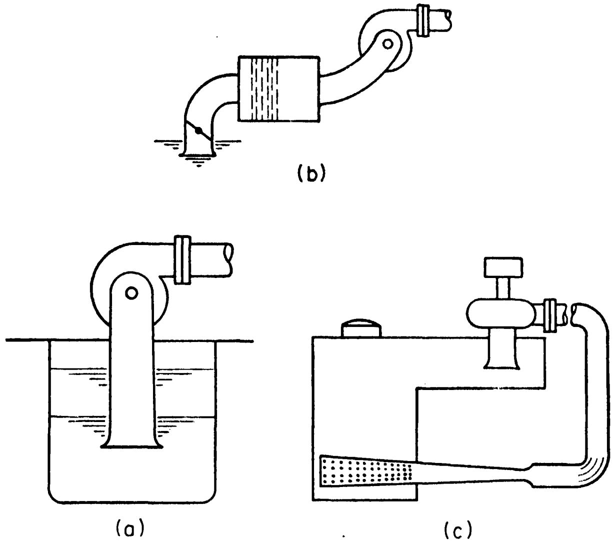


FIG. 71.—Arrangements for determining pump cavitation characteristics.

In the first (Fig. 71*a*), the pump suction pipe simply draws from a sump in the form of a well in which the surface level can be varied in elevation over a fairly large range, thus varying the suction lift.

In the second arrangement (Fig. 71*b*) the suction is taken from a sump of fixed surface level, through a throttle valve followed by an enlarged pipe forming a stilling chamber and containing screens or baffles to dissipate the turbulence from the throttle valve and to distribute the flow evenly, so that the pump takes a flow free from undue turbulence.

In the third arrangement (Fig. 71*c*), the pump is in a closed circuit in which the absolute pressure level can be drawn down to any desired extent without changing the pump head. Various arrangements can be used for carrying it out, one of which is shown by way of illustration.

SECTION 13

WATER HAMMER

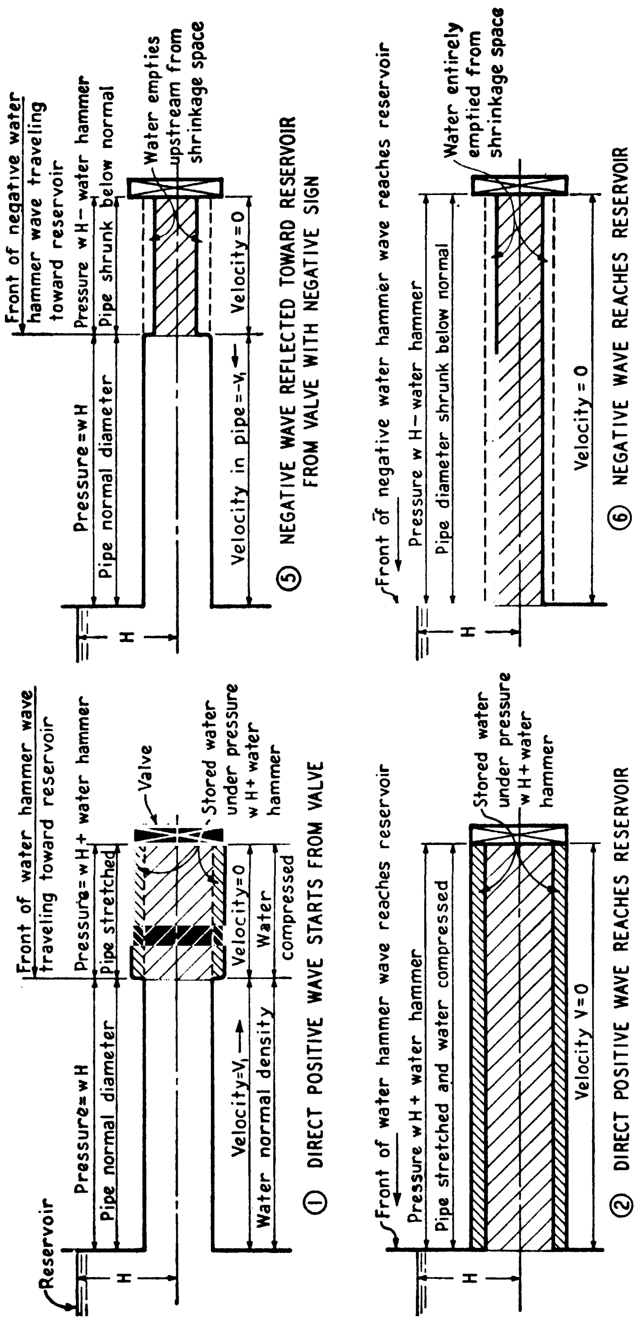
By George R. Rich

1. INTRODUCTION. WATER HAMMER—A TYPICAL TRANSIENT PHENOMENON

Water-hammer effects in closed conduits constitute one particular class of a broad family of electrical and mechanical wave movements which engineers have chosen to designate by the term transient phenomena. These disturbances may be said to mark the transition stage between any two successive steady states of the system. They are initiated by the application of a definite causative force or action, frequently but not necessarily applied with comparative suddenness, and are dissipated down to the level of the second steady state by the operation of some form of damping.

In the case of hydraulic pressure conduits, water hammer is the mechanism immediately responsible for every change in steady-state velocity, gradual or sudden. It is a wave movement propagated with the velocity of sound, usually initiated by a change in setting of the conduit control valve or its equivalent, and is attenuated down to the level of the second steady state by damping in the form of conduit friction. Because water-hammer pressures reach significant magnitudes only in the case of comparatively rapid valve operation, we are prone to lose sight of the fact that water hammer is, nevertheless, the direct means of effecting even the most gradual change in steady-state velocity.

Although it is assumed that most engineers have some acquaintance with water hammer, it may be advantageous, as a means of breaking ground for the essential differential relationships, to review just qualitatively, subject to subsequent mathematical confirmation, the salient steps of the water-hammer cycle for the elementary case of the simple conduit with instantaneous valve closure and with conduit friction and velocity head neglected. Referring to Fig. 1, upon closure of the valve, a wave of positive pressure travels upstream with the velocity of sound (in this particular case converting the conduit velocity to zero), compressing the water and stretching the conduit shell. Upon reaching the reservoir, the pressure wave obtains relief and drops down in intensity to the reservoir pressure. This in turn allows the compressed water stored in the stretched pipe to drop down to reservoir pressure again by means of the passage of a negative pressure wave traveling downstream with the velocity of sound and converting the conduit velocity from zero to minus V_1 in the reverse direction. When this negative wave has reached the closed valve at the downstream end of the conduit, all the water in the conduit is at that instant traveling upstream with a velocity minus V_1. Owing to the inertia of the water, this negative pressure wave traveling downstream will next be reflected at the valve so as to travel upstream with the same negative sign. Behind and downstream of the negative wave so reflected, the conduit velocity will again drop to zero, and the conduit shell will shrink a proportionate amount below the original size so as to furnish the volume of water necessary to maintain the velocity minus V_1 ahead of the traveling wave until the wave reaches the reservoir. When the reflected negative wave reaches the reservoir,



① DIRECT POSITIVE WAVE STARTS FROM VALVE

② DIRECT POSITIVE WAVE REACHES RESERVOIR

⑤ NEGATIVE WAVE REFLECTED TOWARD RESERVOIR FROM VALVE WITH NEGATIVE SIGN

⑥ NEGATIVE WAVE REACHES RESERVOIR

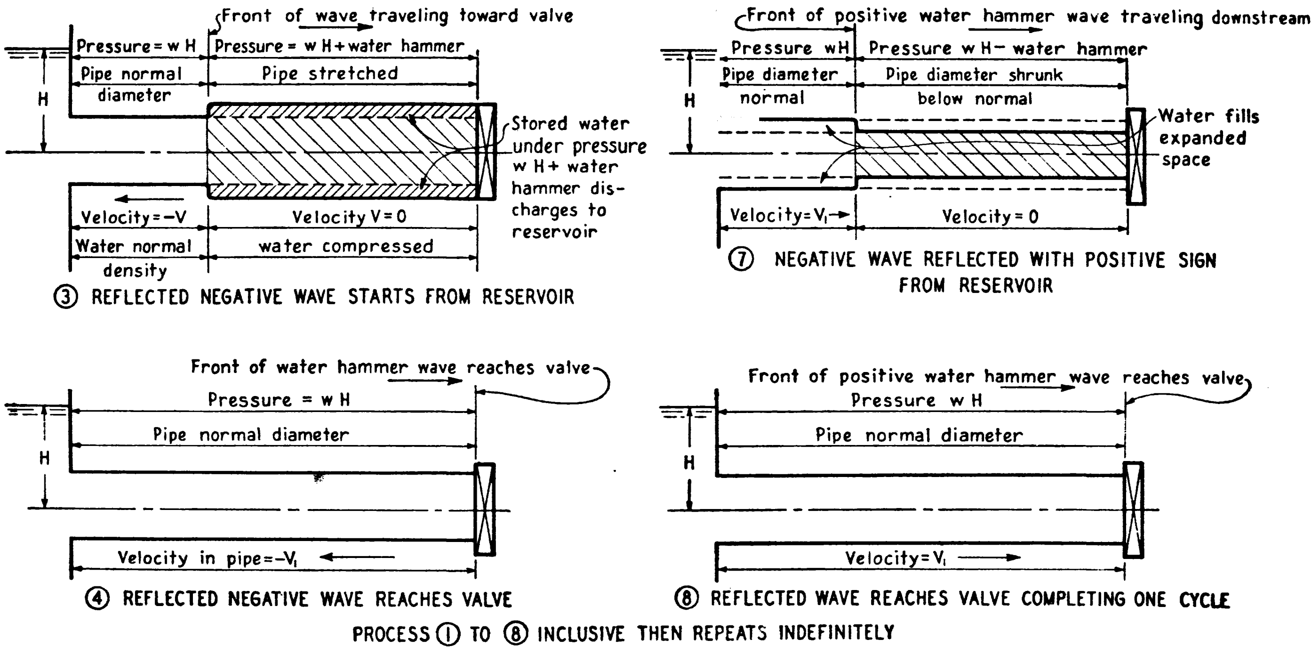


FIG. 1.—Progress of water-hammer wave. Instantaneous valve closure.

the pressure will rise to reservoir level, and a positive pressure wave elevating the conduit pressure to normal reservoir head will be propagated downstream. The velocity upstream from and behind the wave will next be converted to plus V_1. This completes one cycle of the wave motion, and since the effect of friction in the conduit has been neglected, the procedure outlined will be repeated indefinitely and without diminution in intensity.

2. THE BASIC DIFFERENTIAL EQUATIONS

The physical process outlined in Fig. 1 has been so well substantiated by experiment and observation that we are justified in employing the fundamental concept of water hammer as a wave motion, compressing the water and stretching the conduit circumferentially, in formulating the primary differential equations of water hammer.

To facilitate cross reference with current literature, the basic derivations will be made in terms of pressure rather than head, and the origin of coordinates will be taken at the reservoir. Subsequently, in making application of the basic relationships to the various detailed methods of computation, appropriate changes in notation will be made where desirable to facilitate similar cross reference to the literature pertaining to these particular methods.

The following notation will be employed:

D = diameter of pipe, ft.
L = length of pipe, ft.
A = area of pipe, sq ft.
P = total pressure (surge plus steady state), psf.
V = total velocity (surge plus steady state), fps; positive in positive direction of x.
x = distance of section along axis of pipe, ft; measured from origin at reservoir.
K = volume modulus of compression of water, ft units.
b = thickness of pipe walls, ft.
E = modulus of elasticity of pipe wall, ft units.
F = friction force coefficient.
w = weight of unit volume of water, lb/cu ft.
g = acceleration of gravity, ft/sec.^{2}
k = factor of proportionality in establishing linear approximation to conduit friction.
$W = \frac{w}{g}$.
$Q = \frac{1}{K} + \frac{D}{bE}$ (the reciprocal of the equivalent bulk modulus of water and pipe) (shell in combination).
t = time, sec.
$a = \frac{1}{\sqrt{WQ}}$ = water-hammer wave velocity, fps.

Referring to Fig. 2, as the water-hammer wave travels along the conduit, in each element of time ∂t, a length of water ∂x loses a decrement of velocity ∂V. The pressure behind the wave is greater than that ahead of the wave by an increment ∂P. In the figure the sign of ∂x must be plus to agree with the convention that x and V are both positive in the downstream direction. Since P and V depend upon x as well as t, the notation of partial derivatives is employed. A material simplification results from dropping second-order differentials at the outset of the analysis.

$$F = M\frac{\partial V}{\partial t} \qquad \text{Newton's second law}$$

$$\frac{\pi D^2}{4}(P + \partial P) - P = \frac{\pi D^2}{4}\frac{w}{g}\,\partial x\left(-\frac{\partial V}{\partial t}\right)$$

$$-\frac{\partial P}{\partial x} = W\frac{\partial V}{\partial t} \tag{1}$$

Under the effect of the increase in pressure due to water hammer ∂P, the diameter of the conduit increase by an amount ∂D.

$$\partial D = \frac{D^2\,\partial P}{2bE}$$

The volume per unit length of pipe made available by compression of the water under the increased pressure ∂P is

$$\partial\,(\text{volume}) = \frac{\pi D^2\,\partial P}{4K}$$

The total new volume to be filled in length ∂x by the discharge resulting from the

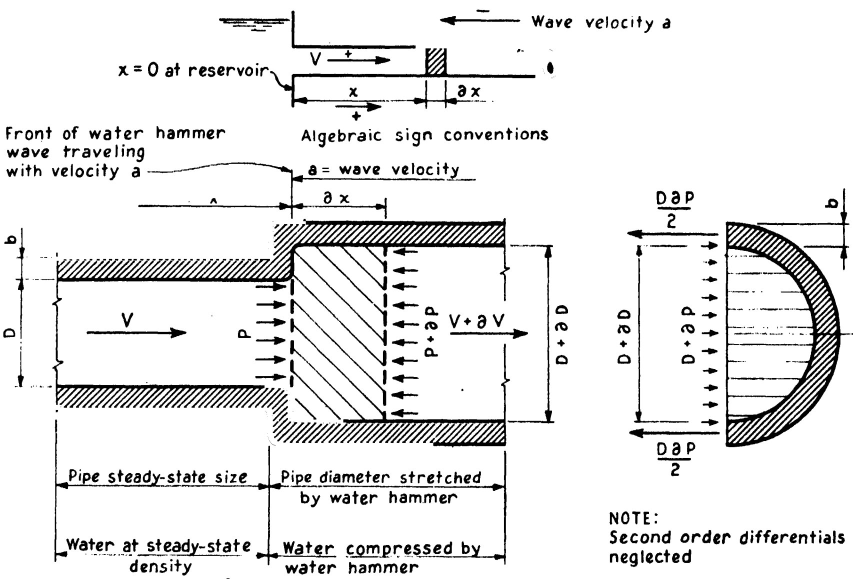


FIG. 2.—Conditions during transit of water-hammer wave.

decrement of velocity ∂V is

$$\left[\frac{\pi D^2\,\partial P}{4K} + \frac{\pi(D + \partial D)^2}{4} - \frac{\pi D^2}{4}\right]\partial x = \frac{\pi D^2\,\partial P\,\partial x}{4K} + \frac{\pi D^3\,\partial P\,\partial x}{4bE}$$

But the volume of water resulting from the decrement of velocity ∂V, accumulated in time ∂t, is $\frac{\pi D^2}{4}\,\partial V\,\partial t$. Equating volumes for hydraulic continuity, and with due regard to sign conventions (Fig. 2), we obtain

$$-\frac{\pi D^2\,\partial P\,\partial x}{4K} - \frac{\pi D^3\,\partial P\,\partial x}{4bE} = \frac{\pi D^2}{4}\,\partial V\,\partial t$$

By definition,

$$Q = \frac{1}{K} + \frac{D}{bE}$$

and

$$-\frac{\partial V}{\partial x} = Q\,\frac{\partial P}{\partial t} \tag{2}$$

Now, if a is the velocity of propagation of the water-hammer wave, the velocity in a length of conduit having a length of adt will be decreased ∂V in the time ∂t.

$$F\,\partial t = M\,\partial V \qquad \text{Newton's second law}$$

$$\frac{\pi D^2}{4}\,\partial P\,\partial t = -\frac{\pi D^2}{4}\frac{wa\,\partial t\,\partial V}{g}$$

The minus sign of a is taken to agree with the sign convention of Fig. 2,

or
$$\partial P = -Wa\,\partial V$$

Substituting in (2)

$$-\frac{\partial V}{\partial x} = -\frac{QWa\,\partial V}{\partial t}$$

but
$$\partial x = a\,\partial t$$

and
$$a^2 = \frac{1}{WQ}$$

or
$$a = \frac{1}{\sqrt{WQ}} \tag{3}$$

By successive differentiation and combination of Eqs. (1) and (2), and using the notation of Eq. (3),

$$\frac{1}{WQ}\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial t^2} \tag{4}$$

$$\frac{1}{WQ}\frac{\partial^2 P}{\partial x^2} = \frac{\partial^2 P}{\partial t^2} \tag{5}$$

From their very form, the mathematical physicist[1] would immediately recognize Eqs. (4) and (5) as representative of a wave motion propagated in the x dimension with a velocity of propagation equal to the square root of $1/WQ$. These identical equations apply not only to water-hammer waves but also to the propagation of elastic stress waves in solid media and a wide variety of other mechanical and electrical transients. In each of these fields the essential character of the wave motion is the same, consisting basically of two subsidiary wave components traveling in opposite directions with the velocity $\sqrt{1/WQ}$ and without mutual interference. In other words, then, the existing fund of established knowledge regarding the general properties of Eqs. (4) and (5) gives us an independent check on the derivation of wave velocity expressed in Eq. (3).

In order to include the effect of conduit friction in our mathematical analysis, it will be necessary to introduce an approximation in order to obtain solvable differential equations. Instead of taking conduit friction proportional to the square of the velocity, we shall develop an equivalent device that permits us to express friction as proportional to the first power of the velocity.

Referring to Fig. 3, the pressure loss in length of conduit ∂x is $wckV_mV\,\partial x$.

The corresponding drop force due to friction in length ∂x is accordingly,

$$\frac{\pi D^2}{4}wckV_mV\,\partial x = \frac{\pi D^2}{4}FV\,\partial x$$

To include friction in Eq. (1) we write

$$\frac{\pi D^2}{4}\Big[(P + \partial P) - P\Big] + \frac{\pi D^2}{4}FV\,\partial x = \frac{\pi D^2}{4}\frac{w}{g}\,\partial x\left(-\frac{\partial V}{\partial t}\right)$$

and
$$-\frac{\partial P}{\partial x} - FV = W\frac{\partial V}{\partial t} \tag{6}$$

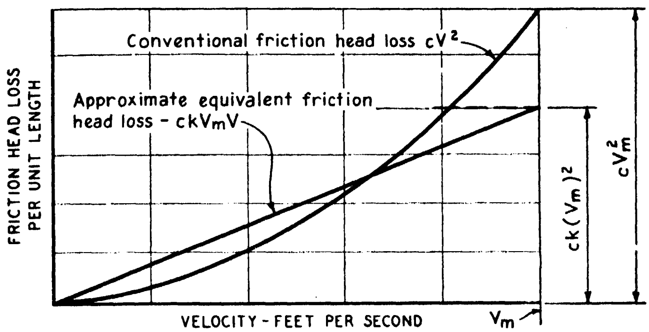
[1] WEBSTER, ARTHUR GORDON, "Partial Differential Equations of Mathematical Physics," Chap. II, pp. 72–135, Teubner, Leipzig, 1933.

The pair of equations to be solved when friction is to be included is, therefore,

$$-\frac{\partial P}{\partial x} - FV = W\frac{\partial V}{\partial t} \tag{6}$$

$$-\frac{\partial V}{\partial x} = Q\frac{\partial P}{\partial t} \tag{2}$$

Equations (1), (2), (3), and (6) form the very foundation of the science of water hammer and are the basis of the development of every known method of computation.



V_m = maximum velocity
k = factor assumed to fix position of best equivalent straight line

Straight line loss for any velocity $V = ck\,(V_m)^2 \dfrac{V}{V_m} = ckV_mV$

Friction force drop per unit length $\partial x = \dfrac{\pi D^2}{4}\, wckV_mV\,\partial x = \dfrac{\pi D^2}{4} FV\partial x$

FIG. 3.—Development of approximate equation for friction force term in water-hammer equations.

3. MECHANISM OF WAVE REFLECTION

With the basic differential relationships of water hammer at our disposal, we are now able to give independent mathematical confirmation to the wave reflection sequence outlined qualitatively in the introduction (Fig. 1). Referring to Fig. 4, the basic equations to be solved are

$$\frac{a^2\,\partial^2 P}{\partial x^2} = \frac{\partial^2 P}{\partial t^2} \tag{5}$$

$$-\frac{\partial P}{\partial x} = W\frac{\partial V}{\partial t} \tag{1}$$

The constants of integration are to be determined to comply with the two boundary conditions:

When $\quad x = 0, \quad P = P_0$

and $\quad x = L \quad V = 0$

Let us note very carefully at this point that no other boundary conditions whatever are adduced and in particular that no assumption whatever has been made regarding the genesis or character of reflections at the reservoir or at the valve. This means that the results we obtain regarding the internal mechanism of water hammer are exclusively the result of rigorous mathematical processes.

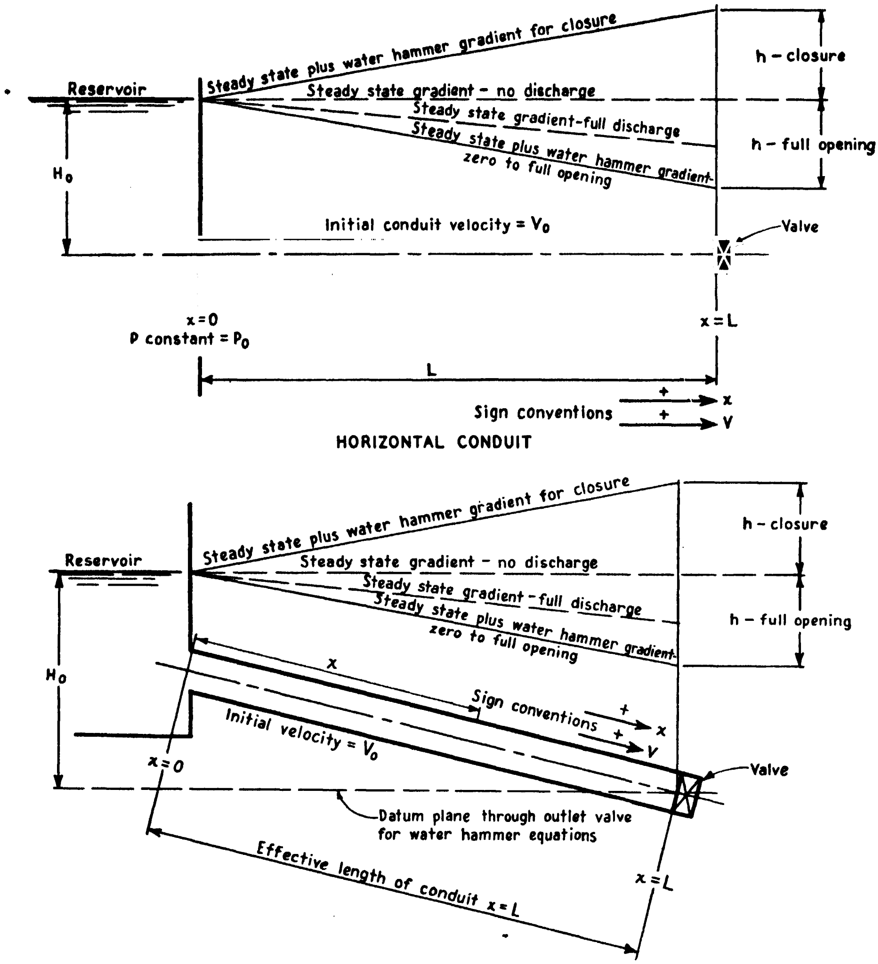


FIG. 4. Simple conduit with instantaneous valve closure.

For manipulative details of the solution the reader is referred to the current literature.[1]

The solution is given below in the form of an infinite summation of two wave components, the time of arrival of each successive component at the section x being denoted by the subscripts.

$$P = P_0 + \frac{waV_0}{g} \sum_{n=1}^{\infty} (-1)^{n+1} [1_{t > \frac{(2n-1)L-x}{a}} - 1_{t > \frac{(2n-1)L+x}{a}}] \tag{7}$$

$$V = V_0\left[1 - \sum_{n=1}^{\infty} (-1)^{n+1} (1_{t > \frac{(2n-1)L-x}{a}} + 1_{t > \frac{(2n-1)L+x}{a}})\right] \tag{8}$$

[1] RICH, GEORGE R., Water-hammer Analysis by the Laplace-Mellin Transformation, *Trans. A.S.M.E.*, 1945, *Paper* 44-A38.

At the gate section $x = L$, the pressure equation becomes

$$P = P_0 + \frac{waV_0}{g}\left[1_{t>0} + 2\sum_{n=1}^{\infty}(-1)^n\, 1_{t>\frac{2nL}{a}}\right] \tag{9}$$

At the reservoir when $x = 0$, the velocity equation becomes

$$V = V_0\left[1_{t>0} - 2\sum_{n=1}^{\infty}(-1)^{n+1}\, 1_{t>\frac{(2n-1)L}{a}}\right] \tag{10}$$

The graphs of Eqs. (9) and (10) are given in Fig. 5 and, as will be seen immediately,

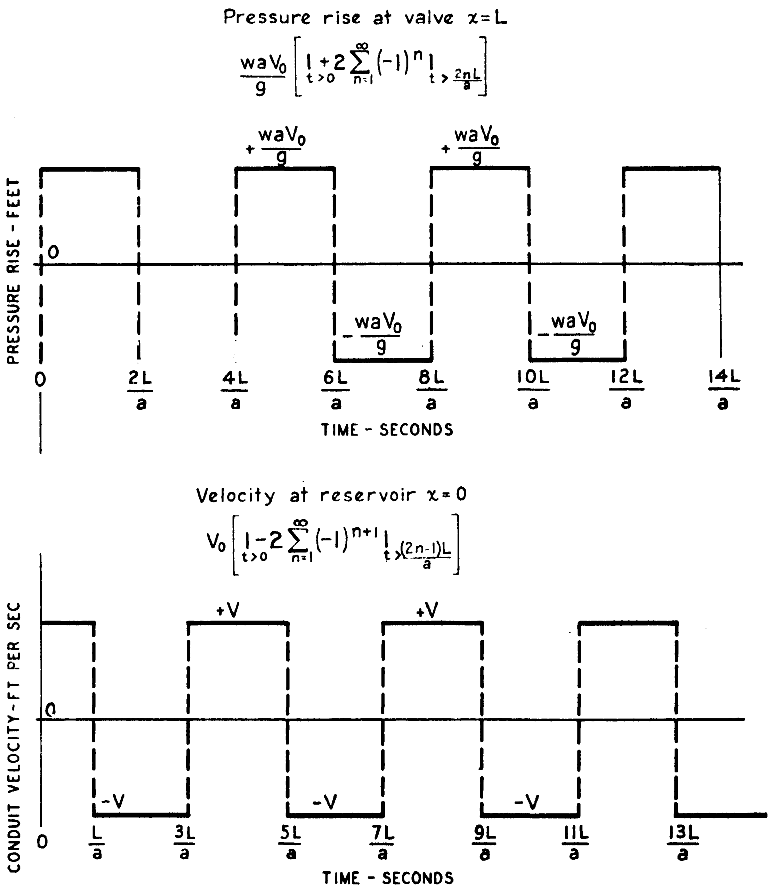


FIG. 5.—Diagrams of pressure and velocity—simple conduit—instantaneous valve closure. Conduit friction neglected.

are a complete confirmation of the physical process described qualitatively in the introduction (Fig. 1).

The sequence of wave propagation is summarized in the general rule that, at the reservoir, pressure waves are reflected with opposite sign and velocity waves with the

same sign. At the control valve, pressure waves are reflected with the same sign and velocity waves with opposite sign.

Since the effect of the damping agent, friction, has not been included in the preceding analysis, the wave pattern shown in Fig. 5 will, as indicated by Eqs. (9) and (10), continue indefinitely with the time without attenuation. To illustrate the man-

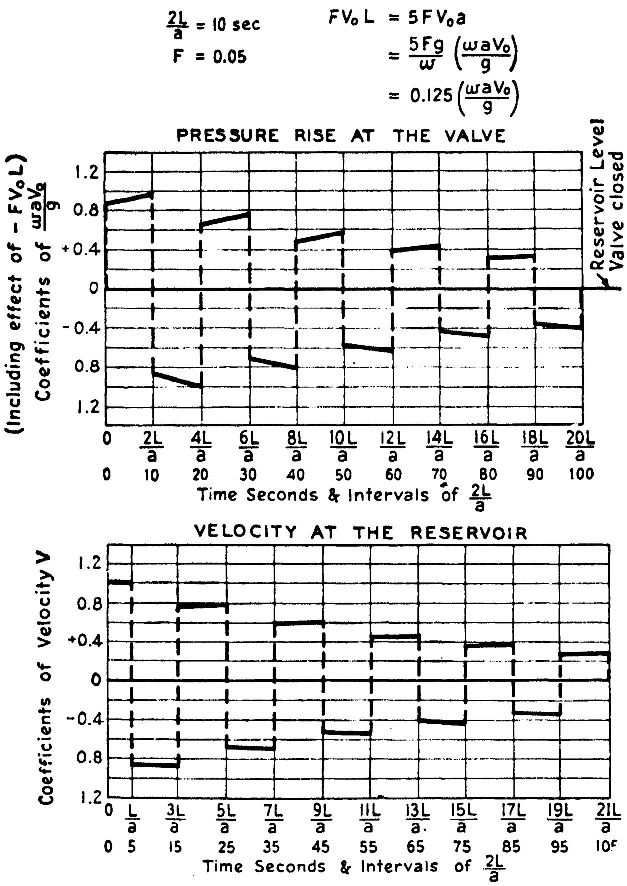


FIG. 6.—Pressure rise and velocity—simple conduit—instantaneous valve closure. Conduit friction included.

ner in which friction operates to damp down the wave motion, Fig. 6 has been calculated, based upon the solution of Eqs. (6) and (2),

$$\frac{\partial P}{\partial x} + FV + W\frac{\partial V}{\partial t} = 0 \tag{6}$$

$$\frac{\partial V}{\partial x} + Q\frac{\partial P}{\partial t} = 0 \tag{2}$$

by the technique described in the current literature.[1] In this particular example, the conduit friction coefficient has been magnified far beyond any normal occurrence in order to accentuate the damping effect and make it more perceptible. In the great

[1] *Ibid.*

majority of cases ordinarily encountered in practice, the rate of attenuation would be so slow as to have no sensible effect upon the magnitude of water-hammer pressures and velocities, and it is of course for this very reason that it is almost invariably neglected in practical computation.

Unfortunately direct analytical solution of the basic differential equations for other than instantaneous closures is not possible when, as is customary, the time rate of decrease of valve opening with efflux velocity proportional to the square root of the steady state plus water-hammer head is specified as one of the boundary conditions.[1] In fact it is because of this very impediment that the seemingly academic case of instantaneous valve closure becomes of such importance from the standpoint of practical detailed computation, by virtue of the assumption that the net actual pressure rise resulting from any gradual noninstantaneous closure may, with sufficient accuracy, be considered as consisting of a series of superimposed elemental square-topped waves, each generated by a small component element of valve movement in a correspondingly short interval of time.

The only difference then remaining for the case of more gradual closures is to take proper account of the effect of the partially open gate valve upon the reflected waves arriving from the reservoir at the downstream terminal section. As would naturally be expected under such conditions, the wave elements from the reservoir will be only partially reflected, the degree of reflection being dependent upon the relative amount of valve opening, the fully closed valve giving complete reflection.

The additional physical principle not yet utilized in our discussion, which controls and establishes the magnitude of wave reflection, is the discharge law corresponding to the setting at the particular instant of the terminal discharge mechanism. In the simplest case, that of a gate valve system in which the nozzle area is reduced according to a given function of the time, frequently a linear relation, the efflux velocity is assumed to follow the simple orifice law. When the terminal mechanism is a pump or turbine, the efflux velocity depends upon the machine speed, as well as the gate opening and total head, and must be taken from performance curves of the machine for conditions corresponding to the particular instant. In this case, algebraic formulation of the discharge law is not feasible.

The practical details of inserting this discharge mechanism law will be developed in greater detail in the illustrative examples given in the subsequent paragraphs. The only point to be emphasized at this juncture is that absolutely no additional compensation in the form of reflection factors is necessary. The truth of this statement can be verified by independent computation using the reflection factor method,[2] and also by deriving formulas for representative cases independently by the method of differential equations.[3] Since this feature has been one of the most troublesome in contemporary literature, it will not be superfluous to add that all derivations of reflection factors at the open control valve reduce simply to restatements of the orifice law and were introduced originally only in the attempt to obviate a portion of the trial-and-error detail in some forms of computation.

We have now forged all the basic tools required and may proceed to their application in various detailed methods of numerical calculation, the first, and in many respects the simplest, being that of arithmetic integration.

[1] *Ibid.* By prescribing the efflux velocity pattern, instead of the time rate of decrease of valve opening, direct analytical solution may be effected and general formulas obtained. Although this method appears to hold promise of future development, it cannot yet be considered to have advanced beyond the stage of research to general acceptance by practicing engineers.

[2] *A.S.M.E.–A.S.C.E. Symposium on Water Hammer*, 1933. GLOVER, ROBERT E., Computations of Water-hammer Pressures in Compound Pipe, p. 66, Eq. (8), *et seq.*; *ibid.*, discussion of F. Knapp, p. 129 following Eq. (2); *ibid.*, High-speed Penstock Design, by A. W. K. Billings, O. H. Dodson, F. Knapp, and A. Santos, Jr., p. 38, Eq. (8), *et seq.*, and Fig. 4, p. 66.

[3] RICH, *op. cit.*

4. METHOD OF ARITHMETIC INTEGRATION

Water-hammer analysis by the method of arithmetic integration requires a comparative minimum of specialized technique and yet affords a direct and incisive means of studying every physical element of the process. To arrange the basic equations in the form most advantageous for step computation, we shall first write Eq. (1), using the notation of heads rather than pressures, and shall replace differentials by finite increments.

$$\Delta V = -g \frac{\Delta h \, \Delta t}{\Delta x} \tag{11}$$

$$Q = A(V_0 + \Sigma \, \Delta V) \tag{12}$$

For convenience, we shall select for the value of interval Δt some submultiple of L/a such as $L/4a$; then Δx will be the length of conduit traversed by the water-hammer wave in time Δt.

For the sake of simplicity in explanation, it will be assumed that the terminal mechanism is simply a gate valve in which the nozzle area is decreased linearly with the time. For the case of nonlinear decrease of valve area, an obvious appropriate modification will be made in the calculation; and for the case of a turbine or pump as a terminal mechanism the same general procedure will be followed except that the trial values of discharge would have to be taken from performance curves at the correct machine speed for the particular instant, head, and gate.

For linear reduction of valve area the discharge at the outlet at any instant will be

$$Q = \frac{Q_0 \times A_v \sqrt{H_0 + \Sigma \, \Delta h}}{A_{v_0} \sqrt{H_0}} \tag{13}$$

assuming constant values of discharge coefficients at all valve openings.

$\Sigma\Delta h$ is the sum of all direct and reflected (positive and negative) pressures up to and including the instant in question.

The mechanism of computation then follows directly from the conception of integration as a process of summation. By trial and error, we simply establish for each successive interval values of h and V that satisfy Eqs. (11), (12), and (13).

With respect to the pertinent question of partial reflections at the valve, we find that by arranging our intervals as even submultiples of L/a we shall, after time $2L/a$ has elapsed, have a reflection arriving from the reservoir at exactly each instant when we are completing the execution of one of our small gate movements. Since friction is neglected, every new elementary square-topped wave that is generated will continue its cycle of reflection according to Fig. 1, forever without diminution, except for the influence of partial reflection.

Now let us carefully examine the illustrative example, Table 1, in complete detail to satisfy ourselves that we actually do make the requisite correction to each reflected wave to compensate for partial reflection at the partly open control valve. For this purpose we select the interval at time 4.17 sec. At that time the wave originally generated at time 0.83 has returned to the valve with negative sign and we multiply it by 2 in column 7, because waves reaching the control valve are reflected upstream with the same algebraic sign.

If it were not for the arrival of this reflection, we should make the computation at time 4.17 sec exactly as we would in the case of the first four intervals. Then, to balance the trial-and-error computation, the value of Δh would be 80 instead of 105 and $\sqrt{H_0 + \Sigma \, \Delta h}$ would be $\sqrt{1{,}281 + 80}$ instead of $\sqrt{1{,}281 + 105 - 124}$. In other words, we have increased the value of Δh by 25 ft, and the negative wave reflected back upstream has the magnitude (62 − 25), or 37 ft instead of 62 ft; that is, about 60 per cent of the wave is reflected.

TABLE 1.—ARITHMETIC INTEGRATION—SIMPLE CONDUIT (FOR LINEAR DECREASE OF VALVE AREA)

Conduit length L = 5,000 ft	Initial conduit velocity V = 12 fps
Conduit diameter = 15 ft	Wave velocity a = 3,000 fps
Conduit area = 177 sq ft	Initial head = 1,000 ft
Valve closure time = 10 sec	Initial Q = 2,120 cfs

For Δt = 0.83 sec,

$$\Delta V = \frac{\Delta h \times 32.2 \times 0.83}{2{,}500}$$

or

$$\text{Column (4)} = \frac{\text{Column (3)} \times 32.2 \times 0.83}{2{,}500}$$

$$\text{Column (8)} = \text{previous } (H_0\Delta h) + \text{Column (3)} + \text{Column (7)}$$

$$\text{Column (10)} = \frac{2{,}120 \times \text{Column (9)} \times \sqrt{\text{Column (8)}}}{\sqrt{1{,}000}}$$

Note on calculation of reflection: Reflected wave component at time t = 4.17 sec equals direct wave component at time t = 0.83 sec × (−2).

Reflected wave component at time t = 7.50 sec equals direct wave component at time t = 0.83 sec × (+2), added to direct wave component at time t = 4.17 sec × (−2).

Reflected wave component at time t = 11.67 sec equals minus twice the sum of the direct wave component at time t = 0.83 sec plus the direct wave component at time t = 1.67 sec; plus twice the sum of the direct wave component at time t = 4.17 sec plus the direct wave component at time t = 5 sec; minus twice the sum of the direct wave component at time t = 7.50 sec plus the direct wave component at 8.33 sec.

(1) Time, sec	(2) Interval Δt, sec	(3) Δh at valve, ft	(4) Δv, fps	(5) V, fps	(6) $Q = 177V$, cfs	(7) $2\Delta h$ reflected from reservoir, ft	(8) Total head $H_0 + h$, ft	(9) Valve opening	(10) Cfs	(11) Column (8)—1,000 ft
0.00	....	.....	....	12.000	2,120	....	1,000.0	1.000	2,120	0
0.83	0.83	62.0	−0.661	11.339	2,010	....	1,062.0	0.916	2,008	62.0
1.67	0.84	65.0	−0.703	10.636	1,885	....	1,127.0	0.834	1,880	127.0
2.50	0.83	72.0	−0.770	9.866	1,745	....	1,199.0	0.750	1,747	199.0
3.33	0.83	82.0	−0.880	8.986	1,592	....	1,281.0	0.667	1,595	281.0
4.17	0.84	105.0	−1.140	7.846	1,390	−124.0	1,262.0	0.583	1,390	262.0
5.00	0.83	110.0	−1.175	6.671	1,180	−130.0	1,242.0	0.500	1,180	242.0
5.83	0.83	110.0	−1.175	5.496	973	−144.0	1,208.0	0.417	971	208.0
6.67	0.84	108.0	−1.171	4.325	764	−164.0	1,152.0	0.333	760	152.0
7.50	0.83	103.0	−1.102	3.223	571	− 86.0	1,169.0	0.250	573	169.0
8.33	0.83	100.0	−1.068	2.155	382	− 90.0	1,179.0	0.167	384	179.0
9.17	0.84	98.0	−1.058	1.907	194	− 76.0	1,201.0	0.083	194	201.0
10.00	0.83	102.5	−1.097	0	0	− 52.0	1,251.5	0	0	251.5
11.67	1.67	Valve closed at time t = 10.0 sec				−230.0	1,021.5	0	0	21.5
13.33	1.66	.....	....	......	.....	−273.0	748.5	0	0	−251.5
15.00	1.67	.....	....	......	.....	230.0	978.5	0	0	− 21.5
16.67	1.67	.....	....	......	.....	273.0	1,251.5	0	0	251.5
18.33	1.66	.....	....	......	.....	−230.0	1,021.5	0	0	21.5
20.00	1.67	.....	....	......	.....	−273.0	748.5	0	0	−251.5
21.67	1.67	.....	....	......	.....	230.0	978.5	0	0	− 21.5
23.33	1.66	.....	....	......	.....	273.0	1,251.5	0	0	251.5

If we chose, we could arrange the computation to show 80 − 37, or 43 ft, as the net head increment; carry it forward as 43 ft in the computation and enter the corresponding reflection $2L/a$ sec later at that value. We deliberately elect to make the entries separately and carry forward the reflection from each newly generated wave forever, simply because experience appears to indicate that we are better able to keep track of waves this second way and that fewer mistakes result. It will be apparent, upon examination, that the two methods of bookkeeping are in every respect equivalent; that is, 105 − 62 = 43 and 80 − 37 = 43.

One of the most important fields of application of water-hammer analysis by the arithmetic integration method is in the computation of speed variation[1] in turbines and pumps during opening or closure of the control valve. In a typical case, that of sudden loss of load by a hydroelectric generating unit, a certain definite interval of time is required to close the turbine gates; and during this time the energy of water flowing through the turbine is absorbed by the WR^2 of the generator and manifested as an increase in speed of rotation. As previously noted, this speed of rotation, head and gate opening at the particular instant determine the so-called setting of the turbine with respect to discharge. A solution of the problem reduces to a matter of trial-and-error balance between water hammer, power input to the machine, WR^2 of the generator, and instantaneous values of head, gate, and discharge.

In handling compound or branched pipes by the arithmetic integration method, the same general form of tabulation is applicable, and the only additional essential difference is that provision must be incorporated in the tabulation to facilitate keeping accurate track of wave reflections. It is believed that this is best arranged to suit the preferences of the particular computer. In this connection it has been the experience of the author that greater accuracy and much better understanding of the computation generally results from writing the actual basic relationships at each junction point rather than placing blind dependence upon general formulas for reflection and transmission coefficients. Only two elementary hydraulic principles are involved at junctions: (1) the total hydraulic pressure (steady state plus surge head) is the same for all branches and (2) to comply with hydraulic continuity, the sum of the discharges of the branches terminating in a junction point is zero, flows toward the junction point being counted positive, and flows away from the junction point being counted negative. The water-hammer wave velocities in each of the component elements terminating in a junction point will naturally reflect the diameters, wall thicknesses, and elastic moduli of the individual branches.

In general practice it has been customary in computation to substitute for the compound pipe a simple pipe of equivalent characteristics determined from the equations:

$$V = \frac{V_1L_1 + V_2L_2 + V_3L_3 + \cdots V_nL_n}{L}$$

$$a = \frac{a_1L_1 + a_2L_2 + a_3L_3 + \cdots a_nL_n}{L}$$

For comparatively slow rates of valve closure, this artifice does not appear to introduce any substantial error, but for rapid closures, particularly those approaching instantaneous closure, there is a marked discrepancy between the true magnitudes of water hammer and the superpressures yielded by use of the above equation.[2] In such cases it will generally be worth while to make more accurate determinations, taking full account of partial reflections at points in the conduit where the wave velocity changes.

[1] Strowger, E. B., and S. L. Kerr, Speed Changes of Hydraulic Turbines and Sudden Changes of Load, *Trans. A.S.M.E.*, 1926, p. 220, Table I.

[2] Rich, *op. cit.* *A.S.M.E.-A.S.C.E. Symposium on Water Hammer*, 1933. Billings, Dodson, Knapp, and Santos, *op. cit.*, p. 58, Fig. 8.

5. ANGUS GRAPHICAL METHOD

The graphical method developed by Prof. R. W. Angus and predicated upon the so-called interlocked series of Allievi furnishes an exceptionally rapid and easily understood means of solving the basic differential equations. To facilitate cross reference with basic literature, the differential equations in Art. 2 are based upon taking $x = 0$ at the reservoir and $x = L$ at the control valve. However, the literature dealing with the Allievi and Angus methods employs $x = L$ at the reservoir and $x = 0$ at the control valve, so that we shall use the latter convention in Arts. 5 and 6. This is equivalent in Eqs. (1) and (2), Art. 2, to replacing x by $L - x$, and since the literature for Arts. 5 and 6 uses the notation of heads rather than pressures, we shall also replace P by wH. This gives the basic equations in the following form:[1]

$$-\frac{\partial H}{\partial x} = \frac{1}{g}\frac{\partial V}{\partial t} \tag{14}$$

$$-\frac{\partial V}{\partial x} = \frac{g}{a^2}\frac{\partial H}{\partial t} \tag{15}$$

As can readily be verified by differentiation, the so-called general integral solution of this pair of partial differential equations[2] is

$$H - H_0 = F\left(t - \frac{x}{a}\right) + f\left(t + \frac{x}{a}\right) \tag{16}$$

$$V_0 - V = \frac{g}{a}F\left(t - \frac{x}{a}\right) - f\left(t + \frac{x}{a}\right) \tag{17}$$

valid for every x along the entire conduit. It is well known that F and f[3] represent two traveling pressure heads propagated in the upstream and downstream directions with constant velocity a and without mutual interference.

Now, in these two preceding equations let us place $x = 0$ to obtain conditions at the control valve. Then, in view of the relatively high velocity of wave propagation, we may, with no practical sacrifice in accuracy, conceive the gate motion to consist of a series of partial gate movements executed at intervals $\frac{2L}{a}, \frac{4L}{a}, \frac{6L}{a}, \frac{8L}{a}$. Since, in the first interval $t_1 < \frac{2L}{a}$, no negative reflection will have returned from the reservoir, the pressure head will be due to the direct wave alone. Now, write Eqs. (16) and (17) to represent conditions at the control valve during these successive intervals.[3]

$$\begin{aligned} H_1 &= H_0 + F_1 \\ H_2 &= H_0 + F_2 - F_1 \\ H_3 &= H_0 + F_3 - F_2 \\ &\cdots\cdots\cdots \end{aligned} \tag{18}$$

$$\begin{aligned} V_1 &= V_0 - \frac{g}{a}F_1 \\ V_2 &= V_0 - \frac{g}{a}(F_1 + F_2) \\ V_3 &= V_0 - \frac{g}{a}(F_2 + F_3) \\ &\cdots\cdots\cdots \end{aligned} \tag{19}$$

[1] ALLIEVI, LORENZO, "The Theory of Water Hammer," translated by Eugene E. Halmos, Rome, 1925, Eqs. IV and V, p. IX.

[2] ANGUS, R. W., Simple Graphical Solution for Pressure Rise in Pipes and Pump Discharge Lines, *Eng. Jour.* of the Engineering Institute of Canada, February, 1935, p. 74, Eqs. (6), (7), and (8). Article 5 is very largely an abstract of this excellent paper.

[3] ALLIEVI, *op. cit.*, p. 6; WEBSTER, *loc. cit.*

Eliminating $F_1, F_2, F_3 \ldots$ from Eqs. (18) and (19), we obtain

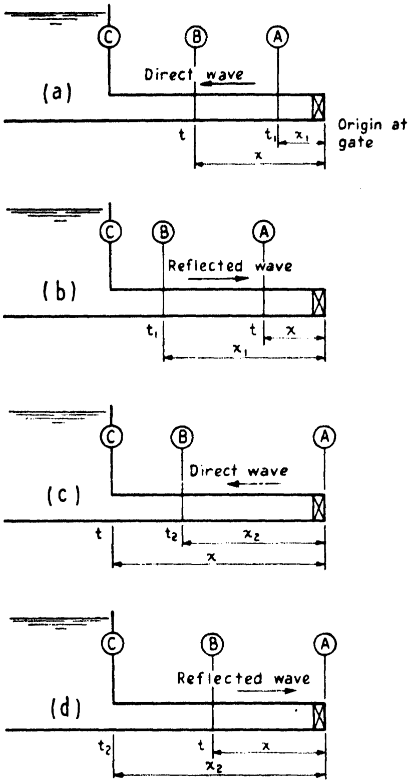


Fig. 7.—Basic notation. Angus graphical method.

$$H_1 - H_0 = \frac{a}{g}(V_0 - V_1)$$

$$H_1 + H_2 - 2H_0 = \frac{a}{g}(V_1 - V_2)$$

$$H_2 + H_3 - 2H_0 = \frac{a}{g}(V_2 - V_3)$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$H_n + H_{n-1} - 2H_0 = \frac{a}{g}(V_{n-1} - V_n) \tag{20}$$

Equation (20) gives the relation between values of pressure and velocity at the start of each successive interval $2L/a$. From the general character of the wave motion and Eqs. (16) and (17) we perceive that the function F, the sum of all direct or positive pressures at the time t taken at the section x along the conduit is identical with the same function F taken at the gate at a time x/a sec earlier. Correspondingly, the sum of all reflected or negative pressures at the section x for time t is the same as the function f at the gate x/a sec later. While Eq. (20) applies only to pressure rise at the valve at the ends of two successive intervals of $2L/a$, Eqs. (16) and (17) apply to any point on the pipe.

First, adding Eqs. (16) and (17), we obtain

$$H - H_0 = -\frac{a}{g}(V_0 - V) + 2F\left(t - \frac{x}{a}\right) \tag{21}$$

Second, subtracting Eqs. (16) and (17),

$$H - H_0 = +\frac{a}{g}(V_0 - V) + 2f_1\left(t + \frac{x}{a}\right) \tag{22}$$

Let A and B be any two sections on the pipe, as shown in Fig. 7a. Equation (21) then gives

$$H_{Bt} - H_{Bo} = -\frac{a}{g}(V_{Bo} - V_{Bt}) + 2F\left(t - \frac{x}{a}\right) \tag{23}$$

$$H_{At1} - H_{Ao} = -\frac{a}{g}(V_{Ao} - V_{At1}) + 2F\left(t_1 - \frac{x_1}{a}\right) \tag{24}$$

Since the conduit is of uniform diameter

$$V_{Ao} = V_{Bo}$$

and, since the velocity head is assumed negligible,

$$H_{Ao} = H_{Bo}$$

It is also apparent from Fig. 7a that

$$t - t_1 = \frac{x - x_1}{a}$$

so that

$$F\left(t - \frac{x}{a}\right) = F\left(t_1 - \frac{x_1}{a}\right)$$

We then obtain by subtracting Eq. (24) from Eq. (23),

$$H_{Bt} - H_{At1} = +\frac{a}{g}(V_{Bt} - V_{At1}) \tag{25}$$

By the same type of reasoning, the reflected wave Eq. (22) and Fig. 7b afford the following:

$$H_{Bt1} - H_{Bo} = +\frac{a}{g}(V_{Bo} - V_{Bt1}) + 2f\left(t_1 + \frac{x_1}{a}\right) \tag{26}$$

$$H_{At} - H_{Ao} = +\frac{a}{g}(V_{Ao} - V_{At}) + 2f\left(t + \frac{x}{a}\right) \tag{27}$$

From Fig. 7b,

$$t - t_1 = \frac{x_1 - x}{a}$$

or

$$t + \frac{x}{a} = t_1 + \frac{x_1}{a}$$

and subtracting Eq. (26) from Eq. (27) gives

$$H_{At} - H_{Bt1} = -\frac{a}{g}(V_{At} - V_{Bt1}) \tag{28}$$

If we are analyzing three points on the pipe, A, B, and C in Fig. 7c, the following equations may be written by means of Eqs. (28) and (25):

Direct wave, Fig. 7c: $$H_{Bt} - H_{Ct2} = -\frac{a}{g}(V_{Bt} - V_{Ct2}) \tag{29}$$

Indirect wave, Fig. 7d: $$H_{Ct} - H_{Bt2} = +\frac{a}{g}(V_{Ct} - V_{Bt2}) \tag{30}$$

Equations (29) and (30) are also applicable if the pipes AB and BC are of different diameters, the only change being in the value of a, which is different for each section, it being as usual assumed that the difference in velocity heads in the two sections is negligible compared to H_0.

Equations (25), (28), (29), and (30), used in conjunction with the law correlating gate motion, head H, and velocity V, permit us to solve all problems.

It is generally more convenient to use ratios H/H_0 and V/V_0 instead of H and V, so that we shall denote the former by h and the latter by v. Following Allievi's notation the quantity $aV_0/2gH_0$ will be designated ρ. Dividing Eq. (25) by H_0 gives

$$\frac{H_{Bt}}{H_0} - \frac{H_{At1}}{H_0} = \frac{aV_0}{gH_0}\left(\frac{V_{Bt}}{V_0} - \frac{V_{At1}}{V_0}\right)$$

or

$$\left.\begin{array}{lll} h_{At1} - h_{Bt} = +2\rho(v_{At1} - v_{Bt}) & \text{Fig. 11}a \\ h_{Bt1} - h_{At} = -2\rho(v_{Bt1} - v_{At}) & \text{Fig. 11}b \\ h_{Ct2} - h_{Bt} = -2\rho(v_{Ct2} - v_{Bt}) & \text{Fig. 12}b \\ h_{Bt2} - h_{Ct} = +2\rho(v_{Bt2} - v_{Ct}) & \text{Fig. 12}a \end{array}\right\} \tag{31}$$

Now the relation between gate setting and pipe velocity is obviously

$$\frac{V}{V_0} = E\sqrt{\frac{H}{H_0}} \qquad \text{or} \qquad v = E\sqrt{h}$$

where E varies only with the gate setting and is usually assumed to vary linearly with the time, starting with $E = 1$ for full gate.

Equations (31) are the loci of two series of parallel lines with slopes equal to $\tan + 2\rho$ and $\tan - 2\rho$. Conditions at the reservoir are always represented by some

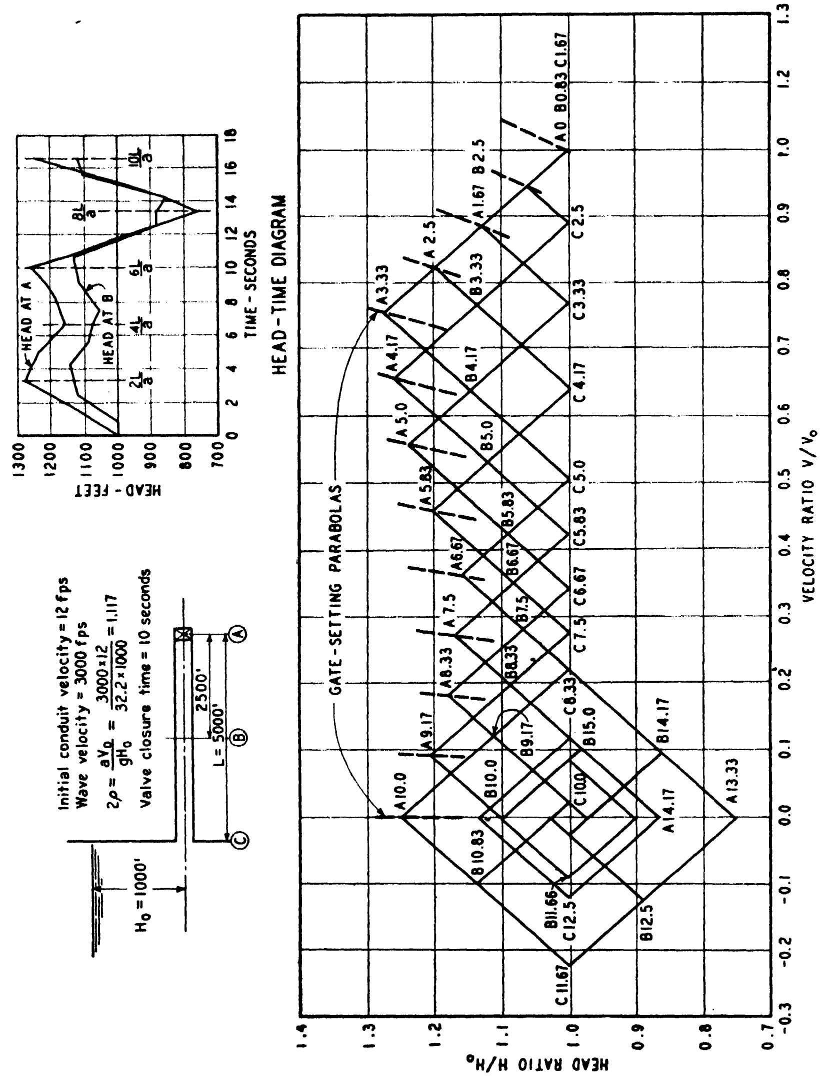


Fig. 8.—Graphical analysis—simple conduit—gradual closure. Conduit friction neglected.

point on the horizontal line $Y = 1$, because there $\frac{H}{H_0} = 1$, if conduit friction is neglected. Conditions at the valve will be given by the intersection of the lines of negative slope with the curves representing the orifice discharge relationship.

Now let us apply the theory we have developed to solution of the identical problem solved by arithmetic integration in Table 1 of the preceding article, regarding the graphical construction, Fig. 8, not as an exercise in drafting but rather as applied

analytic geometry, which enables us to make rapid solution of many pairs of simultaneous equations.

We first plot the parabolas correlating gate opening, head, and velocity for the times $t = 3.33$ sec or $2L/a$ and 6.67 sec or $4L/a$. Initial conditions are represented by the point $h = 1$, $v = 1$, and this will obviously hold true for $B_{0.83}$ and $C_{1.67}$ as well as A_0. Our first equation will be written between $C_{1.67}$ and $A_{3.33}$:

$$h_{C1.67} - h_{A3.33} = -2\rho(v_{C1.67} - v_{A3.33})$$

To plot this equation we draw a straight line of negative slope from the point $h = 1$, $v = 1$, or $C1.67$, to intersect the parabola covering all possible simultaneous values of head and discharge for the gate setting at time $t = 3.33$ sec or $2L/a$, the equation of this parabola being $v = 0.667\sqrt{h}$. The intersection then gives the simultaneous solution of the respective linear and second-degree equations and so establishes the correct relative values of head h and velocity v at $t = 3.33$. We mark the point $A_{3.33}$.

In similar fashion the second equation may conveniently be written between A at time $t = 3.33$ sec and C at 5.00 sec:

$$h_{A3.33} - h_{C5.00} = +2\rho(v_{A3.33} - v_{C5.00})$$

We draw the straight line representing this equation starting from $A_{3.33}$ and produce it downward with a positive slope until it intersects the horizontal axis, which, since conduit friction is neglected, represents all possible relative values of head $h = 1.00$ and velocity v. The intersection will establish conditions at the reservoir and of the line for $t = 5.00$ sec.

We next repeat the process for the equations

$$h_{C5.00} - h_{A6.67} = -2\rho(v_{C5.00} - v_{A6.67})$$

and

$$h_{A6.67} - h_{C8.33} = +2\rho(v_{A6.67} - v_{C8.33})$$

using the intersection of the first of these equations with the gate-setting parabola for $t = 6.67$ sec.

Finally we draw the line of negative slope from point $C8.33$, representing equation

$$h_{C8.33} - h_{A10.00} = -2\rho(v_{C8.33} - v_{A10.00})$$

to intersect the vertical axis, which is the gate-setting parabola for gate opening zero, giving us the relative head ratio at the gate at the instant of closure.

The equations governing the perpetual cycles of afterwaves following valve closure are

$$\begin{aligned}
h_{A10.00} - h_{C11.67} &= +2\rho(v_{A10.00} - v_{C11.67}) \\
h_{C11.67} - h_{A13.33} &= -2\rho(v_{C11.67} - v_{A13.33}) \\
h_{A13.33} - h_{C15.00} &= +2\rho(v_{A13.33} - v_{C15.00}) \\
h_{C15.00} - h_{A16.67} &= -2\rho(v_{C15.00} - v_{A16.67})
\end{aligned}$$

giving the closed diamond-shaped figure with corners $A10.00$, $C11.67$, $A13.33$, $C15.00$, and $A16.67$.

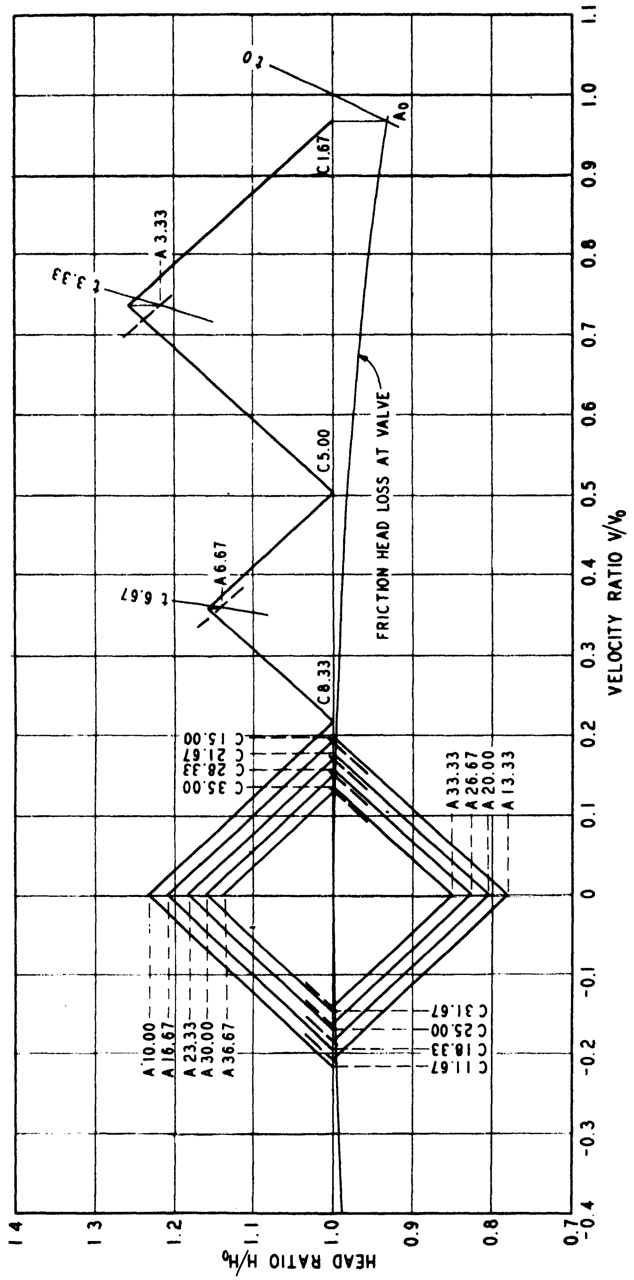
It will also be interesting to see how easily we may now obtain the transient values at the point B midway along the conduit.

The first step is to complete the gate-setting parabolas for all 12 intervals of 0.83 sec. As previously noted, the point $h = 1$, $v = 1$ will represent $A0$, $B0$, $C0$, $B0.83$, $C0.83$ and $C1.67$. To establish $A0.83$, we find the intersection of

$$h_{B0} - h_{A0.83} = -2\rho(v_{B0} - v_{A0.83})$$

with the parabola for $t = 0.83$. $A1.67$ is likewise found from the intersection of

$$h_{B0.83} - h_{A1.67} = -2\rho(v_{B0.83} - v_{A1.67})$$


HEAD RATIO H/H0
VELOCITY RATIO V/V0
FRICTION HEAD LOSS AT VALVE
C 1.67
A 3.33
C 5.00
A 6.67
C 8.33
A0
A 10.00
A 16.67
A 23.33
A 30.00
A 36.67
A 33.33
A 26.67
A 20.00
A 13.33
C 15.00
C 21.67
C 28.33
C 35.00
C 31.67
C 25.00
C 18.33
C 11.67

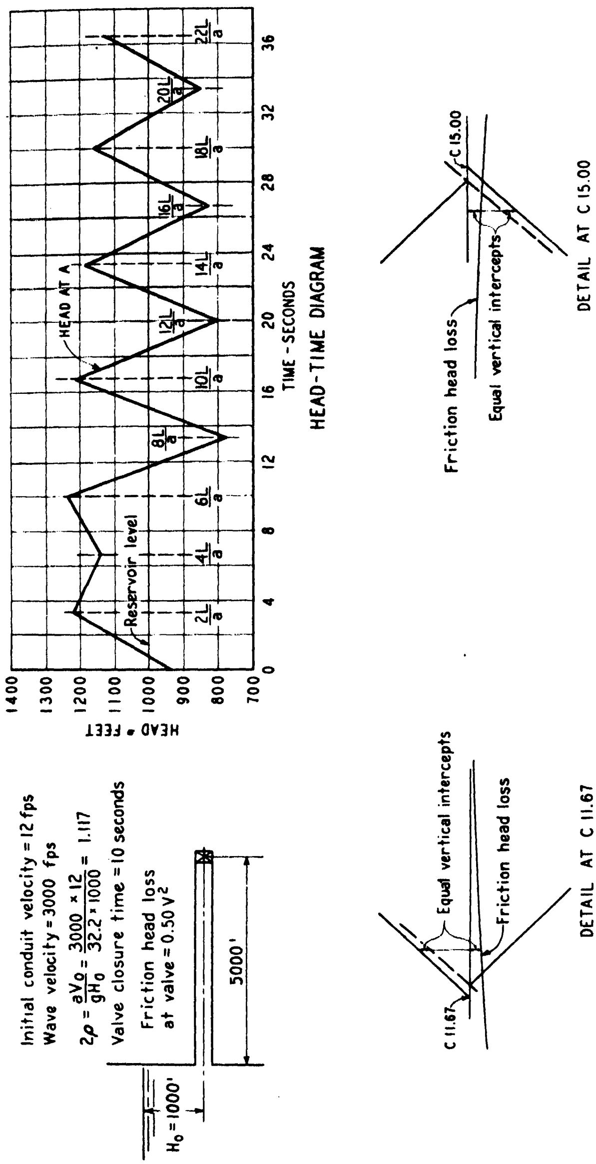


FIG. 9.—Graphical analysis—simple conduit—gradual closure. Conduit friction included.

with the parabola for $t = 1.67$. Also, $A2.50$ is determined by the intersection of

$$h_{C0.83} - h_{A2.50} = -2\rho(v_{C0.83} - v_{A2.50})$$

$C2.50$ if found from the intersection of

$$h_{A0.83} - h_{C2.50} = +2\rho(v_{A0.83} - v_{C2.50})$$

and the horizontal axis.

It is evident that $B1.67$ is the same as $A0.83$ by drawing from $C2.50$:

$$h_{B1.67} - h_{C2.50} = +2\rho(v_{B1.67} - v_{C2.50})$$

$B3.33$ is obtained from the intersection of

$$h_{A2.50} - h_{B3.33} = +2\rho(v_{A2.50} - v_{B3.33})$$

and

$$h_{C2.50} - h_{B3.33} = -2\rho(v_{C2.50} - v_{B3.33})$$

and the remaining points of the charts are obtained by a continuation of the same process.

In the present state of our technique, the direct analytical solution of the basic differential equations is the only method capable of distributing the effect of friction along the conduit in accordance with nature. The graphical method, however, enables us to obtain very satisfactory and useful results by means of the artifice of lumping the cumulative effect of friction at certain selected points, most frequently at the control valve.

As an illustration of the method of application (Fig. 9), let us incorporate the effect of conduit friction in the previous example, assuming for simplicity that the aggregate friction head loss at the valve is represented by the expression $0.50v^2$, where v is the instantaneous value of total velocity at the valve. Next plot a curve of this equation below the horizontal axis, as indicated, so that the vertical intercept will represent the aggregate relative friction-head loss at the valve for various velocity ratios.

Then point $A0$, representing initial conditions at the valve, will be located at the intersection of the valve-setting parabola for the time $t = 0$, and the friction-head loss curve. Point $C1.67$, representing conditions at the reservoir for all times up to $t = 1.67$, will then be located on the horizontal axis directly above A, the intercept representing the aggregate steady-state friction-head loss along the conduit from C to A. But as our basic system of equations has been developed on the basis of neglecting friction, we shall continue to plot our fundamental network of solid lines as representing conditions with friction effect deducted, and shall apply dotted line corrections for friction at all points A. Obviously no such correction will be necessary until we reach the zone of afterwaves for all points C.

The method will be readily understood by explaining the true location of $A3.33$. The basic equation with friction neglected is

$$h_{C1.67} - h_{A3.33} = -2\rho(v_{C1.67} - v_{A3.33})$$

So starting at $C1.67$, produce the requisite solid line of negative slope beyond the intersection with the gate-setting parabola for $t = 3.33$. Then in the vicinity of this intersection transfer the range of intercepts from the friction-head loss curve to give the dotted line. The intersection of the dotted line with the gate-getting parabola gives the true location of $A3.33$.

To proceed in determing $C5.00$ we revert to our fundamental system of solid lines by drawing the vertical line upward from $A3.33$ to intersect the first solid line $C1.67$-$A3.33$. To locate $C5.00$ we draw the line of positive slope from the point so found to

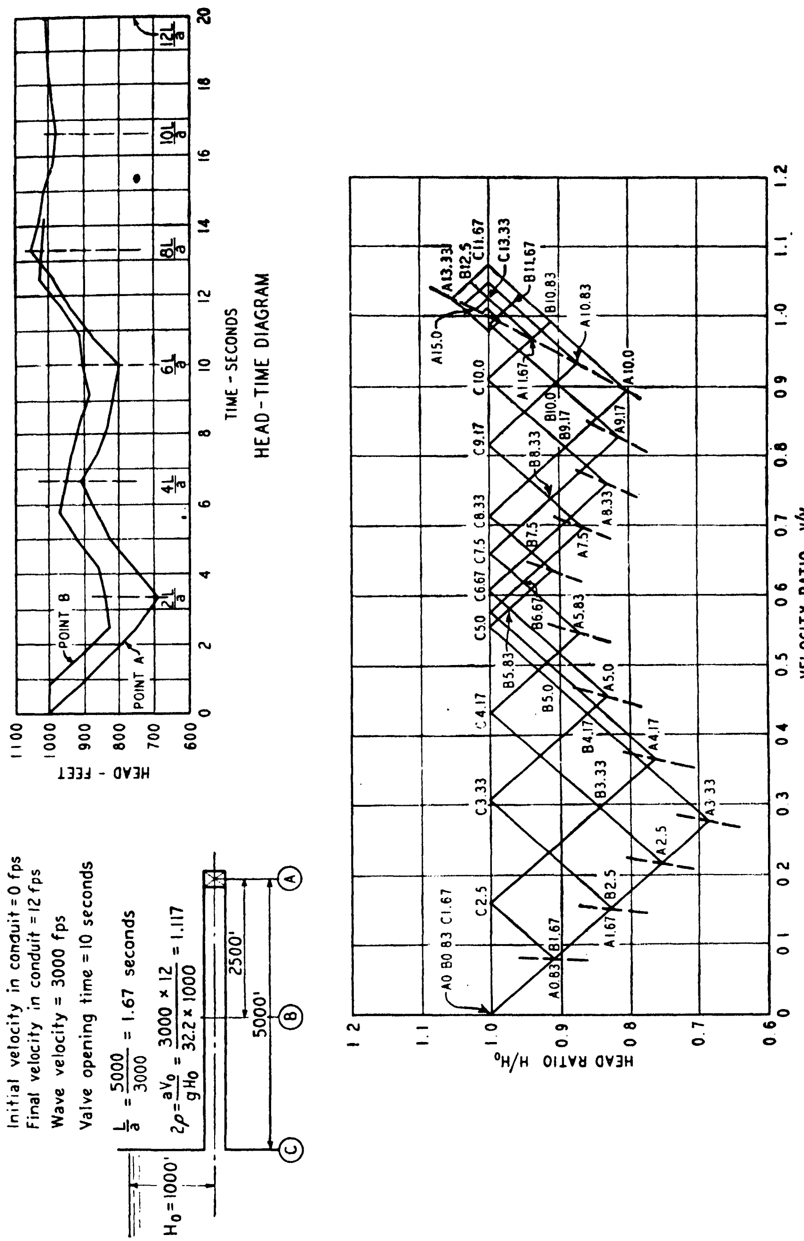


FIG. 10.—Graphical analysis—simple conduit—gradual opening. Conduit friction neglected.

intersect the horizontal axis to give the true position according to the equation

$$h_{A3.33} - h_{C5.00} = +2\rho(v_{A3.33} - v_{C5.00})$$

The remaining points are established in similar fashion.

When we reach the zone of afterwaves, the most convenient place to apply the

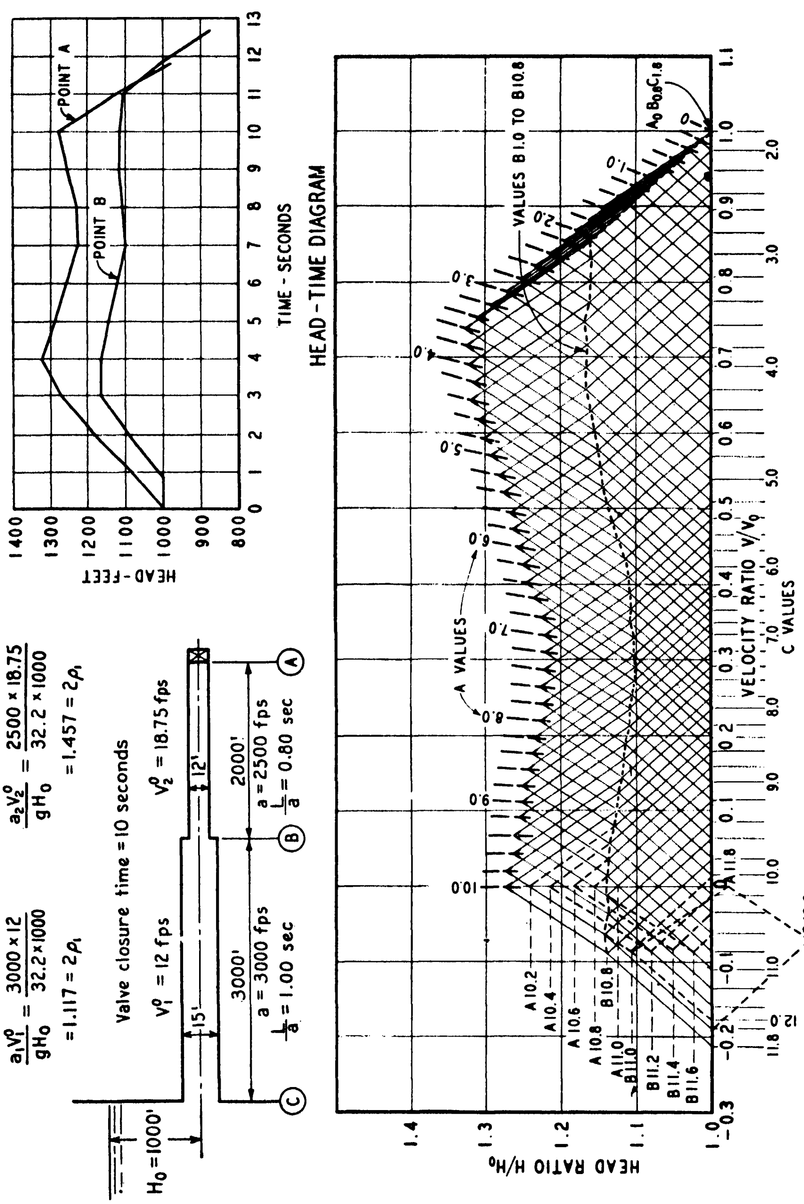


FIG. 11.—Graphical analysis—compound conduit—gradual closure. Conduit friction neglected.

lumped friction correction is to the points C, fairly close to the friction-loss intercept curve. The key suggestion to keep in mind is that friction operates to decrease the conduit velocity at C, whether the conduit flow is in the positive or the negative direction. This explains why the dotted line is drawn below the solid for $C11.67$ and above the solid for $C15.00$. A very interesting exercise to develop better understanding of the method is to check Fig. 6, remembering, of course, to replace the curve cv^2 by the assumed straight-line friction loss.

Figure 10 shows the construction for the case of the simple conduit, friction neglected, under gradual opening; while Fig. 11 illustrates the solution of the compound pipe for gradual closure with friction neglected. It will be noted at once for the latter problem that the effect of differences in wave velocity or diameter of the component sections of the pipe is simply to change the values of slope of the corresponding straight lines. The number of intersections necessary to establish the solution is noticeably greater than would be expected from comparison with the illustrations for this case usually given in textbooks. This is because the latter examples are prefabricated so that the reflection times of the component sections of the conduit are even multiples. In actual practice this is seldom the case, so that Fig. 11 represents the degree of complexity normally to be expected. If only slight changes in the conditions of the given problem are sufficient to make the reflection times of the component sections even multiples, the resultant solution is obviously very greatly simplified; but more often than not changes of substantial magnitude would be necessary, and these could not be effected without introducing serious error. Before proceeding with the analysis of the more involved problems in branched and compound system, the engineer should consult Prof. Angus' excellent article.[1]

6. ALLIEVI CHART METHOD

By a combination of the so-called interlocked series equations and a rather cumbersome graphical device since superseded by the Angus method, the great pioneer investigator, Lorenzo Allievi, developed charts that are probably still the most widely used devices for the rapid solution of the elementary conventional cases of water-hammer analysis. It should be emphasized, however, that these charts have, strictly speaking, a comparatively limited field of applicability. Their derivation is predicated upon the fulfillment of two conditions that in many cases are not even remotely approached. These fundamental assumptions are that the nozzle area at the control valve is decreased or increased linearly with the time and that the efflux velocity at the nozzle at any instant is proportional to the square root of the sum of the steady-state plus water-hammer heads.

Obviously when the terminal mechanism is a centrifugal pump or hydraulic turbine operating through a transient range of speeds, the efflux velocity follows the law of the machine performance curve for the individual mechanism and departs markedly from the conventional orifice law. In commercial turbomachines not only is the discharge dependent upon the instantaneous speed but still further complication results from the fact that the gate motion is characterized by intervals of dead and cushioning time and is accordingly not at all a linear relationship. Results obtained by the application of Allievi charts to such cases may very naturally produce wide discrepancies from the true values of transient pressure and velocity.

Nevertheless, when used within their intended zone of application, the Allievi diagrams do effect an exceedingly great saving in time and labor for determining maximum or minimum point values. Allievi's great ingenuity in the use of his so-called Cartesian synopsis enabled him to summarize the entire field of uniform linear closure by means of two basic parameters, the pipe-line constant $\rho = \dfrac{aV_0}{2gH_0}$ and the valve operation or time constant $\theta = \dfrac{aT}{2L}$. The maximum rise or fall, as the case may be, is then taken from a single reading of the applicable chart, the dotted curved lines on the chart for closure indicating the interval in which maximum point pressure occurs. The charts do not enable us to plot a time history of the water-hammer wave.

[1] ANGUS, ROBERT W., Graphical Analysis of Water Hammer in Branched and Compound Pipes *Trans. A.S.C.E.*, 1939.

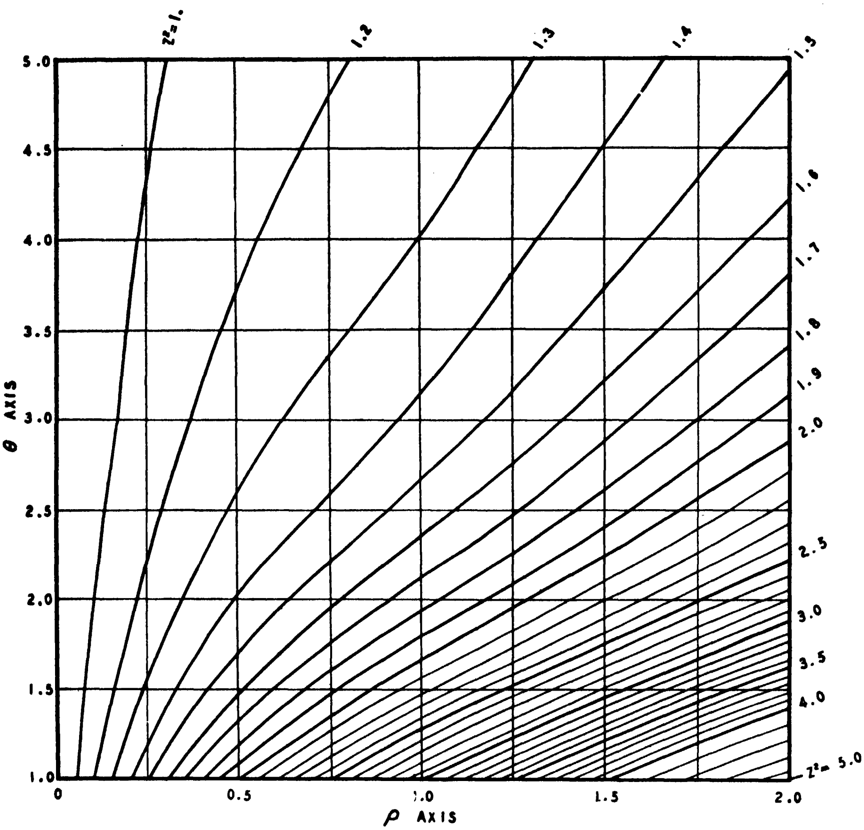


FIG. 12.—Allievi diagram. Maximum pressure rise for uniform gate motion and simple conduits (for small values of ρ and θ).

As the first illustrative example in the use of the Allievi charts, let us establish the maximum values of pressure rise for the case covered by the arithmetic integration method in Table 1 and by the Angus graphical method in Fig. 8.

Simple Conduit—Gradual Closure
Conduit Friction Neglected

Initial head H_0	1,000 ft
Initial velocity in conduit	12 fps
Water-hammer wave velocity	3,000 fps
Valve closure time	10 sec
Length of pipe line	5,000 ft

$$\text{Pipe-line constant } \rho = \frac{aV_0}{2gH_0} = \frac{3{,}000 \times 12}{2 \times 32.2 \times 1{,}000} = 0.558$$

$$\text{Valve operation constant } \theta = \frac{aT}{2L} = \frac{3{,}000 \times 10}{2 \times 5{,}000} = 3.00$$

From Fig. 12 (small values of ρ and θ),

$$Z^2 = \frac{H_0 + h_{\max}}{H_0} = \frac{1{,}000 + h_{\max}}{1{,}000} = 1.280$$

Solving, maximum pressure rise at valve $= h_{\max} = 280$ ft. Since the point $\rho = 0.56$, $\theta = 3.00$ lies

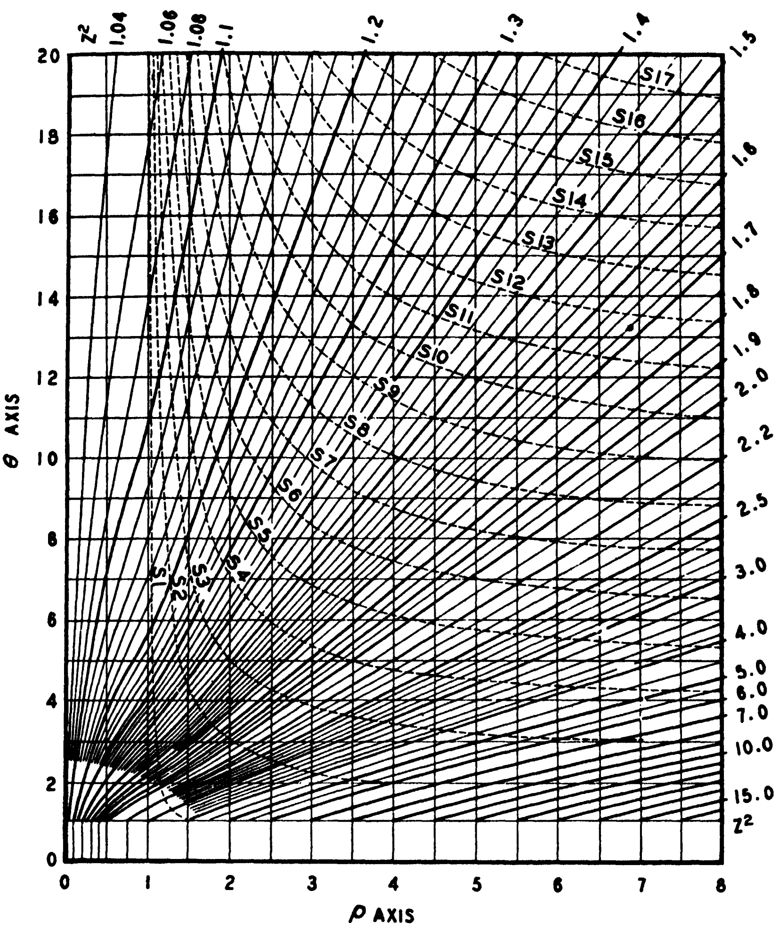


FIG. 13.—Allievi Diagram. Maximum pressure rise for uniform gate motion and simple conduits (for intermediate values of ρ and θ).

to the left of dotted line $S1$, Fig. 12, the maximum value of pressure rise occurs at some time during the first interval $2L/a$, or between $t = 0$ and $t = 3.33$ sec.

This checks very closely the values obtained by the arithmetic integration and graphical methods.

As a typical example of pressure drop during gradual opening by means of the Allievi charts, let us solve the problem given graphically in Fig. 10.

Simple Conduit—Gradual Opening
Conduit Friction Neglected

Initial head H_0	1,000 ft
Final velocity in conduit	12 fps
Water-hammer wave velocity	3,000 fps
Valve opening time	10 sec
Length of pipe line	5,000 ft

$$\text{Pipe-line constant } \rho = \frac{aV_0}{2gH_0} = \frac{3{,}000 \times 12}{2 \times 32.2 \times 1{,}000} = 0.558$$

$$\text{Valve operation constant } \theta = \frac{aT}{2L} = \frac{3{,}000 \times 10}{2 \times 5{,}000} = 3.00$$

From Fig. 15,

$$Z^2 = \frac{H_0 - h}{H_0} = \frac{1{,}000 - h}{1{,}000} = 0.700$$

The pressure drop h is = 300 ft which checks quite closely the value obtained by the graphical method.

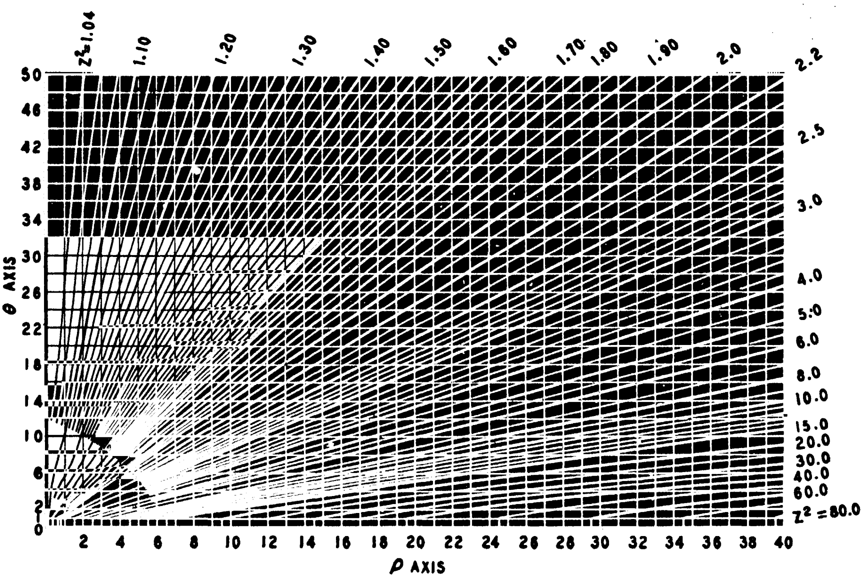


FIG. 14.—Allievi diagram. Maximum pressure rise for uniform gate motion and simple conduits (for large values of ρ and θ).

As a third and final illustrative example in the use of the Allievi charts, let us check the compound pipe analyzed graphically in Fig. 11.

Compound Conduit—Gradual Closure
Conduit Friction Neglected
(See Fig. 11)

Initial head H_0	1,000 ft
Valve closure time	10 sec
V_1	12 fps
V_2	18.75 fps
Wave velocity a_1	3,000 fps
Wave velocity a_2	2,500 fps
Length L_1	3,000 ft
Length L_2	2,000 ft

$$\text{Equivalent conduit velocity } V = \frac{V_1L_1 + V_2L_2}{L_1 + L_2}$$

$$= \frac{12 \times 3{,}000 + 18.75 \times 2{,}000}{3{,}000 + 2{,}000} = 14.7 \text{ fps}$$

$$\text{Equivalent wave velocity } a = \frac{a_1L_1 + a_2L_2}{L_1 + L_2}$$

$$= \frac{3{,}000 \times 3{,}000 + 2{,}500 \times 2{,}000}{3{,}000 + 2{,}000} = 2{,}800 \text{ fps}$$

$$\text{Pipe-line constant } \rho = \frac{aV_0}{2gH_0} = \frac{2{,}800 \times 14.7}{2 \times 32.2 \times 1{,}000} = 0.64$$

$$\text{Valve operation constant } \theta = \frac{aT}{2L} = \frac{2{,}800 \times 10}{2 \times 5{,}000} = 2.80$$

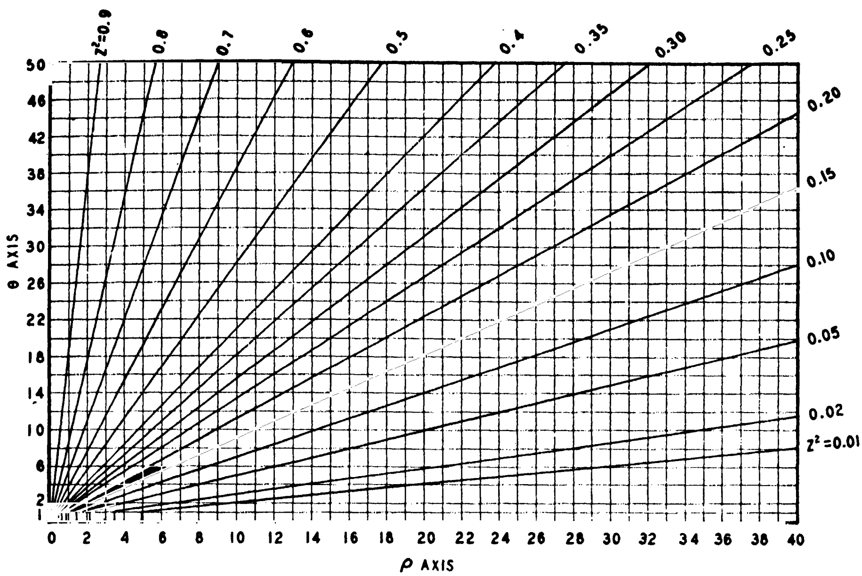


FIG. 15.—Allievi diagram. Maximum fall in pressure for uniform gate motion and simple conduits.

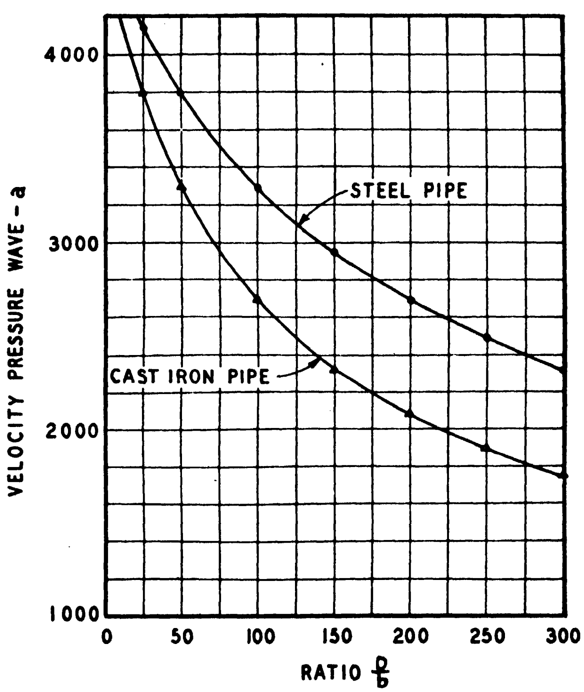


FIG. 16.—Chart for calculation of velocity of pressure wave in cast-iron and steel pipe.

From Fig. 12 (small values of ρ and θ),

$$Z^2 = \frac{H_0 + h_{max}}{H_0} = \frac{1{,}000 + h_{max}}{1{,}000} = 1.320$$

Solving, the maximum pressure rise at the valve is 320 ft and occurs between the time 0 and 3.60 sec, giving a close check on the graphical analysis, Fig. 11.

There is one feature[1] of the Allievi charts that warrants particular emphasis. It will be noted that over the major portion of the diagram, Fig. 12, the radial lines giving value of Z^2 are straight, but that in the lower left-hand corner there is a pronounced break in slope. This means that in many cases closures starting from part gate with proportionally reduced velocity will produce higher water hammer than closure from the fully open position and full discharge velocity. For this reason, it is good practice to test all possible partial closures to establish the true maximum. The following example illustrates the procedure.

Simple Conduit—Gradual Closure
Conduit Friction Neglected

Initial head H_0	1,000 ft
Initial velocity in conduit	100 fps
Water-hammer wave velocity	3,000 fps
Valve closure time	33 sec
Length of pipe line	5,000 ft

First, for complete closure,

$$\text{Pipe-line constant } \rho = \frac{aV_0}{2gH_0} = \frac{3{,}000 \times 100}{2 \times 32.2 \times 1{,}000} = 4.65$$

$$\text{Valve operation constant } \theta = \frac{aT}{2L} = \frac{3{,}000 \times 33}{2 \times 5{,}000} = 9.90$$

From Fig. 12 (small values of ρ and θ),

$$Z^2 = \frac{H_0 + h_{max}}{H_0} = \frac{1{,}000 + h_{max}}{1{,}000} = 1.59$$

$$\text{Pressure rise } h_{max} = 590 \text{ ft}$$

Now, check closure from 10 per cent valve opening:

$$\text{Pipe-line constant } \rho = \frac{aV_0}{2gH_0} = \frac{3{,}000 \times 10}{2 \times 32.2 \times 1{,}000} = 0.47$$

$$\text{Valve operation constant } \theta = \frac{aT}{2L} = \frac{3{,}000 \times 3.3}{2 \times 5{,}000} = 0.99$$

From Fig. 12,

$$Z^2 = \frac{H_0 + h_{max}}{H_0} = \frac{1{,}000 + h_{max}}{1{,}000} = 2.00$$

And the pressure rise h_{max} = 1,000 ft as compared with 590 ft for closure from fully open poisition.

In foot units, the general expression for water-hammer wave velocity is from Eq. (3):

$$a = \frac{1}{\sqrt{WQ}} = \frac{1}{\sqrt{\frac{w}{g}\left(\frac{1}{K} + \frac{D}{bE}\right)}}$$

For steel pipes the value of E is 29,400,000 psi or 42.3×10^8 psf, while the bulk modulus of water is 294,000 psi or 42.3×10^6 psf. $\frac{w}{g} = \frac{62.5}{32.2} = 2.00$ closely enough.

[1] KERR, S. LOGAN, New Aspects of Maximum Pressure Rise in Closed Conduits, *Trans. A.S.M.E.*, 1928, paper HYD-51-3.

The corresponding value of the wave velocity is

$$a = \frac{4{,}660}{\sqrt{1 + \dfrac{D}{100b}}}$$

which is plotted for convenience as the curve for steel pipe, Fig. 16.

For cast-iron pipe, the value of E is 15,000,000 psi or 21.60×10^8 psf, and the corresponding value of the wave velocity is

$$a = \frac{3{,}290}{\sqrt{0.51 + \dfrac{D}{100b}}}$$

the curve of which is plotted as cast iron, Fig. 16.

For concrete pipe with an assumed modulus of 2,500,000 psi $= 3.60 \times 10^8$ psf, the wave velocity becomes

$$a = \frac{1{,}340}{\sqrt{0.085 + \dfrac{D}{100b}}}$$

For a tunnel in solid rock the value of b becomes very large, and consequently the wave velocity approaches the velocity of sound in water, 4,660 fps.

For wood-stave pipe, Strowger[1] has developed the formula

$$a = \frac{4{,}660}{\sqrt{1 + \dfrac{KD}{E_w b + E_s \phi}}}$$

where b = stave thickness, ft.

ϕ = total cross-sectional area of steel bands, sq ft/lin ft of pipe.

E_s = modulus of elasticity of steel bands in tension, 4.23×10^9 psf.

E_w = modulus of elasticity of wood staves.

British Columbia or Douglas fir	2.30×10^8 psf
Redwood or cypress	1.92×10^8 psf
White pine	1.00×10^8 psf

For old deteriorated staves, tests show values as low as 0.60×10^8 psf.

7. FIELD OF APPLICABILITY OF VARIOUS METHODS

Although selection of the method best adapted to fit the various particular problems arising in practice is very largely a matter of preference of the individual engineer, it may be worth while in summary to indicate what the author believes to be the characteristic merits of the various devices.

For all but the cases of instantaneous valve operation, direct analytical solution of the basic differential equations is possible only by prescription of the efflux velocity extinction pattern rather than the conventional time rate of decrease of valve opening with discharge velocity determined from the orifice law. In spite of requiring this departure from tradition and also the command of relatively advanced technique, this method is still believed to be the most effective instrument for research investigation and for establishing in rigorous fashion the fundamental laws of water hammer. Once the manipulative technique has been mastered, the analytical solution does have the

[1] STROWGER, E. B., Water-hammer Problems in Connection with the Design of Hydroelectric Plants, *Trans. A.S.M.E.*, 1944, paper 44-A42.

additional advantage of affording a continuous time history of pressure and velocity in compact and perfectly general formulas. Development along such lines is believed to be incipient and affords definite promise for the future.

The author has found the arithmetic integration method to be particularly incisive in those cases in which the terminal mechanism is a turbomachine and in which, therefore, the discharge characteristics depend upon the instantaneous value of gate and speed. This method also is exceedingly valuable in teaching the computer the internal mechanism of water hammer and the synthesis of total pressures from direct and reflected wave components.

The graphical method, developed very largely on this continent through the efforts of Prof. Angus, is not only one of the greatest savers of time and labor in the hydraulic field, but it has the added advantage that the underlying theory is simple and readily understood. It handles with equal ease cases in which the valve closure is uniform or even nonlinear; and although it may be used effectively for those cases in which the machine speed passes either above or below normal, the author is slightly prejudiced in the case of the latter class in favor of arithmetic integration.

As already outlined, the Allievi charts, in spite of their limitations, continue to be the most popular and widely used of all water-hammer methods; but it should always be borne in mind that they were never intended to apply, even as a rough approach, to more highly specialized cases in which the efflux velocity depends upon the instantaneous setting and speed of the terminal mechanism.

One of the cases occurring most commonly in practice is that of a penstock, surge tank, and pipe line or tunnel. The standard method of analysis is to consider the free surface of water in the tank as an open reservoir and neglect the pressure wave traveling up the pipe line or tunnel. A more accurate determination of the system as a branched conduit problem may readily be effected by means of the graphical method, Ref. 4 in the Bibliography, or by the appropriate Calame and Gaden formulas, Ref. 6.

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SECTION 14

SURGE TANKS

BY GEORGE R. RICH

1. INTRODUCTION

It is evident from Sec. 13 that adverse water-hammer effects may be mitigated easily and inexpensively only in the case of the shorter conduits or in installations in which very slow valve closure is no detriment; but it is likewise apparent that in hydraulic power installations which have any appreciable length of pipe line and which are required to hold frequency within commercial limits even when operating detached from the system, very slow turbine gate movement would not be compatible with practical economical speed regulation. If the expedient of synchronous by-pass valves were employed, which are in themselves a considerable item of expense, the conduit velocity would have to be maintained permanently at a rate sufficiently high to accommodate maximum load demand. In the case of plants having instantaneous load increases in the order of 50 per cent or higher, this last requirement would ordinarily involve a very expensive wastage of valuable water. Provision of a surge tank is the economical solution in such cases. As is well known, the tank has three primary and one concomitant functions: (1) to provide a free reservoir surface close to the terminal discharge mechanism as a quick source of compensating water-hammer reflections to limit penstock and materially reduce main conduit pressures, the initial reflections being negative in the case of closure and positive in the case of opening; (2) to supply the additional water required by the turbine during load demand until the conduit velocity has accelerated to the new steady-state value; (3) to store water during load rejection until the conduit velocity has been decelerated to the new steady-state value; (4) to make certain that the pendulation of water levels following even small as well as large load changes will be quenched positively and rapidly. In the language of the trade, the tank must pass the test for incipient stability.

Because inertia causes water to overtravel beyond the second steady-state position, the characteristic physical action of the surge tank is a pendulation of the water surface levels, although it is possible but usually not economical to obtain aperiodic or deadbeat action. Like water hammer, this mass acceleration in a surge tank is a transient phenomenon; but unlike water hammer, its periodicity is comparatively slow, requiring as much as 30 min or even longer to establish the second steady state for long conduits and large tanks, as compared to about 30 sec or less to damp out the significant portion of the water hammer under conditions commonly encountered in practice.

Because of this basic difference in periodicity, it is fortunately permissible for the purposes of computation to separate these two concurrent phenomena and to treat the surge by itself as a mass-acceleration problem. By this we mean that the velocity changes in the conduit occur so gradually that in computing the mass surge separately we may employ the simple momentum law and neglect the effect of stretching the conduit circumference and compression of the water. The water-hammer effects which are important only during the first 15 or 20 sec, and then principally in the

penstock, and only to a secondary degree in the conduit, are segregated and handled as a detached second problem. Nevertheless, we should always remember that in nature the two actions occur simultaneously and that water-hammer effects, while of important magnitude only at the outset, are in reality the direct causative agent for the mass acceleration in the same fashion that they are the direct means of effecting the velocity changes in a simple pipe, even for the slowest rates of valve movement. As an extension of this same idea, it is sometimes helpful to consider water hammer as a rapid means of telegraphing the valve operation to the reservoir.

2. DIFFERENTIAL SURGE TANK

The salient feature of the differential surge tank invented by R. D. Johnson is the separation of the water-supply or water-storage function from the conduit acceleration or deceleration function, resulting in more rapid, efficient hydraulic action and reflecting sizable economy in tank diameter and capital cost. A very important second feature is the marked damping effect or throttling action afforded by the port arrangement which gives the differential tank pronounced ability to limit and suppress the surges due to synchronous load pulsations.

Accordingly, methods of analysis of the differential surge regulator will be given in detail with but passing reference to simple, restricted orifice, and compressed-air tanks at the close of this section. However, it will soon be apparent to the reader that this seeming deletion will in no way penalize the treatment of such alternative devices. The same principles of design will be found to hold in all cases; and if the analyst is thoroughly conversant with computations for the differential tank, he will have no difficulty with the simple, the restricted orifice, or the compressed-air regulator.

3. BASIC DIFFERENTIAL EQUATIONS

Although the differential equations for the Johnson regulator are not susceptible to direct conventional solution, it will be instructive to develop them as a means of crystallizing the principal elements of hydraulic action, particularly as a prerequisite to forming tabulations for arithmetic integration, the only method capable of effecting an exact solution.

Referring to Figs. 1 and 3 and the notation tabulated in Art. 4, the following equations are true at any instant, proper regard being given to algebraic sign:

Total head on conduit = friction and allied head losses + acceleration head

$$H - h_r = cv^2 + \frac{L}{g}\frac{dv}{dt} \tag{1}$$

Discharge from outer tank = discharge through ports

$$F\frac{dh_t}{dt} = a\sqrt{2g(h_t - h_r)} \qquad (2)^1$$

Turbine discharge Q_w

= conduit discharge Q_c + port discharge Q_t + riser discharge Q_R

$$Q_w = Av + a\sqrt{2g(h_t - h_r)} + A_r\frac{dh_r}{dt} \qquad (3)^1$$

Turbine discharge Q_w = function of head, efficiency, and gate or power

$$Q_w = f(h_r, P_w) \tag{4}$$

[1] The radical will be written $\sqrt{2g(h_r - h_t)}$ when $h_r > h_t$. The discharge coefficient is included in a in accordance with the notation of Art. 4.

By Eq. (4) we mean that the turbine discharge Q_w for any specified power output P_w or gate opening is a function of h_r and the turbine efficiency. Because of the variation in turbine efficiency with the gate, this function is not conveniently expressible in algebraic terms, and the value of Q_w at any instant must, as a practical matter, be selected from turbine performance charts.

The insuperable obstacle to direct conventional solution of the above system is, however, due to the fact that when the friction head loss is taken proportional to the square of the conduit velocity, there is no known solution for the resulting differential equations. In such cases, provided we have a sufficient number of equations to match the number of unknown quantities, the solution may always be effected by step-by-step calculation commonly known as arithmetic integration. The detailed procedure for an illustrative example will be given in Art. 8.

It can be readily appreciated that if we had to depend entirely upon mere guesswork for selecting fortuitous trial combinations of tank size and port area, determination of the correct design of tank by the arithmetic integration method would be hopelessly laborious. Thanks, however, to the genius of the inventor of the differential tank, we are equipped with a powerful method of obtaining very dependable trial values of tank dimensions so that the subsequent computation of performance curves, showing water levels and velocities by arithmetic integration, becomes largely confirmatory in purpose and seldom needs to be carried through more than one trial. The classic article[1] describing this method, which is abstracted in the following sections, will repay detailed study by surge-tank designers.

4. JOHNSON'S EQUATIONS FOR LOAD DEMAND

To avoid confusion, it will possibly be well to emphasize at the outset that the purpose of Johnson's equations is not to afford a means for computing performance

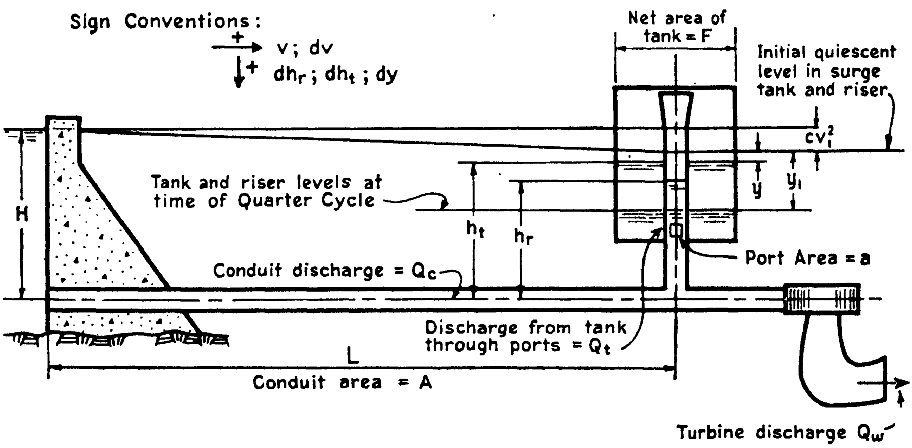

FIG. 1.—Differential surge tank, load-demand condition.

curves of water levels and conduit velocities, but only rapidly to supply reliable trial values of port area, limiting upper and lower tank elevations, and tank and riser diameters for subsequent confirmation of performance by arithmetic integration.

The key to the simplification of Eqs. (1) to (4) in Art. 3, is shown in Fig. 2, in which the actual riser drop curve shown in dotted line is replaced by the assumed heavy solid rectangular curve. This assumption follows very naturally from an inspection of arithmetic integration performance curves for a wide variety of differ-

[1] JOHNSON, RAYMOND D., The Differential Surge Tank, *Trans. A.S.C.E.*, *Paper* 1324, vol. 78, 1915.

ential tanks. For most efficient action, the heel of the characteristic actual riser drop curve will come down initially so as to be level for a few seconds' duration with the subsequent common elevation of both tank and riser curves at the quarter cycle. If the port area is too small in relation to the tank area, the initial heel in the riser drop curve will lie appreciably below the levels at the quarter cycle; conversely, if the port area is too large in proportion to the tank area, the characteristic initial heel of the actual riser drop curve will lie some distance above the levels at the quarter cycle. In the first case, the instantaneous power capacity will be reduced because of excessively low elevation of the heel of the riser drop curve, and in the second instance the power output will be penalized by excessively low elevation of both levels at the quarter cycle. Maximum efficiency results when the riser drop curve has the char-

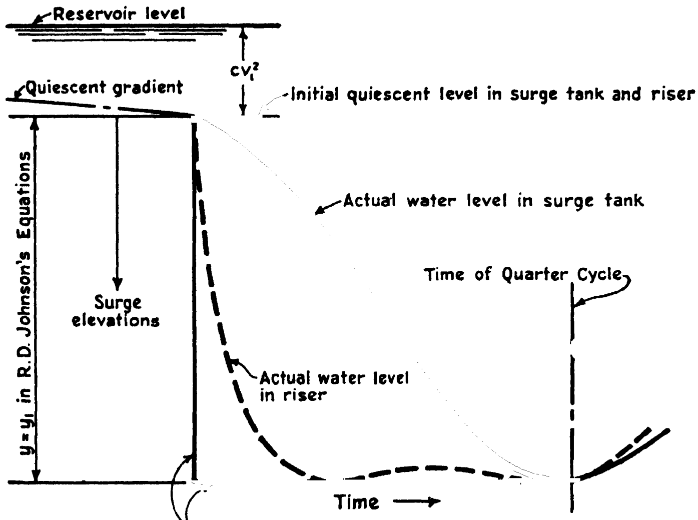

FIG. 2.—Difference between actual and assumed rectangular riser drop curves for load-demand condition.

acteristic form given in Fig. 2, and it is reasonable to expect (and invariably confirmed in subsequent detailed computation) that the substitute rectangular outline gives a close workable approximation to the actual dotted form. This entire question (see Fig. 9) is clarified in detail in the discussion of Johnson's original article by Roy Taylor.[1]

Having established a reasonable and advantageous simplification of shape for the riser drop curve, let us next investigate what the concomitant physical actions in the tank must be to afford the desired rectangular configuration. It is quite obvious that the cross-sectional area of the riser must be considered negligible and equal to zero, and that the inertia of the water in the riser column must be neglected to ensure instantaneous drop of the riser curve to the quarter-cycle level immediately upon opening the turbine gates. Upon further reflection, it is also evident that to maintain the level of water in the riser constantly at the above fixed elevation throughout the quarter cycle, the area of the ports (though usually fixed in actual design) must be assumed to vary at each instant in just the right degree to ensure that, under dropping head on the port orifice, the discharge through the ports plus the increasing discharge from the main conduit is just sufficient to furnish the exact discharge required by the turbine at constant head up to the quarter cycle. Finally, it is quite evident, in

[1] *Ibid.*, Fig. 15, p. 803, discussion by Roy Taylor

view of the foregoing assumptions, that the equations derived on this basis will be valid only up to the time of the quarter cycle.

For convenient reference these assumptions are summarized as follows:

1. The analysis is valid only up to the time of the quarter cycle, that is, the time at which water levels in the riser and main tank approach coincidence.
2. The inertia of the water in the riser column is neglected.
3. In accordance with Fig. 2, it is assumed that upon initiating the opening movement of the turbine gates the water level in the riser drops immediately to its level at the quarter cycle, giving a rectangular shape to the riser drop curve.
4. The cross-sectional area of the riser is considered negligible and equal to zero.
5. The port area, though usually constant in any actual design, is, for the purpose of facilitating solution of the equations, assumed to vary with the time in just the right proportion to give the assumed rectangular shape of riser drop curve.

With these simplifying assumptions, Johnson's equations expressing a relationship between magnitude of surge and diameter of tank are obtained with comparative ease. In accordance with our usual practice, we shall adhere to Johnson's notation to facilitate cross reference with the original article.

The notation is as follows:

t = time in general, sec.
t_a = time of acceleration, sec.
t_r = time of retardation, sec.
T = time to complete one quarter cycle of the oscillation of tank levels, sec.
L = length of conduit, ft.
A = area of conduit, sq ft.
F = net area of tank, that is, in excess of riser area, sq ft.
H = difference in elevation between water surface in reservoir and center line of conduit, ft.
h_r = difference in elevation between water surface in riser and center line of conduit, ft.
h_t = difference in elevation between water surface in tank and center line of conduit, ft.
Q_w = turbine discharge, cfs.
A_r = area of riser, sq ft.
P_w = turbine output, hp.
a = area of restricted opening or port with 100 per cent coefficient of discharge, or the actual area times the discharge coefficient, sq ft.
a_0 = area of restricted opening when $t = 0$, sq ft.
a_1 = area of restricted opening when $t = T$, sq ft.
v = velocity in conduit at any instant, fps.
v_1 = initial conduit velocity before acceleration begins ($t_a = 0$), or = conduit velocity when $t_r = T$, or = constant-draft velocity between surge tank and water wheel, in terms of the conduit velocity during retardation.
v_2 = conduit velocity when $t_a = T$, or = initial conduit velocity before retardation begins ($t_r = 0$), or = constant-draft velocity between surge tank and water wheel, in terms of the conduit velocity during acceleration.
c = a coefficient such that cv^2 = total losses of head in the conduit (these losses may or may not include the velocity head, depending on the location of the surge tank).
y = departure of water level in tank from its initial quiescent position previous to a load change.
y_1 = y for $t = T$ = also, by hypothesis, the amount of initial sudden change of level in standpipe.
p = percentage of velocity change $= \dfrac{v_2 - v_1}{v_2}$ in acceleration.
$r = \dfrac{v_1}{v_2} = 1 - p.$
k = stability factor of surge $= \dfrac{y_1}{c(v_2^2 - v_1^2)}.$
log = natural logarithm to the base e.

$$Z = \sqrt{\frac{y_1}{c} + v_1^2}, \text{ or } y_{1_a} = c(Z^2 - v_1^2).$$

$$Z_1 = \sqrt{v_2^2 - \frac{y_1}{c}}, \text{ or } y_{1_r} = c(v_2^2 - Z_1^2).$$

$$Z_0 = \sqrt{\frac{y_1}{c} - v_2^2}, \text{ or } y_{1_r} = c(v_2^2 + Z_0^2).$$

$$X = \frac{Z}{-} = \sqrt{k(1 - r^2) + r^2}.$$

The resulting simplified differential equations for load demand are as follows:

$$dt = \frac{\frac{L}{g}\,dv}{y_1 - c(v^2 - v_1^2)} \tag{5}$$

$$Av_2\,dt = F\,dy + Av\,dt \tag{6}$$

Solving,[1] we obtain the tank diameter F for any specified surge y_1:

$$F = \frac{AL}{2gcy_{1_a}}\left[\frac{v_2}{Z}\log_e\frac{(Z - v_1)(Z + v_2)}{(Z + v_1)(Z - v_2)} - \log_e\frac{Z^2 - v_1^2}{Z^2 - v_2^2}\right] \tag{7}$$

or

$$F = \frac{AL}{2gc^2v_2^2k(1 - r^2)}\left[\frac{1}{X}\log_e\frac{(X - r)(X + 1)}{(X + r)(X - 1)} - \log_e\frac{k}{k - 1}\right] \tag{8}$$

Equation (8) may be presented in the very convenient form of a chart for rapid computation (Fig. 6). For this purpose, in Eq. (8) let

$$\frac{2gc^2v_2^2F}{AL} = N^2 \tag{9}$$

and to obtain a scale convenient for reading, let

$$100N_a = 100v_2c\sqrt{\frac{2gF}{AL}} \tag{10}$$

Also, by definition,

$$y_{1_a} = k_a c(v_2^2 - v_1^2) \tag{11}$$

Then for any assigned values of y_{1_a} and v_2, we have corresponding values of p, k, and r, and may read the corresponding value of $100N_a$ from the chart (Fig. 6). The required net area of tank is then easily computed from Eq. (10).

In performing the arithmetic integration, it is useful to have an estimate of the time required to reach the quarter cycle. This is given by

$$T_a = \frac{L}{2gcZ}\log_e\frac{(Z - v_1)(Z + v_2)}{(Z + v_1)(Z - v_2)} \tag{12}$$

From Fig. 2 and the simple orifice law, the port area required at the start of the cycle will be

$$a_0 = \frac{A(v_2 - v_1)}{\sqrt{2gy_1}} \qquad \text{(for load demand)} \tag{13}$$

The theoretical[2] port area required at the quarter cycle will be

$$a_1 = \left[\frac{AFy_1}{L}\left(1 - \frac{1}{k}\right)\right]^{1/2} \tag{14}$$

which is always less than a_0.

5. JOHNSON'S EQUATIONS FOR LOAD REJECTION

The key assumption for the development of Johnson's equations for the load-rejection condition illustrated in Fig. 3 is, as would naturally be expected, the same as for load demand except that the water surface elevation in the riser rises (instead of falls) to the common elevation of both tank and riser levels at the quarter cycle.

[1] For complete intermediate steps in the solution, see Johnson, *op. cit.*, pp. 770*ff*.

[2] For details of the derivation see Johnson, *op. cit.*, p. 770, Eqs. (6), (7), (8), and (9).

As a secondary assumption, the depth of water overflowing the riser top as a weir crest is neglected.

Referring to Fig. 3, the fundamental differential equations for load rejection are

$$dt = \frac{\frac{L}{g}\,dv}{y_1 - c(v_2^2 - v^2)} \tag{15}$$

$$Av_1\,dt = F\,dy + Av\,dt \tag{16}$$

For the rejection condition two cases arise, depending upon whether or not the surge rises above the reservoir level, *i.e.*, whether $y_1 < cv_2^2$ or $y_1 > cv_2^2$. In the first

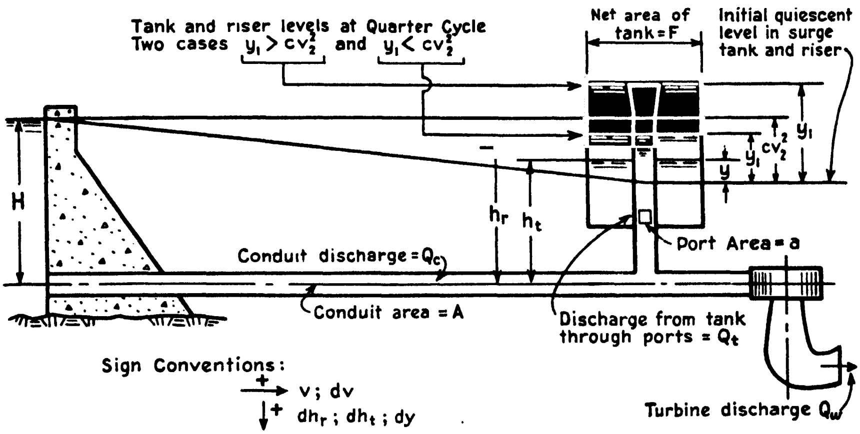

FIG. 3.—Differential surge tank, two cases, load-rejection condition.

instance, if $y_1 < cv_2^2$, the equation for F, upon integration of Eqs. (15) and (16), will lead to the formula

$$F = \frac{AL}{2gcy_{1r}}\left[\log_e \frac{v_2^2 - Z_1^2}{v_1^2 - Z_1^2} - \frac{v_1}{Z_1}\log_e \frac{(v_2 - Z_1)(v_1 + Z_1)}{(v_2 + Z_1)(v_1 - Z_1)}\right] \tag{17}$$

On the other hand, when the surge rises above the reservoir level and $y_1 > cv_2^2$, we have

$$F = \frac{AL}{2gcy_1}\left[\log_e \frac{v_2^2 + Z_0^2}{v_1^2 + Z_0^2} - \frac{2v_1}{Z_0}\left(\text{arc tan}\,\frac{v_2}{Z_0} - \text{arc tan}\,\frac{v_1}{Z_0}\right)\right] \tag{18}$$

For the case of greatest interest in practice, that of complete shutdown with $v_1 = 0$, Eq. (18) becomes

$$F = \frac{AL}{2gcy_1}\log_e \frac{v_2^2 + Z_0^2}{Z_0^2} \tag{19}$$

or, since

$$k_r = \frac{y_1}{cv_2^2}, \tag{20}$$

$$F = \frac{AL}{2gc^2v_2^2k_r}\log_e \frac{k_r}{k_r - 1} \tag{21}$$

we may develop a useful graphical chart for the computation of this equation in the same general fashion as for load demand. In this case let

$$100N_r^2 = 100v_2c\sqrt{\frac{2gF}{AL}} \tag{22}$$

For given values of y_1 and v_2, we derive a corresponding value of k_r and may read $100N_r$ from the chart (Fig. 7). F is then easily obtained from Eq. (22).

From Fig. 3 the port area required at the start of the cycle will be

$$a_0 = \frac{A(v_2 - v_1) - \text{discharge over top of riser}}{\sqrt{2gy_1}} \tag{23}$$

The size of port[1] required at the quarter cycle will be

$$a_1 = v_2 \left[\frac{AFc}{L} (k_r - 1) \right]^{1/2} \tag{24}$$

The port area for the rejection condition will be established by Eq. (24) rather than Eq. (23) because of the discharge capacity over the top of the riser as a weir.

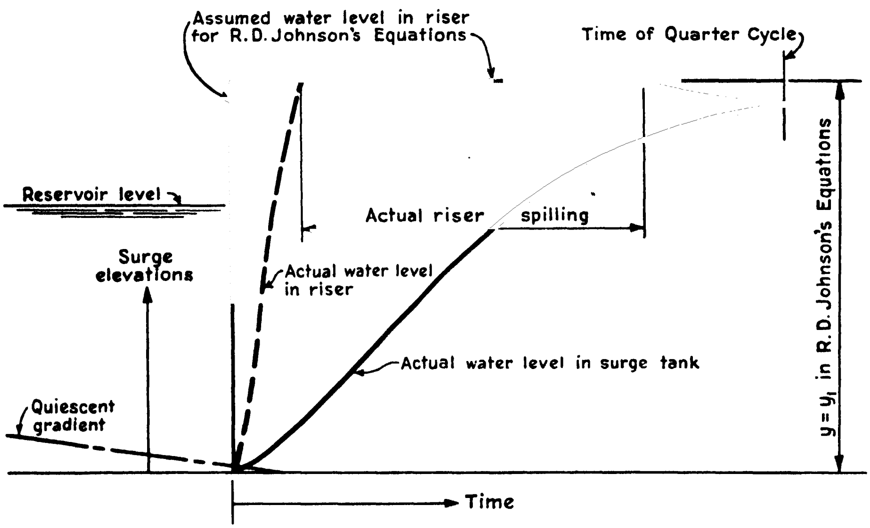

FIG. 4.—Difference between actual and assumed rectangular riser level curves for rejection condition.

The effective size of port for the demand condition, Eqs. (13) and (14), must not exceed that prescribed by Eq. (24) for the rejection condition; otherwise, the water level in the riser during rejection will drop down again before the quarter cycle is reached. This would be contrary to our assumption of constant y_1 in Fig. 3 and might introduce appreciable error into the equations for tank diameter.

6. JOHNSON'S EQUATIONS FOR THE CRITICAL VELOCITY

There are certain combinations of tank design and conduit for which rejection from the maximum full-load velocity does not give the maximum height of surge. In these cases, some lesser velocity v_c, called the critical velocity, will govern the upper limiting elevation of tank.

Let d equal the required height of surge tank above the reservoir level. Then

$$d = cZ_0^2 = c\left(\frac{y_1}{c} - v_2^2\right) \tag{25}$$

and

$$y_1 = c(Z_0^2 + v_2^2) \tag{26}$$

[1] JOHNSON, *op. cit.*, Eq. (33), p. 778.

Substituting Eq. (26) in Eq. (19), we obtain

$$\frac{2gc^2F}{AL} = \frac{\log_e (v_2^2 + Z_0^2) - 2 \log_e Z_0}{Z_0^2 + v_2^2} \tag{27}$$

To find what value of v_2 makes d a maximum, differentiate Eq. (27) with respect to v_2. Place $\frac{dZ_0}{dv_2} = 0$ and solve for v_2. Then this value of $v_2 = v_c$ or

$$v_c = \frac{1}{c}\left[\frac{AL(e-1)}{2gFe}\right]^{1/2} \tag{28}$$

Solving for $d = cZ_0^2$ in Eq. (25)

$$d_{\max} = \frac{AL}{2gceF} \tag{29}$$

Figure 8 affords an interesting summary of the heights of surge resulting from shutdown from conduit velocities other than the critical. The corresponding value of port area for the critical velocity is, from Eqs. (24), (28), and (29),

$$a_1 \text{ (for } v_c) = \frac{A}{\sqrt{2gce}} \tag{30}$$

or

$$a_1 = \sqrt{\frac{AFd}{L}} \tag{31}$$

7. TEST FOR INCIPIENT STABILITY

It was remarked in Art. 1 that one of the requirements of a satisfactory surge tank is that it have the ability to quench positively and rapidly the pendulations of water surface following load changes of whatever magnitude. During the larger load changes, the velocity of flow through the ports is relatively large; there is a very marked difference in elevation between the water surface levels in the main surge tank and riser, with the natural result that comparatively large conduit accelerating or decelerating forces are mobilized. These in turn act with telling effect to throttle and suppress the transient phenomenon.

In contrast, during fairly small load changes, say in the order of 1,000 kw in 50,000, the difference between the main tank and riser levels is likewise small; the velocity through the ports is comparatively insignificant, and the requisite quick suppression of surge must be effected principally by the friction in the conduit. If the tank diameter is sufficiently large, conduit friction will damp down the surge from any small load change before the water levels in the tank and riser have made any appreciable change from quiescence. On the other hand, it can be readily appreciated that the water levels in a tank of too small diameter would rise with greater rapidity, develop a phase lead ahead of conduit friction, with proportionate overtravel of water levels beyond the second steady-state position, and so tend to aggravate, magnify, and perpetuate each small load variation.

For the above reasons, as well as to obtain expressions that are more readily susceptible to mathematical attack, the basic equation for gaging tank stability is predicated upon small load changes as being more critical, since the throttling effect of the ports is then negligibly small. Because of its importance, we proceed to develop in considerable detail the criterion for stability now in universal use, namely, the formula of Dr. Dieter Thoma. For this purpose we assume that the action of riser and ports of the differential tank is negligible and that they may be entirely removed, giving in effect an old-style simple surge tank.

Referring to Fig. 5, we shall assume in the interest of simplicity that the turbine gates are closed and the water levels in the entire hydraulic system stand at reservoir elevation. We then place a very small load on the turbine and, from the character of the resultant differential equations, deduce what the diameter of surge tank must be to ensure the requisite damping effect.

The notation adopted is in general in conformity with Art. 4, and the meaning of the additional new terms will be apparent from the diagram (Fig. 5).

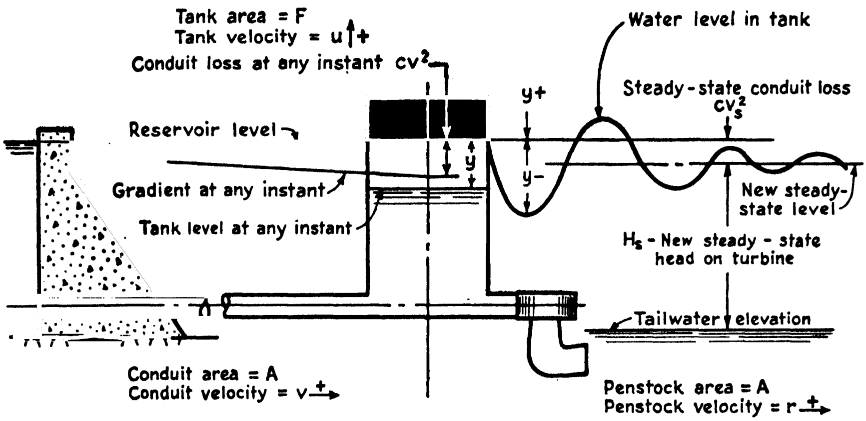

FIG. 5.—Analysis of stability showing effect of placing small load on turbine starting from gates closed and entire system at reservoir level.

With the algebraic sign conventions postulated in Fig. 5, the basic differential equations will have the following form:[1]

$$\frac{L}{g}\frac{dv}{dt} + y + cv^2 = 0 \tag{32}$$

$$Av = Fu + Ar = F\frac{dy}{dt} + Ar \tag{33}$$

Now, if we were to proceed to attack the solution of these two equations directly, according to the conventional procedure, we should again encounter the familiar recurrent difficulty, namely, that the resulting differential equation would be of the form

$$\frac{d^2\phi}{dt^2} + \beta\left(\frac{d\phi}{dt}\right)^2 + \theta = 0 \tag{34}$$

for which there is at present no known solution. Accordingly, we shall be required to introduce sufficient permissible simplifications to transform this equation into the type

$$\frac{d^2\phi}{dt^2} + \beta\frac{d\phi}{dt} + \theta = 0 \tag{35}$$

In essence, then, we must find some legitimate means of expressing the friction head loss according to the first rather than the second power of the conduit velocity, and fortunately for the case of small load changes, this requirement is obtainable without any great difficulty or sacrifice in accuracy. Our second approximation relates to turbine discharge under varying net effective head, but again in this case the assumption of small load changes permits the requisite simplification. Under these condi-

[1] CALAME, J., and DANIEL GADEN, "Theorie des chambres d'equilibre," Paris and Lausanne, 1926.

tions the friction head loss in the conduit at any instant will be an insignificant percentage of the net effective head and the variation in turbine efficiency over the range considered will likewise be negligible.

Accordingly, we may express the penstock velocity as follows:

$$r = r_s \left(\frac{H_s}{H_s + cv_s^2 + y} \right) \tag{36}$$

in which r is the penstock velocity at any instant and r_s is the steady-state penstock velocity. Since, in our case, cv_s^2 will be very small in proportion to H_s, we may write

$$r = r_s \left(\frac{1}{1 + \frac{y}{H_s}} \right) \tag{37}$$

We may then expand the parenthesis by means of the well-known infinite series:

$$\frac{1}{1 + x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \qquad \text{for} \qquad x^2 < 1 \tag{38}$$

Again, because of our assumption of small load change, y will be relatively small and we may drop all terms of this infinite series involving the second and higher powers of y/H_s, giving finally

$$r = r_s \left(1 - \frac{y}{H_s} \right) \tag{39}$$

Substituting this expression in Eq. (33), we obtain

$$v = \frac{F}{A} u + r_s \left(1 - \frac{y}{H_s} \right) \tag{40}$$

and the friction head loss cv^2 becomes

$$c \left(\frac{F}{A} u + r_s - \frac{r_s y}{H_s} \right)^2 = c \left(r_s^2 + \frac{2Fr_s u}{A} - \frac{2r_s^2 y}{H_s} \right) \tag{41}$$

since u^2 and $\left(\frac{r_s y}{H_s} \right)^2$ are very small. With these substitutions, the original differential Eq. (32) will take the form

$$\frac{d^2y}{dt^2} - \left(\frac{Ar_s}{FH_s} - \frac{2cr_s g}{L} \right) \frac{dy}{dt} + \left(\frac{gA}{FL} - \frac{2cr_s^2 gA}{FLH_s} \right) y + \frac{cgAr_s^2}{FL} = 0 \tag{42}$$

From the general theory[1] of ordinary linear differential equations we know that the character of the solution of the above equation will be determined by the complementary function, that is, the solution of the equation

$$\frac{d^2y}{dt^2} - \left(\frac{Ar_s}{FH_s} - \frac{2cr_s g}{L} \right) \frac{dy}{dt} + \left(\frac{gA}{FL} - \frac{2cr_s^2 gA}{FLH_s} \right) = 0 \tag{43}$$

The auxiliary equation for the complementary function will be

$$\omega^2 - \beta\omega + \theta = 0 \tag{44}$$

in which

$$\beta = \frac{Ar_s}{FH_s} - \frac{2cr_s g}{L} \qquad \text{and} \qquad \theta = \frac{gA}{FL} - \frac{2cr_s^2 gA}{FLH_s} \tag{45}$$

[1] GRANVILLE, WILLIAM A., "The Elements of the Differential and Integral Calculus," Chap. 30, p. 431, Ginn and Company, 1911.

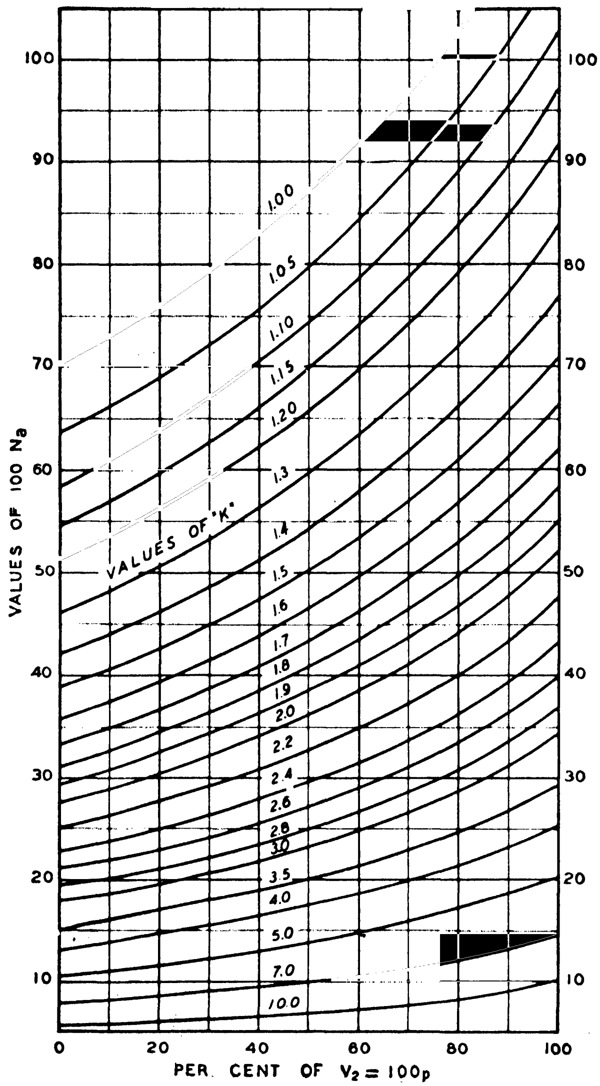

FIG. 6.—Johnson chart, load-demand condition.

The complementary function will then have the form:

$$y = C_1e^{\omega_1 t} + C_2e^{\omega_2 t} \tag{46}$$

in which ω_1 and ω_2 are roots of Eq. (44).

We perceive from inspection of β, θ, and Eq. (43) that there is no possibility of repeated roots for the auxiliary equation. Accordingly, we have two remaining possibilities. A pair of complex roots of the auxiliary equation will afford a solution of the type

$$y = C_1e^{\frac{(\beta+\sqrt{\beta^2-4\theta})t}{2}} + C_2e^{\frac{(\beta-\sqrt{\beta^2-4\theta})t}{2}}$$

or letting $\frac{\sqrt{\beta^2 - 4\theta}}{2} = \sqrt{-1}\,\lambda = i\lambda$,

$$y = C_1 e^{\frac{\beta t}{2}} (\cos \lambda t + i \sin \lambda t) + C_2 e^{\frac{\beta t}{2}} (\cos \lambda t - i \sin \lambda t) \tag{47}$$

Real roots will give solution of the form of Eq. (46).

The case which occurs almost invariably and is of greatest interest in practice is that in which the auxiliary equation has a pair of complex roots, Eq. (47). If 4θ is greater than β^2, and we can see by inspection that this is generally the case, the quantity under the radical sign will be negative and our solution will be periodic. The fundamental requirement in which we are particularly interested, the factor that

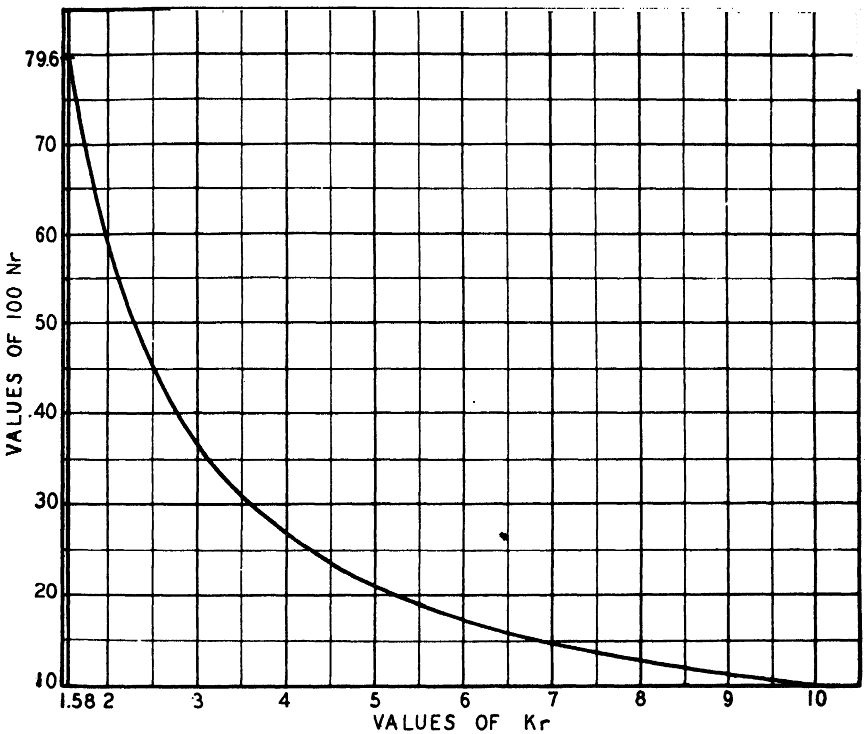

FIG. 7.—Johnson chart, load-rejection condition.

really determines whether our oscillations shall continue to increase indefinitely with the time or decrease progressively with the time, is obviously the algebraic sign of β [Eq. (47)]. The oscillations increase indefinitely with the positive sign and damp down to quiescence for the negative sign of β. In other words, then, β must be negative or

$$\frac{Ar_s}{FH_s} - \frac{2cr_s g}{L} < 0 \tag{48}$$

$$\frac{2cr_s g}{L} > \frac{Ar_s}{FH_s} \tag{49}$$

or

$$F = \frac{AL}{2gcH_s} \tag{50}$$

That is, the minimum area of surge tank, just barely on the theoretical boundary line

between perpetual and damped oscillations, is given by the formula

$$F = \frac{AL}{2gcH} \tag{51}$$

However, it should again be emphasized, and it should be apparent from the above theoretical derivation, that this size of tank is just sufficient to damp disturbances down to quiescence in a time barely short of eternity. To secure satisfactory rapidity of damping, an appreciable increase over this phantom value must be adopted, and the magnitude of this essential increase is based entirely upon experience and the knowl-

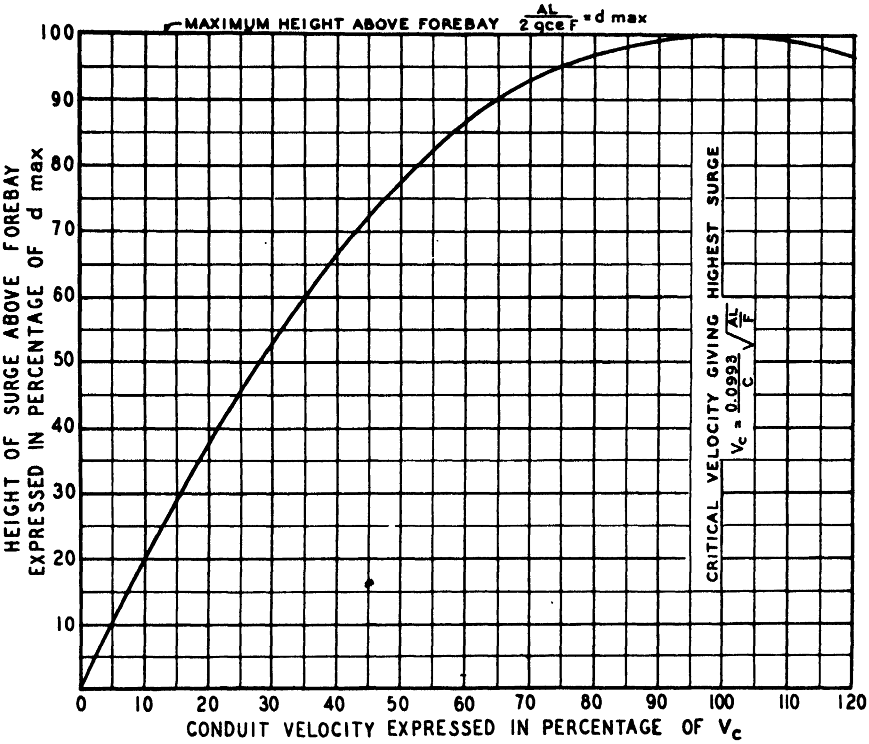

FIG. 8.—Johnson chart, height of surge following rejection of full load.

edge derived from actual practice. It has been found from the consideration of a great range of tanks that this theoretical diameter must be increased at least 25 per cent for differential tanks (in which some benefit derives from the throttling action of the ports even at moderately light loads as compared with the almost infinitesimal load assumed to make our theoretical analysis mathematically workable) and 50 per cent in the case of simple tanks. Experience also dictates that this increase of 25 per cent in the Thoma tank diameter be used in conjunction with a value of H resulting from water levels assumed to be near the limiting bottom elevation of the surge tank. A convenient means of estimating this level in advance is to compute H from the maximum demand surge at the quarter cycle. The factor of 25 per cent also assumes that F be taken as the net area of tank with riser area deducted.

It should be clear from the above derivation that the Thoma formula is not an instrument of hairline precision; it is simply the best rough tool we have been able to

forge for gaging stability up to the present time; and its derivation has been given in considerable detail principally as a means of counteracting the almost universal tendency of designers to attempt to infringe on the 25 and 50 per cent increases essential to ensure practical rapidity of quiescence.

Although the writer has never had occasion in actual practice to use the stability equation covering the aperiodic case, the derivation will be given for the sake of completeness. The basic requirement for aperiodic action is that the roots of Eq. (44) be real. In other words, if β is negative and β^2 greater numerically than 4θ, we obtain

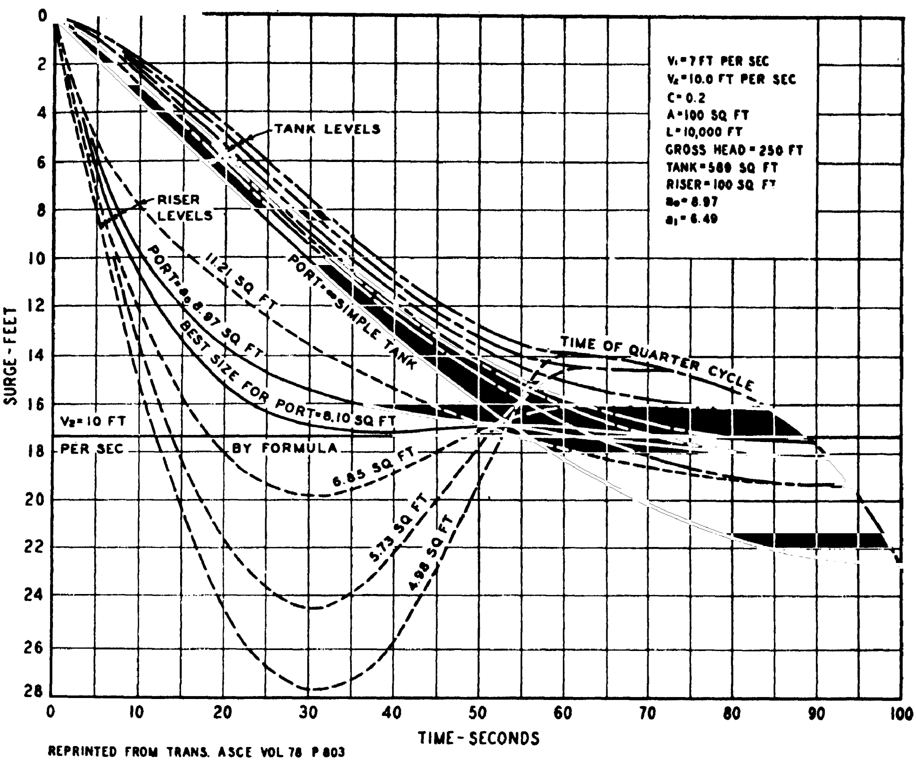

FIG. 9.—Water levels following a load demand with ports of various sizes.

damped aperiodic or deadbeat action; if β is positive and β^2 greater numerically than 4θ, we have aperiodic action with the surge increasing indefinitely with the time.

The basic inequality is

$$\left(\frac{-2cgr_s}{L} + \frac{Ar_s}{FH_s}\right)^2 > 4\left(\frac{gA}{FL} - \frac{2cr_s^2gA}{FL}\right) \tag{52}$$

Solving,

$$F = \frac{AL}{2c^2gH_sr_s^2}\left(H_s - cr_s^2 \pm \sqrt{H_s(H_s - 2cr_s^2)}\right) \tag{53}$$

The positive sign before the radical gives the smallest value of tank diameter to ensure the damped aperiodic or deadbeat condition, while for the negative sign before the radical the surge will be aperiodic and increasing with the time without limit. It will be noted that in Eq. (53) it is necessary to insert the steady-state turbine discharge velocity as well as the steady-state head on the turbine.

8. DESIGN OF APALACHIA SURGE TANK

As a typical illustration of the commercial application of the foregoing principles, determination of the principal elements of design for the surge tank of the Apalachia project of the Tennessee Valley Authority (Paul F. Kruse, consulting engineer) will be given in detail.

One feature of the design specification, the provision to accommodate practically full-load instantaneous demand, is somewhat unusual. The average specification prescribes full-load rejection corresponding to the short-circuit condition; but the most common specified demand is an instantaneous load increase of 50 per cent from half load to full load.

The reason for the large demand prescribed in the case of Apalachia is that this plant serves an area in which close frequency regulation is important for textile manufacturing and other industries requiring close speed control. The Apalachia plant is essentially a seasonal peaking project, and during the off season, when water is accumulating in the reservoir, the generating units are operated floating on the line as synchronous condensers. The governors, however, are so set that in the event that electrical system disturbances cause disconnection of the area load from the main T.V.A. system transmission line, the Apalachia turbine gates will open and the units will absorb the entire local load.

The effect of specifying large load demand not only depresses the limiting bottom elevation of the surge tank but also has a marked influence upon port design. In the conventional tank design the port area required for the relatively smaller demand load will be found to be somewhat less than the theoretical port area required for rejection, and a single port area determined by rejection requirements is selected.

In the case of large demand loads, however, the required port area for demand is appreciably greater than that for rejection, and if a single port size were selected to fit the demand requirement, the result in the case of rejection would be that the water surface in the riser would start dropping down the riser again before the time of the quarter cycle is reached. This recession would give less efficient deceleration of the conduit velocity and subsequently a relatively higher peak elevation of the rejection surge.

This apparent conflict in port area requirements was resolved by incorporation of the port design shown in Fig. 10 and established on the basis of model tests in the hydraulic laboratory. The port orifices, instead of being uniformly rectangular, as in the conventional case, are given a diverging flare in the upward direction. The action of the port is obvious: For conditions of load demand, when the flow is downward from tank into riser, the flow filaments are compressed, flow conditions are improved, and the discharge coefficient for the conditions encountered will be in the order of 0.94. On the other hand, during rejection conditions, the flow will be from riser to tank in the upward direction through the ports. The comparatively rapid rate of flare introduces additional turbulence near the exit of the orifice. Flow conditions are impaired, with the result that the discharge coefficient for operating conditions will be in the vicinity of 0.71. This discrepancy in discharge coefficients was predetermined to match the particular conditions of the Apalachia installation. By changing the rate of divergence of the orifices, the requisite compensation could be obtained to match the requirements of any particular case. This simple device is obviously a marked improvement over some earlier European installations in which this same type of compensation was effected by the use of rectangular orifices, some of which were provided with flap valves to close during rejection and so reduce the rejection port area. As is pointed out in Johnson's basic article, it will be found in the conventional case for load demands up to 50 per cent that the single area determined for the load-off condi-

tion will be entirely satisfactory for load demand. In the following tabulations the principal features of the design will be established, first using Johnson's curves and formulas. Arithmetic integration tabulations will then be given together with the corresponding performance curves, demonstrating the adequacy of the trial design.

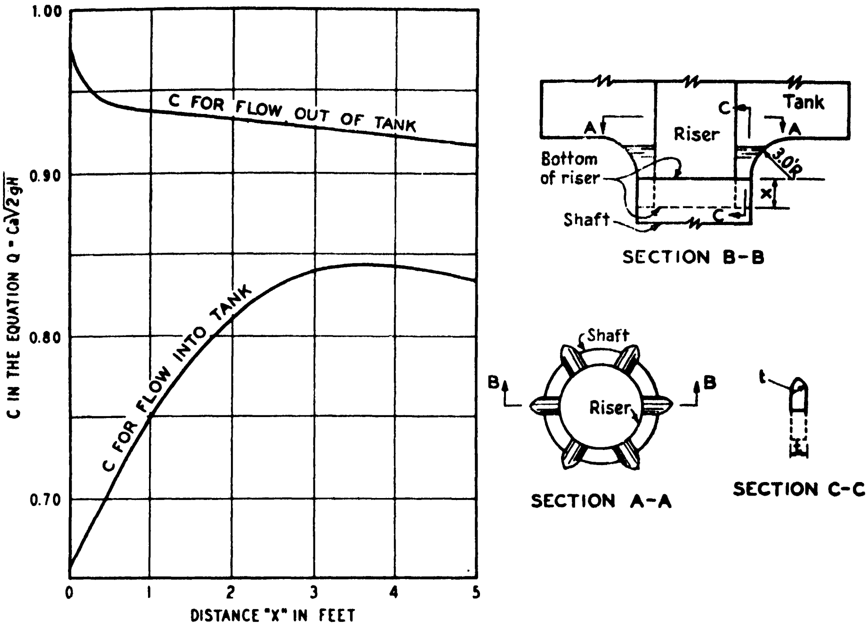

Fig. 10.—Differential surge tank, detail of port.

Design Specifications for Apalachia Surge Tank. *General.*

Headwater	max El. 1282.
	normal El. 1265.
	min El. 1240.
Tailwater	normal El. 840.
Tunnel	41,200 ft of 18-ft-diameter concrete-lined tunnel and 2,000 ft of 16-ft-diameter steel-lined tunnel.
Penstock, tunnel and surge tank	for layout see Fig. 11.
Turbines and generators:	
Turbines	Two 50,000 hp at full gate at 286-ft head, 180 rpm. Best efficiency at 375- to 400-ft head.
Generators	Two 40,000 kva at 0.9 power factor and 60C temperature rise. Assume generator can take the full output of the turbine up to 66,000 hp corresponding to generator rating at 80C temperature rise.
Governors	Net traversing time 5 secs.

Incipient Stability. Check the diameter of tank to meet stability requirements by using a value of $n = 0.010$ in the Manning formula. Use the minimum possible net head as determined from analysis to establish economic diameter.

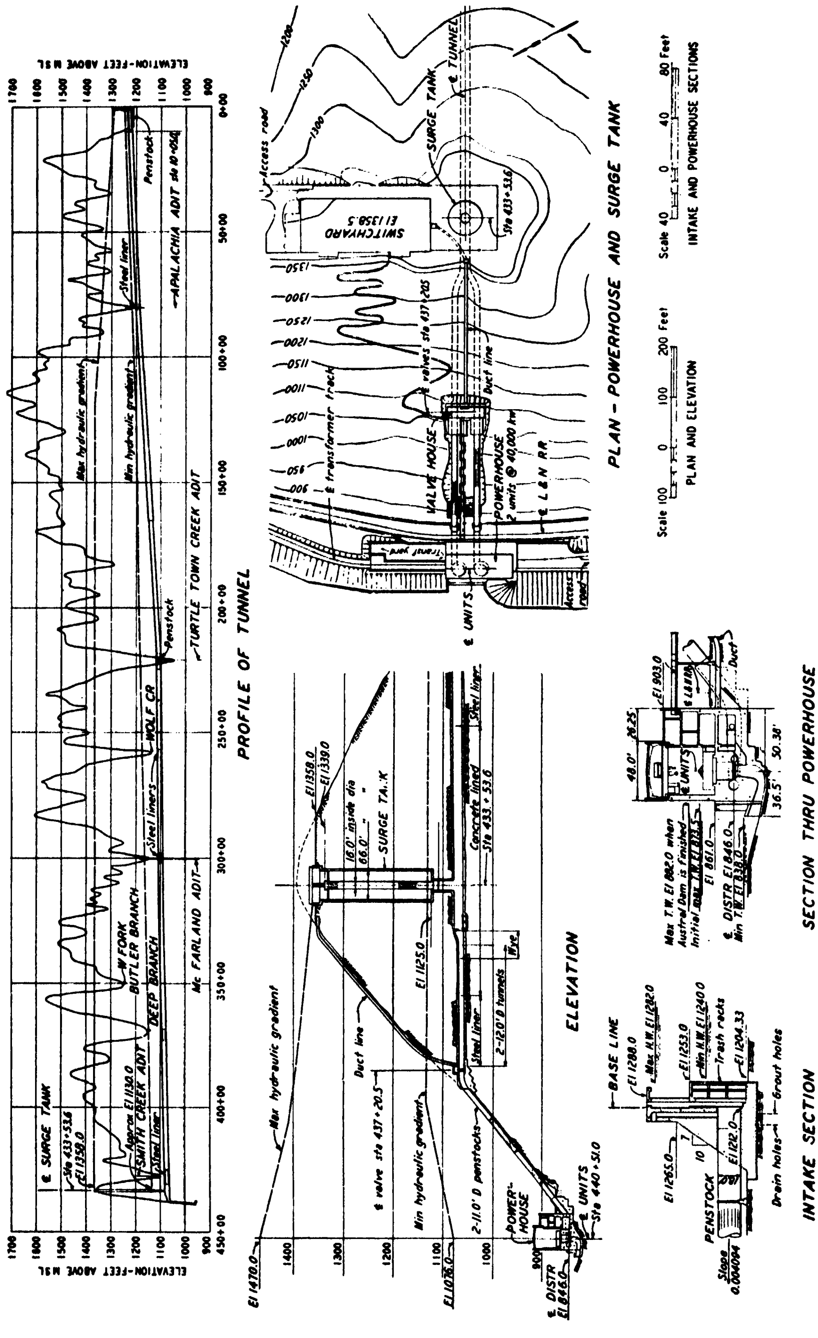

FIG. 11.—General arrangement, Apalachia project.

Load-on. Design tank to supply water for an instantaneous and continuous demand load of 100,000 hp, starting from 0 hp, with pool at minimum level, using a value of $n = 0.013$.

Load-off. Design tank to store water from an instantaneous load rejection of 132,000 to 0 hp with pool at maximum level, using a value cf $n = 0.0115$.

Conduit. The first step is to determine the hydraulic properties of the supply conduit. Using a basic discharge of 3,000 cfs for two units, the head loss and value of c are computed for the three values of n required. With the value of c established, it is possible to compute the head loss for any discharge by one simple operation.

The tunnel as planned is not a uniform section from forebay to tank, which makes it necessary to determine an equivalent conduit.

$$A = \frac{L}{\dfrac{L_1}{A_1} + \dfrac{L_2}{A_2}}$$

$$= \frac{43{,}200}{\dfrac{2{,}000}{201} + \dfrac{41{,}200}{254.5}} = \frac{43{,}200}{172} = 251 \text{ sq ft}$$

For $Q = 3{,}000$ cfs, we have a velocity of 11.95 fps.

By standard hydraulic computations we establish the following properties:

Tunnel from intake to surge tank:

$n = 0.010$,

Head loss = 43.0 ft (includes velocity head at riser)

$$c = \frac{43.0}{(11.95)^2} = 0.303$$

$n = 0.0115$,

Head loss = 54.3 ft (includes velocity head at riser)

$$c = \frac{54.3}{(11.95)^2} = 0.382$$

$n = 0.013$,

Head loss = 69.7 ft (includes velocity head at riser)

$$c = \frac{69.7}{(11.95)^2} = 0.490$$

Tunnel and penstock from surge tank to unit:

$n = 0.010$,

Head loss = 3.5 ft

$$c = \frac{3.5}{(11.95)^2} = 0.0245$$

$n = 0.0115$,

Head loss = 4.3 ft

$$c = \frac{4.3}{(11.95)^2} = 0.030$$

$n = 0.013$,

Head loss = 5.10 ft

$$c = \frac{5.10}{(11.95)^2} = 0.036$$

Tunnel and penstocks from intake to units:

$n = 0.010$,

Head loss = 44.3 ft

$c = 0.312$

$n = 0.0115,$

$$\text{Head loss} = 56.4 \text{ ft}$$
$$c = 0.396$$

$n = 0.013,$

$$\text{Head loss} = 72.6 \text{ ft}$$
$$c = 0.512$$

Riser Diameter. When deciding upon the riser diameter, it should be kept in mind that the area of the riser should be equal to not less than three-fourths of the

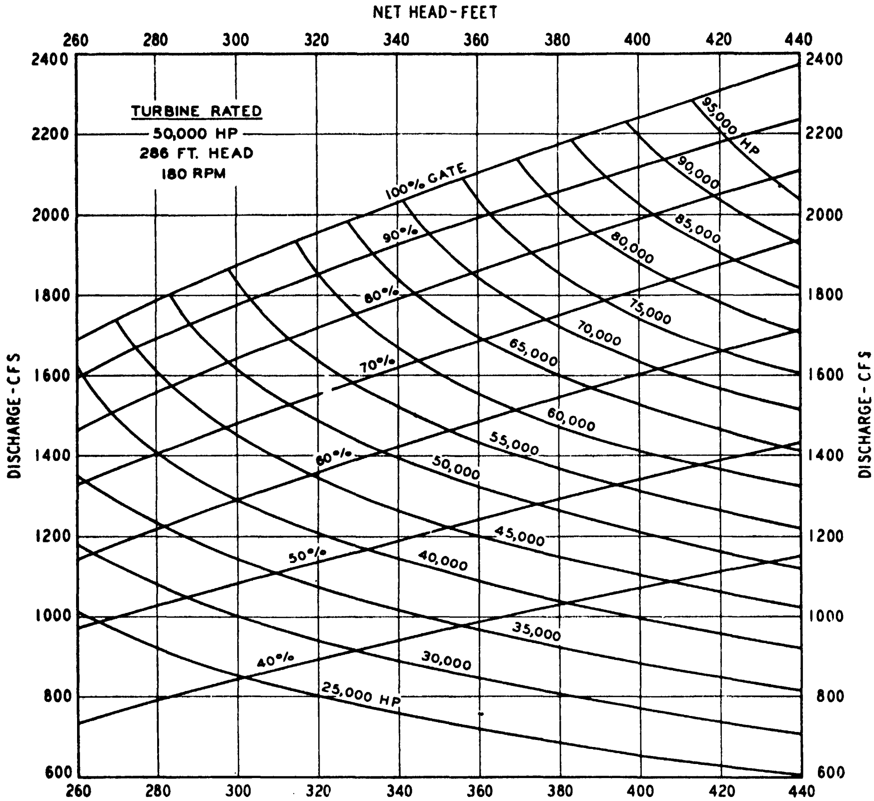

FIG. 12.—Turbine performance chart.

area of the conduit. If the riser is too small, the water level changes in it are so rapid that good governing becomes more difficult.

Assume a riser diameter of 16 ft,

$$R = 201 \text{ sq ft}$$
$$^{201}\!/_{251} = 0.80$$

Economic Diameter of Surge Tank. The determination of the economic diameter of the surge tank might well be called the principal element of the design. Contrary to more or less prevalent impression, selection of the most advantageous diameter does not invariably result from simple direct substitution in the Thoma formula, Eq. (51), using the lowest net head that will give the specified power output, but for best

return on the investment requires establishing a balance between construction cost, magnitude of surge and consequent limitation of instantaneous power at the lowest point of surge, and satisfactory quenching power or stability.

As an aid to judgment in making this determination, we shall require curves showing the relation between tank diameter and magnitude of surge for the demand and rejection condition for various assumed diameters, the capital cost of the corresponding heights and diameters of tank, and finally an estimate of the minimum diameter required to ensure stability. These data for our particular project are presented in Fig. 13; and as a sample of the method of computation we shall give the calculation in detail for one representative point on the load demand curve and one on the load rejection curve, together with a calculation to establish the diameter to ensure an ample margin of stability.

In our case, in which the tank is excavated from solid rock, the surge curves for load rejection do not have a determining effect on the cost of construction, for the reason that less expensive over-all construction expense results from excavating the tank full-bore from the top of hill, rather than excavating a pilot shaft down to the elevation required to clear the top of surge. Accordingly, the cost curve given in Fig. 13 has been calculated on this basis. However, in a plate-steel tank extending above ground, the construction cost curve would reflect stopping the tank just above the calculated surge for load rejection.

Several striking features are at once apparent from Fig. 13:

1. For the turbine whose performance is given by Fig. 12, a tank diameter of 66 ft is the smallest capable of carrying the prescribed continuous demand load of 100,000 hp through each and every point of the surge. For diameters less than 66 ft, the net head on the turbine at the lower levels of the surge is so reduced that even at full gate the power output falls appreciably short of 100,000 hp.
2. If we assume as large a turbine as necessary to give 100,000 hp at 90 per cent efficiency at the reduced heads near the lower levels of the surge, the curve of surge vs. diameter (heavy dotted line in Fig. 13) breaks sharply upward at about the 66 ft diameter.
3. The curve of construction cost also breaks definitely upward between 60 and 66 ft diameter.
4. For the load rejection condition, the slope of the curve of tank diameter vs. surge steepens noticeably at about the 66 ft diameter.

Even before we make any specific computation with regard to stability, it is evident from the above considerations that the performance of the tank becomes noticeably inferior below the region about 66 ft diameter, so that we are naturally led to suspect a positive reduction in quenching power starting at about this diameter. Our judgment is confirmed by the following analysis.

We have emphasized again and again that the Thoma formula is a rough tool in the application of which all the unavoidable errors in estimating should be made to lie on the side of safety, and the first occasion to exercise this caution is in selecting a liberal value for the minimum steady-state operating head on the turbine H. As a generous but not at all prodigal forecast of the stable steady-state capacity likely to be required at minimum headwater, we are, with the assistance of Fig. 12, able to anticipate the need for developing 100,000 hp at 100 per cent gate with net head between 290 and 300 ft, which calls for a steady-state velocity of about 14.5 fps in the conduit. As a second aid to judgment we readily anticipate from Fig. 13 that the bottom of tank will be in the vicinity of El. 1,125 ft so that steady-state tank levels in the vicinity of El. 1,135 are quite possible. Allowing 5 ft for loss in the penstock, we should have a corresponding steady-state head of about 290 ft. The corresponding computation for stability, following the calculation for representative points on the load-on and load-off surge curves, confirms our tentative selection of a tank diameter of 66 ft.

The important conclusion to be drawn from our investigation is that the Thoma formula is not properly an instrument to predetermine and fix the diameter of tank

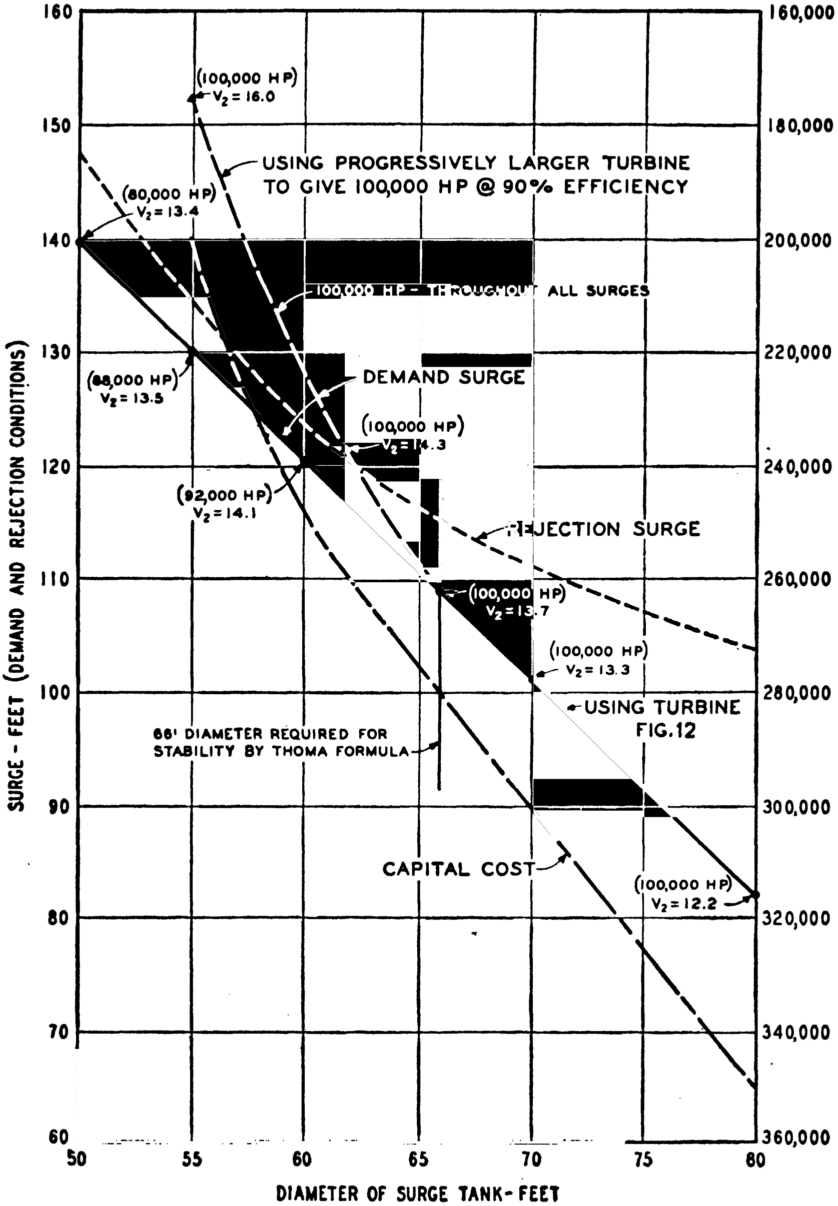

FIG. 13.—Economic diameter of surge tank, Apalachia project.

in advance of other steps in the analysis, but rather a device for final confirmation to sustain and reinforce the indications of the economic determination of tank diameter. Plotting of the economic curves is the essential feature, and the tank diameter selected should be substantially on the side of adequacy.

Load Demand. We shall employ the 66 ft diameter to compute a typical point

on the surge curve for load demand (Fig. 13). Assume a discharge of 3,450 cfs through the conduit at the low point of the surge; $v_2 = 13.70$ fps; $v_1 = 0$.

$$100N_a = 100v_2c\sqrt{\frac{2gF}{AL}}$$
$$= 100(13.70)(0.490)\sqrt{\frac{64.4}{251 \times 43{,}200}}\sqrt{3{,}220} = 93.5$$
$$p = \frac{v_2 - v_1}{v_2}$$

From the load-on chart, Fig. 6, for $p = 1.0$, $K_a = 1.18$,

$$y_{1_a} = K_a c(v_2^2 - v_1^2)$$
$$= 1.18(0.490)(13.70)^2 = 109 \text{ ft}$$

Gross head = El. 1,240 − El. 840	400.0 ft
Velocity head at riser $= \frac{(13.70)^2}{64.4}$	2.9 (energy available to unit)
	402.9 ft
Less surge	109.0
	293.9 ft
Head loss in penstock $= (13.70)^2 (0.036)$	6.7
Net head at low point of surge	287.2 ft
Water surface in tank prior to load change	El. 1,240.0
Surge	109.0
Water surface at low point of surge	El. 1,131.0 ft

From the turbine performance curves, Fig. 12,
For hp = 50,000, Net head = 287.2

Discharge per unit = 1,725 cfs
Conduit discharge = 1,725 × 2 = 3,460 cfs

which checks the assumed discharge.

The arithmetic integration for this case shows the low point of the surge to be at El. 1,130.70 and $v_2 = 13.2 \pm$ fps.

Load Rejection. We shall select the 66 ft diameter to compute a typical point on the surge curve for load rejection (Fig. 13).

First determine the maximum velocity in the tunnel with headwater at its maximum level ($n = 0.0115$).

Try a discharge of 3,300 cfs, $v = 13.15$ fps.

Gross head	442.0 ft
Head loss $= (13.15)^2 (0.396)$	68.5
Net head	373.5 ft

From the turbine performance chart, Fig. 12, the discharge at 66,000 hp for a head of 373.5 ft is 1,650 cfs per unit. This is more than the rated capacity of the generators, but it is less than their assumed capacity at 80C temperature rise.

Use $v_1 = 13.15$ fps, $v_2 = 0$

$$100N_r = 100v_1c\sqrt{\frac{2gF}{AL}}$$
$$= 100(13.15)(0.382)\sqrt{\frac{64.4}{251 \times 43{,}200}}\sqrt{3{,}220} = 69.6$$

From the chart (Fig. 7),

$$K_r = 1.75$$
$$y_{1_r} = K_r c(v_1^2 - v_2^2)$$
$$= 1.75 \times 0.382 \times (13.15)^2 = 115.5 \text{ ft}$$

Headwater elevation	1,282.0 ft
Head loss = $(13.15)^2$ (0.382)	66.0
Water surface elevation in tank prior to surge	1,216.0 ft
Surge	115.5
Elevation of maximum surge	1,331.5 ft

At this point we should compute the critical velocity.

$$v_c = \frac{1}{c}\left[\frac{AL(e-1)}{2gFe}\right]^{1/2}$$
$$= \frac{0.0993}{0.382}\sqrt{\frac{251 \times 43{,}200}{3{,}220}} = 15.2 \text{ fps}$$

The maximum height of surge above forebay for any discharge can be computed

$$d_{\max} = \frac{AL}{2gceF}$$
$$= \frac{251 \times 43{,}200}{64.40(0.382)(2.718)(3{,}220)} = 50.5 \text{ ft}$$

This is the maximum distance above forebay that the tank need be carried for any discharge.

Forebay	El. 1,282.0 ft
$d_{\max}$	50.5
Max point of highest possible surge	El. 1,332.5 ft

Using the shutdown curve, Fig. 8,

$$\frac{v_2}{v_c} = \frac{13.15}{15.2} = 0.865$$

Height of surge above forebay = 0.985(50.5) = 49.7 ft

Forebay	El. 1,282.0 ft
Height above forebay	49.7
Max point of surge	El. 1,331.7 ft

This compares with an elevation of 1,331.5 obtained above. In order to provide ample factor of safety, the top of the riser was placed at El. 1,339 in the tank as constructed.

Stability. Using an assumed conduit velocity of 14.5 fps, we obtain a check on the diameter required to ensure stability as follows:

Min reservoir elevation	1,240.0 ft
Min tailwater elevation	840.0
	400.0 ft
Head loss, reservoir to turbine $(14.5)^2$ (0.512)	108.0
Steady-state head for Thoma formula	292.0 ft

Net tank area,

$$F = \frac{AL}{2gcH}$$
$$= \frac{251 \times 43{,}200}{64.4 \times 0.303 \times 292} = 1{,}900 \text{ sq ft}$$

Riser area	200
Gross tank area	2,100 sq ft
Corresponding diameter	52 ft
25% increase	13.2
Final diameter	65.2 ft

Use diameter of 66 ft.

Port Area. First determine the port area for load-on. (Port area determined for 50,000-hp continuous output.)

At the start of the demand load cycle,

$$a_0 = \frac{A(v_2 - v_1)}{\sqrt{2gy_1}}$$
$$= \frac{251(13.70)}{\sqrt{64.4 \times 109}} = 40.8 \text{ sq ft}$$
$$k = \frac{y_1}{c(v_2^2 - v_1^2)}$$
$$= \frac{109}{0.490 \times (13.70)^2} = 1.19$$

Near the end of the demand cycle, when $v = v_2$,

$$a_1 = \left[\frac{AFy_1}{L}\left(1 - \frac{1}{k}\right)\right]^{1/2}$$
$$= \left[\frac{251 \times 3{,}220 \times 109}{43{,}200}\left(1 - \frac{1}{1.19}\right)\right]^{1/2} = 18.0 \text{ sq ft}$$

The difference between a_0 and a_1 is large.

Using Taylor's[1] empirical equation,

$$a = a_0 - u(a_0 - a_1), \qquad \text{with} \qquad u = 0.15$$
$$= 40.8 - 0.15(40.8 - 18.0) = 37.4 \text{ sq ft}$$

Check the port area for load-off.

$$k = \frac{y_1}{cv_2^2}$$
$$k_r = \frac{115.5}{0.382(13.15)^2} = 1.75$$
$$a_1 = \left[\frac{AFy_1}{L}\left(1 - \frac{1}{k}\right)\right]^{1/2}$$
$$= \left[\frac{251 \times 3{,}220 \times 115.5}{43{,}200}\left(1 - \frac{1}{1.75}\right)\right]^{1/2} = 30.5 \text{ sq ft}$$

If the port area for load-off exceeds 30.5 sq ft, the water in the riser will recede before the end of the quarter cycle and cause the water ultimately to rise to a higher level than is given by the formula. In order to avoid the use of mechanical devices to close off a portion of the port area during rejection, the port shown on Fig. 10 is used.

[1] JOHNSON, *op. cit.*, p. 801.

From Fig. 10 we find the discharge coefficient to be 0.94 for flow out of the tank. This results in a gross area of $\frac{37.4}{0.94} = 39.8$ sq ft (40 sq ft used). The discharge coefficient of 0.71 applies to the same port for flow into the tank and gives a net area of $0.71(40.0) = 28.4$ sq ft, which is satisfactory.

Time. The time required to complete the quarter cycle for load-on is of interest mainly for estimating the number of time intervals which will be required for the arithmetic integration.

For load-on:

$$Z = \sqrt{\frac{y_1}{c} + v_1^2}$$

$$= \sqrt{\frac{109}{0.490} + 0} = 14.9$$

$$T_a = \frac{L}{2gcZ} \log_e \frac{(Z - v_1)(Z + v_2)}{(Z + v_1)(Z - v_2)}$$

$$= \frac{43{,}200}{64.4(0.490)(14.9)} \log_e \frac{(14.9 - 0)(14.9 + 13.70)}{(14.9 + 0)(14.9 - 13.70)} = 293 \text{ sec}$$

By arithmetic integration we find that $T_a = 300$ sec.

For load-off we get:

$$Z_0 = \sqrt{\frac{y_1}{c} - v_2^2}$$

$$= \sqrt{\frac{115.5}{0.382} - (13.15)^2} = 11.4$$

$$T_r = \frac{L}{gcZ_0}\left(\text{arc tan } \frac{v_2}{Z_0} - \text{arc tan } \frac{v_1}{Z_0}\right)$$

$$= \frac{43{,}200}{32.2(0.382)(11.4)}\left(\tan^{-1}\frac{13.15}{11.4} - \tan^{-1}\frac{0}{11.4}\right) = 264 \text{ sec}$$

From the integration we find that $T_r = 260$ sec.

Check by Arithmetic Integration (Tables 1 and 2, curves Fig. 14). The procedure consists of two basic operations: (1) A change in position of the turbine gates immediately changes flow through the turbine. The riser level starts to change, and this develops a head on the port area so water begins to flow in or out of the tank. For a known gate action and an assumed change in riser level, the flow to turbine, riser, and tank can be computed and the total must be the flow in the conduit. (2) The change in riser level results in an accelerating head on the conduit. The change in conduit velocity can be computed for this head and the total flow determined. If this flow checks that from (1), one step of the integration is completed.

Step 1: Fix the time interval as conditions dictate [column (1)].

Step 2: Assume the rise or fall of the water level in the riser during the time interval [column (6)]. This figure is positive when the water is ascending and negative when descending.

Step 3: Determine the discharge through turbines [column (3)] as follows: Estimate discharge through units and compute the head loss in penstock ($h = cv^2$). Compute the net head on the units [column (2)] by deducting from the gross head [column (6) minus tailwater elevation] the head loss in the penstock and adding the velocity head at the riser. Using the net head on the units and the proper gate position, determine the discharge through the units [column (3)] from the performance chart. Average the discharge through the units at the beginning and end of the interval to obtain the volume passed through the units during the interval [column (4)]. If column (6) has been estimated reasonably close, no further changes will be necessary in this step, except to correct the net head on the unit, since the discharges cannot be read with accuracy for small changes of head on the unit.

Step 4: Assume the change in water level in the tank [column (5)] and compute the rate of flow through the ports into the tank by means of $Q = a\sqrt{2gh}$ [column (7)]. The volume of water

stored in the tank can then be computed from the average rate of flow through the ports for the interval [column (8)]. We can then check the assumed change in water level in the tank [column (5)] by dividing the volume flowing into the tank [column (8)] by the value of F. Some cutting and trying are necessary in this step to obtain a balance between the various variables, but it is not troublesome since the water level changes in the tank are relatively sluggish. Columns (5), (7), and (8) are positive for flows into the tank and negative for flows out of the tank.

Step 5: Compute the volume stored in the riser [column (9)] by multiplying the change in water level [column (6)] by the area of the riser. This figure is positive when the riser is filling and negative when it is emptying.

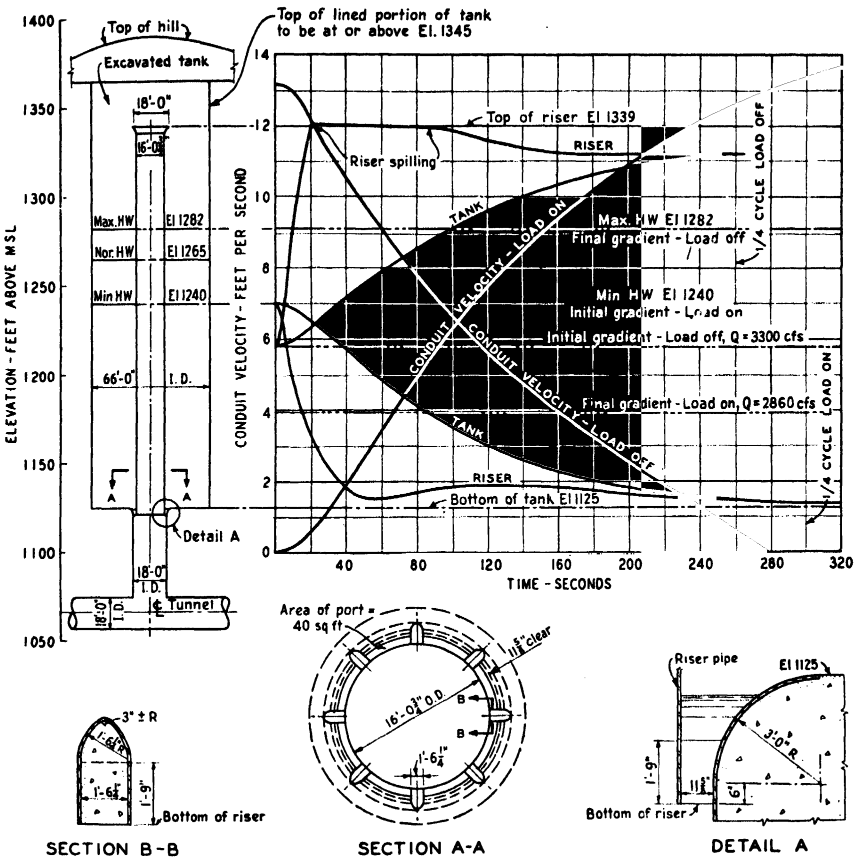

FIG. 14.—Differential surge tank, design data.

Step 6: The head on the conduit [column (10)] is computed by taking the difference between the elevation of the riser water surface and the forebay. This result is positive when the water level in the riser is below the forebay and negative when above.

Step 7: Compute the acceleration head at the end of the interval [column (12)] by adding algebraically the value of the head lost in the conduit [column (11)] to the head on the conduit [column (10)]. Column (11) is obtained by assuming the conduit velocity [column (15)] and computing the head loss from $h = cv^2$. This figure will have to be corrected after the velocity is determined, but again the change in discharge is not sensitive to small head changes so that no serious trouble results if the first guess is close. Column (11) is negative for flows toward the surge tank and positive for flows toward the forebay. Column (12) is positive when the velocity is increasing toward the tank and negative when increasing toward the forebay.

Step 8: Average the acceleration heads at the beginning and end of the interval [column (12)] and obtain the average acceleration head during the interval [column (13)].

TABLE 1.—ARITHMETIC INTEGRATION—DIFFERENTIAL SURGE TANK—APALACHIA PROJECT (132,000-HP LOAD REJECTION)

Headwater elevation = 1282 ft
Tailwater elevation = 840 ft
Port area 40 (0.71) = 28.4 sq ft for flow into tank
Port area 40 (0.94) = 37.6 sq ft for flow out of tank

C = 0.382 from intake to tank
C = 0.030 from tank to turbine
Tank diameter = 66 ft
Riser diameter = 16 ft

Gross area of tank = 3,421 sq ft
Riser area = 201 sq ft
Net area of tank F = 3,220 sq ft

Conduit L = 43,200 ft
Conduit area = 251 sq ft
Governor time = 5 sec

(1) Time, sec	(2) Net head on turbines, ft	(3) Flow through turbines, cfs	(4) Volume through turbines in Δt, cu ft	(5) W.S. elev tank, ft	(6) W.S. elev riser, ft	(7) Flow to tank and riser, cfs	(8) Volume to tank in Δt, cu ft	(9) Volume to riser in Δt, cu ft	(10) Head on conduit, ft	(11) Head lost in conduit, ft	(12) Acceleration head, ft	(13) Avg acceleration head, ft	(14) Flow through conduit, cfs	(15) Conduit velocity, fps	(16) Volume through conduit in Δt, cu ft	(17) Col (4) + col. (8) + col. (9), cu ft	Remarks
0	373.5	3300	0	1,216.00	1,216.00	0	0	0	66.00	66.00	0.00	0.00	3,300	13.15	0	0	132,000 hp, 66% gate
t = 1 1	375.13	2320	2810	1,216.05	1,217.69	970	145	340	64.31	66.00	− 1.69	− 0.85	3,300	13.15	3,300	3,295	90,000 hp, 46% gate
t = 1 2	383.00	1028	1674	1,216.19	1,223.50	2,272	453	1,168	58.50	66.00	− 7.50	− 4.60	3,300	13.15	3,300	3,295	40,000 hp, 20% gate
t = 1.3 3.3	......	....	670	1,216.52	1,236.25	3,300	1,055	2,560	45.75	65.70	− 9.95	− 8.73	3,300	13.14	4,290	4,283	0 hp, 0% gate
t = 1.7 5	......	....	....	1,217.15	1,254.00	3,300	2,040	3,580	38.00	65.70	− 27.70	− 18.83	3,300	13.12	5,600	5,620	
t = 2.0 7	......	....	....	1,218.10	1,271.50	3,270	3,050	3,520	10.50	65.20	− 54.70	− 41.20	3,270	13.06	6,530	6,570	
t = 2.0 9	......	....	....	1,219.20	1,286.50	3,270	3,530	3,020	− 4.50	64.40	− 68.90	− 61.80	3,260	12.97	9,720	6,550	
t = 3.0 12	......	....	....	1,221.05	1,305.50	3,220	5,930	3,820	−23.50	62.60	− 86.10	− 77.50	3,220	12.80	8,720	9,750	
t = 3.0 15	......	....	....	1,223.08	1,320.70	3,170	6,520	3,060	−38.70	61.10	− 99.80	− 92.95	3,170	12.66	9,585	9,580	
t = 3.0 18	......	....	....	1,225.23	1,333.30	3,130	6,920	2,530	−51.30	58.90	−110.20	−105.00	3,130	12.43	9,450	9,450	
t = 2.0 20	......	....	....	1,226.71	1,340.30	3,080	4,800	1,410	−58.30	56.40	−114.70	−112.45	3,060	12.18	6,190	6,210	
t = 20 40	......	....	....	1,244.41	1,341.60	2,646	Weir 10,300 Tank 46,700	262	−59.60	42.90	−101.90	−108.30	2,650	10.56	57,100	57,262	Weir crest- 18 × 3.1416 = 56.6
t = 20 60	......	....	....	1,259.71	1,339.42	2,290	Weir 7,030 Tank 42,750	− 435	−57.42	31.80	− 89.22	− 95.56	2,290	9.13	49,400	49,358	
t = 20 80	......	....	....	1,273.16	1,340.35	1,960	Weir 3,170 Tank 39,100	187	−58.35	23.30	− 81.65	− 85.43	1,970	7.85	42,600	42,457	
t = 8.0 88	......	....	....	1,274.98	1,339.00	1,835	Weir 845 Tank 14,600	− 270	−57.00	20.80	− 77.80	− 79.73	1,840	7.37	15,240	15,175	Spilling stops
t = 12 100	......	....	....	1,281.56	1,338.00	1,670	21,200	− 201	−56.00	17.20	− 73.20	− 75.50	1,680	6.70	21,120	21,000	
t = 20 120	......	....	....	1,291.46	1,333.00	1,410	31,800	−1,005	−51.00	11.95	−62.95	− 68.08	1,420	5.68	31,000	30,795	
t = 20 140	......		....	1,299.86	1,329.00	1,210	27,000	− 804	−47.00	8.44	− 55.44	− 59.20	1,210	4.80	26,200	26,200	

TABLE 2.—ARITHMETIC INTEGRATION—DIFFERENTIAL SURGE TANK—APALACHIA PROJECT (100,000-hp Load Demand)—(Continuous)

Headwater elevation = 1,240 ft
Tailwater elevation = 840 ft
Port area 40 (0.71) = 28.4 sq ft for flow into tank
Port area 40 (0.94) = 37.6 sq ft for flow out of tank
C = 0.490 from intake to tank
C = 0.036 from tank to turbine

Tank diameter = 66 ft
Riser diameter = 6 ft
Gross area of tank = 3,421 sq ft
Riser area = 201 sq ft
Net area of tank F = 3,220 sq ft

Conduit L = 43,200 ft
Conduit area = 251 sq ft
Governor time = 5 sec

(1) Time, sec	(2) Net head on turbines, ft	(3) Flow through turbines, cfs	(4) Volume through turbines in Δt, cu ft	(5) W.S. elev tank, ft	(6) W.S. elev riser, ft	(7) Flow to tank and riser, cfs	(8) Volume to tank in Δt, cu ft	(9) Volume to riser in Δt, cu ft	(10) Head on conduit, ft	(11) Head lost in conduit, ft	(12) Acceleration head, ft	(13) Avg acceleration head, ft	(14) Flow through conduit, cfs	(15) Conduit velocity, fps	(16) Volume through conduit in Δt, cu ft	(17) Col. (4) + col. (8) + col. (9), cu ft	Remarks
0	400.00	0	0	1,240.00	1,240.00	0	0	0	0	0	0	0	0	0	0	0	0 hp. 0% gate
t = 2 2	390.69	2,120	2,120	1,239.76	1,233.25	−2,120	− 770	−1,352	6.75	0.00	6.75	3.37	1	0.005	1	−4	84,000 hp, 40% gate
t = 3 5	371.10	2,580	7,050	1,238.72	1,214.90	−2,570	− 3,351	−3,690	25.25	0.00	25.25	16.00	10	0.041	18	9	100,000 hp, 51% gate
t = 5 10	348.30	2,720	13,250	1,236.04	1,192.50	−2,680	− 8,640	−4,500	47.50	0.00	47.50	36.38	43	0.173	135	110	100,000 hp, 56% gate
t = 5 15	332.80	2,855	13,940	1,232.75	1,177.50	−2,760	−10,575	−3,020	62.50	0.00	62.50	55.00	95	0.378	345	347.5	100,000 hp, 61% gate
t = 5 20	321.00	2,959	14,535	1,229.15	1,166.00	−2,800	−11,575	−2,310	74.00	0.18	73.82	68.16	158	0.632	650	634	100,000 hp, 66% gate
t = 10 30	304.20	3,162	30,650	1,221.45	1,149.00	−2,850	−24,800	−3,420	91.00	0.75	90.25	82.04	312	1.244	2,353	2,430	100,000 hp, 75% gate
t = 10 40	290.60	3,342	33,420	1,213.35	1,137.00	−2,850	−26,100	−2,410	103.00	1.88	101.12	95.68	492	1.961	4,020	4,010	99,000 hp, 85% gate (blocked)
t = 10 50	284.40	3,310	33,100	1,205.24	1,130.70	−2,630	−26,130	−1,265	109.30	3.66	105.64	103.38	686	2.732	5,890	5,865	95,000 hp, 85% gate (blocked)
t = 10 60	284.50	3,300	33,100	1,197.39	1,130.70	−2,420	−25,300	0	109.30	6.02	103.28	104.46	880	3.512	7,830	7,800	95,000 hp, 85% gate (blocked)
t = 20 80	287.40	3,335	66,340	1,183.19	1,134.00	−2,090	−45,800	665	106.00	12.30	93.70	98.49	1245	4.970	21,250	21,205	96,000 hp, 85% gate (blocked)
= 20 100	289.60	3,348	66,830	1,171.09	1,136.00	−1,770	−39,000	402	104.00	19.45	84.55	89.13	1578	6.300	28,230	28,332	98,000 hp, 85% gate (blocked)
= 20 120	290.60	3,340	66,880	1,160.99	1,137.00	−1,460	−32,500	201	103.00	27.60	75.40	79.98	1880	7.490	34,580	34,580	99,000 hp, 85% gate (blocked)
= 20 140	290.60	3,340	66,800	1,152.64	1,137.00	−1,210	−26,700	0	103.00	35.70	67.30	71.35	2130	8.550	40,100	40,100	99,000 hp, 85% gate (blocked)

Turbine gates are blocked at 85% to avoid operation on the rapidly falling portion of the turbine efficiency curve. If the turbine were allowed to open to 100% gate, the result would be a lower required bottom elevation of tank and a reduction of power output at the heel of riser curve with no compensating advantages.

Step 9: Compute the change in discharge in the conduit [column (14)] from the equation $dQ = \frac{H_a g A\, dt}{L}$ and the resulting discharge at the end of the time interval. Check the assumed conduit velocity [column (15)] and make the necessary corrections.

Step 10: Compute the volume of flow through the conduit [column (16)]. Then add the volume of flow through the units to that going into the tank and riser [column (17)]. If columns (16) and (17) are equal or reasonably close, the integration for that time interval is complete. If they are at variance, the necessary corrections must be made.

When the riser is spilling, an additional complication is introduced. In order to determine the exact retarding head which is being applied to the tunnel, it is necessary to compute the height at which the water stands above the top of the riser by use of a weir formula.

The practice of blocking the turbine gates at about 85 per cent as indicated in Table 2 may be considered typical standard practice. If the gates were not so blocked but allowed to open to the 100 per cent position, the surge would be increased quite appreciably owing to the rapid falling away of turbine efficiency after about 85 per cent gate. This would necessitate placing the tank bottom at lower elevation and in addition would result in appreciably lower power output at the heel of the riser drop curve, with no compensating advantages. The output of 95,000 hp at the heel given by Table 2 is usually considered sufficiently close to the specified value of 100,000 hp.

9. OPERATING TESTS

Figure 15 shows the results of applying an instantaneous load demand of 80,000 kw and later, after reasonably quiescent conditions have been established, an instantaneous load rejection of 80,000 kw. These load changes were applied with the

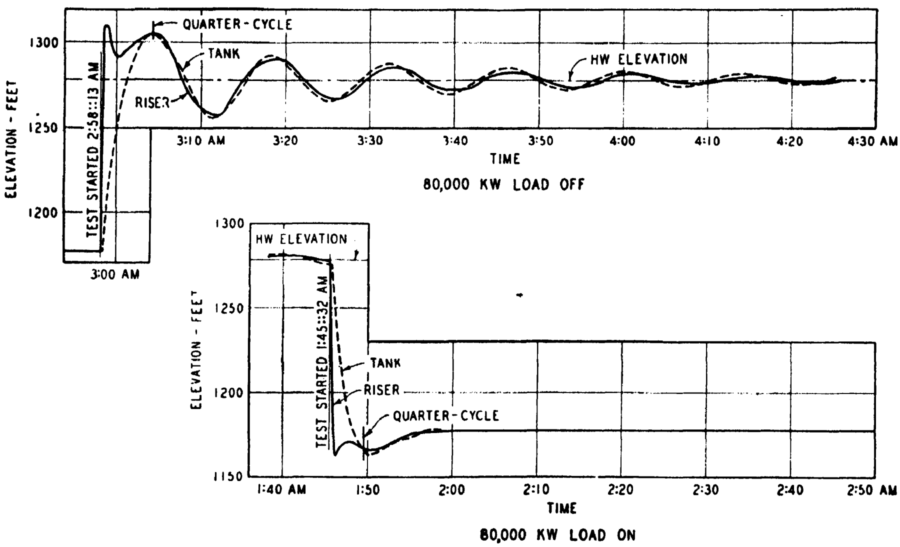

FIG. 15.—Surge tank testing with both units operating, Apalachia project.

Apalachia units synchronized and operating in conjunction with the entire T.V.A. system. The large block of load required for making the test was obtained by prearrangement and interconnection with the Louisville Gas and Electric Company.

There are of course several important differences to be emphasized in comparing these actual performance curves with the demand and rejection conditions for which the surge tank was designed. In the first place, the headwater elevation available was

about El. 1,280 as compared with El. 1,282 for the design rejection condition and 1,240 for the design demand condition; but the demand test load is about 107,000 hp as compared to the design demand load of 100,000 hp. For the demand condition the value of V_2 is about the same so that the demand surge is practically the same in both cases. As would naturally be expected, the final quiescent gradient for demand is about 1,180 for both the arithmetic integration and the operating test. All factors considered, the test does yield substantial assurance that if both units were given an instantaneous load demand aggregating 100,000 hp starting with headwater at El. 1,240, there would be a margin of at least 5 ft between the bottom of tank and the extreme low elevation of water levels.

The load rejection test starts with the tank levels at about El. 1,180 as compared with the design assumption of El. 1,216, a difference of 36 ft. The design case assumed a load rejection of 132,000 hp as compared with 107,000 hp for the test; but this greater demand is compensated to some extent by the fact that in the design case the units were operating at a head about 40 ft greater than that which obtained during test conditions. There was an adverse factor during test conditions caused by the fact that the riser was unable to spill, giving a characteristic recession and subsequent rise in riser levels during the quarter cycle. These considerations will explain why the rejection surge was about 130 ft under test conditions as compared with 120 ft under design conditions. Again, however, the result of the rejection test was a confirmation of the original design and indicated that, if rejection were started with tank levels at El. 1,216, performance would be in accordance with the design curves.

It is interesting to note that in the case of load rejection a very appreciable time interval is required to damp out the major pendulations of water surface in the tank, about one-half hour being necessary for fairly steady conditions while some unimportant pendulations were still observable for an hour after the completion of the test. This is, however, what might reasonably be expected for the case of a 9-mile tunnel of such large dimensions. It will also be observed that, while a periodic disturbance persisted for a comparatively long period of time in the case of load rejection, damping to the quiescent condition required but a brief interval for load demand. This is, of course, due to the fact that under the demand condition the load itself is a means of absorbing excess energy while, in the rejection condition, conduit friction is the only agency affecting the ultimate dissipation. This case parallels to some degree a similar phenomenon observable in the Angus diagrams for water-hammer pressures in the cases of valve closure as compared with valve opening. After the valve has closed, water-hammer pendulations continue for an appreciable time at relatively high intensity; but in the case of opening, the pendulations subsequent to the fully open condition are of small magnitude and commensurate duration.

10. OTHER TYPES OF TANK

It is believed that with the foregoing material on the analysis of differential surge tanks as background, the engineer will have no difficulty in preparing any desired investigation of other types of regulator.

Computation of the simple surge tank by arithmetic integration will be found comparatively easy and similar but shorter than the tabulation given in Art. 8. Because of the aforementioned difficulty with the form of its differential equation, no equations similar to Johnson's expression are available for predetermination of the tank size in relation to the surge, so that for preliminary proportions we have only the judgment gained from experience and the Thoma formula for the diameter required for stability. Before the advent of the modern quick-response governor, proponents of the simple tank claimed as one of its advantages a very gradual drop in the water surface curve, which the turbine governor could follow with comparative ease. With the availa-

bility of modern high-speed governors, however, this advantage is no longer of any great weight in determining selection of types. With respect to synchronous load changes, the simple tank is at a definite disadvantage in comparison with the differential regulator, in which the water supply and conduit acceleration functions are separated. For given performance, the simple tank design is generally a more expensive solution.

The restricted orifice type of tank (Fig. 16) is somewhat similar in action to the differential surge tank, the principal difference being that no riser is provided, and the entrance orifice into the bottom of the tank is relatively smaller than the corresponding

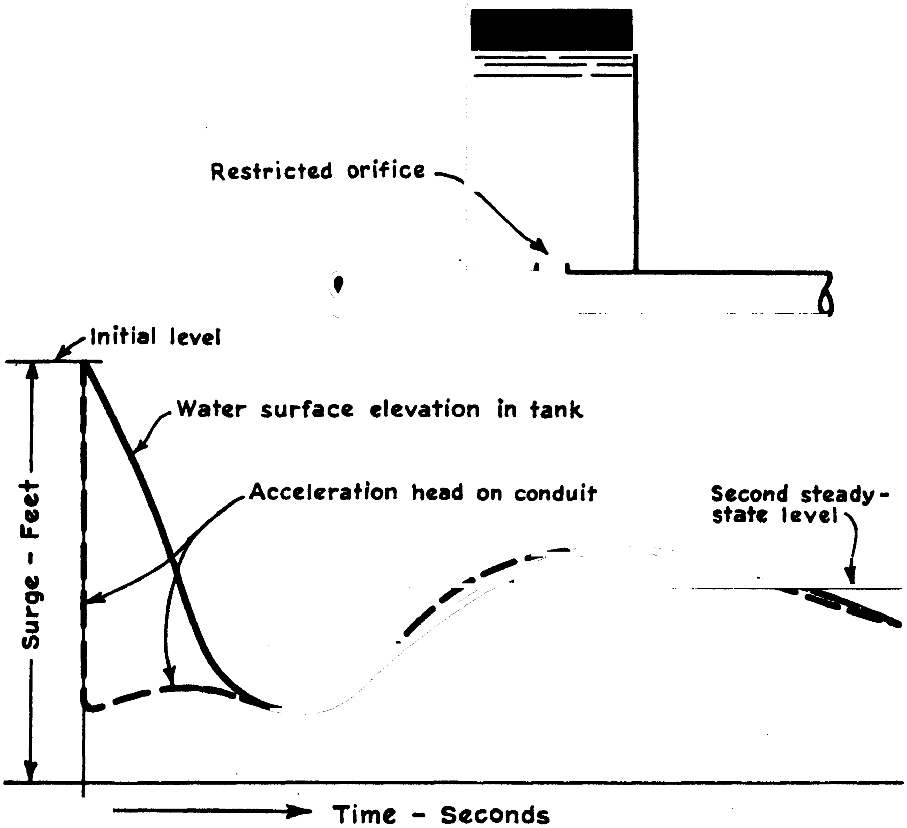

FIG. 16.—Load demand. Restricted orifice surge tank with typical performance curves.

riser diameter of the differential regulator. This means that with respect to the mass-acceleration action the retarding or accelerating head is created at the instant of gate movement—in magnitude just sufficient to store in or supply from the tank the requisite decrement or increment of water corresponding to the load change. This accelerating or retarding head on the restricted orifice then varies from instant to instant as required to give hydraulic continuity. In comparison with the differential tank, the adverse effects of water hammer are increased owing to the reduction in size of the opening into the surge tank and the consequent reduction in discharge relief afforded the conduit.

For very high head plants (those having heads in the order of 1,000 ft or higher), the type of tank shown in Fig. 17 has found favor in some installations both in this

country and in Europe. This type of design finds application in mountainous terrain, in which the various component sections of the regulator may be excavated from solid rock. The design consists basically of comparatively large upper and lower storage chambers connected by an intermediate vertical shaft of relatively small diameter. Upon the application of a load change of any considerable magnitude the water surface moves promptly to either the upper or the lower chamber, as required, creating a large decelerating or accelerating head on the conduit; and it is a comparatively inexpensive matter to excavate the upper and lower chambers of size sufficient to store or supply the volume of water required for the load change. During normal

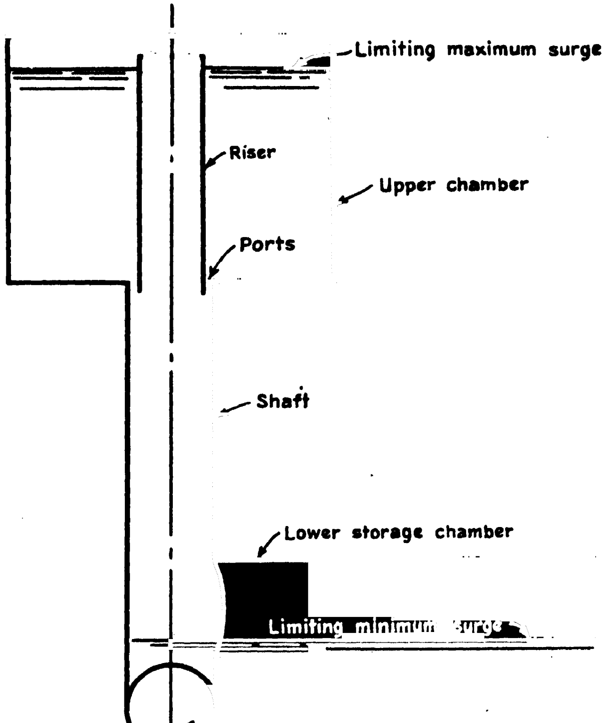

FIG. 17.—Compound surge tank applicable to high head installations.

operation and light load changes, the water surface elevation is in the intermediate shaft; and in view of the very high head, it will be seen from the Thoma formula that the area required for stability in the central shaft is comparatively small. It should be emphasized, however, that, because of stability considerations, this small central shaft can be employed only on the higher head installations, and the whole arrangement is, as a general rule, economical only in cases where it is possible to excavate the regulator from the solid rock.

With the development of large modern hydroelectric generating units of great capacity, the field of applicability of the closed-top tank, in which the surge of water is accompanied by the compression or expansion of a superimposed volume of air, appears to be restricted to a few isolated units in remote locations, in which the difficulty of hauling materials or the lack of headroom is a dominant consideration. In view of the reduced likelihood that the engineer will be called upon to design such

equipment for hydroelectric installations, and the additional fact that they constitute such a specialty, no further discussion of their design will be given in this chapter other than to refer the reader to the literature on the subject.[1]

As would naturally be expected, surge regulators, particularly those of the differential type, not only are employed for hydraulic turbine installations but are equally applicable to the companion reverse case of centrifugal or other pumps feeding long pipe lines employing moderate to high velocities. The only difference in the application is that, upon stoppage of the pump motor by interruption of the electric power supply, there is a reduction in water supply and a diminution of velocity, and hence the initial impulse propagated along the pipe line is a negative instead of a positive pressure wave. Just as in the case of the turbine plants, water-hammer effects are alleviated by providing by means of a surge tank an open reservoir close to the terminal mechanism, capable of propagating back compensating pressure reflections of opposite sign. Again, as in the case of the turbine, it is feasible and advantageous to separate the water hammer and the mass acceleration computations which may be carried out in fashion exactly similar to the material given in Art. 8. In some installations, optimum over-all economy may be secured by eliminating the check valve in the line ahead of the pump and providing for comparatively slow closure of the main valves. In such cases the centrifugal pump not only may slow down upon loss of power supply but, owing to the slow closure of the valve, may run backward as a turbine for a short interval until valve closure is completed. For the solution of such cases, both for water-hammer and mass-acceleration effects, a complete set of performance curves for the pump acting either as a pump or in reverse as a turbine is an essential prerequisite to the calculation

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SECTION 15

SPEED REGULATION AND GOVERNING STABILITY

BY GEORGE R. RICH

1. INTRODUCTION AND SCOPE

The purpose of this section is to furnish background for the calculation and correlation of governor time, generator WR^2, and transient characteristics of the pressure conduit, so as to ensure the degree of speed regulation warranted for the particular project. It is assumed at the outset that the engineer has access to the literature of the governor manufacturers, which gives a very clear and complete account of mechanical construction and accessories.

To meet the increasing demand upon the modern hydraulic engineer for governing stability computations, it was originally intended to present (with the kind authorization of the author and his publishers) simply a translation and abstract of the classic treatise, "Considérations sur le problème de la stabilité des regulateurs de vitesse," by Daniel Gaden.† But in furnishing valued assistance during the progress of this work, Mr. Gaden has virtually rewritten this entire portion of the chapter expressly for American engineers. For this generous cooperation, it is desired to make acknowledgment.

Before proceeding with the analytical work, it appears pertinent to caution against providing refinement in regulation in excess of what is justified economically for the particular project. Improvements in regulation may prove relatively expensive and have a tendency to trespass the boundary of diminishing returns. The actual limits to the regulation must as a general rule be established in collaboration with the operating engineers, who are well aware that investments in speed and frequency control must earn their share of dividends.

2. FLYWHEEL EFFECT FOR THE OPEN-FLUME SETTING

The most logical first step in breaking ground for the development of the working tools for the governing problem is to formulate the basic expression for speed change as a function of load change and flywheel effect, or WR^2.

To reduce the problem to its simplest elements, we shall consider that the turbine is operating in an open-flume setting and that the governor is simply a means of closing or opening the turbine gates uniformly in a prescribed time. We shall employ the following notation:

Δt = time required by governor to move turbine gates to new position, sec.
hp = *instantaneous* change in horsepower delivered *externally* by turbine.
N_0 = speed of rotation of turbine in rpm at instant of external load change.
N = speed of rotation of turbine at end of time Δt, rpm.
WR^2 = polar moment of inertia of rotating masses about axis, pfs.

† Directeur, Ateliers des Charmilles, Geneva. Published by Editions La Concorde, Lausanne, Switzerland.

Then from elementary dynamics,

$$\text{Applied work} = \text{change in kinetic energy}$$

$$\frac{\Delta t \times hp \times 550}{2} = \frac{W}{g}\frac{1}{2}\left[\left(\frac{2\pi RN}{60}\right)^2 - \left(\frac{2\pi RN_0}{60}\right)^2\right]$$

$$N^2 - N_0^2 = \frac{1{,}620{,}000\ \Delta t(hp)}{WR^2} \tag{1}$$

This derivation is fully rigorous and should be employed in all analyses where complete accuracy is essential. However, because of its greater convenience, the following approximation has found almost universal acceptance in the power industry.

Dividing both sides by N_0^2 and factoring,

$$\left(\frac{N + N_0}{N_0}\right)\left(\frac{N - N_0}{N_0}\right) = \frac{1{,}620{,}000\ \Delta t}{C}$$

in which $C = \dfrac{N_0^2(WR^2)}{hp}$ is known as the regulation constant.

If we take $\dfrac{N + N_0}{N_0} = \dfrac{2N_0}{N_0} = 2$ as an approximation, we obtain

$$\omega = \frac{N - N_0}{N_0} = \frac{8.1 \times 10^5\ \Delta t}{C} \tag{2}$$

Equations (1) and (2), particularly (2), find very wide application, but they usually require two additional corrections: the first to compensate for the fact that at any given head there is an upper limit to the speed which the turbine is capable of reaching, namely the so-called runaway speed; the second to make allowance for the effect of water hammer.

To include the effect of turbine runaway limit, let S = the relative speed of the turbine at runaway, referred as a percentage (such as 180) to the normal or synchronous speed.

We next seek an approximate expression that will yield $\omega_{rs} = S - 100$ for the runaway speed as the maximum value, and make commensurate correction for all intermediate values of ω between normal speed and runaway. The following expression will be found to answer the purpose:

$$\omega_{rs} = \frac{\omega}{100 + \dfrac{100\omega}{S - 100}} \tag{3}$$

For example, if the speed rise computed by Eq. (2) is 60 per cent and the runaway speed of the particular turbine is 180 per cent of normal, the corrected speed rise will be

$$\omega_{rs} = \frac{60}{100 + \dfrac{100 \times 60}{180 - 100}} = 34 \text{ per cent}$$

For speed drop for the condition of load on, Eqs. (1) and (2) are usually applied without correction for variation in the slope of the performance curve of the driving couple as a function of the speed of rotation.

Example 1: On the assumption of an open flume setting, and neglecting the effect of the turbine runaway characteristic, compute the percentage speed rise for the following conditions:

Load rejected.	91,500 hp
Synchronous speed.	150 rpm
Gate closure time.	3 sec
Generator WR^2.	53,000,000 pfs

$$\text{Regulation constant } C = \frac{N_0{}^2WR^2}{hp} = \frac{150^2 \times 53{,}000{,}000}{91{,}500} = 13 \times 10^6$$

$$\frac{N_1 - N_0}{N_0} = \frac{8.1 \times 10^5 \,\Delta t}{C} = \frac{8.1 \times 10^5 \times 3}{13 \times 10^6} = 18.5 \text{ per cent}$$

Speed rise = 18.5 per cent

Example 2: Include the effect of turbine runaway speed correction in Example 1, assuming that the runaway speed of the turbine is 180 per cent of normal synchronous speed.

$$\omega_{rs} = \frac{\omega}{100 + \dfrac{100\omega}{S - 100}} = \frac{18.5}{100 + \dfrac{100 \times 18.5}{180 - 100}} = 15 \text{ per cent}$$

Speed rise = 15.0 per cent

3. INFLUENCE OF WATER HAMMER

For the installations encountered in practice, the turbine setting will not be an open flume as contemplated by Eqs. (1) and (2), but will be served by some form of waterway or penstock. Accordingly, reductions or increases in the penstock velocity will be accompanied by a head rise or head drop resulting from the phenomenon of water hammer.

The water-hammer increment (or decrement, as the case may be) will change the power supplied to the turbine according to the $\frac{3}{2}$ power of the resultant relative value of the increased head in accordance with established principles.

For the condition of load rejection, velocity decrease, and pressure rise, the power input to the turbine will become $hp_{wh} = hp_0(1 + h)^{3/2}$ in which $h = \dfrac{H_{wh} - H_0}{H_0}$ (with $h > 0$), and since the speed rise in Eq. (1) varies directly as the power, we obtain

$$\omega_{wh} = \omega(1 + h)^{3/2} \tag{4}$$

Including the effect of both runaway speed and water hammer, we have

$$\omega_{rs+wh} = \omega_{rs}(1 + h)^{3/2} \tag{5}$$

For the condition of load demand, velocity increase, and pressure drop, we also find, by a similar approach,

$$\omega_{wh} = \frac{\omega}{(1 + h)^{3/2}} \tag{6}$$

but with $h < 0$.

The requisite relative values of pressure rise and fall are determined by the methods of the section on water hammer.

Example 3: Include the effect of water hammer in Example 2, assuming the water-hammer effect to be 65 per cent greater than the normal head.

$$\omega_{rs+wh} = \omega_{rs}(1 + h)^{3/2} = 15.0(1.65)^{3/2} = 32.0 \text{ per cent}$$

Speed rise = 32.0 per cent

Compare with the arithmetic integration method, Example 5, which has the same physical data.

Example 4: Compute the percentage speed drop for the following conditions:

Load increase from 51,500 to 91,500 hp	
Synchronous speed.	150 rpm
Gate opening time.	1.5 sec
Generator WR^2.	53,000,000 pfs
Water-hammer drop.	30 per cent ($h < 0$)

$$\text{Regulation constant } C = \frac{N_0{}^2(WR^2)}{hp} = \frac{150^2 \times 53{,}000{,}000}{40{,}000} = 30 \times 10^6$$

$$= \frac{N_1 - N_0}{N_0} = \frac{8.1 \times 10^5 \,\Delta t}{C} = \frac{8.1 \times 10^5 \times 1.5}{30 \times 10^6} = 4 \text{ per cent}$$

$$\omega_{wh} = \frac{\omega}{(1 + h)^{3/2}} = \frac{4}{(1 - 0.30)^{3/2}} = 7 \text{ per cent}$$

Speed drop = 7 per cent

TABLE 1.—SPEED RISE AND WATER HAMMER

150 rpm. 91,500 hp at 330 ft head. Full gate, 135 in. diameter

(Use Appalachia N^2 test curve)

Neglect friction and velocity head

L = 400 ft
Diameter = 14 ft 0 in.
a = 4,660 fps
A = 153.5 sq ft

Closure time = 4 sec
Use uniform equivalent = 3 sec (to allow for dead and cushioning time)
18 intervals at 0.17 = 3.06 sec
WR^2 = 53,000,000 pfs

(1) Interval	(2) Time, sec	(3) Gate, %	(4) ΔV V	(5) ΔH or $145\,\Delta V$	(6) $\Sigma\,\Delta H$	(7) $H_0 + \Sigma\,\Delta H$	(8) Rpm	(9) Rpm_1 12 in.	(10) Q	(11) Efficiency, %	(12) V	(13) Hp_1 12 in.	(14) Hp	(15) Avg hp	(16) Rpm²	(17) $N_2^2 - N_1^2$	(18) ΔT
0	0	100	18.25	0	0	330	150	92.75	2,810	86.5	18.25	0.120	91,500	......	22,500		
1	0.17	94.5	−0.05 18.20	+7.2	+7.2	337.2	153.2	94.2	2,790	87.0	18.20	0.119	93,000	92,250	23,450	950	0.17
2	0.34	89.0	−0.15 18.05	+21.7	+14.5	344.5	156.3	95.0	2,760	88.0	18.05	0.117	94,600	93,800	24,430	980	0.17
3	0.51	83.5	−0.25 17.80	+36.3	+21.8	351.8	159.43	96.0	2,730	88.0	17.80	0.115	95,700	95,150	25,408	990	0.17
4	0.68	78.0	−0.37 17.42	+53.6	+31.8	361.8	162.5	96.2	2,670	90.0	17.42	0.114	98,200	96,950	26,406	998	0.17
5	0.85	72.0	−0.51 16.91	+74.0	+42.2	372.2	165.7	96.8	2,600	90.7	16.91	0.111	100,600	99,400	27,456	1,050	0.17
6	1.02	67.0	−0.65 16.26	+94.0	+51.8	381.8	168.8	96.0	2,510	90.6	16.26	0.106	99,300	99,950	28,561	1,037	0.17
7	1.19	61.0	−0.70 15.56	+102.0	+50.2	380.2	171.95	99.5	2,390	90.0	15.56	0.100	94,000	96,650	29,567	1,006	0.17

8	1.36	56.0	−0.70 14.86	+102.0	+51.8	381.8	174.7	101.0	2,280	90.3	14.86	0.094	88,700	91,350	30,520	953	0.17
9	1.53	50.0	−0.90 13.96	+130.5	+78.7	408.7	177.3	98.5	2,150	89.3	13.96	0.085	89,000	88,850	31,435	915	0.17
10	1.70	44.0	−1.05 12.91	+152.5	+73.8	403.8	179.75	101.0	1,980	87.5	12.91	0.077	79,200	84,100	32,310	875	0.17
11	1.87	39.0	−1.05 11.86	+152.0	+78.2	408.2	182.0	101.5	1,825	86.7	11.86	0.071	74,000	76,600	33,124	814	0.17
12	2.04	33.0	−1.20 10.66	+174.0	+95.8	425.8	183.96	101.0	1,630	83.3	10.66	0.059	65,600	69,800	33,841	717	0.17
13	2.21	28.0	−1.40 9.26	+203.0	+107.2	437.2	185.65	100.0	1,424	80.0	9.26	0.049	56,900	61,250	34,473	632	0.17
14	2.38	22.0	−1.70 7.56	+246.0	+138.8	468.8	187.0	97.0	1,165	76.0	7.58	0.037	47,300	52,100	35,015	542	0.17
15	2.55	17.0	−2.10 5.46	+305.0	+166.2	496.2	188.10	95.0	840	58.8	5.46	0.020	28,000	37,650	35,409	394	0.17
16	2.72	11.0	−2.60 2.86	+377.0	+210.8	540.8	188.3	92.0	440	29.6	2.86	0.005	8,000	18,000	35,596	187	0.17
17	2.89	6.0	−1.88 0.98	+273.0	+62.2	392.2	188.4	107.0	150	0.00	0.98	0.00	0.00	4,000	35,638	42	0.17
18	3.06	0.0	−0.98 0.00	+142.0	+79.8	409.8	188.4	104.5	0.00	0.00	0.00	0.00	0.00	0.00	35,638	0	

$$\text{Interval} = \frac{2L}{a} = \frac{2 \times 400}{4,660} = 0.17 \text{ sec}$$

$$\Delta H = \frac{a\,\Delta V}{g} = \frac{4,660\,\Delta V}{32.2} = 145\,\Delta V$$

$$\text{Work} = \frac{MV^2}{2}$$

$$\text{Avg hp} \times 550 \times \Delta T = \frac{W}{2g}\left[4\pi^2 R^2 \frac{(N_2^2 - N_1^2)}{3,600}\right]$$

$$\Delta T = \frac{16.33(N_2^2 - N_1^2)}{\text{Avg hp}}$$

TABLE 2.—TIME REQUIRED TO PICK UP FULL LOAD
Gate opening time 150 sec
Assume generator synchronized to system at 300 rpm

(1) Interval	(2) Time, sec	(3) Gate opening, %	(4) ΔV / V	(5) ΔH or $145\,\Delta V$	(6) $\Sigma\,\Delta H$	(7) Friction loss	(8) $H_0 + \Sigma\,\Delta H + F$	(9) Q	(10) V	(11) Turbine efficiency, %	(12) Hp
0	0	0	0.00	0	0	0	534	0	0	0	0
1	11.50	0.077	+0.75 0.75	−109	−109	−1	424	59	0.75	Speed, no load	0
2	23.00	0.154	+0.88 1.63	−127	−18	−2	514	129	1.64	48	3,600
3	34.50	0.231	+0.67 2.30	−97	−79	−3	452	182	2.31	68	6,360
4	46.00	0.308	+0.85 3.15	−123	−44	−6	484	250	3.18	79	10,800
5	57.50	0.384	+0.75 3.90	−109	−65	−10	459	304	3.88	85	13,500
6	69.00	0.460	+0.85 4.75	−123	−58	−14	462	372	4.74	87	17,000
7	80.50	0.537	+0.75 5.50	−109	−51	−20	463	433	5.50	91	20,800
8	92.00	0.614	+0.72 6.22	−104	−53	−25	456	487	6.22	92	23,200
9	103.50	0.691	+0.71 6.93	−103	−50	−32	452	544	6.93	92.5	25,800

10	115.00	0.768	+0.70 7.63	−101	−51	−37	446	600	7.63	92.0	28,000
11	126.50	0.845	+0.72 8.35	−104	−53	−45	436	656	8.35	91.0	29,600
12	138.00	0.922	+0.72 9.07	−104	−51	−52	431	712	9.07	89.0	31,000
13	149.50	1.000	+0.72 9.79	−104	−53	−60	421	767	9.79	88.0	32,300
14	161.00	1.00	+0.45 10.24	−65	−12	−60	462	805	10.24	88.0	37,200
15	172.50	1.00	+0.66 10.30	−9	+3	−60	477	810	10.30	88.0	38,600

Tunnel area = 78.5 sq ft
L = 26,850 ft
a = 4,660 fps

$$\Delta H = \frac{a\,\Delta V}{g} = \frac{4{,}660\,\Delta V}{32.2} = 145\,\Delta V$$

$$\frac{2L}{a} = \frac{2 \times 26{,}850}{4{,}660} = 11.50 \text{ sec}$$

4. USE OF ARITHMETIC INTEGRATION

For the more involved studies of transient phenomena accompanying load change, the step-by-step method of arithmetic integration will be found to have wide applicability, and it is believed that the details of the process will be clear from the following illustrative examples. The turbine performance data were supplied by the manufacturers in the form of model test curves, plotted according to the so-called **oak-tree contour arrangement**, giving values of unit power as ordinates, unit speed as abscissas, closed contours as efficiency, and transverse curves to show gate opening. It will be an instructive exercise for the reader to plot curves showing the change in the various variables with the time for the three examples given.

Example 5: Table 1 is a typical, fast, load-rejection study which affords an accurate time history of the transient disturbance. The general mode of attack is to guess simultaneously trial values of V and rpm; when the guess is correct, the values of V in column (11) and Δt in column (17) will check the corresponding values of V in column (3) and Δt in column (1), respectively.

Example 6: This was undertaken (Table 2) to show the length of time required to pick up full load for the case of a plant with a long pressure conduit and no surge tank, but provided with a relief valve for load rejection. Regulation was provided by other plants in the system, but the transient problem is closely allied with the general subject of this chapter. The mode of attack is to guess trial values of ΔV in column (4) and, when the trial value is correct, the value of V in column (10) will check the value of V in column (4). V in column (10) is computed from the total head in column (8) and the turbine efficiency and power in columns (11) and (12). The turbine performance is based upon manufacturers' test curves.

Example 7: This is based (Table 3) upon the same plant as given in Example 6. It is assumed that the turbine gates are stuck in the full open position following rejection of full load for short-circuit condition. Owing to the overspeed characteristic of the turbine, the discharge is reduced as the turbine picks up speed. This in turn produces water hammer just as effectively as if the valve were closed too fast. The fact that the transient disturbance damps down to an equilibrium level indicates that the oscillating system is inherently stable. The general mode of attack is to guess trial values of ΔV in column (3) and rpm in column (8), and verify the trial values in column (12) for V and column (18) for ΔT. For the method of arranging this problem the writer is indebted to W. M. Rheingans, assistant manager, Hydraulic Department, Allis-Chalmers Manufacturing Company. The turbine performance characteristics were obtained from the manufacturer's oak-tree model test curves.

5. BASIC EQUATION OF GOVERNING (Compensation Omitted)

To an increasing degree, the modern hydraulic engineer is called upon to analyze problems in the field of governing stability, by which is meant the selection and correlation of the elements WR^2, governor action, and pressure conduit features so as to ensure rapid damping of the transient disturbance to normal synchronous speed following a load change, the load afterward remaining constant. When due consideration has not been given to stability, there is the possibility of perpetual pendulation of the gate-operating mechanism and of the speed of operation with resultant disruption to power interchange service, electric time keeping, and other related functions of the present-day highly organized electric power industry.

With the ultimate objective of developing a simple, workable test for governing stability, we shall start by formulating the operating equation for a crude rudimentary governor, consisting merely of a speed-responsive element or flyball, a relay valve with suitable ports, and an oil-pressure-operated servomotor for moving the turbine gates. We shall note the defects inherent in such a rudimentary system, and then in succeeding articles we shall add the necessary corrective elements and incorporate their respective functions in the governor-operating equation. For the material presented in the next four articles, the writer is indebted to the classic treatise† of the French engineer, Daniel Gaden.

As the first step, let us rewrite the fundamental flywheel equation [Eq. (1) Art. 2] in

† GADEN, DANIEL, "Considérations sur le problème de la stabilité des regulateurs de vitesse," Editions La Concorde, Lausanne, Switzerland.

TABLE 3.—PRESSURE AND SPEED RISE COMPUTATION

Turbine gates stuck in full gate position[1]

Tunnel area = 78.5 sq ft
L = 26,850 ft
a = 4,660 fps
WR^2 = 4,000,000 pfs

(1) Interval	(2) Time, sec	(3) $\frac{\Delta V}{V}$	(4) ΔH or 145 ΔV	(5) $\Sigma\,\Delta H$	(6) Friction head regained, ft	(7) $H_0 + \Sigma\,\Delta H + F$	(8) Rpm	(9) Rpm_1	(10) Q_1	(11) Q	(12) V	(13) Hp_1	(14) Hp	(15) Avg hp	(16) Rpm^2	(17) $N_1^2 - N_2^2$	(18) ΔT
	0	10.31	0.00	0.00	0	474	300	13.8	37.2	810	....	3.68	38,000	....	90,000		
1	11.50	−1.61 8.70	+233	+233	13	720	582	21.7	25.4	683	8.70	+0.80	+15,500	+26,750	339,000	249,000	11.50
2	23.00	−2.30 6.40	+334	+101	35	610	608	24.6	20.4	504	6.42	−0.56	−8,440	+3,530	369,000	33,000	11.50
3	34.50	−0.57 5.83	+83	−18	39	495	548	24.6	20.4	467	5.81	−0.57	−6,280	−7,360	300,000	69,000	11.50
4	46.00	+0.31 6.14	−45	−27	36	483	518	23.6	22.2	488	6.19	−0.05	−520	−3,405	268,200	31,800	11.50
5	57.50	+0.33 6.47	−48	−21	34	487	516	23.4	22.55	499	6.36	0.00	0.00	−265	265,725	2,475	11.50
6	69.00	+0.10 6.57	−15	+6	32	512	523	23.10	22.95	518	6.60	+0.15	+1,730	+865	273,800	8,100	11.50
7	80.50	−0.10 6.47	+15	+9	33	516	532	23.40	22.50	510	6.49	0.00	0.00	+865	281,900	8,100	11.50
8	92.00	−0.05 6.52	+7	−2	33	505	529	23.50	22.30	502	6.44	−0.05	−568	−284	279,250	2,650	11.50

Interval $\frac{2L}{a} = \frac{2 \times 26{,}850}{4{,}660} = 11.50$ sec $\quad \Delta H = \frac{a\,\Delta V}{g} = \frac{4{,}660\,\Delta V}{322} = 145\,\Delta V \quad$ Work $= \frac{MV^2}{2}$

$$\text{Avg hp} \times 550 \times \Delta T = \frac{W}{2g}\left[4\pi^2R^2\,\frac{(N_1^2 - N_2^2)}{3{,}600}\right] \qquad \Delta T = 1.235\,\frac{(N_1^2 - N_2^2)}{\text{Avg hp}}$$

[1] For the arrangement used in this computation, the writer is indebted to W. J. Rheingans, assistant manager, Hydraulic Department, the Allis-Chalmers Manufacturing Co., Milwaukee, Wis.

differential form; and in order to give complete generality to the results and thus permit the use of tables applicable to all subsequent numerical cases, we shall use relative values for the principal variables such as $\omega = \frac{N - N_0}{N_0}$; $p_0 = \frac{P - P_0}{P_0}$, $h = \frac{H - H_0}{H_0}$, $x = \frac{X - X_0}{X_0}$, and so on.

Symbols for Stability Analysis (Arts. 5 to 8)

Time.

t = general time variable, sec.
t_g = minimum value of time required for governor to move turbine gates from opening at which stability is being investigated, to zero, sec.
T_g = $k\beta t_g$, characteristic time of promptness of governor response, sec.
T_g' = $T_g + k_r\delta' T_d$, modified value of characteristic time of promptness of governor response for a speed-responsive governor with secondary compensation including a dashpot.
$T = \frac{C}{16.2 \times 10^6}$, mechanical inertia parameter which characterizes the effect of rotating masses. The dimension of T is in seconds of time.
$T_r = \frac{j}{j - 3r} T$ = mechanical inertia parameter corrected for effect of first component of water hammer, taking account of turbine efficiency change. The dimension is in seconds of time.
T'' = period of oscillation of governing, sec.
T' = period of free oscillation of water-hammer wave = $2\mu = \frac{4L}{a}$, sec. (Equal to twice the duration of the phase μ, *i.e.*, twice the duration of a direct and reflected transit of the water-hammer wave.)
T_d = characteristic time of dashpot rigidity (or of dashpot throttling action), sec. This time is defined by the equation

$$\omega_r - \omega_p = T_d \frac{d}{dt}(\omega_p - \omega_c)$$

T_{g1} = value of T_g for the first of two generating units operating in conjunction, sec.
T_1 = value of T for the first of two generating units operating in conjunction, sec.
T_{g2} = value of T_g for the second of two generating units operating in conjunction.
T_2 = value of T for the second of two generating units operating in conjunction.

Water Hammer.

a = velocity of water-hammer wave, fps.
g = acceleration of gravity, ft/sec^2.
$\Theta = \frac{LV_0}{gH_0} = \rho\mu$, hydraulic inertia parameter having the dimension of time, sec. This differs from Allievi's notation, in which he uses $\theta = \frac{a(\Delta t)}{2L}$.
$\mu = \frac{2L}{a}$ = duration of one phase of the water hammer (one direct and reflected transit of the wave). μ has the dimension of time in seconds.
L = length of pressure conduit, ft.
ϕ = time interval, measured in phase angle, between $t = 0$ instant at which gate opening change is a maximum $q_0 = q_{0\,\max}$ and $t = t$ instant at which the head change h is equal to h_0, *i.e.*, beginning of first water-hammer phase in Allievi's interlocked series. The dimension of ϕ is in radians or degrees according to the sense.
ϕ_m = phase displacement between gate opening oscillation and head oscillation (water hammer).
$\chi = \frac{T''}{T'}$ = ratio between period of governing oscillation in seconds and period of free water-hammer oscillation in seconds. χ is dimensionless.
ρ = Allievi dimensionless water-hammer parameter = $\frac{aV_0}{2gH_0}$.

Head.

H = general head variable, ft.
H_0 = initial or steady-state head, ft.
h = relative head change = $\frac{H - H_0}{H_0}$, dimensionless.

h_0 = value of h at beginning of first water-hammer phase, $\frac{2L}{a}$ in Allievi interlocked series, dimensionless.

h_n = value of h at beginning of nth water-hammer phase, $\frac{2nL}{a}$ in Allievi interlocked series, dimensionless.

$h_{0\,max}$ = peak value of h_0.

h_r = $h_{0\,max} \cos \phi_m$ = amplitude of (first) head change (water-hammer) component in opposition (180-deg phase difference) to gate opening oscillation q_0; h_r is dimensionless.

h_s = $h_{0\,max} \sin \phi_m$ = amplitude of the (second) head change (water-hammer) component in quadrature (90-deg phase difference) with gate opening oscillation q_0, h_s is dimensionless.

r = dimensionless coefficient denoting first head change component.

s = dimensionless coefficient denoting second head change component.

Velocity.

V = general velocity variable, fps.

V_0 = initial or steady-state velocity, fps.

v = relative velocity change = $\frac{V - V_0}{V_0}$ dimensionless.

v_0 = value of v at beginning of the first water-hammer phase $\frac{2L}{a}$, in the Allievi interlocked series.

v_n = value of v at beginning of nth water-hammer phase $\frac{2nL}{a}$ in Allievi interlocked series.

Power.

P = general term for power, hp (550 ft-lb per sec).

P_0 = initial or steady-state power.

p = relative power change $\frac{P - P_0}{P_0}$ (dimensionless) in absence of, or neglecting, the speed change effect. This power change p is equal to the power change p_0 plus the effect of $\frac{3}{2}h$ of the head change h.

p_0 = relative power change due to the servomotor piston displacement (gate opening change) in absence of, or neglecting, water-hammer and speed change effects; also the relative driving couple change under constant head and at constant speed.

Discharge.

Q = general discharge variable, cfs.

Q_0 = initial or steady-state discharge, cfs.

q = relative discharge change $\frac{Q - Q_0}{Q}$, dimensionless.

q_0 = relative discharge change in the absence of, or neglecting, the water-hammer effect. q_0 is then a measure of gate opening change.

q_{00} = value of q_0 at beginning of first water-hammer phase $\frac{2L}{a}$ in Allievi interlocked series, dimensionless.

$q_{0\,max}$ = peak value of q_0, dimensionless.

q_n = value of q at beginning of nth water-hammer phase $\frac{2nL}{a}$ in Allievi interlocked series.

q_{0n} = value of q_0 at beginning of nth water-hammer phase $\frac{2nL}{a}$ in Allievi interlocked series.

Speed.

N = general speed of rotation, rpm.

N_0 = initial or steady-state speed, rpm.

$\omega = \frac{N - N_0}{N_0}$ = relative speed change, dimensionless.

δ = permanent speed droop = $\frac{N_{max} - N_{min}}{N_0}$ produced by the process of primary compensation and defined by the equation $\omega + \delta p_0 = 0$ for neutral position of relay valve. For full load on $q_0 = +1$, $\omega = -\delta$.

$k_r\delta'$ = temporary speed droop produced by the process of secondary compensation with dashpot.

ω_r = displacement of flyball collar measured to scale of relative speed change ω.

ω_p = displacement of dashpot piston measured to scale of relative speed change ω.

ω_c = displacement of dashpot cylinder measured to scale of relative speed change ω.

General Constants or Parameters.

WR^2 = parameter characterizing inertia of rotating masses; equal to the weight W multiplied by the square of the radius of gyration.

$\therefore C$ = regulation constant $= \dfrac{N_0^2(WR^2)}{hp}$.

X = servomotor piston displacement, ft.

X_0 = servomotor piston displacement during the steady-state condition, ft.

$x = \dfrac{X - X_0}{X_0}$ = relative servomotor piston displacement, dimensionless.

m = acceleration constant in the speed-acceleration responsive type of governor, used to denote effect of acceleration-responsive elements, relative to effect of speed-responsive flyball, sec.

k = inverse value of the slope $1/k$ of the turbine performance curve of relative power vs. relative servomotor piston displacement. $k = \cot \gamma = \dfrac{dx}{dp_0}$, dimensionless.

k_r = factor defining effect of stiffness of dashpot spring on the law of the flyball collar displacement ω_r, in terms of the relative speed change ω. $\omega_r = \omega - k_r(\omega_r - \omega_p)$, dimensionless.

β = on experimental test curve of $\dfrac{dx}{dt}$ vs. ω, β is value of ω, for which the servomotor piston speed reaches its maximum rate: $\left(\dfrac{dx}{dt}\right)_{\max} = \dfrac{1}{t_g}$.

j = coefficient denoting effect of turbine efficiency change: $j = \dfrac{1}{1 - \dfrac{de}{dp_0}}$, $\dfrac{de}{dp_0}$ being the slope of turbine performance curve: relative efficiency vs. relative power.

g = acceleration of gravity, ft/sec².

$u = \dfrac{1 + \rho}{1 - \rho}$, a constant in Allievi series.

n = number of nth successive phase in Allievi interlocked series, such as 0, $\dfrac{2L}{a}$, $\dfrac{4L}{a}$, $\dfrac{2nL}{a}$.

D_e = relative value of difference between turbine driving couple and generator resisting couple.

$c_d = \dfrac{C_d}{C_{0d}}$, ratio between turbine driving couple at any speed, for steady-state gate opening, and the corresponding turbine driving couple at synchronous speed.

$c_r = \dfrac{C_r}{C_{0r}}$, ratio between generator resisting couple at any speed, for steady-state value of absorbed load, and the corresponding generator resisting couple at synchronous speed: $C_{0r} = C_{0d}$.

α_d = value of tangent to c_d performance curve (drawn with relative values).

α_r = value of tangent to c_r performance curve (drawn with relative values).

$\alpha = \alpha_r - \alpha_d$.

δ_* = logarithmic decrement of governing oscillation (gate opening, or speed, or head, etc.).

λ_m, λ_t, = damping coefficients used in formulating the Tables 4 to 6.

m_0; $(T_g'T_r)_0$ = special values used in formulating the Tables 4 to 6.

Variables used in considering stability of combined operation of two generating units:

$w_1 = \dfrac{j_1 - 3r}{j_1}$.

$l_1 = \dfrac{3s\Theta_1}{2j_1}$

$w_2 = \dfrac{j_2 - 3r}{j_2}$.

$l_2 = \dfrac{3s\Theta_2}{2j_2}$.

$$\left(\frac{N + N_0}{N_0}\right)\left(\frac{N - N_0}{N_0}\right) = \frac{1{,}620{,}000(P - P_0)\,\Delta t}{(WR^2)N_0^2} \tag{1}$$

Now let us assume $\dfrac{N + N_0}{N_0} = 2$, giving

$$\omega = \frac{810{,}000P_0p_0\,\Delta t}{(WR^2)N_0^2}$$

This is the expression of the relative speed change which occurs at the end of the time interval Δt during which the relative value of the difference between driving couple and resisting couple varies uniformly from the initial value p_0 to zero, with an average value $p_0/2$. The differential $d\omega$ of the relative speed change, when the instantaneous value (relative) of the difference between driving couple and resisting couple is equal to p_0, must then be written:

$$d\omega = \frac{16.2 \times 10^5 P_0 p_0}{(WR^2)N_0^2}\,dt$$

or
$$\frac{d\omega}{dt} = \frac{p_0}{T} \tag{7}$$
in which
$$T = \frac{1}{16.2 \times 10^5} \frac{(WR^2)N_0^2}{P_0}$$
or
$$T = \frac{C}{16.2 \times 10^5}$$

This is the differential form of the standard flywheel equation in the absence of, or neglecting, water hammer and speed change effect on both of the couples.

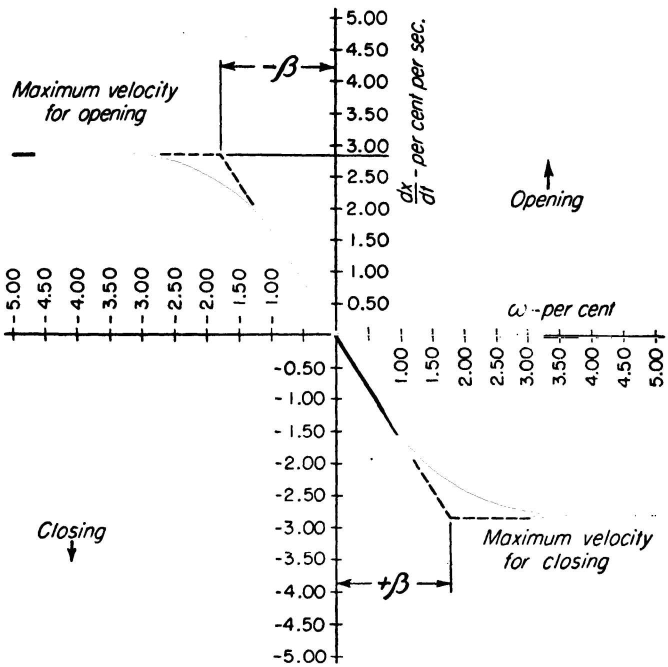

FIG. 1.—Experimental test curve. Relative speed ω versus servomotor piston speed dx/dt.

The mechanical inertia parameter T has the dimension of time in seconds. It is the time which is necessary to bring up the rotating masses of the turbine alternator from rest to the normal synchronous speed, by applying to them the normal rated couple. The value of T depends then upon that of the steady-state power P_0 considered.

Second, in Eq. (8), (*remembering that we are dealing with small oscillations* $\omega < \beta$), let us assume that the speed of travel dx/dt of the servomotor piston varies directly in proportion to the speed change in the generating unit as communicated to the flyball element, inversely proportional to the governor time t_g, for a full opening or closing stroke, and inversely proportional to β, the value of ω on the dx/dt vs. ω test curve, for which the speed dx/dt reaches its maximum rate (Fig. 1). In other words, the velocity of servomotor piston attains its maximum value $1/t_g$ when the speed change ω is equal to β, the relay valve being then wide open. The negative sign has been taken to give positive velocity for an opening stroke.

$$-\frac{dx}{dt} = \frac{\omega}{\beta t_g} \tag{8}$$

Next, construct a curve showing the relation between power p_0 and servomotor piston travel x (relative values) at synchronous speed and under constant head.

From Fig. 2,
$$k = \cot \gamma = \frac{dx}{dp_0}$$

or
$$dx = k\,dp_0 \text{ (in the absence of water hammer)}$$

Substituting in Eq. (8),
$$-\frac{dp_0}{dt} = \frac{\omega}{k\beta t_g}$$

Now let $k\beta t_g = T_g$, the characteristic time of promptness of the governor response, and the above equation becomes

$$-\frac{dp_0}{dt} = \frac{\omega}{T_g} \tag{9}$$

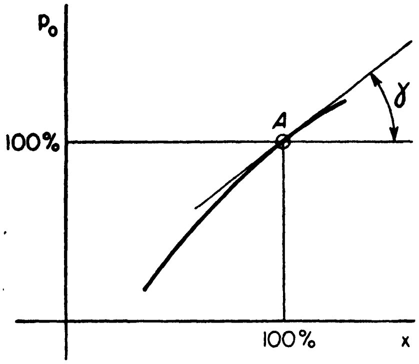

FIG. 2.—Power output versus servomotor piston travel at synchronous speed and in the absence of water hammer.

Example 8: Suppose that we wish to investigate stability at the point of full-gate opening and that at this position the test curve of p_0 vs. x shows the value of k to be 6.00. In addition, the test curve of dx/dt vs. ω shows that the servomotor stroke reaches full velocity when the relative speed change ω becomes 5 per cent; in other words, $\beta = 0.05$. The time for a full opening or full closing stroke of the servomotor is 8 sec. Compute the characteristic time of promptness of the governor response.

$$T_g = k\beta t_g$$
$$k = 6.00 \qquad t_g = 8 \text{ sec} \qquad \beta = 0.05$$
$$T_g = 6.00 \times 8 \times 0.05 = 2.40 \text{ sec}$$

Example 9: Compute T_g for the same conditions as for Example 8 except that the turbine is operating at 0.80 gate at which $k = 3.00$.

$$T_g = 3.00 \times (8 \times 0.80) \times 0.05 = 0.96 \text{ sec}$$

The above definition of the time of promptness T_g is only a schematic one, corresponding to the simplified arrangement of Fig. 3. It is then also the case for Examples 8 and 9. The time of promptness is really defined by the Eq. (9) itself:

$$T_g = -\frac{\omega}{\dfrac{dp_0}{dt}} = -\frac{k\omega}{\dfrac{dx}{dt}} \tag{9}$$

in which, for practical calculation purposes, the ratio $\left(\omega : \dfrac{dx}{dt}\right)$ is to be taken on a *test curve*, as shown on Fig. 1. In practice, the time of promptness T_g can be established *independently* from the governor time t_g, by adjusting the dashpot of the compensating device, as it will be explained subsequently.

Next eliminate p_0 between Eqs. (7) and (9).

$$\frac{d\omega}{dt} = \frac{p_0}{T} \tag{7}$$

$$-\frac{dp_0}{dt} = \frac{\omega}{T_g} \tag{9}$$

$$\frac{d^2\omega}{dt^2} + \frac{\omega}{T_g T} = 0 \tag{10}$$

This is the form of the governing equation for the crude mechanism depicted in Fig. 3. Let us now proceed with the assistance of this equation to analyze the effectiveness of such a governor in maintaining constant speed subsequent to a small load change on the generating unit. The boundary condition ($\omega = 0$ when $t = 0$) in this case is satisfied by the solution:

$$\omega = \omega_{max} \sin \frac{t}{\sqrt{T_g T}} \qquad (11)$$

Consequently the speed will continue to pendulate forever like the trigonometric sine function, or in engineering terms the mechanism is unstable.

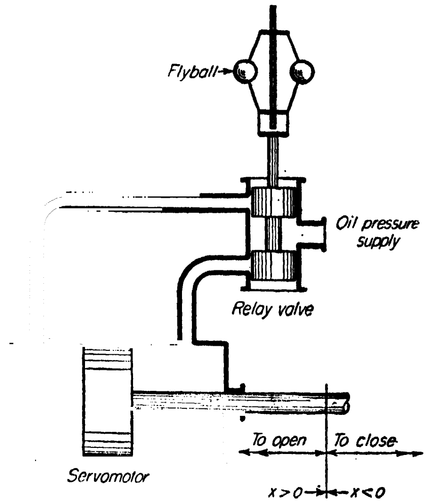

FIG. 3.—Schematic arrangement—speed-responsive governor without compensation.

It will be useful in our subsequent discussion of stability to employ vector diagrams of the type used by electrical engineers to represent a-c phenomena,* and for the applications of this method to governing problems we are again indebted to Daniel Gaden.

For perpetual pendulations representable by the trigonometric functions, the function and its derivative will always differ by a phase angle of 90 deg and the amplitude of the derivative will be that of the function multiplied by ($2\pi : T''$), T'' being the period of the governor oscillations. If this condition is realized in the vector diagram, we know at once that the governing is not stable. On the contrary for *damped* periodic oscillations, the phase angle between the function and its derivative will be greater than 90 deg.

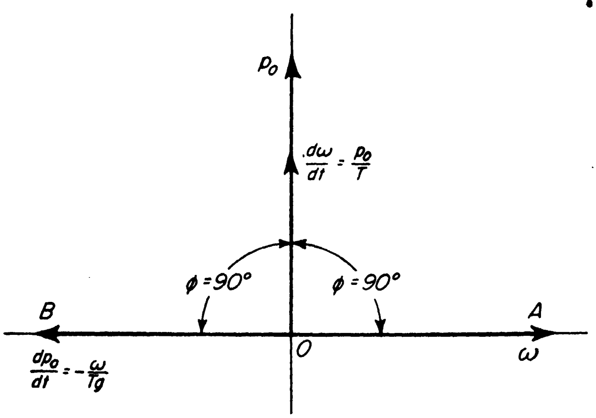

FIG. 4.—Vector diagram speed-responsive-type governor without compensation and water hammer.

The orientation and amplitude (length) of the vectors shown in the diagram Fig. 4 can be verified directly by inserting

$$\omega = \omega_{max} \sin \frac{t}{\sqrt{T_g T}} = \omega_{max} \sin \frac{2\pi t}{T''} \qquad (11)$$

in which $T'' = 2\pi\sqrt{T_gT}$ in the differential Eqs. (7) and (9). The diagram shows that the oscillations are perpetual ($\phi = 90$ deg) and the governing is unstable.

6. BASIC EQUATION OF GOVERNING INCLUDING PRIMARY COMPENSATION

We have seen that the fundamental difficulty with the mechanism analyzed in Art. 5 was due to *overtravel* on the part of the servomotor piston, causing the turbine gates to move too far for the given load change. This in turn caused the speed to change too far in the reverse direction and gave rise to a perpetual condition of hunting or instability. Evidently what is needed is some mechanical means of anticipating where the correct position of servomotor piston should be for a particular load change, and then of starting to close the oil supply ports in the relay valve before the piston

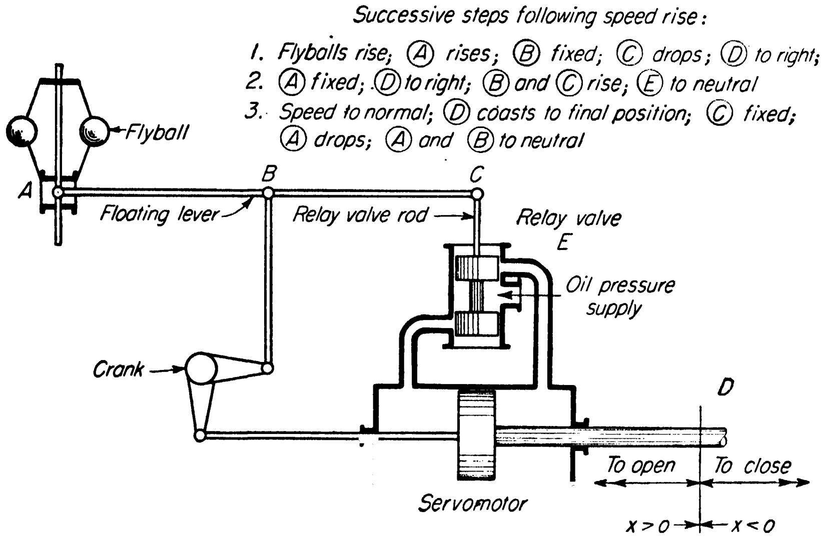

FIG. 5.—Schematic arrangement—speed-responsive governor with primary compensation.

reaches this desired position. The device for accomplishing this result is known as the *primary compensation*, and its basic principles are indicated schematically in Fig. 5.

The important feature to note is that the relay rod connection starts to return the relay valve to the neutral position as soon as the servomotor piston starts to move, thus effecting the requisite anticipation or compensation.

Having devised the mechanical means of anticipating the correct position of the servomotor piston, we proceed to incorporate this primary compensation in the basic equation of governing.

The displacement of the oil-pressure relay valve is now due not only to the speed change as reflected in the flyball movement, but also to the relative gate movement change owing to the action of the primary compensation. As a means of calibrating the effect of this primary compensation, we introduce the notation:

$$\delta = \frac{N_{\max} - N_{\min}}{N_0} \tag{12}$$

and since the position of the relay valve is now a function of the speed change and of the gate movement change x, or in other words, the relative power change p_0,

we may further define the term δ by the equation $\omega + \delta p_0 = 0$ for the neutral position of the oil-pressure relay valve. With this understanding, the expression $(\omega + \delta p_0)$ may be used to replace ω in Eq. 9 as the displacement of the relay valve for small oscillations. For full load on or $p_0 = +1$, $\omega = -\delta$. This term δ is designated as the speed droop of the governor, and its effect is to displace the relay valve farther and farther from the central position as the gate opening increases. This in turn requires the flyball element to draw in progressively more and more to restore the relay valve to the neutral position. In order that the flyball element draw in, the speed level must progressively decrease or droop as the turbine gates move from the closed to the open position.†

Example 10: When the turbine gates are opened from the speed no-load position to the full-gate position, the speed of operation drops from 150 to 143 rpm. The rated speed is 150 rpm. What is the speed droop?

$$\delta = \frac{N_{\max} - N_{\min}}{N_0} = \frac{150 - 143}{150} = 4.7 \text{ per cent}$$

To include the effect of primary compensation, we replace ω in Eq. (9) by $(\omega + \delta p_0)$ giving

$$-\frac{dp_0}{dt} = \frac{1}{T_g}(\omega + \delta p_0) \tag{13}$$

$$\frac{d\omega}{dt} = \frac{p_0}{T} \tag{7}$$

$$\frac{d^2\omega}{dt^2} + \frac{\delta}{T_g}\frac{d\omega}{dt} + \frac{\omega}{T_gT} = 0 \tag{14}$$

This equation has the well-known form:

$$\frac{d^2\Omega}{dt^2} + C_1\frac{d\Omega}{dt} + C_2\Omega = 0$$

in which $\Omega = \omega$, $C_1 = \dfrac{\delta}{T_g}$, and $C_2 = \dfrac{1}{T_gT}$.

The solution depends upon the roots of the auxiliary equation:‡

$$r^2 + C_1r + C_2 = 0$$

$$r = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_2}}{2}$$

In order that the relative speed change ω have the form of a damped *periodic* function, it suffices that $C_1 > 0$; which means that the speed droop must be positive $\delta > 0$.

In order that the relative speed change ω have the form of a damped *aperiodic* function of the form $\omega = \omega_{\max}e^{-rt}$, it is necessary that $C_1 > 0$ and $C_1^2 > 4C_2$.

$$C_1^2 \geq 4C_2$$

$$\frac{\delta^2}{T_g^2} \geq \frac{4}{T_gT}$$

$$\delta \geq 2\sqrt{\frac{T_g}{T}} \tag{15}$$

This means that for the small values of speed droop δ, desired in commercial operation, either the inertia characteristic T must be very large (involving proportionate

† See Cabinet Actuator Governing Equipment for Water Prime Movers, *Bull.* W-100, p. 27, Woodward Governor Company, Rockford, Ill.

‡ GRANVILLE, WILLIAM A., "Elements of the Differential and Integral Calculus," Ginn & Company, 1911, p. 432.

expense) or the promptness of governor response T_g must be very short, which in turn aggravates the effect of water hammer.

Example 11: Assuming that the prescribed value of speed droop is 5 per cent and that T_g the characteristic time of promptness of the governor response $T_g = 0.50$ sec, what must be the value of the regulation constant $C = \frac{N_0^2 WR^2}{hp}$, in order that the relative speed change function, ω, have the form of a damped aperiodic?

$$\delta = 2\sqrt{\frac{T_g}{T}}$$

$$0.05 = 2\sqrt{\frac{0.50}{\frac{C}{16.2 \times 10^6}}}$$

$$0.0025 = \frac{32.4 \times 10^6}{C} \qquad C = 1.30 \times 10^9 \qquad T = 800 \text{ sec}$$

which is about 100 times the available commercial value.

In practice the method of coping with the situation is to accept a damped periodic instead of an aperiodic speed regulation. The requisite rapidity of damping must be secured by an adequate value

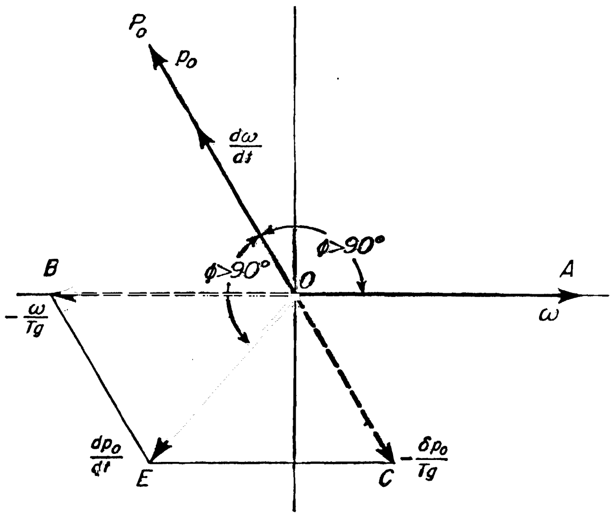

FIG. 6.—Vector diagram—speed-responsive-type governor with primary compensation without water hammer.

of the speed droop. If not sufficient, owing to the small value of the allowable speed droop for commercial operation, it will be necessary to have recourse to a secondary compensation with dashpot or to an acceleration responsive element beside the speed responsive flyball. The effect of these devices upon the basic governing equation will be incorporated in succeeding paragraphs.

But first let us proceed to incorporate the action of the speed droop (resulting from the primary compensation) in the vector diagram, Fig. 6.

The basic equations are (13) and (7). From Eq. (13), we deduce that the vector dp_0/dt is equal to the vector sum of the two vector components $-\omega/T_g$ and $-\delta p_0/T_g$. The first component $-\omega/T_g$, because of the fact that it differs from vector ω, or OA, only in algebraic sign and by a multiplying constant, will lie along OB in opposition to OA and of length equal to $1/T_g$ (OA). The orientation of vector OC representing the second component—$\delta p_0/T_g$ is at first unknown; but since it differs from p_0 only by algebraic sign and a multiplying constant δ/T_g, it must be in opposition to p_0. By Eq. (7) since the algebraic sign is the same on both sides and $d\omega/dt$ is equal to p_0 multiplied by a constant $1/T$, p_0, and $d\omega/dt$ must be in phase and differ only in magnitude. Furthermore, the various variables ω, p_0, and so on will have the same period of oscillation T''. Moreover, the phase angle between any two variables will remain fixed throughout the oscillation, and the phase angle between their two corresponding derivatives will remain fixed at the same angle as between their parent variables.

Consequently the only position of vector OP_0 capable of maintaining the prescribed identical phase

displacement between the vectors ω and p_0, the same as between vectors $d\omega/dt$ and dp_0/dt is at mid-angular position halfway between vectors OA and OE with $\phi > 90$ deg.

Since $\phi > 90$ deg, the oscillation will be damped and the governing will be stable.

7. BASIC EQUATION OF THE SPEED-ACCELERATION RESPONSIVE-TYPE GOVERNOR

Before proceeding to analyze the effect of a secondary compensation including a dashpot, it will be more logical for reasons that will become apparent during the course of our discussion first to study the effect of adding an acceleration-responsive element to the speed-responsive flyball.

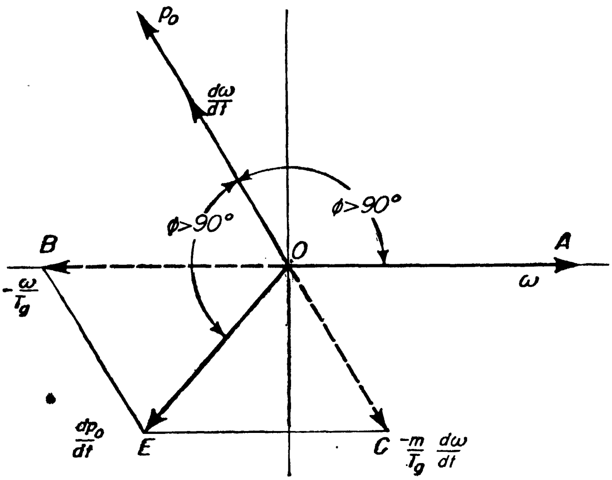

FIG. 7.—Vector diagram—speed-acceleration-responsive-type governor without water hammer.

Equation (9) will then change to the following:

$$-\frac{dp_0}{dt} = \frac{1}{T_g}\left(\omega + m\frac{d\omega}{dt}\right) \tag{16}$$

in which the constant m has the dimension of time in seconds and denotes the effect of the acceleration-responsive element in relation to the speed-responsive flyball effect. Combining Eq. (16) with Eq. (7),

$$\frac{d\omega}{dt} = \frac{p_0}{T} \tag{7}$$

we obtain

$$\frac{d^2\omega}{dt^2} + \frac{m}{T_gT}\frac{d\omega}{dt} + \frac{\omega}{T_gT} = 0 \tag{17}$$

In order that ω be a damped *periodic* function, it is required that $m > 0$. In order that ω be a damped *aperiodic* function, it is required that

$$m > 2\sqrt{T_gT}$$

It will be noted at once that Eq. (17) is identical in form with Eq. (14) except that the term $\frac{m}{T_gT}\frac{d\omega}{dt}$ replaces the term $\frac{\delta}{T_g}\frac{d\omega}{dt}$. The vector diagram Fig. (7) will accordingly be identical with Fig. (6) except that the vector $\frac{\delta p_0}{T_g}$ is replaced by $\frac{m}{T_g}\frac{d\omega}{dt}$, and since

as before $\phi > 90$ deg, the effect of including the acceleration-responsive element is to ensure stability even *without* the primary compensation.

8. BASIC EQUATION OF THE SPEED-RESPONSIVE-TYPE GOVERNOR WITH SECONDARY COMPENSATION (DASHPOT)

Now instead of incorporating an acceleration-responsive element beside the flyball or speed-responsive element, let us interpose a secondary compensation including a spring-loaded, oil-filled dashpot as shown schematically by Fig. 8. A primary com-

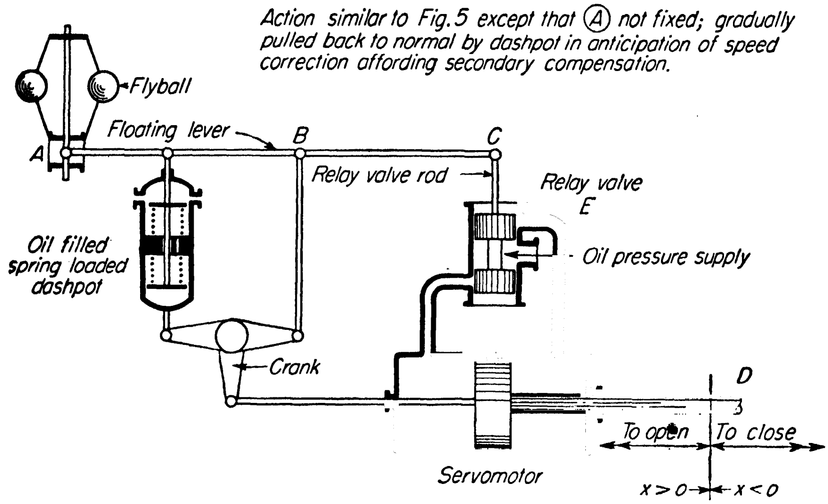

FIG. 8.—Schematic arrangement—speed-responsive governor with primary compensation and secondary compensation in the form of a spring-loaded oil-filled dashpot

pensation is also indicated on the said figure; but for this immediate discussion we shall neglect it and suppose that the point B remains fixed ($\delta = 0$). Then let

ω_r = displacement of the flyball collar at A.

ω_p = displacement of the piston of dashpot.

ω_c = displacement of the cylinder of dashpot.

All these displacements being measured to the scale of the relative speed change ω. According to Eq. (9),

$$-\frac{dp_0}{dt} = \frac{\omega_r}{T_g}$$

The displacement of the flyball collar ω_r will depend upon the difference between the uplift force of the flyball which in turn depends upon the relative speed change ω, and the dashpot reaction which is equal to the deflection of its spring $(\omega_r - \omega_p)$, in other words to the differential movement between the flyball collar A and the dashpot piston, multiplied by a factor k_r. This factor k_r characterizes the effect of the stiffness of the said spring on the law of the flyball collar displacement, in terms of the relative speed change ω:

$$\omega_r = \omega - k_r(\omega_r - \omega_p) \tag{20}$$

On the other hand $(\omega_r - \omega_p)$ depends upon the speed of the differential movement between piston and cylinder of the dashpot, and the time required for the oil to move through the ports from one side of the piston to the other, that is,

$$\omega_r - \omega_p = T_d \frac{d}{dt} (\omega_p - \omega_c) \tag{21}$$

in which T_d is the dashpot constant having the dimension of time in seconds; it characterizes the dashpot rigidity or the dashpot throttling action.

Finally the displacement of the dashpot cylinder, which is attached to the gate motion, must be proportional to the relative power change p_0.

$$\omega_c = \delta' p_0 \qquad \text{or} \qquad \frac{d\omega_c}{dt} = \delta' \frac{dp_0}{dt} \tag{22}$$

On combining these equations and inserting them in the fundamental relationships there results the following:

$$-\frac{dp_0}{dt} - T_d \frac{T_g(1 + k_r)}{T_g'} \frac{d^2p_0}{dt^2} = \frac{1}{T_g'} \left(\omega + T_d \frac{d\omega}{dt} \right) \tag{23}$$

in which
$$T_g' = T_g \left(1 + \frac{k_r \delta' T_d}{T_g} \right) = T_g + k_r \delta' T_d \tag{24}$$

Notice that if the dashpot were entirely rigid,

$$T_d = \infty \qquad \omega_p = \omega_c$$

and in order to restore the neutral position of the flyball collar A and of the oil pressure relay valve $E(\omega_r = 0)$, the following condition ought to obtain according to Eq. (20),

$$\omega + k_r \delta' p_0 = 0$$

$k_r\delta'$ is therefore the speed droop produced by the process of the secondary compensation. However, as the dashpot is never an entirely rigid device, but a sliding one, $k_r\delta'$ corresponds to a temporary speed droop or transient speed droop.

Example 12: Given a governor having secondary compensation in the form of a spring-loaded oil-filled dashpot. The characteristic time of promptness of the governor response without the dashpot is $T_g = 0.50$ sec. The dashpot spring factor k_r is equal to 1.00. The constant δ' is equal to 0.25 (transient speed droop $k_r\delta' = 25$ per cent). The dashpot constant T_d is 2.00 sec. Compute the *modified* time of promptness of the governor response T_g'.

$$T_g' = 0.50 + 1.00 \times 0.25 \times 2 = 1.00 \text{ sec}$$

Equation (23) may then be compared with Eq. (16).

$$-\frac{dp_0}{dt} = \frac{1}{T_g} \left(\omega + m \frac{d\omega}{dt} \right) \tag{16}$$

We have already shown by means of Fig. 7 that the governing according to Eq. (16) is stable. To approach the same result with the secondary compensation, we must consequently seek to render the term in $\frac{d^2p_0}{dt^2}$ in Eq. (23) as small as possible. Obviously this is accomplished by making $T_g(1 + k_r)$ as small as possible, in regard to T_g', that, is by making

$$\delta' T_d \gg T_g \tag{25}$$

Our basic equation then becomes

$$\frac{dp_0}{dt} = -\frac{1}{T_g'} \left(\omega + T_d \frac{d\omega}{dt} \right) \tag{23'}$$

This equation has exactly the same form as Eq. (16) with the constant m replaced by the characteristic time T_d of the dashpot rigidity or dashpot throttling action, and the vector diagram would be similar to Fig. 7, indicating stable governing.

However, the condition (25) can practically never be fully realized, and this is one of the reasons why the speed-responsive type of governor with secondary compensation cannot attain, according to Daniel Gaden, the same performance as the speed-acceleration type of governor, all things being equal. But we shall for our present purpose waive this difference and concede the identity of Eq. (16) and Eq. (23) or (23′).

We must also not forget that the time of promptness is no longer T_g, but

$$T_g' = T_g + k_r\delta' T_d$$

depending upon the secondary compensation characteristics: $k_r\delta'$, the transient speed droop, and T_d, the time of dashpot throttling action.

9. BASIC EQUATIONS INCLUDING WATER HAMMER

With respect to the governing problem, the essential effect of water hammer is to modify the power output of the turbine according to the $3/2$ power of the head. For small values of the relative power change and of the relative head change, we may then write with sufficient accuracy

$$p = p_0 + \tfrac{3}{2}h \tag{26}$$

where p_0 = relative power change due to the gate opening change in the absence of water hammer.
h = relative head change due to water hammer.
p = relative power change due to gate opening change and head change.

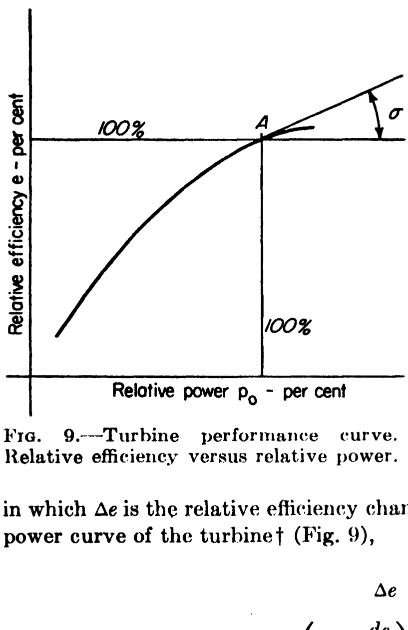

FIG. 9.—Turbine performance curve. Relative efficiency versus relative power.

Moreover, *under constant head*, the gate opening change, which produces the relative power change p_0, leads to a relative discharge change q_0, bound to p_0 by the equation

$$p_0 = q_0 + \Delta e$$

in which Δe is the relative efficiency change. Using the relative efficiency vs. relative power curve of the turbine† (Fig. 9),

$$\Delta e = \frac{de}{dp_0} p_0$$

$$p_0\left(1 - \frac{de}{dp_0}\right) = q_0 \qquad p_0 = jq_0$$

$$j = \frac{1}{1 - \dfrac{de}{dp_0}} = \frac{1}{1 - \tan\sigma}$$

If at the point of steady-state conditions the efficiency curve is ascending, $j > 1$, and if at the said point this curve is descending, $j < 1$.

In analysing the amplitude ratio and the phase displacement between the gate opening oscillation function q_0, and the head oscillation function h, we shall assume

1. That the pendulation is perpetually sustained or undamped. This simplification is permissible because as will be further shown, the amplitude ratio and the phase displacement between the two oscillations remain constant whatever the amplitudes may be (in the limit of small relative amplitudes),

† And of its penstock, scroll case, and draft tube.

especially when these amplitudes decrease progressively as is the case for a damped periodic oscillation.†

2. That the period T'' of the gate oscillation which is also the period of the forced head oscillation, is exactly χ times the period T' of the free water-hammer oscillation, that is to say, 2χ times the water-hammer phase $\mu = \frac{2L}{a}$. It can be proved that the results so obtained have perfect generality even if χ is not an integer† and also whether χ is relatively small or relatively large.‡

3. That the phenomenon has reached the limit of its final form after several cycles during which this final form is progressively established.

In developing the analysis we shall employ the well-known interlocked series of Allievi, in which the subscripts denote conditions at the control gates at successive phase intervals $\frac{2L}{a}, \frac{4L}{a}, \frac{6L}{a}, \cdots \frac{2nL}{a}$, or $\mu, 2\mu, 3\mu, \cdots, n\mu$. We shall accordingly divide the gate-motion period T'' sec into 2χ equal intervals, each corresponding to one water-hammer phase $2L/a$. We shall designate the intervals 0, 1, 2, 3, . . . n, . . . , 2χ.

According to the interlocked series,

$$h_n + h_{n+1} = 2\rho(v_n - v_{n+1})$$

or using relative discharges,

$$h_n + h_{n+1} = 2\rho(q_n - q_{n+1})$$

Now since the effect of water hammer is to increase the discharge according to the ½ power of the head, we may write (using relative values) with sufficient accuracy,

$$q_n = q_{0n} + \tfrac{1}{2}h_n$$

According to the first above-mentioned hypotheses, we shall write that the gate-opening change which is defined by the relative value q_0 of the discharge change in the absence of water hammer, varies like a cosine function, with a period $T'' = 2\chi\mu$:

$$q_{0n} = q_{0\,\max} \cos \frac{2\pi n}{2\chi} + \phi$$

n being the measure of the time calculated in phase intervals μ.

We proceed to write the entire interlocked series:

$$\begin{aligned} h_0 + h_1 &= 2\rho[(q_{00} + \tfrac{1}{2}h_0) - (q_{01} + \tfrac{1}{2}h_1)] \\ h_1 + h_2 &= 2\rho[(q_{01} + \tfrac{1}{2}h_1) - (q_{02} + \tfrac{1}{2}h_2)] \\ h_n + h_{n+1} &= 2\rho[(q_{0n} + \tfrac{1}{2}h_n) - (q_{0n+1} + \tfrac{1}{2}h_{n+1})] \end{aligned}$$

and finally,

$$h_{2\chi-1} + h_{2\chi} = 2\rho[(q_{02\chi-1} + \tfrac{1}{2}h_{2\chi-1}) - (q_{02\chi} + \tfrac{1}{2}h_{2\chi})]$$

according to the third above-mentioned hypothesis and since $h_{2\chi} = h_0$ and $q_{02\chi} = q_{00}$,

$$h_{2\chi-1} + h_0 = 2\rho[(q_{02\chi-1} + \tfrac{1}{2}h_{2\chi-1}) - (q_{00} + \tfrac{1}{2}h_0)]$$

Substituting $q_{0n} = q_{0\,\max} \cos\left(\frac{2\pi n}{2\chi} + \phi\right)$,§ and rearranging,

$$\begin{aligned} h_0(1-\rho) + h_1(1+\rho) &= 2\rho\left[q_{0\,\max}\cos(0+\phi) - q_{0\,\max}\cos\left(\frac{2\pi}{2\chi}+\phi\right)\right] \\ h_1(1-\rho) + h_2(1+\rho) &= 2\rho\left[q_{0\,\max}\cos\left(\frac{2\pi}{2\chi}+\phi\right) - q_{0\,\max}\cos\left(\frac{4\pi}{2\chi}+\phi\right)\right] \\ h_n(1-\rho) + h_{n+1}(1+\rho) &= 2\rho\left[q_{0\,\max}\cos\left(\frac{2\pi n}{2\chi}+\phi\right) - q_{0\,\max}\cos\left(\frac{2\pi(n+1)}{2\chi}+\phi\right)\right] \\ h_{2\chi-1}(1-\rho) + h_0(1+\rho) &= 2\rho\left[q_{0\,\max}\cos\left(\frac{2\pi}{2\chi}(2\chi-1)+\phi\right) - q_{0\,\max}\cos\left(\frac{2\pi}{2\chi}(2\chi)+\phi\right)\right] \end{aligned}$$

† GADEN, *op. cit.*, pp. 127, 129.

‡ *Ibid.*, p. 101.

§ ϕ is the time interval, measured in phase angle, between $t = 0$, when the gate-opening oscillation is maximum ($q_0 = q_{0\max}$), and $t = t$, when the head change $h = h_0$, *i.e.*, the beginning of the first water-hammer phase of the interlocked series.

Now operate on these to determine what value of ϕ makes h_0 a maximum. This will then establish ϕ_m, the phase displacement between the gate opening oscillation and the head oscillation (water hammer). First, divide the above array of interlocked equations by $(1 - \rho)$ and let $\frac{1 + \rho}{1 - \rho} = u$.

$$h_0 = -uh_1 + \frac{2\rho}{1 - \rho} q_{0\,\max} \left[\cos \phi - \cos \left(\frac{\pi}{\chi} + \phi \right) \right]$$

$$h_1 = -uh_2 + \frac{2\rho}{1 - \rho} q_{0\,\max} \left[\cos \left(\frac{\pi}{\chi} + \phi \right) - \cos \left(\frac{2\pi}{\chi} + \phi \right) \right]$$

$$h_n = -uh_{n+1} + \frac{2\rho}{1 - \rho} q_{0\,\max} \left[\cos \left(\frac{n\pi}{\chi} + \phi \right) - \cos \left(\frac{[n + 1]\pi}{\chi} + \phi \right) \right]$$

$$h_{2\chi-1} = -uh_0 + \frac{2\rho}{1 - \rho} q_{0\,\max} \left[\cos \left(\frac{\pi}{\chi} (2\chi - 1) + \phi \right) - \cos \left(\frac{2\pi\chi}{\chi} + \phi \right) \right]$$

Next multiply the equation for h_0 by u^0 or 1; the equation for h_1 by $-u^1$; the equation for h_2 by $+u^2$; the equation for h_n by $\pm u^n$ according as n is even or odd; and finally the equation in $h_{2\chi-1}$ by $-u^{2\chi-1}$. Then add the entire array of equations giving

$$h_0 = \frac{2\rho}{1 - \rho} \frac{q_{0\,\max}}{1 - u^{2\chi}} \sum_{n=0}^{n=2\chi-1} (-1)^n u^n \left[\cos \left(\frac{n\pi}{\chi} + \phi \right) - \cos \left(\frac{[n + 1]\pi}{\chi} + \phi \right) \right]$$

and $$\frac{dh_0}{d\phi} = 0 = \sum_{n=0}^{n=2\chi-1} (-1)^n u^n \left[\sin \left(\frac{[n + 1]\pi}{\chi} + \phi \right) - \sin \left(\frac{n\pi}{\chi} + \phi \right) \right]$$

or $$\sum_{n=0}^{n=2\chi-1} (-1)^n u^n \left[\sin \frac{(n + 1)\pi}{\chi} \cos \phi + \cos \frac{(n + 1)\pi}{\chi} \sin \phi - \sin \frac{n\pi}{\chi} \cos \phi - \cos \frac{n\pi}{\chi} \sin \phi \right] = 0$$

giving

$$\tan \phi_m = \frac{\displaystyle\sum_{n=0}^{n=2\chi-1} (-1)^n u^n \left[\sin \frac{n\pi}{\chi} - \sin \frac{(n + 1)\pi}{\chi} \right]}{\displaystyle\sum_{n=0}^{n=2\chi-1} (-1)^n u^n \left[\cos \frac{(n + 1)\pi}{\chi} - \cos \frac{n\pi}{\chi} \right]}$$

or by simplifying,

$$\tan \phi_m = -\frac{1}{\rho} \cot \frac{\pi}{2\chi} \tag{27}$$

and

$$h_{\max} = \frac{2\rho}{1 - \rho} \frac{q_{0\,\max}}{1 - u^{2\chi}} \sum_{n=0}^{n=2\chi-1} (-1)^n u^n \left[\cos \left(\frac{n\pi}{\chi} + \phi_m \right) - \cos \left(\frac{(n + 1)\pi}{\chi} + \phi_m \right) \right]$$

or by simplifying,

$$h_{\max} = 2q_{0\,\max} \cos \phi_m \tag{28}$$

It appears from Eqs. (27) and (28) that the phase displacement ϕ_m and the amplitude ratio ($h_{\max}$: $q_{0\,\max}$) are independent of the value of the amplitude as mentioned before.

Equation (27) expressing the phase difference ϕ_m between the gate opening oscillation and the head oscillation (water hammer) has been calculated for several repre-

sentative values of χ and ρ and summarized for convenient reference in the charts Figs. 11 and 12. Figures 13 and 14 calculated on a similar basis summarize Eq. (28) by giving the ratio of $\frac{h_{\max}}{q_{0\,\max}}$ for various combination of χ and ρ.

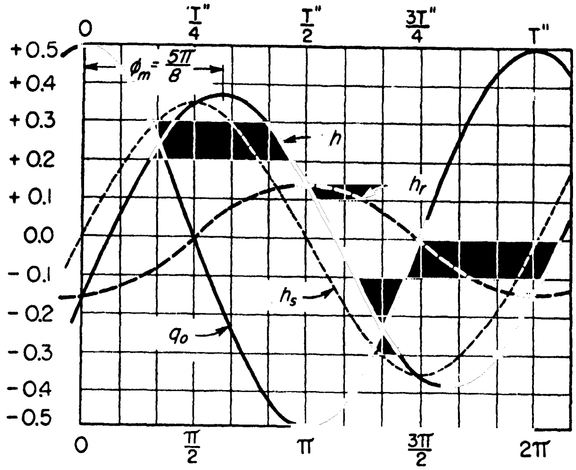

FIG. 10.—Forced head oscillation h caused by a gate oscillation of 5 per cent for $\rho = 1$ and $\chi = 4$.

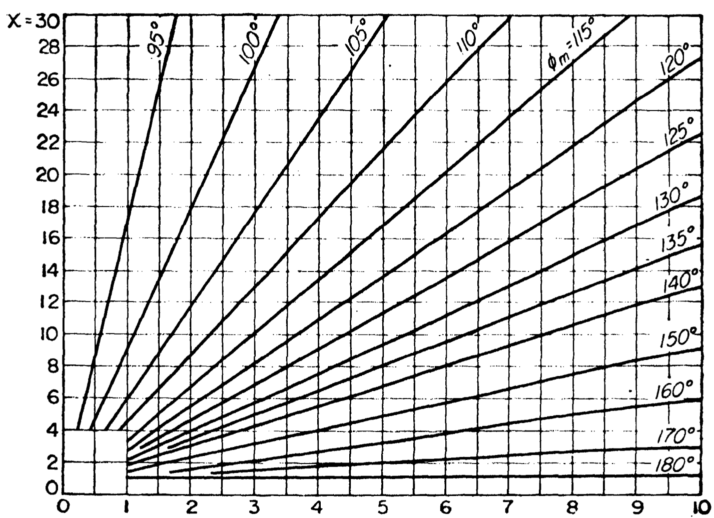

FIG. 11.—Values of phase displacement ϕ_m between gate opening oscillation and head oscillation (water hammer) for larger values of χ and ρ.

For reasons that will develop as the discussion proceeds, we shall find it advantageous to resolve the relative head change h (which is a trigonometric sine function) into two components (which are also trigonometric sine functions) as indicated in Fig. 15.

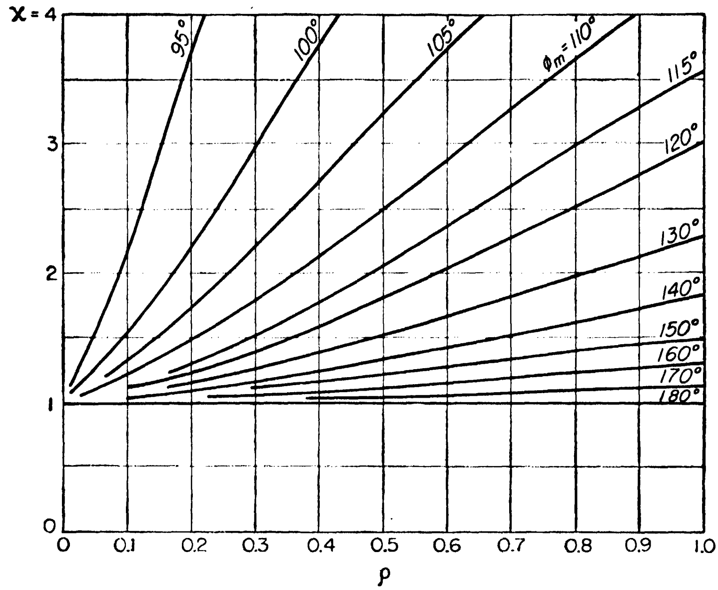

FIG. 12.—Values of phase displacement ϕ_m between gate-opening oscillation and head oscillation (water hammer) for smaller values of χ and ρ.

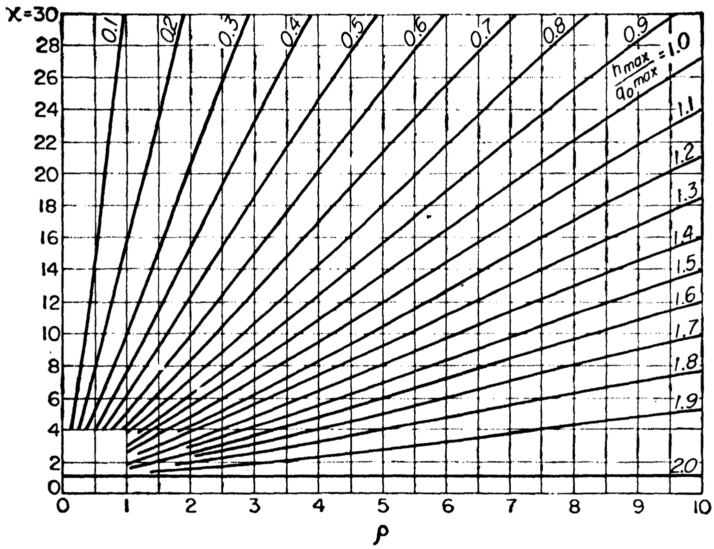

FIG. 13.—Values of amplitude ratio $h_{max}/q_{0\,max}$ between head oscillation (water hammer) and gate opening oscillation for larger values of χ and ρ.

The first component h_r is in opposition with the gate oscillation q_0, that is to say, in phase with $-q_0$; its amplitude is equal to $h_{max} \cos \phi_m$. The second component h_s is in quadrature with and backward, that is to say, in phase with $-\dfrac{dq_0}{dt}$; its amplitude is equal to $h_{max} \sin \phi_m$.

Using for further convenience the coefficients r and s, we shall express these two components as follows:

	Instantaneous Value	Maximum Value (Amplitude)
First component.	$h_r = -r(2q_0)$	$h_{r\,\max} = 2rq_{0\,\max}$
Second component.	$h_s = -s\Theta \dfrac{dq_0}{dt}$	$h_{s\,\max} = s\Theta \dfrac{2\pi}{T''} q_{0\,\max} = s \dfrac{\pi\rho}{\chi} q_{0\,\max}$

in which $\Theta = \rho\mu = \rho \dfrac{T''}{2}$ is a hydraulic inertia parameter having the dimension of time

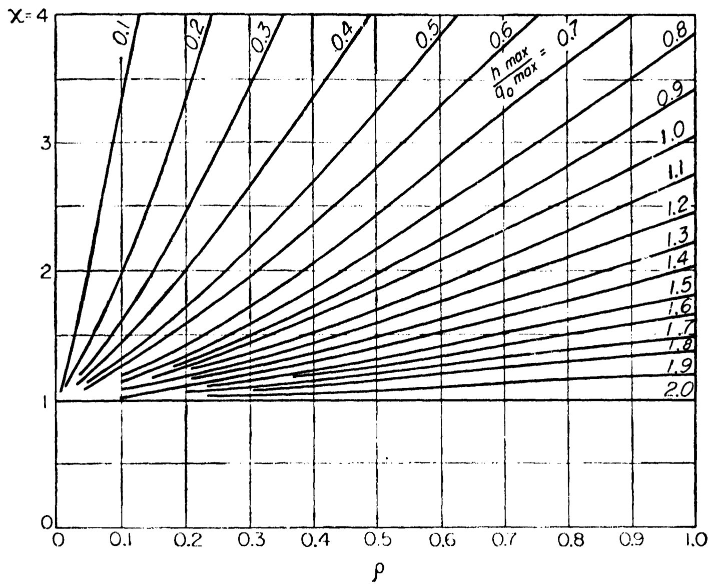

FIG. 14.—Values of amplitude ratio $h_{\max}/q_{0\,\max}$ between head oscillation (water hammer) and gate opening oscillation for smaller values of χ and ρ.

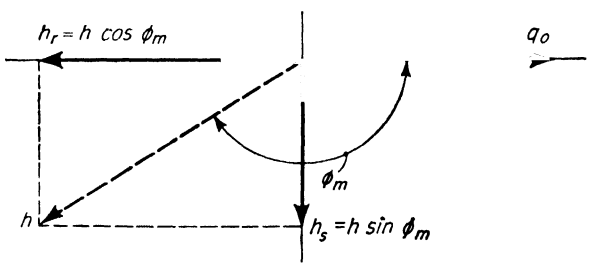

FIG. 15.—Vector diagram showing decomposition of relative head oscillation h into components $h_r = h \cos \phi_m$ and $h_s = h \sin \phi_m$.

in seconds. Θ is the time necessary to bring up the water masses contained in the pressure conduit from the rest to the flowing velocity V_0, under the impulse of a pressure difference equal to the head H_0:

$$\Theta = \frac{LV_0}{gH_0}$$

According to Eq. (28),

$$r = \frac{1}{2}\frac{h_{r\,\max}}{q_{0\,\max}} = \frac{1}{2}\frac{h_{\max}}{q_{0\,\max}}\cos\phi_m = \cos^2\phi_m \tag{29}$$

$$s = \frac{\chi}{\pi\rho}\frac{h_{s\,\max}}{q_{0\,\max}} = \frac{\chi}{\pi\rho}\frac{h_{\max}}{q_{0\,\max}}\sin\phi_m = \frac{2\chi}{\pi\rho}\sin\phi_m\cos\phi_m \tag{30}$$

The values of the coefficients r and s for any combination of χ and ρ may be selected directly from Figs. 16 to 19.

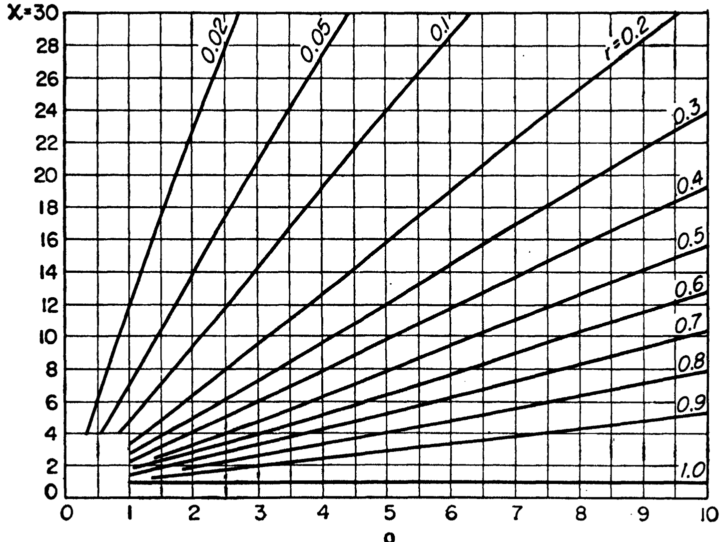

FIG. 16.—Coefficient r—denoting the first component h_r of the head oscillation for larger values of χ and ρ.

Example 13: χ is the ratio between period T'' of the governor oscillation in seconds and the period T' of the free water-hammer oscillation in seconds. χ is dimensionless. $\rho = \dfrac{aV_0}{2gH_0}$, the Allievi, dimensionless water-hammer parameter. Suppose $\chi = 20$ and $\rho = 5$. What is the phase displacement ϕ_m between the governor or gate opening oscillation and the head oscillation?

From Fig. 11, for $\chi = 20$, $\rho = 5$,

$$\phi_m = 112°$$

Example 14: For the condition of Example 13, what is the value of the ratio $\dfrac{h_{\max}}{q_{0\,\max}}$? From Fig. 13, for $\chi = 20$, $\rho = 5$,

$$\frac{h_{\max}}{q_{0\,\max}} = 0.74$$

Example 15: For $\chi = 3.5$ and $\rho = 0.8$, what is the value of the phase displacement ϕ_m and the value of the ratio $\dfrac{h_{\max}}{q_{0\,\max}}$?

From Fig. 12, for $\chi = 3.5$, $\rho = 0.8$,

$$\phi_m = 111°$$

From Fig. 14, for $\chi = 3.5$, $\rho = 0.8$,

$$\frac{h_{\max}}{q_{0\,\max}} = 0.72$$

Example 16: Determine the values of r and s for the condition $\chi = 16$, $\rho = 7$.

From Fig. 16, for $\chi = 16$, $\rho = 7$,

$$r = 0.33$$

From Fig. 18, for $\chi = 16$, $\rho = 7$,

$$s = 0.68$$

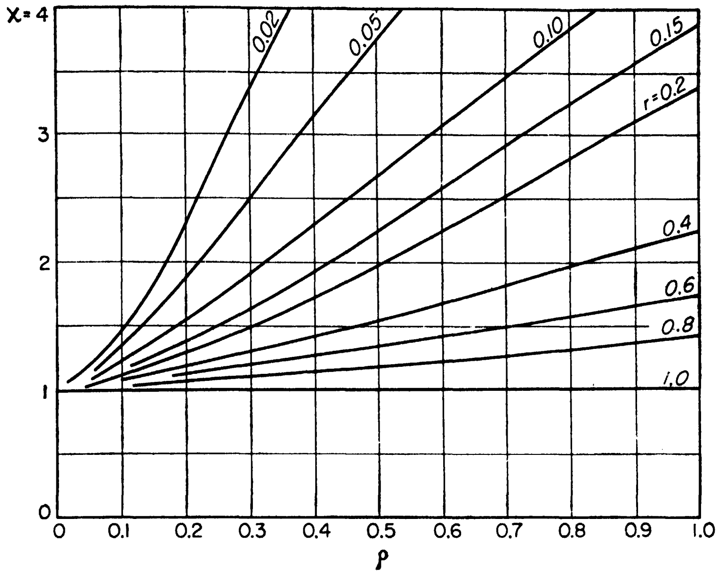

FIG. 17.—Coefficient r—denoting the first component h_r of the head oscillation for smaller values of χ and ρ.

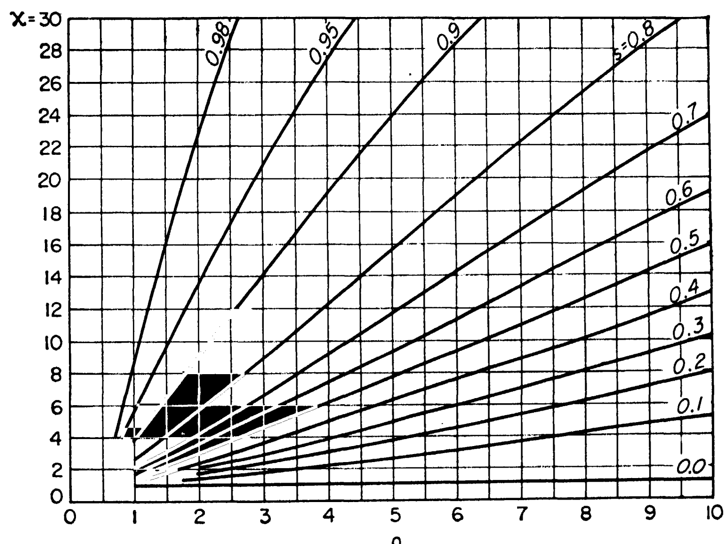

FIG. 18.—Coefficient s—denoting the second component h_s of the head oscillation for larger values of χ and ρ.

Example 17: Determine the values of r and s for the condition $\chi = 2.5$, $\rho = 0.4$.
From Fig. 17, for $\chi = 2.5$, $\rho = 0.4$,

$$r = 0.078$$

From Fig. 19, for $\chi = 2.5$, $\rho = 0.4$,

$$s = 1.066$$

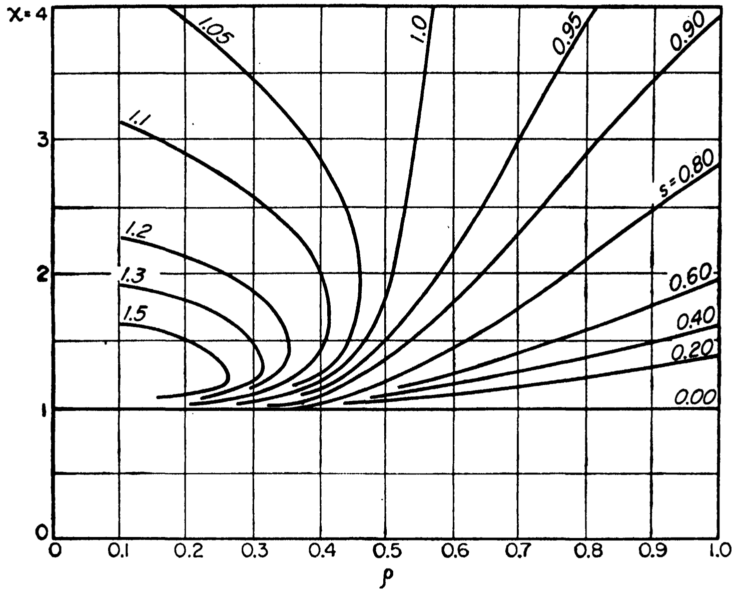

FIG. 19.—Coefficient s—denoting the second component h_s of the head oscillation for smaller values of χ and ρ.

In the presence of water hammer, the standard flywheel equation (7) must now be written:

$$\frac{d\omega}{dt} = \frac{p}{T} \tag{7}$$

in which, according to Eq. (26),

$$p = p_0 + \tfrac{3}{2}h = p_0 + \tfrac{3}{2}h_r + \tfrac{3}{2}h_s \tag{31}$$

Effect of the First Component h_r. Taking account of this single component, Eq. (26) becomes

$$p = p_0 + \frac{3}{2}h_r = p_0 - \frac{3}{2}(2rq_0) = p_0\left(1 - \frac{3r}{j}\right)$$

and Eq. (7),

$$\frac{d\omega}{dt} = \frac{p_0}{T}\left(1 - \frac{3r}{j}\right) \tag{32}$$

The effect of the water-hammer component h_r is accordingly to change the mechanical inertia parameter T in the ratio $\frac{j}{j - 3r}$, in other words to increase the inertia parameter T (since $r > 0$) to the effective value $T_r = T\frac{j}{j - 3r}$. This is manifestly a benefit from the standpoint of stability.

Effect of the Second Component h_s Added to That of the First One. Taking account of both of the components, Eq. (26) becomes

$$p = p_0\left(1 - \frac{3r}{j}\right) + \frac{3}{2}h_s = p_0\left(1 - \frac{3r}{j}\right) - \frac{3}{2}s\Theta\frac{dq_0}{dt}$$

or

$$p = p_0\left(1 - \frac{3r}{j}\right) - \frac{3}{2}\frac{s}{j}\Theta\frac{dp_0}{dt} \tag{33}$$

and Eq. (7),

$$\frac{d\omega}{dt} = \frac{p_0}{Tr} - \frac{3}{2}\frac{s}{j}\frac{\Theta}{T}\frac{dp_0}{dt} \tag{34}$$

in which

$$T_r = T\frac{j}{j - 3r}$$

is the fictitious value of the mechanical inertia parameter due to the influence of the first component h_r.

We now proceed to combine the flywheel equation (34) with the equations of governor movement comprising three cases: a speed-responsive governor with primary compensation; a speed- and acceleration-responsive governor without primary compensation; a speed-responsive governor with secondary compensation in the form of a dashpot or, in other words, with temporary compensation.

Case I. The basic equation of governor movement is

$$\frac{dp_0}{dt} = -\frac{\omega + \delta p_0}{T_g} \tag{13}$$

Combining Eqs. (34) and (13),

$$\frac{d^2\omega}{dt^2} + \frac{1}{T_g}\left(\delta - \frac{3s}{2j}\frac{\Theta}{T}\right)\frac{d\omega}{dt} + \frac{\omega}{T_gT_r} = 0 \tag{35}$$

In order that this equation represent a *damped* periodic speed oscillation, it is required that†

$$\delta T > \frac{3s}{2j}\Theta \tag{36}$$

Case II. The basic equation of governor movement is

$$\frac{dp_0}{dt} = -\frac{1}{T_g}\left(\omega + m\frac{d\omega}{dt}\right) \tag{16}$$

Combining Eqs. (16) and (34),

$$\frac{d^2\omega}{dt^2} + \frac{m}{T_gT_r}\frac{1 - \frac{3s}{2j}\frac{\Theta}{m}\frac{T_r}{T}}{1 - \frac{3s}{2j}\frac{m}{T_g}\frac{\Theta}{T}}\frac{d\omega}{dt} + \frac{1}{T_gT_r}\left(\frac{1}{1 - \frac{3sm\Theta}{2jT_gT}}\right)\omega = 0 \tag{37}$$

In order that this equation represent a *damped* periodic speed oscillation,† it is required that the factors of $d\omega/dt$ and ω be both positive, that is,

$$m > \frac{3s}{2j}\Theta\frac{T_r}{T}$$

or

$$m > \frac{3s}{2j}\left(\frac{j}{j - 3r}\right)\Theta \qquad \text{and} \qquad T_gT > \frac{3s}{2j}m\Theta$$

When the oscillation is just barely suppressed in time equal to infinity

$$m = \frac{3s}{2j}\left(\frac{j}{j - 3r}\right)\Theta \tag{38}$$

Substituting this value of m in the inequality

$$T_gT > \frac{3s}{2j}m\Theta$$

† GRANVILLE, *op. cit.*, p. 434.

there results

$$T_gT > \left(\frac{3s}{2j}\right)^2 \left(\frac{j}{j - 3r}\right) \Theta^2 \tag{39}$$

Case III. As previously indicated, this case is *approximately* the same as Case II, except that the acceleration constant m is replaced by the characteristic time T_d of dashpot rigidity or dashpot throttling action.

According to Eqs. (38) and (39) the value of the characteristic time T_d must then be

$$T_d > \frac{3s}{2j} \left(\frac{j}{j - 3r}\right) \Theta \tag{40}$$

and the criterion of stability becomes

$$T_g'T > \left(\frac{3s}{2j}\right)^2 \frac{j}{j - 3r} \Theta^2 \tag{41}$$

with the understanding that T_g' is very much greater than T_g.

Effect of the Speed Change upon the Difference between the Resisting and the Driving Couples. Before proceeding with the construction of vector diagrams confirming the criteria for incipient instability in cases involving water hammer, it is desirable to mention here one of the most important stabilizing factors, namely, the effect of the speed change upon the difference between the resisting couple of the electric generator and the hydraulic couple driving the turbine.

Writing the standard flywheel equation in the form

$$\frac{d\omega}{dt} = \frac{p}{T} \tag{7}$$

in which

$$p = p_0 + \tfrac{3}{2}h \tag{26}$$

We have assumed that the difference D_c between these two couples corresponds only to the power change p, due to the gate opening change ($p_0 = x:k$) and to the effect $\frac{3}{2}h$ of the head change h (water hammer). In other words, we have neglected the effect of the speed change ω upon both of the couples. But this last effect is often very important.

Using again the notation of relative values, let $c_d = \dfrac{C_d}{C_{0d}}$ in which C_d is the turbine driving couple at any speed for the gate opening steady state and C_{0d} is the turbine driving couple at normal synchronous speed for the said gate opening. $c_r = \dfrac{C_r}{C_{0r}}$ in which C_r is the generator resisting couple at any speed for the steady-state value of the absorbed load and C_{0r} is the generator resisting couple at normal synchronous speed for the said load.

By definition $C_{0d} = C_{0r} = C_0$, and both couples are then measured with the same unit C_0. On the diagram of Fig. 20 are represented the curves c_d and c_r in terms of ω; the coefficients α_d and α_r are the slopes of those curves at point A corresponding to the steady-state condition.

For the small oscillations assumed in our investigations of stability, we may write with sufficient accuracy

$$c_d = (1 + p)(1 + \alpha_d\omega)$$

or, neglecting terms of second order,

$$c_d = 1 + p + \alpha_d\omega$$

The value of the coefficient α_d depends upon the specific speed of the turbine. It may vary from $\alpha_d = -1.1$ (low specific speed) to $\alpha_d = -0.6$ (high specific speed).

Similarly, we can write for the resisting couple

$$c_r = 1 + \alpha_r \omega$$

The value of the coefficient α_r depends upon the nature of the load.

If the load is composed of a-c electric motors (load depending upon frequency), the value of the coefficient α_r depends upon the torque-speed characteristic of the driven machines:

$\alpha_r = 0$ for constant torque machines;

$\alpha_r = +2$ for turbomachines such as centrifugal pumps, fans, turbocompressors.

If load is composed of ohmic resistance (load independent of frequency), such as electric boilers, arc ovens, electrolyzers, and lamps, or, of d-c circuits fed by means of rectifiers, the value of the coefficient α_r depends solely upon the voltage-frequency characteristic of the voltage regulation. If the voltage constant, independent of frequency (speed), $\alpha_r = -1$ and that is the only case in which α_r may be equal to α_d:

$$\alpha = \alpha_r - \alpha_d = 0$$

In the other cases $\alpha = \alpha_r - \alpha_d$ is generally positive. The expression of the difference between the resisting and the driving couples must then be written

$$D_c = p - \alpha\omega = p_0 + \tfrac{3}{2}h - \alpha\omega$$

and the flywheel equation (34)

$$\frac{d\omega}{dt} = \frac{p_0}{T_r} - \frac{3s}{2j}\frac{\Theta}{T}\frac{dp_0}{dt} - \frac{\alpha\omega}{T} \qquad (42)$$

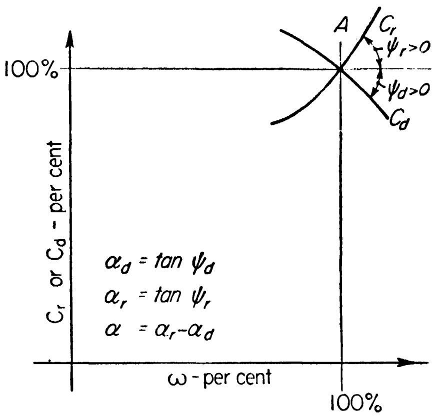

FIG. 20.—Turbine-generator performance curve—showing relative values of driving couple and resisting couple in terms of relative speed.

Following the type of analysis previously employed, the criterion for incipient stability in Case II (speed-acceleration responsive governor) becomes

$$T_gT > \frac{\left(\frac{3s}{2j}\right)^2 \frac{T_r}{T}\Theta^2}{1 + \frac{\alpha 3s}{2j}\frac{\Theta}{T}\frac{T_r}{T}} \qquad (43)$$

in which

$$\frac{T_r}{T} = \frac{j}{j - 3r}$$

In other words, the minimum value of the product T_gT given by Eq. (39) assuming $\alpha = 0$ is to be divided by

$$1 + \alpha\frac{3s}{2j}\frac{\Theta}{T}\frac{T_r}{T}$$

It is then evident that for positive values of α† the effect of the speed change upon the difference between the resisting and the driving couples greatly favors the stability.

On the other hand, experimental checking of the speed regulation stability, by loading the generator on a hydraulic resistance, appears to be a very severe test, when

† This means that the resisting couple curve cuts the driving couple curve at the point of steady-state conditions, by passing *over* in direction of increasing speed (Fig. 20).

the voltage is maintained constant independently of the frequency, by quick-acting voltage regulation ($\alpha_r = -1$, $\alpha \cong 0$). In fact, except for some special cases this test is valueless; since at least a part of the load is in practice generally composed of a-c electric motors ($\alpha_r \geq 0$) and for the other part of the load which is independent of frequency, it is always possible to have recourse to a judicious adjustment of the voltage-frequency characteristic of the voltage regulation, in order to secure $\alpha_r > -1$ and $\alpha > 0$. If the voltage change is $\pm n$ per cent for a frequency change of ± 1 per cent, the change of the ohmic load reaches $\pm 2n$ per cent and the change of the resisting couple $\pm(2n - 1)$ per cent so that $\alpha_r = 2n - 1$.

Vector Diagrams. We are now in position to confirm the criteria for incipient stability by means of appropriate vector diagrams, and we shall start (Fig. 21) with the

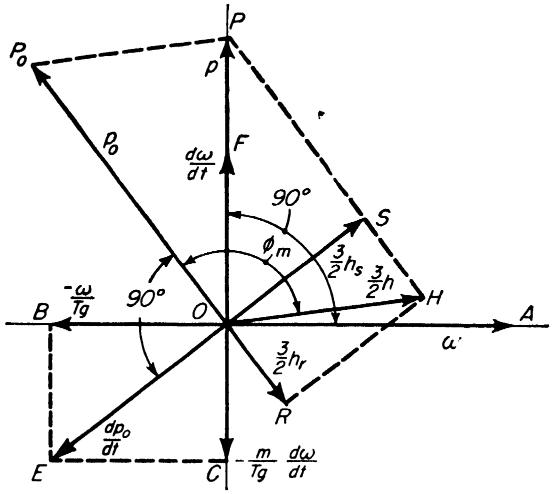

FIG. 21.—Vector diagram—speed-acceleration-responsive governor including water hammer.

diagram for Case II, the speed-acceleration responsive governor when water hammer is present. The diagrams will be drawn to indicate that the system is just on the brink of instability, and the corresponding criteria deduced from the geometry of the diagrams:

Vector $\overline{OA}$ represents the speed-change oscillation.
Vector $\overline{OB}$ represents the tachymetric effect upon the relay valve.
Vector $\overline{OC}$ represents the accelerometric effect upon the relay valve.
Vector $\overline{OE}$ represents the oscillation of the relay valve displacement.
Vector $\overline{OP_0}$ represents the power-change oscillation due to the gate opening oscillation.
Vector $\overline{OH}$ represents the power-change oscillation due to the head-change oscillation (water hammer).
Vector $\overline{OP}$ represents the total power-change oscillation.
Vector $\overline{OF}$ represents the acceleration (derivative of speed).

As in the previous simpler cases, the length and orientation of the vectors follow from the basic differential equations for the particular case under investigation. The fundamental assumption of incipient instability is responsible for locating ω and $d\omega/dt$, and p_0 and dp_0/dt at 90 deg phase difference.

To Eq. (16) corresponds the vectorial operation:

$$\overline{OE} = \overline{OB} + \overline{OC}$$

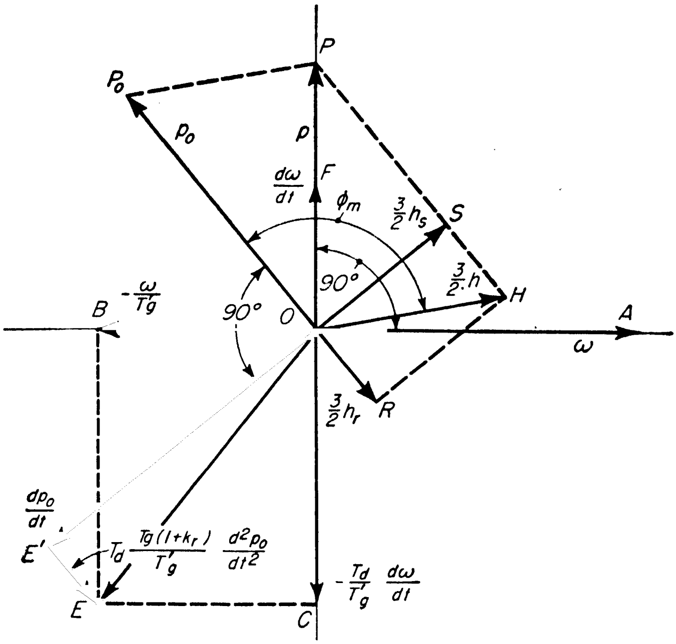

FIG. 22.—Vector diagram—speed-responsive governor with secondary compensation (dash-pot) including water hammer.

and the manner of passing from vector OE to vector $\overline{OP}_0$:

$$\overline{OP}_0 = \frac{T''}{2\pi} \overline{OE}\dagger$$

To Eq. (26) corresponds the vectorial operation:

$$\overline{OP} = \overline{OP}_0 + \overline{OH}$$

and to Eq. (7) (page 736) the manner of passing from vector $\overline{OP}$ to vector $\overline{OA}$:

$$\overline{OF} = \frac{1}{T}\overline{OP} \qquad \overline{OA} = \frac{T''}{2\pi}\overline{OF}\dagger$$

From elementary geometry,

$$\frac{OP_0 - OR}{OP} = \frac{OB}{OE}$$
$$\overline{OE}^2 = \overline{OB}^2 + \overline{OC}^2$$
$$\overline{OP}^2 = (OP_0 - OR)^2 + \overline{OS}^2$$

By appropriate substitutions and simple calculations, we obtain

$$m = \frac{3s}{2j}\Theta\frac{T_r}{T} \qquad \text{and} \qquad T_gT = \left(\frac{3s}{2j}\right)^2\frac{T_r}{T}\Theta^2$$

which confirms the criterion for incipient stability according to Eq. (39).

The diagram in Fig. 22, corresponding to Case III, differs from the diagram of Fig.

† Relation between the amplitude of the sine function and the amplitude of its derivative.

21 in a single feature. The relay valve displacement is no longer represented by vector $\overline{OE}$ but by vector $\overline{OE'}$, which is obtained by the vectorial operation:

$$\overline{OE'} + \overline{E'E} = \overline{OB} + \overline{OC}$$

corresponding to Eq. (23). Owing to the effect of vector $\overline{E'E}$, vector $\overline{OE'}$ is displaced backward with respect to vector $\overline{OE}$, which is an adverse effect as regards stability. With a view to reducing this disadvantage to a minimum, we must seek to render the length of vector $\overline{E'E}$ as short as possible and therefore make T_g' very much greater than $T_g(1 + k_r)$, that is to say, $\delta' T_d$ very much greater than T_g, which confirms the condition (25).

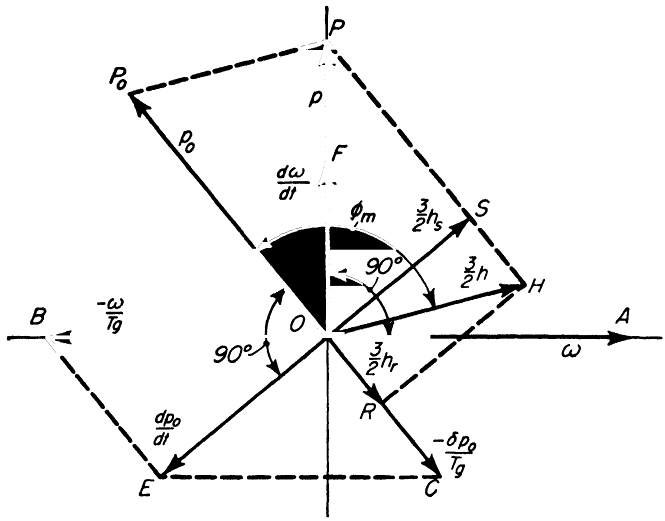

FIG. 23.—Vector diagram—speed-responsive governor with primary compensation only including water hammer.

For Case I and the diagram of Fig. 23, in which vector $\overline{OC}$ represents the compensation effect upon the relay valve, we deduce from the geometry of the figure,

$$\frac{\overline{OP}_0 - \overline{OR}}{\overline{OP}} = \frac{\overline{OE}}{\overline{OB}}$$

$$\overline{OB}^2 = \overline{OE}^2 + \overline{OC}^2$$

$$\overline{OP}^2 = (\overline{OP}_0 - \overline{OR})^2 + \overline{OS}^2$$

Substituting the appropriate vectors, we confirm the criterion for incipient stability according to Eq. (36):

$$\delta = \frac{3s}{2j}\frac{\Theta}{T}$$

Finally, we draw Fig. 24, the vector diagram for the case of a speed-acceleration responsive governor (Case II) including the effect of speed change upon the difference between the resisting and driving couples, and the influence of water hammer.

Examination of the different diagrams shows clearly that every effect leading to an advance of vector $\overline{OP}$ (total power change oscillation) or of vector $\overline{OD}$ (oscillation of the difference between resisting and driving couples), with respect to vector $\overline{OA}$ (speed-

change oscillation), in the trigonometric direction of increasing phase angles, improves stability, *i.e.*, the accelerometric effect upon the relay valve, the compensation effect upon the relay valve, and the effect of the speed change upon the difference between resisting and driving couples, with the understanding that $\alpha > 0$.

On the contrary, every effect leading to a backward rotation of the above vectors imperils stability, the most important influence of this kind being the water-hammer effect.

Experimental Statistics. From the foregoing analyses of Cases II and III, which comprise the important practical applications, we may conclude that for given values of the coefficient j (effect of the efficiency change) and α effect of the speed change upon

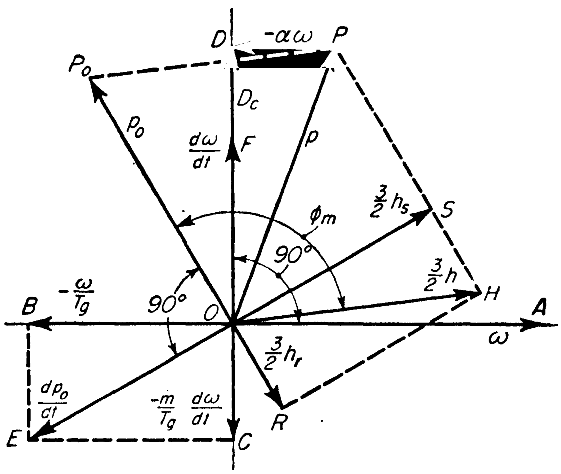

FIG. 24. Vector diagram—speed-acceleration-responsive governor including water hammer and the effect of speed change upon the difference between resisting and driving couples.

the difference between resisting and driving couples) governing stability depends upon five parameters, each having the dimensions of time in seconds:

For the rotating masses:

1. The time T, representing the specific inertia of the rotating masses. The value of T is normally between 2 and 8 sec, except in cases where a separate flywheel is provided in addition to the generator rotor.

For the pressure conduit:

2. The time Θ, representing the effect of water hammer. The value of Θ in practice is usually between 0.5 and 3 sec.

3. The time $T' = 2\mu$, representing the period of the free water-hammer oscillations, *i.e.*, two complete forward and backward transits of the water-hammer wave along the conduit or $4L/a$. The value of T' in practice lies usually between a few hundredths of a second and about 10 sec, seldom more.

For the governor:

4. The time m, measuring the importance of the acceleration factor in a speed-acceleration responsive governor. For a speed-responsive governor with secondary compensation in the form of a spring-loaded oil-filled dashpot, the time m is replaced by the time T_d defining the rigidity of the dashpot or its throttling action.

5. The time T_g measuring the promptness of the governor response. For the speed-acceleration responsive type, T_g depends essentially upon the dimensions of the oil-pressure relay valve. For the speed-responsive type with secondary compensation by dashpot, the value of T_g' depends primarily upon the dashpot rigidity T_d and the temporary speed droop $k_r\delta'$.

Since all installations having the same values for these five parameters will exhibit the same degree of stability, for the same turbine and load characteristics (values of j and α), it will be possible eventually as sufficient statistical data become available to prepare a chart (similar to the specific speed vs. head charts now employed so advantageously in turbine selection) permitting experimental determination of the value of the coefficient K in the general criterion of stability:†

$$T_gT > K(\tfrac{3}{2}\Theta)^2$$

directly deduced from inequality (39) or (41)

$$T_gT > \frac{s^2}{j(j-3r)}\left(\frac{3}{2}\Theta\right)^2$$

This criterion can then be used to check governing stability with the same ease that the Thoma formula is employed to check surge-tank stability.

Until such experimental results can be collected in sufficient volume and without neglecting their importance *in order to verify the analytic calculation*, fairly dependable values of the coefficient K can be computed on the basis of Eq. (37), which is rigorously valid for Case II (speed- and acceleration-responsive governor) and approximately for Case III (speed-responsive governor with secondary compensation).

Calculation of the Coefficient K of the General Criterion of Stability. The differential Eqs. (37) and (17) are of familiar form from vibration theory:‡

$$\frac{d^2\omega}{dt^2} + C_1\frac{d\omega}{dt} + C_2\omega = 0$$

in which, for Eq. (17),

$$C_1 = \frac{m}{T_gT} \qquad \text{and} \qquad C_2 = \frac{1}{T_gT}$$

and for Eq. (37),

$$C_1 = \frac{m}{T_gT_r}\,\frac{1 - \dfrac{3s}{2j}\dfrac{\Theta}{m}\dfrac{T_r}{T}}{1 - \dfrac{3s}{2j}\dfrac{m\Theta}{T_gT}} \qquad \text{and} \qquad C_2 = \frac{1}{T_gT_r}\,\frac{1}{1 - \dfrac{3s}{2j}\dfrac{m\Theta}{T_gT}}$$

Both equations were established neglecting the effect of speed change upon the difference between resisting and driving couples ($\alpha = 0$), the first one (17) in absence of and the second one (37) in presence of water hammer.

Their general solution is

$$\omega = Ce^{-\delta_*\frac{t}{T''}}\cos\left(\frac{2\pi}{T''} - \psi\right)$$

in which C and ψ are constants of integration.

$$T'' = \frac{4\pi}{\sqrt{4C_2 - C_1^2}}, \text{ the period of the oscillations.}$$

$$\delta_* = \frac{2\pi C_1}{\sqrt{4C_2 - C_1^2}}, \text{ their logarithmic decrement.}$$

In the case of Eq. (17), the values of this period and of this decrement are then

$$T'' = 2\pi\sqrt{T_gT}\,\frac{1}{\sqrt{1 - \frac{1}{4}\left(\frac{m}{\sqrt{T_gT}}\right)^2}}$$

† The fraction $\frac{3}{2}$ remaining in this expression of the criterion of stability reflects the assumption that at constant speed, the driving couple varies with the power $\frac{3}{2}$ of the head.

‡ PAGE, LEIGH, "Introduction to Theoretical Physics," D. Van Nostrand Company, Inc., 1941, pp. 74–82.

$$\delta_* = \pi \frac{\dfrac{m}{\sqrt{T_gT}}}{\sqrt{1 - \dfrac{1}{4}\left(\dfrac{m}{\sqrt{T_gT}}\right)^2}}$$

In the case of Eq. (37) we have already shown that in order to damp the oscillations, it is required that

$$m > \frac{3s}{2j}\frac{j}{j - 3r}\Theta \tag{38}$$

that is,

$$m = \lambda_m \frac{s}{j - 3r}\left(\frac{3}{2}\Theta\right) \tag{44}$$

with $\lambda_m > 1$ and

$$T_gT > \left(\frac{3s}{2j}\right)^2\left(\frac{j}{j - 3r}\right)\Theta^2 \tag{39}$$

that is,

$$T_gT = \lambda_t \frac{s^2}{j(j - 3r)}\left(\frac{3}{2}\Theta\right)^2 \tag{45}$$

or

$$T_gT_r = \lambda_t \frac{s^2}{(j - 3r)^2}\left(\frac{3}{2}\Theta\right)^2 \tag{45'}$$

with

$$\lambda_t > 1.\dagger$$

According to Eq. (45) the coefficient K of the criterion of stability is equal to

$$K = \lambda_t \frac{s^2}{j(j - 3_r)} \tag{46}$$

Now using the above defined coefficients λ_m and λ_t, we can express the factors C_1 and C_2 as follows:

$$C_1 = \frac{m}{T_gT_r}\frac{\lambda_t(\lambda_m - 1)}{\lambda_m(\lambda_t - \lambda_m)} \qquad C_2 = \frac{1}{T_gT_r}\frac{\lambda_t}{\lambda_t - \lambda_m}$$

or

$$C_1 = \frac{m_\Theta}{(T_gT_r)_\Theta} \qquad C_2 = \frac{1}{(T_gT_r)_\Theta}$$

in which,

$$m_\Theta = m\frac{\lambda_m - 1}{\lambda_m} \qquad (T_gT_r)_\Theta = T_gT_r\frac{\lambda_t - \lambda_m}{\lambda_t} \tag{47}$$

The effect of water hammer then operates:

1. To reduce the real value of the characteristic time m of the acceleration factor to the fictitious value m_Θ.
2. To reduce the real value of the product T_gT (time of promptness multiplied by time of mechanical inertia) to the fictitious value $(T_gT_r)_\Theta$, in Eq. (47) in which $T_r = \dfrac{j}{j - 3r}T$.

Operating in similar fashion on Eq. (17), we easily obtain the following expressions of the period and of the decrement:

$$T'' = 2\pi\sqrt{(T_gT_r)_\Theta}\frac{1}{\sqrt{1 - \dfrac{1}{4}\left[\dfrac{m_\Theta}{\sqrt{(T_gT_r)_\Theta}}\right]^2}}$$

$$\delta_* = \pi\frac{\dfrac{m_\Theta}{\sqrt{(T_gT_r)_\Theta}}}{\sqrt{1 - \dfrac{1}{4}\left[\dfrac{m_\Theta}{\sqrt{(T_gT_r)_\Theta}}\right]^2}}$$

† Hence for T_gT to be $> \dfrac{3s}{2j}m_\Theta$ (see Case II). It is also required that $\lambda_t > \lambda_m$.

But according to Eq. (47),

$$\frac{m\Theta}{\sqrt{(T_gT_r)_\Theta}} = \frac{m}{\sqrt{T_gT_r}}\,\frac{\lambda_m - 1}{\lambda_m}\sqrt{\frac{\lambda_t}{\lambda_t - \lambda_m}}$$

and according to Eqs. (44) and (45′),

$$\frac{m}{\sqrt{T_gT_r}} = \frac{\lambda_m}{\sqrt{\lambda_t}}$$

so that

$$\frac{m\Theta}{\sqrt{(T_gT_r)_\Theta}} = \frac{\lambda_m - 1}{\sqrt{\lambda_t - \lambda_m}}$$

In consequence of

$$R = \sqrt{1 - \frac{1}{4}\left[\frac{m\Theta}{\sqrt{(T_gT_r)_\Theta}}\right]^2} = \sqrt{1 - \frac{1}{4}\left(\frac{\lambda_m - 1}{\sqrt{\lambda_t - \lambda_m}}\right)^2} \tag{48}$$

and

$$T'' = \frac{2\pi}{R}\sqrt{(T_gT_r)_\Theta} \qquad \delta_* = \frac{\pi}{R}\,\frac{\lambda_m - 1}{\sqrt{\lambda_t - \lambda_m}} \tag{49}$$

Using Eqs. (47) and (45′),

$$(T_gT_r)_\Theta = (\lambda_t - \lambda_m)\frac{s^2}{(j - 3r)^2}\left(\frac{3}{2}\Theta\right)^2$$

so that

$$T'' = \frac{3\pi}{R}\,\frac{s}{j - 3r}\sqrt{\lambda_t - \lambda_m}\,\Theta \tag{50}$$

and

$$\chi = \frac{3\pi}{2R}\,\frac{s}{j - 3r}\sqrt{\lambda_t - \lambda_m}\,\rho\dagger \tag{51}$$

(*a*) Let us assume (1) that $j = 1$, that is to say, the typical point of the steady-state condition is on the flat portion of the turbine efficiency curve, and (2) that the system is just on the brink of instability; in other words that the oscillations of gate opening change, head change, driving couple change, speed change, and so on are all sustained or undamped, so that

$$\delta_* = 0 \qquad \lambda_m = 1 \qquad R = 1$$

$$\chi = \frac{3\pi}{2}\,\frac{s}{1 - 3r}\sqrt{\lambda_t - 1}\,\rho \tag{52}$$

Considering a given value of ρ, we can now compute by trial, for different values of λ_t and by means of Eqs. (27), (29), (30), or of Figs. 16 to 19,

1. The values of χ, r and s satisfying Eq. (52).
2. The values of the coefficient K of the criterion of stability.

$$K = \lambda_t\frac{s^2}{1 - 3r} \tag{46′}$$

Choosing for each value of ρ, the particular value of λ_t corresponding to the minimum value of the coefficient K, we finally arrive at the results condensed in Table 4. According to the figures of this table, the curve of the diagram Fig. 25 has been drawn, taking as ordinates, the ratio $(K_{\rho-\rho}:K_{\rho-10})$ between the value of K for the considered value of ρ, and the value $K = 1.66$, for $\rho > 10$; as abcissas, at a logarithmic scale, the values of $1/\rho$ and of ρ. Underneath are also indicated the approximate values of the head corresponding to the said values of ρ.

The curve of Fig. 25 shows that the value of the coefficient K varies very little

† For $\chi = \frac{T''}{2\mu}$ and $\frac{\Theta}{2\mu} = \frac{1}{2}\frac{LV_0}{gH}$, $\frac{a}{2L} = \frac{1}{2}\frac{aV_0}{2gH} = \frac{1}{2}\rho$.

TABLE 4

ρ	10	2.5	1.5	1.0	0.5	0.3
λ_t	1.255	1.255	1.27	1.30	1.45	1.58
x	37.3	9.33	5.78	4.08	2.53	1.86
s	0.848	0.857	0.876	0.905	1.021	1.195
r	0.153	0.150	0.148	0.142	0.121	0.102
$\frac{s}{1-3r}$	1.56	1.56	1.57	1.58	1.60	1.72
$K\delta_{*-0}$	1.66	1.68	1.75	1.86	2.37	3.25
$\frac{K_{\rho-\rho}}{K_{\rho-10}}$	100 %	101 %	105 %	112 %	143 %	196 %

in the scale of the low heads, which is an important statement. This value being $K = 1.66$ for a head in the vicinity of 30 ft, its variation is only 5 per cent up to a head of about 330 ft and 10 per cent up to a head of about 550 ft.

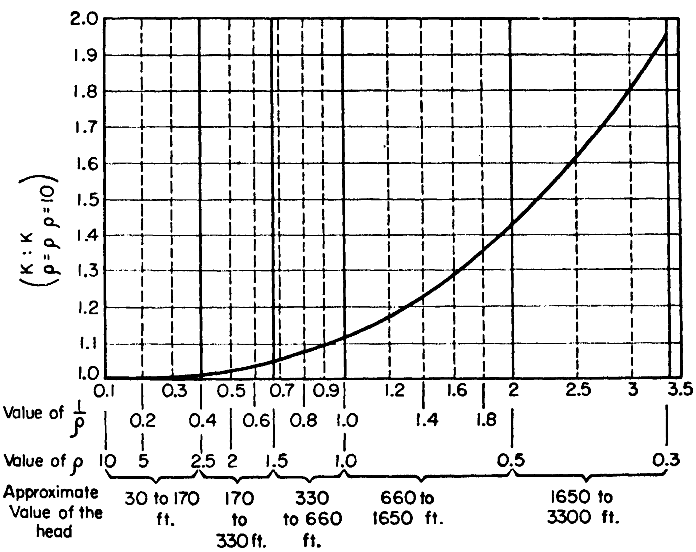

FIG. 25.—Relative value (in comparison with the value for $\rho \geq 10$) of the coefficient K of the criterion of stability in terms of ρ – the Allievi water-hammer parameter (that is to say, particularly in terms of the head).

The value of the fraction $\frac{s}{1-3r}$ mentioned in the sixth line of Table 4 is to be used to compute the most convenient value of the characteristic time m(or T_d) of the acceleration factor, by means of Eq. (44), with $\lambda_m = 1$ and $j = 1$,

$$m = \frac{s}{1-3r}\left(\frac{3}{2}\Theta\right)$$

This value of m corresponds fortunately to the minimum value of the coefficient K.

(b) If the oscillations should be damped, i.e., with a logarithmic decrement $\delta_* = 1.38$ (such amplitude being then the quarter of the former $e^{1.38} \cong 4$), j being still equal to 1, the calculation of the value of K will be carried on as follows:

1. By means of Eqs. (48) and (49), compute a series of couples of values of λ_m and λ_t, corresponding to $\delta_* = 1.38$.
2. Considering a given value of ρ, compute by trial for different couples λ_m, λ_t, the values of χ, r, and s satisfying Eq. (51),† using for this purpose Eqs. (27), (29), (30) or Figs. 16 to 19.
3. Compute the values of the coefficient K, by means of Eq. (46′).
4. Choose, for each value of ρ, the minimum of the calculated values of K.

The results of these operations are given by Table 5. The value of the expression

TABLE 5

ρ	10	2.5	1.5	1.0	0.5	0.3
λ_t	1.405	1.41	1.425	1.465	1.66	1.80
λ_m	1.196	1.198	1.204	1.215	1.268	1.303
χ	35.9	9.03	5.575	3.965	2.48	1.822
r	0.162	0.159	0.157	0.150	0.1265	0.109
s	0.838	0.849	0.867	0.900	1.022	1.205
$\lambda_m \frac{s}{1-3r}$	1.95	1.95	1.97	1.99	2.085	2.30
In % of Table 4 $\frac{s}{1-3r}$	125 %	125 %	125 %	125 %	130 %	134 %
$K\delta_{*=1.38}$	1.92	1.945	2.025	2.16	2.78	3.87
$\frac{K\delta_{*=1.38}}{K\delta_{*=0}}$	115.8 %	115.8 %	115.8 %	116.2 %	117 %	119 %

$\lambda_m \frac{s}{1-3r}$ given in the seventh line of Table 5 is to be used to compute the most convenient value of the time m (or T_d), by means of Eq. (44), with $j = 1$,

$$m = \lambda_m \frac{s}{1-3r}\left(\frac{3}{2}\Theta\right)$$

The figures of Table 6 were established in the same way, for the case in which the value of the logarithmic decrement is equal to $\delta_* = 0.69$ (such amplitude being then half of the former $e^{0.69} = 2$).

The comparison between Tables 5 and 6 and Table 4 leads us to conclude that in order to damp the oscillations with a logarithmic decrement equal to $\delta_* = 0.69$ or to $\delta_* = 1.38$, the values of the coefficient K of the criterion of stability, computed on the basis of the assumption of sustained oscillations, should be increased in an approximately constant proportion of 8 to 10 per cent and 16 to 19 per cent, respectively. This statement is valid over the entire range of head values from 30 ft ($\rho \cong 10$) to 3,000 ft ($\rho \cong 0.3$). Concerning the characteristic time m of the acceleration factor, it should be increased according to the indication of the eighth lines of Tables 5 and 6, in a proportion which is also nearly constant and respectively equal to 13 to 17 per cent or to 25 to 34 per cent.

† But with $j = 1$.

TABLE 6

ρ	10	2.5	1.5	1.0	0.5	0.3
λ_t	1.34	1.35	1.36	1.39	1.575	1.735
λ_m	1.105	1.108	1.109	1.115	1.143	1.165
χ	36.5	9.23	5.70	4.01	2.515	1.856
r	0.157	0.154	0.152	0.146	0.123	0.1025
s	0.842	0.854	0.872	0.903	1.022	1.194
$\lambda_m \frac{s}{1-3r}$	1.76	1.76	1.78	1.79	1.85	2.01
In % of Table 4 $\frac{s}{1-3r}$	113%	113%	113%	113%	115%	117%
$K\delta_{*-0.69}$	1.80	1.82	1.90	2.02	2.59	3.58
$\frac{M\delta_{*-0.69}}{M\delta_{*-\theta}}$	108%	108%	108%	108.5%	109%	110%

(*c*) Let us now return to the case of sustained oscillations, corresponding to the limit of stability $\delta_* = 0$, with the purpose of studying the effect of efficiency change $j \neq 1$.

The procedure is the same as that explained in paragraph (*a*), except that Eq. (51) with $\lambda_m = 1$ and $R = 1$,

$$\chi = \frac{3\pi}{2}\frac{s}{j - 3r}\sqrt{\lambda_t - 1}\,\rho \tag{53}$$

is to be used instead of Eq. (52) and that Eq. (46),

$$K = \lambda_t \frac{s^2}{j(j - 3r)} \tag{46}$$

is to be used instead of Eq. (46′).

The results of the calculation are illustrated by the curves of Fig. 26 having as abcissas, the value of the coefficient j defining the rate of efficiency change at the representative point of steady-state conditions, in other words, the slope of the turbine efficiency curve; as ordinates, the ratio $(K_{j-j}:K_{j-1})$ between the values of K computed taking account of the effect of the efficiency change ($j \neq 1$) and the values of K computed in paragraph (*a*), Fig. 25, neglecting the said effect ($j = 1$).

The three curves of Fig. 26, which are not far apart, have been drawn for $\rho = 10$ (case of low head), $\rho = 1.5$ (case of medium head) and $\rho = 0.3$ (case of high head). Their examination leads to the following comments.

When the turbine efficiency curve is ascending, *i.e.*, when $j > 1$, the value of the coefficient K of the criterion of stability becomes smaller than for $j = 1$. This statement is logical because in this case the discharge change is smaller than the output change, so that the water hammer effect is less important. This evidently improves stability.

On the contrary, when the turbine efficiency curve is descending ($j < 1$), the value of the coefficient K becomes greater than for $j = 1$. In this case the discharge change is in fact greater than the output change, so that the water-hammer effect is more important. The conditions for stability are then more severe.

Moreover, the effect of efficiency change is a little less marked in case of high heads than in case of low and medium heads, the slope of the curves $\rho = 10$ or $\rho = 1.5$ being greater than that of the curve $\rho = 0.3$.

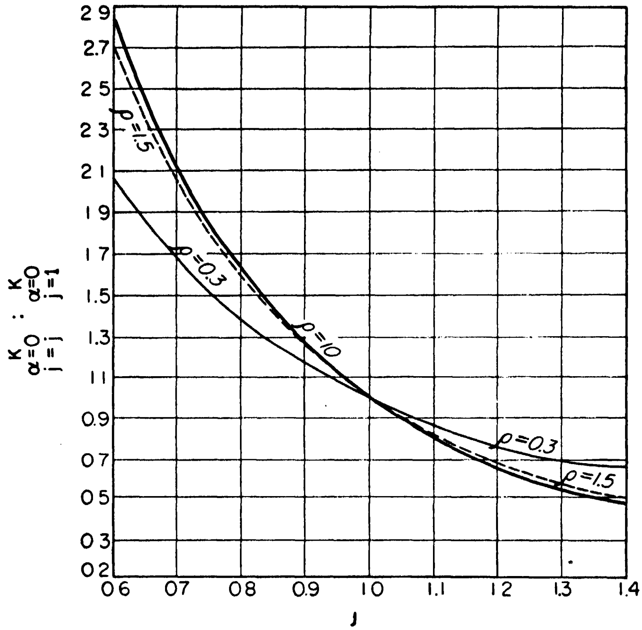

FIG. 26.—Relative value (in comparison with the value for $j = 1$) of the coefficient K of the criterion of stability in terms of j — the parameter defining the effect of efficiency change, for different values of the water-hammer parameter.

(*d*) To take account of the effect of speed change upon the difference between resisting and driving couples, we have to use the basic equation of governor movement (Case II):

$$\frac{dp_0}{dt} = -\frac{1}{T_g}\left(\omega + m\frac{d\omega}{dt}\right) \tag{16}$$

The flywheel equation which as seen before must be then written

$$\frac{d\omega}{dt} = \frac{p_0}{T_r} - \frac{3s}{2j}\frac{\Theta}{T}\frac{dp_0}{dt} - \frac{\alpha\omega}{T} \tag{42}$$

Combining these two equations, we easily obtain a single equation of the same form as Eq. (37):

$$\frac{d^2\omega}{dt^2} + C_1\frac{d\omega}{dt} + C_2 = 0$$

in which C_2 has the same value as in the said equation, but

$$C_1 = \frac{m}{T_gT_r}\,\frac{1 - \dfrac{3s}{2j}\dfrac{\Theta}{m}\dfrac{T_r}{T} + \alpha\dfrac{T_gT_r}{mT}}{1 - \dfrac{3s}{2j}\dfrac{m\Theta}{T_gT}}$$

Operating as explained for Eq. (37), the following expressions can be computed for the case of sustained oscillations ($\lambda_m = 1$, $\delta_* = 0$):

1. For the characteristic time m (or T_d) of the acceleration factor, instead of Eq. (38) for Eq. (37):

$$m = \left(\frac{3s}{2j}\,\Theta - \alpha T_g\right)\frac{j}{j - 3r} \tag{54}$$

2. For the coefficient K of the criterion of stability, instead of Eq. (46) for Eq. (37):

$$K = \lambda_t \frac{s^2}{j(j - 3r)} \frac{1}{1 + \dfrac{3s}{2(j - 3r)} \lambda_t \alpha \dfrac{\Theta}{T}} \tag{55}$$

3. For the period of the oscillations, instead of Eq. (50) with $R = 1$, for Eq. (37):

$$T''' = 3\pi \frac{s}{j - 3r} \sqrt{\lambda_t - 1}\, \frac{\Theta}{\sqrt{1 + \dfrac{3s}{2(j - 3r)} \lambda_t \alpha \dfrac{\Theta}{T}}} \tag{56}$$

4. For the ratio $\chi = (T''':T') = (T''':2\mu)$, instead of Eq. (51) with $R = 1$, for Eq. (37):

$$\chi = \frac{3\pi}{2} \frac{s}{j - 3r} \sqrt{\lambda_t - 1}\, \frac{\rho}{\sqrt{1 + \dfrac{3s}{2(j - 3r)} \lambda_t \alpha \dfrac{\Theta}{T}}} \tag{57}$$

As we have already studied the effect of efficiency change in paragraph (*c*), let us now neglect the said effect and assume that $j = 1$. It appears from Eq. (55), (56), and (57) that the parameter to be used for the present study is $\alpha \dfrac{\Theta}{T}$ composed of the coefficient α defining the effect of speed change upon the difference between resisting and driving couples; the ratio $\dfrac{\Theta}{T}$ between characteristic times of hydraulic and mechanical inertias.

The results of the calculation are illustrated by the curves of Fig. 27, having as abscissas, the values of the parameter $\alpha \dfrac{\Theta}{T}$; as ordinates, the ratio $(K_{\alpha=\alpha}:K_{\alpha=0})$ between the values of K computed taking account of the effect of speed change upon the difference between resisting and driving couples $\alpha \neq \Theta$, and the values of K computed in paragraph (*a*) (Fig. 25) neglecting the said effect $\alpha = 0$.

These curves were computed for severe values of ρ, but down to $\rho = 1.5$ they all overlap and diverge slightly only for $\rho < 1.5$, as shown in dotted line for $\rho = 0.3$.

Figure 27 confirms the statement already established. The effect of speed change upon the difference between resisting and driving couples favors stability when $\alpha > 0$ (the value of the coefficient K becoming smaller), that is to say, when the resisting couple curve cuts the driving couple curve, at the point of steady-state conditions, by passing *over* in the direction of increasing speed (Fig. 20). We may now add that this advantageous influence, being defined by the value of the parameter $\alpha \dfrac{\Theta}{T}$, is of increasing importance according as the value of the hydraulic inertia parameter Θ becomes large, and the value of the mechanic inertia parameter T becomes small.

Example 18: Compute, for the conditions of Example 5, the minimum value of the time of promptness (maximum governing rapidity), which assures governing stability in the case of a speed-acceleration responsive governor and in the case of a speed-responsive governor with secondary compensation, in absence of or neglecting (1) the effect of the primary compensation $\delta = 0$ and (2) the effect of the speed change upon the difference between the resisting and the driving couples $\alpha = 0$.

The calculation is based on the following values:

$P = 91{,}500$ hp	$N = 150$ rpm	$H = 330$ ft
$L = 400$ ft	$V_0 = 18.25$ fps	$WR^2 = 53 \times 10^6$ pfs
$a = 4{,}660$ fps	$\rho = 4$†	$\mu = 0.172$ sec
$\Theta = 0.688$ sec	$T = 8$ sec	

The diagram of Fig. 28 shows the turbine output curve established as for Fig. 2; the slope of this curve at its upper end corresponds to $k = 3$. The diagram of Fig. 29 shows the turbine efficiency curve established as for Fig. 9; the slope of this curve at its upper end corresponds to $j = 0.77$.

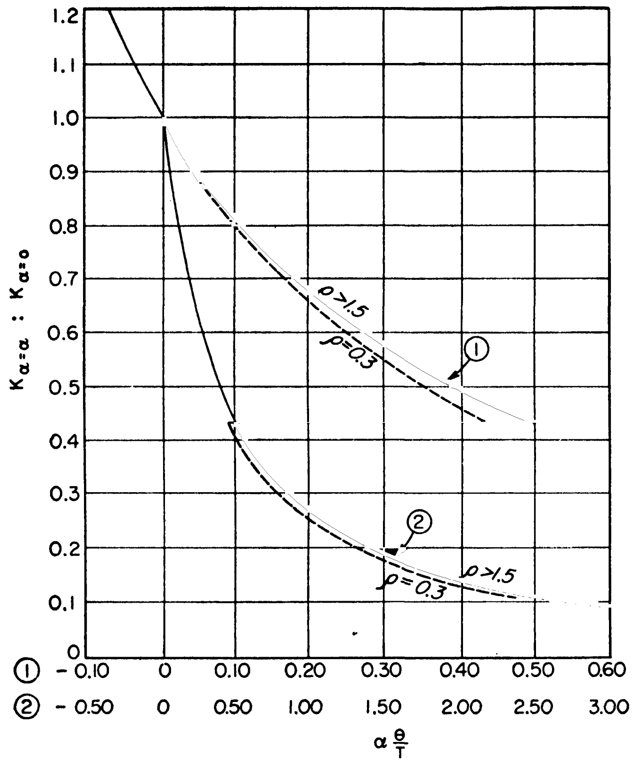

FIG. 27.—Relative value (in comparison with the value for $\alpha = 0$) of the coefficient K of the criterion of stability in terms of $\alpha \dfrac{\Theta}{T}$ parameter in which the coefficient $\alpha = \alpha_r - \alpha_d$ measures the effect of the speed change upon the difference between the resisting and the driving couples.

According to Fig. 25, $K_{\rho=4} : K_{\rho=10} = 1$. In the case of sustained oscillations and neglecting first the effect of efficiency change, we may then write, for a speed-acceleration responsive governor:

$$T_g T = 1.66(\tfrac{3}{2}\Theta)^2 = 1.78 \text{ sec}^2$$

$$T_g = \frac{1.78}{8} = 0.223 \text{ sec} \qquad \text{time of promptness}$$

$$m = \frac{3}{2}\Theta \frac{s}{1 - 3r}$$

$$m = \tfrac{3}{2} \times 0.688 \times 1.56‡ = 1.61 \text{ sec} \qquad \text{acceleration constant}$$

† The value of ρ is higher than these normally computed for a head of 330 ft, because, with the very short penstock in proportion to the head, it was possible to adopt a high velocity V_0.

‡ According to Table 4, for $\rho = 4$, $\dfrac{s}{1 - 3r} = 1.56$.

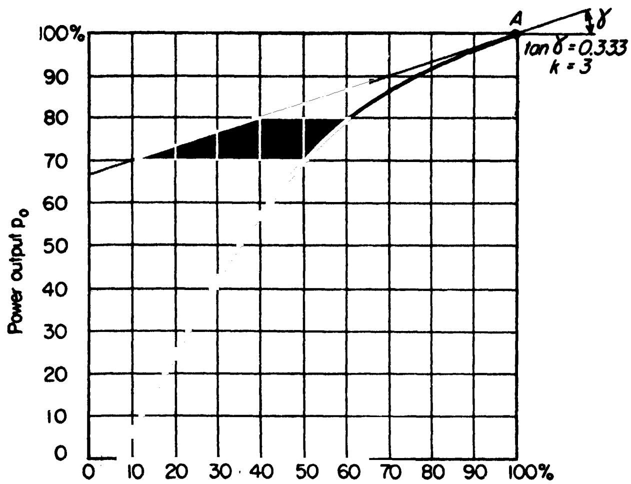

Fig. 28.—Servomotor piston travel x.

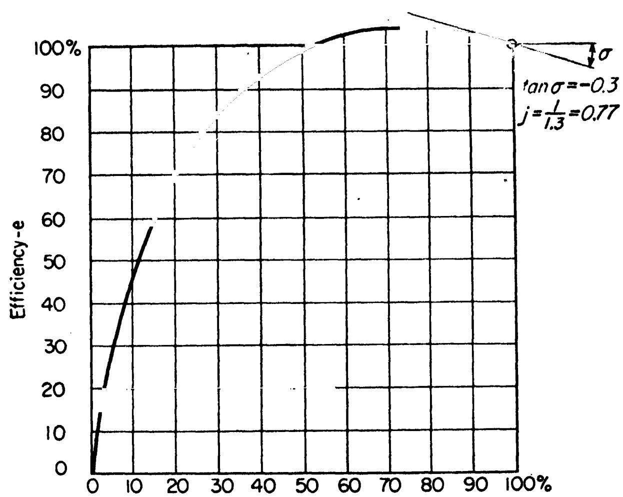

Fig. 29.—Power output p_0.

Assuming that condition (25) is fully reached, we may also compute for a speed-responsive governor with secondary compensation:

$$T_g' = 0.223 \text{ sec} \qquad \text{time of promptness}$$
$$T_d = 1.61 \text{ sec} \qquad \text{time of dashpot rigidity}$$
$$k_r\delta' = \frac{0.223}{1.61} = 13.9 \text{ per cent} \qquad \text{transient speed droop}$$

In both cases the period of the governor oscillations is about the same and equal to

$$T'' = 3\pi \frac{s}{1 - 3r} \Theta \sqrt{\lambda_{t-1}} \quad , \quad \frac{s}{1 - 3r} = 1.56 \qquad \lambda_t = 1.255\dagger$$
$$T'' = 5.10 \text{ sec}$$

Introducing now the effect of efficiency change, we can read on Fig. 26, for $\rho = 4$ and $j = 0.77$,

$$K_{j=0.77} : K_{j=1} = 1.7$$

† See Table 4.

Consequently,

$$T_g = 0.223 \times 1.7 = 0.38 \text{ sec}\dagger$$

Adding a margin of 8 per cent in order to damp the oscillations, with a logarithmic decrement of $\delta_* = 0.69$,

$$T_g = 0.41 \text{ sec}\ddagger$$

This means that, in the presence of a frequency change of ± 0.1 cycles/sec (0.166 per cent of 60 cycles/sec), the governor will be able to decrease or to increase the turbine output at the rate of 0.166 per cent in 0.41 sec, or 0.405 per cent in 1 sec, or 370 hp in 1 sec.

This governing rapidity (promptness) is quite favorable, owing to the small length of the penstock (in proportion to the head) and to the sufficiently large value of the mechanical inertia WR^2 defined by $T = 8$ sec.

Example 19: Compute for the conditions of Example 18, the minimum value of the permanent speed droop which assures governing stability, in case of a speed-responsive governor with primary compensation, assuming that the time of promptness should be the same as calculated for Example 18.

Neglecting first the effect of efficiency change ($j = 1$), we compute by trial for $T_g = 0.223$ sec (see Example 18), according to Eq. (35) and Fig. 16:

$$T'' = 2\pi\sqrt{T_gT_r} \qquad T_r = \frac{j}{j - 3r}\,T$$

$$T' = 0.344 \text{ sec} \qquad T'' = 9.15 \text{ sec} \qquad \chi \cong 27 \qquad \rho = 4 \qquad r = 0.05 \qquad T_r = 9.4 \text{ sec}$$

$$9.15 = 2\pi\sqrt{0.223 \times 9.4} = 9.15$$

in consequence of which, according to Fig. 18,

$$s = 0.95 \qquad \delta = \frac{3s}{2}\frac{\Theta}{T} = \frac{3 \times 0.95 \times 0.688}{2 \times 8}$$

$$\delta = 12.2 \text{ per cent}$$

Now taking account of the effect of efficiency change ($j = 0.77$), we compute in the same fashion for $T_g = 0.38$ sec (see Example 18):

$$T'' = 2\pi\sqrt{T_gT_r} \qquad T_r = \frac{j}{j - 3r}\,T$$

$$T' = 0.344 \text{ sec} \qquad T'' = 11.6 \text{ sec} \qquad \chi \cong 34 \qquad \rho = 4 \qquad r = 0.035 \qquad T_r = 9 \text{ sec}$$

$$11.6 = 2\pi\sqrt{0.38 \times 9} = 11.6$$

$$s = 0.975 \qquad \delta = \frac{3s}{2j}\frac{\Theta}{T} = \frac{3 \times 0.975 \times 0.688}{2 \times 0.77 \times 8}$$

$$\delta = 16.4 \text{ per cent}$$

In spite of the favorable conditions of Example 18, as mentioned before, these values of permanent speed droop are much too high and cannot be accepted in practice. Consequently it is not possible in this case to assure governing stability, by means of primary compensation alone.

† We may also compute by trial for $\rho = 4$, $j = 0.77$:

$$\chi = \frac{3\pi}{2}\frac{s}{j - 3r}\rho\sqrt{\lambda_t - 1} \qquad s = 0.9 \qquad r = 0.1 \qquad \lambda_t = 1.26$$

$$K = \lambda_t\frac{s^2}{j(j - 3r)} \qquad \chi = 18.5 \qquad T'' = 6.35 \text{ sec} \qquad K = 2.82$$

$$T_g \text{ or } T_g' = \frac{K}{T}\left(\frac{3}{2}\Theta\right)^2 = 0.38 \text{ sec} \qquad \text{time of promptness}$$

$$m \text{ or } T_d = \frac{3}{2}\Theta\frac{s}{j - 3r} = 1.98 \text{ sec} \qquad \text{acceleration constant or time of dashpot rigidity}$$

$$k_r\delta'\frac{T_g'}{T_d} = \frac{0.38}{1.98} = 19 \text{ per cent} \qquad \text{transient speed droop}$$

‡ The characteristic time m of the acceleration factor or T_d of the dashpot rigidity should be simultaneously increased to

$$m \text{ or } T_d = 1.98 \times 1.13 = 2.24 \text{ sec}$$

so that in the case of a speed-responsive governor with secondary compensation, the transient speed droop should reach

$$k_r\delta' = \frac{0.41}{2.24} = 18.5 \text{ per cent}$$

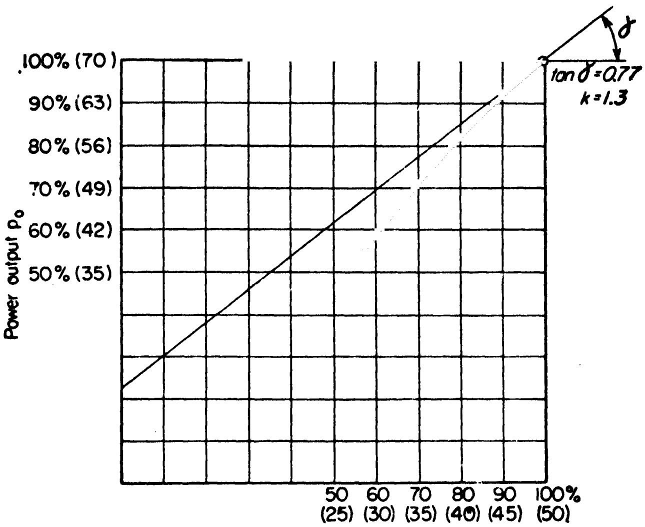

FIG. 30.—Servomotor piston travel x.

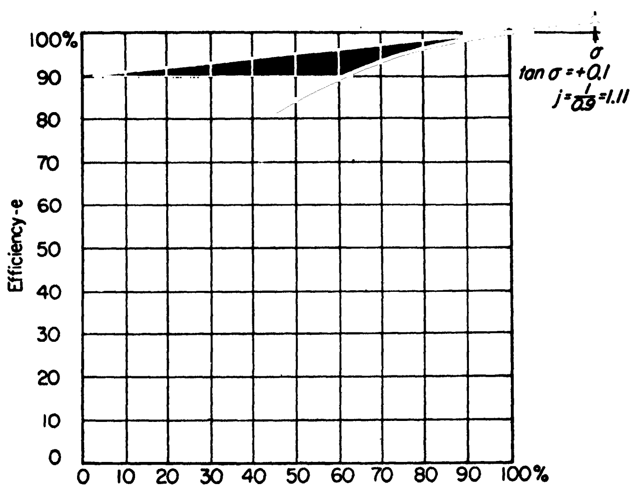

FIG. 31.—Power output p_0.

***Example* 20:** Check the stability of governing for the conditions of Example 18, but at 70 per cent of full load (50 per cent of full servomotor piston travel, see Fig. 28), the adjustment of the governor remaining *the same* as calculated for Example 18.

The diagram of Fig. 30 shows the turbine output curve established as for Fig. 2; the slope of this curve at its upper end corresponds to $k = 1.3$. The diagram of Fig. 31 shows the turbine efficiency curve established as for Fig. 9; the slope of this curve at its upper end corresponds to $j = 1.11$.

Neglecting first the effect of efficiency change, we have computed for Example 18: $T_g = 0.223$ sec at full load. To compute the time of promptness for 70 per cent load with the same governor adjustment, we shall reduce the aforesaid value: (1) in the proportion 50/100, the piston stroke for the new steady conditions being 50 per cent, (2) in the proportion of the values of the coefficient $K = \frac{1.3}{3}$ depending upon the slope of the output curve.

$$T_g = 0.223 \times \frac{50}{100} \times \frac{1.3}{3} = 0.0482 \text{ sec}$$

To compute the mechanical inertia parameter T, we shall increase the former value $T = 8$ sec in the proportion 100/70 of the loads:

$$T = 8^{100}/_{70} = 11.4 \text{ sec}$$

To compute the hydraulic inertia parameter Θ, we shall modify the former value $\Theta = 0.688$ sec (1) in the proportion 70/100 of the loads, (2) in the proportion 100/104 of the efficiencies (see Fig. 29).

$$\Theta = 0.688 \times {}^{70}/_{100} \times {}^{100}/_{104} = 0.462 \text{ sec}$$

In the same way,

$$\rho = 4 \times {}^{70}/_{100} \times {}^{100}/_{104} = 2.7$$

According to Fig. 25, the value of the coefficient K of the criterion of stability

$$T_g T > K(\tfrac{3}{2}\Theta)^2$$

is equal to $K = 1.66 \times 1.01 = 1.68$. As

$$0.0482 \times 11.4 < 1.68 \times \tfrac{3}{2}(0.462)^2$$
$$0.55 < 0.805$$

The criterion of stability is not satisfied owing to the greater value of the slope of the output curve ($k = 1.3$ instead of $k = 3$) and the time of promptness should be increased.

Now taking account of the effect of efficiency change, we have computed for Example 18: $T_g = 0.38$ sec at full load, so that the time of promptness for 70 per cent load, with the same governor adjustment, is equal to

$$T_g = 0.38 \times \frac{50}{100} \times \frac{1.3}{3} = 0.0825 \text{ sec}$$

According to Fig. 25 ($K\rho_{=2.7}:K\rho_{=10} = 1.01$) and Fig. 26 ($j = 1.11$; $\rho = 2.7$; $K_{j=1.11}:K_{j=1} = 0.8$) the value of the coefficient K of the criterion of stability is equal to

$$K = 1.66 \times 1.01 \times 0.8 = 1.34$$

As

$$0.0825 \times 11.4 > 1.34(\tfrac{3}{2}0.462)^2$$
$$0.94 > 0.645$$

The criterion of stability is satisfied, owing to the effect of the efficiency change ($j = 1.11$ instead of $j = 0.77$) and in spite of the greater value of the slope of the output curve ($k = 1.3$ instead of $k = 3$).

Example 21: Check the stability of governing for the conditions of Example 18, but with the following modifications: (1) The length of the penstock is increased to $L = 2{,}000$ ft. (2) In order to reduce the head losses, in spite of this penstock length, the velocity is reduced to $V_0 = 11.1$ fps. (3) The penstock being of metallic construction, the velocity of the water-hammer waves is equal to $a = 2{,}960$ fps. (4) To correspond to the most economical construction, the inertia of the rotor of the generator is reduced to $WR^2 = 36.5 \times 10^6$ pfs.

Consequently,

$$\rho = 1.55 \qquad \Theta = 2.1 \text{ sec} \qquad T = 5.5 \text{ sec}$$

According to Fig. 25 ($K\rho_{=1.55}:K\rho_{=10} = 1.05$) and Fig. 26 ($K_{j=0.77}:K_{j=1} = 1.7$):

$$T_g T = 1.66 \times 1.05 \times 1.7(\tfrac{3}{2}\Theta)^2$$
$$T_g = \frac{1.66 \times 1.05 \times 1.7}{5.5}\left(\frac{3}{2}2.1\right)^2$$
$$T_g = 5.4 \text{ sec}\dagger$$

† We may also compute by trial for $\rho = 1.55$ and $j = 0.77$:

$$\chi = \frac{3\pi}{2}\frac{s}{j - 3r}\rho\sqrt{\lambda_t - 1} \qquad s = 0.92 \qquad r = 0.098 \qquad \lambda_t = 1.27$$

$$K = \lambda_t \frac{s^2}{j(j - 3r)} \qquad \chi = 7.5 \qquad T'' = 20 \text{ sec} \qquad K = 2.98$$

$$T_g \text{ or } T_g' = \frac{K}{T}\left(\frac{3}{2}\Theta\right)^2 = 5.4 \text{ sec} \qquad \text{time of promptness}$$

$$m \text{ or } T_d = \frac{3}{2}\Theta\frac{s}{j - 3r} = 6.1 \text{ sec} \qquad \text{acceleration constant or time of dashpot rigidity}$$

$$k_r\delta' = \frac{T_g'}{T_d} = 89 \text{ per cent} \qquad \text{transient speed droop}$$

Notice the high value of the transient speed droop, which would cause prohibitively large frequency deviations.

Adding a margin of 8 per cent in order to damp the oscillations with a logarithmic decrement of $\delta_* = 0.69$,

$$T_g = 5.85 \text{ sec}$$

This means that, in the presence of a frequency deviation of ± 0.1 cycle/sec (0.166 per cent of 60 cycles/sec), the governor will be able to decrease or to increase the turbine output at the rate of $\frac{0.166}{5.85} = 0.0284$ per cent in 1 sec or 26 hp in 1 sec.

This governing rapidity (promptness) is quite unfavorable. For a large hydroelectric unit of 91.500 hp, the poor regulation capacity corresponding to an output change of only 26 hp/sec is not practically acceptable, except when the unit is not required to accomplish frequency control. For this purpose its action must be much quicker, considering the rate of change of load demand on the distribution system.

This calculation was made neglecting the effect of speed change upon the difference between resisting and driving couples $\alpha = 0$. Let us now assume that the load is essentially ohmic and that the voltage regulation is accomplished according to the following law:

$$\frac{u \text{ (voltage change in per cent)}}{\omega \text{ (frequency change in per cent)}} = 1$$

that is,

$$\frac{p}{\omega} = 2 \qquad \frac{C_r}{\omega} = 1 \qquad \alpha_r = +1$$

Moreover taking account of the specific speed of the turbine,

$$\alpha_d = -1, \qquad \text{so that} \qquad \alpha = +2$$

We can then read on the diagram Fig. 27, for $\alpha \frac{\Theta}{T} = 2 \frac{2.1}{5.5} = 0.765$:

$$K\alpha_{-2} : K\alpha_{-0} = 0.31$$

Consequently, the approximate value of the time of promptness is equal to

$$T_g = 5.85 \times 0.31 = 1.81 \text{ sec}$$

In the presence of a frequency deviation of ± 0.1 cycle/sec, the rate of change of the turbine output becomes 0.0925 per cent in 1 sec, or 84 hp in 1 sec.

The effect of the speed change upon the difference between resisting and driving couples allows an increase in the governing rapidity to a higher value which may be acceptable in this particular case. The actual rate of change of the consumer's load is the essential basis for judgment.

Governing Stability of Two (or Several) Generating Units. As the final topic in our abridged summary of the essentials of governing stability, we shall consider the case of two generating units coupled in parallel, so that their relative speed changes ω are necessarily the same. We shall try to discuss, at least approximately, the criterion of stability which is to be satisfied in order to prevent the gate opening changes of both units to oscillate *in phase*, in the undamped condition. For this purpose we shall select the speed-acceleration responsive type of governor, although the method of attack applies equally well to all types. The following method can easily be extended to the case of several generating units.

Using the subscript 1 for the terms corresponding to the first unit and subscript 2 for the terms corresponding to the second, we may write their respective equations (16) of governor movement as follows:

$$\frac{dp_{01}}{dt} = -\frac{1}{T_{g1}}\left(\omega + m_1 \frac{d\omega}{dt}\right) \tag{58}$$

$$\frac{dp_{02}}{dt} = -\frac{1}{T_{g2}}\left(\omega + m_2 \frac{d\omega}{dt}\right) \tag{59}$$

We notice here that, if the total power output at the steady-state condition is

$$P_0 = P_{01} + P_{02}$$

with

$$P_{01} = k_1 P_0 \qquad P_{02} = k_2 P_0 \qquad k_1 + k_2 = 1$$

the relative value p_0 (in regard to the total power output P_0) of the total output change, due to the governor movements (neglecting water-hammer and speed change effects upon driving and resisting couples) of both units, is equal to

$$\begin{aligned} p_0 &= \frac{(P_1 + P_2) - (P_{01} + P_{02})}{P_{01} + P_{02}} \\ &= k_1 \frac{P_1 - P_{01}}{P_{01}} + k_2 \frac{P_2 - P_{02}}{P_{02}} \\ &= k_1 p_{01} + k_2 p_{02} \end{aligned} \tag{60}$$

Considering only the speed change effect upon the governor movements, we may then write:

$$\begin{aligned} \frac{dp_0}{dt} &= k_1 \frac{dp_{01}}{dt} + k_2 \frac{dp_{02}}{dt} \\ &= -\frac{k_1}{T_{g1}} \omega - \frac{k_2}{T_{g2}} \omega \\ &= -\frac{1}{T_g} \omega \end{aligned} \tag{61}$$

in which T_g is the resulting time of promptness of response of both governors acting in concert. It can be calculated by the following formula deduced from the above equations:

$$\frac{1}{T_g} = \frac{k_1}{T_{g1}} + \frac{k_2}{T_{g2}} \tag{62}$$

The flywheel equations (34) of both units, considered independently one from the other, may be written (neglecting the speed change effect upon the couples):

$$\frac{d\omega}{dt} = \frac{1}{T_1}\left(w_1 p_{01} - l_1 \frac{dp_{01}}{dt}\right) \tag{63}$$

$$\frac{d\omega}{dt} = \frac{1}{T_2}\left(w_2 p_{02} - l_2 \frac{dp_{02}}{dt}\right) \tag{64}$$

in which (1) $w = \frac{j - 3r}{j} = \frac{T}{T_r}$ (dimensionless) measures the importance of the relative power output change component *in phase* with the gate opening change. The value of w depends upon the first component of the water hammer, characterized by the coefficient r, and upon the rate of change of the turbine efficiency, characterized by the coefficient j; (2) $l = \frac{3s}{2j}\,\Theta$ (having the dimension of time) measures the importance of the relative power output change component, in quadrature with the gate opening change. The value of l depends upon the second component of the water hammer, characterized by the coefficient s, upon the hydraulic inertia parameter Θ, and also upon the rate of change of turbine efficiency, characterized by the coefficient j.

We have already shown that the criterion of stability of each unit separately considered, is

$$T_{g1}T_1 > \frac{l_1^2}{w_1} \qquad T_{g2}T_2 > \frac{l_2^2}{w_2} \tag{39}$$

But when both units are coupled together, the flywheel equation must be written

$$\frac{d\omega}{dt} = \frac{1}{T}(k_1 p_1 + k_2 p_2) \tag{65}$$

in which T is the resulting value of the inertia parameter, which characterizes the effect of the rotating masses of both units. In order to calculate this resulting value T, we

shall for the time being neglect the water-hammer effect and return to the defining formula of the inertia parameter, given when establishing Eq. (7):

$$T = \frac{1}{16.2 \times 10^5} \frac{(WR^2)_1 N_{01}^2 + (WR^2)_2 N_{02}^2}{P_{01} + P_{02}}$$
$$= \frac{1}{16.2 \times 10^5} \left[\frac{(WR^2)_1 N_{01}^2}{\frac{P_{01}}{k_1}} + \frac{(WR^2)_2 N_{02}^2}{\frac{P_{02}}{k_2}} \right]$$
$$T = k_1 T_1 + k_2 T_2 \tag{66}$$

Now taking account of the water-hammer effect, the flywheel equation (65) becomes

$$\frac{d\omega}{dt} = \frac{k_1}{T}\left(w_1 p_{01} - l_1 \frac{dp_{01}}{dt}\right) + \frac{k_2}{T}\left(w_2 p_{02} - l_2 \frac{dp_{02}}{dt}\right) \tag{67}$$

Combining Eqs. (58), (59), and (67) and operating as explained for Case II, page 735, we arrive at the following criterion of stability:

$$1 - \frac{k_1}{T_{g1}T}\frac{l_1^2}{w_1} - \frac{k_2}{T_{g2}T}\frac{l_2^2}{w_2} > 0 \tag{68}$$

$$T_{g1}T_{g2}T > T_{g2}k_1 \frac{l_1^2}{w_1} + T_{g1}k_2 \frac{l_2^2}{w_2} \tag{69}$$

$$T_{g1}T_{g2}k_1T_1 + T_{g1}T_{g2}k_2T_2 > T_{g2}k_1 \frac{l_1^2}{w_1} + T_{g1}k_2 \frac{l_2^2}{w_2} \tag{70}$$

or

$$T_gT > k_1 \frac{T_g}{T_{g1}} \frac{l_1^2}{w_1} + k_2 \frac{T_g}{T_{g2}} \frac{l_2^2}{w_2}$$

This criterion of stability (70) of both units coupled together will be surely satisfied if

$$T_{g1}T_{g2}k_1T_1 > T_{g2}k_1 \frac{l_1^2}{w_1} \qquad \text{or} \qquad T_{g1}T_1 > \frac{l_1^2}{w_1}$$

and

$$T_{g2}T_{g1}k_2T_2 > T_{g1}k_2 \frac{l_2^2}{w_2} \qquad \text{or} \qquad T_{g2}T_2 > \frac{l_2^2}{w_2}$$

that is, if the individual criteria of stability of the two units separately considered are both satisfied.

On the contrary, the criterion of stability (70) of both units coupled together will certainly never be satisfied, if the individual criteria of stability of the two units separately considered are both unsatisfied.

In summary, if the individual criterion of stability of one unit (unit No. 1) separately considered is not satisfied, the criterion of stability (70) of both units coupled together may, however, be satisfied, when the individual criterion of stability of the other unit (unit No. 2) separately considered is substantially satisfied: $T_{g2}T_2 \gg \frac{l_2^2}{w_2}$. This is especially the case when the instability of the first unit is not too pronounced ($T_{g1}T_1$ not very much smaller than $\frac{l_1^2}{w_1}$ and when its contribution to the total power output is not too great (k_1 not too large).

From further detailed study we may finally reach three very important generalizations, concerning the governing stability of several units operating in parallel:

1. If the governing of each of the units is individually stable, the governing of all of them operating in parallel (their governors acting then in concert) will certainly be also stable.

2. If the governing of each of the units is individually unstable, the governing of all of them operating in parallel will certainly be also unstable.

3. If the governing of an important fraction of units operating in parallel (above all when this fraction corresponds to the major part of the total power output) presents a liberal margin of stability, coupling to them one or several units which are individually unstable may not disturb the governing stability of the whole. This is especially the case when the instability of these last units is not too pronounced and when their contribution to the total power output is not too great.

Example 22: Let us first consider a hydroelectric unit No. 1 and its speed-acceleration responsive governor (without primary compensation), the whole corresponding to the following characteristics:

The Allievi dimensionless parameter	$\rho_1 = 10$
The hydraulic inertia parameter having the dimension of time	$\Theta_1 = 1$ sec
The period of the free oscillations of the water-hammer waves, $T' = 2\mu = 2\dfrac{\Theta}{\rho}$	$T_1' = 0.2$ sec
The characteristic time of promptness of governor response	$T_{g1} = 0.72$ sec
The mechanical inertia parameter having the dimension of time	$T_1 = 6$ sec
The acceleration constant of the governor	$m_1 = 2.93$ sec

We shall assume that $j = 1$ (no efficiency change) and $\alpha = 0$ (no speed change effect upon the difference between the couples). In order to solve Eq. (37), we shall also assume for the period of the governor oscillations:

$$T_1'' = 7.18 \text{ sec}\dagger$$

that is,

$$\chi_1 = 35.9$$

We may then compute:

For the water-hammer coefficients r and s:

$$r_1 = 0.162 \qquad s_1 = 0.838$$

For the coefficient w and the time l:

$$w_1 = 1 - 3r_1 = 0.514 \qquad l_1 = \tfrac{3}{2}s_1\Theta_1 = 1.257 \text{ sec}$$

For the factors C_1 and C_2 of Eq. (37):

$$C_{11} = \frac{w_1}{T_{g1}T_1}\,\frac{m_1 - \dfrac{l_1}{w_1}}{1 - \dfrac{m_1 l_1}{T_{g1}T_1}} = 0.385$$

$$C_{21} = \frac{w_1}{T_{g1}T_1}\,\frac{1}{1 - \dfrac{m_1 l_1}{T_{g1}T_1}} = 0.803$$

For the period of the governor oscillations T'':

$$T_1'' = \frac{4\pi}{\sqrt{4C_{21} - C_{11}^2}} = 7.18 \text{ sec}$$

This result confirms the above-mentioned values ($T_1' = 7.18$ sec, $\chi_1 = 35.9$) which can be now accepted as rigorously correct.

For the logarithmic decrement of governor oscillations, δ_{*1}:

$$\delta_{*1} = \frac{2\pi C_{11}}{\sqrt{4C_{21} - C_{11}^2}} = 1.38$$

Consequently the governing of unit No. 1, independently considered, affords a liberal margin of stability.

Let us then consider a second unit No. 2, with the following characteristics: $\rho_2 = 1.5$; $\Theta_2 = 1.5$ sec; $T_2' = 2$ sec; $T_{g2} = 1.18$ sec; $T_2 = 7.5$ sec; and $m_2 = 3.54$ sec.

In a similar way, we can compute on the basis of

$$\begin{aligned} T_2'' &= 11.56 \text{ sec} & \chi_2 &= 5.78 \\ r_2 &= 0.148 & s_2 &= 0.876 \\ w_2 &= 0.556 & l_2 &= 1.97 \text{ sec} \\ C_{12} &= 0 & C_{22} &= 0.294 \end{aligned}$$

from which we confirm

$$T_2'' = 11.58 \text{ sec}$$

and calculate

$$\delta_{*2} = 0$$

† This value is to be chosen and then verified by further calculation (successive approximations).

The governing of unit No. 2 is consequently just on the brink of instability.

Let us now put both units in parallel, assuming that No. 1 produces 75 per cent ($k_1 = 0.75$) of the total power output and No. 2 produce 25 per cent ($k_2 = 0.25$), so that the resulting value of the mechanical inertia parameter reaches:

$$T = (0.75 \times 6) + (0.25 \times 7.5) = 6.375 \text{ sec}$$

In order to solve Eq. (37), we shall assume for the time being, for the oscillations in phase of both governors, the period:

$$T'' = 8.16 \text{ sec}; \qquad \text{that is to say,} \qquad \chi_1 = 40.8; \qquad \chi_2 = 4.08$$

We can then compute:

$$\begin{array}{llll} r_1 = 0.130 & s_1 = 0.869 & r_2 = 0.272 & s_2 = 0.770 \\ w_1 = 0.610 & l_1 = 1.303 & w_2 = 0.184 & l_2 = 1.730 \end{array}$$

$$C_1 = \frac{\dfrac{k_1 w_1}{T_{g1}T_1}\left(m_1 - \dfrac{l_1}{w_1}\right) + \dfrac{k_2 w_2}{T_{g2}T_2}\left(m_2 - \dfrac{l_2}{w_2}\right)}{1 - \dfrac{k_1 m_1 l_1}{T_{g1}T_1} - k_2 \dfrac{m_2 l_2}{T_{g2}T_2}} = 0.249$$

$$C_2 = \frac{\dfrac{k_1 w_1}{T_{g1}T_1} + \dfrac{k_2 w_2}{T_{g2}T_2}}{1 - k_1 \dfrac{m_1 l_1}{T_{g1}T_1} - k_2 \dfrac{m_2 l_2}{T_{g2}T_2}} = 0.620$$

$$T'' = \frac{4\pi}{\sqrt{4C_2 - C_1^2}} = 8.13 \text{ sec}$$

This result is sufficient confirmation of the above-mentioned value ($T'' = 8.16$ sec) and we can now accept as rigorously correct $T'' \cong 8.15$ sec.

$$\delta_* = \tfrac{1}{2} T'' C_1 = 1.01$$

Conclusion. The governing of both units coupled together is stable. The period of the oscillations $T'' = 8.15$ sec is greater than the period $T_1'' = 7.18$ sec of unit No. 1, independently considered, but smaller than the period $T_2'' = 11.58$ sec of unit No. 2, independently considered. Whereas the governing of unit No. 2 is just on the brink of instability, $\delta_{*2} = 0$, and thanks to the margin afforded by the governing of unit No. 1, $\delta_{*1} = 1.38$, the logarithmic decrement of the governor oscillations, when both units operate in parallel, reaches $\delta_* = 1.01$, that is to say, the amplitude is 2.75 ($e^{1.01} = 2.75$) times smaller.

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SECTION 16

NAVIGATION LOCKS

By George R. Rich

INTRODUCTION

From the standpoint of the hydraulic engineer determination of proportions for filling and emptying systems is the principal problem in the design of navigation locks. The fundamental difficulty is to reconcile two conflicting requirements: (1), that the lock fill quickly, in the order of 12 to 15 min, to avoid penalizing traffic and (2), that the resultant disturbance in the lock chamber not be sufficient to cause dangerously high hawser stresses with the attendant possibility of breaking a ship from its moorings so as to collide with and wreck the main lock gates.

Owing principally to the effect of the ship in lockage upon the subdivision and partial reflection of the regimen of translatory waves, by means of which filling of the chamber is effected, the practical problem of filling system design is not susceptible to complete and final determination by analytical methods alone, but confirmation must be obtained by the familiar process of reduced-scale model tests in the hydraulic laboratory. However, it would certainly be a mistake to infer that analytical methods have no place in lock design. By isolating certain of the major controlling elements and studying them separately, we establish a more rational and effective basis for trial designs and obtain incisive tools for the planning and interpretation of model tests.

The fundamental physical action in lock-filling systems affords still another instance of wave motion with respect to both water hammer in the conduits and waves of translation, which, as previously noted, constitute the mechanism by which filling of the chamber is accomplished.

For reasons that will appear during the course of our discussion, slow opening or closing of the lock valves is essential to ensure quiet lockage. Accordingly, the hydraulic design of the filling conduits may be treated as a mass-acceleration problem, *i.e.*, stretching of the conduit walls and compression of the water may be disregarded without appreciable error because the magnitude of the sub and superpressures due to water hammer is not significant. The objective in conduit design is primarily to ensure that the filling time is satisfactorily short.

The translatory action in the chamber, while not affecting to any perceptible extent the hydraulic action of the filling conduits, will be found to measure the degree of disturbance to the vessel in lockage; and although the presence of this vessel does complicate all but the initial stages of the wave regimen, the analytical approach is valuable in affording us improved insight into the causes of excessive hawser stress. This second coordinate part of the basic problem has as its objective the insurance that disturbance to the vessel in lockage, as measured in the hydraulic laboratory by model hawser stresses, will not be objectionably great.

We have solved the entire problem completely when an economic and physical balance is effected between these two fundamental but somewhat conflicting tendencies.

BASIC MECHANISM OF LOCK CHAMBER FILLING

When water is admitted to or released from the lock chamber by any of the generally accepted filling systems, the mechanism immediately responsible for the actual change in water surface elevation is a procession of translatory waves traversing the length of the chamber. Before proceeding to the analytical expression of the pertinent physical laws, we may find it helpful to obtain in advance some intuitional introduction from the simplified apparatus depicted in Fig. 1.[1] The lock chamber is rectangu-

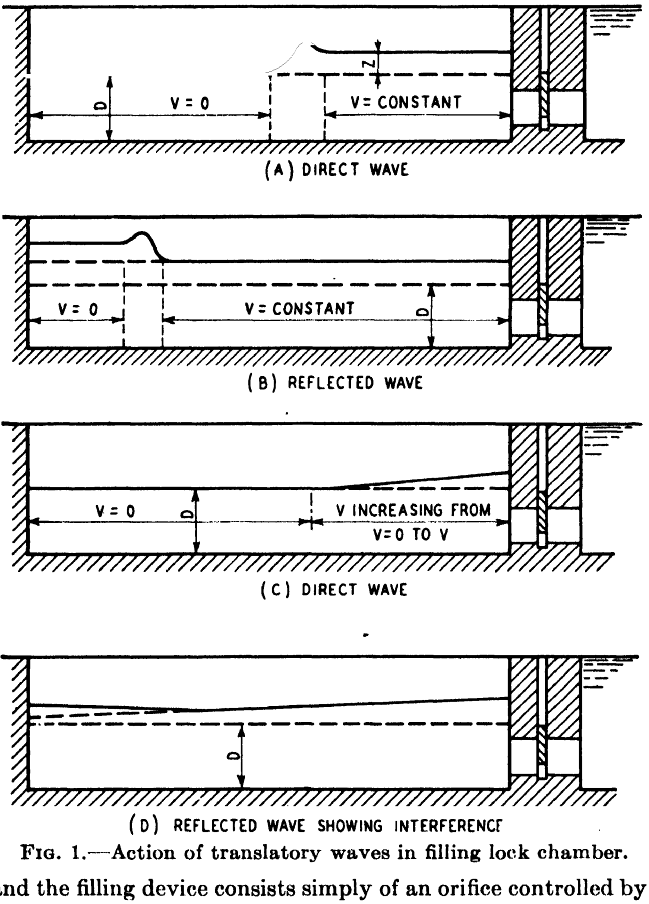

FIG. 1.—Action of translatory waves in filling lock chamber.

lar in form and the filling device consists simply of an orifice controlled by a rectangular valve. For the initial case it will be assumed that there is no ship in the chamber and that the lock valve is opened instantaneously.

We should then observe a definite intumescence on the water surface propagated with comparatively slow, readily followed velocity in the direction of the lock gates. As shown by the diagram, the velocity in the chamber behind the wave would be converted from zero to the value V. Upon reaching the lower gate the wave would be reflected in the opposite direction, increasing the depth in the chamber by a second increment and reconverting the chamber velocity behind the wave from V to zero. In other words, the head increments are reflected at the ends of the chamber with positive algebraic sign, and the velocity increments with negative algebraic sign.

[1] RICH, GEORGE R., Basic Hydraulics of Water Storage Projects, *Civil Eng.*, August, 1944, p. 352.

Now it is readily apparent that, if a ship were placed in lockage under such conditions, it would be driven with considerable violence first in the direction of the lower and then in the direction of the upper lock gates by an unbalanced force proportional to the crest height Z.

Our natural impulse is to alleviate such adverse conditions by substituting a slow uniform valve motion for the sudden opening employed initially, and we should then observe the marked improvement indicated by Fig. 1(C) and 1(D). In the first place the water surface exhibits, instead of the vertical-faced bore wave, a relatively flat slope with very beneficial interference effects subsequent to the first reflection. The unbalanced force now acting upon the vessel is greatly reduced in magnitude and the

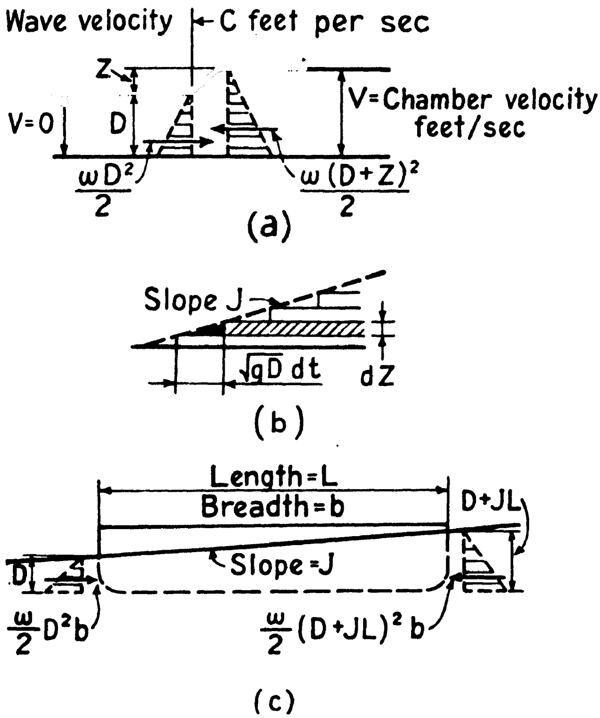

FIG. 2.—Development of unbalanced force on ship in lockage.

ship motion, although still in the nature of a characteristic longitudinal shuttling, is markedly decreased in frequency. Obviously slow uniform valve motion is one of the prerequisites of quiet lockage.

But, by means of Fig. 2, we can place this entire matter upon a much firmer foundation than mere intuition. We first obtain the familiar equation for wave velocity C by means of the momentum equation: In every second the passage of the wave changes the velocity of a mass of water $\frac{w}{g}C(D + Z)$ from zero to V, through the action of the unbalanced force $\frac{w}{2}[(D + Z)^2 - D^2]$, or

$$Ft = M(V - 0) \tag{1}$$

$$\frac{w}{2}[(D + Z)^2 - D^2] \times 1 = \frac{w}{g}CV(D + Z) \tag{2}$$

In addition, from considerations of hydraulic continuity, the quantity of water per unit width of chamber and above the dotted line, Fig. 1(*a*), is CZ. But this must be continuously supplied by flow over the entire chamber cross section behind the wave in amount $(D + Z)V$, or

$$CZ = (D + Z)V \qquad \text{or} \qquad Z = \frac{DV}{C} \tag{3}$$

Combining Eqs. (2) and (3) and noting that in the cases we shall encounter Z is very small in proportion to D, we obtain

$$C = \sqrt{gD} \tag{4}$$

Figure 2(*b*) will enable us to express the relationship between the surface slope and the chamber velocity and acceleration.

For a vertical front wave,

$$Z = \frac{(D + Z)V}{\sqrt{gD}} = \frac{DV}{\sqrt{gD}} \tag{5}$$

In addition, we may reason that the sloping surface in the chamber is in reality the envelope curve of a series of differential vertical-front waves each of height dZ, and that in time dt each differential wave travels a distance $\sqrt{gD}\,dt$, since the velocity of propagation is $\sqrt{gD}$. From the geometry of the figure, the slope at any instant is

$$J = \frac{dZ}{\sqrt{gD}\,dt} \tag{6}$$

But from Eq. (5)

$$dZ = \frac{D\,dV}{\sqrt{gD}}$$

Substituting in (6),

$$J = \frac{1}{g}\frac{dV}{dt} \tag{7}$$

For the slope in terms of the chamber flow Q we write

$$Q = bDV, \qquad dV = \frac{dQ}{bD}, \qquad \text{and} \qquad J = \frac{1}{gbD}\frac{dQ}{dt} \tag{8}$$

Finally, by means of Fig. 2(*c*) let us determine the quantitative relationship between the unbalanced force on the ship and the slope of the water surface in the chamber:

$$\text{Unbalanced force on the ship} = \frac{wb}{2}(D^2 + 2DJL + J^2L^2 - D^2)$$

or, since J^2L^2 is a small quantity of secondary order,

$$\text{Unbalanced force} = wLbDJ \tag{9}$$

But the ship displacement is with sufficient accuracy for small values of slope,

$$wLbD = K \tag{10}$$

or

$$\text{Unbalanced force} = KJ \tag{11}$$

and substituting J from Eq. (8),

$$\text{Unbalanced force} = \left(\frac{K}{gbD}\right)\frac{dQ}{dt} \tag{12}$$

The presence of a vessel in the lock will naturally effect a subdivision and reflection of the elementary wave motion upon which these simple equations are predicated, and engineers who are sufficiently interested can pursue the details further in the literature.[1] However, upon detailed examination we should find that these additional refinements introduce no point of conflict with the deductions we shall now make solely upon the basis of the simple parent equations. The implications of Eqs. (7), (8), (11), and (12) really constitute the fundamental principle of ship lockage: To secure the flat slope of chamber surface essential for quiet filling, the initial opening of the lock valves should be very small, and the discharge into the chamber should increase during the earlier stages of filling and decrease during the later stages of filling at a slow uniform rate.

From Eqs. (7) and (8) we also derive a second conclusion of far-reaching importance: The filling conduits may be made of large cross-sectional area since no limitation from the standpoint of quiet lockage is imposed upon the magnitude of the chamber velocity or flow. Only the acceleration dV/dt or dQ/dt need be kept small to ensure flat surface slopes. For the very high lifts in which impulse effects of the inflow are of important magnitude, the larger conduits also tend in the direction of reduced velocities and reduced impulse forces. Aside from this consideration of provision for baffling to avoid excessive impulse forces at the very high lifts, there is no valid objection to designing filling conduits to function as short loops of large cross-sectional area located entirely in the gate blocks, and dispensing with the port and lateral systems frequently installed in the side walls.

Our first natural objection to a slow valve opening rate is that the increased time required for filling and emptying the chamber would penalize traffic. However, owing to the fact that we use conduits of relatively great cross section in conjunction with slow valve motion, we are able to regain practically all of this time. During the later stages of filling, when the available head difference is comparatively small, conduits of large cross section enable us to carry large discharges even at the lower velocities.

Finally, we draw a very important general deduction from Eq. (3) which states the wave increment height:

$$Z = \frac{DV}{C} \tag{3}$$

But

$$V = \frac{\beta\sqrt{2gh}}{bD}$$

in which h is the head difference at any instant.

Substituting in (3) we obtain

$$Z = \frac{D\beta\sqrt{2gh}}{bD\sqrt{gD}} = \frac{\beta}{b}\sqrt{\frac{2h}{D}} \tag{13}$$

From Eq. (13), we conclude that the filling operation rather than the emptying process imposes the heavier burden upon the lock design, for the reason that at the start of filling the chamber depth is much lower than at the start of emptying.

The chief value of the foregoing equations and discussions is to afford us an insight into the basic hydromechanics of lockage, so that we may select the type of filling system best adapted to the exigencies of each particular lock. We shall probably never have occasion to employ the formulas given for the numerical computation of hawser stress, as these are determined with comparative ease and greater dependability as a part of the laboratory model tests to confirm the hydraulic design of the filling

[1] RICH, GEORGE R., The Hydromechanics of Ship Lockage, *Military Engr.*, July–August, 1932, pp. 364–369.

conduits. With a clear understanding of the mechanism of filling in the chambers, we may now proceed to develop the basic principles of design of the various forms of filling conduits for admitting water to the chamber.

TYPES OF LOCK-FILLING SYSTEMS

From the many arrangements that have been used as a means of controlling the admission of water to the lock chamber, three types have been selected to illustrate

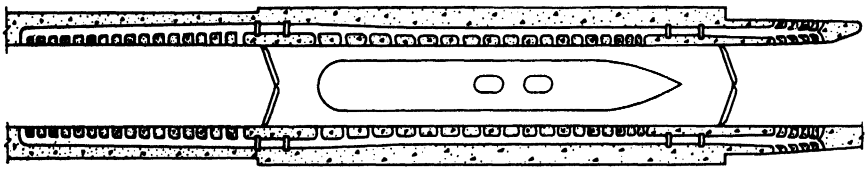

FIG. 3.—Longitudinal filling conduits with lateral ports, Kentucky project, T.V.A.

the fundamental principles of design: The first type, which has found outstanding favor in American practice, is illustrated by Fig. 3 and consists basically of main longitudinal header conduits in the side walls of the lock with numerous short lateral ports delivering discharge to the chamber. No doubt the aim of the originators of that scheme was to distribute the inflow uniformly over the entire chamber and thus

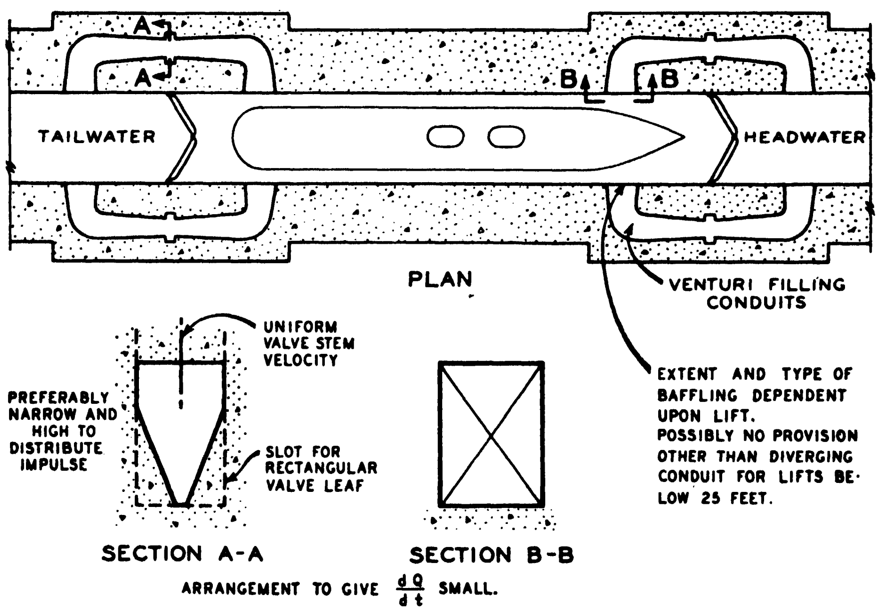

FIG. 4.—Venturi-loop conduit filling system.

eliminate what they believed to be the principal source of disturbance to the vessels in lockage. But as will be shown analytically in the course of our subsequent discussion, owing to the influence of the inertia of water in the conduits and the effect of contraction at the port connections, uniform port spacing does not afford anything like uniform discharge. In fact, in the latest examples of this type of design we have the rather anomalous incorporation of marked variation in port spacing to effect the requisite degree of freedom from turbulence. In the light of the best modern theory,

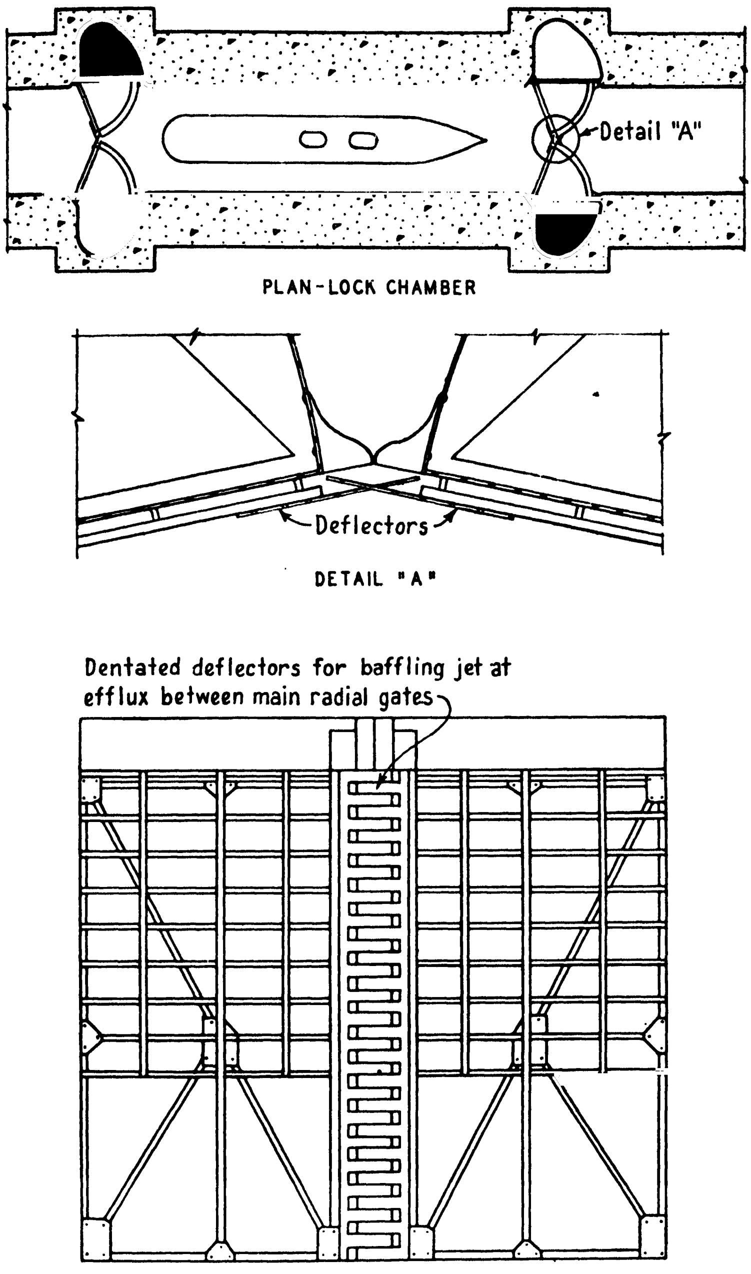

FIG. 5.—Filling chamber by cracking main radial lock gates, Passamaquoddy project.

the longitudinal culvert and lateral port arrangement finds its principal justification as a device for baffling the impulse effects of the inflowing discharge, which reach serious proportions in high-lift installations.

For lifts of moderate height (15 to 35 ft), European practice favors the selection of the so-called Venturi-loop filling system (Fig. 4) located entirely in the terminal masonry sections of the lock structure. In the opinion of the author, this type represents the best utilization of scientific principles and deserves more widespread adoption in America. It may be the means of effecting sizable economies in side-wall construction, particularly in the lower medium head range, where it permits the use of sheet piling for side walls, eliminating the necessity for a cofferdam during construction.

The loop principle also permits ready incorporation of the basic requirements for quiet lockage. By employing a trapezoidal cross section at the valves, we obtain the desired small initial valve opening, and by using flared conduits of large cross-sectional area we materially reduce the impulse forces resulting from the velocity of inflow into the chamber. A second advantage of the Venturi construction is that it operates to reduce the size and expense of lock valves.

For lifts in the low-head range, 15 ft or lower, the most advantageous method of admitting water is by cracking the main lock gates (Fig. 5). This is accomplished by imparting a very slow steady initial opening rate to the main gate drive, and after filling is accomplished the mechanism automatically increases the gate opening speed to normal. Some rudimentary form of baffling device, such as indicated, will be found advantageous in reducing disturbances to the vessel in lockage.

The literature abounds with many intermediate variations of the three foregoing fundamental types, but all the principles necessary for their execution will be found embodied in the material to be presented in subsequent paragraphs.

FILLING SYSTEM AS AN ORIFICE

In making comparisons of trial designs, it is of the greatest usefulness to have a simple rapid means of arriving at reasonably close proportions of the various competing types. In practice this is accomplished by means of assigning representative discharge coefficients to various classes of filling systems and then making the computation on the basis of the simple orifice law. The following coefficients are a fairly representative average:

System	Discharge Coefficient[1]
Longitudinal conduit and lateral ports	0.85–0.95
Short Venturi loops	0.75–0.85
Cracking main lock gates	0.80[2]
Simple rectangular orifices	0.60–0.80

[1] Rich, George R., Hydraulics of Lock-filling Systems, *Military Engs.*, January–February, 1933, pp. 60–64.

[2] Based upon the area of water between the main lock gates at equalization of levels.

The analytical background for applying these discharge coefficients is simply the old school exercise of filling a rectangular tank under a diminishing head. We shall employ the following notation:

t = filling (or emptying) time, sec.
A = horizontal area of lock chamber, sq ft.
F = cross-sectional area of filling conduits taken at the valves.
h = head difference between lock chamber and upper pool at any instant, ft.
h_1 = lift, ft.
g = acceleration of gravity, ft/sec^2.
C = discharge coefficient summarizing all conduit and valve losses.
T_1 = time required to open lock valves, sec.

For any short interval of time dt we may write

$$-A\,dh = CF\sqrt{2gh}\,dt \tag{14}$$

h being taken positive in the increasing direction.

In the first case let us suppose that the lock valves are opened instantaneously. Then, integrating Eq. (14), we obtain for the total time of filling:

$$t = \frac{-A}{CF\sqrt{2g}}\int_0^{h_1}\frac{dh}{\sqrt{h}} = \frac{2A\sqrt{h_1}}{CF\sqrt{2g}} \tag{15}$$

In the second case, assume that the valves are so operated as to reduce the valve area linearly with respect to the time, and that the valve is fully opened before equalization of levels is effected. We then have

$$-A\,dh = \frac{t}{T_1}CF\sqrt{2gh}\,dt \tag{16}$$

or

$$t\,dt = -\frac{AT_1}{CF\sqrt{2g}}\frac{dh}{\sqrt{h}} \tag{17}$$

If h_2 is the head difference remaining to be equalized after the valve is completely opened,

$$\int_0^{T_1} t\,dt = \frac{-AT_1}{CF\sqrt{2g}}\int_{h_2}^{h_1}\frac{dh}{\sqrt{h}} \tag{18}$$

or

$$T_1 = \frac{4A(\sqrt{h_1} - \sqrt{h_2})}{CF\sqrt{2g}} \tag{19}$$

By Eq. (15) the time required to fill the head difference h_1

$$T_2 = \frac{2A\sqrt{h_2}}{CF\sqrt{2g}} \tag{20}$$

The total filling time then becomes

$$T_1 + T_2 = T_1 + \frac{2A\sqrt{h_2}}{CF\sqrt{2g}} = \frac{T_1}{2} + \frac{2A(\sqrt{h_1} - \sqrt{h_2})}{CF\sqrt{2g}} + \frac{2A\sqrt{h_2}}{CF\sqrt{2g}}$$
$$= \frac{T_1}{2} + \frac{2A\sqrt{h_1}}{CF\sqrt{2g}} \tag{21}$$

Equation (21) states that the total filling time is equal to the sum of one-half the time necessary to open the lock valves plus the time that would be required for filling under instantaneous valve opening.

There still remains a third case of fairly frequent occurrence in Venturi-loop designs, namely, that in which equalization of levels is effected before the valves are completely opened. In this case let T_2 be the time at which the levels are equalized. We then write Eq. (18)

$$\int_0^{T_2} t\,dt = \frac{-AT_1}{CF\sqrt{2g}}\int_0^{h_1}\frac{dh}{\sqrt{h}} \tag{22}$$

$$\frac{T_2^2}{2} = \frac{AT_1}{CF\sqrt{2g}}\left[2\sqrt{h}\right]_0^{h_1}$$

$$T_2^2 = \frac{4AT_1}{CF\sqrt{2g}}\sqrt{h}$$

$$T_2 = 2\sqrt{\frac{AT_1\sqrt{h}}{CF\sqrt{2g}}} \tag{23}$$

These equations represent the limit of refinement usually found practical for the advance calculation of the longitudinal culvert and port system. Reliance for final performance is placed almost entirely on laboratory model tests. However, in the case of Venturi-loop systems, it is entirely practical to make a more refined calculation, taking full account of acceleration head and a more accurate evaluation of friction and related losses in the tubes. Accordingly, we shall devote the next article to the detailed analysis of the Venturi system. Then, in order to provide a more accurate understanding of what takes place in the longitudinal culvert and port type, we shall present the analytical approach and performance curves for a simplified system. Computation for a practical installation of 10 or more laterals would be hopelessly laborious, out of all proportion to the benefit received.

VENTURI-LOOP SYSTEM

Because relatively slow valve motion is prerequisite to quiet lockage, the analysis of the Venturi-loop lock-filling system may, like the surge-tank problem, be reduced to the consideration of friction and acceleration head effects, neglecting water hammer. It is then essentially a mass-acceleration exercise in which compression of the water and stretching of the conduit walls may be neglected without significant error. The basic principle is that at any instant of time the total instantaneous head difference between the upper pool and the lock chamber is expended in overcoming frictional and other allied losses that are finally dissipated in the form of heat, and in accelerating water in the conduit which is manifested as a change in velocity.

Losses of the frictional type may be calculated by conventional methods and include the following components: head loss at entrance, conduit friction loss in the flared and straight portions of the tube, head loss due to enlargement of sections in the flared section following the straight portion of tube, head loss at exit, and varying head loss due to obstruction at the partially open valve. All the foregoing losses may be conveniently summarized as a constant factor applied to the instantaneous velocity at the cross section of the straight portion of the conduit.

Acceleration head losses are of the familiar form readily obtained from the momentum equation:

$$F = M\frac{dV}{dt}$$

$$F\,dt = M\,dV$$

For a length of conduit dx:

$$wAh\,dt = \frac{wA\,dx}{g}\,dV$$

$$h = \frac{dx}{g}\frac{dV}{dt} \tag{24}$$

In the flared portion of the tube the velocity is expressed as a ratio of the velocity in the straight portion; for example (Fig. 6),

$$V_x = \frac{AV}{A_x} = \frac{AV}{A + \frac{(A_1 - A)x}{L_1}} = \frac{AL_1V}{AL_1 + (A_1 - A)x}$$

$$h_1 = \int_0^{L_1} \frac{AL_1\,dV\,dx}{[AL_1 + (A_1 - A)x]\,dtg} = \frac{AL_1\,dV}{g\,dt}\int_0^{L_1}\frac{dx}{AL_1 + (A_1 - A)x} \tag{25}$$

Now, integrating with respect to x, we obtain the instantaneous loss of head due to giving the water in the flared tube L_1 an acceleration increment corresponding to the increment dV in the straight portion of the tube:

$$h_1 = \frac{AL_1\,dV}{(A_1 - A)\,dtg} \log_e \frac{A_1}{A} \tag{26}$$

Similarly, for the flared portion L_2,

$$h_2 = \frac{AL_2\,dV}{(A_2 - A)\,dtg} \log_e \frac{A_2}{A} \tag{27}$$

And finally for the straight portion L,

$$h = \frac{L}{g}\frac{dV}{dt} \tag{28}$$

Adding Eqs. (26) to (28), we obtain an expression of the form

$$h_a = \phi \frac{dV}{dt} \tag{29}$$

and the total head

$$h = \phi \frac{dV}{dt} + \beta V^2 \tag{30}$$

Also, by hydraulic continuity,

$$A_c\,dh = AV\,dt \tag{31}$$

where A_c is the area of lock chamber.

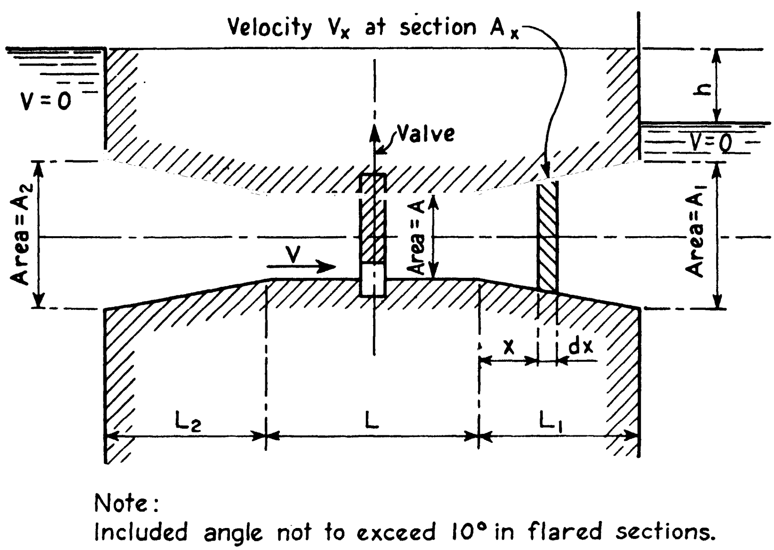

FIG. 6.—Analysis of Venturi-loop filling system.

We seek a simultaneous solution of Eqs. (30) and (31) to give the time t. Since the resulting differential equation

$$\frac{d^2h}{dt^2} + \frac{\beta A_c}{\phi A}\left(\frac{dh}{dt}\right)^2 - \frac{hA}{\phi A_c} = 0$$

is not solvable by any means known at present, we resort to the familiar device of arithmetic integration. In Eq. (31) for a small assumed value of Δh for a given time interval Δt, we compute the average value of velocity during the interval V. We next

TABLE 1.—ARITHMETIC INTEGRATION–VENTURI-LOOP CONDUITS—KENTUCKY PROJECT (SYSTEM E)

Headwater elevation = 370 ft
Tailwater elevation = 305 ft
Valve opening rate = 2.4 fpm
V = velocity in conduit near valve
Area of lock chamber = 110 × 675 = 74,250 sq ft

Area of conduit at valve = 202 sq ft
V_a = average velocity in conduit near valve
Length of straight portion of tube L = 19.83 ft
Length of flared portion of tube L_1 = 93.33 ft
Length of flared portion of tube L_2 = 101.08 ft

Column (4): $202 V_a \, \Delta t = \dfrac{74{,}250 \Delta h}{2}$

$$\Delta h = \frac{404 \times 50 \times V_a}{74{,}250}$$

$$\Delta h = \frac{V_a}{3.67}$$

Column (10): $\Delta h = \dfrac{5.45 \Delta V}{\Delta t}$

$$\Delta V = \frac{50 \Delta h}{5.45}$$

$$\Delta V = 9.18 \Delta h$$

(1) Time, sec	(2) Velocity V, fps	(3) Average velocity V_a, fps	(4) $\Delta h = V_a/3.67$, ft	(5) *W.s.* elev in chamber, ft	(6) Head on conduit, ft	(7) Valve and friction loss, ft	(8) Acceleration head, ft	(9) Avg acceleration head, ft	(10) $\Delta V = 9.18\Delta h$, fps	(11) V, fps
0	0	0	0	305.00	65.00	0	0	0	0	0
t = 50 50	10.03	5.015	1.36	306.36	63.64	61.45	2.19	1.095	10.03	10.03
t = 50 100	15.53	12.78	3.48	309.84	60.16	61.13	−0.97	0.61	5.58	15.61
t = 50 150	18.38	16.96	4.62	314.46	55.54	53.97	1.57	0.30	2.77	18.38
t = 50 200	20.00	19.19	5.22	319.68	50.32	51.536	−1.216	0.177	1.62	20.00
t = 50 250	21.75	20.88	5.70	325.38	44.62	43.04	1.58	0.185	1.70	21.70
t = 50 300	25.08	23.42	6.39	331.77	38.23	39.07	−0.844	0.368	3.38	25.08
t = 50 350	26.45	25.77	7.02	338.79	31.21	30.08	1.13	0.145	1.34	26.42
t = 50 400	28.10	27.28	7.45	346.24	23.76	24.52	−0.764	0.183	1.68	28.10
t = 50 450	25.50	26.80	7.30	353.54	16.46	16.25	0.210	−0.277	−2.54	25.56
t = 50 500	21.20	23.35	6.37	359.91	10.09	11.26	−1.17	−0.48	−4.38	21.18
t = 50 550	15.35	18.28	4.99	364.90	5.10	5.90	−0.80	−0.64	−5.87	15.31
t = 50 600	9.50	12.43	3.39	368.29	1.71	2.26	−0.55	−0.63	−5.79	9.52
t = 50 650	4.22	6.86	1.87	370.16	−0.16	−0.45	−0.61	−0.58	−5.32	4.20
t = 50 700	−1.35	1.43	0.39	370.55	−0.55	−0.046	−0.60	−0.605	−5.55	−1.35
t = 50 750	−2.95	−2.15	−0.59	369.96	0.04	0.22	0.26	−0.17	−1.56	−2.91
t = 50 800	0.20	−1.38	−0.38	369.58	0.42	0.00	0.42	0.34	3.11	0.20
t = 50 850	2.10	1.15	0.31	369.89	0.11	0.11	0	0.21	1.93	2.13
t = 50 900	0.80	1.45	0.40	370.29	−0.29	0.01	−0.30	−0.15	−1.38	0.75

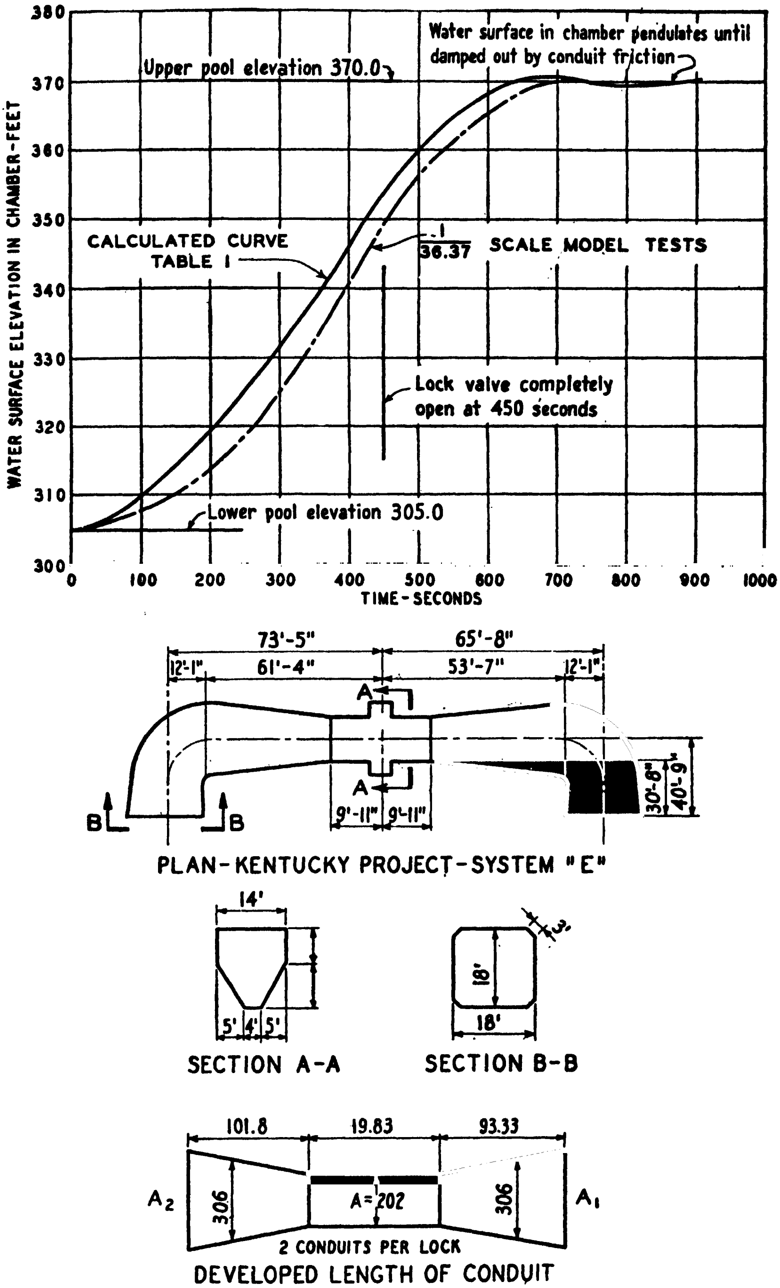

FIG. 7.—Lock-filling curve for Venturi-loop system by arithmetic integration and by model test (see Table 1).

calculate ΔV from the expression $\frac{V_1 + \Delta V}{2} = V$, V_1 being the velocity at the end of the preceding interval.

Equation (31) is then applied as a test at the end of the interval: h being equal to $h_1 + \Delta h$; V to $V_1 + \Delta V$, where the subscript 1 denotes values at the end of the preceding interval. Table 1 is a typical example of such calculation.

This tabulation affords a ready answer to a question frequently raised by hydraulic engineers in connection with all lock-filling systems, namely, why filling always continues some distance beyond upper pool level. Referring to the line of integration at time $t = 650$ sec, it will be noted that, when the water surface elevation in the chamber reaches 370.16 ft, the velocity in the filling conduits is still quite appreciable, 4.22 fps. Accordingly, owing to the inertia of water in the conduits, the chamber level continues to rise until this velocity is decelerated. Because of the superelevation of water in the chamber, the flow then reverses in direction until the water surface elevation in the chamber drops sufficiently to decelerate this reversed flow and initiate flow in the normal direction from the pool to the chamber. As shown by the computation, this pendulation of chamber levels continues until damped down by conduit friction.

LONGITUDINAL CONDUIT AND LATERAL PORT SYSTEM

Because of the exceedingly great labor involved, and in view of the fact that laboratory model tests must be conducted in any event to determine hawser stresses, it would be impractical to undertake the analytical determination of design for a longi-

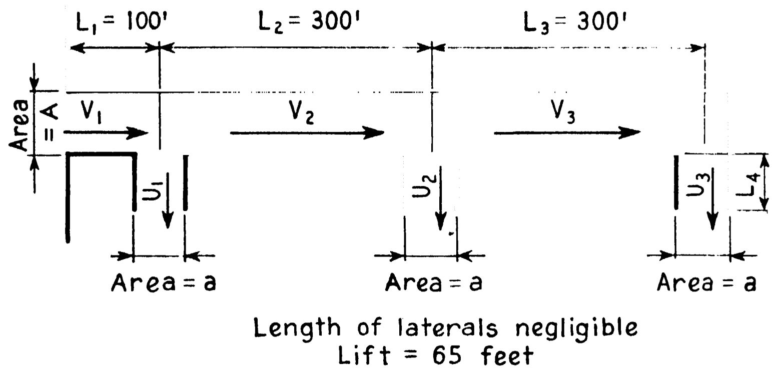

FIG. 8.—Elementary filling system. Longitudinal conduit with short laterals.

tudinal conduit system with the usual great number of laterals. The chief value of the analytical approach in this case is that it affords a rational explanation why uniformly spaced ports do not give uniform distribution of water inflow along the longitudinal axis of the chamber, and why the laterals adjacent to the lower lock gates frequently come into action more quickly than laterals adjacent to the valves or upper lock gates. Analytical methods enable us to understand why differential port spacing with the closer spacing at the upper gates is the optimum arrangement for quiet lockage.

For the above purpose it will be entirely sufficient to employ the vastly simplified manifold system indicated in Fig. 8, consisting of a longitudinal main conduit with but three laterals. Our plan of attack is (1) to develop pertinent equations, neglecting the effect of contractions which cause head loss at the entrance to the laterals, (2) to incorporate the effect of such contractions in a second system of equations, (3) to

calculate a numerical example by each of these two methods and thus throw into bold relief the profound effect of what might, at first inspection, appear to be a mere trivial refinement.

Just as in the case of the Venturi-loop system, we are again dealing essentially with a mass-acceleration problem in which the head difference is expended in overcoming friction and allied losses and in accelerating the conduit velocity. The valve opening rate is necessarily slow to ensure quiet lockage, so that compression of the water and stretching of the conduit wall may, without significant error, be neglected, *i.e.*, we ignore water-hammer effect.

As in the case of the Venturi-loop system, we shall denote by βV^2 and θU^2 all losses of head manifested eventually as lost energy in the form of heat and by ϕV the summation of all acceleration head losses. We shall employ the symbol V for the main conduit, U for the laterals, and appropriate subscripts will identify the particular branch under consideration.

In the analysis we assume that the water surface in the chamber is a horizontal plane at all times, so that the head difference from upper pool to chamber is the same by paths through each lateral successively. We proceed to write equations for head difference and relationships dictated by hydraulic continuity.

$$h = \beta_1 V_1^2 + \theta_1 U_1^2 + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_4}{g}\frac{dU_1}{dt} \tag{32}$$

$$h = \beta_1 V_1^2 + \beta_2 V_2^2 + \theta_2 U_2^2 + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_2}{g}\frac{dV_2}{dt} + \frac{L_4}{g}\frac{dU_2}{dt} \tag{33}$$

$$h = \beta_1 V_1^2 + \beta_2 V_2^2 + \beta_3 V_3^2 + \theta_3 U_3^2 + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_2}{g}\frac{dV_2}{dt} + \frac{L_3}{g}\frac{dV_3}{dt} + \frac{L_4}{g}\frac{dU_3}{dt} \tag{34}$$

$$AV_1 - AV_2 = aU_1 \tag{35}$$

$$AV_2 - AV_3 = aU_2 \tag{36}$$

$$AV_3 = aU_3 \tag{37}$$

$$\frac{A_c}{2}\,dh = AV_1\,dt \tag{38}$$

If we proceed to solve these seven equations for the seven unknown variables in the conventional manner, we once again encounter the familiar differential equation of type

$$\frac{d^2h}{dt^2} + C_1\left(\frac{dh}{dt}\right)^2 + C_2 h = 0$$

for which no solution is yet known. Accordingly, we employ the device of arithmetic integration with the arrangement given in Table 2. The method of attack is to work with Eqs. (32), (35), and (38) until V_2 in Eq. (35) reaches a significant magnitude. We then add to the calculation Eqs. (33) and (36) and continue until V_3 reaches a significant magnitude in Eq. (36). From then on, we employ all seven equations.

The results of the example of Table 2 are given in Fig. 9, from which it will be noted that the laterals do not all come into action at once, but that there is quite an appreciable time lag between the initial functioning of the successive ports and that they come into play in the order of their distance from the upper lock gates.

In the analysis thus far, we have assumed that the flow in all portions of the system is axial, *i.e.*, we have made no allowance for the effect of contractions at the entrance to the laterals. We shall now incorporate this feature in Eqs. (32) to (38).

In this connection there are two possibilities depending upon whether the lateral is long or short. In either case the cross-sectional area of the discharge will be the same at the plane of intersection of the two tubes and will be equal to $a \cos \alpha$.

In the first case, that of the short lateral, the V component will be expended on

TABLE 2.—ARITHMETIC INTEGRATION–LONGITUDINAL CONDUIT WITH LATERALS

Headwater elevation = 370 ft — Friction loss L_1 (including entrance loss) = $0.010V_1^2$
Tailwater elevation = 305 ft — Friction loss L_2 = $0.005V_2^2$
Valve opening time = 450 sec — Friction loss L_3 = $0.005V_3^2$
Lock chamber area = 74,250 sq ft

Discharge at U_1, U_2, and U_3 assumed axial, with orifice discharge coefficients all equal to 0.80.

	Time, sec		
	30	60	90
Time, sec	30	60	90
Velocity V_1, fps	13.613	18.32	19.78
Average velocity V_{1a}, fps	6.807	15.97	19.05
$\frac{400V_{1a}\,\Delta t}{74{,}250} = \Delta h$, ft	1.098	2.58	3.08
W.S. elevation in chamber, ft	306.098	308.68	311.76
Friction loss in L_1 includes valve obstruction head loss, ft	56.72	59.80	52.50
Average acceleration head $= \frac{L_1}{g}\frac{\Delta V_1}{\Delta t}$, ft	1.412	0.488	0.151
Acceleration head for L_1, ft	2.82	−1.84	2.14
Head on port U_1, ft	4.36	3.36	3.60
Velocity through U_1, fps	16.73	14.70	15.22
Velocity $V_2 = \frac{AV_1 - aU_1}{A}$, fps	5.248	10.97	12.17
Friction loss L_2, ft	0.14	0.60	0.74
Average acceleration head $= \frac{L_2}{g}\frac{\Delta V_2}{\Delta t}$, ft	1.627	1.776	0.37
Acceleration head for L_2, ft	3.25	0.30	0.44
Head on port U_2, ft	0.97	2.46	2.42
Velocity through U_2, fps	7.89	12.56	12.48
Velocity $V_3 = \frac{AV_2 - aU_2}{A}$, fps	1.303	4.69	5.93
Friction loss L_3, ft	0.01	0.11	0.18
Average acceleration head $= \frac{L_3}{g}\frac{\Delta V_3}{\Delta t}$, ft	0.41	1.052	0.38
Acceleration head for L_3, ft	0.82	1.28	−0.52
Head on port U_3, ft	0.14	1.07	2.76
Velocity through U_3, fps	3.00	8.30	13.30
Volume $U_1 + U_2 + U_3$ in Δt, cu ft	41,400	94,800	114,800
Volume to chamber in $\Delta t = \frac{74{,}250}{2}\,\Delta h$, cu ft	40,900	95,600	114,200

producing pressure upon the side wall of the lateral, and the discharge into the chamber will be at efflux velocity U_1 and cross-sectional area $a \cos \alpha = \frac{aU_1}{\sqrt{U_1^2 + V_1^2}}$. In other words, in Eq. (35) we replace a by $\frac{aU_1}{\sqrt{U_1^2 + V_1^2}}$; in Eq. (36) we replace a by $\frac{aU_2}{\sqrt{U_1^2 + V_2^2}}$; and in Eq. (37) we replace a by $\frac{aU_3}{\sqrt{U_3^2 + V_3^2}}$. In performing the arithmetic integration, we employ values of U_n and V_n from the previous time interval as a first trial and then refine the procedure to effect the requisite balance. For the very first interval, the first trial will naturally be made with the values a and A. In the second case, that of long laterals, the discharge will, after passing the contraction, expand and fill the tube so that the head loss is properly computed for the standard case of a

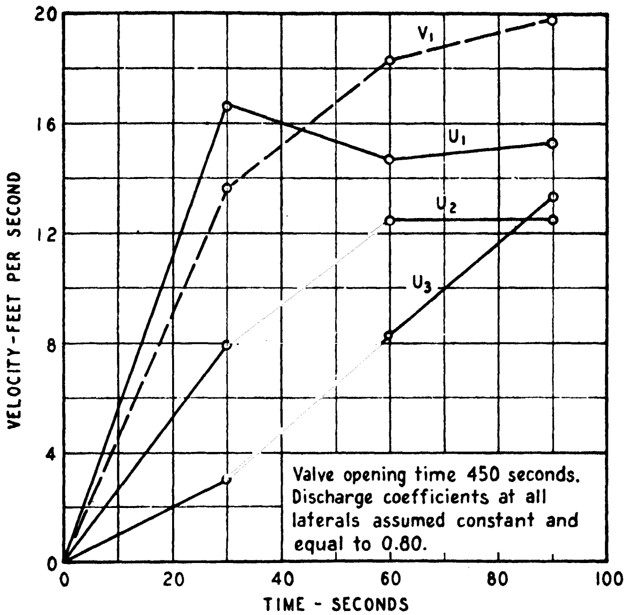

FIG. 9.—Showing contraction at laterals.

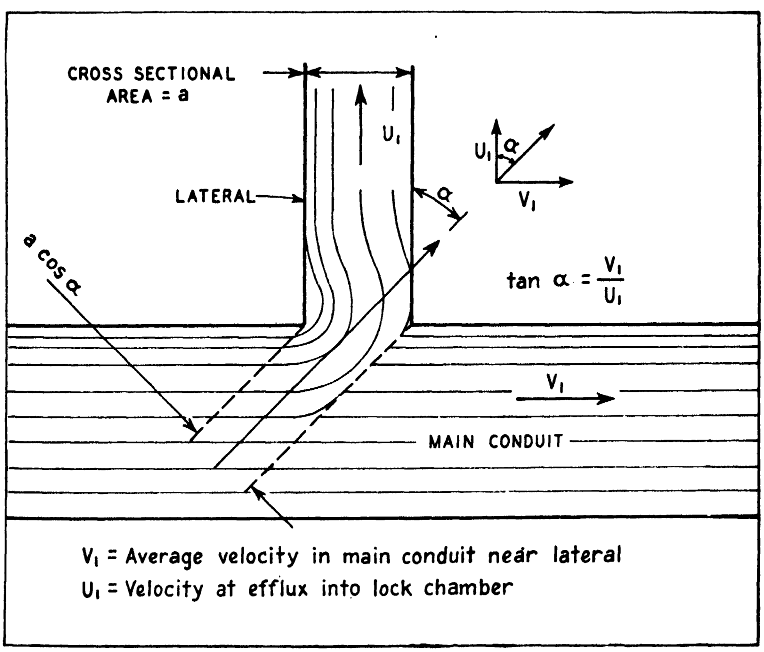

FIG. 10.—Performance chart. Elementary filling system. Longitudinal conduit for three short laterals.

sudden increase in cross section of conduit from $a \cos \alpha$ to a. If the velocity is U_1 at efflux into the chamber, it will be $\frac{U_1}{\cos \alpha}$ just after making the right-angle turn, and the component V_1 will, as before, be expended in producing pressure upon the side wall of the lateral. The head loss in the lateral chargeable to effecting the turn will be

$$\frac{\left(\frac{U_1}{\cos \alpha} - U_1\right)^2}{2g} = \frac{U_1^2}{2g}\left(\frac{1}{\cos \alpha} - 1\right)^2$$

in which, as before, $\cos \alpha = \frac{U_1}{\sqrt{U_1^2 + V_1^2}}$. The head loss is, accordingly,

$$\frac{U_1^2}{2g}\left(\frac{\sqrt{U_1^2 + V_1^2}}{U_1} - 1\right)^2 \quad \text{for the first lateral}$$

$$\frac{U_2^2}{2g}\left(\frac{\sqrt{U_2^2 + V_2^2}}{U_2} - 1\right)^2 \quad \text{for the second lateral}$$

$$\frac{U_3^2}{2g}\left(\frac{\sqrt{U_3^2 + V_3^2}}{U_3} - 1\right)^2 \quad \text{for the third lateral}$$

These values are incorporated in the coefficients θ_1, θ_2, and θ_3 in Eqs. (32) to (34). In making the first trials of the arithmetic integration, values of U_n and V_n from the preceding interval may be employed and then refined for the second and succeeding trials.

It will be evident from this derivation that the contraction losses for the first ports may be so large relatively that the ports farthest removed from the lock valves come into action first. This has frequently been observed in actual prototype operation. Table 3 gives a sample of the arithmetic integration process for the case of short laterals. The corresponding operations for long laterals follow directly from the derivation. The chief value of the foregoing analysis is to demonstrate that uniform port spacing for the system of main longitudinal conduits with lateral branches cannot be expected to give uniform discharge into the lock chamber.

From a study of Tables 2 and 3 it will be apparent that the degree of contraction at the entrance to each lateral port depends upon the relative magnitude of conduit and port velocities at the successive laterals. Although there is quite a marked difference in the valve opening rates for the two cases shown in this table, this does not result in any relative difference in the distribution of flow through the various laterals. However, let us consider for a moment the common case of a long main conduit with 10 or 15 laterals. When many laterals are used in this fashion, the cross-sectional area of each will be small in proportion to the main conduit area. Consequently, the major portion of the discharge will still be undelivered to the chamber after passing the first and second laterals. In addition, this velocity will cause a proportionately great drop in pressure head at the first two orifice outlets so that the head on these orifices will be small and the efflux velocity through the first two ports will be very small. Our formula for contraction coefficient for this case of a large conduit velocity and small port velocity will then give a very large contraction at the first few ports, so that the major initial flow will be delivered by the downstream ports. It has been observed in some model tests for high lifts and during early stages of filling that the flow in the first two laterals would actually reverse and be in the direction from the chamber to the main conduit. To carry out this demonstration by arithmetic integration for 15 ports would obviously be a task of prohibitive proportions; but that the mechanism of filling under such conditions would proceed substantially as just indicated above will be evident from the behavior of the integration process in Table 3.

TABLE 3.—ARITHMETIC INTEGRATION–LONGITUDINAL CONDUIT WITH LATERAL PORTS, VARIATION IN CONTRACTION AT PORTS CONSIDERED

Headwater elevation = 370 ft — Friction loss L_1 (including entrance loss) = $0.010V_1^2$
Tailwater elevation = 305 ft — Friction loss L_2 = $0.005V_2^2$
Lock chamber area = 74,250 sq ft — Friction loss L_3 = $0.005V_3^2$
Effect of difference in contraction at port entrance included in computation.

	Valve opening time, 450 sec	Valve opening time, 90 sec
Time, sec	30	30
Velocity V_1, fps	13.604	23.15
Average velocity V_{1a}, fps	6.81	11.58
$\frac{400V_{1a}\,\Delta t}{74{,}250} = \Delta h$, ft	1.10	1.87
W. S. elevation in chamber, ft	306.10	306.87
Friction loss in L_1 includes valve obstruction head loss, ft	56.64	47.75
Average acceleration head $= \frac{L_1}{g}\frac{\Delta V_1}{\Delta t}$, ft	1.408	2.398
Acceleration head for L_1, ft	2.82	4.79
Head on port U_1, ft	4.44	10.59
Velocity through U_1, fps	16.92	26.10
Discharge coefficient $\frac{U_1}{\sqrt{U_1^2 + V_1^2}}$	0.777	0.748
Velocity $V_2 = \frac{AV_1 - aU_1}{A}$, fps	5.38	10.93
Friction loss L_2, ft	0.15	0.60
Average acceleration head $= \frac{L_2}{g}\frac{\Delta V_2}{\Delta t}$, ft	1.670	3.40
Acceleration head for L_2, ft	3.34	6.80
Head on port U_2, ft	0.95	3.19
Velocity through U_2, fps	7.82	14.32
Discharge coefficient $\frac{U_2}{\sqrt{U_2^2 + V_2^2}}$	0.823	0.795
Velocity $V_3 = \frac{AV_2 - aU_2}{A}$, fps	1.36	3.82
Friction loss L_3, ft	0.01	0.07
Average acceleration head $= \frac{L_3}{g}\frac{\Delta V_3}{\Delta t}$, ft	0.42	1.185
Acceleration head for L_3, ft	0.84	2.37
Head on port U_3, ft	0.10	0.75
Velocity through U_3, fps	2.54	6.95
Discharge coefficient $\frac{U_3}{\sqrt{U_3^2 + V_3^2}}$	0.880	0.878
Volume $U_1 + U_2 + U_3$ in Δt, cu ft	40,900	69,500
Volume to chamber in $\Delta t = \frac{74{,}250}{2}\,\Delta h$, cu ft	40,900	69,500

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SECTION 17

IRRIGATION

BY IVAN E. HOUK

Irrigation is one of the oldest arts of civilization. The controlled application of water to land for agricultural purposes began in the Old World long before the time of Christ. Nile River water has been used in irrigating lands in northern Egypt for 5,000 years. Irrigation in southwestern United States, although crudely practiced by the Indians in prehistoric times, really began with the invasion of the Rio Grande Valley by the Spaniards about the end of the sixteenth century. Modern irrigation developments began when the Mormons settled Utah in 1847. Since that time, many irrigation communities, including thriving municipalities as well as prosperous farm lands, have gradually developed in all the arid and semiarid states west of the central Mississippi Valley. The 1930 census showed that irrigated farms produced approximately 29 per cent of the three billion dollars worth of crops grown in the 17 Western states in 1929.[1]

Large rice areas are irrigated in southeastern Texas, Louisiana, and Arkansas, and some truck gardens in eastern United States; but the total areas included in the latter are small in comparison with the extensive areas irrigated in the West.

In determining the feasibility of a contemplated irrigation project, three fundamental questions must be carefully considered: (1) Are the project lands suited to agricultural use from topographic and soil-structure viewpoints? (2) Are available water supplies sufficient to meet the irrigation demands? (3) Will the cost of the necessary engineering works per acre of irrigable land be low enough to justify construction?

These basic questions are treated briefly in the following discussions. However, the greater part of this section is devoted to the physical aspects of irrigation work, such as water supply, water requirements, water losses, consumptive use, and irrigation methods. The principles governing the design and construction of irrigation works required in a contemplated development are discussed in the sections on dams, spillways, canals, flumes, pipe lines, and tunnels, as well as in the succeeding section on irrigation structures.

LAND CLASSIFICATION

Land classification, a matter that unfortunately did not receive adequate attention in some of the earlier irrigation developments, is now being carefully considered in planning new projects.[2] Such classifications must be made by engineers working in cooperation with agricultural and soil experts. Some lands, which may be easily and economically watered, may not merit inclusion in the irrigable acreage because of unproductive soil composition, undesirable soil texture, difficult drainage possibilities, presence of undesirable soluble salts, or danger of developing alkali surfaces under continued irrigation. Other lands, which may cost more to irrigate, may

[1] Irrigation of Agricultural Lands, Fifteenth Census of the United States, 1930, Bureau of the Census, U.S. Dept. Comm., 1932.

[2] Johnston, W. W., Classification of Irrigable Lands, *Proc. A.S.C.E.*, May, 1942, pp. 667–682.

deserve inclusion because of ideal soil characteristics, easy drainage, and high prospects of developing into permanent and prosperous productive areas.

Alkali. In irrigation work, the term *alkali* is used to mean any soluble salts that may be brought to the surface by capillary soil moisture movements and precipitated as the moisture evaporates. The most common alkali compounds are the sodium sulphates, chlorides, and carbonates. Other alkali compounds are the potassium and magnesium carbonates, chlorides, and sulphates, and the calcium chlorides and nitrates. Sodium carbonates, which cause the decomposition of organic matter and the formation of a dark-colored crust at the surface of the ground, are called *black alkali*. Sodium carbonate (sal soda), sodium bicarbonate (baking soda), and sodium chloride (table salt) are especially detrimental to plant growth, the first two being the more objectionable and more difficult to remove from the soil. Noncaustic compounds that do not deflocculate the soil, such as sodium chloride and sulphate, are often called *white alkali*.

The proportions of alkali salts that the soil may contain and still be profitably irrigated vary with the character of the soil, fertility, drainage, methods of irrigating, and crops grown, as well as with the kinds of salts present. The danger of nonalkali lands developing alkali surfaces under continued irrigation depends on the permeability of the soil, quantity of water applied, drainage facilities, and saline content of the irrigation water. Engineers classifying alkali lands, or areas that may develop alkali surfaces, should consult agricultural experiment station bulletins and government reports on the subject also books on soils and soil alkali.[1]

Most plants can better resist the detrimental effects of alkali if grown on soils of high fertility. Consequently, the liberal use of green or barnyard manures may assist in overcoming alkali troubles when the salts are not too concentrated. With a few exceptions, grasses, small grains, alfalfa, and root crops are more resistant to alkali than corn, beans, peas, melons, and fruits.

The presence of noticeable quantities of alkali on the ground surface or considerable proportions of salts in the soil may not be serious if the ground is permeable, can be economically drained, and sufficient water can be applied to maintain percolation toward drainage outlets. However, in such cases care must be exercised in rediverting the drainage flow farther down the valley, inasmuch as the water may contain more salts than are desirable for further irrigation use. During 1933, the total dissolved salts in the flow of the Rio Grande between Leasburg, N. M., and Fort Quitman, Tex., was increased from 0.79 to 2.72 tons/acre-ft, owing to the concentrated salt content of the return flow from irrigated lands in the Rio Grande Valley.[2]

If the lands are properly drained, objectionable quantities of alkali can sometimes be leached out of the soil by flooding the surface during the winter months, when the available water supplies are not needed for irrigating nonalkali areas. In the case of impermeable soils, it may be desirable to increase the permeability by adding calcium sulphate (gypsum) or aluminum sulphate, the former being the cheaper and more generally obtainable.[3] Further discussions of alkali problems are included under a subsequent heading on Quality of Water.

Selenium Soils. Selenium soils may need to be excluded from the irrigable acreage since selenium, unless leached from the soil, may be transferred to the vegetation in proportions great enough to injure animal life. Such soils, which derive their selenium content from sulphide minerals, occur in several western states, including Nebraska, Wyoming, and South Dakota. Selenium areas thus far investigated have

[1] HILGARD, E. W., "Soils," The Macmillan Company, 1918; and HARRIS, F. S., "Soil Alkali," John Wiley & Sons, Inc., 1920.

[2] SCOFIELD, CARL S., The Salinity of Irrigation Water, Smithsonian Report for 1935, pp. 275–287.

[3] SCOFIELD, CARL S., The Movement of Water in Irrigated Soils, *Jour. Agr. Research*, Mar. 1, 1924, pp. 617–693.

usually been found where the soils were formed from cretaceous shales. The harmful limits of the selenium content have not been definitely determined. According to Horace G. Byers,

"The amount of selenium absorbed from seleniferous soils by plants is apparently dependent on (1) the quantity of selenium and its distribution in the soil profile; (2) the kind of plant; (3) the portion of the plant examined; and (4) the soil composition, especially its available sulphur content. It is also believed that variation will also result from seasonal variation in rainfall."[1]

Air Drainage. Air drainage should be studied in classifying lands where appreciable areas may be adapted to fruit or winter vegetable culture. Experience has shown that crops of such nature, grown on low-lying lands where wind movements are obstructed by surrounding hills and consequently are not adequate to provide satisfactory air drainage, may suffer severe damage from frost at times when similar crops, grown on near-by sloping topography where wind movements provide ample air drainage, may suffer no damage at all. For instance, orchards on the sloping hillsides of the Roza and Tieton divisions, Yakima project, Washington, frequently are undamaged by frost when similar orchards on low-lying lands near the town of Yakima are severely damaged. Many comparable examples occur in the citrus-fruit sections of Arizona and California, as well as in other sections of the West. Consequently, it may sometimes be desirable to include sloping hillside regions in the irrigable areas, even though the preparation of land and conveyance of water may cost considerably more per acre than in the low-lying, more level divisions of the projects.

WATER SUPPLY

The basic source of water supply for plant growth is precipitation, whether it falls naturally on the farm lands during the irrigation season or on other portions of the earth's surface during earlier periods and is later brought artificially to the fields by pumping or gravity methods. The direct source of irrigation supply may be diversion from unregulated stream flow, release from storage reservoirs, rediversion of return flow, appropriation of spring or artesian-well discharges, or pumping from streams, lakes, or underground storage.

The relatively small quantities needed during the early years of irrigation history were mostly diverted from the unregulated flow of creeks and rivers. During later years, the increased demands for irrigation water caused by the construction of new projects and the changing character of crops grown exhausted natural flow supplies in many western streams. Consequently, it became necessary to build storage reservoirs, install pumping plants, harness flowing wells, and resort to other methods of obtaining needed supplies. According to the 1930 census, about 66 per cent of the total areas irrigated in the United States in 1929 were supplied by gravity diversions from stream flow; about 20 per cent by pumping from streams, lakes, and wells; and about 14 per cent by gravity, pumping, or combined gravity and pumping from miscellaneous sources.

Surface Supplies. Surface-water supplies in western streams are dependent, mostly, on winter precipitation in mountainous regions. Although flood flows, caused by heavy rainfalls during other seasons of the year, are sometimes stored and later released for irrigation purposes, the principal source of surface supplies is the spring and summer melting of winter snow accumulations in the higher mountain ranges. Part of the runoff from the melted snow runs along the ground, directly into the drainage channels. The remainder soaks into the soil where it percolates into the

[1] BYERS, HORACE G., Selenium Occurrence in Certain Soils in the United States with a Discussion on Related Topics, *U.S. Dept. Agr., Tech. Bull.* 482, 1935; also second, third, and fourth reports, *U.S. Dept. Agr., Tech. Bulls.* 530, 601, and 702, 1936, 1938, and 1940.

TABLE 1.—ANNUAL DEPTH OF SNOWFALL AT SOME MOUNTAIN STATIONS IN WESTERN UNITED STATES[1]

State	Station	Elev.	Length of record, years	Annual snowfall, in.	Mountains
Arizona	Alpine	8,500	17	70.6	White
	Bright Angel	8,400	6	119.3	Kaibab
California	Bear Valley Dam	6,700	9	117.4	San Bernardino
	Tamarack	8,000	21	451.4	Sierra Nevada
Colorado	Savage Basin	11,522	15	400.2	San Miguel
	Silver Lake	10,200	18	281.3	Front Range, Rocky
Idaho	Roland	4,150	10	230.1	Bitter Root
Montana	Babb, (near)	4,461	20	88.0	Glacier National Park
Nevada	Eureka	6,500	31	80.9	Diamond
New Mexico	Anchor Mine	10,600	8	317.2	Cimarron
	Nogal	8,000	12	69.8	Capitan
Oregon	Crater Lake	6,475	8	377.7	Cascades
	Greenhorn	6,250	5	288.1	Blue
Utah	Silver Lake	8,700	10	346.0	Wasatch
Washington	Mount Baker Lodge	4,200	4	477.5	Mount Baker
	Paradise Inn	5,550	11	591.3	Mount Rainier
Wyoming	Dome Lake	8,821	20	218.6	Big Horn
	Thumb	7,772	7	215.4	Yellowstone National Park

[1] Compiled from Climatic Summary of the United States, to 1930, U.S. Weather Bureau.

TABLE 2.—AVERAGE RAINFALL DURING IRRIGATION SEASON AND MEAN ANNUAL PRECIPITATION AT SOME WEATHER STATIONS IN WESTERN UNITED STATES[1]

State	Station	Years of record	Elev.	Irrigation season, months	Avg rain during irr. seas., in.	Mean annual prec., in.
Arizona	Yuma W.B.	53	141	Jan.–Dec.	3.47	3.47
	Phoenix	38	1,108	Jan.–Dec.	7.78	7.78
California	San Bernardino	65	1,172	Jan.–Dec.	16.12	16.12
	Fresno	54	287	Feb.–Oct.	5.28	9.39
	Orland	53	254	Mar.–Oct.	5.89	17.33
Colorado	Grand Junction	44	4,602	Apr.–Oct.	5.69	8.83
	Rocky Ford	47	4,177	Apr.–Oct.	10.51	12.45
Idaho	Boise	72	2,739	Apr.–Oct.	5.73	13.10
Montana	Augusta	38	4,071	May–Sept.	9.65	14.88
	Glendive	47	2,090	May–Sept.	9.81	14.62
Nevada	Fallon	39	3,965	Mar.–Oct.	2.68	4.66
New Mexico	Albuquerque	58	5,105	Mar.–Oct.	6.42	8.06
Oregon	Klamath Falls	44	4,100	Apr.–Sept.	3.54	12.50
South Dakota	Orman	30	2,933	May–Sept.	10.29	15.31
Texas	El Paso	57	3,778	Feb.–Nov.	8.18	9.16
Utah	Ogden	65	4,458	May–Oct.	6.14	15.98
Washington	Omak	27	850	May–Sept.	3.51	10.36
	Yakima	27	1,071	Apr.–Oct.	3.30	8.15
Wyoming	Fort Laramie	57	4,715	May–Sept.	8.07	13.04
	Powell	29	4,389	Apr.–Oct.	4.96	5.87

[1] Compiled, principally, from Annual Summaries of Climatological Data for 1935, U.S. Weather Bureau.

underground reservoir or feeds underground channels which later discharge as springs along the surface watercourses. Such springs help maintain the low water stream flow during the late summer and fall months. During recent years the low water discharges of some western streams have been appreciably increased by return flow from irrigated areas near the stream channels.

Tables 1, 2, and 3 give some precipitation data for western United States, compiled from the records of the U.S. Weather Bureau. Table 1 gives the annual depth of snowfall at some of the higher mountain stations where unusually heavy snowfalls are experienced; Table 2 gives the annual precipitation and total rainfall during the irrigation season at some stations located on or near large irrigated areas; and Table 3 gives the mean monthly precipitation at the stations listed in Table 2.

TABLE 3.—MEAN MONTHLY PRECIPITATION AT STATIONS LISTED IN TABLE 2[1]

Station	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Yuma, W.B.	0.45	0.41	0.34	0.10	0.04	0.02	0.18	0.50	0.35	0.26	0.29	0.53
Phoenix	0.80	0.77	0.68	0.40	0.12	0.07	1.07	0.95	0.75	0.47	0.80	1.00
San Bernardino	3.30	3.07	2.71	1.39	0.66	0.09	0.03	0.15	0.21	0.70	1.28	2.53
Fresno	1.73	1.43	1.58	0.95	0.44	0.08	0.01	0.01	0.21	0.57	0.93	1.45
Orland	3.43	2.92	2.29	1.12	0.76	0.33	0.02	0.02	0.45	0.90	1.98	3.11
Grand Junction	0.60	0.58	0.76	0.83	0.81	0.40	0.61	1.17	0.92	0.95	0.57	0.63
Rocky Ford	0.22	0.29	0.56	1.49	1.98	1.52	2.46	1.43	0.74	0.89	0.46	0.41
Boise	1.73	1.44	1.35	1.18	1.43	0.92	0.24	0.19	0.53	1.24	1.28	1.57
Augusta	0.67	0.44	0.91	1.07	2.35	2.81	1.62	1.29	1.58	0.86	0.60	0.68
Glendive	0.52	0.44	0.86	1.10	2.20	3.34	1.74	1.27	1.26	0.84	0.50	0.55
Fallon	0.60	0.50	0.46	0.41	0.56	0.27	0.13	0.20	0.26	0.39	0.31	0.57
Albuquerque	0.40	0.32	0.45	0.60	0.60	0.49	1.41	1.24	0.82	0.81	0.46	0.46
Klamath Falls	1.94	1.36	1.15	0.90	0.92	0.67	0.29	0.22	0.54	1.02	1.74	1.75
Orman	0.51	0.30	0.72	1.47	2.70	2.53	2.22	1.42	1.42	1.09	0.44	0.49
El Paso	0.46	0.41	0.36	0.26	0.33	0.58	1.99	1.70	1.25	0.80	0.50	0.52
Ogden	1.67	1.68	1.85	1.66	1.72	0.80	0.42	0.75	0.99	1.46	1.29	1.69
Omak	1.50	0.83	0.53	0.60	0.83	1.00	0.62	0.48	0.58	0.69	1.44	1.26
Yakima	1.29	0.99	0.38	0.47	0.63	0.59	0.32	0.17	0.51	0.61	0.85	1.34
Fort Laramie	0.44	0.45	0.68	1.67	2.37	1.75	1.66	1.17	1.12	0.85	0.40	0.48
Powell	0.18	0.11	0.18	0.51	0.96	1.06	0.65	0.52	0.84	0.42	0.26	0.18

[1] Compiled from Annual Summaries of Climatological Data for 1935, U.S. Weather Bureau.

Table 4 gives the maximum, minimum, and mean yearly runoff for some rivers in western United States. Although the data included in the table were abstracted from several government bulletins, as indicated by the footnotes, most of the original records were probably compiled by the U.S. Geological Survey.

Return Flow. During recent years, return flow, sometimes called *unused water* or *return water*, has become an important factor in irrigation water supply. When a project is first operated, most of the water absorbed by the soil, not used by plant growth or lost by soil moisture evaporation, percolates into the deeper subsoil strata. Later, as the surface of the ground-water storage rises, much of the deep percolation returns to the natural stream channels, either through artificial drains or by direct seepage, the latter sometimes being called *invisible return flow*. Detailed investigations have shown that on a well-supplied project, which has been operated 20 years or more, 30 to 60 per cent of the diverted supply may be expected to come back to the stream as return flow.

According to a published symposium, the total inflow to the North Platte River from irrigated lands between Whalen, Wyo., and Bridgeport, Neb., during 1925 to

TABLE 4.—MAXIMUM, MINIMUM, AND MEAN YEARLY RUNOFF FOR SOME RIVERS IN WESTERN UNITED STATES

State	Station	River	Years of record	Drainage area, sq miles	Total yearly runoff, acre-ft			Reference
					Maximum	Minimum	Mean	
Arizona	Yuma	Colorado	24	242,000	25,975,000	7,959,000	16,606,000	1
	Roosevelt	Salt	16	5,756	3,226,000	240,900	1,072,000	1
California	Oroville	Feather	28	3,640	9,340,000	1,190,000	4,630,000	3
	Red Bluff	Sacramento	25	9,300	15,400,000	2,970,000	8,200,000	3
	Newman	San Joaquin	18		4,780,000	198,000	1,720,000	3
Colorado	Fruita	Upper Colorado	16	23,800	8,122,000	4,243,000	6,365,000	1
	Julesburg	South Platte	24	22,919	1,110,000	93,100	419,000	5
Idaho	King Hill	Snake	13		10,900,000	7,090,000	9,100,100	2
	Arrowrock	Boise	11	2,230	2,530,000	986,000	1,840,000	2
Montana	Intake	Yellowstone	21	67,901	15,400,000	5,530,000	10,700,000	6
	Near Vandalia	Milk	9	21,833	1,310,000	52,600	653,000	6
Nevada	Palisade	Humboldt	7	5,010	538,000	86,000	304,000	4
New Mexico	San Marcial	Rio Grande	26	30,000	2,420,000	240,000	1,200,000	1
	Comstock	Pecos	21		1,007,900	159,300	453,700	1
Oregon	The Dalles	Columbia	45	237,000	222,000,000	93,800,000	151,000,000	2
	Mecca	Deschutes	12		4,170,000	3,130,000	3,635,000	2
	Umatilla	Umatilla	20	2,130	819,000	188,000	450,000	2
Texas	Austin	Colorado	24	34,200	5,171,000	359,000	1,802,000	1
	Three Rivers	Nueces	6	15,600	1,431,000*	16,300	513,000	1
Utah	Preston	Bear	24	4,500	1,648,320	401,000	1,006,620	4
	Devils Gate	Weber	10	1,090	758,000	180,000	420,000	4
Washington	Okanogan	Okanogan	13	7,740	2,920,000	1,550,000	2,120,000	2
	Union Gap	Yakima	23	3,550	4,690,000	1,570,000	3,330,000	2
Wyoming	Guernsey	North Platte	11	19,483	2,620,000	1,460,000	1,782,000	5
	Green River	Green	15	7,670	2,102,600	656,000	1,392,000	1

1. *U.S. Dept. Agr., Tech. Bull.* 185.
2. *U.S. Dept. Agr., Tech. Bull.* 200.
3. *U.S. Dept. Agr., Tech. Bull.* 379.
4. *U.S. Dept. Agr., Dept. Bull.* 1340.
5. *House Doc.* 197, 73d Cong., 2d Sess.
6. *House Doc.* 238, 73d Cong., 2d Sess.

* Ten months.

1927, amounted to about 65 per cent of the total diversions.[1] Although some part of the inflow was caused by surface waste and precipitation on the irrigated lands, the major part of the total must have been return flow. Measurements made in California, during 1924 to 1929, showed that the average annual return flow, expressed as a percentage of the annual gross diversions, could be estimated at 42.5 per cent in the Sacramento Valley, 35 per cent in the San Joaquin Valley, 15 per cent in the delta uplands, and 14 per cent in the Mokelumne Valley; they also showed that 75 per cent of the return flow in the Sacramento Valley could be expected to return during the irrigation season from April to October.[2]

Figure 1, prepared by the late Dr. Fortier from measurements made by R. L. Parshall and others, shows the gradual increase in return flow to the South Platte River between Kersey and Julesberg, northeastern Colorado, during the 40-year period from 1890 to 1920. Figure 2 shows the monthly distribution of return flow on the Garland Division of the Shoshone project, northern Wyoming, based on measurements made in 1922 and 1923. Average monthly irrigation deliveries to the farms are

[1] WILLIS and MURPHY, Return Water and Drainage Recovery from Irrigation—A Symposium, *Trans. A. S. C. E.*, **94**, 327–344, 1930.

[2] Variation and Control of Salinity in Sacramento-San Joaquin Delta and Upper San Francisco Bay, *Calif. Dept. Pub. Works, Bull.* 27, p. 148, 1931.

added for comparative purposes. A valuable digest of return flow in western river valleys, together with a comprehensive discussion of the legal problems involved in reclaiming and reappropriating such water supplies, has recently been published as a government bulletin.[1]

Transmountain Diversions. In some cases, deficient water supplies for irrigation projects have been augmented by tunnel diversions from other drainage areas. The Strawberry Tunnel, northeastern Utah, Gunnison Tunnel, western Colorado, and Laramie-Poudre Tunnel, north central Colorado, may be cited as specific examples.[2]

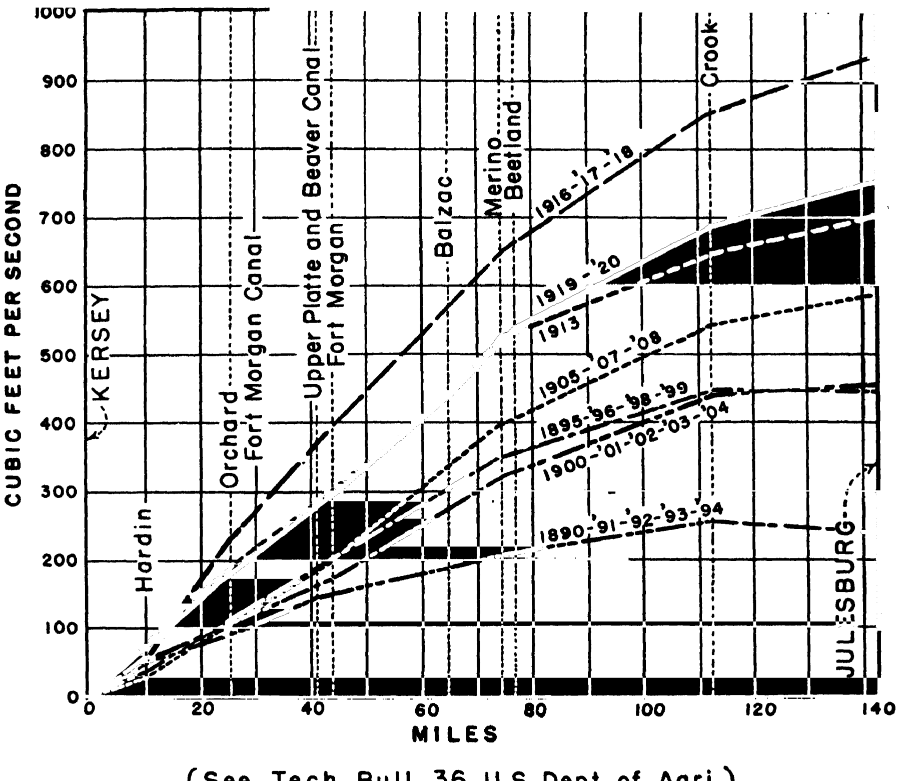


FIG. 1.—Return flow to South Platte river, Kersey to Julesburg, Colo.

Some additional transmountain diversions are now being built, or are being planned for construction, along the front range of the Rockies in eastern Colorado and in other sections of the West.

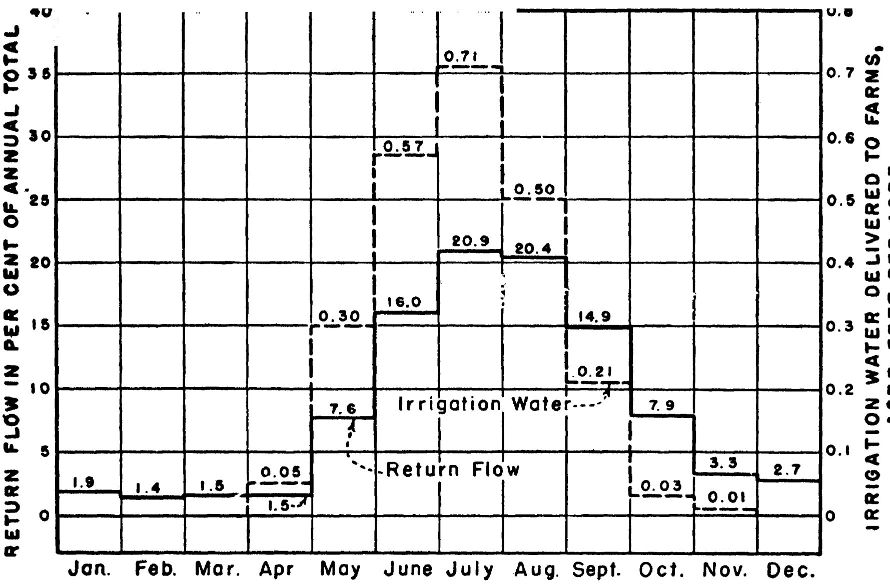
Underground Supplies. Underground water storage constitutes an important source of irrigation supply in many places, particularly along valley floors where extensive gravel deposits occur at relatively shallow depths, as in northeastern Colorado, southern California, and many other irrigated sections of western United States. Underground storage is also important in regions where pervious rock formations are bounded by impervious geological strata, as in the Roswell area of south-

[1] HUTCHINS, WELLS A., Policies Governing the Ownership of Return Waters from Irrigation, *U.S. Dept. Agr., Tech. Bull.* 439, 1934.

[2] DEBLER, VAN NORMAN, and FOLLANSBEE, Trans-mountain Water Diversions—A Symposium, *Trans. A. S. C. E.*, **94**, 345–376, 1930.

eastern New Mexico where about 200,000 acre-ft of irrigation water is obtained annually from artesian wells.[1]

Underground water supplies, whether located in surface gravel deposits or deep geological formations, are maintained by percolation inflow from rainfall, melting snow, running streams, lakes, or other water accumulations somewhere on the ground surface. Consequently any continued draft on the underground storage at rates greater than the inflow may cause serious depletion of the supply. The Roswell artesian area, which originally included about 663 sq miles, was reduced to 499 sq miles by 1916 and to 425 sq miles by 1925, owing to the extensive irrigation use which began



(See H. Doc. 256, 73rd. Cong., 2nd Sess., 1934.
Irrigation deliveries added for comparison.)

FIG. 2.—Monthly distribution of return flow, Garland Division, Shoshone Project, Wyo.

about 1905. Engineers endeavoring to estimate available ground-water supplies should refer to the comprehensive treatment prepared by O. E. Meinzer.[2]

Artesian well supplies are sometimes seriously reduced by leakage into porous strata lying between the ground surface and the water-bearing formations. During recent years, material improvements in some underground supplies have been made by plugging abandoned wells. State engineer Thomas M. McClure estimates that such operations in New Mexico have saved approximately 56,000 acre-ft per season.[3] In southern California, where water supplies are limited, underground storage in surface gravel deposits is sometimes increased by spreading excess surface runoff over porous areas where it can percolate into the gravel formations.[4]

[1] FIEDLER and NYE, Geology and Ground-water Resources of the Roswell Artesian Basin, New Mexico, *U.S. Geol. Survey Water Supply Paper* 639.

[2] MEINZER, O. E., Outline of Methods for Estimating Ground-water Supplies, *U.S. Geol. Survey Water Supply Paper* 638-C, 1931.

[3] MCCLURE, THOMAS M., Plugging Old Artesian Wells to Stop Underground Water Loss, *Eng. News-Record*, **116**, 425–427, 1936.

[4] MITCHELSON and MUCKEL, Spreading Water for Storage Underground, *U.S. Dept. Agr.. Tech. Bull.* 578, 1937.

Use of Sewage.—Municipal sewage is used to irrigate small areas of farm lands near several western cities, the sewage being delivered to the lands directly from outfall lines or as effluents from treatment plants. Although the total quantities of sewage produced by all western cities constitute but relatively small proportions of the total water supplies used for irrigation in the West, the use of such sewage discharges may constitute important supplemental supplies in localities where natural stream flows are limited. The combined discharge of the sanitary sewers in Denver during the 10-year period from 1927 to 1936, prior to the construction of a treatment plant, averaged 70 per cent of the total water consumption in the city. Where sewage discharges or treatment-plant effluents are delivered to natural stream channels, such flows automatically become available for irrigation use at downstream locations. The results of an extensive survey of sewage irrigation in western United States was published in 1939.[1]

Quality of Water. Water-supply studies for irrigation projects should include chemical analyses of the available supplies. Ordinarily, headwater runoff and flood flows are sufficiently low in salt content to be satisfactory for irrigation purposes. However, unfortunate exceptions have sometimes been experienced in the case of headwater runoff. Low water discharges, downstream from large irrigated areas, may be objectionable owing to high salt contents in return flow, as previously mentioned. Underground supplies may also be objectionable because of the presence of undesirable chemicals dissolved from the geological strata through which the water percolates.

Methods of analyzing water supplies and interpreting results were discussed by Carl S. Scofield in reporting on a recent investigation conducted in southern California.[2] He found that the critical concentration, in parts per million, was 0.5 to 1.0 for boron, 142 to 335 for chloride, and 192 to 480 for sulphate, also that the critical percentage of sodium, obtained by dividing the milligram equivalent of sodium by the sum of the milligram equivalents of calcium, magnesium, and sodium, was between 50 and 60. He reported that the nitrate content was seldom harmful and that the carbonate and bicarbonate constituents had less agricultural significance than the other chemicals considered. The water was considered safe for general use when the boron concentration was below the limit of tolerance, the sodium percentage less than 60, and the specific electrical conductance, in reciprocal ohms at 25C, less than 100.

In a subsequent investigation at the Newlands Field Station, western Nevada, values of the percentage of chloride were determined for both subsoil moisture and irrigation supply. The percentage of chloride was obtained by dividing the sum of the anions into the value of the chloride multiplied by 100. The percentage of chloride was considered to be important as an indication of the changes in the quality of the water that may occur as the result of the precipitation from solution of the bicarbonate and sulphate constituents. At the Newlands Station, the percentage of chloride in the subsoil water was about the same as in the irrigation supply.[3]

In considering the quality of irrigation water, it must be remembered that the salt content of the soil solution increases with continued irrigation, owing to the continued evaporation of moisture from the soil surface and the relatively small proportion of dissolved salts absorbed by the plant roots. The concentration of the soil solution is seldom less than twice the concentration of the water applied, even when considerable deep percolation takes place. Usually, the soil solution is four to eight times

[1] HUTCHINS, WELLS A., "Sewage Irrigation as Practiced in the Western States, *U.S. Dept. of Agr., Tech. Bull.* 675, 1939.

[2] South Coastal Basin Investigation—Quality of Irrigation Waters, *Calif. Dept. Pub. Works, Bull.* 40, 1933.

[3] SCOFIELD, MOON, and KNIGHT, Subsoil Waters of Newlands Field Station, *U.S. Dept. Agr Tech. Bull.* 533, 1936.

as concentrated as the irrigation water.[1] Consequently it is often desirable, when practicable, to flush the concentrated soil solution beyond the root zones by flooding the land during the winter months when plant growth is dormant and soil moisture evaporation a minimum.

Table 5 gives some analyses of the water in the Colorado River and certain tributaries, selected from detailed tabulations prepared by C. S. Howard. Data for the Colorado River flow in January, 1927, and May, 1928, were included because of the relatively high and low sodium contents during low water and flood discharges, respectively. The author concluded that "The usefulness of the water of the Colorado and its tributaries for irrigation is wholly dependent on the drainage of the irrigated land. With good drainage and liberal use of the water no trouble should be experienced from the ordinary constituents of the water. Without adequate drainage the soil may be seriously damaged."

The boron content, which is especially important in walnut and citrus-fruit irrigation, was not included in the published data from which Table 5 was prepared. However, samples of Colorado River water, taken weekly at Yuma, Ariz., for a period of one year, showed an average boron content of 0.19 ppm.[2]

TABLE 5.—ANALYSES OF FLOW IN COLORADO RIVER AND CERTAIN TRIBUTARIES[1]
(Parts per million)

Chemicals		Green River at Green River, Utah, 1930[2]	San Juan River at Goodridge, Utah, 1930[2]	Colorado River at Grand Canyon, Arizona			
Name	Symbol			Jan. 1, 2, and 4–10, 1927	May 11 and 13–20, 1928	1930[2]	5-year mean 1925–1930[2]
Silica	SiO_2	14	16	16	15	16	17
Iron	Fe	0.12	0.10	0.15	0.24	0.11	0.22
Calcium	Ca	59	75	143	54	81	73
Magnesium	Mg	23	17	68	14	26	23
Sodium	Na	56	50	245	20	85	74
Potassium	K	3.8	3.5	12	6.1	5.0	5.4
Bicarbonate	HCO_3	179	159	289	145	184	165
Sulphate	SO_4	177	214	557	96	252	220
Chloride	Cl	26	11	248	17	62	53
Nitrate	NO_3	1.2	1.7	22	0.78	3.4	2.4
cfs	.....	6,290	2,380	3,940	79,300	18,500	21,800

[1] Compiled from C. S. Howard, Quality of Water of the Colorado River, *U.S. Geol. Survey, Water Supply Papers* 636-A and 638-D.

[2] Means are for years ending Sept. 30, calculated by weighting different analyses according to discharge.

Effects on plant growth caused by boron contents of soils and irrigation waters have been studied extensively by the Bureau of Plant Industry.[3] Small boron contents in soil solutions are desirable in order to obtain normal plant development. If the boron content is less than about 0.1 to 0.5 ppm, plant diseases may result. Heart

[1] SCOFIELD, CARL S., The Salinity of Irrigation Water, Smithsonian Report for 1935, pp. 275–287.

[2] SCOFIELD and WILCOX, Boron in Irrigation Waters, *U.S. Dept. Agr., Tech. Bull.* 264, 1931.

[3] EATON, FRANK M., Boron in Soils and Irrigation Waters and Its Effects on Plants, *U.S. Dept. of Agr., Tech. Bull.* 448, 1935.

EATON, FRANK M., and L. V. WILCOX, The Behavior of Boron in Soils, *U.S. Dept. of Agr., Tech. Bull.* 696, 1939.

EATON, FRANK M., RAY D. MCCALLUM, and MILES S. MAYHUGH, Quality of Irrigation Waters of the Hollister Area of California, *U.S. Dept. of Agr., Tech. Bull.* 746, 1941.

rot and dry rot of sugar beets, top rot of tobacco, brown heart of turnips, cracked stem of celery, and certain core defects in apples are caused by insufficient amounts of available boron. However, if the boron concentration is more than about 0.5 to 5.0 ppm, injurious effects on plant development may be anticipated. Some experiences indicate that most plants withstand high boron concentrations better in cool humid climates than in climates conducive to high rates of transpiration.

Conditions that determine the suitability of a water supply for use in irrigating crops are so complex and variable that no general limits of permissible chemical analyses can be given. A supply that has proved satisfactory for irrigating a certain crop may prove unsatisfactory for a different crop, even when grown on the same soil structure. A supply that is generally suitable for irrigation use on one soil structure may be generally unsuitable on another. In many cases, adequate drainage provisions and frequent leaching operations may permit the use of a water supply that otherwise would be objectionable. If appreciable depths of rainfall occur during the nongrowing season, sufficient leaching of the soil may be provided by climatic conditions. The quality of each water supply must be studied as a special problem, giving careful consideration to the crops contemplated, soil texture, drainage conditions, leaching possibilities, and nongrowing-season precipitation, as well as to the chemical analysis of the water.[1]

A detailed method of studying the quality of irrigation water, the problems involved in mixing different water supplies, and the changes in quality caused by irrigation use was proposed in 1941. The method is based on an ingenious use of geochemical charts.[2]

Water Rights. Existing water rights must be carefully considered in water-supply studies for contemplated irrigation projects. In some cases, it may be necessary to evaluate and purchase prior rights in order to secure an adequate supply.[3] Controversies over water rights have had a prominent place in irrigation history, not only between different irrigation projects but also between different states. The latter have in some cases led to interstate compacts regarding the allocation of stream flow. The Colorado River Compact, executed at Santa Fe, N. M., on Nov. 24, 1922, and later ratified by all the seven states involved except Arizona, is the outstanding example of interstate water agreements.[4]

Water-supply Data. Data regarding available water supplies may be secured from various Federal, state, and municipal agencies. The records and publications of the U.S. Weather Bureau and the U.S. Geological Survey constitute the principal sources of information regarding precipitation and runoff data, respectively. The water-supply papers published by the latter organization also contain much valuable information on underground water supplies. State agencies that may be able to supply data include engineering offices, water departments, agricultural experiment stations, and universities. Other agencies that may be able to supply data include irrigation districts, special commissions, and municipal water departments. Water-supply data may also be secured from the offices of the U.S. Army engineers and the U.S. Department of Agriculture. Data regarding water rights are usually obtained from the offices of the state engineers.

Snow Surveys. Carefully planned snow surveys, cooperatively conducted by Federal, state, and other agencies, are now being used in forecasting available water

[1] KELLY, W. P., Permissible Composition and Concentration of Irrigation Water, *Trans. A. S. C. E.*, **106**, pp. 849–861, 1941.

[2] HILL, RAYMOND A., Salts in Irrigation Water, *Proc. A. S. C. E.*, June, 1941, pp. 975–990.

[3] FIELD, JOHN E., Evaluation of Water Rights, *Trans. A. S. C. E.*, **94**, 247–294, 1930. Also see BALDWIN, HYATT, and BACON, Administrative Water Problems—A Symposium, *Trans. A. S. C. E.*, **94**, 295–329, 1930.

[4] Colorado River Development, *Sen. Doc.* 186, 70th Cong., 2d Sess., 1929, pp. 30–40.

supplies in several western states. The surveys include accurate measurements of snow depths and water contents, improved sampling tubes and weighing scales being used. The measurements are preferably made about 50 to 100 ft apart, along carefully selected courses in the upper sections of the drainage areas where the major portions of the water supplies originate. The surveys are made late in the spring, just before appreciable surface runoff begins.[1]

WATER REQUIREMENTS

The depth of water required to produce satisfactory crop yields varies with the soil texture and composition, kinds of crops grown, preparation of field surfaces, methods and frequency of irrigation, rainfall characteristics, and general climatic conditions. Other things being equal, the amount of water that must be artificially applied to the land depends on the deficiency in rainfall during the growing season. In semiarid regions, where appreciable depths of precipitation sometimes occur during the summer months, irrigation demands may be relatively small in some years and relatively high in others. In arid regions, where but little or no rainfall occurs during the growing season, irrigation demands are more nearly constant from year to year, inasmuch as the total depths of water required to produce given crops on given soil areas, under given climatic conditions, are approximately the same.

During the earlier years of irrigation, the term *duty of water* was commonly used to denote the relation between the quantity of irrigation water applied and the area irrigated. The term was used in two ways: (1) as the actual volume delivered to the land per unit area, and (2) as the area irrigated by a given rate of flow, usually in cubic feet per second. However, during recent years there has been a growing tendency to replace the term duty of water by the term *water requirement* or *irrigation requirement*, except in legal procedure, where the original term has usually been retained because of its use in laws relating to water rights.

In the following discussions, water requirement is used to mean the total quantity of water per unit area needed to produce satisfactory crop yields, whether supplied wholly by irrigation or by irrigation and precipitation combined. The term irrigation requirement is used to mean the total quantity per unit area that must be applied artificially, in addition to the natural precipitation. The use of the word *net* before either term means that the quantity applies to farm areas and does not include conveyance losses or waste. Water requirements and irrigation requirements, as well as duty of water, are now commonly given in acre-feet per acre, although they might, logically, be given simply as feet of depth, just as rainfall is commonly given as inches of depth.

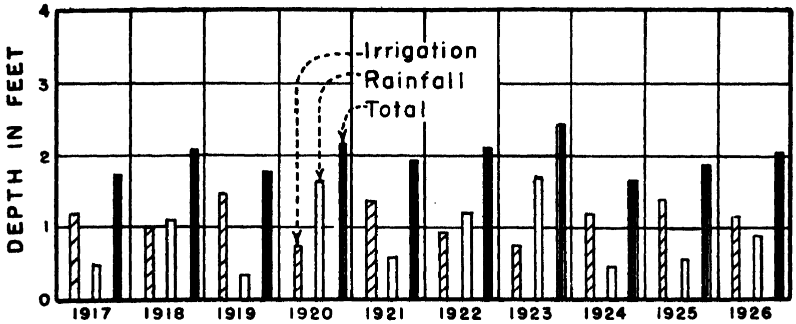
Economic Requirements. Water requirements of irrigated lands in western United States were exhaustively studied by the late Samuel Fortier and Arthur A. Young of the Bureau of Agricultural Engineering, U.S. Department of Agriculture. The results of their researches, recently published as a series of government bulletins, included estimates of the average monthly and seasonal requirements that should ultimately prove satisfactory in raising diversified crops in the various sections of the west, essentially the same as are being grown today, but under improved, economical methods of irrigation. In general, the estimates assumed that the land surfaces would be properly prepared, water supplies adequately regulated, unnecessary waste eliminated, and some deep percolation recovered by pumping.

Tables 6 to 10 give Fortier's and Young's estimates of seasonal requirements, grouped according to principal drainage basins. Table 6, applying to the Great Basin, where precipitation usually is not an important factor, gives the net water

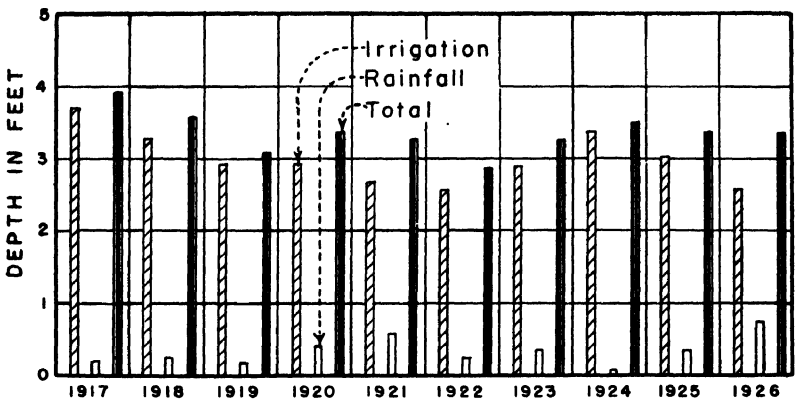
[1] CHURCH, J. E., Principles of Snow Surveying as Applied to Forecasting Stream Flow, *Jour. Agr. Research*, U.S. Dept. Agr., July 15, 1935.

requirements, including seasonal rainfall as well as irrigation supply. The other tables give the net irrigation requirements, exclusive of precipitation. Neither the net water requirements nor the net irrigation requirements include allowances for transmission or other losses between diversion works and farms.

Although the seasonal requirements given in Tables 6 to 10 are really ideal requirements, usually exceeded in present-day irrigation practice in localities where water is plentiful, they are useful in showing the minimum quantities that should be provided



BELLE FOURCHE PROJECT - IRRIGATION SEASON, MAY TO SEPT.



YUMA PROJECT - IRRIGATION SEASON, JAN. TO DEC.

(Plotted from data in Trans. Am. Soc., C.E., Vol. 94, pages 1199 and 1222)

FIG. 3.—Irrigation deliveries, seasonal rainfall, and total use of water.

in the different sections of the West in order to secure satisfactory crop production. However, they are not essential for profitable production if available water supplies are not sufficient to provide such depths. Some irrigation districts in California have been operating profitably for several years with supplies somewhat lower than those shown in Table 10.[1]

Use on Federal Projects. Considerations of the actual use of water on Federal irrigation projects are valuable in studying contemplated irrigation developments. Such projects cover a wide range of location, soil structure, crops, irrigation methods, and climatic conditions. Table 11 gives a summary of pertinent data for Bureau of

[1] Irrigation Requirements of California Lands, *Calif. Dept. Pub. Works, Bull.* 6, 1923.

TABLE 6.—SEASONAL NET WATER REQUIREMENTS IN THE GREAT BASIN[1]

Division number	Description of division	States	Irrigation season	Seasonal water requirement, acre-ft/acre
1	Bear River basin	Idaho and Utah	May–Oct.	2.0
2	Utah Lake and Great Salt Lake valleys south of Weber River basin	Utah	May–Oct.	2.2
3	Sevier River basin	Utah	May–Oct.	2.1
4	Irrigable lands of southwestern Utah	Utah	Apr.–Oct.	1.8
5	Irrigable lands of southern Nevada	Nevada	Apr.–Oct.	1.7
6	Antelope Valley and Mohave River areas[2]	Calif. and Nev.	Mar.–Oct.	1.8
7	Mono, Owens, and Inyo-Kern valleys[2]	California	Mar.–Oct.	2.1
8	Walker River basin	Nev. and Calif.	Apr.–Oct.	2.0
9	Truckee and Carson River basins	Nev. and Calif.	May–Oct.	2.1
10	Humboldt, Quinn, and White River basins	Nevada	May–Sept.	2.0
11	Honey Lake basin	Nev. and Calif.	Apr.–Sept.	1.7
12	Malheur Lake, Harney Lake, and other basins	Oregon	Apr.–Sept.	1.5

[1] Compiled from Samuel Fortier, Irrigation Requirements of the Arable Lands of the Great Basin, *U.S. Dept. Agr., Dept. Bull.* 1340, 1925.
[2] See Irrigation Requirements of California Lands, *Calif. Dept. Pub. Works, Bull.* 6, 1923.

TABLE 7.—SEASONAL NET IRRIGATION REQUIREMENTS IN THE MISSOURI AND ARKANSAS RIVER BASINS[1]

Division number	Description of division	State	Irrigation season[2]	Seasonal irrigation requirements, acre-ft/acre
1	Northeastern Montana	Montana	May–Aug.	1.40
2	North Central Montana	Montana	May–Aug.	1.50
3	Central Montana	Montana	May–Aug.	1.70
4	Upper Missouri River basin	Montana	May–Aug.	1.60
5	Upper Yellowstone River basin	Montana	May–Aug.	1.90
6	Southeastern Montana	Montana	May–Sept.	1.95
7	Big Horn River basin	Wyoming	May–Sept.	1.65
8	Yellowstone and Missouri River basins	Wyoming	May–Sept.	1.70
9	Upper Platte River basin	Wyoming	May–Sept.	1.60
10	Northeastern Colorado	Colorado	Apr.–Sept.	2.05
11	North Central Colorado	Colorado	Apr.–Sept.	2.20
12	South Central Colorado	Colorado	Apr.–Sept.	2.10
13	Southeastern Colorado	Colorado	Apr.–Oct.	2.30
14	Western Kansas	Kansas	Apr.–Oct.	1.75
15	Central Nebraska	Nebraska	Apr.–Oct.	1.25
16	Western Nebraska	Nebraska	Apr.–Oct.	2.00
17	Western South Dakota	South Dakota	Apr.–Oct.	1.50
18	Western North Dakota	North Dakota	May–Sept.	1.35

[1] Compiled from Samuel Fortier, Irrigation Requirements of the Arid and Semi-arid Lands of the Missouri and Arkansas River Basins, *U.S. Dept. Agr., Tech. Bull.* 36, 1928.
[2] Some winter irrigation in October.

Reclamation projects, including average elevations, predominating soil characteristics, acreages irrigated, crops raised, seasonal precipitation, and average quantities of irrigation water delivered to the farms during the years considered. Figure 3 shows the yearly irrigation deliveries, seasonal rainfall, and total use of water on the Yuma and Belle Fourche projects of southwestern Arizona and western South Dakota, where irrigation seasons and climatic conditions are greatly different.

TABLE 8.—SEASONAL NET IRRIGATION REQUIREMENTS IN SOUTHWESTERN UNITED STATES[1]

Division number	Description of division	State	Irrigation season	Seasonal irrigation requirements, acre-ft/acre
1	Imperial Valley	California	Jan.–Dec.	3.10
2	Southern Nevada	Nevada	Jan.–Dec.	2.90
3	Southwestern Arizona	Arizona	Jan.–Dec.	3.00
4	Northwestern Arizona	Arizona	Mar.–Oct.	2.30
5	Navajo country	Arizona	Mar.–Oct.	2.30
6	Southeastern Arizona	Arizona	Feb.–Nov.	2.60
7	San Juan basin	New Mexico	Apr.–Sept.	2.20
8	Western New Mexico	New Mexico	Apr.–Oct.	1.70
9	Rio Grande basin	New Mexico	Jan.–Dec.	2.60
10	Pecos River basin	New Mexico	Jan.–Dec.	2.40
11	Northeastern New Mexico	New Mexico	Feb.–Nov.	1.60
12	Upper Rio Grande basin	Texas	Jan.–Nov.	2.40
13	Pecos River basin	Texas	Jan.–Nov.	2.25
14	West Central Texas	Texas	Jan.–Dec.	1.60
15	Lower Rio Grande basin	Texas	Jan.–Dec.	1.75
16	Upper Nueces and Colorado River basins	Texas	Jan.–Dec.	1.30
17	Upper Brazos and Red River basins	Texas	Jan.–Dec.	1.10
18	Eastern Panhandle	Texas	Mar.–Oct.	1.35
19	Western Panhandle	Texas	Mar.–Oct.	1.65
20	Panhandle	Oklahoma	Apr.–Oct.	1.25
21	Western Oklahoma	Oklahoma	Apr.–Oct.	1.00
22	San Luis basin	Colorado	May–Sept.	1.80
23	San Juan basin	Colorado	Apr.–Sept.	1.90
24	Yampa and White River basins	Colorado	May–Aug.	1.35
25	Upper Colorado River basin	Colorado	Apr.–Sept.	1.70
26	Virgin River basin	Utah	Feb.–Nov.	2.25
27	San Juan basin	Utah	Apr.–Sept.	2.10
28	Green River basin	Utah	Apr.–Oct.	2.00
29	Uintah basin	Utah	Apr.–Sept.	1.75
30	Green River basin	Wyoming	May–Aug.	1.60

[1] Compiled from Fortier and Young, Irrigation Requirements of the Arid and Semi-arid Lands of the Southwest, *U.S. Dept. Agr., Tech. Bull.* 185, 1930.

The data in Table 11 show that the average seasonal deliveries on Bureau of reclamation projects varied from 0.65 acre-ft/acre on the Milk River project, northeastern Montana, where heavy soils predominate, to 7.01 acre-ft/acre on the King Hill project, southern Idaho, where very light soils predominate. Total depths of water used per season, including rainfall, varied from 1.56 ft on the Milk River project to 7.36 ft on the King Hill project. Omitting the Umatilla, Uncompahgre, and King Hill projects, where waste and return flow were unusually large, the total depths used did not exceed 4 ft, except on the Grand Valley project, western Colorado, where the total amounted to 4.09 ft.

TABLE 9.—SEASONAL NET IRRIGATION REQUIREMENTS IN THE COLUMBIA RIVER BASIN[1]

Division number	Description of division	States	Irrigation season	Seasonal irrigation requirement, acre-ft/acre
1	Snake River Valley	Idaho	Apr.–Oct.	2.5
2	Upper Snake River Valley	Idaho	Apr.–Sept.	2.3
3	Jackson Lake and Upper Snake River basin	Wyo. and Idaho	May–Sept.	1.7
4	Southwestern Idaho and northern Nevada	Idaho and Nev.	Apr.–Sept.	1.9
5	Salmon River basin	Idaho	May–Aug.	2.0
6	Northern Idaho	Idaho	May–Sept.	1.5
7	Bitter Root and Missoula River basins	Montana	Apr.–Nov.	2.1
8	Flathead Lake and River basins	Montana	Apr.–Sept.	1.8
9	Owyhee and Malhuer River basins	Oregon	Apr.–Sept.	2.4
10	Northeastern Oregon	Oregon	Apr.–Sept.	2.0
11	Lower Umatilla, John Day, Deschutes, and Hood River basins	Oregon	Apr.–Oct.	2.5
12	Central Oregon	Oregon	May–Aug.	2.4
13	Yakima and Wenatchee River basins	Washington	Apr.–Nov.	2.6
14	Southeastern Washington	Washington	Apr.–Oct.	2.1
15	Northeastern Washington	Washington	Apr.–Oct.	2.2
16	Okanogan River basin	Washington	Apr.–Nov.	2.3
17	Lower Columbia River basin	Washington	May–Sept.	1.3
18	Willamette River basin	Oregon	May–Sept.	1.2
19	Puget Sound District[2]	Washington	May–Sept.	1.4

[1] Compiled from Fortier and Young, Irrigation Requirements of the Arid and Semi-arid Lands of the Columbia River Basin, *U.S. Dept. Agr., Tech. Bull.* 200, 1930.
[2] Not in Columbia River basin.

TABLE 10.—SEASONAL NET IRRIGATION REQUIREMENTS IN THE PACIFIC SLOPE BASINS[1]

Division number	Description of division	States	Irrigation season	Seasonal irrigation requirement acre-ft/acre
1	Umpqua, Coquill, and lower Rogue River basins	Oregon	Apr.–Sept.	0.85
2	Upper Rogue River basin	Oregon	Mar.–Sept.	1.50
3	Klamath Lake and River basins	Ore. and Calif.	Apr.–Sept.	2.00
4	Pacific slope, northwestern California	California	Apr.–Oct.	1.40
5	Pit River basin	California	Apr.–Sept.	1.60
6	Feather, Yuba, and American River basins	California	Mar.–Nov.	1.50
7	Sacramento Valley	California	Mar.–Oct.	2.10
8	Sacramento-San Joaquin Delta	California	May–Sept.	2.00
9	San Francisco Bay basin	California	Mar.–Nov.	1.50
10	Salinas River basin	California	Mar.–Oct.	1.70
11	Santa Maria, Santa Inez, and Santa Clara basins	California	Jan.–Dec.	1.60
12	San Joaquin valley	California	Feb.–Oct.	2.30
13	West Slope of Sierras, east of San Joaquin	California	Feb.–Nov.	1.70
14	East Slope of Coast Range, west of San Joaquin	California	Feb.–Oct.	1.80
15	Antelope and Victor valleys	California	Mar.–Oct.	1.90
16	Los Angeles, San Gabriel, and Lower Santa Ana basins	California	Jan.–Dec.	1.70
17	Upper Santa Ana Valley	California	Jan.–Dec.	1.80
18	San Diego County	California	Jan.–Dec.	1.40

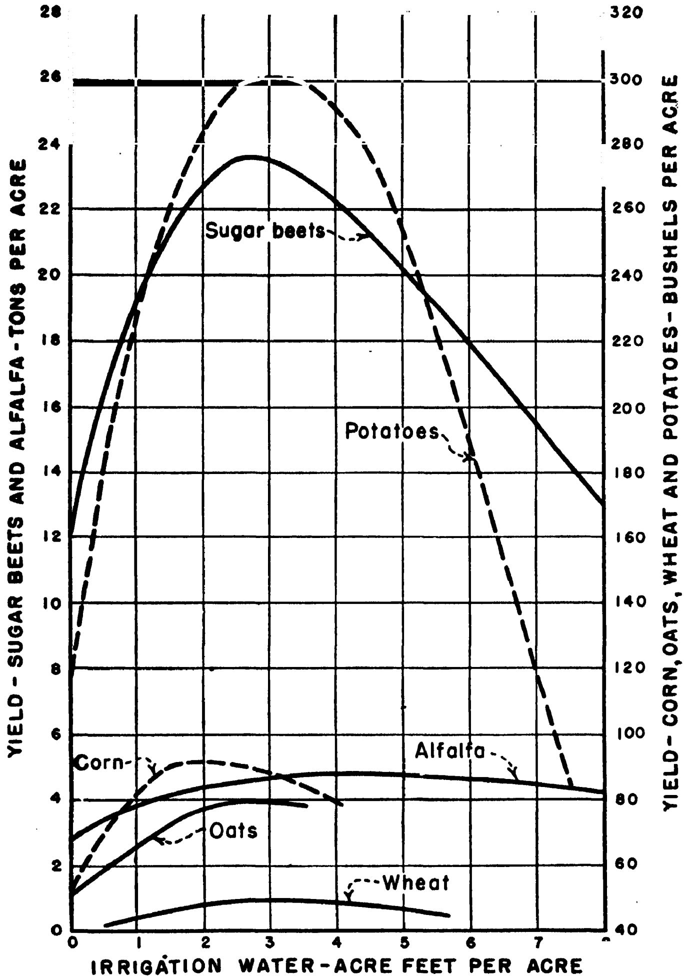
[1] Compiled from Fortier and Young, Irrigation Requirements of the Arid and Semi-arid Lands of the Pacific Slope Basins, *U.S. Dept. Agr., Tech. Bull.* 379, 1933.

TABLE 11.—AVERAGE USE OF WATER ON FEDERAL IRRIGATION PROJECTS[1]

State	Project	Average elevation	Years considered	Area irrigated, acres	Predominating character of soils	Crops grown, % of total area: Alfalfa, hay, and pasture	Small grain	Furrow crops	Trees	Irrigation season	Precipitation, ft: Before growing season	During growing season	Water delivered to farms, acre-ft/acre	Water delivered plus growing season prec., ft
Ariz.-Calif.	Yuma	120	1917–1926	51,950	Medium	39	3	58	..	Jan.–Dec.	0.33	0.33	3.01	3.34
California	Orland	250	1917–1926[2]	14,554	Light	53	5	19	23	Jan.–Nov.	1.02	0.37	3.17	3.55
Colorado	Grand Valley	4,700	1916–1925	10,139	Heavy	36	24	33	7	Apr.–Nov.	0.27	0.48	3.61	4.09
	Uncompahgre	5,500	1917–1926	61,178	Medium	47	25	25	3	Apr.–Oct.	0.26	0.55	5.76	6.31
Idaho	Boise	2,500	1917–1925[3]	145,616	Light	53	31	13	3	Apr.–Oct.	0.43	0.38	3.60	3.98
	King Hill	2,750	1921–1927	6,460	Very light	75	7	11	6	Apr.–Nov.	0.41	0.35	7.01	7.36
	Minidoka, S. Side Pumping	4,200	1917–1926	44,945	Medium	50	28	22	..	Apr.–Oct.	0.32	0.53	2.54	3.07
Montana	Huntley	3,000	1917–1926	19,406	Heavy	42	34	24	..	May–Sept.	0.37	0.63	1.39	2.02
	Lower Yellowstone	1,900	1917–1926	17,540	Heavy	45	33	22	..	May–Sept.	0.33	0.71	1.34	2.05
	Milk River	2,200	1917–1926	16,793	Heavy	73	22	5	..	Apr.–Oct.	0.18	0.91	0.65	1.56
	Sun River, Fort Shaw	3,700	1917–1926	7,650	Heavy	75	20	5	..	May–Oct.	0.23	0.60	1.54	2.14
	Sun River, Greenfields	3,700	1919–1926	9,867	Medium	23	75	2	..	May–Oct.	0.28	0.65	1.28	1.93
Nevada	Newlands	4,000	1917–1926	38,808	Medium	85	11	4	..	Mar.–Nov.	0.15	0.25	2.88	3.13
New Mexico	Carlsbad	3,100	1917–1926	22,535	Medium	33	4	63	..	Jan.–Nov.	0.06	0.86	2.36	3.22
N. M.-Tex.	Rio Grande	3,700	1919–1926	96,847	Medium	37	8	53	2	Feb.–Dec.	0.06	0.60	2.89	3.49
Oregon	Klamath	4,100	1917–1926	43,325	Medium	77	21	2	..	Apr.–Sept.	0.68	0.18	1.43	1.61
	Umatilla	470	1916–1925	10,970	Light	85	1	6	8	Mar.–Nov.	0.40	0.35	5.02	5.37
South Dakota	Belle Fourche	2,800	1917–1926	45,164	Heavy	62	22	16	..	May–Sept.	0.48	0.86	1.22	2.08
Washington	Okanogan	1,000	1921, 1923 1925	5,260	Light	9	..	3	88	May–Sept.	0.51	0.42	2.60	3.02
	Yakima, Sunnyside	800	1917–1926	91,726	Medium	57	7	21	15	Mar.–Oct.	0.27	0.21	3.29	3.50
	Yakima, Tieton	1,500	1917–1926	27,607	Light	44	12	11	33	Apr.–Sept.	0.44	0.16	2.51	2.67
Wyoming	Shoshone, Frannie	4,150	1922–1926	7,963	Heavy	72	17	11	..	Apr.–Oct.	0.11	0.39	2.19	2.58
	Shoshone, Garland	4,400	1917–1926	32,380	Medium	59	28	13	..	Apr.–Nov.	0.09	0.33	2.38	2.71
Wyo.-Neb.	North Platte	4,100	1919–1926	107,694	Medium	36	26	38	..	May–Sept.	0.47	0.81	2.23	3.04

[1] Abstracted from E. B. Debler, Use of Water on Federal Irrigation Projects, *Trans. A. S. C. E.*, **94**, 1223–1224, 1930.
[2] 1918, 1920, and 1924 omitted because of heavy water shortage.
[3] 1924 omitted because of water shortage.

Crop Requirements. Other things being equal, the depths of water required by different crops vary with the kind of crop. Water requirements of the same crop, grown in different localities, vary, principally, with the climatic and soil conditions. Variations for different crops, grown under similar climatic and soil conditions, are caused by differences in transpiration rates, lengths of growing season, methods of



(From curves in Bull. 173 Utah Agri. Exp. Sta.)

FIG. 4.—Yield of crops in Cache Valley, Utah.

land treatment, methods of applying water, and frequency of irrigations. Yearly water requirements of forage crops, such as alfalfa, which may be cut several times a year, naturally are greater in regions where long growing seasons permit a greater number of cuttings. Crops usually require greater quantities of water when grown on light sandy soils than when grown on heavy clay soils, since more water is lost by percolation beyond the root zones in the case of the more permeable soils. Given

TABLE 12.—WATER REQUIREMENTS OF CROPS IN THE MISSOURI AND ARKANSAS RIVER BASINS, INCLUDING RAINFALL[1]

Crop	Number of tests	Water requirement, acre-ft/acre		Crop	Number of tests	Water requirement, acre-ft/acre	
		Average low	Average high			Average low	Average high
Alfalfa and other forage	648	1.94	2.62	Peas	168	1.46	1.94
Potatoes	350	1.38	1.70	Buckwheat	3	1.05	1.30
Cantaloupes	10	1.50	2.30	Corn	70	1.23	1.83
Tomatoes	6	2.10	2.80	Kafir corn	15	1.43	1.57
Sugar beets	128	1.60	2.50	Millet	14	0.81	0.94
Sunflowers	16	1.20	1.40	Milo maize	27	1.09	1.70
Apples	4	2.10	2.60	Sorghum	26	1.06	1.47
Wheat	542	1.36	1.80	Beans	4	1.30	1.60
Oats	409	1.35	1.81	Flax	50	1.47	1.86
Barley	335	1.33	1.82				

[1] From Samuel Fortier, Irrigation Requirements of the Arid and Semi-arid Lands of the Missouri and Arkansas River Basins, *U.S. Dept. Agr., Tech. Bull.* 36, 1928.

TABLE 13.—WATER REQUIREMENTS OF CROPS IN THE SOUTHWEST, INCLUDING RAINFALL[1]

Crop	Number of tests	Water requirement, acre-ft/acre		Crop	Number of tests	Water requirement, acre-ft/acre	
		Lowest general average	Highest general average			Lowest general average	Highest general average
Alfalfa	369	3.47	5.08	Millet	5	0.91	1.09
Barley	3	1.24	1.83	Oats	2	1.90	2.09
Table beets	28	0.87	1.37	Onions	4	0.73	1.52
Sugar beets	5	1.77	2.72	Peas	8	1.21	1.56
Broomcorn	9	0.97	1.15	Potatoes	12	1.59	2.04
Cabbage	21	0.94	1.49	Rhodes grass	12	3.49	4.43
Cauliflower	6	1.43	1.77	Snap beans	9	0.83	1.44
Carrots	6	1.27	1.60	Spinach	12	0.80	1.07
Corn	42	1.44	1.99	Sorghum	34	1.69	2.08
Cotton	103	2.35	3.51	Soybeans	36	1.66	2.81
Emmer	6	1.19	1.87	Sudan grass	25	2.88	3.16
Flax	3	1.23	1.59	Sugar cane	41	3.48	4.56
Feterita	8	0.97	1.10	Sweet potatoes	3	1.77	2.25
Kafir	16	1.32	1.54	Tomatoes	17	0.95	1.42
Lettuce	49	0.72	1.35	Wheat	46	1.46	2.24
Milo	35	0.96	1.67				

[1] From Fortier and Young, Irrigation Requirements of the Arid and Semi-arid Lands of the Southwest, *U.S. Dept. Agr., Tech. Bull.* 185, 1930.

crop yields usually require smaller quantities of water when produced on soils of high fertility.

Most crops have an optimum water requirement, *i.e.*, a seasonal depth of application that will produce a maximum yield. Optimum requirements of different crops grown on loam soils in Cache Valley, Utah, are indicated by the summits of the curves in Fig. 4, a diagram showing yields produced by different depths of irrigation water,

TABLE 14.—WATER REQUIREMENTS OF CROPS IN THE COLUMBIA RIVER AND PACIFIC SLOPE BASINS, INCLUDING RAINFALL[1]

Crop	Location	Years	Number of tests or tracts	Water requirement, acre-ft/acre	
				Lowest	Highest
Valencia oranges	Orange and Los Angeles Cos., Calif.	1929–1931	211	2.54	2.91
Navel oranges	San Diego, Tulare, and Los Angeles Cos., Calif.	1923–1931	82	2.69	3.15
Lemons	San Diego, Orange, and Los Angeles Cos., Calif.	1923–1931	68	2.10	2.90
Avocados	Orange County, Calif.	1930–1931	28	2.35	2.58
Grapes	Kern County, Calif.	1930	11	3.12	5.20
Walnuts	Orange and Los Angeles Cos., Calif.	1927–1931	182	2.38	4.37
Pears	Rogue River Valley, Ore.	1930–1932	7	1.25	1.50
Apples	Snake River Valley, Idaho	1914–1915	10	1.56	3.77
Orchard fruits[2]	Wenatchee, Wash.	1908, 1910, 1911	11	2.08	2.83
Cotton	Shafter, Calif.	1926–1930		2.12	3.46
Rice	Sacramento Valley, Calif.	1916–1917	[3]	4.10	8.32
Rice	Sacramento Valley, Calif.	1924	[3]	4.25	5.77

[1] Compiled from *U.S. Dept. Agr., Tech. Bull*. 200 and 379, 1930 and 1933.
[2] Mostly apples, some peach, pear, cherry, and plum trees.
[3] Data given are for clay and clay-adobe soils.

plotted from similar curves prepared by F. S. Harris.[1] The optimum requirement in the Cache Valley seems to be fairly well defined in the case of potatoes and sugar beets but less definite in the case of alfalfa and small grain. Additional and more elaborate experimental data on crop requirements obtained near Logan, Utah, although varying greatly in yield and applied water, showed substantially the same general relations as the Cache Valley investigation.[2] Furthermore, curves somewhat similar to those in Fig. 4 were obtained at the Brooks Experiment Station, Alberta, Canada.[3]

Optimum water requirements vary with the soil and climatic conditions as well as with the kind of crop. In the case of most grains, increases in depths of applied water beyond the optimum requirement cause increases in straw and fodder but decreases in kernel yields.[4]

Mixed grasses, most garden products, and a few field crops require only relatively small quantities of water. Orchard fruits, cereals, and citrus products require medium quantities, Alfalfa, Rhodes grass, sugar cane, rice, and sugar beets require

[1] The Duty of Water in Cache Valley, Utah, *Utah. Agr. Exp. Sta., Bull.* 173, 1920.
[2] PITTMAN and STEWART, Twenty-eight Years of Irrigation Experiments Near Logan, Utah, 1902–1929, Inclusive, *Utah Agr. Exp. Sta., Bull.* 219, 1930.
[3] SNELSON, W. H., Irrigation Practice and Water Requirements for Crops in Alberta, *Irrigation Series, Dept. Int., Canada, Bull.* 7, 1930.
[4] WIDTSOE and MERRILL, The Yields of Crops with Different Quantities of Irrigation Water, *Utah Agr. Exp. Sta., Bull.* 117, 1912.

relatively large quantities. Rice crops require continuous flooding during the irrigation season and consequently use large quantities of water by surface evaporation as well as by transpiration. If grown on loam or sandy soils, they also lose large quantities by percolation. Consequently, clay or adobe soils are preferable for rice production.

Tables 12, 13, and 14 give some data on water requirements of different crops, abstracted from the previously mentioned bulletins by Fortier and Young. Table 12 gives the requirements of crops in the Missouri and Arkansas river basins; Table 13 gives the requirements of crops in the southwest; and Table 14 gives the requirements of some special crops in the Columbia River and Pacific Slope basins. Inasmuch as the quantities of water used on the different fields investigated varied greatly, even in the same basins, average low and high requirements are given in the tabulations.

CONVEYANCE LOSSES AND WASTE

Losses of water between diversion works and irrigated farms, although to some extent unavoidable, should be kept as low as possible. Conveyance losses occur by evaporation from canal water surfaces, leakage at canal structures, and seepage into the bed and banks of the canals. Seepage losses are more important than either evaporation or leakage losses.

Waste of irrigation water in transit occurs at spillways and wasteways along the canals and at the lower ends of the channels. Wastage is not objectionable if the flow returns to the river channel so that it can be rediverted for use farther downstream. Better canal operation and greater flexibility in control can often be secured by diverting larger quantities than needed. Consequently, such procedure is frequently followed on projects where ample water supplies are available.

Evaporation Losses. Evaporation losses vary with the area of the canal water surface and the prevailing rate of evaporation. They may be practically eliminated by constructing closed conduits. However, such losses usually are not great enough or of sufficient value to warrant the expense of such construction. For instance, a canal surface 20 ft wide, evaporating water at a rate of a ½ in. a day, would lose about 2 acre-ft/day in a length of 20 miles, a quantity equivalent to a continuous flow of about 1 cfs.

Seepage Losses. Seepage losses vary with the size and length of the canal and the nature of the soil through which it is excavated. Such losses are greatest where sandy or other permeable soils predominate. Sometimes the losses can be materially reduced by diverting water when the river flow contains large proportions of silt. However, the most efficient method of reducing seepage, although also the most expensive, is to line the channel with concrete. In some parts of California, where water supplies are scarce, the construction of concrete pipe lines and concrete-lined canals has reduced conveyance losses to less than 5 per cent of the diversions.[1]

Data on Losses and Waste. Careful measurements, made on irrigation projects in Idaho, showed that small farm ditches, carrying less than 1 cfs, may lose half their flow in a length of 1 mile. The measurements made on canals carrying 10 to 3,000 cfs, through sections of different soil texture, showed seepage losses per mile varying from less than 0.1 to 10.8 per cent of the flow.[2] Measurements made on canals in the Salt River Valley, Arizona, showed an average seepage rate of approximately 0.34 acre-ft per acre of wetted area per 24 hr.[3]

[1] Irrigation Requirements of California Lands, *Calif. Dept. Pub. Works, Bull.* 6, 1923.

[2] BARK, DON H., Experiments on the Economic Use of Irrigation Water in Idaho, *U.S. Dept. Agr., Bull.* 339, 1916; also FORTIER, SAMUEL, Concrete Lining As Applied to Irrigation Canals, *U.S. Dept. Agr., Bull.* 126, 1914.

[3] SMITH, G. E. P., The Use and Duty of Water in the Salt River Valley, *Ariz. Agr. Exp. Sta., Bull.* 120, 1927.

Table 15 shows the average conveyance losses and waste on Bureau of Reclamation projects in western United States, expressed as percentages of total diversions. Total lengths of canals and laterals and lengths of lined or enclosed sections are also included in the tabulation. Data on predominating soil characteristics are given in Table 11. The canal and lateral losses are seen to vary from a minimum of 13 per cent of the diversions on the Uncompahgre project, western Colorado, to a maximum of 48 per cent on the Carlsbad project, southeastern New Mexico.

TABLE 15.—CANAL LOSSES AND WASTE ON BUREAU OF RECLAMATION PROJECTS[1]

State	Project	Length of canals and laterals		Gross diversions, acre-ft/acre	Per cent of diversions		
		Total miles	Lined or enclosed, miles		Canal and lateral losses	Waste	Delivered to farms[2]
Ariz.-Calif.	Yuma	336	...	10.75	14	58	28
California	Orland	135	89	4.95	27	9	64
Colorado	Grand Valley	180	7	10.31	43	22	35
Colorado	Uncompahgre	470	11	7.48	13	10	77
Idaho	Boise	1,004	37	5.15	28	2	70
Idaho	King Hill	96	43	13.21	..	..	53
Idaho	Minidoka, South Side Pumping	275	...	4.38	39	3	58
Montana	Huntley	232	...	4.09	36	30	34
Montana	Lower Yellowstone	202	...	3.83	44	21	35
Montana	Milk River	275	...	1.44	36	19	45
Montana	Sun River, Fort Shaw	99	...	4.05	36	26	38
Montana	Sun River, Greenfields	190	...	2.72	31	22	47
Nevada	Newlands	319	...	6.40	41	14	45
New Mexico	Carlsbad	45	11	5.11	48	6	46
N. M.-Tex.	Rio Grande	485	10	9.96	32	39	29
Oregon	Klamath	240	2	2.75	39	9	52
Oregon	Umatilla	173	157	10.04	32	18	50
South Dakota	Belle Fourche	547	58	2.35	33	15	52
Washington	Okanogan	68	39	3.66	29	..	71
Washington	Yakima, Sunnyside	602	125	4.70	23	7	70
Washington	Yakima, Tieton	335	86	3.39	24	2	74
Wyoming	Shoshone, Frannie	166	...	5.92	42	21	37
Wyoming	Shoshone, Garland	279	4	4.33	38	7	55
Wyo.-Neb.	North Platte	1,154	...	4.55	43	8	49

[1] Abstracted from E. B. Debler, Use of Water on Federal Irrigation Projects, *Trans. A. S. C. E.*, **94**, 1223–1224, 1930.

[2] See Table 11 for deliveries in acre-feet per acre and additional pertinent data.

Waste on the Bureau of Reclamation projects varies from a minimum of 2 per cent on the Boise project, southwestern Idaho, to a maximum of 58 per cent on the Yuma project, southeastern California and southwestern Arizona. The relatively high waste on the Yuma project is partly due to the use of water for power development and partly to the operation of the canals at high levels in order to reduce silt deposition. Wastage on some of the other projects is high because ample supplies are available and the excess diversions can be returned to the streams for rediversion to lower areas. Quantities given as waste do not include water used in sluicing sand and silt deposits at points of diversion.

IRRIGATION LOSSES AND WASTE

When irrigation water is applied to the fields, losses occur by evaporation and percolation, and waste occurs as surface runoff. The waste ceases as soon as the supply is shut off and the field surfaces drained. However, percolation and evaporation continue for some time. Percolation continues until the quantity of water that has entered the ground is reduced to the moisture-holding capacity of the soil. Evaporation continues until the losses, together with transpiration depletions, reduce the soil moisture to the point beyond which further capillary movements are negligible. Since evaporation losses can hardly be separated from plant transpiration in conducting field investigations, the two are usually considered together and designated *evapotranspiration* or *consumptive use*, a quantity discussed later.

Surface Waste. Surface waste from individual fields varies with the nature of the soil, slope of ground surface, method of preparing the land, and depth of irrigation. Naturally, larger quantities of water run off the fields during the greater depths of irrigation. Waste is more difficult to control on steep slopes than on flat slopes and more important on clay soils than on sandy soils, since either a flat slope or a sandy soil is more conducive to a higher rate of soil absorption. Waste from individual fields should be kept as low as possible, although some surface runoff is not serious if collected and used on lower areas. Surface waste can be practically eliminated, or effectively recovered, by constructing adequate levees, or drainage ditches, along the lower edges of the fields. From the practical viewpoint, the best way to minimize surface waste is to charge the farmers for all water delivered and to increase the rates in the case of excessive use.

Table 16 shows some average values of surface waste, measured on typical irrigated fields in Idaho and Utah. The shallow, medium, and heavy irrigations in the Sevier Valley investigations were 2-, 3½, and 5-in. depths in the case of the potato crops; and 3- to 4½-in., 6-in., and 8- to 10-in. depths, respectively, in the case of the alfalfa and sugar-beet crops.

Average quantities of water actually lost to beneficial use by surface waste on large irrigation projects are considerably smaller than the quantities lost on individual

TABLE 16.—AVERAGE SURFACE WASTE FROM FIELDS
(Percentages of applied quantities)

Crop	Southern Idaho 1910–1913,[1] average waste		Coal Creek Valley, Utah, 1917–1919,[2] yearly average waste, different soils	Sevier Valley, Utah, 1915–1920,[3] yearly average waste		
	Clay loam	Gravelly soil		Shallow irrigation	Medium irrigation	Heavy irrigation
Alfalfa	19.1	1.8	13–16	6–28	6–24	11–25
Grain	25.3	2.3	6–21			
Potatoes	....	...	12–25	1–17	9–28	12–28
Sugar beets	....	...	.....	3–26	17–33	20–35

[1] BARK, DON H., Experiments on the Economical Use of Irrigation Water in Idaho, *U.S. Dept. Agr., Bull.* 339, 1916.

[2] FIFE, ARTHUR, Duty of Water Investigations on Coal Creek, Utah, *Utah Agr. Exp. Sta., Bull.* 181, 1922.

[3] ISRAELSEN and WINSOR, The Net Duty of Water in Sevier Valley, *Utah Agr. Exp. Sta., Bull.* 182, 1922.

fields. Investigations in the Cache La Poudre Valley, northern Colorado, showed an average surface runoff equal to 6 per cent of the quantity applied.[1] An allowance of 10 per cent of the total quantity delivered to the land will usually be an ample provision for surface waste in considering a large irrigation project as a whole.

Percolation. Although the quantities of irrigation water lost by percolation to depths below the root zones vary with the depths of irrigation and distances the water must travel in reaching the lower edges of the fields, they are dependent primarily, on the texture of the soil. Percolation losses are comparatively unimportant on heavy clay or adobe soils but may be of serious magnitude on loose sandy soils or on shallow permeable loams underlain by open gravel formations. Such losses, although not wholly avoidable, may be minimized by using shallow depths of irrigation and laying out systems of farm laterals so that the water need not travel long distances across the field surfaces, the aim being to supply enough water to moisten the soil sufficiently for plant growth without adding enough to cause deep percolation.[2]

Some percolation is desirable, at times, in order to flush saline soil solutions beyond the root zones. However, large percolation losses during the irrigation season are undesirable, not only from the viewpoint of water conservation, but also because they leach out valuable plant food and cause the ground-water level to rise. If the ground water rises too close to the soil surface, plant roots may be rotted, soil surfaces may become alkaline, and low-lying irrigable lands may be changed into untillable seeped areas. Excessive use of water during the early history of irrigation caused the development of large areas of alkali and water-logged lands on many western projects, in some cases necessitating the construction of expensive drainage systems. Sometimes pumping plants can be installed to keep the ground-water levels from rising to dangerous heights, the pumped water being conveyed to other areas for further irrigation use.

Quantitative measurements of deep percolation under field conditions are not practicable. Some estimates have been made on the basis of soil sampling, allowances being deducted for surface waste, soil moisture evaporation, and plant transpiration from the depths delivered to the land. However, such estimates are only rough approximations. Accurate quantitative measurements of percolation may be made with small tanks or lysimeters. Although the results of such investigations are not directly applicable to field conditions, they are useful in studying the general conditions of percolation. Experimental measurements made in Idaho, using 6-ft tanks filled with porous soil and planted with alfalfa, showed that percolation could be practically eliminated, without reducing crop yield, by supplying 1.65 ft of water in 10 irrigations instead of 6.6 ft in 7 irrigations.[3] The percolation caused by the heavy irrigations amounted to 83.5 per cent of the supply. Similar tank experiments, made at Umatilla, Ore., showed percolation quantities varying from 12 per cent of the supply in the case of silt soils to 25.5 per cent of the supply in the case of coarse sandy soils, alfalfa being grown in the tanks in both instances.[4]

CONSUMPTIVE USE

The consumptive use of water is the actual quantity of water used by plant growth, plus losses by soil moisture and water surface evaporation. Studies of consumptive use on irrigated lands have been made from the viewpoints of small tanks, individual farm areas, separate irrigation projects, and entire river valleys. Sometimes the

[1] HEMPHILL, ROBERT G., Irrigation in Northern Colorado, *U.S. Dept. Agr., Bull.* 1026, 1922.

[2] SMITH, G. E. P., Use and Waste of Irrigation Water, *Ariz. Agr. Exp. Sta., Bull.* 101, 1925.

[3] BARK, DON H., Experiments on the Economical Use of Irrigation Water in Idaho, *U.S. Dept. Agr., Bull.* 339, 1916.

[4] DEAN, H. K., Work of the Umatilla Field Station in 1923, 1924, and 1925, *U.S. Dept. Agr., Cir.* 422, 1927.

Table 17.—Monthly, Seasonal, and Annual Consumptive Use of Water in Sacramento-San Joaquin Delta*

Crop or water using agency	Footnote reference	Monthly consumption, feet depth or acre-feet per acre												Total consumptive use, ft	
		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Growing season	Year
Alfalfa	1	(0.06)	(0.08)	0.10	0.30	0.40	0.50	0.65	0.55	0.50	0.20	(0.10)	(0.07)	3.20	3.51
Asparagus	2	0.05	0.05	0.05	0.05	0.08	0.14	0.40	0.68	0.55	0.42	0.12	0.10	2.69	2.69
Beans	2	(0.06)	(0.08)	(0.08)	(0.16)	(0.20)	0.14	0.24	0.58	0.37	(0.09)	(0.07)	(0.05)	1.33	2.12
Beets	2	(0.06)	(0.08)	(0.08)	0.13	0.32	0.51	0.61†	0.53†	0.20†	(0.13)	(0.10)	(0.07)	2.30	2.82
Celery	2	(0.04)	(0.04)	(0.04)	(0.08)	(0.10)	0.10	0.10	0.20	0.25	0.30	0.20	0.05	1.20	1.50
Corn	2	(0.04)	(0.04)	(0.04)	(0.08)	(0.10)	0.24	0.85	0.84†	0.40†	0.10	(0.10)	(0.07)	2.43	2.90
Fruit	1	(0.04)	(0.04)	(0.04)	0.18	0.32	0.50	0.57	0.40	0.23	0.07	(0.07)	(0.05)	2.27	2.51
Grain and hay	2	(0.04)	(0.04)	0.07	0.60	0.83	0.20	(0.14)	(0.23)	(0.21)	(0.14)	(0.07)	(0.05)	1.70	2.62
Onions	2	(0.04)	(0.04)	0.08	0.13	0.27	0.49	0.43	0.20	(0.16)	(0.13)	(0.10)	(0.07)	1.60	2.14
Pasture	3	0.08	0.10	0.20	0.25	0.25	0.25	0.25	0.25	0.20	0.15	0.10	0.08	2.16	2.16
Potatoes	2	(0.06)	(0.08)	(0.08)	(0.16)	0.15	0.38	0.52	0.30	0.15	(0.09)	(0.07)	(0.05)	1.50	2.09
Seed	3	(0.06)	(0.08)	(0.08)	0.10	0.25	0.50	0.50	0.50	0.35	0.10	(0.10)	(0.07)	2.30	2.69
Truck	3	(0.06)	(0.08)	0.10	0.10	0.25	0.50	0.45	0.45	0.30	0.15	0.10	(0.07)	2.40	2.61
Tules	4	0.16	0.09	0.30	0.74	1.10	1.28	1.53	1.32	1.18	0.98	0.59	0.36	9.63	9.63
Bare land	2	0.04	0.04	0.04	0.08	0.10	0.13	0.14	0.13	0.11	0.09	0.07	0.05	1.02	1.02
Open water surface	4	0.08	0.13	0.23	0.34	0.60	0.76	0.84	0.78	0.60	0.33	0.14	0.08	4.91	4.91
Willows	5	0.05	0.03	0.09	0.22	0.33	0.38	0.46	0.40	0.35	0.29	0.18	0.10	2.88	2.88
Idle land with weeds	6	0.07	0.09	0.10	0.19	0.24	0.31	0.33	0.28	0.19	0.15	0.12	0.08	2.15	2.15

* From Variation and Control of Salinity in Sacramento-San Joaquin Delta and Upper San Francisco Bay, *Calif. Dept Pub. Works, Bull.* 27, 1931.

† Includes additional use of water by weeds.

Figures in parentheses show estimated consumptive use on cropped areas during dormant season.

1. From experiments in adjacent areas.
2. From recent cooperative experiments by Division of Water Resources and U.S. Department of Agriculture.
3. Estimated by comparison with similar crops.
4. From data of recent cooperative experiments and other agencies, modified by Charles H. Lee.
5. From data of recent cooperative experiments and other agencies, modified by Charles H. Lee. Use based upon willows in large groves, with additional 10 per cent for isolated trees.
6. Revised data for idle land below elevation 5.0, U.S. Geological Survey datum, see Sacramento-San Joaquin Water Supervisor's Report for Year 1932, *Calif. Dept Pub. Works*, June, 1933. Use on unirrigated lands above elevation 5.0 is considered zero.

values estimated for the larger areas have been based on actual irrigated acreages. Other times, they have been based on gross acreages, including seeped lands and other uncropped areas as well as the cropped areas. Sometimes they have been calculated for the growing season, other times, for the full year. Consequently, methods of compilation must be carefully considered before drawing conclusions from published information.

Data on the consumptive use of water on irrigated lands are difficult to determine in the field because of uncertainties regarding changes in underground storage. Accurate determinations should include not only differences between inflow and outflow of applied water but also proper allowances for precipitation, changes in the moisture content of the surface soils, and differences in volume in water stored below the ground-water level. Probably the most accurate determinations of consumptive use thus far made have been the experimental results obtained in small tanks, where percolation losses as well as water supplies were carefully measured. However, there is always some question as to how such results apply to large field areas. Probably the average consumptive use on a large irrigated area is somewhat smaller than that shown by tank measurements.[1]

Tank Data. Table 17 gives some data on monthly, seasonal, and annual consumptive use of water for different surface conditions in the Sacramento-San Joaquin Delta, based on 6 years of tank experiments conducted by the U.S. Department of

TABLE 18.—CONSUMPTIVE USE OF WATER ON EXPERIMENTAL FIELD PLOTS AND FARMS[1]
(Acre-feet per acre)

Crop	Cache Valley, Utah; most profitable yields		Snake River Valley, Idaho; data by Lewis		Cache La Poudre Valley, Colorado; data by Hemphill		Brooks Experiment Station, Alberta;[2] data by Snelson, maximum yields	
	Data by Widtsoe	Data by Harris	High production	Average production	Maximum yield	Average yield	High fertility soils	Average, all soils
Alfalfa	3.3	4.6	2.89	2.55	5.8	3.7	3.03	3.03
Alfalfa seed	...	...	....	....	...	...	1.42	1.42
Peas	...	...	1.66	1.36	...	...	2.05	2.05
Beans	...	...	1.20	1.20				
Wheat	2.4	2.5	1.78	1.18	3.1	2.1	1.85	1.98
Oats	2.5	3.1	1.84	1.45	3.6	1.4	1.87	1.86
Barley	...	...	1.68	1.58	3.4	2.3	1.52	1.74
Flax	...	...	....	....	...	...	1.70	1.70
Potatoes	2.2	2.2	1.63	1.60	4.2	3.3	1.70	1.65
Mixed grasses	...	...	....	....	...	...	....	1.63
Clover	...	...	2.14	1.54				
Sugar beets	2.5	2.4	....	....	3.6	3.0	1.58	1.60
Corn	2.5	2.2	0.96	1.29	...	...	1.37	1.37

[1] Compiled from Progress Report of the Duty of Water Committee of the Irrigation Division, *Trans. A. S. C. E.*, **94**, 1358 to 1361, 1930, except as noted.

[2] SNELSON, W. H., Irrigation Practice and Water Requirements for Crops in Alberta, *Dept. Interior, Canada, Irrig. Series, Bull.* 7.

[1] Water Losses under Natural Conditions from Wet Areas in Southern California, *Calif. Dept. Pub. Works, Bull.* 44, 1933.

Agriculture in cooperation with the state of California. The data include consumptive use for bare soil, tules, willows, and open water surfaces as well as for cultivated crops. Yearly values vary from 1.02 ft for bare land to 9.63 ft for tules. Seasonal values for crops vary from 1.20 ft for celery to 3.20 ft for alfalfa.

Estimates made by the California State Engineer's Office, based on the data in Table 17, showed an average growing season consumptive use of 2.20 acre-ft/acre, and an average annual consumptive use of 2.65 acre-ft/acre, for the total area of irrigated crops in the delta during the years 1929 to 1932.[1] Corresponding estimates for the total gross area, including nonirrigated crop areas, open water surfaces, tules, willows, and other noncropped areas, were 2.62 and 2.97 acre-ft/acre, respectively.

Farm Consumptive Use. Table 18 gives some data on farm consumptive use of water by different crops, estimated from experimental measurements at small plots and typical farm areas. Although percolation losses could not be determined at any of the locations investigated, soil conditions were reported to be such as to render such losses negligible or of minor importance in the case of the Utah and Alberta measurements. Proper allowances for percolation probably would reduce the most of the values given for the Cache La Poudre Valley of Colorado.

TABLE 19.—SEASONAL CONSUMPTIVE USE OF WATER IN VALLEY IRRIGATION[1]

State	Location	Area, thousands of acres	Average elev.	Mean annual temp., deg F	Precipitation, in.		Length of growing season, days	Irrigation water, acre-ft/acre		Consumptive use,[2] acre-ft/acre	
					Mean annual	Growing season		Headgate diversion	Delivery to land	Irrigation water	Including grow. seas. rainfall
Colorado.....	North Park, North Platte River basin	120	8,200	38	10	2	60	a	a	1.00	1.2
Colorado.....	Cache la Poudre Valley	200	5,000	48	14	7	135	1.9	1.5	1.25	1.8
Colorado.....	South Platte and tributary valleys	1,100	4,600	49	14	7	145	2.0[3]	...	1.25[3]	1.8
Colorado.....	South Platte, Kersey to Julesburg	229	4,100	48	14	9	145	2.7	1.8	1.15	1.9
Idaho........	Mason Creek area, Boise project	13.6	....	50	11	3	155	4.1	...	1.20	1.4
Idaho........	Part of Boise project	49.1	2,500	50	11	3	155	....	3.6	1.30	1.6
Utah.........	Sevier Basin, Sevier to Gunnison	65	5,200	48	8	2	110	....	...	1.30	1.5
Wyoming.....	Little Laramie, Hatton to Two Rivers[3]	28	7,300	40	14	4	95	b	b	0.90	1.2
Wyoming.....	Little Laramie, Hatton to Two Rivers[4]	28	7,300	40	14	4	95	b	b	1.00	1.3

[1] Data compiled by R. I. Meeker, abstracted from Progress Report of the Duty of Water Committee of the Irrigation Division, *Trans. A. S. C. E.*, **49**, 1372 and 1373, 1930.

[2] No allowances for changes in soil moisture or ground-water storage.

[3] Twelve-year record, 1912–1923.

[4] Three-year record, 1912–1914.

a. Chiefly meadows, freely flooded.

b. Chiefly meadows, heavily flooded, compact area.

[1] Sacramento-San Joaquin Water Supervisor's Report for Years 1933 and 1934, *Calif. Dept. Pub. Works*, June, 1935.

Valley Consumptive Use. Table 19 gives some data on the seasonal consumptive use of water in valley irrigation, compiled by R. I. Meeker. Although the estimates were carefully made, they included no allowances for changes in soil moisture or ground-water storage or for losses from seeped areas. Estimates are given for seasonal irrigation deliveries and for irrigation deliveries plus growing season precipitation. Values of consumptive use estimated on the latter basis vary from 1.2 acre-ft/acre in the Little Laramie Valley, Wyoming, to 1.9 acre-ft/acre in the South Platte Valley between Kersey and Julesburg, Colo. The Bureau of Agriculture Engineering, U.S. Department of Agriculture, in a recent comprehensive investigation of water utilization in the Upper Rio Grande Valley, concluded that the annual consumptive use of water averages 1.5 ft in the San Luis Valley of Colorado and 3.2 ft along the main stream between the Colorado-New Mexico boundary and Fort Quitman, Tex.[1] These figures include losses on noncropped areas as well as use of water on irrigated areas.

A comprehensive study of the consumptive use of water in twenty valleys located throughout the United States, including humid areas as well as arid and semiarid areas, was recently reported by engineers of the Bureau of Reclamation.[2] The valleys chosen for the study included areas varying from sea-level elevations to altitudes of approximately 10,000 ft above sea level, raising diversified crops ranging from cotton and citrus fruits to meadow hay and alpine forests. The study showed that the average annual consumptive use varies approximately as the accumulated daily maximum temperatures above 32F during the growing season. Average annual values of consumptive use for the 20 valleys varied from 1.30 acre-ft per acre in Wagon Wheel Gap, Colorado, to 3.19 acre-ft per acre in the San Jacinto River valley, southeastern Texas.

SUMMARY OF IRRIGATION WATER

Conditions affecting available supplies, diversions, conveyance, waste, use, and losses of water on irrigation projects are so variable that no definite recommendations can be made for use in considering new developments. Each contemplated project is a problem in itself—a problem that must be analyzed on the basis of its local soil, topographic, hydrologic, agricultural, and climatic conditions. However, it may be pertinent to summarize, briefly, the usual seasonal limits involved in the operation of existing enterprises. The following list gives such limits for large irrigation projects in western United States, comparable with those supervised by the Federal government.

Diversions	2–10	acre-ft/acre
Canal losses	15–40	per cent of diversions
Canal waste	5–35	per cent of diversions
Deliveries to farms	30–70	per cent of diversions
Deliveries to farms	1–7	acre-ft/acre
Surface waste	5–10	per cent of deliveries
Percolation losses	5–60	per cent of deliveries
Return flow	20–60	per cent of diversions
Consumptive use	1–3.5	acre-ft/acre

The preceding limits may not apply to areas where water is scarce and where unusual precautions are taken to conserve supplies, as, for instance, in the citrus-fruit region of southern California.

LAND PREPARATION

In the case of old lake beds and other comparatively flat lands where crops may be irrigated by flooding methods, as in the mountain meadows of Colorado and Wyoming,

[1] Regional Planning, Part VI, Upper Rio Grande, National Resources Committee, February, 1938, pp. 293–427.

[2] LOWRY, ROBERT L., Jr., and ARTHUR F. JOHNSON, Consumptive Use of Water for Agriculture, *Proc. A. S. C. E.*, April, 1941, pp. 595–616.

the land may be placed under cultivation without much preliminary preparation. However, in most new developments, considerable surface preparation is necessary before the irrigation water can be efficiently applied to the fields.

Preliminary land preparations include clearing of trees and brush, removal of stumps, large roots, and boulders, and grading the fields to even surfaces. All knolls must be cut down and all depressions filled, so that water may be spread uniformly over all parts of the land. Areas having thin soil coverings may need to be cut and terraced in narrow strips. Accurate topographic surveys are needed for use in planning and carrying out land preparations, in order to secure satisfactory leveling and uniform slopes in the directions the water is to be run across the soil surfaces. Scrapers of the Fresno type, either tractor or team drawn, are often used in the grading work. Large steel or timber constructed floats are frequently used in the final leveling operations. Suitable equipment for land preparation can be secured from various irrigation machinery firms. After the preliminary preparations are completed, borders, checks, levees, and drainage ditches may be located and built to suit the contemplated methods of irrigation.

IRRIGATION METHODS

Irrigation methods vary in different sections of the country, owing to differences in topographic conditions, soil characteristics, climatic environment, kinds of crops grown, adequacy of water supply, and various special considerations. Frequencies and depths of irrigation during the growing season vary, particularly, with the soil texture, crops grown, and size of field irrigated. No definite rules can be given for the application of water on different projects. Each farmer should apply enough water to moisten the soil sufficiently for plant growth, without causing undesirable surface runoff or deep percolation, and should irrigate again before the plants begin to show the need for additional moisture. Bulletins describing methods that have been found satisfactory for different crops in different parts of the country may be secured from the U.S. Department of Agriculture and the agricultural experiment stations at the various state colleges.

New grass crops, where the seed is lightly covered, require frequent shallow irrigations after planting, in order to keep the soil from drying out and stopping germination. Irrigation after planting, before the plants attain heights of 3 in. or more, is not considered advisable for other farm crops, especially if the irrigation water carries appreciable silt loads which may form crusts on the soil surfaces. On projects where the winter precipitation is not sufficient to moisten the soil adequately, an application of irrigation water just before planting is advisable for such crops, in order to facilitate germination and permit the plants to become well established before applying additional water. In the case of some loam soils which absorb water readily and have a relatively high moisture-holding capacity, fall irrigation may be practiced for the same purpose. Fall irrigation was not found to increase crop yields in the case of the heavy clay soils on the Belle Fourche project, western South Dakota.[1] Late summer irrigation is sometimes useful in destroying weeds and supplying green manure, the ground being irrigated after harvesting and then plowed later in the fall, after the weed seeds have germinated and the weeds attained some height.

Usual Application Methods. The usual methods of applying water on large irrigation projects may be grouped under two general headings, furrow and flooding, *i.e.*, the water is either applied in furrows or allowed to flood the entire soil surface. The corrugation method is simply a modified furrow method in which relatively small and shallow furrows are excavated from 12 to 36 in. apart, depending on soil condi-

[1] FARRELL and AUNE, Effect of Fall Irrigation on Crop Yields at Belle Fourche, South Dakota, *U.S. Dept. Agr., Bull.* 546, 1917.

tions. Flooding methods may be used on large natural areas, in large basins enclosed by dikes, on long strips bounded by borders, or on small areas separated by rectangular or contour checks. Flooding water may be supplied from ditches along the edges of the fields, from farm laterals, from flumes or pipe lines, or from movable, jointed pipes which can be unjointed and the water discharged at different locations on the field. Furrows may be excavated along straight lines or along contours. They may be supplied from ditches, flumes, or pipes. From the viewpoint of water conservation, furrow methods are more desirable than flooding methods, since smaller quantities of water are lost by evaporation.

Flooding methods are often used for hay, alfalfa, and small grain crops. Furrow methods are used for corn, potatoes, orchard fruits, asparagus, and similar crops. Corrugation methods are often used for cereals, alfalfa, and sugar beets, especially on soils that crust when drying. In irrigating alfalfa and pasture lands in southern Idaho, the corrugation method has been found advisable during the first season and flooding methods satisfactory thereafter.[1] The method of flooding between checks is being increasingly used in irrigating orchard fruits in California. Flooding between borders and checks is largely practiced in irrigating cotton in the Salt River Valley, Arizona. Continuous submergence of the fields, beginning about a month after the plants emerge from the ground, is necessary in the case of rice cultivation. Flooding methods are desirable in leaching alkali lands and in flushing concentrated soil solutions beyond the root zones.

Special Application Methods. Subirrigation methods, *i.e.*, methods of applying irrigation water below the ground surface instead of above, are sometimes used on lands where sandy or other permeable soils overlie impervious soil formations, careful control of ground-water levels being thus permitted. Such methods are successfully used in some parts of the Sacramento-San Joaquin Delta, California, the Egin Bench, Upper Snake River Valley, Idaho, the San Luis Valley, south central Colorado, and in a few additionl areas of limited extent in western United States. The irrigation water, supplied in small ditches about 50 to 200 ft apart, soaks into the porous soil and raises the ground-water level high enough to moisten the root zone by capillary action. Subirrigation methods, in which water is supplied through pipes, buried in the ground, have been tried but have not proved satisfactory, owing to the clogging of the pipes by roots and the fact that the pipes must be laid relatively close together, the cost of installation being thereby increased. Subirrigation methods reduce evaporation losses, eliminate soil baking, and avoid the necessity of constructing surface borders and checks.

Spray irrigation methods are sometimes used in watering lawns, truck gardens, berries, and orchard crops in the humid sections of the country as well as in irrigating especially valuable crops in the West. In using such methods, the water is either sprayed from pipes set in the ground, from portable nozzles, or from horizontal, overhead pipe systems supported by posts. Spray irrigation practically eliminates surface runoff and deep percolation but increases evaporation. Several government bulletins on spraying methods and equipment have been published.[2]

The Division of Irrigation of the University of California has investigated the hydraulic characteristics of rotating sprinklers, the loss of water by evaporation, the hydraulic characteristics of sprinkler lines, the cost of applying water by sprinkling, and the general success of sprinkling as a method of irrigation.[3]

[1] "Experiments with Legume Crops under Irrigation," *Idaho Agr. Expt. Sta., Bull.* 94, 1917; and WELCH, J. S. "The Management of Irrigated Grass Pastures," *Idaho Agr. Exp. Sta., Bull.* 95, 1917.

[2] MITCHELL and STAEBNER, Spray Irrigation in the Eastern States, *U.S. Dept. Agr., Farmers' Bull.* 1529, 1927.

STAEBNER, F. E., Supplemental Irrigation, *U.S. Dept. Agr., Farmers' Bull.* 1846, 1940.

[3] CHRISTIANSEN, J. E., Hydraulics of Sprinkling Systems for Irrigation, *Proc. A. S. C. E.*, January, 1941, pp. 107–125.

RESULTS OF IRRIGATION

Tables 20 and 21, compiled from census reports,[1] show the results of irrigation in the 19 Western states, including Arkansas and Louisiana. Table 20 shows the growth in number of farms irrigated, total areas irrigated, and total investments in irrigation enterprises during the 50-year period from 1890 to 1940. Areas are given to the nearest 1,000 acres, and investments to the nearest 1,000 dollars. Table 21 shows the farms reported, areas irrigated, and yields per acre for some important crops produced on irrigated lands during 1939. Areas are given to the nearest 1,000 acres.

TABLE 20.—GROWTH OF IRRIGATION IN THE WESTERN STATES, 1890 TO 1940

Census year	Farms irrigated		Area irrigated		Investments	
	Number	Increase, %	1,000 acres	Increase, %	Amount, $1,000	Increase, %
1940	291,655	10.0	21,004	7.4	1,052,049	17.8
1930	265,147	19.0	19,548	1.9	892,756	28.0
1920	222,789	36.9	19,192	33.0	697,657	117.0
1910	162,723	43.0	14,433	86.4	321,454	359.2
1900	113,849	110.3	7,744	108.4	70,011	137.1
1890	54,136	.....	3,716	.....	29.534	

TABLE 21.—SOME IMPORTANT CROPS PRODUCED ON IRRIGATED LANDS IN THE WESTERN STATES DURING 1939

Kind of crop	Farms reported, number	Area irrigated, 1,000 acres	Yield per acre
Cereals:			
Barley	45,750	978	34.6 bushels
Corn	42,711	456	29.7 bushels
Oats	35,676	454	38.3 bushels
Rice	8,447	848	51.5 bushels
Rye	1,139	17	16.4 bushels
Sorghum	6,518	145	30.4 bushels
Wheat, winter	12,797	354	23.2 bushels
Wheat, spring	46,273	552	28.3 bushels
Miscellaneous:			
Alfalfa seed	8,899	219	2.2 bushels
Beans—navy, kidney, etc	19,745	449	23.2 bushels
Cotton	15,538	762	1.16 bales
Irish potatoes	39,498	360	221.0 bushels
Sugar beets	29,213	626	13.07 tons
Hay:			
Alfalfa	136,096	3,652	2.64 tons
Sweet clover	3,934	58	1.45 tons
Clover and/or timothy	13,757	534	1.37 tons
Small grain	17,862	242	1.47 tons
Wild	11,555	1,648	0.93 tons
Orchards, vineyards, and nuts	86,576	1,374	

[1] Irrigation of Agricultural Lands, Sixteenth Census of the United States, 1940, Bureau of the Census, U.S. Dept. Comm., 1942.

The total area irrigated in the Western states in 1930, about 19½ million acres, amounted to about 73 per cent of the total area irrigated in North America, and to about 10 per cent of the total area irrigated throughout the world. Results of irrigation in foreign countries were discussed in a book on "Use of Water in Irrigation," by the late Samuel Fortier, also in a government bulletin published by the Department of Commerce.[1]

Results of irrigation enterprises cannot be evaluated solely on the basis of areas irrigated and values of crops grown. Proper consideration must be given to the community developments which accompany the construction of irrigation works and the growth of prosperous agricultural areas. Many of the thriving cities and towns in western United States, with their millions of dollars of residential, commercial, and industrial valuations, have attained their present status largely as a result of the successful development of irrigation enterprises.

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SECTION 18

IRRIGATION STRUCTURES

By Ivan E. Houk

Structures on irrigation projects include dams, spillways, intakes, canals, chutes, drops, checks, turnouts, wasteways, floodways, flumes, siphons, tunnels, culverts, pipe lines, desilting works, drainage systems, pumping plants, measuring devices, and other features. Dams vary from low diversion weirs to high storage structures. Distribution channels vary from small farm ditches, where the flow can be controlled by movable canvas barriers, to huge supply canals carrying several thousand cfs. Some sections of the All-American Canal, recently built to carry water to Imperial Valley, southern California, have a bottom width of 160 ft, a permissible flow depth of 22 ft, and a capacity of 15,000 cfs. Other irrigation structures vary almost as widely.

Hydraulic features of reservoirs, dams, spillways, conveyance channels, and other major structures are described in other sections. The following discussions are confined to irrigation structures not treated elsewhere and to such special considerations as may be necessary from the viewpoint of irrigation. Except as otherwise noted, designs included as illustrations are presented through the courtesy of the Bureau of Reclamation.

DIVERSION WEIRS

Irrigation projects supplied by gravity diversions from natural streams usually require the construction of weirs near upper limits of irrigable lands, the purpose being to raise river surfaces high enough to permit controlled diversion of required canal discharges. When two or more feasible sites are available, the weir location should be determined on the basis of economic considerations. Ordinarily, adoption of a site farther upstream permits a reduction in height of weir, since the canal can usually be built on a grade somewhat flatter than the river slope. However, savings resulting from a reduced height may be offset by costs of extending the canal upstream.

Types of Diversion Weirs. Since canal flows are relatively small proportions of total stream discharges during flood periods, diversion weirs must be of overflow or open-dam types or provided with by-pass channels capable of carrying flood discharges. If by-pass channels are not feasible, the dams must either be capable of carrying flood flows over their crests or be provided with enough gates or collapsible elements to pass flood discharges without damage. Weirs designed with gates to pass flood flows are sometimes called *barrages*. They are usually built in bays, separated by piers and surmounted by operating bridges. Flow between piers may be controlled by collapsible shutters; by Stoney, radial, roller, or drum gates; or, in comparatively low weirs of inexpensive design, by horizontal removable flashboards.

Overflow weirs may vary from low temporary cobble and brush barriers, resembling beaver dams, to costly concrete arch or gravity structures. Intermediate designs may be built of logs, piles, cribs, rock, timber, steel, masonry, or reinforced concrete. Overflow weirs of permanent construction are usually provided with gate-controlled

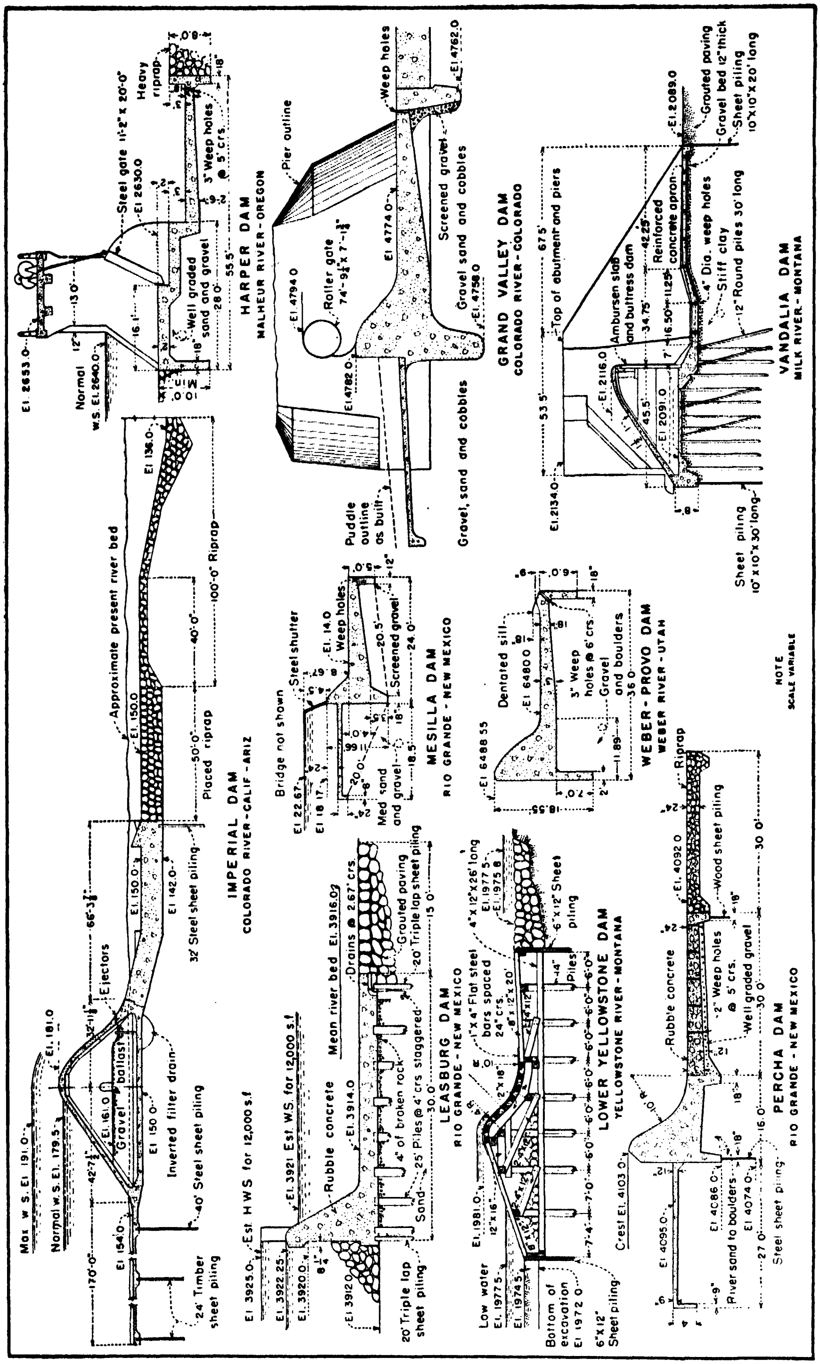

FIG. 1.—Diversion weirs on earth foundations.

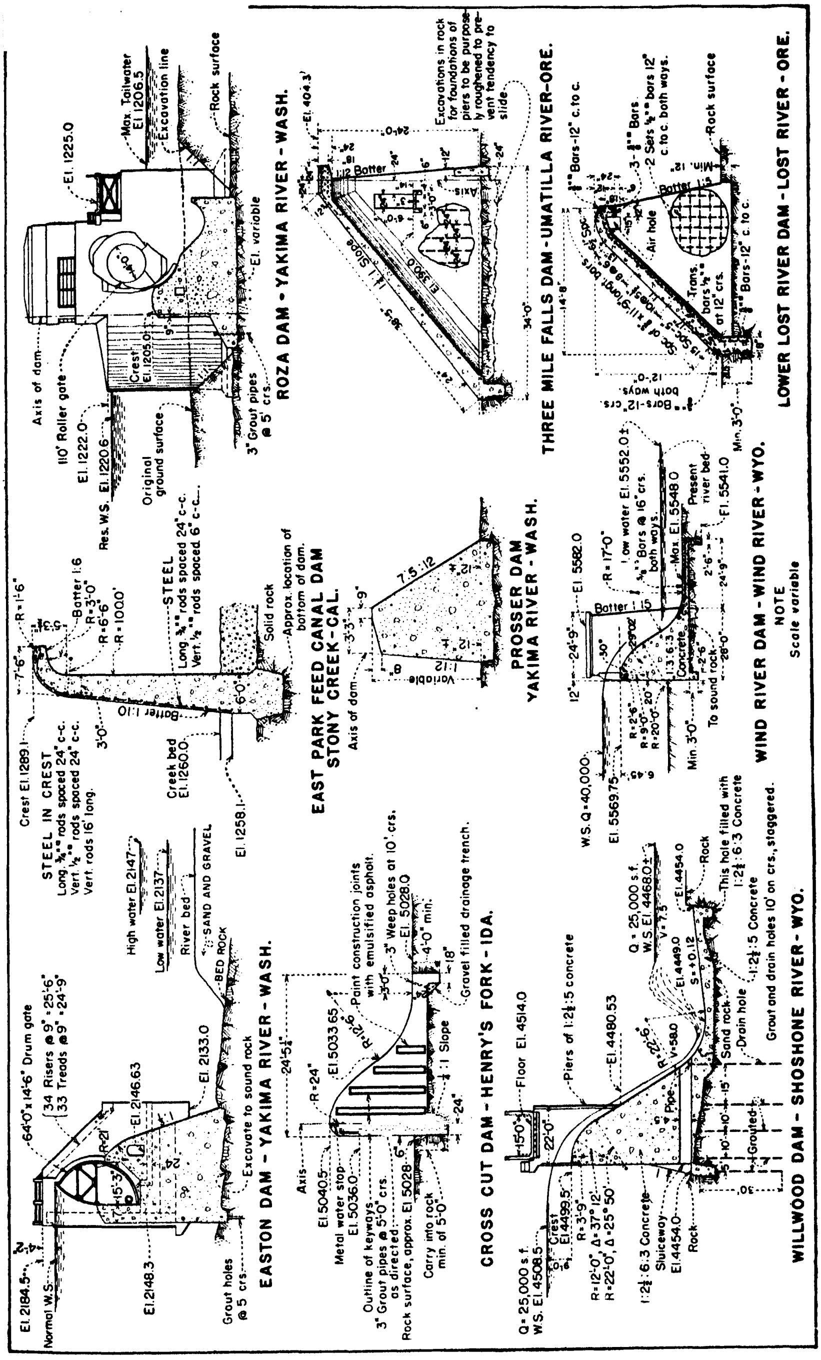

FIG. 2.—Diversion weirs on rock foundations.

sluiceways, so that upstream sand deposits can be flushed out periodically. Fish ladders, or other provisions for passing fish over the dam, may be required on streams where fishing is important, as in Northwestern United States. Earth embankments, extending to high ground, are often built at the ends of weirs, in both overflow and open-dam types.

The most desirable type of diversion weir for a given site depends on height of weir, foundation conditions, stream-flow characteristics, permissible upstream flooding, available construction materials, and amount of expense justified. Overflow types are desirable from the viewpoint of floating material, since they offer less obstruction to passage of ice, logs, brush, and miscellaneous debris. Gate installations are advantageous from the viewpoint of silt problems, since deposits above the weir are scoured downstream during flood periods. Gate installations are also advantageous from the viewpoint of canal operation, since they permit some regulation of river surfaces at intake structures.

Design of Diversion Weirs. Diversion weirs should be designed as dams. Concrete weirs on rock foundations should be made safe against failure by sliding, overturning, crushing, or rupture by tension. Analyses should include adequate allowances for uplift, impact, and dynamic forces as well as horizontal water pressures, vertical water loads, ice forces, and dead-load weights. Weirs on sand, gravel, and other earth foundations should be made safe against the same forces and should also provide ample bearing resistance, percolation distance, and protection against erosion. A comprehensive investigation of more than 200 masonry dams on earth foundations, including analyses of piping and percolation movements, has recently been published.[1] Detailed methods of design are discussed in the sections on dams, spillways, and gates.

Figure 1 shows some condensed cross sections of diversion weirs built on earth foundations. Figure 2 shows some cross sections of weirs built on rock foundations.

RIVER INTAKES

In rivers having stable beds and low-water discharges considerably greater than diversion requirements, intake structures sometimes may be built without constructing diversion dams. However, such cases are rare. Ordinarily, intake structures are appurtenant parts of diversion weirs and are built at one or both ends of the weirs. Sometimes they are constructed short distances upstream, as when water is diverted into tunnel sections instead of open channels. Since obstruction of river flow causes silt deposition above the weir, sand sluices must often be constructed in connection with intakes, especially when weirs are of the overflow type. Screens, to keep fish out of the canals, may be required at intakes, depending on the fishing characteristics of the stream.

Design of Intakes. Intakes should be designed to draw water from river channels with minimum entrance losses and as little disturbance as possible. Temporary timber structures may be built on small projects, but permanent concrete construction usually is desirable. Ordinarily, a permanent structure consists of one or more bays, controlled by radial or vertical slide gates, the bays being separated by piers and surmounted by an operating platform. Vertical slide gates are suitable for small intakes; but radial gates are usually more economical and easier to operate. Stoney or roller gates may be used at large diversion structures. Overpour types have the advantage of drawing water from river surfaces, but undershot types are less likely to become clogged by sediment.

In diversions to concrete-lined conduits, where water is carried at relatively high velocities, total areas of gate openings are usually made about the same as conduit areas. In diversions to earth canals, where water is carried at relatively low veloci-

[1] LANE, E. W., Security from Under-seepage, Masonry Dams on Earth Foundations, *Trans. A. S. C. E.*, **100**, 1235–1351, 1935.

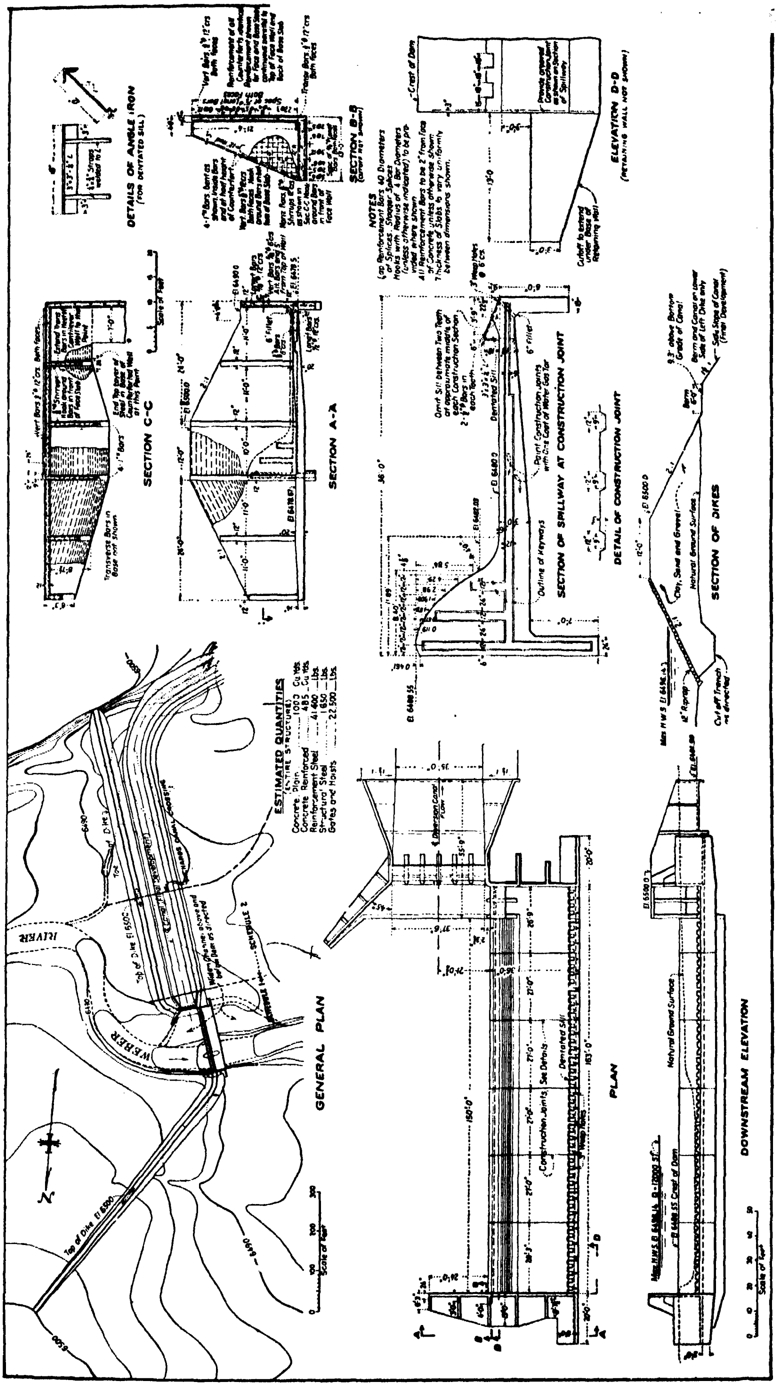

FIG. 3.—Intake structure at Weber-Provo Diversion Canal, Salt Lake Basin project, Utah.

ties, total gate areas are usually based on velocities of about 5 fps. Consequently, they usually vary from about 50 to 75 per cent of canal areas. However, full gate openings can be utilized only during periods of low river flow and high irrigation demand. At other times, water is diverted through partial gate openings, and gate velocities may be considerably higher than 5 fps.

A transition structure, consisting of a concrete floor and diverging curved or warped side walls, should connect the gate section with the head of the canal. The length of the transition should be great enough to quiet entrance disturbances and permit the gradual adjustment of flow to canal dimensions. The gate sill, with connecting transition floor, is usually placed at the same elevation as the canal bed. Portions of the abutment walls and piers upstream from the gates should be rounded to reduce entrance losses. Figure 3 shows an intake structure recently built on the Salt Lake Basin project, northern Utah.

Sand Sluices. Sand sluices are usually located at ends of diversion weirs, just below or adjacent to intake structures, so as to draw water from the front of the intake gates. Sluiceway sills should be placed somewhat lower than intake sills. The size and number of sluice gates required depend on the silt-carrying characteristics of the stream.

DISTRIBUTION SYSTEM

The distribution system includes conduits, conveyance structures, regulating works, protective elements, and miscellaneous features needed to deliver water to all parts of the irrigable area. On projects where irrigable lands are located on both sides of the river, or where water is diverted at more than one point, the distribution system may include two or more separate units. When parts of the irrigable area cannot be supplied by gravity flow, the system may also include pumping units, consisting of pumping plants, pressure pipe lines, and various incidental structures required by such methods of distribution.

On some gravity projects, as where water supplies are limited, distribution systems may consist principally of pipe lines, or closed conduits, with their necessary appurtenant works. However, distribution systems on most gravity projects consist, primarily, of open canals and appurtenant canal structures.

Conduits on a gravity project usually include main canals, branch canals, laterals, sublaterals, such additional ditches as may be necessary to convey water to individual farm units, and such waste canals as may be needed to carry undelivered water back to natural channels. Conveyance structures may include chutes, drops, transition sections, inverted siphons, lined or unlined tunnels, flumes, aqueducts, closed conduits, or other crossings at railways, highways, or drainage courses. Regulating works usually consist of canal checks, turnouts, and major gate structures such as division works, large lateral intakes, and wasteways. Protective elements include automatic spillways, tributary flood overchutes, settling basins, and sand traps. Miscellaneous features include drainage inlets, drainage culverts, farm and highway bridges, measuring structures, and other incidental construction.

Naturally, features for two or more purposes are often combined in one structure. For instance, box culverts, checks, radial gate wasteways, and siphon spillways were combined in one structure at two locations on the Casper Canal, Kendrick project, Wyoming. Furthermore, a single feature may serve dual purposes. Wasteways, although essentially of a regulating nature, may also constitute protective structures, since they may be used to sluice sand and silt deposits off canal beds, to divert canal flow during emergencies, or to discharge excess drainage waters which enter canal sections during storm periods. Various elements of the distribution system are discussed on the following pages.

CONDUITS

Conduits on most gravity projects are canals, excavated through earth formations, but may include lined or unlined sections through tunnels or open rock cuts or along side-hill locations where one bank may be formed by an earth fill or a concrete retaining wall. Pipe, flume, and tunnel conduits, which are often used, are discussed in Sec. 10.

Canals should be lined with concrete or selected earth materials when excavated through porous sand or gravel deposits, where relatively high seepage losses may occur and where such losses cannot be adequately reduced by running silty water. Earth lining has been used effectively for preventing seepage on several government canals, the lining being placed about 8 in. thick and consisting of a mixture of clay, sand, and gravel. Concrete lining may be needed where steep longitudinal slopes require conveyance of water at relatively high velocities or in deep cuts where savings in excavation costs, resulting from carrying water at high velocity, overbalance lining costs. Lined canals are usually preferable from operation and maintenance viewpoints, except that maintenance costs may be relatively high for concrete-lined canals in cold climates.

Canal Location. Main canals are usually located along higher edges of irrigable areas but may follow interior ridges, depending on local topographic conditions. Branch and lateral canals may follow interior ridges or may be located at approximately right angles to general ground slopes. Canal locations should follow roads or property lines whenever possible. On the Garland Division, Shoshone project, northern Wyoming, the main canal parallels a railroad right of way which runs diagonally across the irrigable area, lateral canals branching off on each side at 1.5- to 3.0-mile intervals and running approximately parallel around the comparatively uniform basin slopes. The basic criterion for canal location is that maximum water surfaces in laterals and sublaterals must be high enough to permit deliveries to farm units.

In planning a canal system for a proposed irrigation project, approximate office locations of principal canals are made on topographic maps. Final locations are then made in the field, where the engineer can give more adequate consideration to local requirements.

Canal Capacities. Capacities for which irrigation canals must be designed depend on conveyance losses, irrigable areas served, and maximum quantities of water needed during the irrigation season. Main and branch canals are usually operated continuously during the growing season. Lateral and sublateral canals are sometimes operated on the rotation plan. In such cases, the smaller canals must be designed for relatively greater capacities per unit area served.

Main and branch canals on large irrigation projects, where only small proportions of the lengths are lined, are usually designed on the basis of 1 cfs serving from 50 to 100 acres. On projects where large proportions of the lengths are lined, 1 cfs may serve 100 to 150 acres, depending on local soil, topographic, agricultural, and seasonal rainfall conditions. Capacities of laterals, per unit area served, usually should be 10 to 25 per cent greater than main canal capacities. Sublateral and farm-ditch capacities usually should be 25 to 50 per cent greater. Farm-ditch capacities should seldom be less than two irrigating heads.

Canal Cross Sections. Unlined earth sections should ordinarily be provided with side slopes of 1½:1 to 2:1, depending on earth materials. Extremes of 1:1 and 2½:1 have been used. Side slopes of 1¾:1 and 2:1 are being used on the All-American Canal. Rock sections and lined earth sections may be provided with steeper side slopes. Relatively large canals are often designed with berms between excavated sections and waste banks. When canal sides are in fill and the excavated earth is not

uniformly satisfactory, core banks of selected fine materials should be specified along the inner slopes, extending from natural ground levels to a minimum height of 12 in. above maximum canal water surfaces. In certain cases, it may be desirable to compact the core material by sprinkling and rolling. Freeboard provisions should vary with size of canal, nature of canal banks, possible variation in water surface during full operation, and extent of damages that may result from breaks in canal banks. Concrete linings usually should be carried 9 to 24 in. above maximum canal water surfaces. Table 1 gives principal dimensions and flow conditions for a few canals recently constructed on Federal irrigation projects.

TABLE 1.—DESIGN OF SOME IRRIGATION CANALS RECENTLY CONSTRUCTED BY THE BUREAU OF RECLAMATION

Canal	Project	Wetted perimeter	Bottom width, ft	Side slopes	Flow depth, ft	Kutter's n	Velocity, fps	Discharge, cfs	Freeboard ft.[1]
Ogden-Brigham	Ogden River	Concrete	4	1¼:1	2.48	0.014	4.82	85	0.75
S. Branch, Kittitas Div.	Yakima	Concrete	5	1¼:1	3.80	0.014	5.94	220	1.00
Main	Vale	Concrete	12	1¼:1	6.81	0.014	4.30	600	1.20
Black Canyon, Payette Div.	Boise	Concrete	12	1¼:1	9.18	0.014	4.81	1,042	1.80
Yakima Ridge, Roza Div.	Yakima	Concrete	14	1¼:1	11.20	0.014	7.02	2,200	1.80
Main, Kittitas Div.	Yakima	Lined rock	12	½:1	9.83	0.014	7.94	1,320	1.25
S. Laterals, Succor Creek Division	Owyhee	Earth	6	1½:1	2.60	0.025	2.57	66	1.90
N. Canal, Dead Ox Flat Division	Owyhee	Earth	12	1½:1	4.40	0.0225	2.52	206	2.90
Milner-Gooding	Minidoka	Earth	30	1½:1	5.20	0.025	2.83	556	3.00
Black Canyon, Payette Div.	Boise	Earth	24	1½:1	10.40	0.0225	2.53	1,042	4.60
Yakima Ridge, Roza Div.	Yakima	Earth	55	1½:1	12.00	0.0225	2.51	2,200	4.00
All-American	Boulder Canyon	Earth	118	2:1	14.70	0.020	3.50	7,600	6.00
All-American	Boulder Canyon	Earth	160	1¾:1	20.60	0.020	3.75	15,155	6.00
Main, Grav. Ext. Div.	Minidoka	Rock	20	¼:1	5.30	0.035	10.10	1,140	
Yuma	Yuma	Rock	60	½:1	8.46	0.030	3.68	2,000	Deep cut
All-American	Boulder Canyon	Rock	69	¾:1	20.10	0.035	6.00	10,155	Deep cut
All-American	Boulder Canyon	Rock	94	¾:1	22.80	0.035	6.00	15,155	Deep cut
Milner-Gooding	Minidoka	Rock and gunite	16	¼:1	5.60	0.025	5.57	550	1.00

[1] To top of concrete in lined canals.

TABLE 2.—VALUES OF KUTTER'S ROUGHNESS FACTOR FOR USE IN DESIGNING IRRIGATION CANALS

Wetted perimeter	Canal description	Roughness factor, range
Concrete	Sections free from curvature	0.013–0.014
Concrete	Sections containing curvature	0.015–0.017
Concrete and gunite	Lined bottom with gunited rock side slopes	0.020–0.025
Concrete and rock	Retaining wall and lined bottom, one rock side	0.020–0.025
Concrete and rock	Retaining wall, unlined bottom and one rock side	0.025–0.030
Rock	Main, branch, and large lateral canals	0.030–0.035
Rock	Small canals, rough excavation	0.035–0.040
Earth	Main, branch, and large lateral canals	0.020–0.025
Earth	Small laterals and farm ditches	0.025–0.030

Canal cross sections may be designed by Kutter's formula, using roughness factors n selected from Table 2. All-American Canal sections were designed for roughness factors of 0.020 to 0.025 in unlined earth sections, 0.014 in concrete-lined sections, and 0.035 in unlined rock sections. Proper factors for unlined or partly lined rock sections depend on size of canal, smoothness of excavation, and proportion of wetted perimeter lined with concrete. Sometimes flow in rock canals can be economically improved by lining the bottom with concrete and guniting the side slopes. Gunite may also be used in repairing damaged linings.[1] Many comprehensive tables for use in designing canal cross sections have been published.[2]

CONVEYANCE STRUCTURES

Conveyance structures are needed where canal grades or cross sections change abruptly or where canals cross highways, railways, or drainage courses. Chutes or drops are needed when canal flows are suddenly dropped to lower levels; transitions are needed when cross sections suddenly change, as from an earth to a lined, closed, or pipe section, or vice versa; and inverted siphons, flumes, or aqueducts are needed at crossings. Chutes and drops are often used as wasteways as well as in lowering canal sections.

In permanent construction, which is always desirable if economically feasible, conveyance structures are usually built of reinforced concrete, gunite, steel, or treated timber. Wood staves, treated or untreated, are frequently used in permanent siphon, pipe, or flume installations. Untreated timber may be used in temporary construction.

Chutes. Chutes may be pipes, flumes, or open lined channels. A typical chute usually includes an intake structure, designed to control upstream water surfaces and regulate inflow; a relatively long inclined section in which the greater portion of the drop takes place; and an outlet structure designed to destroy the excessive energy developed in the inclined section. Chutes are often used on relatively steep slopes where a single drop or series of drops would be more expensive or less desirable from other viewpoints. Concrete, wood-stave, or steel pipes or flumes are used for the smaller discharges. Wood-stave pipes or flumes, used as chutes, may leak, owing to intermittent operation. Open, concrete-lined sections are usually preferable for the larger discharges.

Velocities in long inclined portions of open chutes are generally greater than critical, the drop through critical stage occurring in the inlet section. Consequently, surface curves in inclined portions usually belong in the class where neutral depths are less than critical depths and actual depths are intermediate between critical depths and neutral depths, the neutral depth being the depth at which the surface slope parallels the bottom slope.[3] Water surfaces along inclined portions of chutes should be determined by back-water or drop-down calculations, proper allowances being made for changes in velocity head as well as friction losses. Friction losses in concrete-lined chutes may be computed by Manning's or Kutter's formula, using roughness factors of 0.012 to 0.014, depending on smoothness.

Narrow, deep, rectangular, or trapezoidal cross sections are sometimes desirable for relatively small discharges; but wide shallow sections are usually desirable for relatively large discharges. Semicircular cross sections are sometimes used in open-lined chutes as well as in flume chutes.

[1] Reeves, A. B., Concrete Rehabilitation Work on the Uncompahgre Project, *Jour. Am. Concrete Inst.*, January–February, 1937, pp. 303 to 310.

[2] "Hydraulic and Excavation Tables," 6th ed., Bureau of Reclamation, 1934.

[3] Woodward, Sherman M., Theory of the Hydraulic Jump and Backwater Curves, Part III, Technical Reports, The Miami Conservancy District, Dayton, Ohio, 1917.

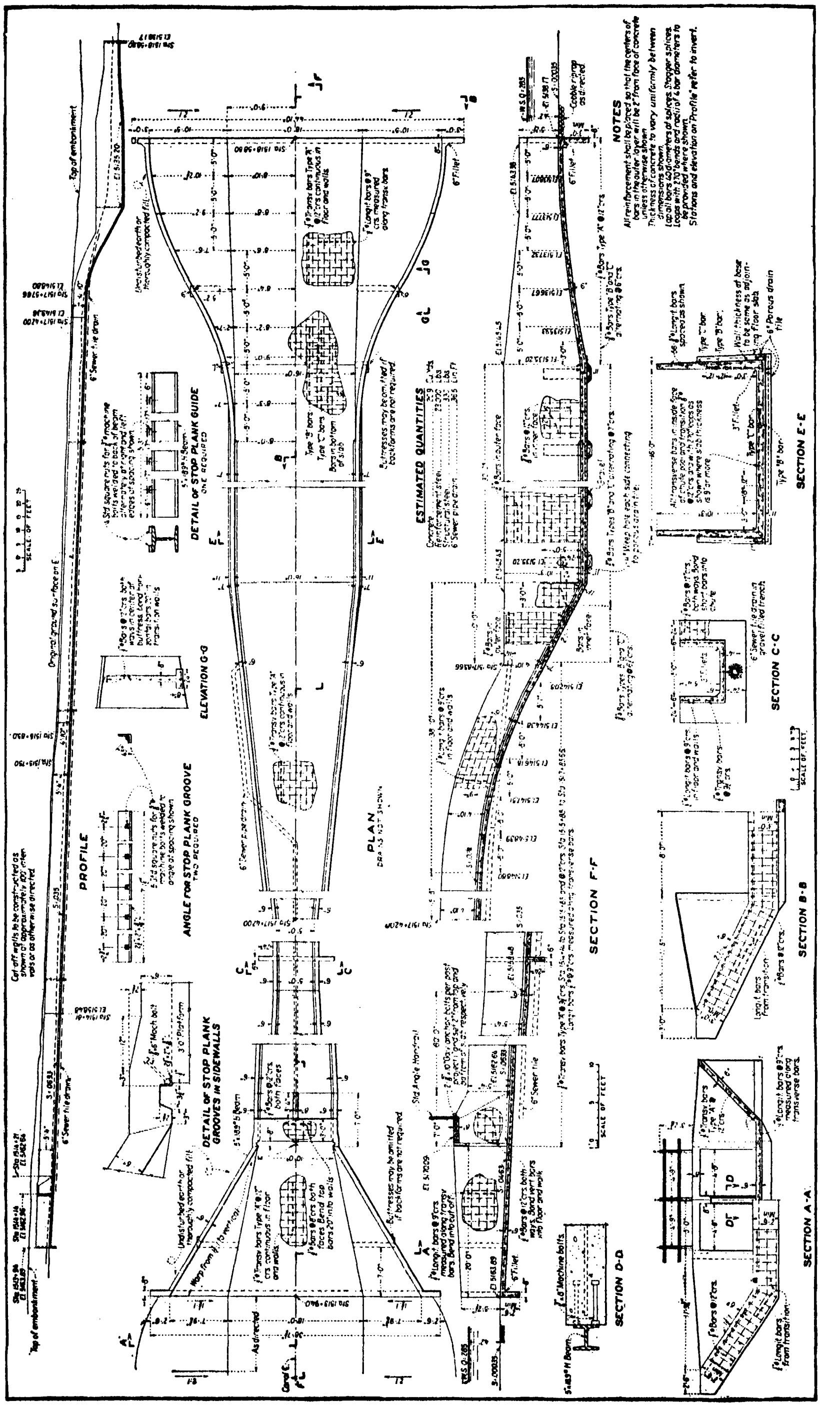

FIG. 4.—Open lined chute on Pilot Canal, Riverton project, Wyoming.

Open chutes, carrying water at high velocities, often contain waves and agitated water surfaces, even though hydraulic jumps occur at the outlets. Consequently, ample freeboard must be provided in inclined sections. Additional freeboard provisions are required in outlet structures because of the splashing troubles which may be

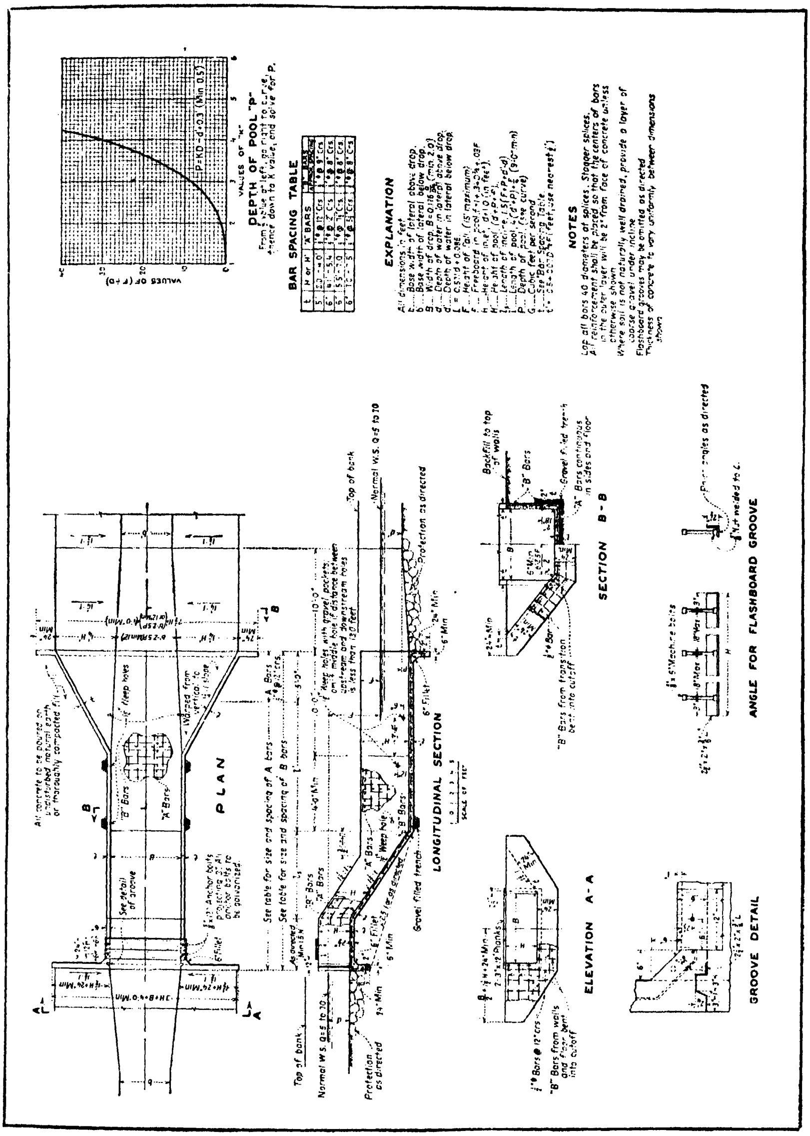

FIG. 5.—Rectangular inclined drop for capacities of 5 to 70 cfs.

experienced and because the volume of the water may be increased from 10 to more than 25 per cent, owing to air entrained in the jump transformations.

The outlet structure is usually some form of stilling basin. It may be provided with a stepped floor, baffle walls, baffle piers, or other features designed to facilitate

energy dissipation. Model tests are often desirable in designing outlet sections for chutes carrying large discharges. The inlet structure may be provided with stop planks, check gates, a rectangular weir, a Cippoletti weir, or a control section, a control

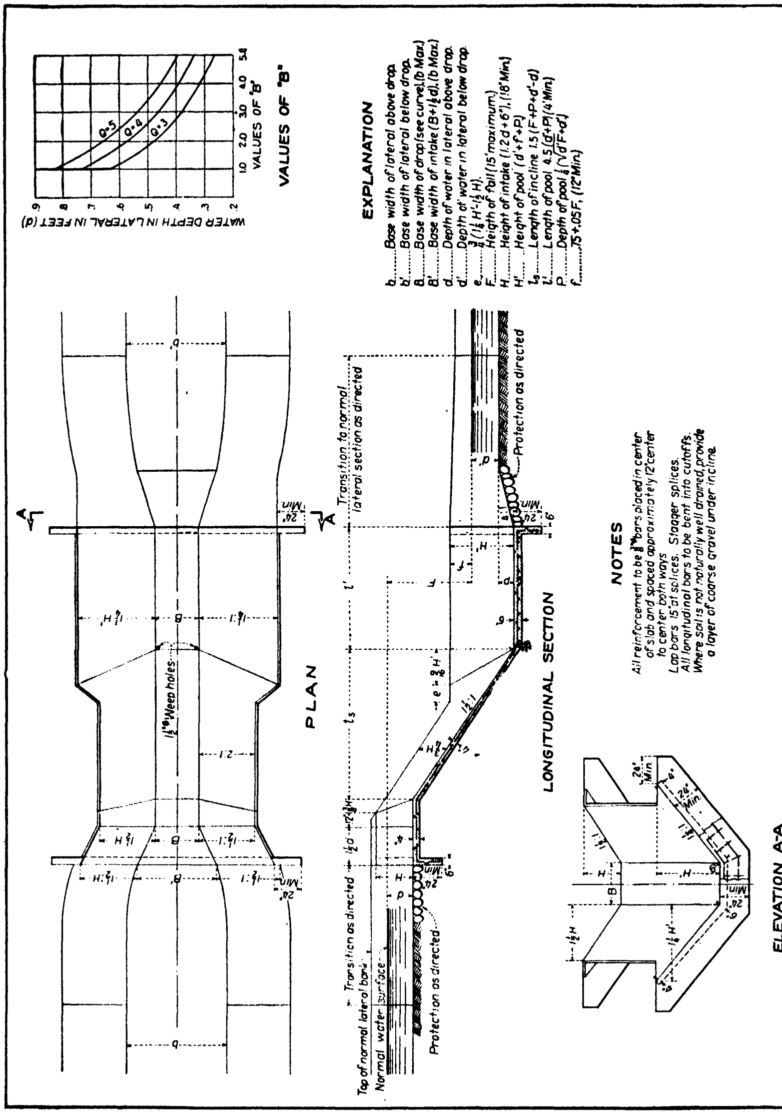

FIG. 6.—Trapezoidal inclined control section drop, maximum capacity 5 cfs.

section being one designed so that flow passes through at critical depth. Figure 4 shows an open lined chute, recently built on the Pilot Canal, Riverton project, Wyoming.

Drops. Drops are similar to chutes and fulfill essentially the same purposes. They are used when the required lowering of the water surface is relatively small and can be effected in a comparatively short distance. A great many vertical notched drops were built during early irrigation work. However, during recent years, there

has been a growing tendency to abandon such structures in favor of short, inclined, concrete drops.

Figure 5 shows the design of a rectangular inclined drop which is sometimes used on Federal irrigation canals for discharges up to 70 cfs and grade lowerings up to 15 ft. Figure 6 shows the design of a trapezoidal inclined control section drop which is sometimes used for relatively small discharges and grade lowerings up to 15 ft. The control section drop has been found satisfactory for single installations. However, when several are installed close together on the same slope, the relatively high velocities caused by drawdown curves may necessitate continuous concrete linings between drops, in which case chute construction probably would be preferable. Radial gates are being used to control water surfaces at upstream ends of drops in the All-American Canal, all but one of which are power drops.

Transitions. Transition sections are built to conserve head. Their purpose is to minimize losses where the velocity is increasing and to recover as much head as possible where the velocity is decreasing. They are used at inlets and outlets of flumes, inverted siphons, and closed conduits, as well as at places where the shape of the canal cross section suddenly changes. In open conduits, they are usually concrete sections with gradually converging or diverging side walls, but may also include gradually changing bottom grades. In closed conduits, gradual transformations are made at tops and bottoms of sections as well as at sides. Sharp angles should be avoided in all major transition structures. Transitions in small lined ditches and at ends of small flumes and turnouts may include straight bottom grades and straight converging or diverging side walls; but all large transitions should be designed with curved or warped section transformations.

The proper length of a transition depends on the relative change in shape of section, initial velocity, and velocity change, the longer transitions being required for the higher velocities and greater velocity changes. Outlet transitions, where velocities are decreasing and flow conditions relatively unstable, should be 10 to 20 per cent longer than inlet transitions, where velocities are increasing and flow conditions relatively stable. Curves in alignment, either within or close to transition sections, have disturbing effects that tend to increase hydraulic losses. Laboratory measurements in small rectangular channels, containing 180-deg curves with inner radii equal to channel widths, have shown curve losses as great as two-tenths the velocity head.[1] However, irrigation conduits seldom, if ever, contain such pronounced curvature.

In carefully designed, warped, transition structures, free from curves in alignment, hydraulic losses are probably less than 0.1 the difference in velocity head at the inlet and less than 0.2 the difference in velocity head at the outlet.

As a general rule, designs for transition structures should include the following allowances for hydraulic losses, including friction, the exact allowance to be made in a given case depending on the size of the transition and the care exercised in design.

1. Inlet transitions, free from curves in alignment, 0.1 to 0.3 the velocity head of the smaller cross section.
2. Outlet transitions, free from curves in alignment, 0.2 to 0.5 the velocity head of the smaller cross section.
3. Inlet and outlet transitions, curved alignments, add 0.05 to 0.1 the head to values considered applicable for straight alignments.

Careful hydraulic calculations should be made in planning all important transition structures, in order to secure efficient designs and avoid complications that may be experienced if flow occurs at or near critical depths.[2] Ample freeboard should be provided at the outlet, so that the water surface will not rise above any part of the structure in case the velocity head recovered should be greater than assumed.

[1] YARNELL and WOODWARD, Flow of Water around 180-degree Bends, *U.S. Dept. Agr., Tech. Bull.* 526, 1936.

[2] SCOBEY, FRED C., The Flow of Water in Flumes, *U.S. Dept. Agr., Tech. Bull.* 393, 1933.

Figure 7 shows well-designed, warped transitions constructed between lined earth and rock sections on the Kittitas Main Canal, Yakima project, Washington. Figure 8 shows warped inlet and outlet transitions at a monolithic reinforced-concrete siphon recently built on the Heart Mountain Canal, Shoshone project, Wyoming.

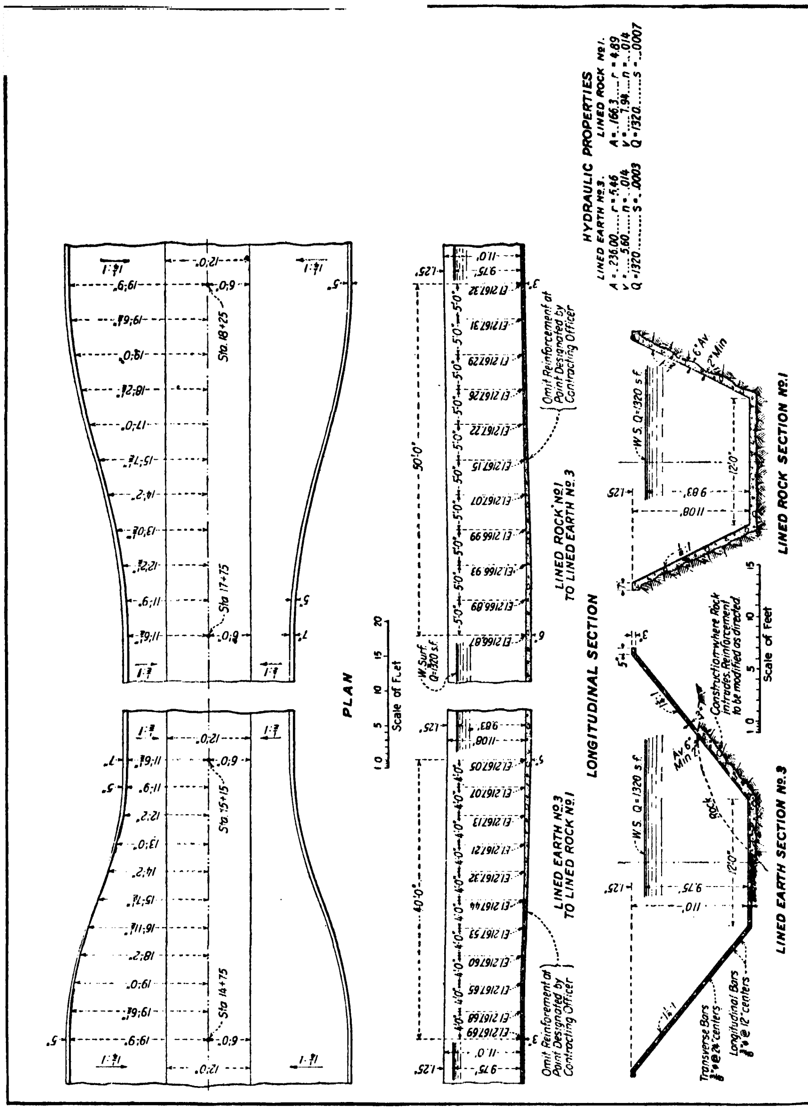

FIG. 7.—Warped transition sections, Kittitas Main Canal, Yakima project, Washington.

Inverted Siphons. Inverted siphons are used to carry canal discharges under highways, railways, streams, or across relatively long and deep drainage courses where construction of flumes or other aqueducts at continuous canal grades would be too expensive or ojectionable for other reasons. They usually consist of reinforced-concrete inlet and outlet transitions with connecting barrels of reinforced concrete,

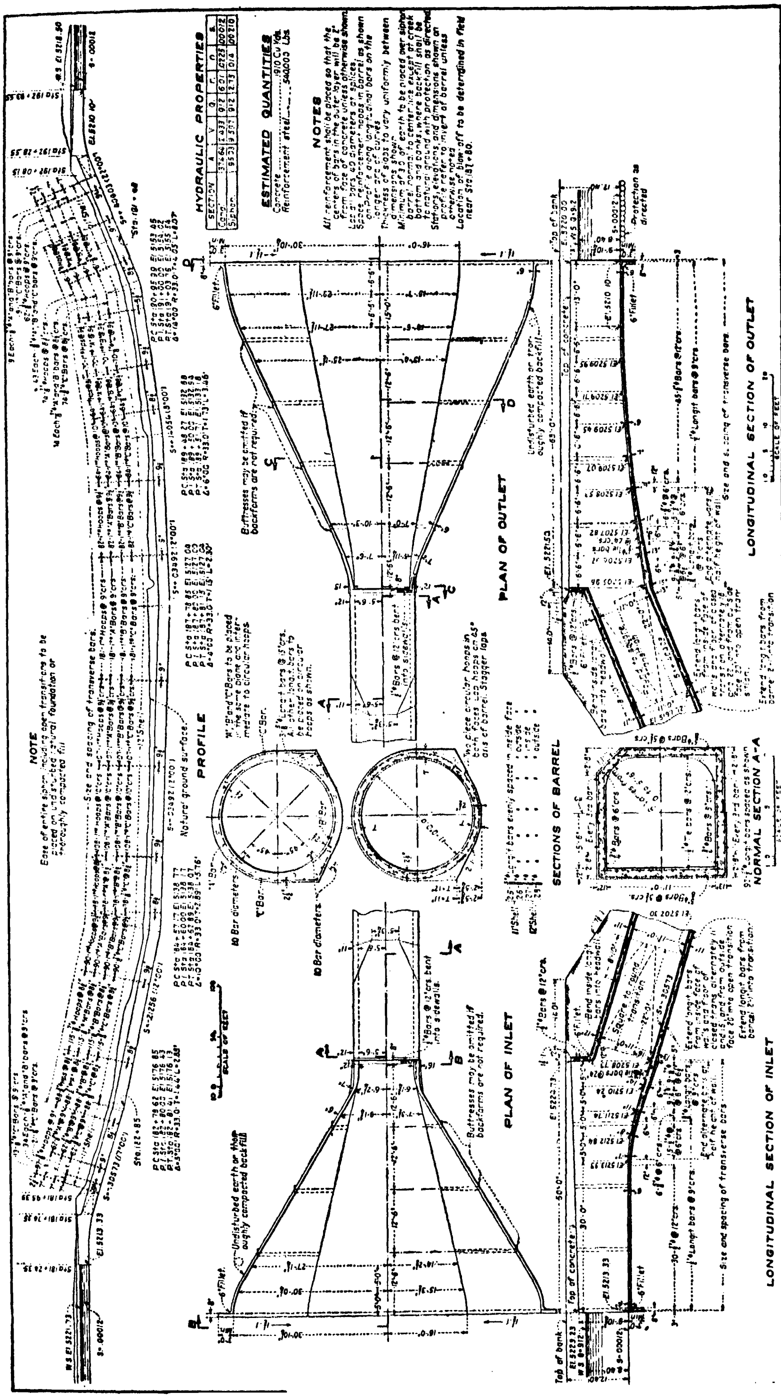

FIG. 8.—Reinforced-concrete siphon, Heart Mountain Canal, Shoshone project, Wyoming.

wood staves, or steel. Precast concrete pipes are sometimes used for barrels in the smaller installations. They have been used up to 120 in. in diameter. Monolithic reinforced concrete is usually preferred for relatively large siphons built mostly in

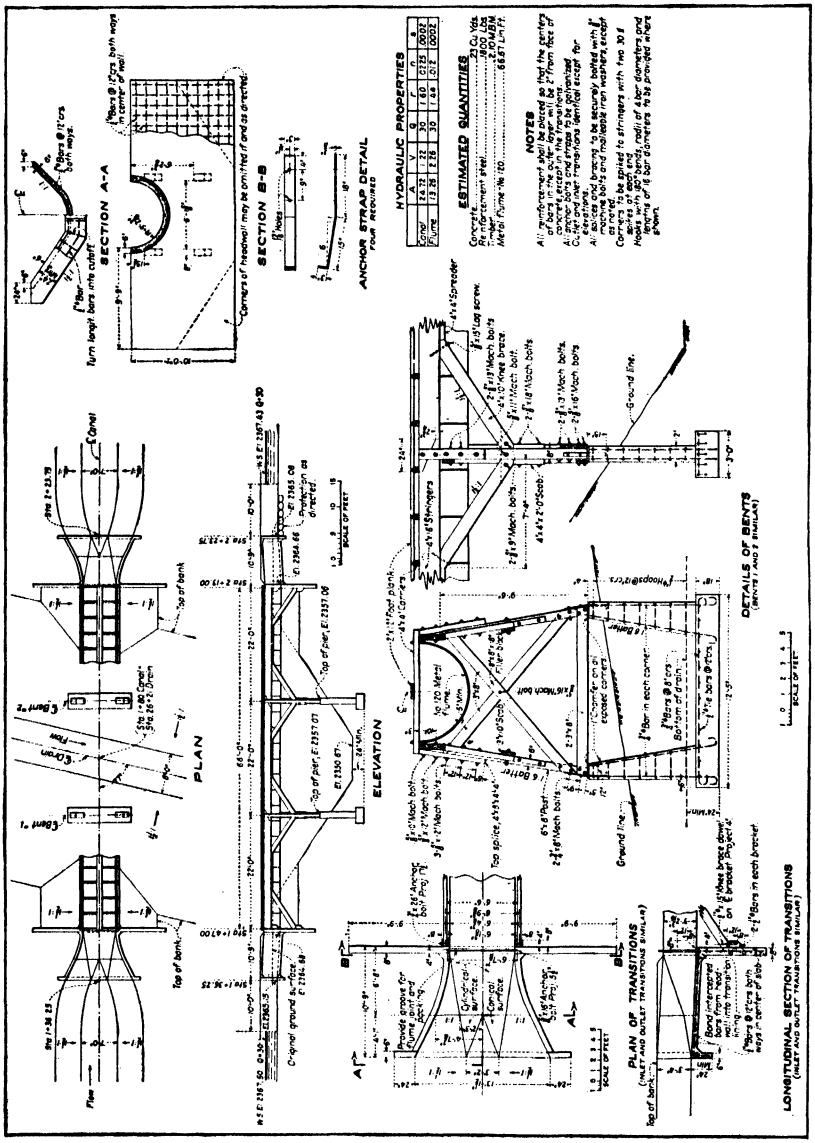

FIG. 9.—Metal flume over Sage Creek main drain, Owyhee project, Idaho.

excavation. If the use of concrete is being considered, soil formations along the proposed location should be investigated for alkali conditions. Steel or wood-stave pipes are often used when siphons are built aboveground or are carried across central portions of canyons on bridges or trestles.

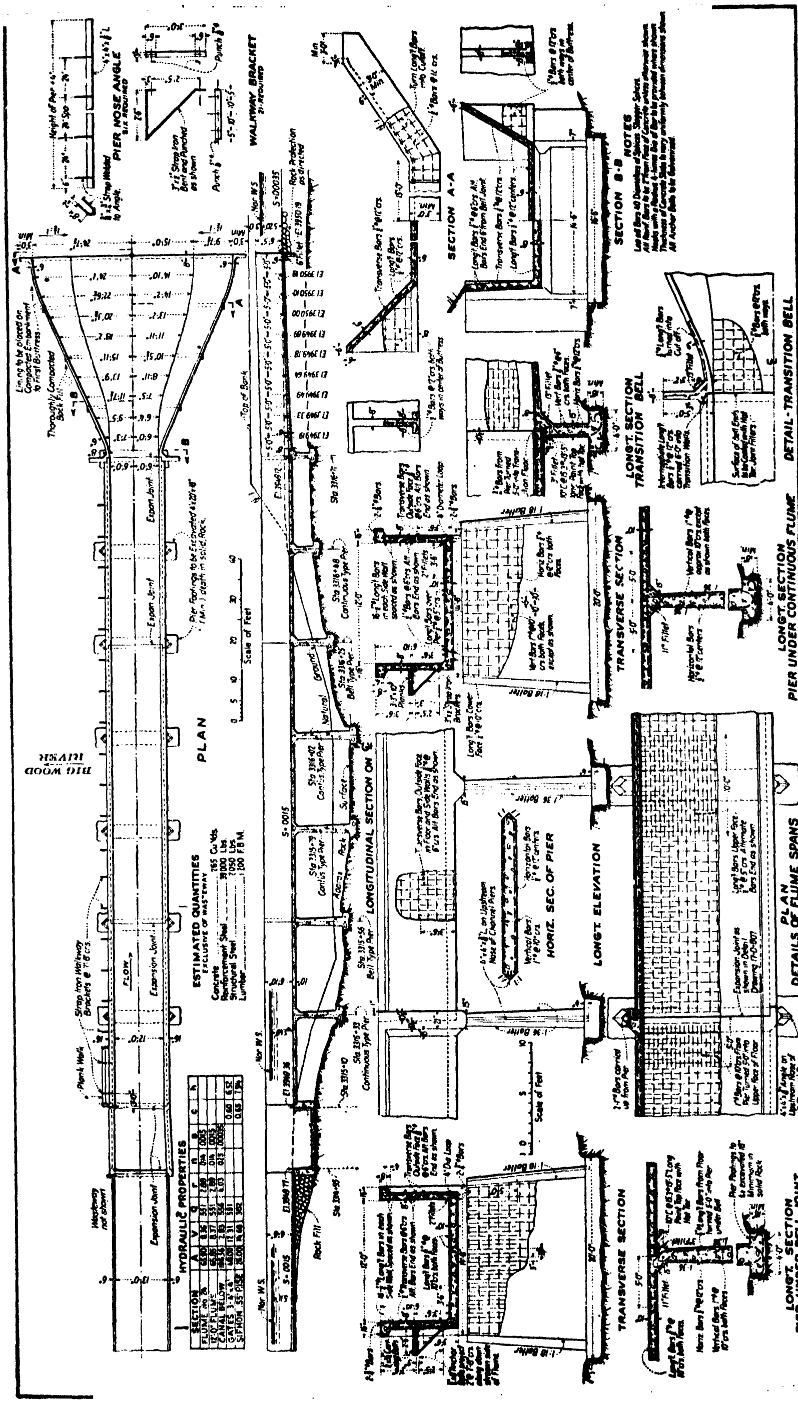

FIG. 10.—Reinforced concrete aqueduct over Big Wood River, Minidoka project, Idaho.

Wood-stave barrels may have the disadvantage of leaking after periods of nonuse, especially if built of untreated timber. Steel barrels may have the disadvantage of paint scaling and rusting, on the inside as well as the outside, when not carrying water. If the siphon is to be drained during the nonirrigation season, a blowoff valve should be installed near the bottom of the barrel. Provision of stop plank grooves at the inlet may be advisable, so that flow through the siphon can be shut off during the priming of the canal, after which the grooves may be filled with planks, to provide smooth continuous flow surfaces. Figure 8 shows the detailed design of the Heart Mountain Canal siphon mentioned above. Inverted siphons at road crossings are similar to those at drainage crossings, but usually operate under lower heads and are often made square or rectangular in cross section.

Flume Crossings. Flume crossings at canal grades may include timber, wood-stave, metal, or reinforced-concrete aqueducts. They may be supported on piles, pile bents, structural-steel bents on concrete pedestals, concrete piers, structural-steel trusses, or other construction. Timber-box flumes, supported on piles or pile bents, are often used to carry farm deliveries; metal flumes, supported on pile or structural-steel bents, are frequently used to carry lateral discharges; and large metal or reinforced-concrete flumes, supported on concrete piers, steel trusses, or other construction, are used to carry main canal, branch canal, and large lateral discharges.

Practically all flumes connecting canal sections should be provided with inlet and outlet structures, (see discussion of transitions). Inlet grades should be fixed so that ample head will be available to overcome transition losses and increase the velocity to flume requirements. In relatively small flumes, such as those required for farm deliveries, outlet losses need no special consideration, except that ample freeboard should be provided to prevent overlapping of the banks or outlets. Riprap or other protection against scour may be needed for short distances beyond outlet transitions, depending on flume velocities and composition of canal beds. Flume sections should usually be designed so that velocities are less than critical and so that ample freeboard is available. Semicircular flumes, free from curves in alignment, are usually provided with freeboards varying from 6 to 10 per cent of the diameter, the larger percentages being used for the smaller flumes, and no freeboards of less than 3 in. being used in any installations. Curved flumes require greater freeboards, especially along outer edges of curves.

Figure 9 shows a No. 120 metal flume built over the Sage Creek main drain, Succor Creek division, Owyhee project, Idaho. Figure 10 shows a reinforced-concrete aqueduct, built to carry the Milner-Gooding Canal over the Big Wood River, Minidoka project, southern Idaho.

Additional material on flumes is presented in Sec. 10.

Other Crossings. Canal crossings under highways or railroads, where canal grades can be continued without interruption, may consist of pipes or concrete box culverts through road embankments or of open channels with various types of overhead bridges. Such structures are designed to conform with usual highway or railway specifications.

REGULATING STRUCTURES

Regulating structures are built where canal water surfaces or discharges must be controlled. Checks are built to raise canal surfaces so that deliveries can be made to relatively high lands. Major gate structures are built where main canals divide into branch canals, usually called *division works*, at large lateral intakes and at wasteways. Wasteway structures often include automatic spillway elements, briefly mentioned in the discussion of protective structures, and also gate-controlled wasteway

elements. Turnouts are used at lateral, sublateral, and farm-ditch intakes, where deliveries are made through embedded conduits or cuts in canal banks.

Temporary regulating works and minor control structures may be constructed of timber, but permanent works of major dimensions are usually built of reinforced con-

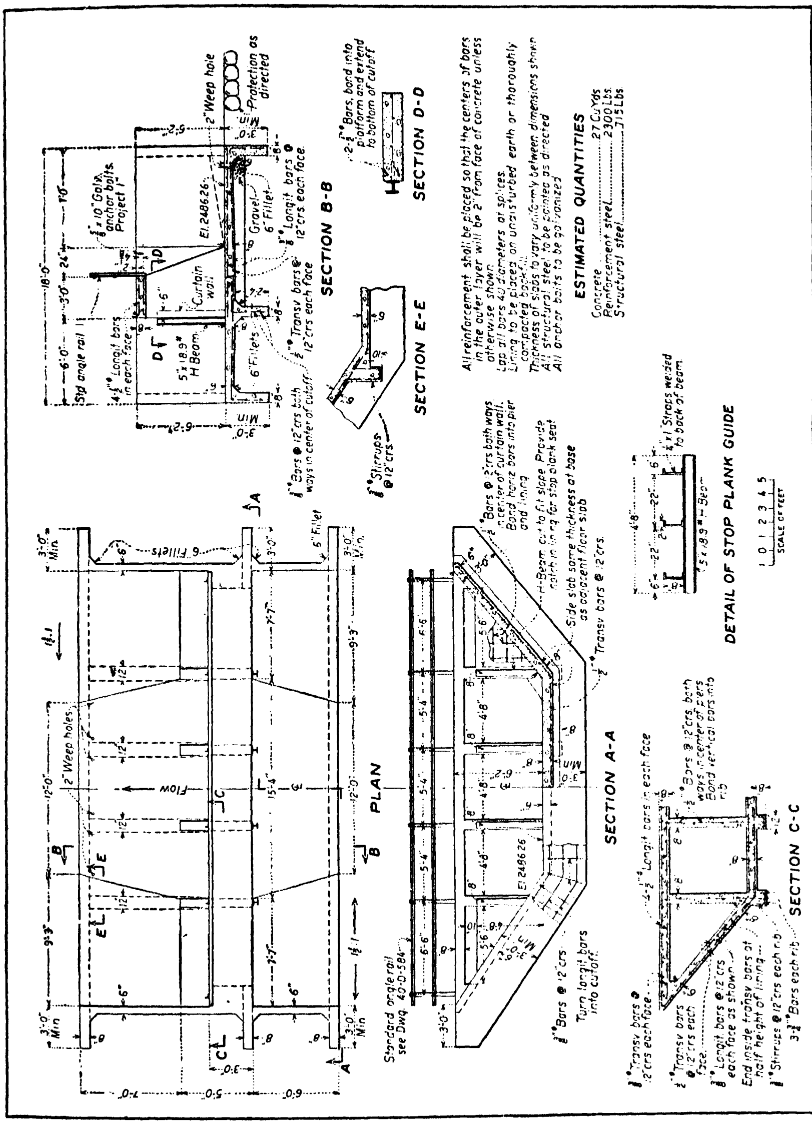

FIG. 11.—Concrete check on South Canal, Succor Creek division, Owyhee project, Idaho.

crete. Metal, wood, or concrete conduits may be used in turnouts. Water surfaces and discharges are regulated by timber flashboards or various types of steel gates.

Checks. Most check structures on irrigation canals of ordinary dimensions are designed for flashboard regulation. However, some checks have been built with

radial or other gate controls. Radial gates are being specified for check structures on the All-American Canal.

Checks on canals of ordinary size are usually short concrete-lined sections provided with piers, about 6 ft apart, to hold flashboards in place, and with narrow footbridges from which flashboards may be inserted, or removed, as required. Bottom widths of canals are widened at check sections, to compensate for contraction caused by piers. Some riprap protection may be needed below concrete floors of checks, depending on velocities and composition of canal bed. Figure 11 shows a concrete check recently constructed on the South Canal, Succor Creek division, Owyhee project.

Division Works and Intakes. Division works and large lateral intakes are similar to river intakes, but are subjected to less uncontrolled variation in water surfaces at upstream ends. They usually consist of one or more bays where flow is controlled by gates. Stoney or roller gates may be used in structures of unusual size; but radial or vertical slide gates are commonly installed in structures of ordinary dimensions. Radial gates are now more frequently used in regulating canal discharges than formerly, owing, largely, to recent improvements in gate seals.

In division structures, gates are usually installed at the head of each branch canal, so that the division of flow can be regulated at all times and so that flow in any branch can be shut off when desired. At lateral intakes, main canal water surfaces may be controlled by checks.

Total gate openings at lined canal intakes are made approximately the same area as canal flow sections, since the canals are designed to carry water at relatively high velocities. At earth canal intakes, total gate openings are usually based on a velocity of about 5 fps. However, actual gate velocites are often higher, owing to operation at partial openings. Reinforced-concrete transition structures are provided below the gates, so that velocities can become adjusted to canal requirements before leaving transition sections. Transition side walls and floors may also be needed above the gates, depending on the design of canal sections and layout of gate structures.

Hydraulic losses through gate openings are seldom controlling factors in designing large gate structures. When gates are operated at full openings, entrance losses are simply transition losses. When operated at partial openings, available heads are not being fully utilized, so that increased losses due to gate contractions are not important. Figure 12 shows the South Branch Canal headworks, Kittitas Main Canal, Yakima project, Washington.

Wasteways. Wasteways are built at locations where excess canal discharges are diverted into natural drainage courses. They include gate structures and such artificial wasteway channels as may be necessary to reach natural channels. Ordinarily, wasteway capacities should be the same as maximum discharges expected in canal sections. Natural channels should be capable of carrying maximum canal wastage together with such direct flood drainage as may be expected. Wasteway gate structures are similar to division works and large lateral intakes, except that they may be provided with somewhat larger gate openings, especially if they are to be used as sand sluices or as outlets for flood flows entering canal sections.

When wasteways are used for sand sluicing, gate sills are placed low enough and wasteway channels made large enough, to produce drawdown curves and scouring velocities in canal sections above the gates. In such cases, check structures should be built in the canals, just below the wasteways, and some concrete or riprap protection against abnormal erosion should be provided just above and below the gate structures. Sand deposition above the wasteways may be aided by enlarging canal sections and depressing canal beds, thus forming sand traps.

Wasteway structures are often built in combination with automatic spillway elements. In such cases, wasteway gate openings may be somewhat smaller than

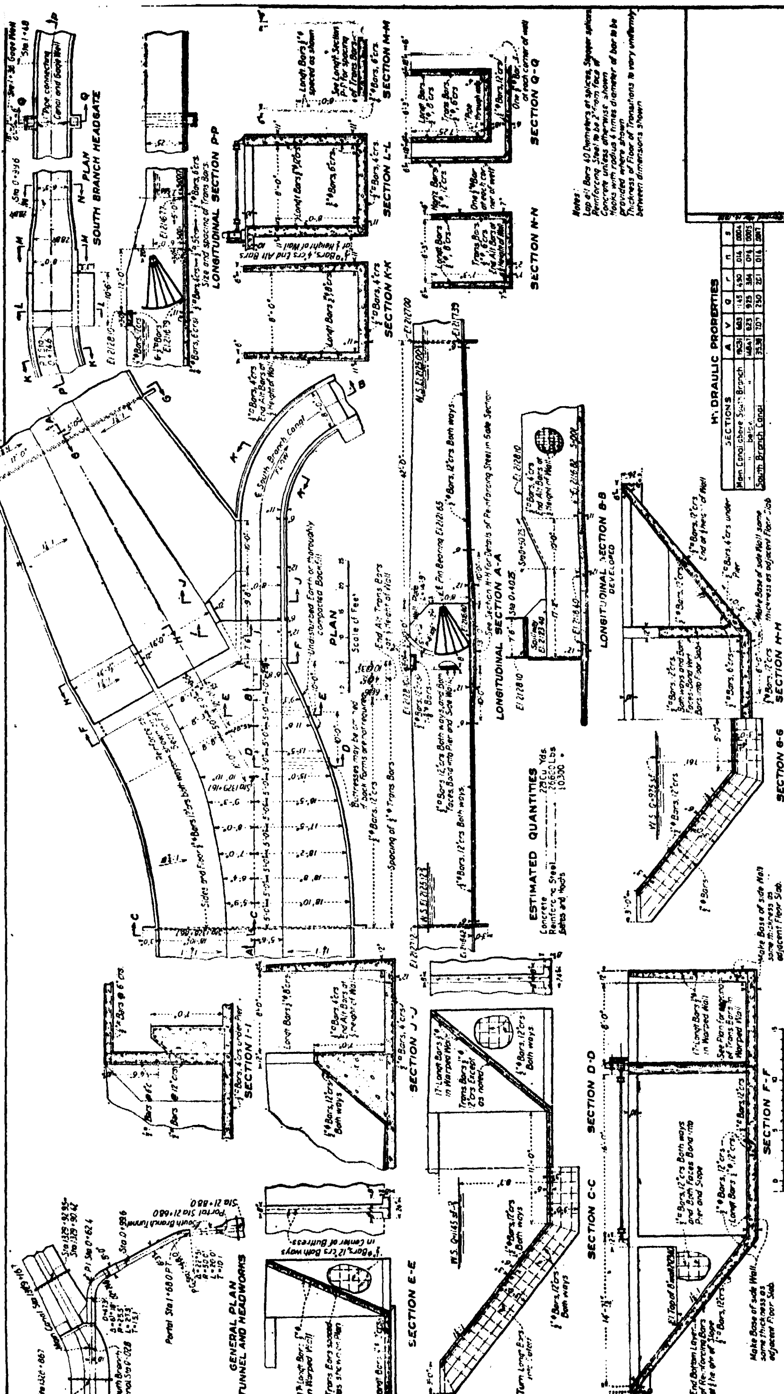

FIG. 12.—South Branch Canal headworks, Kittitas Main Canal, Yakima project, Washington.

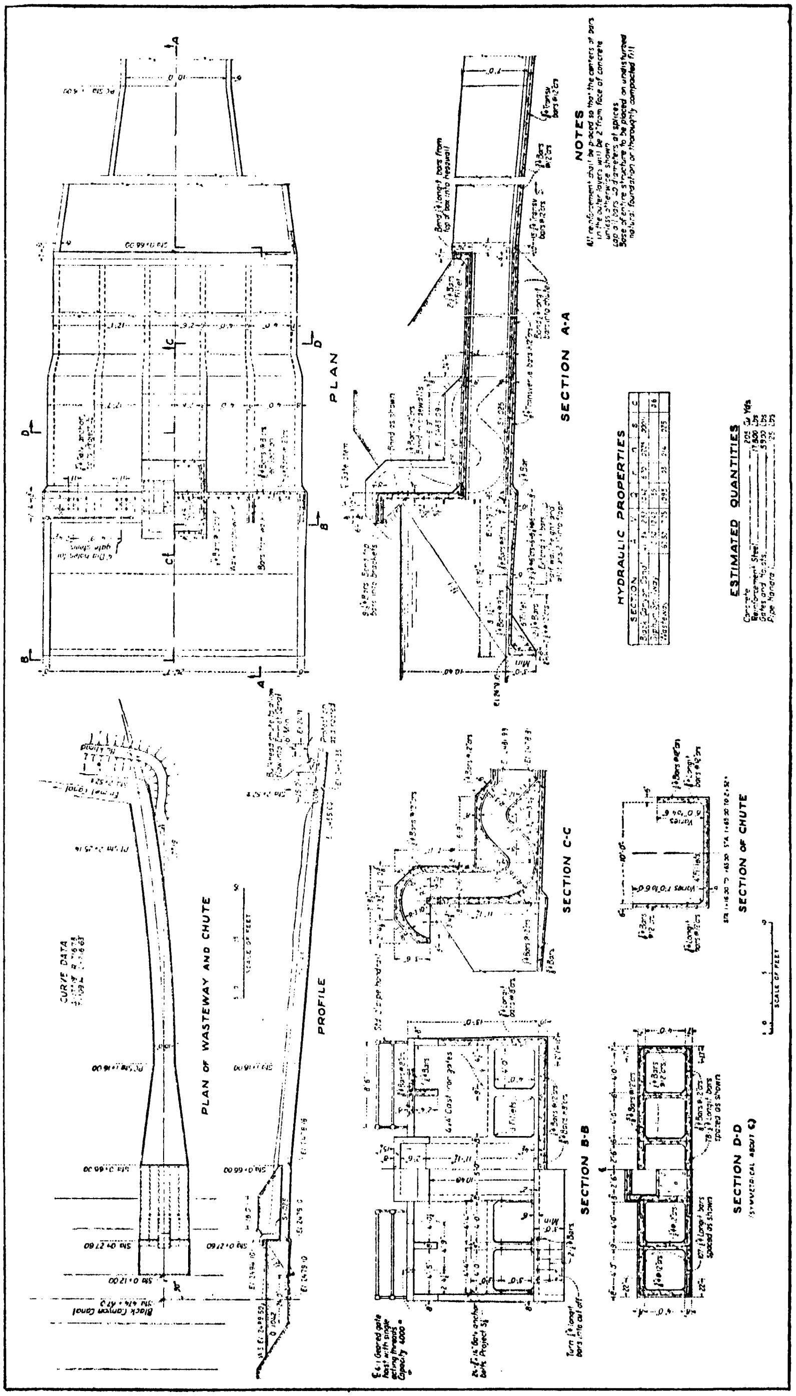

FIG. 13.—Wasteway structure with siphon spillway, Boise project, Idaho.

otherwise would be necessary. Figure 13 shows a slide-gate wasteway structure, including a siphon spillway unit, recently built on the Black Canyon Canal, Boise project, Idaho.

Turnouts. Turnouts are used where relatively small proportions of canal discharges are withdrawn. When built on large lateral or main canals, they usually

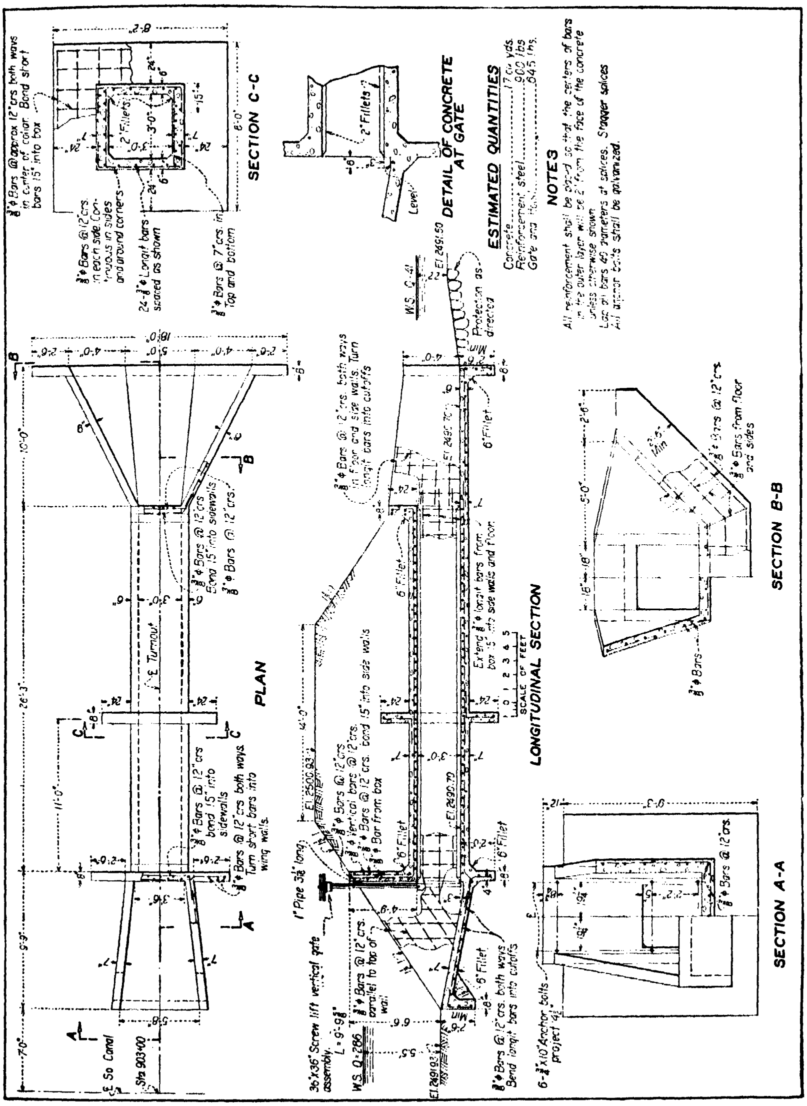

FIG. 14.—Concrete box turnout, Owyhee project, Idaho.

consist of reinforced-concrete inlets and head walls at the canal sides, with connecting, single- or double-barrel, pipe or box culvert conduits running through canal embankments to heads of ditches. Head-wall openings are generally provided with slide-gate controls, vertical in earth canals but often sloping in lined canals. In the latter case,

canal linings may be thickened, locally, to form head walls. Pipes through embankments are usually metal, wood, precast concrete, or monolithic concrete. Box culvert conduits are built of reinforced concrete. Transition sections are provided at outlet ends of conduits, and riprap protection is usually needed below the transition.

Gate sills are generally placed at elevations of canal beds, or somewhat lower, with concrete floors and converging concrete side walls extending from canal beds to gates. Tops of conduit outlets are usually placed at approximately the same elevations as water surfaces in ditches. However, some submergence at the outlet end may be necessary to ensure the occurrence of the hydraulic jump within the conduit. In relatively short conduits, air needed in jump transformations can be drawn from outlet ends. Relatively long conduits should be provided with air inlets, just below the intake gates.

Ordinarily, conduit sizes are determined on the basis of a 5-fps velocity at the outlet ends. Control gates are usually operated at partial openings, and gate velocities may be calculated by the formula $V = 0.75 \sqrt{2gh}$, where h is the head on the center of the opening. Friction losses in conduit sections may be neglected except in unusually long turnouts. Figure 14 shows a concrete box turnout, recently built on the Succor Creek South Canal, Owyhee project, Idaho.

Special designs, involving larger conduits, different gate controls, or entirely different construction may be necessary for unusually large turnouts or where available head is limited, as when water must be drawn from the upper part of the canal to supply relatively high areas. Radial-gate controls were specified for the larger turnouts on the All-American Canal, where capacities are greater than about 100 cfs.

In farm diversions from small lateral ditches, turnouts are often controlled by small slide gates or stop planks, set in timber or concrete frames, with no connecting pipe or culvert sections. Sometimes small lateral turnouts consist of timber or concrete distribution boxes, provided with gates or stop-plank grooves at the sides, so that farm deliveries can be made to small ditches or pipes running in one, two, or three directions.[1] Movable canvas dams are often used in turning water out of farm ditches.

PROTECTIVE STRUCTURES

Protective structures are built to prevent damages to the distribution system by flooding or silt deposition. Automatic spillways are built to prevent canal water surfaces from overflowing canal banks when discharges are suddenly checked or when flood flows enter canals. Overchutes are sometimes built to carry tributary flood discharges across canals at places where aqueducts, flume crossings, or inverted siphons are impracticable, to keep flood flows from entering canal sections. Settling basins are built to remove the heavier sand and silt loads in river diversions, which otherwise would settle to the canal beds and impair the efficiency of the system. Sand traps are built at places where local sand troubles are pronounced.

Automatic Spillways. Automatic spillways may be uncontrolled overflow crests, crests equipped with automatic gates, or enclosed crests designed for siphon operation. They are built at locations along canal sides, where flow can be wasted into natural drainage courses. They are often built in combination with other structures, as shown in Fig. 13. Hydraulic designs for spillways are discussed in another section.

Overchutes. An overchute is usually some type of flume running transversely over the canal. It is provided with inlet and outlet sections, consisting of erosion-protected floors and training walls of masonry or riprapped embankments, designed to prevent tributary flood flows from overtopping canal banks and discharging into the

[1] MARR, JAMES C., The Corrugation Method of Irrigation, *U.S. Dept. Agr. Farmers' Bull.* 1348, 1931; also FORTIER, SAMUEL, The Border Method of Irrigation, *U.S. Dept. Agr. Farmers' Bull.* 1243, 19?7.

canal section. It may or may not be provided with a stilling basin at the outlet end, depending on local soil and topographic conditions. Overchutes are used where ordinary types of canal crossings are impracticable and where heavy loads of sand, gravel, and debris carried by tributary floods would soon clog culverts under canal sections.

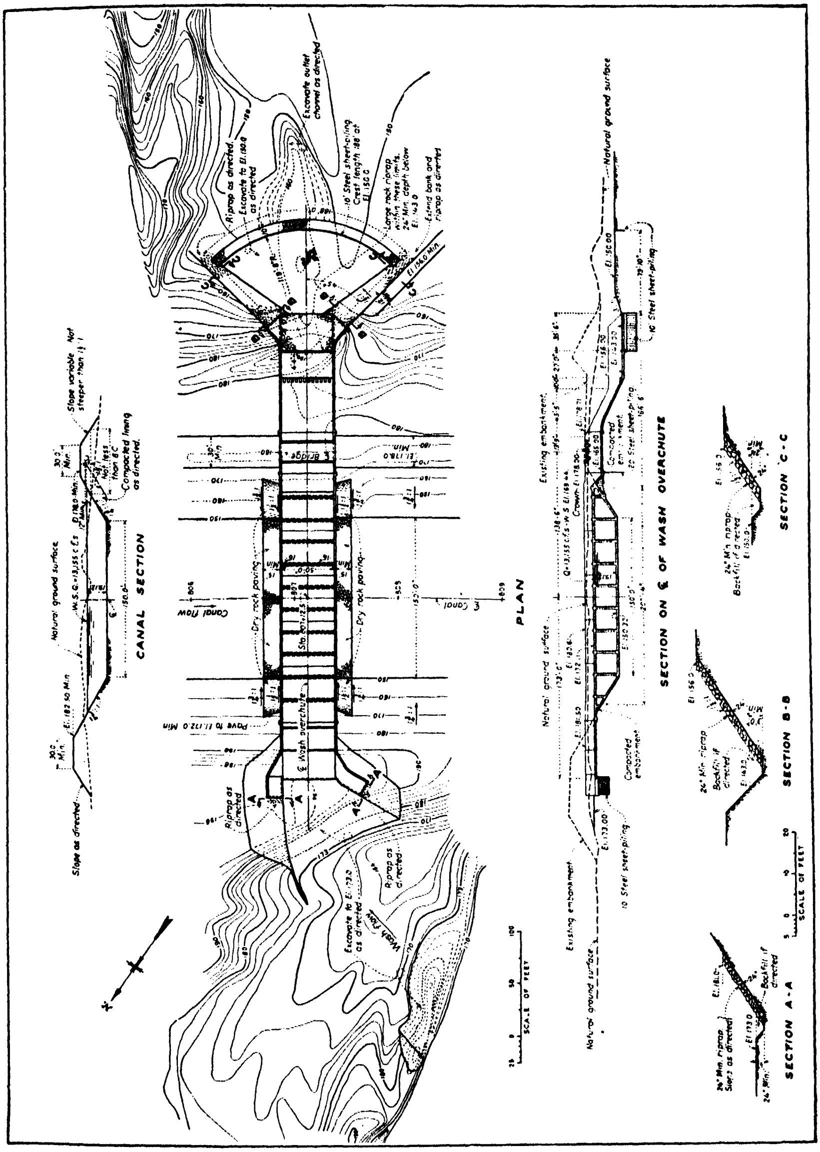

Fig. 15.—Overchute on All-American Canal near Yuma, Ariz.

Thus far, extensive use of overchutes has not been necessary on irrigation projects. However, several reinforced-concrete structures of this type were built on the All-American Canal. Figure 15 shows an installation near Yuma, Ariz.

Settling Basins. Settling basins, properly planned and constructed, are effective means of removing bed loads, suspended sand loads, and some of the heavier silt loads. However, only relatively small proportions of the finer silt particles carried in suspension can be removed by settling-basin operation. Such particles are usually

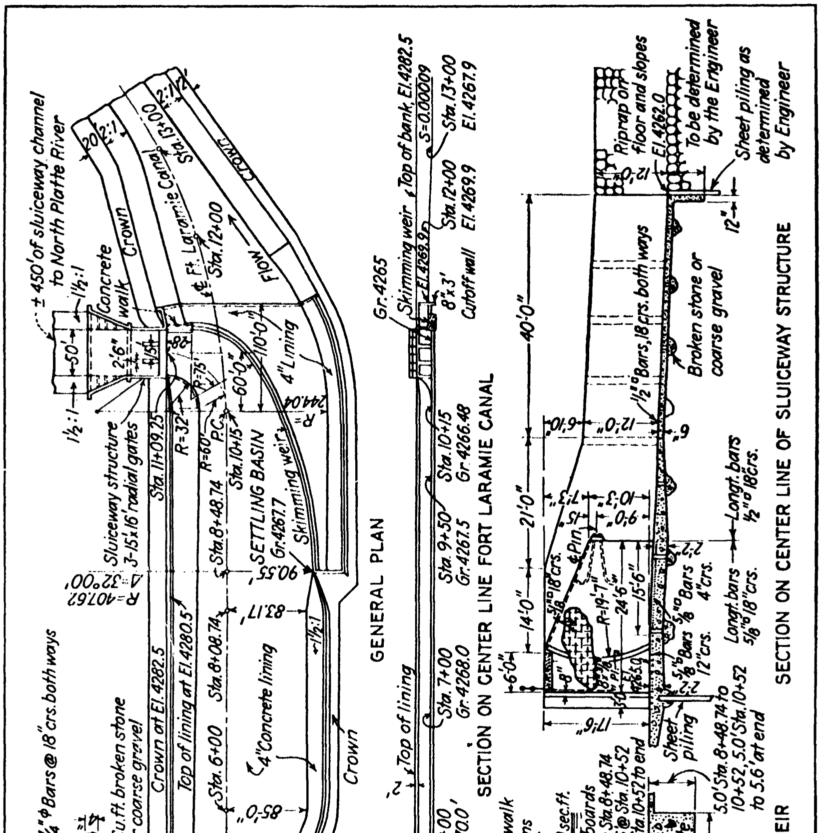

Fig. 16.—Desilting works on Fort Laramie Canal, North Platte project, Wyoming.

carried through the basins and later deposited on canal beds or farm lands. Ordinarily, settling basins should be built just below diversion structures, where deposited materials can be sluiced back to river channels. When properly designed basins are built at such locations, major sand-removal structures are seldom required at other places along the canal system.

A typical settling basin usually has a skimming weir at the downstream end, over which canal flow is withdrawn; a gate-controlled sluiceway at one end of the weir, for sluicing deposited material off the floor of the basin; and a sluiceway channel, to carry the sluicing discharge back to the river at relatively high velocities. The skimming weir should be provided with gates or flashboards, so that canal flow can be shut off during slucing operations. The sides and floor of the basin should be riprapped, or lined with concrete, to prevent excessive erosion during sluicing operations. In order to minimize interference with canal operation, it usually is desirable to design the sluiceway gates and wasteway channel so that velocities of 10 to 15 fps can be attained in the settling basin during sluicing periods.

The size of the settling basin is determined by the canal discharge and the maximum velocity that can be maintained through the basin with the sluice gates closed, without carrying sediment over the skimming weir. Since it is seldom practicable to remove all deposited material during a sluicing period, the presence of some sediment on the basin floor must be considered in calculating basin velocities during normal canal operation. In most cases, it probably is desirable to proportion basin dimensions so that mean velocities do not exceed 1 fps during normal full canal discharge. However, effective results are being secured at the Fort Laramie Canal basin, North Platte project, Wyoming, where the sediment is mostly sand and where the basin was designed for a maximum velocity of 1.25 fps.[1] Figure 16 shows the Fort Laramie installation.

Sand Traps. Sand traps are sometimes built at inverted siphon inlets or at other places along canals where sand can be sluiced into natural drainage channels. They are needed when sand cannot be removed at the headworks, when sand is eroded from the bed and banks of the canal sections, and when sand is carried into canals by local drainage.

A sand trap may consist of a short depressed length of canal bed above a check structure, with wasteway gates at one side of the canal, as mentioned under wasteways. However, most efficient traps are usually provided with curved cross channels in the depressed bed, designed to guide the sandy flow toward the sluice gates.[2] Gates may be operated intermittently, at such intervals as may be necessary to sluice out sand deposits; or they may be set at partial openings, adjusted to waste water continuously during periods of heavy sand transportation.

MISCELLANEOUS STRUCTURES

Miscellaneous structures not discussed on the preceding pages may include drainage inlets, culverts, farm bridges, highway bridges, measuring structures, and other incidental construction. Inlets are provided at drainage courses where tributary runoff does not carry appreciable sediment and does not exceed discharges that may be safely admitted to canal sections. Culverts are built at drainage courses where tributary runoff cannot be safely admitted to canals and where inverted siphons, canal aqueducts, or other types of crossings are not required. Farm and highway bridges are built where needed for transportation purposes. Measuring structures are built where canal discharges must be gaged to serve as a basis for water charges or for other reasons.

Drainage Inlets. Drainage inlets may be pipes or open channels running through canal banks. Riprap or concrete protection against erosion should be provided at both ends. Automatic flap gates should be provided at outlet ends when pipes are placed below canal water surfaces. Open channel inlets should be lined with concrete

[1] HOUK, IVAN E., Sand Control Works at Fort Laramie Canal Intake, *Eng. News-Record*, **100**, pp. 922–926, 1928.

[2] HOUK, IVAN E., Denver's Concrete Sand Trap, *Public Works*, October, 1925, pp. 378–379.

or riprap, depending on inlet dimensions and local bank formations. Inlets may terminate at the upper edge of the canal lining when discharging into lined canals, but should be carried down the side slope to the bottom of the canal when discharging into

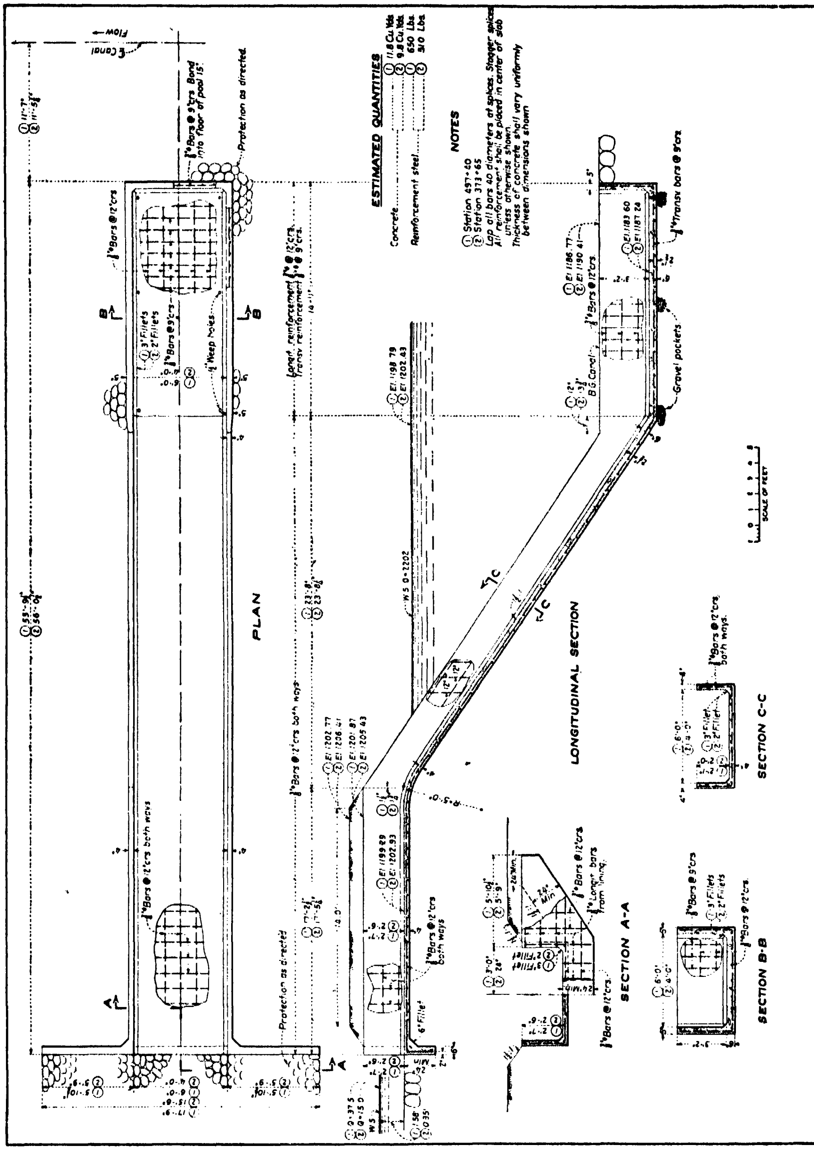

FIG. 17.—Drainage inlets, earth sections of Yakima Ridge Canal, Yakima project, Washington.

earth canals. Figure 17 shows drainage inlets recently installed in earth sections of the Yakima Ridge Canal, Yakima project, Washington.

The principal problem involved in designing drainage inlets is the determination of discharge capacities needed. Since inlets are used only for relatively small drainage

areas, the problem can best be solved by studying maximum rainfall and runoff conditions and necessary storm-sewer capacities, discussed in other sections. Rainfall intensity and frequency data are helpful in conducting such studies.[1]

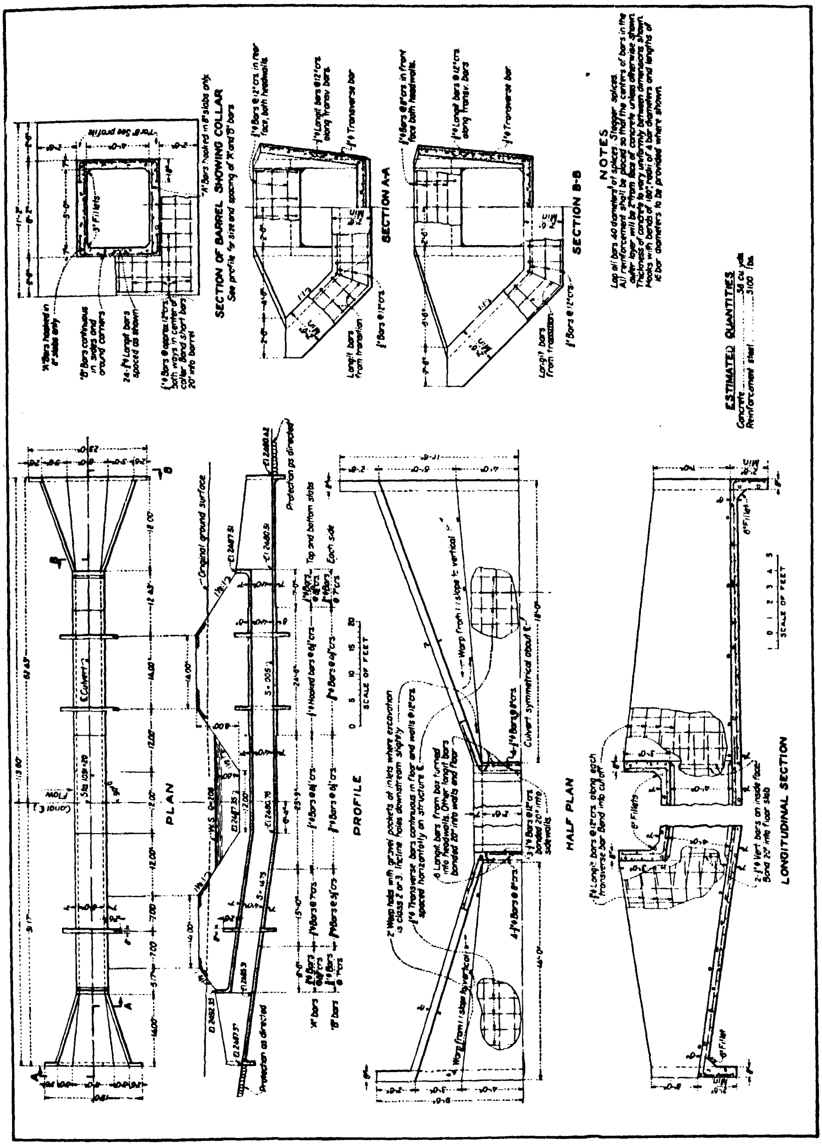

FIG. 18.—Box culvert on South Canal, Succor Creek division, Owyhee project, Idaho.

Drainage Culverts. Drainage culverts under irrigation canals may be single- or multiple-barrel, pipe, box, or arched sections. Such structures are essentially the same as drainage culverts under railroads and highways. Figure 18 shows a single-

[1] YARNELL, DAVID L., Rainfall Intensity-frequency Data, *U.S. Dept. Agr., Misc. Pub.* 204, 1935.

barrel box culvert, built on the South Canal of the Succor Creek division, Owyhee project, Idaho.

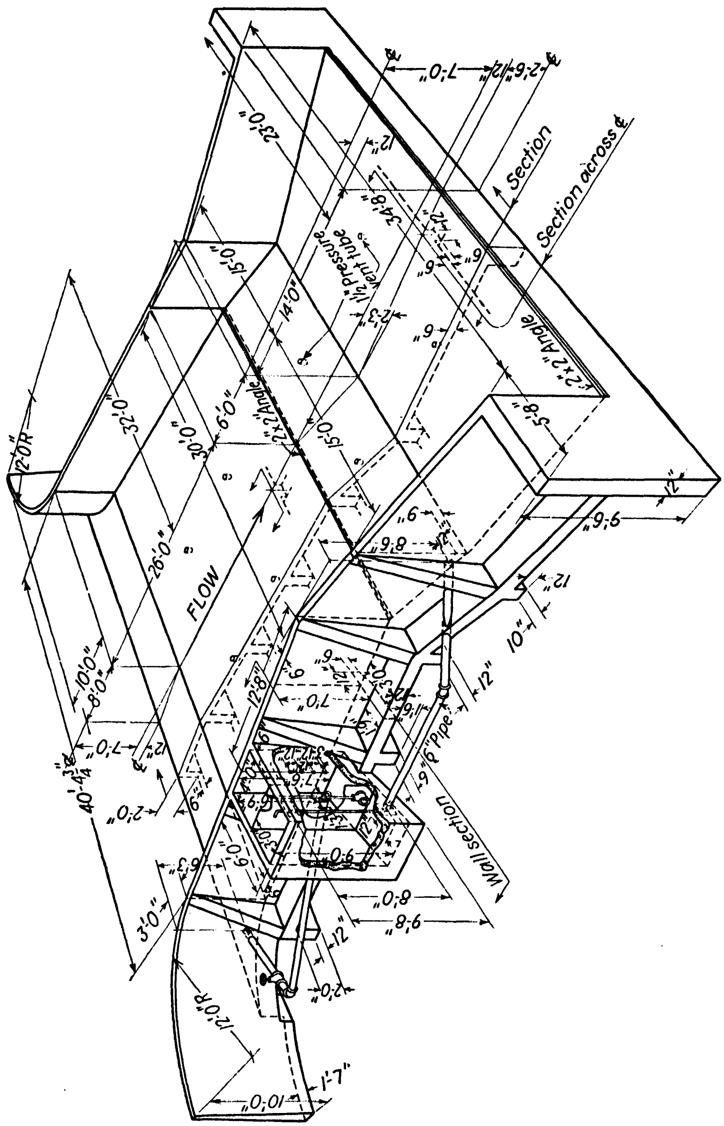

Fig. 19.—Reinforced-concrete Parshall measuring flume, 30-ft throat. (*Courtesy, Ralph L. Parshall.*)

Drainage culverts are similar to drainage inlets in that the most difficult problem involved in their design is the determination of proper discharge capacities. However, culverts usually serve somewhat larger tributary areas than inlets. Consequently, runoff records at near-by stream gaging stations are more helpful in arriving at an

adequate solution. It seldom is advisable to design culvert waterways for unprecedented cloudburst floods which may occur at long intervals.[1]

Farm and Highway Bridges. Farm bridges are usually timber-trestle structures. Highway bridges may be of steel or reinforced concrete. Bridges on state or interstate

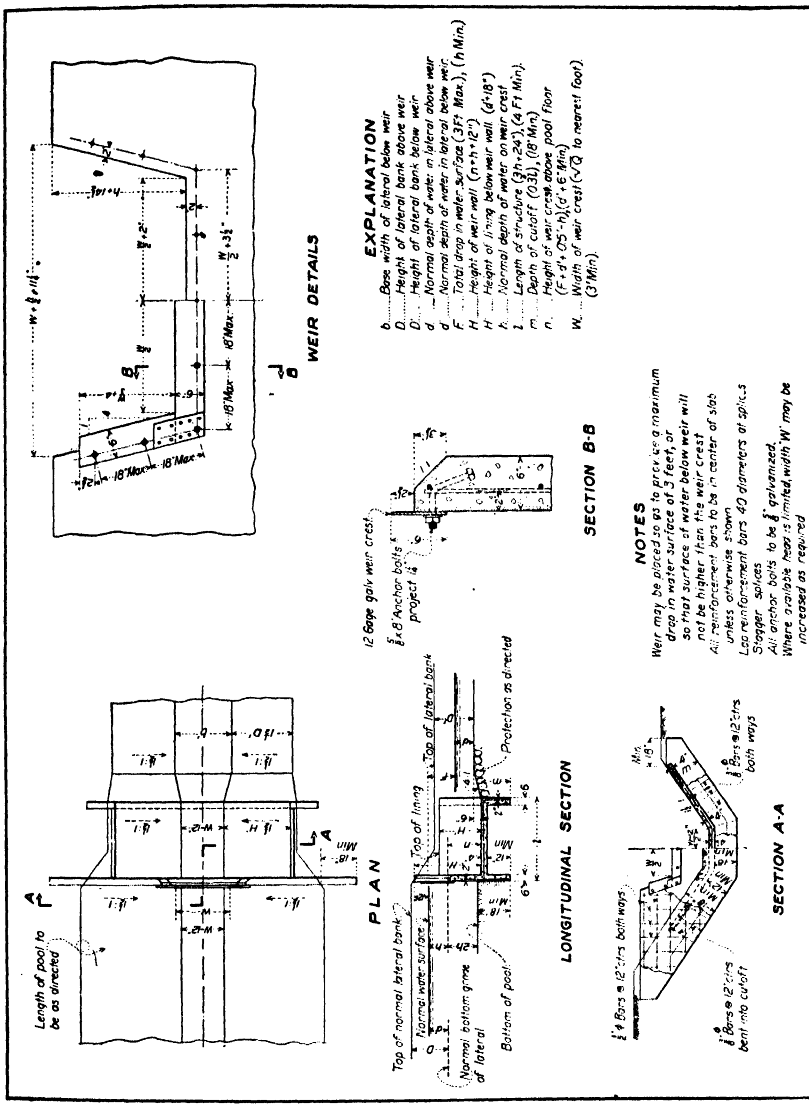

FIG. 20.—Measuring weir, capacity 3 to 35 cfs.

thoroughfares are designed in accordance with state or Federal specifications. No important hydraulic problems are involved in such designs. Bridge floors should be placed above maximum canal surfaces. When piers are located in flow sections, it may be desirable to widen the canals to compensate for pier obstructions.

[1] HOUK, IVAN E., Hydraulic Design of Bridge Waterways, *Eng. News-Record*, **88**, pp. 1071–1075, 1922.

Measuring Structures. Measuring structures are widely used in determining discharges of farm ditches, sublaterals, and lateral canals. Discharges of diversion canals, main canals, and branch canals are usually determined by current-meter gagings. However, measuring flumes are being used in some main canals carrying as much as 2,000 cfs. Farm deliveries and other relatively small discharges are measured by weirs, orifices, meter gates, commercial meters, and Parshall measuring flumes.[1] The Parshall measuring flume is also used in measuring relatively large discharges. The 40-ft reinforced-concrete flume, installed on the Fort Lyon Canal, southeastern Colorado, in December, 1928, is capable of measuring discharges exceeding 2,000 cfs.

The Parshall measuring flume is of the Venturi type. It consists of a converging upstream section, a downward sloping throat, and an upward sloping, diverging, downstream section. This flume has three advantages: (1) a relatively high degree of accuracy, (2) an inappreciable reduction in accuracy when subjected to a high degree of submergence, and (3) a freedom from silt and sand accumulations.[2] It operates most efficiently when installed in a straight section of canal, where flow conditions are relatively uniform.

Figure 19 shows the design of a reinforced-concrete Parshall measuring flume having a 30-ft throat width. Figure 20 shows a concrete weir with a galvanized-iron crest, sometimes used in measuring discharges of 3 to 35 cfs.

Commercial meter gates, frequently used in gaging small discharges, are special types of submerged orifices. Commercial meters, also used in gaging small discharges, are usually operated by ditch flows. They are designed to record total quantities passing the meters. The Dethridge meter, developed in Australia, consists of a revolving drum with wide projecting vanes. It is installed in a rectangular concrete section, designed with a curved bottom, so that the vanes revolve with a uniform clearance of about ¼-in. at the sides and bottom. It is practically the only type of meter used in Australia for measuring discharges less than 5 cfs.[3]

Fish-control Structures. Fish screens may be needed at canal intakes; and fish ladders, or other means of facilitating fish migrations, may be needed at diversion weirs. Several kinds of fish screens have been used; but none has been wholly satisfactory. Electrical screens seldom can be operated to control all sizes of fish in the stream. Stationary mechanical screens soon become clogged by debris and moss, so that frequent cleanings are required. Probably the most satisfactory mechanical fish screen thus far developed is the removable rotating type, protected by trashracks. Fish screens are not operated during the winter season.

Mechanical elevators, to aid fish migrations, have been provided at a few high dams. Fishways at weirs less than about 70 ft high are usually of the ladder type. Fish ladders consist of successive pools, beginning in the white water below the weir and rising on average slopes not greater than about 1:8. Proper pool sizes depend on the number and size of the migrating fish. Holes should be provided in the walls of the pools, so that small fish can pass through without jumping. From 5 to 20 cfs of water is diverted through the ladder during the migrating season.[4]

PUMPING INSTALLATIONS

Pumping installations are used in delivering water to irrigable areas that cannot be economically reached by gravity systems. Large areas are usually supplied from central pumping plants, cooperatively installed and operated. Most large plants

[1] CHRISTIANSEN, J. E., Measuring Water for Irrigation, *Calif. Agr. Expt. Sta., Bull.* 588, 1935; also PARSHALL, R. L., Measuring Water in Irrigation Channels, *U.S. Dept. Agr., Farmers' Bull.* 1683, 1932.

[2] PARSHALL, R. L., Parshall Flumes of Large Size, *Colo. Agr. Expt. Sta., Bull.* 386, 1932; also PARSHALL, R. L., *Colo. Agr. Expt. Sta., Bull.* 336, 1928.

[3] LEWIS, A. D., Irrigation in Australia, Irrig. Dept., Union of South Africa, Pretoria, 1935.

[4] BAKER and GILROY, Problems in Fishway Construction, *Civil Eng.*, December, 1933, pp. 671–675.

pump water from surface sources, *i.e.*, from canals, rivers, or lakes. Farm plants are usually installed and operated by individual landowners. They generally pump from underground sources, but sometimes draw water from surface supplies. In all cases, pumped water is delivered to the highest part of the irrigable area, so that further distribution can be made by gravity.

Pumping Equipment. The equipment needed for an irrigation pumping plant includes pumps, power units, suction pipe, discharge pipe, and such auxiliary equipment as may be necessary. Suction and discharge pipes should be of ample size, so that water can be moved at relatively low velocities and friction losses kept as low as possible. Pumps should be placed as close to water-supply surfaces as practicable, to keep suction heads within allowable limits, preferably less than 20 ft and never more than 25 ft. Discharge pipes should be as short and straight as possible.

Great care should be exercised in selecting the pumping unit. Pump efficiencies vary with head, discharge, and speed of operation, as well as with size and type of pump. Consequently, all operating conditions must be carefully considered before making a selection. A plant designed for a high efficiency during full-load operation may be relatively inefficient when operating at small partial loads. Pumps used in irrigation pumping plants are usually screw, centrifugal, reciprocating plunger, or deep-well turbine types. Air-lift pumps are seldom used because of low efficiencies.

Power equipment may be operated by electricity, gas, or crude oil. Electricity is generally considered the most satisfactory source of power; but internal-combustion engines are sometimes more economical for large low-head plants. Diesel engines are used in many rice-irrigation plants in the Gulf states.[1] Gas and crude-oil engines are used in farm plants where electric power is not available. Internal-combustion engines are relatively inefficient when brake horsepower is compared with theoretical fuel power; but this disadvantage may be overbalanced by low fuel costs. Electric motors constitute highly efficient methods of power transmission. They are used in small farm plants as well as in practically all large, high-head, cooperative plants. Windmills are sometimes used to pump well water for irrigating small farm areas.[2]

Agricultural experiment stations in several western states have conducted comprehensive field investigations of farm pumping plants. Anyone designing farm plants should refer to such publications.[3]

Plant Efficiencies. Plant efficiencies depend on pumping units, power equipment, and load conditions. Large, electrically operated, pumping plants, properly designed for use in cooperative irrigation enterprises, should have plant efficiencies of 70 to 75 per cent. However, small farm plants seldom have efficiencies higher than about 60 per cent, even when operating under the most favorable conditions. Tests of about 90 small plants in California showed average efficiencies of 49.8 per cent for plants equipped with centrifugal pumps, 40.5 per cent for plants equipped with deep-well turbine pumps, and 44.5 per cent for plants equipped with deep-well screw pumps.[4]

Power Requirements. The power requirement for an irrigation pumping plant varies with the plant efficiency, discharge pumped, and total pumping head. The total pumping head includes static head, pipe friction, loss at pipe bends, and such other losses as may be included in the piping system. In a plant pumping from a

[1] GREGORY, W. B., Modern Rice Irrigation System, *Mech. Eng.*, May, 1937, pp. 359 and 360.

[2] FULLER, P. E., The Use of Windmills in Irrigation in the Semi-arid West, *U.S. Dept. Agr., Farmers' Bull.* 866, 1917.

[3] SMITH, G. E. P., Motor Driven Irrigation Pumping Plants and the Electrical District, *Ariz. Agr. Exp. Sta., Bull.* 99, 1924. CODE, W. E., Cost of Pumping for Irrigation in Colorado, *Colo. Agr. Exp. Sta., Bull.* 387, 1932. CODE, W. E., Equipping a Small Irrigation Pumping Plant, *Colo. Agr. Exp. Sta., Bull.* 433, 1936. ROHWER, CARL, Putting Down and Developing Wells for Irrigation, *U.S. Dept. Agr. Circ.* 546, 1940; also Small Irrigation Pumping Plants, *U.S. Dept. Agr. Farmers' Bull.* 1857, 1940.

[4] JOHNSTON, C. N., Principles Governing the Choice of, Operation, and Care of Small Irrigating Pumping Plants, *Calif. Agr. Exp. Sta., Circ.* 312, 1928; also see Cost of Irrigation Water in California, *Calif. Dept. Pub. Works, Bull.* 36, 1930.

surface supply, static head is the difference between water-surface elevations at intake and discharge ends. In a plant pumping from an underground supply, the depth of drawdown required to bring water through the soil to the suction pipe must be added to the difference in water-surface elevations. The discharge required varies with the size of area irrigated and depth of application needed.

The power required in an irrigation pumping plant is expressed in horsepower, 1 hp being the energy required to lift 550 lb of water 1 ft in 1 sec. Consequently, the theoretical power required to pump 2 cfs against a total head of 15 ft is

$$P = \frac{2 \times 62.5 \times 15}{550} = 3.41 \text{ hp}$$

If the plant has an efficiency of 50 per cent under such load conditions, the actual power requirement is 3.41 divided by 0.50, or 6.82 hp.

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SECTION 19

WATER SUPPLIES

By Thomas R. Camp

The treatment of the subject of water supplies in Secs. 19, 20, and 21 is limited to the material that is not dealt with elsewhere in this book in detail. Such duplication as does appear is due to the desirability of emphasizing differences in practice and differences in factors controlling the methods of design.

WATER CONSUMPTION

Uses of Water. The uses of water are generally classified as domestic, commercial, industrial, public, and agricultural. Domestic use includes all water used in and around residences. The amount of domestic consumption varies with the standard of living but is proportional to the resident population. Commercial use includes water used in business or commercial districts by persons who are not residents of the district. Commercial use is itinerant domestic and light manufacturing use and cannot be stated conveniently in terms of the population of a business district. Such use is best estimated in terms of the floor area of the buildings in the district. Industrial use is for manufacturing purposes, and the amount of such use bears no relation to the population of an industrial district. Public use of water is for fire fighting, street and sewer flushing, and for unmetered public buildings. Waste due to leakage and other causes, frequently a substantial portion of the total supply, is sometimes classed as public use. Agricultural use is for irrigation purposes. Such use is unimportant for municipal supplies in regions of good rainfall but must be taken into account in arid regions.

Per Capita Consumption. The average per capita consumption for a waterworks is the total consumption for a year divided by the population and the number of days in the year. Sometimes the average per capita consumption is based upon the population of the community and sometimes upon the number of consumers. The population and number of consumers may differ greatly in some cities.

The average consumption in the United States varies from about 35 to as high as 500[1] U.S. gal per capita per day (gpcpd) with a mean value for the whole country of about 115. In strictly residential communities, the consumption is usually less than 100 gpcpd; but it is higher, the higher the standard of living. Some of the factors affecting the amount of consumption are climate, class of consumer, industrial use, quality of water, pressure in the distribution system, cost[2] of water, sewer facilities, and extent of metering of consumers' services. For a given city, the per capita consumption usually increases as the population[2] increases.

The quantity of water required for strictly domestic[3] use in residential communities of the United States has been estimated at 15 gpcpd for lower class districts to about

[1] Folwell, Maximum Daily and Hourly Water Consumption in American Cities, *Jour. N.E.W.-W.A.*, **46**, 393, 1932. See also Report of Committee on Water Consumption, *Jour. A.W.W.A.*, **2**, 181, 1915.

[2] Metcalf, Effect of Water Rates and Growth of Population upon per Capita Consumption, *Jour. A.W.W.A.*, **15**, 1, 1926.

[3] "Water Works Practice," A.W.W.A., 1929, p. 427.

50 gpcpd for high-class districts. These figures do not take into account necessary public use of water in residential areas which amounts to 5 to 10 gpcpd and leakage or water unaccounted for which amounts to 20 to 30 per cent of the quantity delivered to the system.

The quantity of water required for commercial and industrial use bears no relation to the population. In large American cities, the amount of commercial and industrial consumption has been estimated at 15 to 65 per cent[1] of the total consumption. Brush[2] has estimated the average daily consumption in the borough of Manhattan at approximately 300 gpd per 1,000 sq ft of floor area. Measurements of water consumption for various types of buildings in Manhattan are given in Table 1.

TABLE 1.—COMMERCIAL WATER CONSUMPTION IN MANHATTAN PER 1,000 SQ FT OF FLOOR AREA

	U.S. gallons per day
Hotels	600–1100
Office buildings	100– 500
Department stores	100– 400
Apartment hotels	200– 400
Average for Manhattan	300

The agricultural use of water for sprinkling lawns and gardens has been estimated[3] to be as much as 25 per cent of the total yearly demand for some cities in the arid regions of the southwestern part of the United States. The agricultural demand is for the summer months only, and during the month of maximum demand the water used amounts to about 30 gpd per 100 sq ft of lawn area or, in some cases, to as much as two-thirds of the total demand.

The growing use of summer air conditioning in the United States is creating a new demand for water which is of considerable magnitude. This demand is for refrigerating purposes. According to Lewis and Polderman,[4] the average demand per person supplied with air-conditioning equipment in 1935 was 18 gpcpd for the whole year, and the maximum demand for this purpose was 130 gpcpd. These figures are based upon 1935 practice in which 60 per cent of the installations made use of evaporative condensers and cooling towers. If all installations were equipped with evaporative condensers and cooling towers for recirculating the cooling water, the corresponding demand would be 2 gpcpd average for the year and 15 gpcpd maximum. It was estimated that nearly 6 per cent of the population of large cities in the United States was provided with summer air conditioning prior to 1936, over 20 per cent of which was installed in 1935.

Fluctuations in Demand. Fluctuations in per capita consumption from day to day and throughout any one day are of great importance in design.

The average consumption per day based upon a year's record is referred to as the average daily consumption. The maximum consumption during any one day in the year is called the maximum daily or the maximum 24-hr consumption. The peak consumption during any moment of the year, excluding fire drafts, is called the peak or maximum hourly consumption. Similarly, the minimum consumption during any one day is called the minimum daily or minimum 24-hr consumption; and the lowest momentary rate of consumption during the year is called the extreme minimum or minimum hourly consumption.

Records of maximum daily and peak consumption are not available for many water works. From questionnaires sent to 654 United States municipalities, Folwell[5]

[1] "Water Works Practice," A.W.W.A., 1929, p. 430.
[2] BRUSH, Floor Area Unit as a Basis for Estimating Consumption, 32d *Proc. A.W.W.A.*, 35, 1912.
[3] VEATCH, N. T., JR., Use of Water in Arid and Semi-arid Cities, *Jour. A.W.W.A.*, **23**, 937, 1931.
[4] LEWIS and POLDERMAN, Modern Air Conditioning, *Jour. A.W.W.A.*, **28**, 1181. 1936.
[5] FOLWELL, *loc. cit.*

found that only 94 had reliable figures on the maximum hourly consumption. Estimates of the minimum daily and extreme minimum consumption are almost completely lacking.

Folwell found that the maximum daily consumption ranged from 120 to 240 per cent of the average daily consumption with an average ratio for the whole country of 176 per cent. For cities of more than 100,000 population, the average ratio was 153 per cent. A study[1] of the water consumption of 67 Massachusetts cities and towns in 1910 indicated a range in the maximum daily consumption of 119 to 368 per cent of the average daily consumption. The average ratio of maximum daily to average consumption for all 67 communities was 198 per cent.

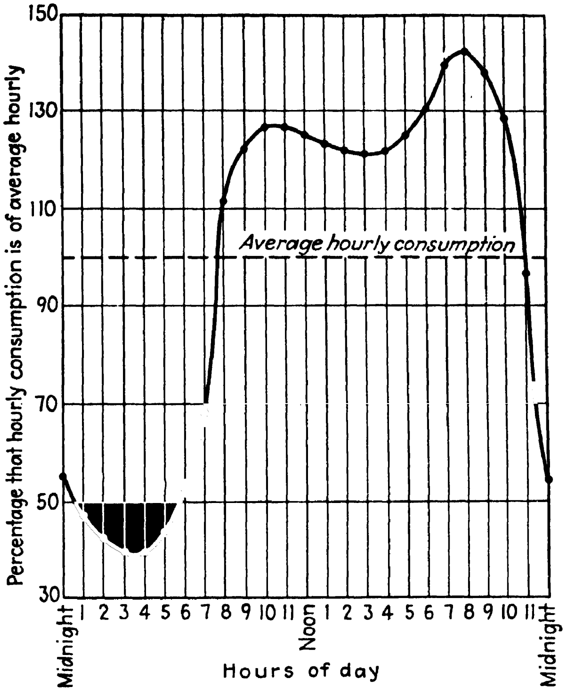


FIG. 1.—Hourly variations on days of large consumption at Evanston, Ill. (*Hansen, Jour. A.W.W.A.*, **15**, 360, 1926.)

The maximum hourly or peak consumption for all cities for which records were available in Folwell's study ranged from 140 to 874 per cent of the average daily consumption with an average ratio of about 310 per cent.

In general, it may be said that fluctuations are greatest in smaller cities, but the local factors affecting the demand are much more important than size of community. In estimating future demands for a city, it is not safe to neglect local factors.

Fluctuations in demand below the average are in general much less than fluctuations above the average. The extreme minimum draft is never zero even for the smallest system, as there is always some leakage or waste. Some idea of the minimum rates of demand may be obtained from sewage-flow records. The ratio of the peak flow to the extreme minimum flow of sewage in any one year is frequently as great as 8:1 and sometimes exceeds 12:1 for small communities. In interpreting these flows

[1] *Jour. N.E.W.W.A.*, **27**, 29, 1913.

in terms of water consumption, the ground-water infiltration into the sewers and the leakage from water mains are large sources of error which should be considered.

The hourly variations of consumption on the days of large demand are of importance in design inasmuch as these variations affect the storage requirements in distribution and filtered water reservoirs. A typical curve of hourly fluctuations is shown on Fig. 1.

Fire Demand. The quantity of water used for fire fighting in relation to the total consumption for the year is negligible, but the heavy demands for brief periods of time during fires greatly influence the design of distribution systems and storage reservoirs. The rate of fire flow for the entire system required by the National Board of Fire Underwriters[1] is given by the following empirical formula:

$$Q = 1{,}020 \sqrt{P}\ (1 - 0.01 \sqrt{P}) \tag{1}$$

in which Q is the rate of fire flow in U.S. gpm, and P the population in thousands. The formula is applicable to populations up to 200,000 in which case 12,000 gpm is required. For populations greater than 200,000, an additional flow of 2,000 to 8,000 gpm is required for a second fire.

The Underwriters recommend that sufficient water be available to sustain the fire flow and the maximum daily flow for at least 4 hr for a population of 1,000, 5 hr for 1,500, 6 hr for 2,000, 7 hr for 3,000, 8 hr for 4,000, 9 hr for 5,000, and 10 hr for a population of 6,000 or more.

Distribution systems should be so designed as to make it possible to concentrate the fire flow given by the preceding equation at any point in the high-value commercial or industrial areas of the city at a time when the normal draft is equal to the maximum daily consumption. In residential districts, according to the Underwriters:

"The required fire flow depends upon the character and congestion of the buildings. Sections where buildings are small and of low height and with about one-third of the lots in a block built upon require not less than 500 gpm; with larger or higher buildings, up to 1,000 gpm is required, and where the district is closely built, or buildings approach the dimensions of hotels or high-value residences, 1,500 to 3,000 gpm is required with up to 6,000 gpm in densely built sections of 3-story buildings."

TABLE 2.—WATER CONSUMPTION FOR TYPICAL LARGE FIRES

City, date	Population nearest U.S. census	N.B.F.U. requirements			Actual use		
		Fire flow, gpm	Duration, hr	Extra water, million gal	Fire flow, gpm	Duration, hr	Extra water, million gal
Salem, Mass.,[1] June 25–26, 1914	43,697	6,000	10	3.6	15,000	13	16
Fall River, Mass.,[1] Feb. 2–3, 1928	115,274	10,000	10	6	18,000	12	20
Nashua, N.H.,[1] May 4–5, 1930	31,463	5,000	10	3	6,500	19	5
Norfolk, Va.,[1] June 7, 1931	129,710	10,000	10	6	8,500	14	3.6
Chicago, Ill.[2] May 19–21, 1934	3,376,438	20,000	10	12	70,000	56	69.5

[1] MOWRY, Water Consumption during Fires, *Jour. N.E.W.W.A.*, **46**, 93, 1932.
[2] GAYTON, The Chicago Stock Yards Fire, May 19, 1934, *Jour. A.W.W.A.*, **27**, 803, 1935.

The water used for fighting several typical conflagrations is shown in Table 2 with the duration of each fire and the maximum fire flow. For comparison, the

[1] "Standard Schedule for Grading Cities and Towns of the United States with Reference to Their Fire Defenses and Physical Conditions," National Board of Fire Underwriters, edition of 1942.

requirements of the National Board of Fire Underwriters are also given. In all but one of the examples, the maximum fire flow and quantity of water used exceeded the Underwriters' standards.

Forecasting Demand. As a preliminary to the design of water works or extensions to an existing plant, it is necessary to estimate future demands and agree upon one or more suitable *design periods*. The design period for a large water supply which requires many years to complete may be more than 50 years. The design period may be only 10 years for pumps, filter units, and other items that may be added with facility in a short time or that have short lives or high rates of obsolescence.

Demands are estimated from the per capita consumption and the population, due account being taken of industrial and commercial use and fire demand. For extensions to distribution systems, population density is an important factor. Zoning regulations tend to stabilize population densities and hence to increase the reliability of estimates of future population density.

The following table contains the rates of flow, exclusive of fire flow. which may be of importance in the design of water facilities.

TABLE 3.—DISCHARGE TABLE FOR WATER WORKS DESIGN

	Extreme min consumption	Min daily consumption	Average daily consumption	Max daily consumption	Peak consumption
Beginning of design period	*A*	*B*	*C*		
End of design period	..	..	*D*	*E*	*F*

For a system provided with no storage, facilities must be provided for all rates of flow from *A* to *F* in the table and in addition thereto for the fire flow. If elevated storage is provided for hourly fluctuations in demand, pumps and filters may operate at constant rates during any one day. In this case, items *B* to *E* are significant for design. The average daily consumption is of importance in estimating costs of operation.

Population Estimates. The prediction of future population is at best only a guess. The accuracy of the prediction if done intelligently is greatest with large populations and small periods of forecast. The normal growth curve of a large population is one of increasing rate of growth for a time followed by a decreasing rate of growth. The rate of growth of the United States is now decreasing, and the population, according to Dublin[1] and Lotka, is approaching a maximum of about 148,000,000 which will be reached about 1970.

The best basis upon which to estimate future population trends of a community is its past development. If the curve of population up to the present time is comparatively smooth and no major changes in the factors affecting population growth are anticipated, the curve may be projected into the future. If the city is free to extend its boundaries, there is no way of knowing whether the rate of growth during the design period will continue at approximately the present rate, will decrease, or will increase. In a particular case, the probability of boundary extensions and the effect of such extensions on the population may be taken into account in projecting the curve. The extend to which a given area is built up should be considered in estimating the future population growth of that area. Curve *A* of Fig. 2 is a straight-line projection of the growth curve of Milwaukee from 1910. The actual population of the city in 1920, 1930, and 1940 is shown by crosses.

[1] TYLOR, Population Trends and the Growth of Cities, *Civil Eng.*, May, 1933, p. 279.

The projected growth curve of a community may be estimated by comparison with the past growth of other cities. The past growth curves of the cities selected for the comparison should be somewhat similar to the growth curve of the community prior to the time when their populations were the same as the present population of the community. This similarity being the case, it is assumed that their growths subsequent to this time will be an indication of the trends to be expected in the future growth of the community. Curve *B* of Fig. 2 is a projected growth curve of Milwaukee from 1910 determined by this method. Experience indicates that future populations predicted by this method are on the average somewhat too high, because of the fact that the rate of growth of the United States as a whole is declining.

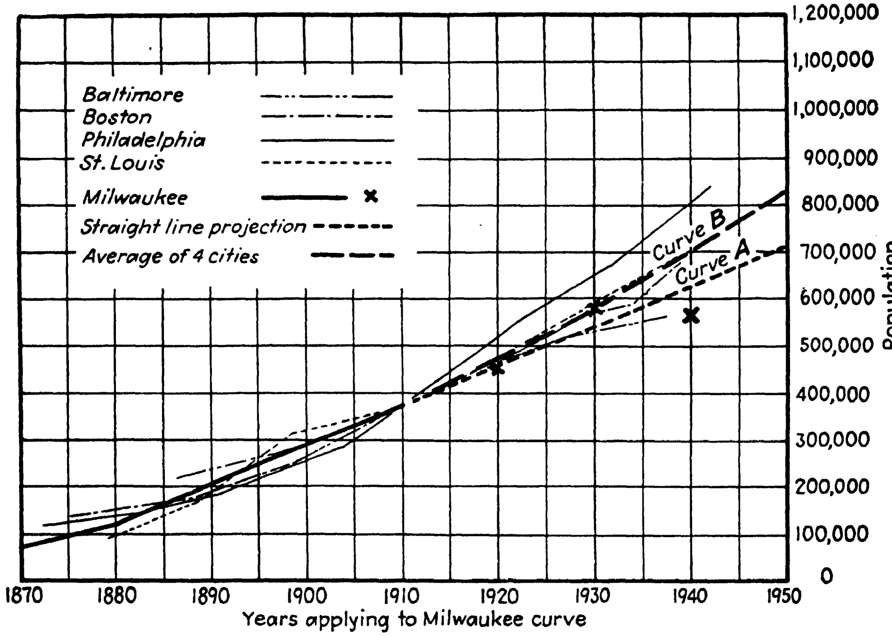


FIG. 2.—Forecast of population for Milwaukee, 1910.

The density of population for whole cities varies in the United States from less than 10 to about 80 persons per acre.[1] The density of population varies widely, however, from district to district within a city. The density in a high-class residential district fully built up may be less than five persons per acre, depending upon the size of the lots. In a middle-class residential district, fully built up with single-family houses, the density will be 15 to 30 persons per acre. In a densely built up residential area consisting of two-family houses and six-family apartment buildings, the density will be 30 to 100 persons per acre.[2] In districts built up with five and six-story apartment buildings, the density will run from about 300 persons per acre for high-class apartments to 1,000 persons per acre for tenements. The population density in industrial and mercantile districts will usually be 10 to 30 persons per acre. The preceding figures are for fully built up districts. They take into account the area occupied by streets and alleys but do not account for parks, lakes, and cemeteries.

[1] METCALF and EDDY, "American Sewerage Practice," McGraw-Hill Book Company, Inc., vol. I, 1928, p. 186.

[2] BABBITT, "Sewerage and Sewage Treatment," John Wiley & Sons, Inc., 1940, p. 29.

In order to estimate the future population of a portion of a city or district, the character of occupancy and degree of population saturation must be taken into account. Such estimates are essential to the design of distribution works. For very small areas, say 25 acres or less, it is unsafe to estimate future populations at less than the saturation values even for short design periods. For larger areas, the degree of population saturation will decrease as the size of the area increases. If the character of the various districts in a city is controlled by zoning regulations, estimates of future population density will be much more reliable than if no such regulations exist. Even for a city with a zoning ordinance, the growth rate of small areas within the city is so difficult to predict that for design purposes higher growth rates should usually be assumed than for the city as a whole. The smaller the area of the district, the higher should be the assumed growth rate.

SOURCES OF WATER SUPPLY

Classification. Water supplies are classified as *surface* and *ground-water supplies.* Surface supplies may be divided into two groups: class *a*, those from large rivers or lakes which must be pumped into the distribution systems, and class *b*, those from smaller upland streams which require storage reservoirs and aqueducts or pipe lines for delivery, usually by gravity, to the distribution systems. *Ground-water supplies* are obtained from wells, springs, and filter galleries.

In 1934, there were about 7,100 public water-supply systems in the United States serving 80 million people.[1] About three-fourths of these people are served by surface supplies and the remainder by public ground-water supplies. Some 40 million persons, or approximately one-third the total population, use private well supplies.

Reliability of Source of Supply. An important consideration in the selection of a new source of water supply is its reliability. A new supply should be capable of furnishing an adequate quantity of water continuously with a minimum danger of interruption due to breakdown or other causes.

The relative order of reliability[2] from the standpoint of quantity of water is substantially as follows:

1. A surface or ground-water supply from a practically inexhaustible source distributed to and throughout the city by gravity
2. A gravity source of supply that is inadequate at times and therefore requires storage reservoirs
3. A never-failing source that requires pumping
4. A source of supply that requires both storage reservoirs and pumping

Effects of Source of Supply upon Water Quality. The quality of water is determined by its content of living organisms and by its content of mineral and organic matter. Living organisms may be present in suspension and in colloidal dispersion. Mineral and organic matter may be in solution, in colloidal dispersion, and in suspension. Practically all the foreign matter in water is collected as the water flows over the surface of the ground or through the soil and rocks. Rain water is saturated with the gases of the atmosphere, but it contains few other impurities except in areas where the atmosphere is charged with smoke, industrial fumes, or dust.

Water that flows over the surface of the ground collects silt and particles of organic matter in suspension. A small portion of these materials will go into solution. Surface waters also contain bacteria from the topsoil and other microorganisms and aquatic life which feeds upon the organic matter in the water. If the watershed is inhabited, the water may also contain industrial wastes and sewage together with the living organisms and disease germs which are associated with such wastes.

[1] Report of National Resources Board, Dec. 1, 1934.

[2] BABBITT and DOLAND, "Water Supply Engineering," McGraw-Hill Book Company, Inc., 1939, p. 9. National Board of Fire Underwriters, Standard Schedule for Grading Cities and Towns, 1930 edition.

Water that percolates through the soil or rocks dissolves the more soluble of the minerals with which it comes in contact. The quantity of such minerals held in solution depends upon their abundance in the soil and the amount of carbon dioxide dissolved in the water. Most of the carbon dioxide in ground waters is obtained from the topsoils in which this gas is continuously being generated by bacterial action. Ground water is practically free from suspended matter, for such material is effectively strained out in the pores of the soil. Ground water is usually free from sewage and industrial wastes but may contain traces of wastes and some bacteria if they are free to percolate through crevices or large soil pores to the ground water near wells or springs.

Water from surface supplies is a mixture of surface runoff and ground water. The quantity of dissolved substances in surface waters depends upon the relative content of ground water as well as the solubility of the substances with which the water has come in contact. River waters of the class *a* group tend to be turbid, sometimes colored, and are usually polluted with sewage and industrial wastes. Lake waters of the class *a* group are usually clearer than river waters but are also subject to pollution. Waters of the class *b* group are usually quite clear and pure and are suitable in their natural state for human consumption. They are, however, subject to pollution from transient population and cannot be considered safe without protective measures. The protection may be by regulation of the watershed by patrolling and placarding, by closing it entirely to public trespass, or by treating the water.

When surface waters are stored in impounding reservoirs, their quality is usually changed considerably. The first and most noticeable effect is the clarification of the water due to the settling of suspended particles. Color is also reduced owing to bleaching by sunlight and the coagulation and settling of colloidal color particles. From the standpoint of safety of the water for drinking purposes, storage results in a reduction of the bacteria, particularly pathogenic organisms. Storage of one to two months will render a water practically free from typhoid bacilli if they are present in the inflowing water.[1] The extent to which these improvements in quality take place is dependent principally upon the length of the storage period.

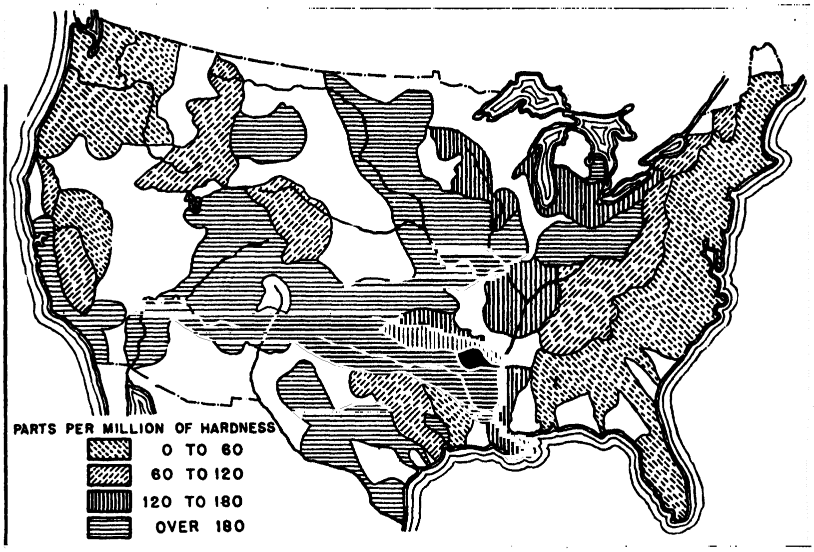
The impounding of surface waters also results in impairments to their quality which are particularly troublesome in new reservoirs.[1,2] Bacterial decomposition of the organic matter on the floor of the reservoir is accompanied by a depletion of the dissolved oxygen in the water and the formation of carbon dioxide. The corrosiveness of the water is increased by the carbon dioxide, and it also causes the solution of deleterious minerals such as iron and manganese and the hardness producing minerals calcium and magnesium from the soil of the reservoir floor. The extent to which these changes take place depends upon the amount of organic matter available for decomposition on the reservoir floor and the character of the mineral matter in the soil and rocks.

If the reservoir is deep enough so that wave action is insufficient to destroy the stratification of the water because of temperature and consequent density variations, the carbon dioxide content will be greatest at the bottom and the oxygen content least. The variation in chemical composition with depth is accompanied by a variation in the character and numbers of microorganisms. In regions where the water temperature passes through that of maximum water density (4C) in the spring and fall, turnovers occur in deep reservoirs which mix the water from top to bottom and stir up the bottom sediment. During these turnover periods, which last from a few days to several weeks, the water is of poor quality.

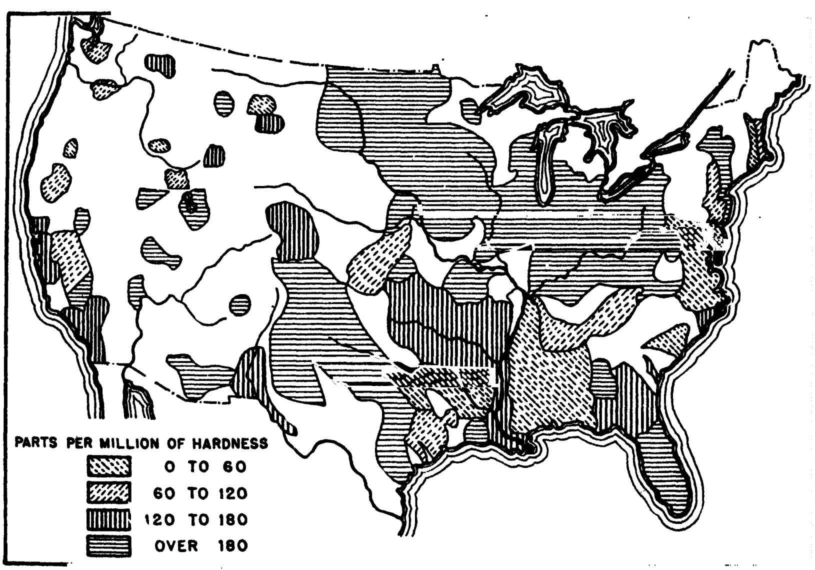
During periods of high rainfall, the total amount of suspended matter in surface waters increases because of soil erosion. The concentration of suspended matter in

[1] EDDY, The Quality of Impounded Water Supplies, *Jour. N.E.W.W.A.*, **46**, 396, 1932.
[2] CATLETT, Problems Arising from Impounded Water Supplies, *Jour. A.W.W.A.. North Carolina Section*, **5**, 32, 1927.

runoff from heavily wooded watersheds may decrease with increase in runoff, but both concentration and total amount of suspended matter will increase with the runoff



a



b

FIG. 3*a*.—Hardness of surface-water supplies. *b*. Hardness of ground-water supplies.

from cultivated and deforested watersheds. During periods of drought, the concentration of dissolved minerals in surface waters increases. Powell[1] shows an

[1] Report of National Resources Board, Dec. 1, 1934, p. 313.

increase in the hardness of the Delaware River water due to low runoff at one point of as much as fivefold.

The quality of ground water from a given source is not affected greatly by rainfall, and the mineral content changes only slightly from season to season. This is particularly true of deep-well supplies. The quality of ground water is affected by its abundance. Where the water is plentiful, it is usually not excessively hard; where it is sparse, it is likely to be highly mineralized. Most deep-well and artesian supplies are quite hard. Most shallow-well waters are relatively soft. They are also more subject to dilution and interchange with surface runoff and are therefore more likely to be polluted and infected than are deep-well waters.

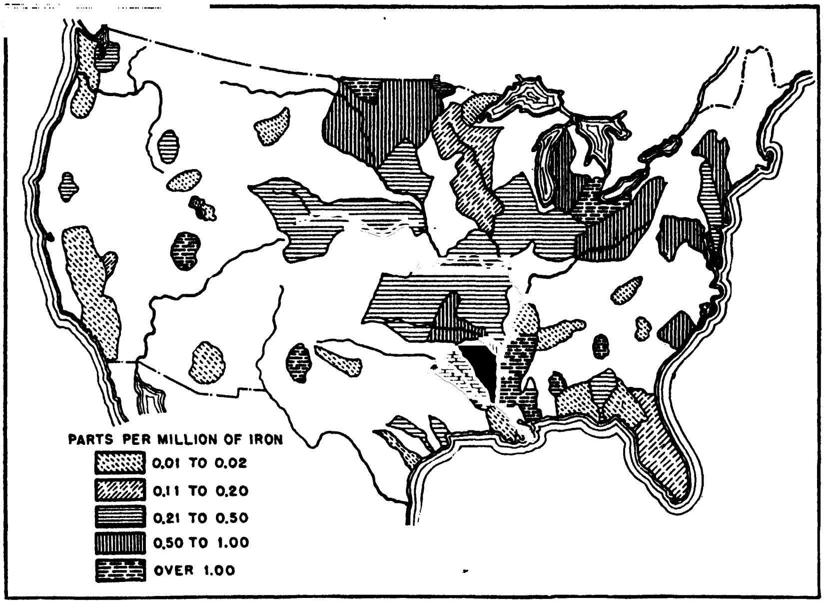


Fig. 3c.—Iron in ground-water supplies. (*Jansson, Jour. A.W.W.A.*, **27**, 368, 1935.)

Dissolved mineral matter has little effect upon the potability of water. It is a serious economic factor, however, for other domestic and many industrial uses. The average hardness of surface- and ground-water supplies and the iron content of ground waters of the United States are given by Powell in the 1934 Report of the National Resources Board and are shown in Fig. 3. In the areas that are unshaded, there were insufficient data available.

SURFACE-WATER SUPPLIES

Yield. The quantity of water that may be drawn continuously from a stream or lake depends upon the area of the watershed, the topography, vegetation, rainfall, climate, and amount of storage. The maximum quantity of water that may be drawn continuously after deducting losses due to evaporation from the proposed reservoir surface, leakage through and under dams, and necessary withdrawals for riparian owners downstream is called the *safe yield.* The estimated safe yield must exceed the estimated future demand if a proposed water supply is to be adequate.

The safe yield of a source of supply can be estimated only from records of the past. The most reliable information from which to predict yield is a hydrograph of the

stream for a long period of years at the location of the proposed intake or dam. The hydrograph should be of sufficient duration to include a period of extreme drought. If no such hydrograph exists, it is necessary to supply equivalent data by adjusting hydrographs made at other points near by or by computing runoff from rainfall records. The methods that may be used for estimating runoff are described in Sec. 25. Surface supplies are more frequently taken from small upland streams than from large rivers both because of the superior quality of the water and because of the savings to be had in pumping costs. Hydrographs are seldom available for the smaller streams, although rainfall records are usually available at near-by stations. Hence, the computation of runoff from watersheds by means of the rainfall-loss method is more commonly desirable for water supplies than for water-power projects.

If the minimum runoff from a watershed is insufficient to satisfy the estimated demand but the average runoff over a period including the driest period of record is

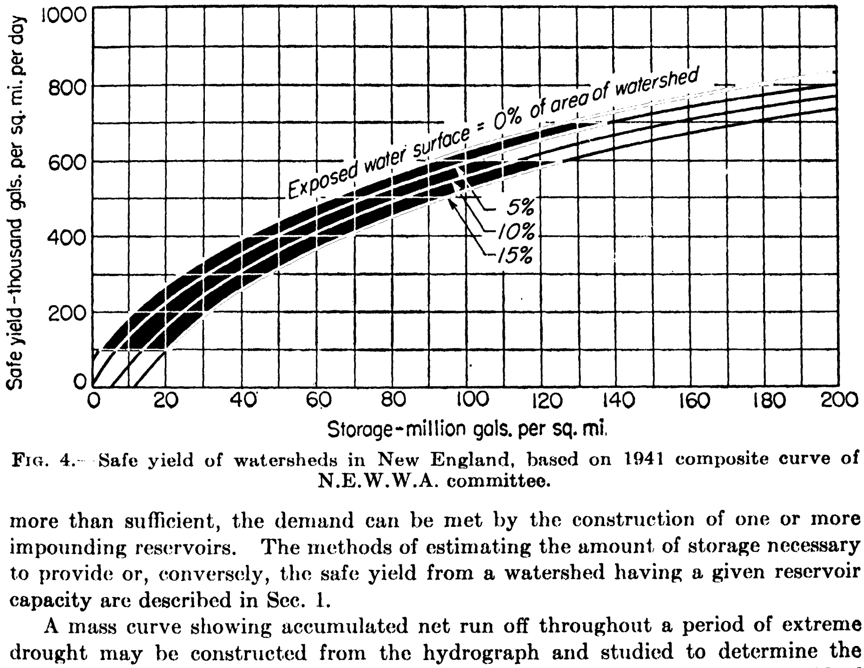


FIG. 4.—Safe yield of watersheds in New England, based on 1941 composite curve of N.E.W.W.A. committee.

more than sufficient, the demand can be met by the construction of one or more impounding reservoirs. The methods of estimating the amount of storage necessary to provide or, conversely, the safe yield from a watershed having a given reservoir capacity are described in Sec. 1.

A mass curve showing accumulated net run off throughout a period of extreme drought may be constructed from the hydrograph and studied to determine the relation of storage volume to safe yield. A comprehensive report[1] of the yield of drainage areas in New England was made by the Committee on Yield of Drainage Areas of the New England Water Works Association for the extreme drought which started in June, 1908. The Committee on Rainfall and Yield of Watersheds in New England in its second[2] (1940) and third[2] (1941) progress reports studied the yields in the light of subsequent records and presented curves of safe yield for various amounts of storage and water surface area. Figure 4 shows average curves based on the 1941 composite curve of the N.E.W.W.A. Committee.

Selection of Sites[3] for Impounding Reservoirs. The proper location of an impounding reservoir for a water supply is determined primarily by the existence of suitable dam sites. It is also influenced by the quality of the water that may be had from the

[1] *Jour. N.E.W.W.A.*, December, 1914.
[2] *Jour. N.E.W.W.A.*, September, 1945, pp. 285 and 310.
[3] See also EDDY, *op. cit.*

reservoir, the size of the watershed, and the distance of the reservoir from and its elevation with relation to the point of distribution. To be acceptable, a proposed reservoir site should have a tributary drainage area which, with the storage capacity it is possible to impound, will produce a satisfactory yield. Moreover, it should be possible to construct the works and supply water of acceptable quality to the city at less cost than from another available site.

A reservoir site that is capable of supplying pure water that may be distributed with little or no treatment is ordinarily preferable to a proposed supply that will require continuous treatment, other factors being equal. It should be noted, however, that water-quality requirements are becoming so exacting and the difficulties in the prevention of pollution are so great that few supplies will be both safe and satisfactory for future American communities without some kind of treatment. Many supplies that have been selected because no treatment was thought necessary will in the future require treatment plants. Such problems as taste and odor control, correction of corrosiveness, reduction of iron and manganese content, and reduction of color, which arise occasionally in the operation of nearly all waterworks reservoirs, are difficult to meet without filtration plants.

The density and distribution of population upon a watershed are important factors with regard to pollution and possible infection of the supply. A watershed containing scattered dwellings located at some distance from the main streams is obviously preferable to one upon which there are sewered municipalities and industries. Regional planning of water resources resulting in the early selection of future reservoir sites is desirable. Population and industrial development on the watersheds can thus be better controlled.

Waters from streams in limestone regions are not so desirable as water from a catchment area in a noncalcareous region because of hardness. It is desirable that a watershed should be free from considerable amounts of swamp land. Swamps contribute to high color in the water and also produce extensive growths of microorganisms which may occasionally seed the reservoir. Hilly drainage areas with considerable farm land are undesirable because they produce water high in turbidity during heavy rainfall.

The shape of a proposed reservoir should not be favorable to short-circuiting of runoff to the intake. Narrow reservoirs with their major axes in the direction of the prevailing winds are subject to this danger. Large reservoirs with storage capacity for a year or more are not so subject to troubles from microorganisms as small ones. If the banks of the reservoir are steep and the water is deep around the edge, fluctuations in water level will have little effect upon the water quality. If large portions of the reservoir are so shallow, however, that they are uncovered by the lowering of the water surface and are subsequently submerged, difficulties are likely to ensue from decaying vegetable matter, color, and microorganisms.

The amount of storage to provide at a given reservoir site depends upon the height to which the dam can be built economically, the storage required for the desired yield, the capacity of the watershed to replenish the reservoir following droughts, and sometimes the effect of volume of storage and fluctuations in reservoir level on the water quality. It is frequently cheaper to build more than one reservoir on a watershed than to store the required volume of water behind a single dam. In this case, the intake is usually located at the lower reservoir, and only during dry periods is this reservoir replenished by opening gates in the upper dams.

If a reservoir is constructed with too much capacity, the watershed may be incapable of filling it and of delivering water to the city at the same time. It is desirable, however, to have sufficient capacity so that it will never be necessary to draw the water level too low during a drought. The water remaining in a reservior

after the level has been drawn down severely is usually high in color, microorganisms, and carbon dioxide. Moreover, the actual time of storage is so low that short-circuiting may occur during freshets.

Preparation of Reservoir Sites. The site of a proposed reservior should be cleared of all trees and brush, and a marginal strip around the water's edge should be grubbed. This marginal strip should extend above the high water level and to some depth below the average water level. If funds are available, it may be desirable to riprap the steeper banks in order to prevent the growth of weeds and water plants and to protect the banks from erosion due to wave action.

Plants that grow around the water's edge with their roots submerged, sometimes to depths of 20 ft below the water surface, may impart tastes and odors[1] to the water. It is usually too expensive to riprap all the areas upon which such growths will develop. Areas that are too shallow and are subject to exposure when the water level is lowered should be filled in or stripped of fertile topsoil to prevent growths. Sand or gravel is best for filling as clay will impart turbidity to the water. If the expense of preventing weed growths is too great, weeds that cause trouble may be killed after the reservoir is in operation by lowering the water level to expose the roots. The dead weeds may then be burned off and removed.

Swampy areas on the reservoir site should be excavated and the muck removed. If this is too expensive, the deposits may be covered with gravel or sand. All habitations on the site should be destroyed and removed. Barnyards, privies, and cesspools should be cleaned and manure piles removed.

It was formerly considered good practice to strip or remove the top layer of soil from the entire reservoir area in order to eliminate organic matter subject to bacterial decomposition. This policy was inaugurated by the late Desmond FitzGerald for the Boston water supply after exhaustive studies.[2] The Wachusett Reservoir, constructed about 1900, was thoroughly stripped. The advantage of stripping is that a clean basin for storing water is provided immediately. Experience indicates that after 10 or 15 years the quality of the water is about the same whether the reservoir was stripped or not. The great expense for stripping is not usually justified today, since the money can better be spent for adequate purification works which will usually be required in the future in any event.

For other details with regard to the construction and operation of reservoirs, see Sec. 1.

Watershed Control. In addition to the cleaning up of the reservoir site itself, it is desirable to remove possible sources of pollution and contamination on the watershed. In order to accomplish this, it is frequently necessary to purchase the property upon which such sources of contamination exist. Future control of the watershed is best effected by ownership of the entire area. When complete ownership is too expensive or otherwise impractical, marginal strips around the reservoir and other strategic areas including principal tributary streams may be acquired.

All habitations upon these marginal areas should be removed, and farm lands and other deforested lands should be planted to trees. Evergreens are superior to deciduous trees for this purpose, because they contribute less color to the water and because leaves from deciduous trees frequently blow into the water. Reforestation of the areas contiguous to reservoirs is advantageous in several respects. It reduces the amount of silt and clay that would otherwise be washed into the reservoir, and it also results in retarding runoff and thus reduces flood flows.

The sanitary quality of the water can be protected to some extent by the enforcement of regulations established by health departments. These regulations usually

[1] ARNOLD, Weed Growths in Reservoirs, *Jour. A.W.W.A.*, **27**, 1684 1935.
[2] EDDY, *op. cit.*

prohibit direct acts of pollution and set distances limiting the proximity of sources of pollution to the streams and reservoirs. Methods of enforcement include the posting of the regulations at points on the watershed and patrolling and policing the watershed.

Dams. Descriptions of types of dams, spillways, and other appurtenances and methods of design are given in Secs. 2 to 9. Certain features of intakes and outlet works that are peculiar to waterworks are discussed in the following article.

In the design of spillways for water-supply dams, the flood flow per square mile of drainage area desirable to provide for is usually greater than for water-power projects because so many water supplies are taken from small mountain streams. The records of floods in many parts of the world[1] indicate the possibility in almost any location of runoffs in excess of 2,000 cfs/sq mile for catchment areas less than 10 sq miles. Some excessive floods[2] have exceeded 4,000 cfs/sq mile on even larger catchment areas. Corrections for storage above the spillway crest during flood flows will reduce the discharge passing over the spillway. In the selection of the maximum spillway head and the length of the spillway, consideration should be given to the effect of fluctuations in water level upon the quality of the water.

Intakes. Intake structures for surface supplies consist of some form of conduit with protective works and screens at the open end and gates or valves for regulating the flow. In the location and design of intake works, both reliability of operation and water quality are important considerations. The factors that influence reliability and the quality of the water at the intake differ with the character of the body of water from which the supply is taken. Four general classes of intakes may be differentiated as follows:

1. Intakes in large rivers
2. Intakes in small streams requiring diversion dams
3. Intakes in large shallow lakes or reservoirs
4. Intakes in large deep lakes or reservoirs

Intakes in large rivers should be so located that the ports are submerged at all stages of the river to a sufficient depth to avoid trouble with ice cakes or floating debris and to preclude the entraining of air. The ports should also be several feet above the bottom of the stream so that sand and gravel being transported on the bottom will not be drawn into the intake. In order to meet these requirements, it is usually necessary to locate the intake in the deepest part of the stream and away from the shore, particularly if the river is subject to large fluctuations in stage. Under these conditions, the water must be conveyed from the intake to the shore through a pipe laid upon the river bottom or through a tunnel constructed under the bottom.

For small water supplies, submerged pipe lines are usually used with some type of submerged crib or a screened bellmouth at the open end. The intake at Steubenville,[3] Ohio, on the Ohio River is an example of this type. For large supplies, tunnels are required; and in this case, the intake itself must be of the exposed type in order that the intake shaft may be used in constructing the tunnel. Intake towers are usually constructed of masonry circular in plan or similar in shape to a bridge pier. The top of the masonry structure must be above the maximum high water, and the structure must be designed to withstand impact due to floating debris in times of flood. One or more chambers consisting of vertical shafts open at the top are provided in the interior of the structure which connect with the tunnel below. The intake ports are constructed through the walls of the structure to the interior chamber. The masonry

[1] GUTMAN, New Cloudburst Flood Formula, *Eng. News-Record*, **117**, 474, 1936. *Eng. News-Record*, **117**, 826, 1936.

[2] *Eng. News-Record*, **117**, 655, 1936. See also JARVIS, Flood Flow Characteristics, *Trans., A. S. C. E.*, **89**, 1003, 1926.

[3] *Eng. Record*, **38**, 360, 1898.

structure is usually surmounted by an operating house. The intake ports should be provided with gates which are preferably mounted within the interior chamber. Screens may also be provided in the chamber, so that cleaning of the screens and operation of the gates may be performed by attendants within the operating house. At least one port and gate should be provided at the bottom of the interior chamber to be used for flushing sediment out of the chamber. If the intake is not too far out in the river, access to the tower from the shore should be provided by means of a bridge. The intakes at St. Louis[1] and Cincinnati[2] are typical of this general class.

If the river stage does not fluctuate too much, it is sometimes possible to construct the intake on the shore of the river. The necessity for a tunnel or subaqueous pipe line is thus avoided, and the design of the intake structure is much simplified. Typical intakes of this type are the Queen Lane Intake[3] at Philadelphia and the intake at Omaha,[4] Neb.

Most intakes in large rivers are so low with relation to the distribution system that pumping is required. The pumping stations are usually constructed on the river bank adjacent to the intake. The site should be on high ground which is accessible during flood stage for the bringing in of fuel and other supplies. The pumps must be placed so that the suction head, including friction, is not more than 15 or 20 ft when the river is at its lowest stage. This requirement frequently necessitates the construction of very deep pump wells with the pumps located many feet below high water level. If the pumps are located in a dry well, as is usually the case with larger stations, the walls of the building must be made watertight to an elevation above the maximum flood stage. The walls and floor of the pump well must be designed to withstand the water pressure during flood flows, and the whole structure must be designed against flotation. In cases where the intake can be built at the bank of the river, it is sometimes economical and otherwise desirable to combine the intake and pump station in a single structure.

Intakes in small streams frequently require the construction of small diversion dams for the double purpose of providing a sufficient depth of water at all flows to divert water into the intake port and a settling period in order to reduce the turbidity of the water. A small period of quiescent flow will also permit suspended leaves and wood either to rise to the surface or to sink if they have become waterlogged; and it will also favor the formation of sheet ice in cold weather and thus reduce the difficulties from anchor and frazil ice. Supplies of this type are usually small gravity supplies located at some distance from the distribution systems. It is not usually feasible to keep an attendant at the intake or even to make daily inspections. It is therefore desirable to design the intake so that it will function reliably with a minimum of attention. Water from this class of supplies contains a great deal more suspended vegetable matter such as leaves, pine needles, moss, and twigs than does water from large rivers. The problem of screening is particularly important, therefore, especially in the autumn.

Both bar racks and mesh screens are frequently necessary with this type of intake, the former to protect the latter from heavy floating objects. The opening in bar racks when used alone cannot be made small enough for effective removal of small suspended particles. If hand raking is required, the racks should have a slope of 2 to 4 in. horizontal to 12 in. vertical, and a suitable raking platform should be provided above high water. The bars of the rack should be supported top and bottom at the downstream edge of the bars so that the tines of the rake may penetrate between the bars to the full depth of the bars.

[1] *Eng. News*, **26**, 4, 1891; and **61**, 340, 1914. *Eng. Record*, **25**, 319, 1892.
[2] *Eng. News*, **40**, 354, 1898.
[3] *Proc. Eng. Club, Philadelphia*, **13**, 245, 1897.
[4] *Eng. News*, **74**, 342, 1915.

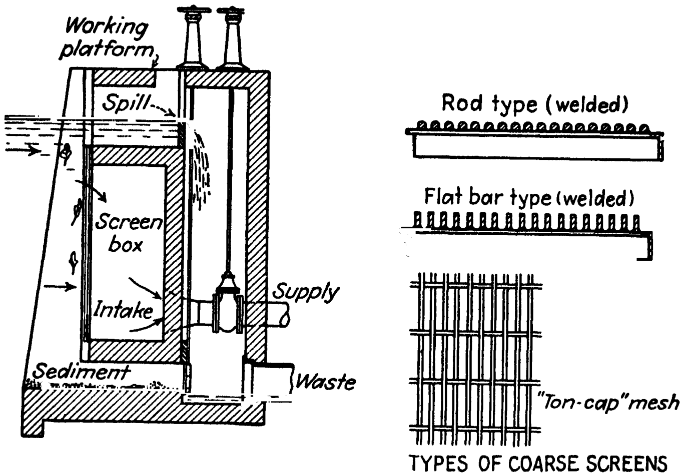


FIG. 5a.—A small intake structure.

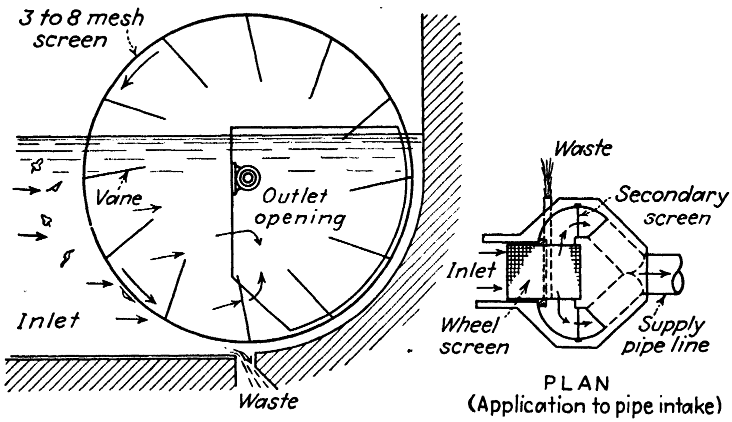


FIG. 5b.—Self-cleaning leaf screen.

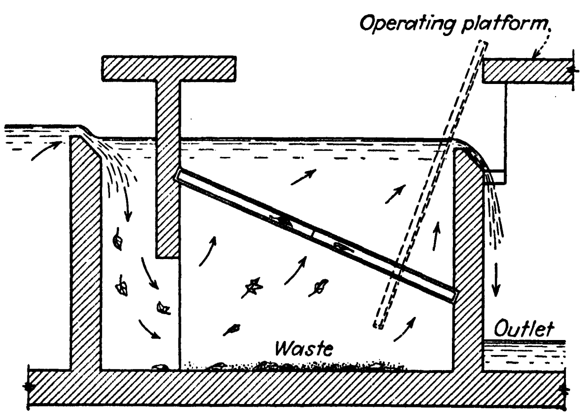


FIG. 5c.—Upward flow screen for fine screening.

Screens should have not less than two and sometimes as many as eight meshes to the inch, depending upon the character of the floating matter in the water. Screens should be of corrosion-resistant metal, easily removable, and, if hand cleaning is necessary, provisions should be made for washing them with a hose stream. The Associated Factory Mutual Fire Insurance Companies recommend[1] a screen area of 1 sq in./gpm, which corresponds to a velocity of not more than 3½ in./sec. Low velocities and small openings are necessary to prevent the entrance of fish. Figure 5 shows some screen arrangements for small supplies proposed by Cunningham.[2] In Fig. 5*a*, the screen is placed vertically so that leaves drifting against the screen will tend to slide vertically upward or downward off the screen, the direction depending upon their density. In order to encourage this tendency toward self-cleansing, the velocity through the screen must be very low. In Fig. 5*b*, the drum screen revolves because of the water impinging upon the vanes and thereby throws the screenings off into the waste. A constant water level is necessary for the satisfactory operation of this screen. In Fig. 5*c*, the screen is ordinarily cleaned by washing it down with a hose stream, but if brushing is necessary the screen may be tilted as shown.

An unusual type of intake on a small stream is the Ware River intake[3] of the Boston Metropolitan Water Supply. Water is delivered from this intake through a vertical shaft 18 ft 8 in. in diameter to the Quabbin Aqueduct tunnel about 250 ft below. Water is taken into the shaft by means of nine automatic siphons having a maximum capacity of 3,100 cfs. The flow in the Ware River is allowed to pass downstream up to a maximum of 131 cfs, everything in excess of this amount up to the capacity of the intake being diverted during normal operation. The water level in the small reservoir is kept practically constant at 1.0 ft below the crest of the diversion dam, except when the stream flow exceeds the capacity of the works at which time water flows over the dam. Operation is entirely automatic.

Intakes in large lakes or reservoirs should be located in the deepest water economically available. In the case of large natural lakes, such as the Great Lakes, it is seldom feasible to locate an intake in water that is more than 30 or 40 ft deep because of the excessive distance of deep water from the shore. In large lakes, wind and convection currents will keep the water circulated and mixed to depths as great as this. In large impounding reservoirs, however, the most desirable location for an intake is usually at or near the dam where greater depths can frequently be had. Moreover, impounding reservoirs are usually not so exposed to wind action as are large natural lakes, and the zone of circulation is therefore of less depth. Below the zone of circulation in a deep reservoir or lake is a small region of transition in which the temperature changes rapidly with depth, and below this region is a zone of stagnation in which there is very little circulation of water. In the bottom zone, the chemical and biological character of the water may vary considerably with depth.

Intakes in large shallow lakes are comparatively simple in design. If the supply is a small one, a subaqueous pipe line with a submerged crib or bellmouth at the open end may be used. In the Great Lakes region where ice is a major difficulty, the ports should be kept about 25 ft under the surface. The new intake for Erie,[4] Pa., consists of a 72-in. riveted steel pipe extending some 6,000 ft out into the lake to a submerged timber crib. The crib is about 40 ft square and 17 ft 6 in. deep. The top of the crib is 6 ft above the lake bottom and 25 ft under mean lake level. The crib is divided into nine pockets about 12 ft square. The four corner pockets were filled with rock to sink

[1] FLINN, WESTON, and BOGERT, "Waterworks Handbook," McGraw-Hill Book Company, Inc. 1927, p. 82.

[2] CUNNINGHAM, Waterworks Intakes and the Screening of Water, *Jour. A.W.W.A.*, **23**, 258, 1931.

[3] KENNISON, Boston Metropolitan Water Supply Extension, *Jour. N.E.W.W.A.*, **48**, 147, 1934. KENNISON, Ware River Intake Shaft and Diversion Works, *Civil Eng.*, August, 1934, p. 388.

[4] CAMPBELL, The New Auxiliary Water Plant at Erie, Pa., *Jour. A.W.W.A.*, **25**, 771, 1933.

the crib. The inlet manifold with three ports is built through the walls of the crib. The ports are located in pockets of the crib 4 ft below the top and are protected by coarse bar gratings and by wooden gratings with 2-in. openings over the top of the pockets. The crib is protected from storms by crushed stone laid around it.

For large supplies such as for Chicago and Cleveland, exposed cribs several miles from shore are used. The location of the cribs is determined by the desired depth of the ports and the proximity of pollution. The new W. E. Dever intake[1] is the eighth exposed crib built for the city of Chicago. The crib consists of two concentric steel cylinders 40 ft and 90 ft in diameter. They are 45 ft high and are connected by 16 radial diaphragms. There are eight intake tubes 8 ft high by 10 ft wide connecting the two shells. The steelwork was erected ashore and then floated to the site, buoyancy being obtained by sealing the intake tubes at both ends with temporary bulkheads and by constructing auxiliary buoyancy chambers between the shells. The crib was sunk into position in 34 ft of water by opening the valves in the buoyancy chambers. The outer steel cylinder was equipped with a cutting edge to make an effective seal in the clay of the lake bottom. After the steelwork was in position, the space between the shells was filled with concrete, the water was pumped from the interior chamber, and the tunnel shaft was constructed.

The tunnel shaft is usually extended up into the interior chamber of the crib and equipped with screens at the top. The intake ports should be submerged about 25 ft and may be equipped on the inside, as at Buffalo, with gates. The crib should be surmounted with an operating house which should be provided with living quarters for the attendants and a boat landing. In Chicago, the crew consists of four men normally on each crib with two additional men in winter because of ice difficulties.

The design of Great Lakes cribs is controlled largely by the requirements of protection against storms and waves and by ice difficulties. The large diameters are dictated by the necessity of stability during storms. Ice difficulties are minimized by placing the ports deep and keeping entrance velocities down to 3 or 4 in./sec. Vortices in the ports should be avoided if possible. Screens may have to be raised in winter to prevent clogging with frazil. Steam or compressed air may also be used to blow the ice from the screens. Dynamite charges of one-fourth to one-half stick have been used to dislodge ice from the crib ports. For some intakes, breakwaters have been constructed for protection. Murphy[2] recommends steam piping around valves, gates, and screens to keep their temperature high enough to prevent freezing.

The new Detroit intake[3] is protected from ice by a lagoon about 2,400 ft long and of such cross section that the velocity of approach to the intake is about 3 in./sec for about 2 hr. Under these conditions, it is expected that sheet ice will form and prevent the formation of frazil and anchor ice. The port velocity at Detroit is about 1 fps.

Deep reservoirs should be provided with intakes or gate houses designed so that water may be taken at several different depths. The advantages[4] of this provision are:

1. Avoidance of microorganisms which live near the surface and require sunlight, many of which produce objectionable odors in water
2. Avoidance of surface water at too warm a temperature during the summer
3. Avoidance of too much turbidity during wind storms by drawing from the surface
4. Avoidance of troublesome plankton by drawing from a depth at which they are not prevalent
5. Avoidance of too much carbon dioxide by drawing near the surface
6. Reduction in the quantity of coagulant required by drawing bottom water high in iron
7. Avoidance of too much iron, manganese, color, and hardness by drawing near the surface

[1] *Eng. News-Record*, **110**, 737, 1933.
[2] *Can. Eng.*, Feb. 19, 1920, p. 233.
[3] *Eng. News-Record*, **107**, 494, 1931.
[4] See "Water Works Practice," A.W.W.A., 1929, p. 165.

A common type of deep reservoir intake is in the form of a masonry tower exposed above the water surface and housed for the gate and screen hoisting machinery. In order that the tower may be in deep water, it is frequently necessary that it be some distance from the shore. Sometimes bridges are built from the shore or the dam to the intake, but if it is too far out boats are used by the operators. In order to make the tower accessible during all weather, it is sometimes built into the dam. If the dam is of earth, this procedure may require expensive retaining walls in order to provide a vertical face for the intake ports. Ports should be at two or three different levels, depending upon the water depth. Gates and screens should be inside the tower for protection. In important works, emergency gates or stop plank grooves should be provided in front of the sluice gates to permit the unwatering of the intake well for repair to service gates. It is also desirable, in order to increase reliability, that the intake well be divided into two parts equipped with duplicate gates, screens, and outlet pipe. The screens are commonly placed in an interior wall across the direction of flow. Blowoff gates and pipes should be provided at the bottom of the tower for flushing sediment out of the tower and silt from the reservoir around the base of the tower. Intakes of this type can usually be constructed in the dry, but the walls and gates must sometimes be designed to withstand very high pressures.

Pipe Lines and Aqueducts. Descriptions of types of pipe lines and aqueducts and methods of design are given in Sec. 10. For pipe lines in distribution systems, see Sec. 20.

GROUND-WATER SUPPLIES

Types of Collecting Works. Rain water which percolates into the soil beyond the reach of vegetation collects in the pores and fissures and flows, usually in the general direction of the slope of the ground surface, toward outlet points. The water-bearing strata, called *aquifers*, include formations of soil and sand, porous sandstone, alluvial deposits of sand and gravel, porous lava flows; glacial drift, and limestone containing fissures.

The upper surface of the ground water is called the *ground-water table.* Flow through the soil is in the direction of the slope of the ground-water table. The water table rises during rainy seasons and falls during droughts. Excessive draft of ground water from wells also lowers the water table. Figure 6 illustrates the position of the ground water and shows several different types of collection works.

A well constructed in water-bearing soil which receives its supply from rainfall in the immediate vicinity of the well is called a *shallow well.* A *filter gallery* is a shallow well constructed along the shore of a river or lake. If the draft from a filter gallery is high enough to lower the water table at the gallery below the water surface in the adjoining body of water, a portion of the water is drawn therefrom. A well that penetrates through an impervious stratum to an aquifer below is called a *deep well.* A deep well into an aquifer in which the water is under pressure is called an *artesian well.* The water supply to an artesian aquifer is from rainfall at a distant point at a higher elevation, such as at A, Fig. 6. The surface to which water would rise due to the pressure in an artesian aquifer is called the *piezometric surface.* If the piezometric surface is above the ground surface at an artesian well, the well is called a *flowing well.*

A number of different types[1] of springs are encountered in nature. An *artesian spring* is one in which the water issues under artesian pressure through a fissure in the impervious rock overlying the aquifer. Springs formed at points where the ground-water table outcrops are called *gravity springs.* A gravity spring in which the water flows to the surface from permeable material over the outcrop of impermeable material is called a *contact spring.* A *fracture* or *fissure spring* is a gravity spring in which the

[1] MEINZER, *U.S. Geol. Survey Water Supply Paper* 494.

water issues from relatively large openings in rock. A *depression spring* is a gravity spring due to a depression of the ground surface below the water table. During the dry seasons of the year, depression springs may go dry owing to the lowering of the water table.

Works for the collection of water from springs consist mainly of collecting basins, from which the water may be pumped to the distribution system, and safeguards against the contamination of the spring water.

Hydraulics of Artesian Wells. In order to study the flow of water into wells and filter galleries, it is necessary to make several simplifying assumptions. One assumption is that the aquifer is homogeneous and hence that no appreciable error results from the assumption of a constant permeability coefficient. In most water-bearing forma-

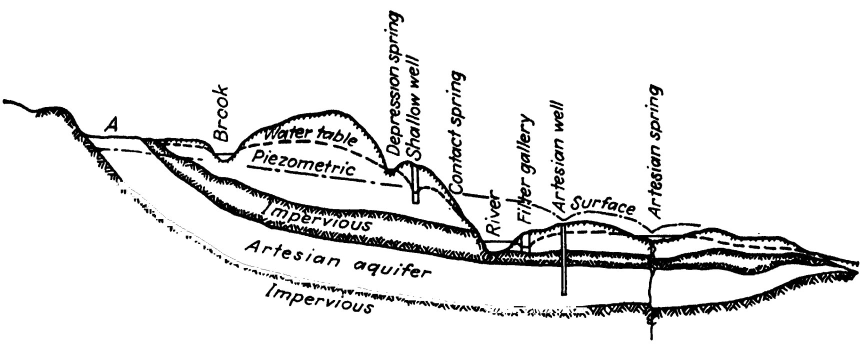


FIG. 6.—Hypothetical section showing relation of ground water to topography.

tions, the pores are so small that laminar flow takes place, and therefore Darcy's law is assumed to hold. Darcy's law is as follows:

$$Q = kAS \tag{2}$$

in which Q = discharge through the area A of the aquifer perpendicular to the direction of flow

S = hydraulic gradient, in this case the potential gradient since the velocity head is negligible

k = coefficient of permeability

The apparent velocity Q/A is not the velocity through the pores but the velocity of a solid column of water through the cross section A. In filtration, it is the velocity of approach to the sand bed or the rate of filtration. The permeability k is therefore the corresponding velocity with a hydraulic slope of unity. The factors that determine the value of k are discussed in Sec. 21.

For the development of the theory for artesian wells, it is further assumed that the aquifer is of substantially constant thickness and is approximately horizontal and that the well penetrates to the bottom of the aquifer. Flow is assumed to take place into the well uniformly throughout the thickness of the aquifer. Under these conditions, flow is horizontal and an analysis may be made in terms of two-dimensional potential theory. The method followed is substantially that of Slichter[1] and takes no account of the elastic deformation of the water and the aquifer.

For steady radial flow only, the velocity at any point P_1 (Fig. 7) at a distance ρ_1 from the center of the well is

$$v = \frac{c}{\rho_1} = -k\frac{d\Phi_1}{d\rho_1} \tag{3}$$

[1] SLICHTER, *U.S. Geol. Survey, 19th Annual Report*, Part II.

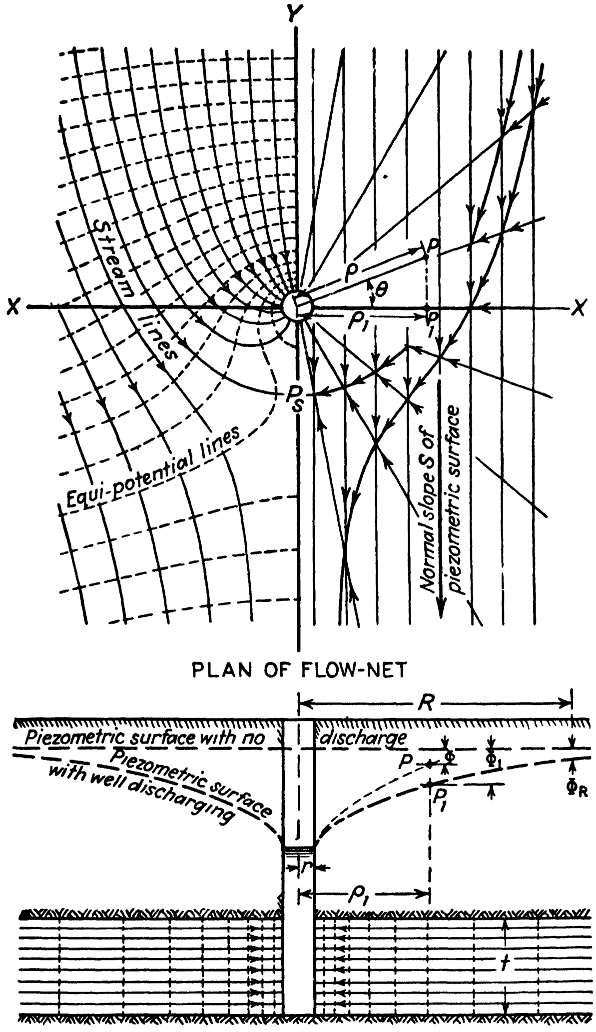


FIG. 7.— Flow net into an ideal artesian well.

in which Φ_1 is the drawdown of the piezometric surface at point P_1 and c a constant. Integrating between the wall of the well of radius r and the distance R, the drawdown at P_1 is found to be

$$\Phi_1 = h - \frac{h - \Phi_R}{\ln \frac{R}{r}} \ln \frac{\rho_1}{r} \tag{4}$$

in which h is the drawdown at the well, and Φ_R the drawdown at a distance R from the center of the well.

It has been customary to consider a distance R of such a value that the drawdown Φ_R is substantially zero. A value of 200 m has been arbitrarily taken for R by some

authorities, and R has become known as the *radius of the circle of influence.* It is obvious that R becomes very large as Φ_R approaches zero; and a wide discrepancy in the value of Φ will result from Eq. (4) for values of R between 200 m and the maximum. Many unsuccessful attempts have been made to evaluate the radius of the circle of influence by experimental measurements on wells.[1] A more satisfactory method of procedure is to select a value of R for which Φ_R has a finite value of sufficient magnitude to be measured with accuracy.

If the flow into a well is radial only, the piezometric surface for the condition of no flow into the well is horizontal and the water in the aquifer is still. Hence when the well is discharging, the water stored in the aquifer is being drawn upon and the piezometric surface is falling. Therefore, steady conditions cannot exist with radial flow only. The existence of steady flow into a well is evidence of a sloping piezometric surface.

With a sloping piezometric surface, the flow net for a single well is as shown in Fig. 7. In this case, the potential at any point P with reference to the piezometric surface at the well for the condition of no draft from the well is

$$\Phi = h - \frac{h - \Phi_R}{\ln \frac{R}{r}} \ln \frac{\rho}{r} - S(\rho - r) \sin \theta \tag{5}$$

in which S = normal slope of the piezometric surface with no flow into the well, taken from the direction of Y.
θ = angle between the radius vector ρ and the X-axis.
Φ_R = value of Φ at a distance R along the X-axis from the center of the well.

The velocity at point P is, from Eqs. (3) and (5),

$$v = \frac{k}{\rho}\left(\frac{h - \Phi_R}{\ln \frac{R}{r}} + S\rho \sin \theta\right) \tag{6}$$

The discharge into the well is

$$Q = \rho t \int_0^{2\pi} v \, d\theta = 2\pi k t \frac{h - \Phi_R}{\ln \frac{R}{r}} \tag{7}$$

in which t is the thickness of the aquifer. The discharge has the same value as for the case of a level piezometric surface.

It will be noted from the flow net in Fig. 7 that the stream lines for this case are not radial and the equipotential lines are not circles. The circle of influence conception is therefore erroneous. Some of the ambiguous results obtained in efforts to measure the radius of the circle of influence may be explained by the neglect to take into consideration the slope of the normal piezometric surface. The flow net may be constructed as shown in Fig. 7 by sketching the stream lines as resultants of the stream lines from the radial-flow and the parallel-flow systems. The equipotential lines cross the stream lines at right angles.

The location of the stagnation point P_s in Fig. 7 may be found from Eq. (6) by making $v = 0$ and $\theta = -\frac{\pi}{2}$. The distanct to P_s is

$$\rho = \frac{1}{S} \frac{h - \Phi_R}{\ln \frac{R}{r}} \tag{8}$$

[1] BALCH, Hydraulics of Deep Wells, *Jour. A.W.W.A.*, **22**, 727, 1930.

Many artesian wells do not penetrate through the full thickness of the aquifer, and many wells are provided with casings which are perforated or provided with screens for only a portion of the thickness of the aquifer. In such cases, the stream lines are not horizontal in the vicinity of the well. They converge vertically and thus increase the velocity in the vicinity of the well. The drawdown at the well is greater than for the ideal well of Fig. 7. The effect of vertical convergence of the stream lines is shown in Fig. 8. The equations developed above do not hold good for this case in the region where vertical convergence takes place, but they are approximately correct for points beyond this region. Muskat[1] gives a solution for the case of a well that partly penetrates an artesian aquifer. If a well penetrates through an impervious stratum to the top of an infinitely deep artesian aquifer, the flow pattern, except in the immediate vicinity of the well, is that of a point sink. Solutions of this

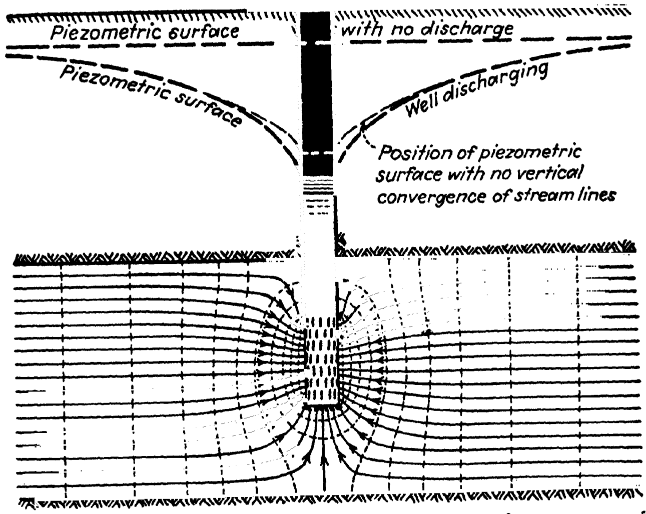


FIG. 8.—Flow net into an artesian well showing effect of vertical convergence of stream lines.

case are given by Forchheimer[2] and Dachler.[3] There is evidence[4] that the flow from deep artesian wells is influenced by the elastic expansion of the water and the aquifer when the pressure is reduced.

Hydraulics of Shallow Wells. No accurate analysis of the hydraulics of shallow wells appears to have been made because of the difficulty of establishing the boundary conditions for this case of three-dimensional potential flow. A hypothetical vertical section through a shallow-dug well with the flow net shown approximately is given in Fig. 9. It will be noted that some of the flow seeps into the well above the water surface. The stream lines are tangent to the wall of the well above the water surface and perpendicular to it below the water surface.

An approximate analysis may be made of the drawdown of the ground-water table in the vertical plane perpendicular to the direction of the normal slope of the water table. The analysis assumes horizontal stream lines and is therefore of no value near

[1] MUSKAT, M., "The Flow of Homogeneous Fluids through Porous Media," McGraw-Hill Book Company, Inc., 1937, Chap. V.

[2] FORCHHEIMER, P., "Hydraulik," 3d ed., B. G. Teubner, Leipzig, 1930, p. 77.

[3] DACHLER, R., "Grundwasserströmung," Verlag Julius Springer, Berlin, 1936, p. 26.

[4] JACOB, C. E., *Proc. A.S.C.E.*, May, 1946, p. 629.

the well where vertical convergence of the stream lines is great. In Fig. 10, the bottom of the aquifer is assumed to be horizontal and the stream lines substantially horizontal beyond the distance R_1 from the well. In the vertical plane perpendicular to the general direction of flow of the ground water, the radial velocity into the well is the same as for a horizontal water table. Therefore, the velocity

$$v = \frac{c}{\rho D} = k\frac{dD}{d\rho} \tag{9}$$

in which D is the thickness of the saturated aquifer at the distance ρ from the center

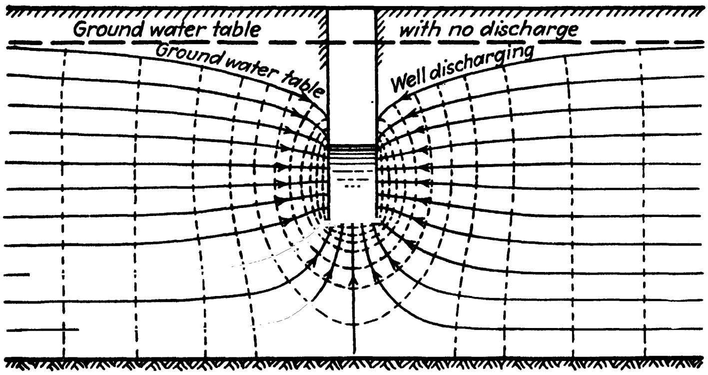


FIG. 9.—Flow net into a shallow-dug well.

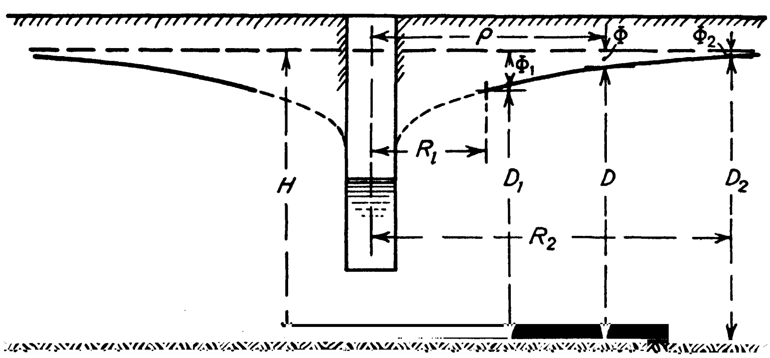


FIG. 10.—Flow into shallow well in plane normal to general direction of ground-water flow.

of the well. Integrating (9) between R_1 and R_2, where the values of D are known to be D_1 and D_2, respectively, it is found that

$$v = \frac{k}{2\rho D}\frac{D_2^2 - D_1^2}{\ln \frac{R_2}{R_1}} \tag{10}$$

From (9) and (10),

$$D = \sqrt{D_1^2 + (D_2^2 - D_1^2)\frac{\ln \frac{\rho}{R_1}}{\ln \frac{R_2}{R_1}}} \tag{11}$$

The amount of flow into a shallow well is not influenced by the general slope of the water table, and the radial velocity in the vertical plane perpendicular to the general direction of flow is the average radial velocity. Therefore the discharge is

$$Q = 2\pi\rho Dv = \pi k \frac{D_2^2 - D_1^2}{\ln \frac{R_2}{R_1}} \tag{12}$$

The foregoing equation may be used for evaluating the permeability coefficient k of the aquifer from observations of the discharge at the pumped well and the drawdown in two observation wells distance R_1 and R_2 from the pumped well.[1] All three wells must be located in the plane perpendicular to the direction of slope of the normal ground-water table. Equation (12) is of no value in predicting the capacity of the pumped well for a given drawdown at that well.

The analysis of flow into infiltration galleries of various shapes differing from that of the shallow well treated above is much more complicated. If solutions are required, they may be produced approximately by making graphical constructions of the flow nets, or by determining the flow nets by means of electrical models.

Specific Yield and Safe Yield. The specific yield or specific capacity of a well is the discharge per foot of drawdown at the well. In determining the well yield by test, it is assumed that the potential in the well is the same from the water surface to the bottom. If the well is small in diameter and being pumped at a high rate, it is necessary to correct for friction loss and velocity head in the casing in determinations of the drawdown.

For artesian wells, it may be noted from Eq. (7) that the yield is not directly proportional to the drawdown h. If R is assumed constant, Φ_R will increase as h is increased, but not at the same rate. If the value of R is taken such that Φ_R is very small as compared with h, the yield is very nearly directly proportional to the drawdown. For artesian wells, therefore, it is usually assumed that the specific yield is constant within the working limits of the drawdown.

The specific yield for shallow wells decreases as the drawdown is increased. In Eq. (12), $D_2^2 - D_1^2$ may be written $(D_2 - D_1)(D_2 + D_1)$. The first term $D_2 - D_1$ approximates the drawdown at the well if R_1 is small and R_2 large. With increase in the drawdown, the term $D_2 + D_1$ is decreased, the result being a decreased specific yield with increased drawdown.

The *maximum safe yield* of a well or well field is limited by the capacity of the aquifer to supply water without suffering a continuous lowering of the water table or piezometric surface. The maximum safe yield is limited, therefore, by the rate at which the ground water is replenished by rainfall. The seasonal fluctuation in safe yield from shallow well supplies is more marked than for artesian supplies.

If a well or well field is not developed to the full capacity of the aquifer, the maximum yield is limited by the maximum permissible drawdown at the well and by the size and method of construction of the well. For fields of shallow tubular wells, the maximum permissible drawdown may be limited by the suction head on the pumps or by the depth of the wells. For economy of pumping, the drawdown should be kept to a minimum. For greatest over-all economy, the cost of pumping against the additional head due to drawdown should be balanced against the cost of securing additional yield by improving the method of construction of the well. In practice, such an economic balance is difficult to obtain because it is not possible to predict accurately the yield of a well before it is constructed.

For a given drawdown, the yield increases slightly with increase in diameter. To secure the greatest yield, it is also desirable for the well to penetrate the full thickness

[1] Lohman, *Jour. A.W.W.A.*, **26**, 211, 1934.

of the aquifer and thus reduce vertical convergence of the stream lines near the well. Another method of increasing the yield for a given drawdown is to surround the well screen with gravel packing having a higher permeability than the material of the aquifer which is replaced by the gravel.

The specific yield of wells ranges from about 5 gpm for small shallow driven wells to over 100 gpm for large-diameter gravel-wall deep wells.

Interference of Wells. When two or more wells are so close together that the yield of each of the wells for a given drawdown is decreased when the other wells are discharging, the wells are said to interfere with one another. The effects of interference may be studied by constructing graphs of the flow nets into the wells or by means of electric models. Analytical solutions are not available except for the simplest cases.

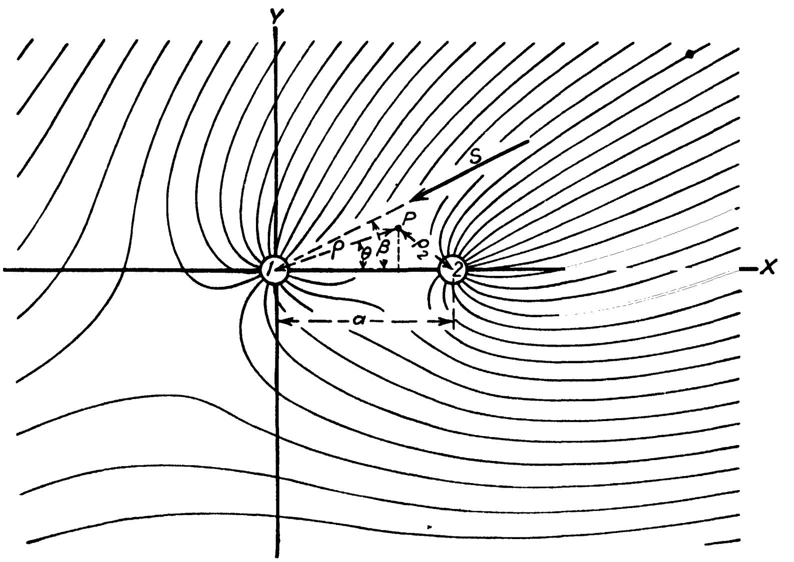


FIG. 11.—Interference of two similar ideal artesian wells with a sloping piezometric surface and the same drawdown for each well.

The case of interference of two similar ideal artesian wells with a sloping piezometric surface is given below. In the development, it is assumed that the drawdown and discharge are the same for each well. In Fig. 11, well 1 is used as a reference point and well 2 is located at a distance a along the X-axis from the origin. With no flow into the wells, the piezometric surface has a slope S from the direction of β with the X-axis. The stream lines into the two wells are as shown in Fig. 11.

The potential at any point P due to the sloping piezometric surface is

$$\Phi_s = S(\rho - r)\cos(\beta - \theta) \tag{13a}$$

The potential at P due to the flow into well 1 and well 2 may be obtained by integrating Eq. (3) and is

$$\Phi_1 = -\frac{c_1}{k}\ln\rho + C_3 \tag{13b}$$

and

$$\Phi_2 = -\frac{c_2}{k}\ln\rho_2 + Sa\cos\beta + C_4 \tag{13c}$$

The total potential at P is the sum of Eqs. (13), and since

$$\rho_2 = \sqrt{\rho^2 - 2a\rho \cos \theta + a^2} \tag{14}$$

the potential at P is

$$\Phi = -\frac{c_1}{k} \ln \rho - \frac{c_2}{2k} \ln (\rho^2 - 2a\rho \cos \theta + a^2) + Sa \cos \beta + S(\rho - r) \cos (\beta - \theta) + C_5 \tag{15}$$

When $\rho = r$, $\Phi = h$, from which relation C_5 may be evaluated. The other constants c_1 and c_2 may be evaluated by setting $\rho = R$ and $\theta = \pi$ at which point $\Phi = \Phi_R - S_R \cos \beta$, and by setting $\rho = R + a$ and $\theta = 0$ at which point $\Phi = \Phi_R + S(R + a) \cos \beta$. When the values of the constants c_1, c_2, and C_5 are inserted in Eq. (15), the potential at point P becomes

$$\Phi = h - \frac{h - \Phi_R}{\ln \dfrac{R(R + a)}{r(r + a)}} \left(\ln \frac{\rho}{r} + \frac{1}{2} \ln \frac{\rho^2 - 2a\rho \cos \theta + a^2}{r^2 - 2ar \cos \theta + a^2} \right) + S(\rho - r) \cos (\beta - \theta) \tag{16}$$

In the preceding equation, the value of r is assumed to be small as compared with the value of a. The radial velocity toward well 1 at any point P is, from Eqs. (3) and (16),

$$v = k \frac{h - \Phi_R}{\ln \dfrac{R(R + a)}{r(r + a)}} \left(\frac{1}{\rho} + \frac{\rho - a \cos \theta}{\rho^2 - 2a\rho \cos \theta + a^2} \right) - kS \cos (\beta - \theta) \tag{17}$$

The discharge into the well is

$$Q = \rho t \int_0^{2\pi} v \, d\theta = 2\pi kt \frac{h - \Phi_R}{\ln \dfrac{R(R + a)}{r(r + a)}} \tag{18}$$

To illustrate the use of the foregoing equation, consider two 6-in. wells 200 ft apart being pumped with a drawdown such that $h - \Phi_R = 10$ ft when $R = 600$ ft. When both wells are being pumped, the discharge from each, from Eq. (18), is $Q = 1.091(2\pi kt)$. When only one well is being pumped, the discharge, from Eq. (7), is $Q = 1.283(2\pi kt)$. The effect of interference in this case is thus to reduce the capacity of the wells by about 15 per cent.

It may be noted that the piezometric slope S does not appear in Eq. (18). The interference is therefore independent of the general direction of flow in the aquifer and of the magnitude of the slope of the piezometric surface.

An analysis of the interference of a large number of ideal artesian wells arranged in a row with equal spacing is presented by Slichter.[1] Muskat[2] presents an extensive discussion of multiple-well systems.

Methods of Construction of Wells.[3] Wells are classified as *dug*, *driven*, and *drilled* wells, depending upon the method of construction. Shallow wells are sometimes constructed as *dug wells*, the methods used being similar to the methods used in excavating any open shaft. *Driven wells* are shallow wells constructed by driving a pipe of small diameter into the water-bearing formation. The pipe is usually of heavy wrought iron 2½ in. or less in diameter and equipped with a well point and strainer at the bottom. Driven wells[4] are usually developed in gangs or fields for municipal supplies, the wells being spaced 25 to 100 ft apart and connected to a com-

[1] SLICHTER, *op. cit.*
[2] MUSKAT, *op. cit.*
[3] *U.S. Geol. Survey Water Supply Papers* 255, 1910, and 257, 1911.
[4] *Jour. N.E.W.W.A.*, **47**, 5, 1933; and **50**, 149, 197, 207, and 219, 1936.

mon suction header. As the wells are pumped by suction, they cannot be used when the water table is more than about 15 ft below the surface. The useful depth of driven wells is 20 to about 60 ft. A typical driven well field is shown in Fig. 12.

Deep wells are sunk by several different methods, depending upon the character of material penetrated and the locality. Some of the important methods are described below.

The *standard* or *oil-well method* is used for wells 4 to 12 in. in diameter through both earth and rock to depths sometimes exceeding 5,000 ft. Drilling is done by a

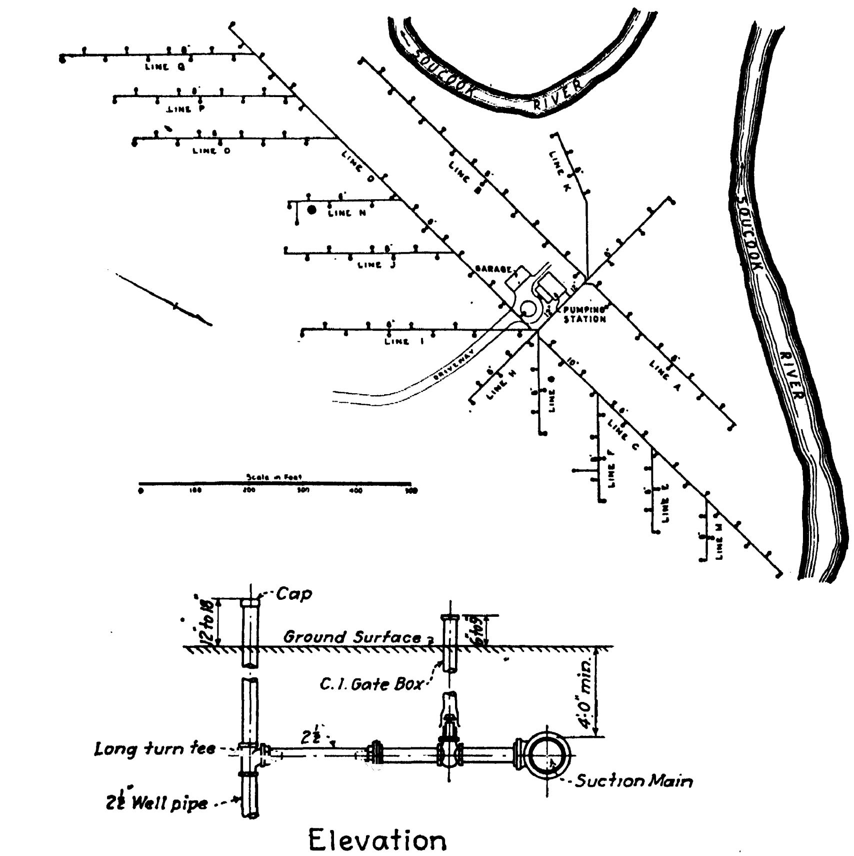


Fig. 12.—Pembroke well system and details, Concord, N.H. (*Sanders and Marston, Jour N.E.W.W.A.*, **47**, 5, 1933.)

heavy drill bit suspended into the hole upon a rope attached at the upper end to the walking beam of a derrick. The bit is alternately lifted and dropped by machinery, the material being thus broken into small fragments for subsequent bailing with a bucket. Steel pipe casing of slightly smaller diameter than the hole is lowered or driven into the hole as drilling proceeds down to an impervious stratum. At this point, the casing is sealed or cemented at the bottom against the passage of water downward around the outside of the casing to the well, the well being thus protected from contamination by shallow ground water and surface water.

Drilling is then continued to the aquifer. If the well penetrates water-bearing strata containing water of unsatisfactory quality or deposits of loose material, these

strata are also sealed off by casing of smaller size lowered through the upper casing. If the equifer is of loose granular material, it is penetrated by the casing which is perforated or equipped with a well screen at the bottom. Tubular strainers or screens of corrosion-resistant metal closed at the lower end are usually set by lowering to the bottom of the well through the casing. The casing is then pulled up to the height of the strainer, and the annular space between the bottom of the casing and the top of the strainer is sealed with a lead packing ring.

The *California* or *stovepipe method* is used for wells 6 to 38 in. in diameter through earth or alluvial deposits. The casing consists of two sizes of lap-riveted or welded-steel cylinders, one of which just fits over the other, the joints of the outer cylinders butting at the mid-points of the inner cylinders. The cylinders are usually made 4 ft long and of 12- to 8-gage steel, 8 gage being used for diameters greater than 20 in. The casing is forced down by means of hydraulic jacks, the casing being fitted with a drive pipe and starter shoe at the bottom. As the casing is forced down, the material within is excavated by standard methods or by means of a mud scow alternately raised and dropped.

When the well has been driven through the aquifer to impervious material, the casing is perforated for the depth of the aquifer by a specially designed tool. Stovepipe casing cannot be pulled back for setting screens. If it is desired to set a screen at the bottom, the stovepipe casing must be stopped at the level proposed for the top of the screen. The screen is then lowered through the casing to the bottom and is sunk into position by bailing the material from within the screen.

The *hydraulic rotary method*[1] has practically superseded all other methods except the California method in areas where the formations are of unconsolidated sand and clay. A hollow-drill rod driving a hollow drill is rotated and lowered into the hole while a slurry composed of water and mud or clay (driller's mud) is pumped through the rod and drill into the hole. The mud plasters the side of the hole and seals the pores, preventing the leakage of the pumped water out into the formation. The velocity of the rising slurry can thus be kept high enough to lift the cuttings to the surface. The mud which is plastered into the sides of the hole combined with the lateral pressure of the slurry in the hole prevents caving of the walls while the casing is being placed.

It is possible with this method to set 2,000 or 3,000 ft of casing without the necessity of reducing the diameter. Bores 10 to 60 in. in diameter have been constructed. The larger diameters are sometimes drilled by first sinking a small hole and then reaming to large size. The slurry is circulated continuously during the reaming process and during the setting of the casing. If pumping of the mud is stopped, the casing may lodge against the side of the hole. The casing is usually of riveted or welded steel or of wrought iron. Sometimes sections of the casing are preperforated to serve as screens, but usually a separate strainer of corrosion-resistant metal is set by the bailing-in method. The driller's mud which has been circulated behind the casing during the setting of the pipe serves to seal the well from water percolating down from the surface.

Large important wells in unconsolidated sand and clay are now frequently made of the gravel-wall type. They may be constructed by either the stovepipe or hydraulic rotary method. A large conductor pipe or caisson 6 to 12 in. larger than the well casing is first sunk down to a cutoff in clay or to a sufficient depth to seal the well from surface water. The well casing is then placed concentrically within the conductor pipe and sunk through the water-bearing strata. If the stovepipe method is used, the well casing is equipped at the bottom with a starter shoe several inches greater in diameter than the pipe. The gravel is fed into the annular space between the hole

[1] Lanham and Allen, *Jour. A.W.W.A.*, **24**, 1519, 1932.

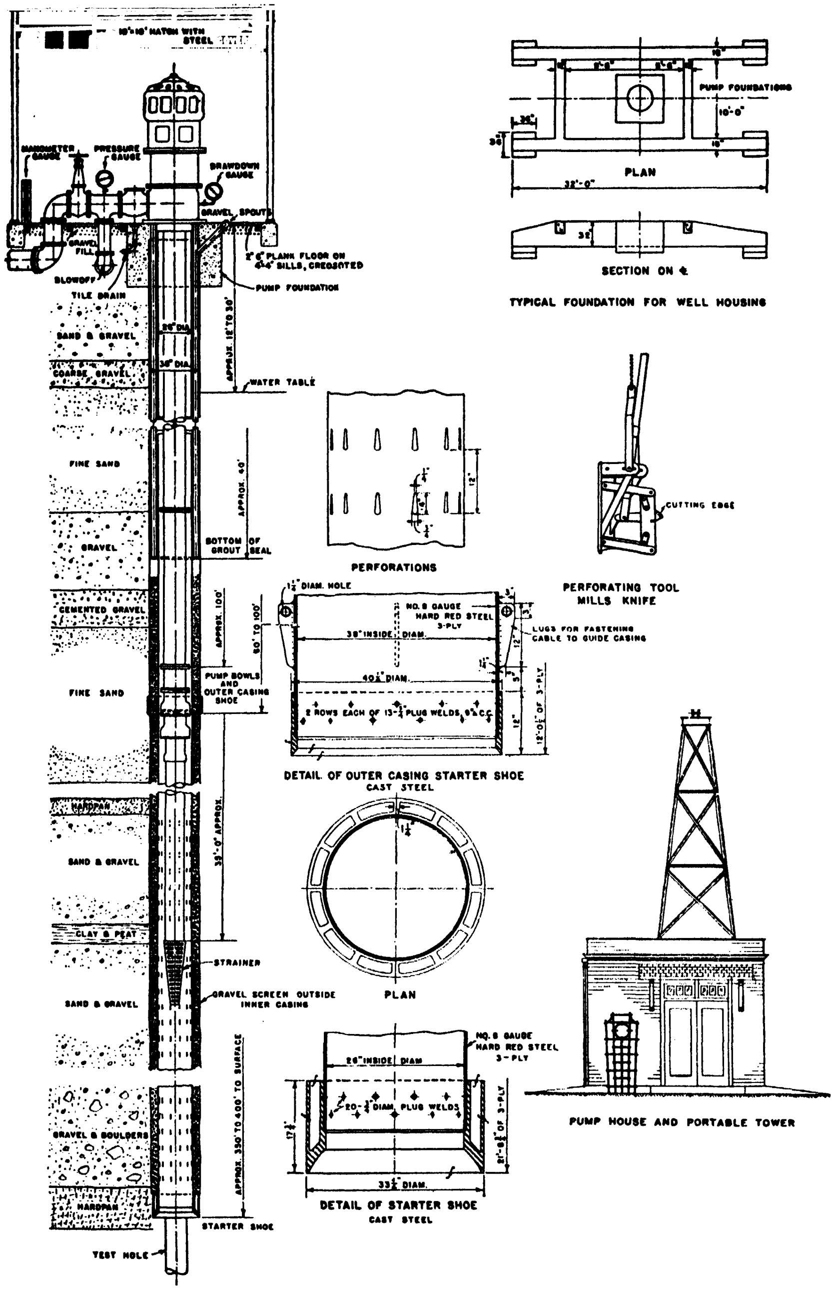


FIG 13.—Gravel-wall well for Tacoma, Wash. (*Kunigk, Jour. A.W.W.A.*, **26**, 846, 1934.)

and the outside of the casing. When the starter shoe penetrates to the desired depth, the bottom of the casing is sealed. The casing is then perforated at the proper levels and the well is developed by swabbing[1] and pumping from it at a high rate. In this process, the fine sand surrounding the casing is dislodged and pumped out. The gravel is fed in from above to replace the sand. Figure 13 shows a typical well of this type.

When the hydraulic rotary method is used for a gravel-wall well, the well casing is usually fitted with a screen at the bottom. The well may be developed in the same manner as described above, but it is important that agitation be sufficiently violent to dislodge the driller's mud from the wall of the hole. This is a difficult procedure and requires great skill on the part of the driller. It is particularly difficult when the well is underreamed to large diameter in order to provide more space for the gravel, for in this case the velocity of the water flowing through the mud wall during the developing process may not be sufficient to wash out the mud. Figure 14 shows a gravel-wall well constructed by the hydraulic rotary method.

Methods for developing this type of well as described by Schweitzer[2] include the use of a circular nozzle device placed around the screen in the annular space between the screen and the mud wall. Nozzles spaced about 6 in. apart and directed against the mud wall are fed through a 3-in. pipe from above which also serves to support the device and to permit of its being raised as the hole is graveled. Water is pumped at high velocity through the nozzles against the mud wall which is about 3 in. in front of the nozzles. At the same time, gravel and water is pumped though a second pipe discharging just above the nozzles. A third pipe is suspended a short distance above the device, and through it is pumped by suction the native sands and mud dislodged by the nozzles.

The screen openings for gravel-wall wells should be large enough to permit the fine sand to pass readily, never less than about ⅛ in. The gravel should be large enough so that the particles cannot clog or pass through the screen openings and small enough to flow readily into position. The size used varies from ¼ to about 1 in., depending upon the character of the formation and the size of the space through which the gravel must be fed. According to Schweitzer, for formations of coarse sand 1.0 mm or more in diameter, a single envelope of gravel not less than ¼ in. in size is sufficient. For fine sand, most of which is less than 0.4 mm in size, a double envelope is desirable. In this case, a 2 to 6 in. thickness of ¼- to ⅜-in. gravel should be placed next to the screen, and a coarse sand 2 to 3 mm in size should be placed between the gravel and the native sand.

Sanitary Precautions. Wells should be so located[3] as not to be subject to surface drainage of flood water and seepage from cesspools, privies, and sewers. A minimum distance from sources of pollution of 50 to 100 ft is sometimes specified, but it must be kept in mind that the effectiveness of removal of polluting matters from seeping water depends upon the character of the formation through which the water percolates as well as the length of the path. The importance of safeguards against contamination cannot be overemphasized. Many cases of typhoid have been traced to polluted well supplies.[4]

In order to minimize the possibility of pollution, the casing, or caisson in the case of graveled wells, should penetrate into an impervious formation and be sealed off at the bottom. In the hydraulic rotary method, the driller's mud between the casing and the hole is sometimes relied upon as a seal. When the stovepipe method is used, it is desirable to seal the casing by a cement plug at the bottom or by pumping cement

[1] See KUNIGK, *Jour. A.W.W.A.*, **26**, 850, 1934.
[2] *Jour. A.W.W.A.*, **26**, 712, 1934.
[3] KLASSEN and FERGUSON, *Jour. A.W.W.A.*, **24**, 875, 1932.
[4] MERCKEL, *Jour. A.W.W.A.*, **26**, 338, 1934.

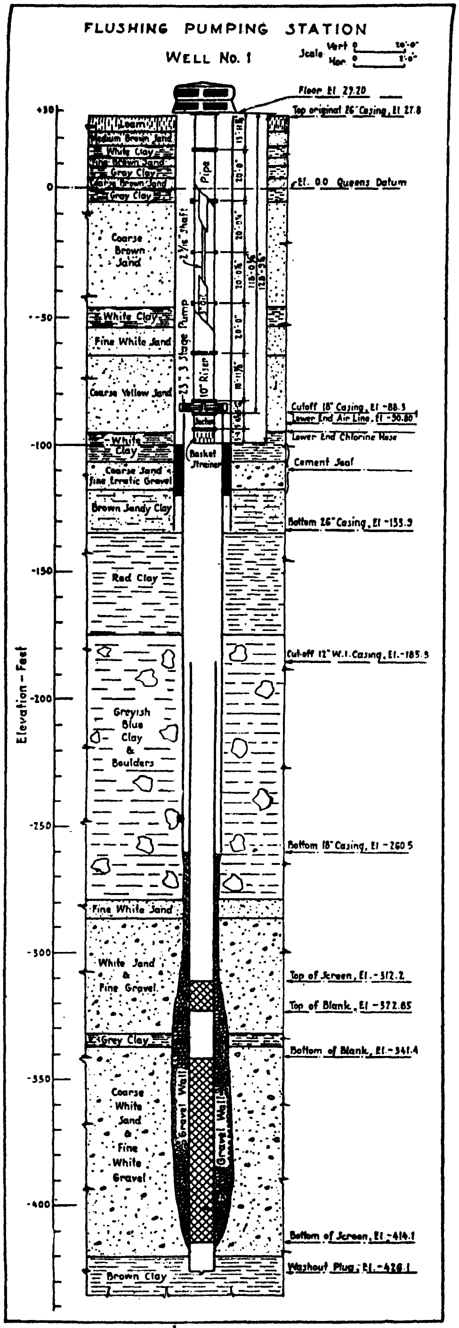


Fig. 14.—Gravel-wall well at Flushing, N. Y. (*Laase, Jour. A.W.W.A.*, **24**, 1530, 1932)

grout through the casing and forcing it up around the casing on the outside. Several methods of cementing are described by Fiedler.[1] Pollution may enter a well through seams in the casing or holes caused by corrosion. It is important to make the casing watertight and to use corrosion-resistant metal. Schweitzer[2] describes a well that was lined with cement in such a manner that protection is afforded by the cement lining even if the casing is perforated by corrosion.

Driven wells are particularly subject to pollution from the surface. The best practical safeguard is the proper location of the well field.

Shallow wells near the sea are subject to contamination with salt water if the water table is too low. For each foot of elevation of the water table above mean sea level, the fresh water extends about 38 ft below sea level. Below the fresh ground water is a heavier layer of salt water.

The top of a well should be sealed against the possibility of contamination from the surface, and the top of the well or pump-house floor should be drained away from the well. Before a well is placed in service, it is desirable to disinfect it by chlorination.

Well Pumps. Driven well fields are usually pumped by centrally located horizontal centrifugal pumps equipped with a sand chamber on the main suction pipe. Small-bore deep wells are pumped by reciprocating deep-well pumps or by air lifts. Large deep wells are usually pumped by deep-well turbine pumps. Descriptions of well pumps are given in Sec. 12.

The range of capacities of various types of deep-well pumps for casings up to 12 in. inside diameter as given by Gordon[3] is shown in Table 4. The figures quoted for air-lift pumps are for submergences ranging from 40 to 70 per cent and for taper discharge pipes. For large-diameter wells, turbine pumps have been used to capacities exceeding 5,000 gpm.

TABLE 4.—RANGE OF CAPACITIES OF VARIOUS TYPES OF DEEP-WELL PUMPS
Discharge in U.S. gallons per minute

Interior diameter of well casing, in.	4	6	8	10	12
Air-lift	11–38	70–240	160–540	310–1,050	560–1,875
Deep-well turbine	.....	40–350	60–450	120–1,150	120–1,800
Deep-well plunger	30	100	200	345	490

According to Gordon, over-all efficiences from motor input to water delivered range from 20 to 35 per cent for air-lift pumps and around 65 per cent for plunger pumps. Over-all efficiencies for turbine pumps range from about 50 for the smaller installations to about 70 for large pumps. Air lifts are adapted to crooked wells, to wells discharging large amounts of sand and to installations where reliability is of more importance than efficiency. There are no submerged moving parts subject to wear in an air-lift installation.

[1] FIEDLER, *Jour. A.W.W.A.*, **22**, 924, 1930.
[2] SCHWEITZER, *Jour. A.W.W.A.*, **29**, 91, 1937
[3] GORDON, *Eng. News-Record*, **108**, 620, 1932

SECTION 20

WATER DISTRIBUTION

BY THOMAS R. CAMP

The distribution system of a waterworks consists of the pipes, valves, hydrants, and appurtenances used for distributing the water; the elevated tanks and reservoirs used for fire protection and for equalizing pressures and pump discharges; and the consumer service pipes and meters. For administrative purposes, booster pump stations and treatment works located within the bounds of the distribution system are sometimes classed as distributing works.

A distribution system should be so designed that an adequate supply of water is available to the consumers and for fire protection at all times at a minimum of cost. It should also be so constructed and operated that the chances for contamination of the water after it has entered the system are reduced to a minimum. Since most distribution systems have developed with the growth of the community served, the problem of designing a complete new system seldom arises except for small towns. The principles involved in the design of a complete system may be employed in the design of extensions and reinforcing mains with modifications to suit each individual case.

The twofold function of distribution systems as designed in America requires (1) the relatively uniform distribution of water throughout the system for ordinary use, and (2) the concentration at any point in the system of high rates of flow for fire extinction. The investment[1] in the distributing system varies from over 90 per cent of the total waterworks investment for small communities supplied from wells to about 40 per cent for large cities; whereas the percentage of the cost of the distribution system chargeable to fire protection varies from about 60 for small towns to about 4 or less for large cities.

DESIGN OF PIPE DISTRIBUTION SYSTEMS

Pressure and Flow Requirements for Normal Consumption. The pressure required in the mains for normal domestic consumption depends upon the height of the buildings, the maximum instantaneous rate of flow through the house service pipes, and the friction losses in meters, house services, plumbing, and fixture outlets.

The maximum instantaneous rate of flow through a house service pipe depends upon the character and number of plumbing fixtures in the building and the probability of their simultaneous use. The approximate flow and pressure requirements for some common domestic fixture outlets are shown in Table 1.

In studies of the rate of flow through house meters at Hartford, Griswold and Gentner[2] found that the maximum instantaneous rate of flow for one-, two-, and three-family houses having tank-type water closets did not exceed 10 gpm. The peak flow to one-, two-, and three-family houses having water closets equipped with flushometers was less than 30 gpm. The peak flow to be used in design of the house water piping and for selection of meters is, according to Cleverdon,[3] about sixteen times the average

[1] See "Water Works Practice," A.W.W.A., 1929, p. 297.
[2] GRISWOLD and GENTNER, *Jour. N.E.W.W.A.*, **46**, 288, 1932.
[3] CLEVERDON, "Plumbing Engineering," Pitman Publishing Corporation, 1937, p. 79.

TABLE 1.—FLOW REQUIREMENTS TO HOUSE PLUMBING FIXTURES

Fixture	Excellent flow, gpm	Pressure at outlet (faucets wide open), psi
Lavatory faucets, single	4	4
Bathtub faucets, single	6	5
Combination bathtub faucets	8	5
Sink faucets	6	5
Shower heads	6	3
Shower mixing valves	6	30
Water closets, tank type	5	5
Water closets with flush valves	30	25
Garden hose and nozzle	6	30 at hydrant

daily consumption figured on a floor-area basis for small private houses and about thirty times the average daily consumption for apartment houses, with three to six apartments. Cleverdon's estimates are for buildings equipped with tank-type water closets.

A substantial loss of pressure is required at high flows through house meters, and the pressure loss varies considerably with different makes of meters. The Standard Specifications for Cold-Water Meters adopted in 1941 and 1947 by the American Water Works Association and the New England Water Works Association limits the pressure loss to 15 psi for meters 1 in. and smaller and to 20 psi for larger meters at the maximum flow capacity (see Table 13). The limiting pressure losses at other rates of flow may be estimated by assuming that the pressure loss varies as the square of the discharge. The estimated maximum loss of head through meters for residences equipped with tank toilets is thus 3.6 psi, and for residences equipped with flushometer toilets is 12 psi, unless the flushometers are fed from tank storage within the building.

If the pressure loss in the house service pipe and water lines within the building is assumed at 20 psi, the total pressure loss from the main to a residence with tank toilets at peak flow of 10 gpm is about 28 psi. The pressure required at the street level for excellent flow to a three-story building is therefore about 42 psi. Experience indicates that flow is adequate for residential areas if the pressure is not reduced below 35 psi. Flushometers and shower mixing valves cannot be used satisfactorily with such low pressures.

In business districts, the required pressure for normal use depends directly upon the height of the buildings. Buildings up to about 20 stories in height may be conveniently served directly from the pressure in the street main in many cases. The pressure required at the street for a 20-story building will be about 120 psi. Such high pressures are not ordinarily desirable in the mains or in plumbing systems because of leakage and waste, but they frequently exist in the lower sections of a district in which there are marked differences in ground elevation. Very tall buildings are usually served with their own pumping equipment, and the building water piping may be divided into several pressure zones.

The capacity of the pipes in a distribution system should be sufficient to deliver the maximum domestic flow together with the fire flow. The domestic consumption loads for each district in the distribution system are computed from the estimated population and estimated maximum daily consumption per capita. For design purposes, the maximum daily consumption at the end of the design period should be used. In grading cities, the Underwriters[1] assume that "the maximum daily consumption

[1] "Standard Schedule for Grading Cities and Towns of the United States with reference to their Fire Defenses and Physical Conditions," National Board of Fire Underwriters, edition of 1942.

is 50 per cent in excess of the average, but in all cases of application the actual average consumption for the year previous shall be taken as the average consumption, and the maximum consumption for any 24 hours in the past 3 years taken as the maximum consumption, unless conditions have so changed that this maximum will not occur again." Industrial consumption loads must be estimated from the needs of the various manufacturing plants within the district. Commercial loads may be estimated in terms of floor area as indicated in Sec. 19. Peak commercial loads on the mains in the distribution system will be much less than the peak used by Cleverdon for plumbing design. These loads will range from two to eight times the average daily consumption figures given in Table 1 of Sec. 19.

Flow and Pressure Requirements for Fire Protection. The fire flow for the whole system and for each district within the system, as required by the Underwriters,[1] is given by Eq. (1) and the paragraphs that follow it in Sec. 19. The Underwriters recommend that when pumping engines are used a residual pressure at the hydrants of not less than 10 to 20 psi should be maintained with fire flow plus domestic flow. All hydrants should have at least one engine connection (steamer outlet). If streams are to be used direct from hydrants, a residual pressure of 75 psi is required for high-value districts, 60 psi where not more than 10 buildings exceed three stories and in closely built residential districts, and 50 psi in village mercantile districts where buildings do not exceed two stories and in thinly built residential sections.

In districts in which the mains are designed for a normal pressure just sufficient to give adequate domestic flow to homes, it is evident that there will be an insufficient pressure for fire fighting without mobile pumpers. According to the American Water Works Association,[2] it is desirable that a normal pressure of 60 to 75 psi be maintained on a distribution system for the following reasons:

"*a*. It will supply ordinary consumption for buildings up to 10 stories in height.

"*b*. Gives effective sprinkler service in buildings of 4 or 5 stories.

"*c*. Permits direct hydrant-service for a few hose streams, insuring quicker operation by the fire department.

"*d*. Allows a larger margin of fluctuation in local pressures in meeting sudden drafts, and offsets losses due to partial clogging or excessive length of service pipes."

An additional advantage of such a high pressure is that it will permit the wider use of flushometers and shower mixing valves. The advantages of higher pressures, however, should in every case be weighed against the additional pumping cost, for all the water used must be pumped against the additional head.

Four methods are in use for supplying pressure for fire streams, as follows:

1. The maintenance of sufficient pressure on the mains at all times for direct hydrant service for hose streams.
2. The use of emergency fire pumps to boost the pressure in the distribution system during fires.
3. The use of mobile pumping engines which take suction from the hydrants.
4. The use of a separate high-pressure distribution system for fire protection only.

The first method is not ordinarily economical for large communities but is usually the best method for villages not provided with full-time fire departments and mobile pumpers. The second method is applicable to villages requiring higher pressures for fire fighting than are desirable for normal consumption and in which there are no mobile pumpers. It is more economical but less reliable than the first method. The third method is preferable for all communities large enough to maintain modern and well-trained fire departments. The fourth method is in use only in portions of some of the large cities. Separate high-pressure systems are usually supplementary to the

[1] *Idem.*
[2] "Water Works Practice," A. W. W. A., 1929, p. 298.

TABLE 2.—FIRE-STREAM AND HOSE DATA

Nozzle pressures by Pitot, psi	1: Discharge, gpm	1: Pressure loss 100 ft 2½-in. hose, psi	1: Vertical reach, ft	1: Horizontal reach, ft	1⅛: Discharge, gpm	1⅛: Pressure loss 100 ft 2½-in. hose, psi	1⅛: Vertical reach, ft	1⅛: Horizontal reach, ft	1¼: Discharge, gpm	1¼: Pressure loss 100 ft 2½-in. hose, psi	1¼: Vertical reach, ft	1¼: Horizontal reach, ft	1⅜: Discharge, gpm	1⅜: Pressure loss 100 ft 3-in. hose, psi	1⅜: Vertical reach, ft	1⅜: Horizontal reach, ft	1½: Discharge, gpm	1½: Pressure loss 100 ft 3-in. hose, psi	1½: Vertical reach, ft	1½: Horizontal reach, ft
20	132	4.8	35	37	167	7.3	36	38	206	10.6	36	39	250	5.8	36	40	298	8.1	37	42
25	148	5.8	43	42	187	8.9	44	44	230	13.1	45	46	280	7.2	45	47	333	10.1	46	49
30	162	6.8	51	47	205	10.5	52	50	253	15.5	52	52	307	8.6	53	54	365	11.9	54	56
35	175	7.9	58	51	221	12.1	59	54	273	17.8	59	58	331	9.9	60	59	394	13.7	62	62
40	187	8.9	64	55	237	13.8	65	59	292	20.0	65	62	354	11.2	66	64	422	15.5	69	66
45	198	9.9	69	58	251	15.3	70	63	309	22.2	70	66	376	12.5	72	68	447	17.3	74	71
50	209	10.9	73	61	265	16.8	75	66	326	24.7	75	69	396	13.8	77	72	472	19.1	79	75
55	219	11.9	76	64	277	18.3	79	69	342	27.2	80	72	415	15.1	81	75	494	20.8	83	78
60	229	12.8	79	67	290	19.8	83	72	357	29.6	84	75	434	16.4	85	77	517	22.6	87	80
65	238	13.8	82	70	301	21.3	86	75	372	31.7	87	78	451	17.6	88	79	537	24.3	90	82
70	247	14.8	85	72	313	22.9	88	77	386	33.9	90	80	469	18.8	91	82	558	26.0	92	84
75	256	15.8	87	74	324	24.5	90	79	399	36.1	92	82	485	20.0	93	84	578	27.8	94	86
80	264	16.7	89	76	335	26.1	92	81	413	38.6	94	84	500	21.2	95	86	596	29.5	96	88
85	272	17.7	91	78	345	27.7	94	83	425	40.8	96	87	516	22.5	97	88	614	31.2	98	90
90	280	18.7	92	80	355	29.3	96	85	438	43.1	98	89	531	23.8	99	90	633	32.9	100	91

main distribution system and thus give added fire protection in high-value districts. Pressures of 150 to 300 psi are used in high-pressure systems.

The effective reach of fire streams for smooth nozzles and for various nozzle pressures, as determined by Freeman,[1] is shown in Table 2. The corresponding values of the discharge and pressure loss in 2½- and 3-in. best quality rubber-lined fire hose, as given by the National Board of Fire Underwriters,[2] are also shown. Nozzle pressures are velocity heads in pounds per square inch at the nozzle tips, measured by means of Pitot tubes. The effective reach is the distance in feet from the nozzle at which streams will do effective work with a moderate wind blowing. It will be noted that the vertical reach ranges from about 78 per cent of the nozzle velocity head at 20 psi to only 46 per cent at 90 psi. The maximum practical vertical reach is therefore about 100 ft, and the streams are more effective if the nozzles are elevated.

For inside hand lines,[3] 2½-in. hose lines with 1⅛-in. shutoff nozzles are ordinarily used. When the nozzle pressure exceeds about 60 psi, the jet reaction is too great for the nozzle to be held by hand. For fighting large fires from the outside, 1½-in. nozzles are usually used with 3-in. hose lines or siamesed 2½-in. lines. Such nozzles are usually fixed in position and require pressures of 65 to 80 psi.

For residential areas, 175-gal fire streams are standardized as satisfactory, and for business districts of ordinary character 250-gal streams. For direct hydrant service in residential areas, it may be noted from Table 2 that hydrant pressures of about 65 psi are required with standard streams and 500-ft hose lines. For direct hydrant service in business areas with 250-gal standard streams and 300-ft lines, hydrant pressures of about 95 psi are required. Since such high pressures on the system are normally inadvisable, mobile pumping engines are usually employed for important fires. The ordinary capacity of a pumper is 750 gpm, which should be supplied by one hydrant.

General Arrangement of Pipe System. The location of the small distributor pipes in a distribution system is controlled by the location of the consumers and by the location of property requiring fire protection. The pipes are usually laid in the streets at some standardized position between curbs. In the case of very wide streets, however, it is sometimes cheaper to install a main behind the curb on each side of the street because of the saving in service pipes. The system should be gridironed with connecting pipes laid on the cross streets at intervals not exceeding about 600 ft whether or not there are consumers on the cross streets. Dead ends should be avoided in order to minimize troubles from corrosion and from organic growths. Moreover, a pipe fed from both ends has a capacity equivalent to two pipes.

A large system will consist of supply mains, arteries, and secondary feeders spaced at intervals of about 3,000 ft in the grid system and preferably looped. The approximate location of the feeders will be determined largely by the distribution of the consumers and high-value property.

For fire protection, the Underwriters[4] specify a minimum size of main of 6 in. for residential areas and 8 in. for high-value districts if cross-connecting mains are not more than 600 ft apart. On principal streets and for all long lines not cross-connected at frequent intervals, 12-in. and larger mains are required by the Underwriters.

Gate valves[4] should be so located that no single case of breakage in the pipe system, exclusive of arteries, shall require more than 500 ft of pipe to be shut from service in high-value districts, or more than 800 ft in other sections, or shall require the shutting down of an artery. The valves should be located at street intersections in standardized positions so that they can be readily found in case of pipe breakage. All small distributors branching from larger pipes should be equipped with valves, although the

[1] FREEMAN, *Trans. A. S. C. E.*, **21**, 303, 1889.

[2] "Fire Engine Tests and Fire Stream Tables," National Bureau of Fire Underwriters, 4th ed., 1931.

[3] GOLDSMITH, *Jour. A.W.W.A.*, **22**, 17, 1930.

[4] "Standard Schedule for Grading Cities and Towns of the United States with reference to their Fire Defenses and Physical Conditions," National Board of Fire Underwriters, edition of 1942.

larger pipes need not have valves at each such branch. At intersections of large pipes, a valve in each branch is desirable. Large supply mains should be gated about once a mile and should be provided with air valves at high points and blowoffs at low points. Arteries should be gated so that not more than ¼ mile within the system will be affected by a break.

Hydrants should be located at street intersections where they are accessible from four directions. They should be so spaced that no hose line need exceed 500 to 600 ft. The spacing will vary from about 150 ft in high-value districts of large cities to about 600 ft in suburban residential districts. The requirements[1] of the Underwriters for hydrant distribution are given in Table 3.

TABLE 3

Required Fire Flow, gpm	Average Area per Hydrant, sq ft
1,000	120,000
2,000	110,000
3,000	100,000
4,000	90,000
5,000	85,000
6,000	80,000
7,000	70,000
8,000	60,000
9,000	55,000
10,000	48,000
11,000	43,000
12,000	40,000

Hydrants are required to have not less than two 2½-in. hose outlets, standard 4½-in. suction outlets where necessary, and to be connected to the main with pipe not smaller than 6 in. and gated. Hydrants shall be able to deliver 600 gpm with a loss of not more than 2.5 psi in the hydrant and a total loss of not more than 5 psi between the street main and the outlet.

The depth to which pipes should be laid is controlled by the cover required for protection against structural failure due to street traffic loads and for protection against freezing. In the southern part of the United States where there is little or no frost penetration into the ground, a minimum cover of 18 to 24 in. has been used. For protection against wheel loads of heavy trucks, the amount of cover required increases with the size of pipe and is greater for steel than for cast-iron mains. For important mains, the amount of cover to be used should be determined after an investigation of the stresses produced by wheel loads. Less cover is required for pipes under concrete pavements than for pipes under more resilient pavements and dirt roads. The depth of cover required for protection against freezing may be estimated from Table 4, which shows the frost penetration and the depth of cover used in cities with widely divergent climatic conditions.

The Pressure Table and Pressure Districts. The pressure table, or piezometric surface, of a distribution system is the imaginary surface above the ground to which the water would rise in piezometers connected into the pipes. The pressure at any point in the system corresponds to the height of the pressure table above the ground. If the pressure table is controlled by elevated tanks or reservoirs within the system, it may be called a *fixed* pressure table. If there are no open surface tanks within the system, the

[1] "Standard Schedule for Grading Cities and Towns of the United States with reference to their Fire Defenses and Physical Conditions," National Board of Fire Underwriters, edition of 1942.

elevation of the pressure table may be varied at will by varying the rate of inflow to the system above or below the rate of consumption. Such a system has a *variable* pressure table.

TABLE 4

	Charleston,[1] S. C.	Washington[2] Suburban Sanitary District	Wilmington,[2] Del.	Portland,[3] Me.	Chicago,[2] Ill.	Peterborough,[4] Ont.	Sudbury,[4] Ont.	Ottawa,[4] Canada
Frost penetration, ft	....	2½	3	4	Clay, 5-5½ Sand, 5-6	Gravel with pavement, 6-7 Gravel no pavement, 5-6 Clay or sand, 5	7	Paved streets, 7½
Depth of cover, ft	2 to 2½	4	4	5-ft trench	Pipes 12 in. and smaller, 5½ large feeders, 2	....	7-ft trench	7

[1] GIBSON, *Jour. A.W.W.A.*, **22**, 1315, 1930.
[2] HECHMER, WILLS, and GAYTON, *Jour. A.W.W.A.*, **28**, 841, 837, 849, 1936.
[3] FULLER, *Jour. N.E.W.W.A.*, **50**, 300, 1936.
[4] DOBBIN, MARTINDALE, and MACDONALD, *Jour. A.W.W.A.*, **26**, 1160, 1159, 1166, 1934.

The pressure table slopes in the general direction of the flow of the water because of head losses in the pipes, and the slopes are greatest where friction losses are greatest. The slopes increase with the consumption and are greatest during fire flows, particularly in the immediate vicinity of a fire. The fluctuation in the elevation of the pressure table is greatest at points most remote from the source of supply to the system or from equalizing tanks. The shape of the pressure table for any particular condition of flow may be illustrated by a contour map.

When the flow approaches zero, the pressure table approaches a horizontal plane, which may be conveniently referred to as the static pressure table. A fixed static pressure table is a horizontal plane at the water surface of the equalizing reservoir. Actually a fixed static pressure table varies somewhat owing to the fluctuation in the elevation of the water level in the equalizing tank. In order to minimize this fluctuation, equalizing reservoirs should be shallow. A variable static pressure table is a horizontal plane at the elevation corresponding to the pressure at the pumps. The elevation of this plane may be raised or lowered by increasing the rate of inflow above or reducing it below the rate of consumption. It may be held constant by continuously varying the rate of inflow to correspond to the demand. Because of the difficulty of controlling pressures by changes in the pumping rate, most modern systems have equalizing reservoirs. Such reservoirs, when provided with sufficient capacity, permit the pumps to operate at a constant rate throughout the daily pumping period and provide fire storage. In large systems, better equalization of pressures is obtained by having several reservoirs or elevated tanks distributed at strategic points in the system.

In communities located in hilly country with large differences in ground elevation and for very large systems, two or more pressure districts may be required. Each pressure district has its own pressure table, and the distribution system in each district operates independently. Connections may be made between the districts by means of mains equipped with gate valves normally closed or equipped with automatic pressure-reducing valves. High-pressure fire systems are usually not connected with the distribution system carrying the domestic supply. Complete separation of

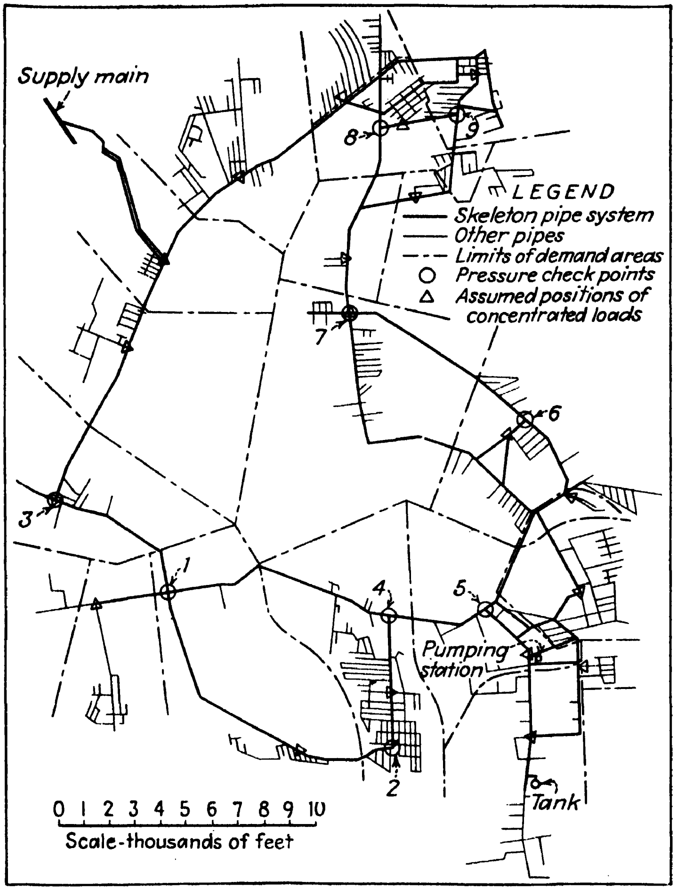


FIG. 1.—Water distribution system, Warwick, R.I.

the two systems makes it possible to use a cheap, nonpotable water or salt water in the high-pressure fire system.

General Procedure in Design. In the design of a distribution system, the following general procedure may be used.

1. *Preliminary Layout.* A preliminary layout of all the pipes is prepared on a suitable map of the community. A contour map showing all street and lot lines is preferable. The location of all existing buildings, with heights shown, is helpful. The layout should include the distributing reservoirs and elevated tanks, with their water surface elevations indicated if they are fixed arbitrarily or by the topography. The desired residual pressures for peak flows at critical points in the system stated

in terms of the elevation of the pressure table at these points should be shown. A tentative division of the system into two or more pressure zones may be made if required. Pipe sizes may next be assumed in accordance with the Underwriters' requirements or the designer's judgment. These sizes are checked or corrected as a result of the hydraulic computations and economic analysis that follow.

2. *Skeleton Systems.* If practicable, the system is skeletonized for the hydraulic computations by eliminating all the smaller pipes in which the flow is negligible for a

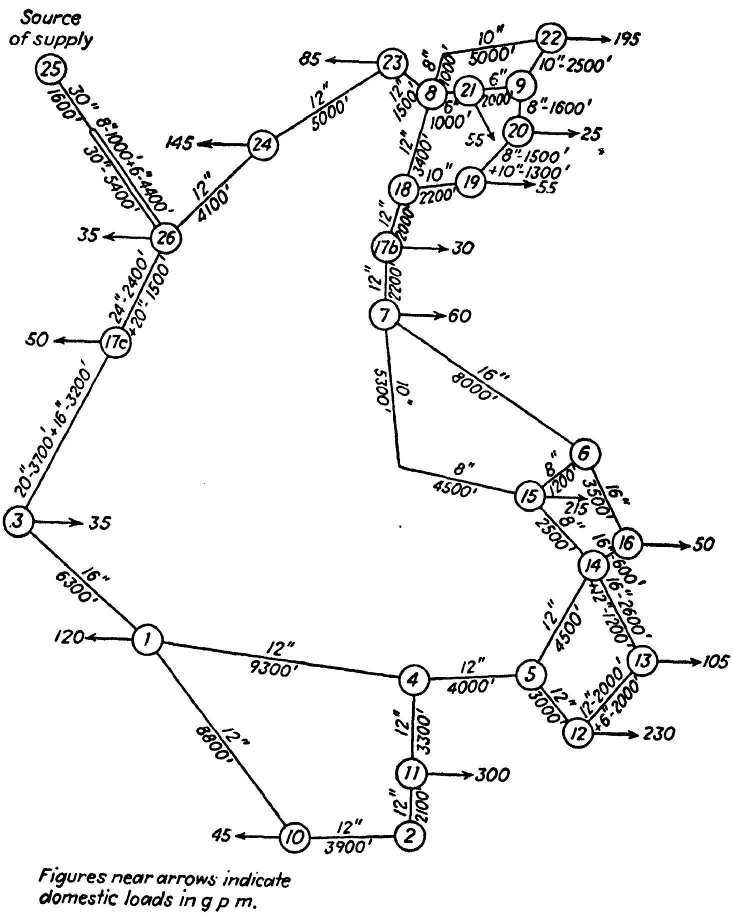


FIG. 2.—Simplified skeleton distribution system at Warwick, R.I.

particular assumed position of the fire load. Figure 1 shows the skeleton framework for a small system. The system must be examined for several positions of the fire load. Hence several skeleton frameworks may be required. These systems will be similar with regard to the larger mains but may differ in the inclusion of smaller pipes immediately surrounding the fire. The purpose of the simplified framework is to reduce the number of variables in order to make the hydraulic computations practicable. In skeletonizing the system, it is desirable to examine the magnitude of the errors caused by the neglect of smaller pipes. When the error is too great for a particular element in the skeleton framework, it is desirable to use an equivalent pipe for this

element which is of sufficient size to take account of the smaller pipes. In some cases, also, the skeleton system may be simplified by substituting a single equivalent pipe for several elements.

For many systems, the small differences in the size of the pipes or the difficulty of evaluating the errors due to the neglect of the smaller pipes may make skeletonizing impractical. In such cases, approximate hydraulic analyses may be required since the accurate methods available are too laborious.

3. *Computation of Loads.* In order to simplify the computations, the draft from the system for ordinary use is assumed to be concentrated at relatively few take-off points

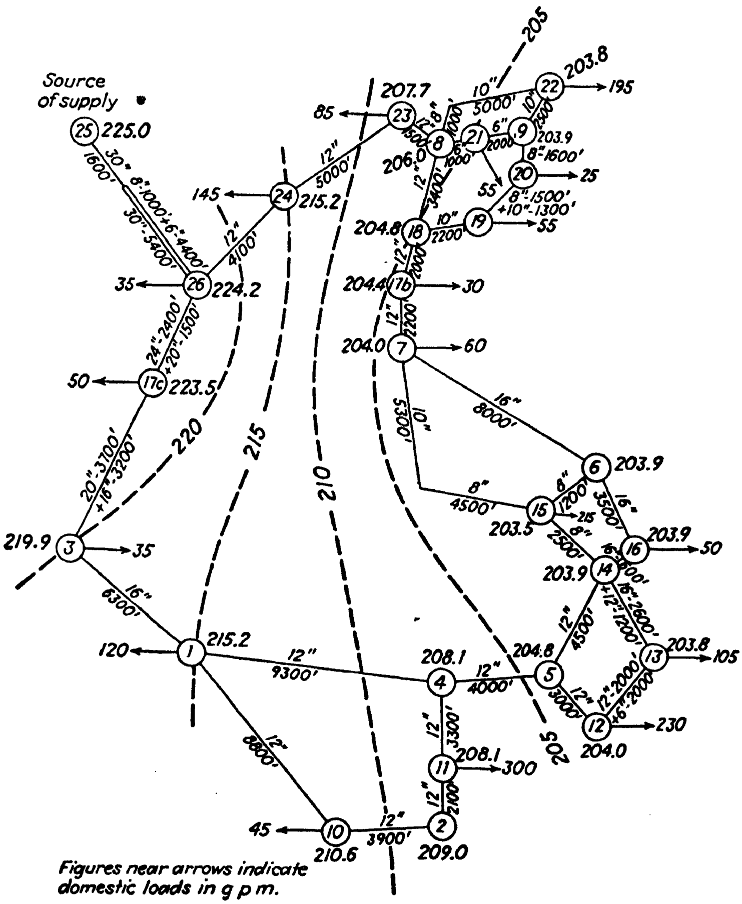


FIG. 3.—Pressure contours for domestic consumption. (Hazen-Williams C = 110.)

on the network. The loading points are taken at pipe junctions where feasible in order not to increase the number of elements in the system. Fire loads are applied at junctions also. The elevation of the pressure table corresponding to the minimum desired résidual pressure for each loading point should be determined. When the loading points are selected, it is necessary to divide the system into districts, one to each loading point, in order to compute the loads. The district boundaries are arbitrarily selected, but some of the bounds may be conveniently located on natural bounds between districts of different types or on natural fire breaks. The number of

loading points used is also arbitrarily selected, but the accuracy of the hydraulic computations increases with the number of districts selected. Figure 1[1] shows the assumed loading points and district boundaries for a small distribution system. Figure 2[1] shows the simplified skeleton system with pipe sizes and lengths and domestic loads indicated.

4. *Hydraulic Analysis.* Hydraulic computations are made to determine the discharge and head loss in each pipe element of the system for the domestic loads only and

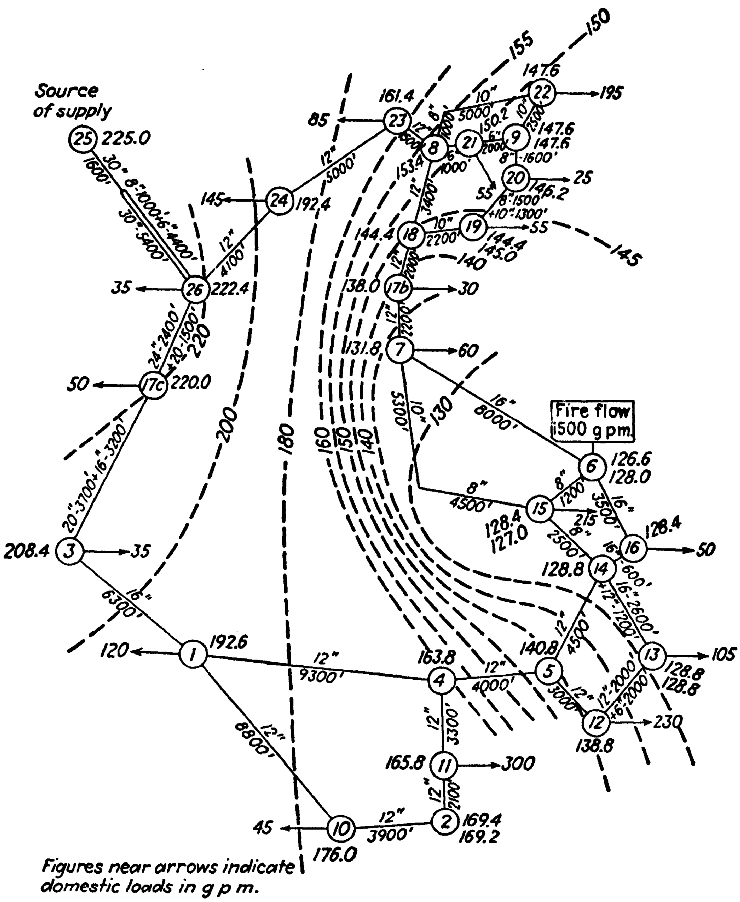


FIG. 4.—Pressure contours for fire at point 6. (Hazen-Williams $C = 110$.)

for the combined domestic and fire loads for each position of the fire load to be investigated. From these computations, the shape of the pressure table may be determined for each loading condition. Figure 3 shows the pressure contours for the Warwick system in feet above sea level for domestic loads only. Figure 4[1] shows the contours for the fire load at point 6. Methods for making the hydraulic analysis are described in the following articles.

5. *Correction of Pipe Sizes.* The adjustment of pipe sizes to secure the desired pressures at critical points and best economy is a trial-and-error process requiring the

[1] COLLINS and JONES, Bachelor's Thesis, M.I.T., 1933. CAMP and HAZEN, Hydraulic Analysis by Electric Analyzer, *Jour. N.E.W.W.A.*, **48**, 383, 1934.

alternate use of economic and hydraulic analyses. Several adjustments may be required before the best sizes are found. Methods for determining economic pipe sizes are described below.

Hydraulic Analysis of Pipe Networks. The relation between the head loss and discharge for any pipe or system of pipes in which there is turbulent flow may be expressed as follows:

$$h = kQ^x \tag{1}$$

in which h is the head loss and Q the corresponding discharge. The coefficient k is a constant for the pipe or system and may be computed for a single pipe directly from the friction formula used. For the Hazen-Williams formula which is widely used in America, the value of the constant k if h is in feet and Q in cubic feet per second is

$$= \left(\frac{1{,}594}{C}\right)^{1.85} \frac{l}{d^{4.87}} \tag{2}$$

in which C = Hazen-Williams coefficient.
l = pipe length, ft.
d = diameter, in.

For the Chézy formula, also widely used, with Manning's value of the Chézy coefficient,

$$k = 2.65(1{,}000n)^2 \frac{l}{d^{5.33}} \tag{3}$$

in which n is the Manning coefficient of roughness and the other symbols are the same as above. The value of the exponent x is equal to the reciprocal of the exponent of the hydraulic slope in the friction formula used. For the Hazen-Williams formula, x is 1.85; for the Chézy formula, x is 2.00. If Q is expressed in gallons per minute, the value of k from Eqs. (2) and (3) must be divided by 450^x.

The value of k for the Hazen-Williams formula may be obtained quickly for each element from Fig. 5 by finding the head loss corresponding to any discharge and dividing it by Q^x. The same procedure may be used with charts for other friction formulas.

1. *Analytical Relations for Compound Pipes.* The hydraulic problem in connection with pipe networks consists of solving for the distribution of flow and head loss in the individual elements for a given total discharge or for a given total head loss. For each element, there are two unknowns the discharge and head loss, and for the system as a whole one unknown the head loss or the discharge. Hence the number of unknowns equals twice the number of elements plus one.

The equations required for the solution of the unknowns are of three types and arise from three laws[1] as follows:

1. The head loss varies as some power of the discharge, Eq. (1).
2. The algebraic sum of the discharge rates toward any junction point is zero.
3. The total head loss between any two points in the system is the algebraic sum of the head loss of all the elements along any route between the points, and the total head loss is the same by all routes.

Figure 6 shows three simple compound pipe systems and the equations required for their analysis. It will be noted that only two types of equations are actually required for the solution of series and parallel systems since the other equations are identities but that all three types of equations are required for the analysis of a complex system. For any network, the number of equations available is sufficient for a solution. To solve for the unknowns, the equations must be solved simultaneously, but the direct solution of the large number of simultaneous equations involved in distribution systems is impractical for all but the simplest systems. Trial-and-error methods are used in practice.

[1] CAMP and HAZEN, *op. cit.*

2. *Equivalent Pipes.* A simple system consisting of two or more elements and having one inlet and one outlet point may be replaced by an equivalent pipe. The head

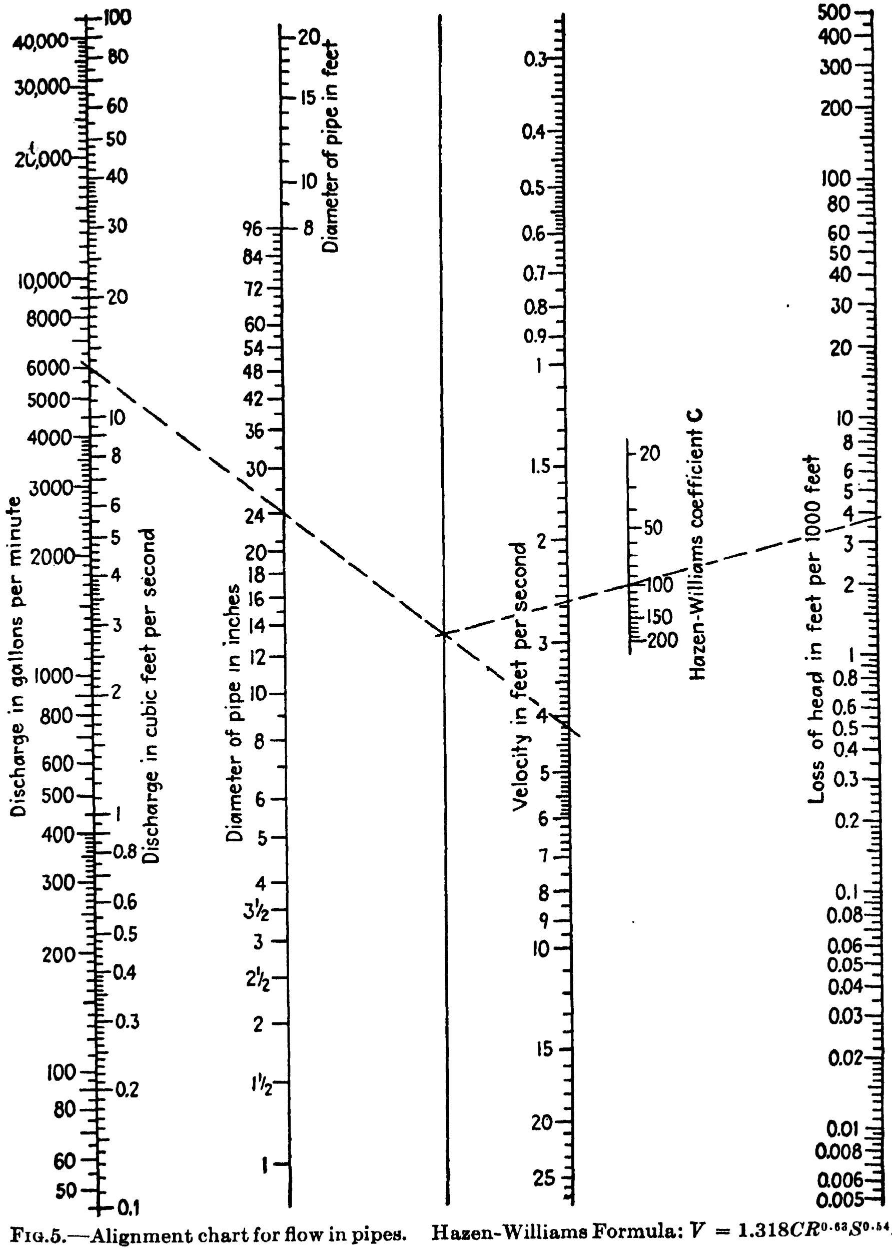


FIG. 5.—Alignment chart for flow in pipes. Hazen-Williams Formula: $V = 1.318CR^{0.63}S^{0.54}$.

loss through an equivalent pipe for any given flow is the same as for the replaced system. Equivalent pipes may be found readily by means of flow charts such as Fig. 5 for simple series and parallel systems such as Fig. 6*a* and *b*.

To find an equivalent pipe for a simple series, assume a discharge rate and find the head loss for each element. Any pipe which has for the assumed discharge a head loss equal to the sum of the losses in all the elements is an equivalent pipe. Any size of pipe may be selected provided the proper length is chosen to give the desired head loss. For example, in Fig. 6*a*, for $C = 100$, and an assumed discharge rate of 800 gpm, $h_1 = 39$ and $h_2 = 10$ from Fig. 5. Hence $h = 49$, and 7,350 ft of 10 in. or 2,510 ft of 8 in. are equivalent pipes. It will be noted that if the discharge is given, a simple series may be solved directly from a chart; but if the total head loss is given, an equivalent pipe must first be found before the discharge can be solved for by means of the chart.

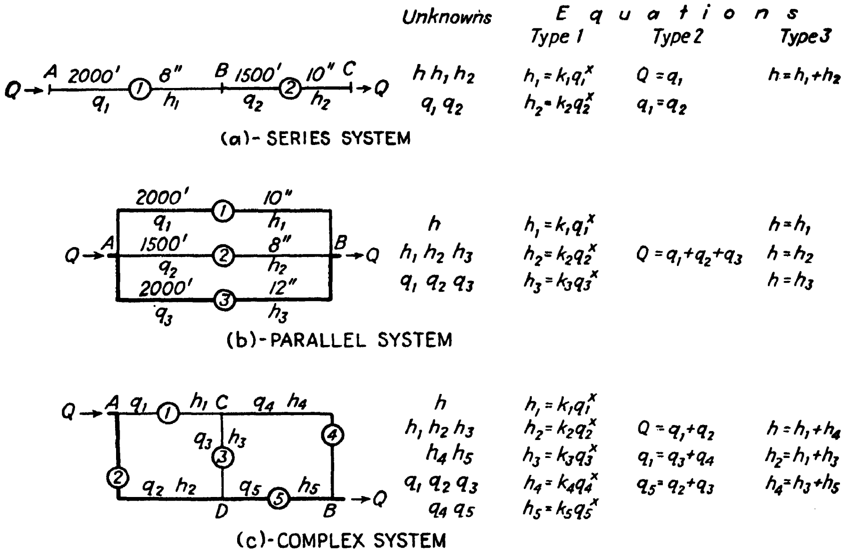


Fig. 6.—Simple compound pipes.

To find an equivalent pipe for a simple parallel system, assume a total head loss and find the corresponding discharge for each element. Any pipe that has for the assumed head loss a discharge equal to the sum of the discharge rates for all the elements is an equivalent pipe. For example, in Fig. 6*b*, for $C = 100$, and an assumed head loss of 30 ft, $q_1 = 1{,}250$, $q_2 = 810$, and $q_3 = 2{,}010$ gpm from Fig. 5. Hence $Q = 4{,}070$ gpm, and 2,190 ft of 16-in. pipe is an equivalent pipe. It will be noted that if the head loss is given a simple parallel system may be solved directly from a chart, but if the total discharge is given an equivalent pipe is first required.

An equivalent pipe for a complex system such as Fig. 6*c* cannot be found directly by the foregoing methods, for neither the head nor the discharge for any element is known when either the head or discharge for the whole system is known. Howland and Aldrich[1] have developed a trial-and-error method for solving complex systems using Freeman's[2] graphical method.

The equivalent pipe method is a convenient device for reducing the number of elements and thereby simplifying the framework of a distribution system. When used

[1] Howland, Expansion of the Freeman Method for the Solution of Pipe Flow Problems, *Jour. N.E.W.W.A.*, **48**, 408, 1934. Aldrich, Solution of Transmission Problems of a Water System, *Trans. A. S. C. E.*, **103**, 1579, 1938.

[2] Freeman, *Jour. N.E.W.W.A.*, **7**, 1892.

for this purpose, it is not necessary to find an equivalent pipe for the replaced elements but only to find the value of the constant k of Eq. (1) which corresponds to the equivalent pipe. For example, the equivalent pipes of the series in Fig. 6*a* all have the same value of k as follows: $k = (49/800^{1.85}) = 20.9 \times 10^{-5}$ for the discharge in gallons per minute. Similarly, the value of k for all equivalent pipes of the parallel system (Fig. 6*b*) is 0.625×10^{-5}.

3. *Hydraulic Network Solutions.* Five methods are available for making solutions of the simultaneous equations involved in the hydraulic analysis of distribution systems:

1. Uncontrolled trial-and-error method
2. Freeman graphical method[1]
3. Hardy Cross method[2]
4. Electric network analyzer method[3]
5. Hydraulic-model method[4]

In the first three methods, a trial distribution of either the flow or the head loss is made throughout the system, the corresponding head or discharge is computed or measured for each of the elements, and adjustments are then made in the distribution of values. The process is repeated until a set of values is obtained which approximately satisfy all three laws described above under Analytical Relations

In the uncontrolled method, the adjustments in the assumed values are made arbitrarily. The convergence of errors by this method is therefore slow and uncertain, and it is impractical for most systems. The same may be said of the Freeman method.

The fourth method consists of the use of an electric network analyzer in which electric resistors are connected together in such a manner that each element of the hydraulic system is represented by a resistor in proper position. The analyzer is an electric model of the distribution system in which voltage represents head loss and current the discharge, with a suitable scale ratio selected for each. When the method was first developed, ordinary resistors were used with which the voltage drop is proportional to the first power of the current instead of the x power as required by the hydraulic system. Because of this defect, it was not possible to represent each element of the hydraulic system by a constant electrical resistance; and trial-and-error solutions were obtained by adjusting the resistance in all the resistors so that the relation of voltage to current in each resistor satisfied the analagous head-discharge relation as indicated by Eq. (1). Satisfactory solutions could usually be obtained with three adjustments of the resistors. Lamps have now been developed with suitable filaments for use as resistors such that upon heating the filaments the resistance changes automatically to represent a hydraulic element without subsequent adjustment and results can be read as soon as the loads are applied. Figure 4 represents a hydraulic analysis of a distribution system by means of an electric-network analyzer.

The hydraulic-model method consists of representing the distribution system by means of a small-scale hydraulic model. The elements may be represented by constrictions in rubber tubes produced by means of pinch cocks, in which case the value of x in Eq. (1) will be about 1.75 but may be higher or lower depending upon the amount of throttling. The elements may also be represented by orifices inserted in pipe or rubber tubing, in which case x will have a value of nearly 2.0; or they may be represented by short tubes inserted in rubber tubing, the ratio of length to diameter of

[1] ALDRICH, *op. cit.*
[2] CROSS, Analysis of Flow in Networks of Conduits or Conductors, *Eng. Exp. Station. Univ. Ill., Bull.* 286, November, 1936. DOLAND, *Eng. News-Record*, **117**, 475, 1936.
[3] COLLINS and JONES, *op. cit.* CAMP and HAZEN, *op. cit.* MCILROY, Doctor's Thesis, M.I.T., 1946.
[4] CAMP, Hydraulics of Distribution Systems—Some Recent Developments in Methods of Analysis, *Jour. N. E. W. W. A.*, **57**, 334, 1943.

short tube being selected to produce the required value of x. Results may be read directly on hydraulic models as soon as the loads are applied.

The Hardy Cross method is a trial-and-error method in which the adjustments to be made in the assumed values are computed and are therefore controlled. Convergence of errors is often rapid, and sufficient precision in the results can ordinarily be had by three adjustments. Two methods may be used: the method of balancing heads or the method of balancing flows. The method of balancing heads is as follows:

1. Assume any distribution of discharge.
2. Compute the head loss in each element by means of Eq. (1): $h = kq_0^x$.
3. With due attention to sign, compute the total head loss around each elementary closed circuit: $\Sigma h = \Sigma kq_0^x$.
4. Compute also for each elementary circuit without reference to sign the sum: $\Sigma xkq_0^{(x-1)}$.
5. To balance the head in each circuit (so that $\Sigma kq^x = 0$), set up a counterbalancing flow equal to

$$\Delta = \frac{\Sigma kq_0^x \text{(with due attention to direction of flow)}}{\Sigma xkq_0^{(x-1)} \text{(without reference to direction of flow)}} \tag{4}$$

6. Compute the revised flows, and repeat the process until the desired accuracy is obtained.

The flow correction Δ for each circuit places the heads for that circuit substantially in balance if Δ is small. Since some elements of each circuit are common to other circuits, however, the balance of heads in each circuit is disturbed by subsequent adjustments in other circuits. Hence several traverses of the system are required before satisfactory precision is obtained. The proof of the method is as follows:

$$q = q_0 + \Delta$$

in which q = actual discharge for any element.
q_0 = assumed discharge.
Δ = required flow correction.

Then

$$kq^x = k(q_0 + \Delta)^x = k(q_0^x + xq_0^{(x-1)}\Delta + \cdots)$$

The remaining terms in the preceding expansion may be neglected if Δ is small as compared with q_0. For a single circuit,

$$\Sigma kq^x = 0$$

and from above,

$$\Sigma kq^x = \Sigma kq_0^x + \Delta \Sigma xk_0^{(x-1)}$$

Therefore,

$$\Delta = -\frac{\Sigma kq_0^x}{\Sigma xkq_0^{(x-1)}} \tag{4}$$

If Δ is large compared with q_0, Eq. (4) does not give a close approximation of the value of Δ because of the neglect of the terms beyond the second term in the expansion. This neglect is not usually important, however, particularly if subsequent adjustments bring rapid convergence.

Figure 3 represents a hydraulic analysis of the Warwick system by means of the Hardy Cross method. Figure 7 shows the skeleton framework with the value of the constant $k = 10^5$ for each element indicated on the line representing the element. The figures nearest each element represent the flow in the element corresponding to the assumed distribution of discharge. The underlined figures near each element represent the adjusted discharge. The computations are shown in Table 5.

There appears to be nothing inherent in either the electric analyzer method with ordinary resistors or the Hardy Cross method which will consistently produce convergence of the errors toward zero with subsequent adjustments. Some networks

have been studied by the Hardy Cross method with which convergence in many of the loops does not occur. In such a case, the designer may have to be content with an approximate hydraulic analysis such as the contour and circle methods.[1]

Economic Size of Pipes.[2] The economic pipe sizes for a distribution system are those sizes for which the total cost of the system, fixed plus operating, is a minimum. The fixed cost consists of the first cost plus the depreciation; and the operating cost consists of the cost of pumping. The fixed cost may be limited to the cost of the

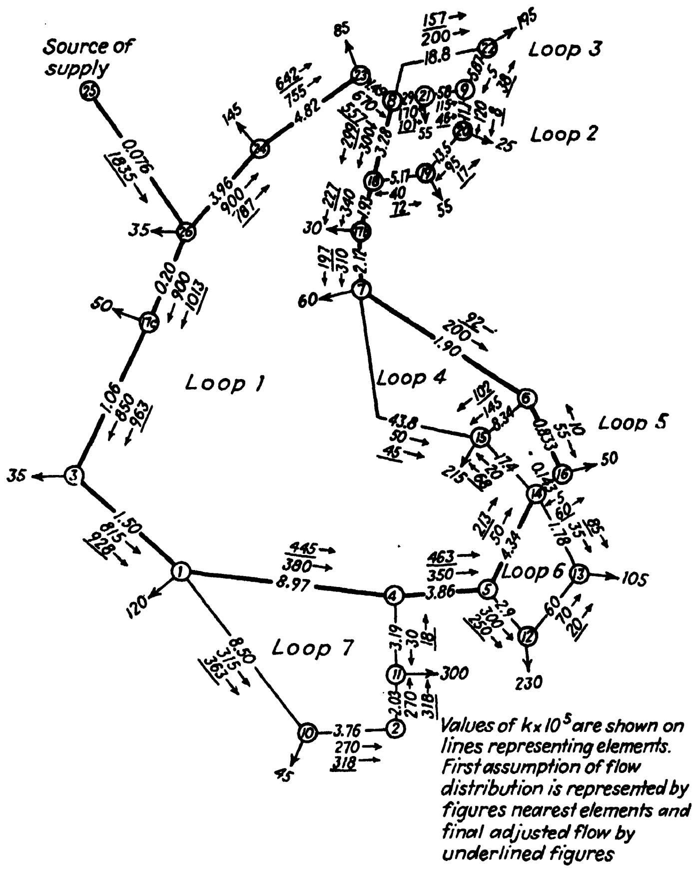


FIG. 7.—Computation of domestic flow distribution by Hardy Cross method. (Hazen-Williams $C = 110$.)

piping system on the assumption that the cost of distributing reservoirs and elevated tanks is not a function of pipe sizes. Precision in the selection of economic pipe sizes is unnecessary and unwarranted because (1) relatively large variations in pipe size in the immediate range of the economic size change the total cost but slightly, (2) commercial sizes must be used whether or not they are the computed economic sizes, (3) accurate predictions of cost and flow over the design period cannot be made, and (4) the computed economic size for any element differs with the loading conditions on the system.

[1] TURNEAURE and RUSSELL, "Public Water Supplies," John Wiley & Sons, Inc., 1924, pp. 721–725. BARDOE, *Eng. News-Record*, **93**, 517, 1924. TYLER, *Water Works and Sewerage*, August, 1939, p. 285.

[2] CAMP, Economic Pipe Sizes for Water Distribution Systems, *Trans. A. S. C. E.*, **104**, 190, 1939.

TABLE 5.—COMPUTATION OF DOMESTIC FLOW DISTRIBUTION IN WARWICK, R. I., DISTRIBUTION SYSTEM BY HARDY CROSS METHOD (Hazen-Williams $C = 110$)

Element	10^5k	First adjustment				Second adjustment				Third adjustment				Final values			
		$q_0^{0.85}$	$10^5kq_0^{0.85}$	q_0' gpm	h' ft	$q_0^{0.85}$	$10^5kq_0^{0.85}$	q_0'' gpm	h'' ft	$q_0^{0.85}$	$10^5kq_0^{0.85}$	q_0''' gpm	h''' ft	$q^{0.85}$	$10^5kq^{0.85}$	q gpm	h ft
Loop 1																	
26-24	3.96	324	1,284	+900	+11.55	292	1.157	+ 799	+9.25	288	1,141	+ 788	+9.00	288	1,141	+ 787	+8.98
24-23	4.82	278	1,340	+755	+10.12	248	1.195	+ 654	+7.82	243	1,171	+ 643	+7.53	242	1,166	+ 642	+7.49
23-8	1.45	252	365	+670	+ 2.45	220	319	+ 569	+1.82	216	313	+ 558	+1.75	215	312	+ 557	+1.74
8-18	3.28	128	420	+300	+ 1.26	113	371	+ 262	+0.97	120	394	+ 280	+1.10	127	416	+ 299	+1.24
18-17*b*	1.93	142	274	+340	+ 0.93	105	203	+ 239	+0.49	101	195	+ 228	+0.44	101	195	+ 227	+0.44
17*b*-7	2.12	131	278	+310	+ 0.86	94	199	+ 209	+0.42	89	189	+ 198	+0.37	89	189	+ 197	+0.37
7-6	1.90	90	171	+200	+ 0.34	45.6	87	+ 90	+0.08	46.6	89	+ 92	+0.08	46.6	89	+ 92	+0.08
6-16	0.833	30.1	25	+ 55	+ 0.01	2.5	2	+ 3	+	6.5	5	− 9	−	7.1	5.9	− 10	−0.
16-14	0.143	3.9	1	+ 5	+	26.3	4	− 47	−	32	5	− 59	−0.003	32.5	4.6	− 60	−0.003
14-5	4.34	27.8	121	− 50	− 0.06	87	378	− 191	−0.72	94	408	− 211	−0.86	95.1	413	− 213	−0.88
5-4	3.86	145	560	−350	− 1.96	180	695	− 451	−3.14	184	710	− 462	−3.28	184	710	− 463	−3.29
4-1	8.97	156	1,399	−380	− 5.32	176	1,578	− 438	−6.91	178	1,596	− 444	−7.10	178	1,596	− 445	−7.11
1-3	1.50	294	441	−815	− 3.59	330	495	− 916	−4.53	333	500	− 927	−4.64	334	501	− 928	−4.65
3-17*c*	1.06	309	328	−850	− 2.79	340	360	− 951	−3.42	343	364	− 962	−3.50	345	366	− 963	−3.53
17*c*-26	0.20	324	65	−900	− 0.58	355	71	−1.001	−0.71	357	72	−1,012	−0.73	357	72	−1,013	−0.73
			7,072		+13.22		7.114		+1.42		7,152		+0.16		7,176		+0.15
		$\Delta = \frac{-13.22 \times 10^5}{1.85 \times 7{,}072} = -101$				$\Delta = \frac{-1.42 \times 10^5}{1.85 \times 7{,}114} = -11$				$\Delta = \frac{-0.16 \times 10^5}{1.85 \times 7{,}152} = -1$				$\text{Error} = \frac{+0.15 \times 10^5}{1.85 \times 7{,}176} = +1.1$ gpm			
Loop 2																	
8-21	29.0	78.7	2,281	+170	+ 3.88	57.0	1,653	+ 117	+1.93	52.8	1,531	+ 109	+1.67	50.2	1,456	+ 101	+1.47
21-9	58.0	56.4	3,272	+115	+ 3.76	33.4	1,937	+ 62	+1.20	29.7	1,723	+ 54	+0.93	25.9	1,503	+ 46	+0.69
9-20	11.1	58.6	650	+120	+ 0.78	31.1	345	+ 57	+0.20	17.0	189	+ 28	+0.05	5.9	65	+ 8	+0.005
20-19	13.5	48.0	648	+ 95	+ 0.61	19.0	256	+ 32	+0.08	2.5	34	+ 3	+0.001	11.1	150	− 17	−0.025
19-18	5.17	23.0	119	+ 40	+ 0.05	14.4	75	− 23	−0.02	28.7	148	− 52	−0.08	37.9	196	− 72	−0.14
18-8	3.28	90.0	295	−199	− 0.59	109.0	357	− 251	−0.90	120.0	394	− 279	−1.10	127.0	416	− 299	−1.24
			7,265		+ 8.49		4,623		+2.49		4,019		+1.47		3,786		+0.76
		$\Delta = \frac{-8.49 \times 10^5}{1.85 \times 7{,}265} = -63$				$\Delta = \frac{-2.49 \times 10^5}{1.85 \times 4{,}623} = -29$				$\Delta = \frac{-1.47 \times 10^5}{1.85 \times 4{,}019} = -20$				$\text{Error} = \frac{+0.76 \times 10^5}{1.85 \times 3{,}786} = +11$ gpm			
Loop 3																	
8-22	18.8	90.0	1,693	+200	+ 3.39	86.3	1,623	+ 190	+3.08	78.2	1,468	+ 169	+2.48	73.3	1,378	+ 157	+2.16
22-9	5.87	3.9	23	+ 5	+	3.9	23	− 5	−	15.9	93	− 26	−0.02	22.0	129	− 38	−0.05
9-21	58.0	28.7	1,665	− 52	− 0.87	19.5	1,131	− 33	−0.37	20.0	1,160	− 34	−0.39	25.9	1,503	− 46	−0.69
21-8	29.0	53.0	1,536	−107	− 1.64	45.0	1.305	− 88	−1.15	45.4	1,317	− 89	−1.17	50.2	1,456	− 101	−1.47
			4,917		+ 0.88		4,082		+1.56		4,038		+0.90		4,466		−0.05
		$\Delta = \frac{-0.88 \times 10^5}{1.85 \times 4{,}917} = -10$				$\Delta = \frac{-1.56 \times 10^5}{1.85 \times 4{,}082} = -21$				$\Delta = \frac{-0.90 \times 10^5}{1.85 \times 4{,}038} = -12$				$\text{Error} = \frac{-0.05 \times 10^5}{1.85 \times 4{,}466} = -0.6$ gpm			

Loop 4																	
7-6	1.90	49.7	95	+99	+0.09	41.0	78	+79	+0.06	46.3	88	+91	+0.08	46.6	89	+92	+0.08
6-15	8.34	69.0	576	+145	+0.83	44.5	371	+87	+0.32	50.2	419	+101	+0.42	50.9	424	+102	+0.43
15-7	43.8	27.8	1,217	−50	−0.61	32.0	1,403	−59	−0.83	25.9	1,134	−46	−0.52	25.4	1,113	−45	−0.51
			1,888		+0.31		1,852		−0.45		1,641		−0.02				0
		$\Delta = \frac{-0.31 \times 10^5}{1.85 \times 1,888} = -9$				$\Delta = \frac{+0.45 \times 10^5}{1.85 \times 1,852} = +13$				$\Delta = \frac{+0.02 \times 10^5}{1.85 \times 1,641} = +1$				Error = 0			
Loop 5																	
6-16	0.833	25.9	21	−46	−0.010	5.9	5	−8	....	7.1	6	−10	....	7.1	5.9	−10	−0.
16-14	0.143	48.3	7	−96	−0.007	31.5	5	−58	−0.003	32.5	5	−60	−0.003	32.5	4.6	−60	−0.003
14-15	17.4	12.7	221	+20	+0.044	36.5	635	+69	+0.438	36.2	627	+68	+0.427	36.2	630	+68	+0.43
15-6	8.34	65.0	542	−136	−0.738	50.0	417	−100	−0.417	50.9	426	−102	−0.435	50.9	424	−102	−0.43
			791		−0.711		1,062		+0.018		1,064		−0.005				0
		$\Delta = \frac{+0.711 \times 10^5}{1.85 \times 791} = +49$				$\Delta = \frac{-0.018 \times 10^5}{1.85 \times 1,062} = -1$				$\Delta = \frac{-0.005 \times 10^5}{1.85 \times 1,064} = 0$				Error = 0			
Loop 6																	
5-14	4.34	71.0	308	+151	+0.465	91.0	395	+202	+0.798	95.0	412	+212	+0.875	95.1	413	+213	+0.88
14-13	1.78	20.5	37	+35	+0.013	39.2	70	+75	+0.052	43.2	77	+84	+0.065	43.6	78	+85	+0.066
13-12	60.0	37.0	2,220	−70	−1.554	18.0	1,080	−30	−0.324	13.3	798	−21	−0.168	12.8	768	−20	−0.154
12-5	2.9	128.0	371	−300	−1.113	113.0	328	−260	−0.852	110.0	319	−251	−0.802	109.0	316	−250	−0.790
			2,936		−2.189		1,873		−0.326		1,606		−0.03				0
		$\Delta = \frac{+2.189 \times 10^5}{1.85 \times 2,936} = +40$				$\Delta = \frac{+0.326 \times 10^5}{1.85 \times 1,873} = +9$				$\Delta = \frac{+0.03 \times 10^5}{1.85 \times 1,606} = +1$				Error = 0			
Loop 7																	
1-4	8.97	190.0	1,704	+481	+8.200	180.0	1,614	+449	+7.250	178.0	1,596	+445	+7.100	178.0	1,596	+445	+7.11
4-11	3.19	18.0	57	+30	+0.017	8.8	28	−13	−0.004	11.7	37	−18	−0.007	11.7	37.3	−18	−0.007
11-2	2.03	116.0	235	−270	−0.635	132.0	268	−313	−0.839	134.0	272	−318	−0.866	134.0	272	−318	−0.866
2-10	3.76	116.0	436	−270	−1.176	132.0	496	−313	−1.553	134.0	504	−318	−1.605	134.0	504	−318	−1.605
10-1	8.50	133.0	1,130	−315	−3.560	148.0	1,258	−358	−4.51	150.0	1,275	−363	−4.63	150.0	1,275	−363	−4.63
			3,562		+2.846		3,664		+0.34		3,684		0				0
		$\Delta = \frac{-2.846 \times 10^5}{1.85 \times 3,562} = -43$				$\Delta = \frac{-0.34 \times 10^5}{1.85 \times 3,664} = -5$				$\Delta = 0$				Error = 0			

Two general classes of systems are encountered in practice, *viz:* (1) gravity systems in which the elevations of the distributing reservoirs and therefore the total available lost head is fixed arbitrarily or by the topography, and (2) systems with pumped supplies in which the total available head loss is not fixed arbitrarily or by the topography and may therefore be established for best over-all economy. In the first class, the pumping cost, if any, is not a function of the pipe sizes and therefore need not be considered as a part of the total cost. In the second class, a portion of the pumping cost attributable to pipe friction is a function of the pipe sizes and is therefore a part of the total cost. There are several types of systems in the second class. Some of the cases in the second class described below have no practical importance, but the fundamental theory is necessary to the development of methods of determining the economic sizes for the cases generally met with in practice.

For each case, the pipe sizes may be determined for only one loading condition at a time. For other positions of the fire load, the flow distribution changes and other points may become critical as to residual pressure. In practice, it is necessary to select for any element a commercial pipe size that comes nearest to satisfying the computed sizes for all loading conditions in which the element is an important feeder.

1. *Cost As a Function of Pipe Diameter.* The first cost of laying cast-iron pipe in cents per lineal foot, based upon cost data from D. H. Maury,[1] may be expressed by the following empirical equation:

$$CL = Bd = [(0.06 + 0.02D)W + 0.19L + 0.007Y]d \tag{5}$$

in which D = depth of trench, ft.
W = wage rate for common labor, cts per hour.
L = cost of lead, cts per pound.
Y = cost of yarn, cts per pound.
d = pipe diameter, in.

The costs given by the preceding equation are for ordinary earth excavation without pumping, sheeting, rock, or other unpredictable costs. One joint is assumed for every 11 ft of pipe. The equation includes a sufficient margin for contractor's profit, engineering, insurance, and similar costs. The unpredictable costs and the cost of hydrants are affected little by pipe size and may be safely ignored.

The first cost of materials in cents per lineal foot of pipe including cast-iron pipe, valves, fittings, and pipe lining is given by the following empirical equation:

$$CM = Fad^{1.55} \tag{6}$$

in which a is the cost of cast-iron pipe in cents per pound with freight allowed. The values of F for costs of different types of pipe at the trench, including contractor's profit, engineering, etc., are given in Table 6.

TABLE 6.—VALUES OF F IN $CM = Fad^{1.55}$

Type	Unlined pipe		Pipe lined with cement or Bitumastic Enamel	
	Pipe only	Pipe with 1 valve, 1 tee and 1 cross for each 500 ft	Pipe only	Pipe with 1 valve, 1 tee and 1 cross for each 500 ft
Class 100	1.50	2.05	1.65	2.20
Class 150	1.80	2.35	1.95	2.50
Class *B*	2.10	2.65	2.25	2.80
Class *C*	2.40	2.95	2.55	3.10

[1] *Eng. News-Record,* **88,** 779, 1922. Also BABBITT and DOLAND, "Water Supply Engineering," McGraw-Hill Book Company, Inc., 1939, p. 392.

The annual cost of pumping in cents is given by

$$ACP = 236pqh \tag{7}$$

in which p = cost in cents of raising 1 million gal 1 ft.
q = discharge, cfs.
h = head, ft.

The annual cost of pumping against friction in cents per linear foot of pipe is, from the Hazen-Williams formula and Eq. (7),

$$ACP = 236p\left(\frac{1{,}594}{C}\right)^{1.85}\frac{q^{2.85}}{d^{4.87}} \tag{8}$$

2. *Case* I. *Gravity Systems.* The economic pipe sizes for a gravity system will be one set of sizes which utilizes all the available head in friction when delivering the peak flow to each critical point in the system. There are an infinite number of ways in which the total head may be distributed among the pipes in each series, but only one of these ways will produce a minimum total cost for the series.

Let the first cost of a series of gravity pipe lines be

$$c = c_1 + c_2 + c_3 + \cdots = f_1(h_1) + f_2(h_2) + f_3(h_3) + \cdots$$

in which c_1, c_2, c_3 . . . equals the cost of elements 1, 2, 3, etc., and $f_1(h_1)$, $f_2(h_2)$, $f_3(h_3)$ . . . equals these costs expressed as functions of the head loss in each element. The cost of any two connecting pipes in the series may be made a minimum for any given total head for these two pipes by equating to zero the first derivative of the cost with respect to the head loss in either element, as follows:

$$\frac{d(c_1 + c_2)}{dh_1} = f_1'(h_1) - f_2'(h_2) = 0$$

since $-dh_1 = dh_2$. Hence for a minimum cost of any two connecting elements,

$$\frac{dc_1}{dh_1} = \frac{dc_2}{dh_2}$$

For a minimum cost of the whole series,

$$\frac{dc_1}{dh_1} = \frac{dc_2}{dh_2} = \frac{dc_3}{dh_3} = \cdots \tag{9}$$

and

$$h_1 + h_2 + h_3 + \cdots = H \tag{10}$$

in which H is the total head available for friction loss in the series.

For cast-iron pipes in a gravity series, the first cost is

$$c = (B_1d_1 + Fad_1^{1.55})l_1 + (B_2d_2 + Fad_2^{1.55})l_2 + \cdots \tag{11}$$

From the Hazen-Williams formula,

$$d = 16.5\frac{q^{0.38}l^{0.205}}{C^{0.38}h^{0.205}} \tag{12}$$

By substituting this value of d for each pipe in Eq. (11) and differentiating as in Eq. (9), the following result is obtained:

$$3.38B_1\frac{q_1^{0.38}}{C_1^{0.38}S_1^{1.205}} + 24.5Fa\frac{q_1^{0.59}}{C_1^{0.59}S_1^{1.318}} = 3.38B_2\frac{q_2^{0.38}}{C_2^{0.38}S_2^{1.205}} + 24.5Fa\frac{q_2^{0.59}}{C_2^{0.59}S_2^{1.318}} \tag{13}$$

This equation gives the relation for best economy of the hydraulic slopes of any two connecting pipes in a series. An approximate solution may be obtained for Eq. (13) by assuming that the exponent of S in every term is 1.25, as follows:

$$\frac{S_1}{S_2} = \left(\frac{3.38B_1 \dfrac{q_1^{0.38}}{C_1^{0.38}} + 24.5Fa \dfrac{q_1^{0.59}}{C_1^{0.59}}}{3.38B_2 \dfrac{q_2^{0.38}}{C_2^{0.38}} + 24.5Fa \dfrac{q_2^{0.59}}{C_2^{0.59}}} \right)^{0.8} \tag{14}$$

Results obtained by means of Eq. (14) will usually vary by not more than 2 per cent from values obtained by the more exact Eq. (13). From Eq. (14), it will be noted that for best economy the hydraulic slope should be steeper for the pipe with the greater discharge.

In a gravity network, each main feeder will be a part of several pipe series, and its computed economic size will not be the same for all series nor will it be the true value as obtained by considering the network as a whole. Nevertheless, since standard pipe diameters must be used, the standard size that comes nearest to satisfying all series will probably be the most economical.

3. *Case* II. *Single Pipe. Pumped Supply with Constant Discharge.* If the system consists of a single pipe discharging at a constant rate at the lower end, the total annual cost of pipe and pumping against friction per lineal foot of pipe is

$$AC = Brd + Fard^{1.55} + 236p \left(\frac{1{,}594}{C}\right)^{1.85} \frac{q^{2.85}}{d^{4.87}} \tag{15}$$

in which r is the annual rate of interest plus depreciation for the pipe. When the first derivative of this cost with respect to the diameter is equated to zero, the following relation is obtained for the economic size of pipe:

$$Brd^{5.87} + 1.55Fard^{6.42} = 1{,}150p \left(\frac{1{,}594}{C}\right)^{1.85} q^{2.85} \tag{16}$$

An approximate solution of this equation may be obtained as follows by assuming that the exponent of d in both terms is 6.15:

$$d_{ec} = \frac{27.4}{C^{0.30}} \left[\frac{p}{r(B + 1.55Fa)} \right]^{0.163} q^{0.46} \tag{17}$$

The coefficient has been adjusted to 27.4 to compensate for the change in exponents. Equation (17) gives values of the economic pipe size within 3 per cent of the values obtained by Eq. (16).

4. *Case* III. *Pipe Network. Pumped Supply with Constant Discharge Throughout.* If the system consists of a network of pipes with a number of take-offs, the pressure at only one take-off point may be made to correspond with the desired residual pressure. At each of the other take-off points, there is an excess of pressure, and there is therefore a power loss corresponding to the draft from the system and this excess pressure. Some of the power loss at take-offs is due to the topography of the city and is not influenced by pipe sizes; but some of it is properly chargeable to the pipes inasmuch as larger pipes downstream from a take-off point would reduce the excess pressure at the point.

In the simple network represented by Fig. 8, the excess head at b is all chargeable to element 1. The portion of the excess head at c, e, and f, corresponding to the lost head in elements 2, 4, and 5, respectively, is chargeable, respectively, to those pipes. The excess head at d is due to pipes 1 and 2, and the difference between the total head charged to these pipes and the excess at d is chargeable to pipe 3. Since this value is negative in the example, the take-off power charge for pipe 3 is a credit. To formulate a rule: the head corresponding to the take-off power loss chargeable to any pipe is the

head lost by friction in the pipe or the difference between the excess head at the upper end of the pipe and the sum of the heads chargeable to the pipes downstream, whichever is the smaller.

The take-off power loss chargeable to element 1 (Fig. 8) is due to the draft from the system at b and k and a portion of the draft from points c, d, e, m, and n. Since the loss due to the take-off at c is chargeable to elements 6 and 7 as well as to elements 1 and 2, it is convenient to apportion the loss by distributing the draft at c between the two routes. A similar distribution of the discharge at other points should be made between the routes downstream from these points. To formulate a rule: the discharge cor-

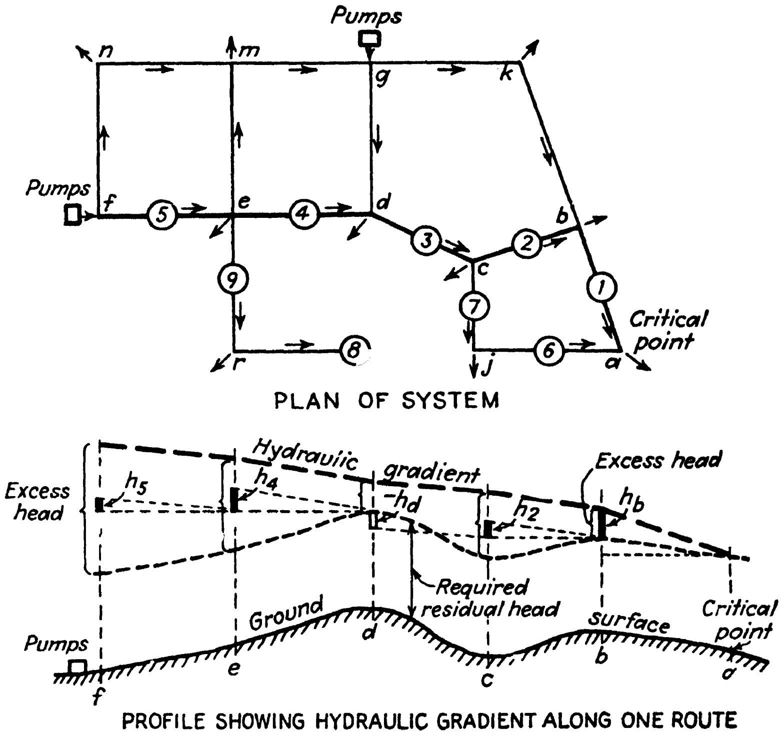


FIG. 8.—Case III. Pumped supply with constant discharge throughout.

responding to the take-off power loss chargeable to any pipe is the sum of the drafts from the system at all points upstream from the pipe, a suitable apportionment of drafts being made between parallel routes.

For any pipe of a network through which water is pumped at a constant rate, the total annual cost of pipe and power loss due to pipe friction and take-offs is, per lineal foot of pipe,

$$AC = Brd + Fard^{1.55} + 236p\left(\frac{1{,}594}{C}\right)^{1.85}(q + mQ)\frac{q^{1.85}}{d^{4.87}} \tag{18}$$

in which q = discharge through the pipe, cfs.

Q = total take-off discharge chargeable to the pipe, cfs, estimated as described above.

m = ratio of the take-off head to the head lost by friction in the pipe, the take-off head being estimated as described above.

By following the procedure used in the derivation of Eq. (17), the economic diameter of pipe for any element in a network through which water is pumped at a constant rate is

$$d_{ec} = \frac{27.4}{C^{0.30}} \left[\frac{p(q + mQ)}{r(B + 1.55Fa)} \right]^{0.163} q^{0.30} \tag{19}$$

It should be noted that an elevated tank (although it would serve no useful purpose) may be inserted into any part of a system of this type without changing the method for determining the economic pipe sizes, provided the water surface in the tank conforms to the pressure table established by the economic pipe sizes. If the water

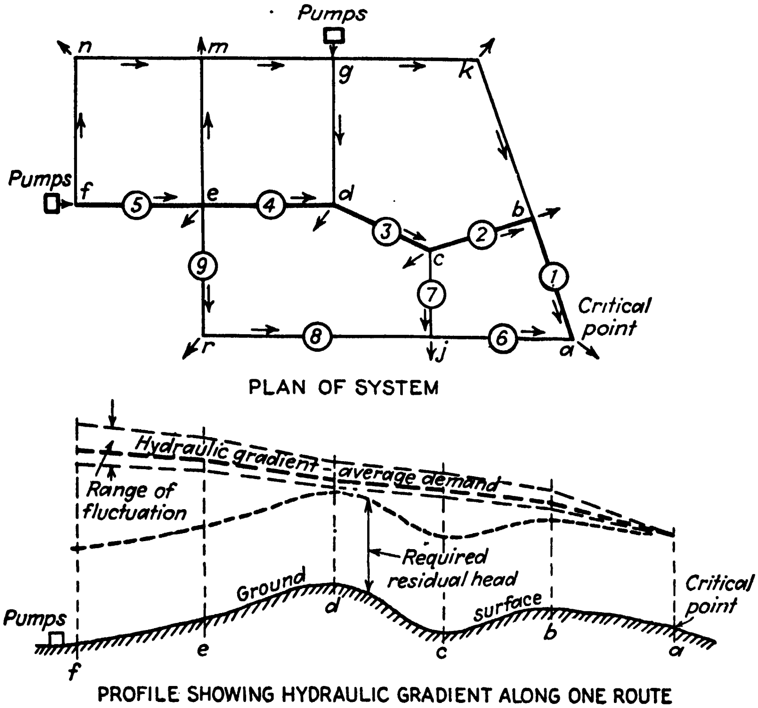


FIG. 9.—Case IV. Pumped supply with varying demand and varying pumping head. No storage in system.

surface in the tanks does not conform, the system becomes a gravity system and pumping costs do not influence the determination of economic pipe sizes.

5. *Case* IV. *Pipe Network. Pumped Supply with Varying Demand and Varying Pumping Head. No Storage in System.* Estimates may be made of the expected variation in demand throughout the design period for the system as a whole. The following analysis is based upon the assumption that the flow is distributed throughout the system in the same proportions at all times. Fire flows are excluded since they account for a negligible part of the yearly pumping cost. If throughout the design period, the discharge and head loss for any element vary such that q_a, q_b, q_c, etc., equals the discharge for times t_a, t_b, t_c, etc., and h_a, h_b, h_c, etc., equals the corresponding friction heads, the average annual cost of the friction loss is

$$ACP = 236p\left(q_a h_a \frac{t_a}{t} + q_b h_b \frac{t_b}{t} + q_c h_c \frac{t_c}{t} + \cdots\right)$$
$$= 236pqh\left(\frac{q_a h_a}{qh}\frac{t_a}{t} + \frac{q_b h_b}{qh}\frac{t_b}{t} + \frac{q_c h_c}{qh}\frac{t_c}{t} + \cdots\right)$$

or

$$ACP = 236pAqh \tag{20}$$

in which q and h = average discharge in the element over the design period and the corresponding friction head.

t = design period.

$$A = \left(\frac{q_a h_a}{qh}\frac{t_a}{t} + \frac{q_b h_b}{qh}\frac{t_b}{t} + \cdots\right)$$

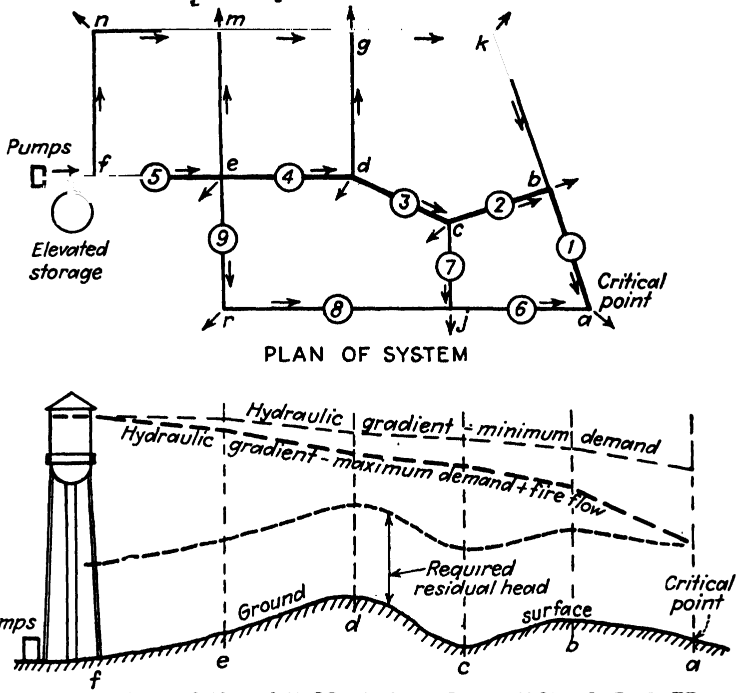


FIG. 10.—Case V. Pumped supply with varying demand and constant pumping head. Elevated storage at pumps.

For the Hazen-Williams formula,

$$A = \left(\frac{q_a}{q}\right)^{2.85}\frac{t_a}{t} + \left(\frac{q_b}{q}\right)^{2.85}\frac{t_b}{t} + \left(\frac{q_c}{q}\right)^{2.85}\frac{t_c}{t} + \cdots \tag{21}$$

The value of A will usually lie between 2 and 4, the value increasing with the amount of fluctuation in the demand.

The economical diameter of any pipe in a system with a varying discharge and pumping head is

$$d_{ec} = \frac{27.4}{C^{0.30}}\left[\frac{pA(q + mQ)}{r(B + 1.55Fa)}\right]^{0.163} q^{0.30} \tag{22}$$

in which q is the average discharge through the pipe throughout the design period, and m and Q are values corresponding to q estimated as described for Eq. (18).

6. *Case* V. *Pipe Network. Pumped Supply with Varying Demand and Constant Pumping Head. Elevated Storage at Pumps.* In this case, the pumping head will be fixed by the peak flow through the system, including fire flow, and will be maintained constant by a distributing reservoir or elevated tank placed near the pumps with its water-surface elevation established for best economy. The annual cost of pumping is not affected by the rate of pumping.

The power loss chargeable to each pipe for friction and take-offs is determined from the maximum friction head and from the average discharge over the design period. The elevation of the tank is established by the maximum friction head through the system.

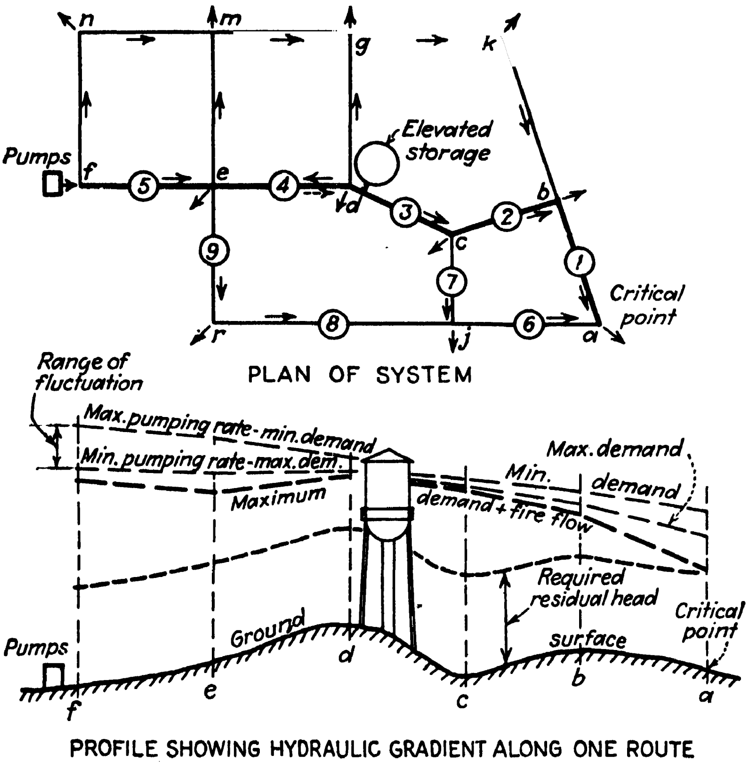


FIG. 11.—Case VI. Pumped supply with varying demand and varying pumping head. Elevated storage remote from pumps.

The economical diameter of any pipe in the system for this case is

$$d_{ec} = \frac{27.4}{C^{0.30}} \left[\frac{p(q + mQ)}{r(B + 1.55Fa)} \right]^{0.163} q_{max}{}^{0.30} \tag{23}$$

in which q_{max} is the peak flow, including fire flow, in the pipe, and other quantities are the same as for Eq. (22).

7. *Case* VI. *Pipe Network. Pumped Supply with Varying Demand and Varying Pumping Head. Elevated Storage Remote from Pumps.* In this case, the elevation of the distributing reservoir will be determined by the maximum friction loss through the pipes delivering water from the reservoir to the critical point. This will occur with the maximum draft from the system, including a fire load at the critical point.

The pumps would presumably be operating at their maximum capacity during a fire, as this practice would make for the greatest economy in the design of the system.

In a system of this type, the variation in discharge for normal demand is not similar in all pipes in the system. In some of the pipes, as in pipe 4 of Fig. 11, a reversal of flow takes place. For others, such as pipes 1, 2, and 3, the flow is always in the same direction for normal demand. Although the pumping head varies, the manner of variation is determined by both the rate of pumping and the demand, which are not the same. The increased cost of pumping due to variable head is chargeable to friction loss in the pipes between the pumps and the elevated storage and is substantially independent of the pipes downstream from the elevated storage.

Three types of pipes may be distinguished in a system of this class, in so far as the manner of determination of the economic sizes is concerned, as follows:

1. Pipes downstream from the elevated storage on direct routes to the critical point, such as pipe 2 (Fig. 11). These pipes may be considered as falling in case V and their sizes determined by Eq. (23).
2. Pipes on direct routes between the pumps and elevated storage, such as pipe 5 (Fig. 11). These pipes may be considered as falling in case IV and their sizes determined by Eq. (22). The values of A and q can be approximated only very roughly since they are influenced by both demand and pumping rates.
3. Pipes that partake of the nature of both types 1 and 2 above, such as pipe 4, which is downstream from the storage for the fire load at a and is en route from pumps to storage for minimum demand. Both Eqs. (22) and (23) may be used for estimating the economic size of pipes of this type. Equation (22) should be given greater weight for pipe 4, since the influence of this pipe on fluctuations of pumping head is greater than its influence upon the established elevation of the storage. About equal weight should probably be given to both equations for finding the size of pipe 8.

In the application of Eq. (22) to the determination of the size of pipes in which there is a reversal of flow, such as in pipe 4, the effect of the reversal is to increase the range of fluctuation of the pumping head. Since negative friction losses correspond with savings in pumping head, however, they should be credited to the pipe by using negative values of the corresponding terms in Eq. (21) for the determination of A. A will thus be small for pipes in which there is a reversal of flow for normal demands.

PIPES AND MATERIALS[1]

Cast-iron Pipe and Fittings. Cast iron is the most widely used material for the mains of distribution systems for sizes up to about 30 in. It is usually stocked in sizes up to 48 in. and may be ordered in standard size up to 84 in. In sizes above 30 in., steel pipe and prestressed reinforced-concrete pipe compete favorably with cast iron, and in smaller sizes cement-asbestos pipe competes favorably.

A.W.W.A. standard bell-and-spigot class B and C pipe has been widely used in distribution systems, but for smaller sizes in new construction its use is being abandoned in favor of lighter and stronger pipe made by newer processes. A.W.W.A. standard pipe is cast in vertical sand molds. Three new types of cast-iron pipe described by Federal Specifications WW-P-421 are being widely used. Type I of these specifications, exemplified by de Lavaud pipe, is centrifugally cast in metal contact molds; type II, of which Monocast pipe is an example, is centrifugally cast in sand-lined molds; and type III, of which McWane pipe is an example, is horizontally cast in green sand molds. The sizes, weights, and thickness of pipes commonly used in distribution systems are given in Table 7. The weight and thickness of class 150 pipes are the same for the same laying lengths for all three types covered by the Federal specifications, and nominal values are given in the table.

For distribution systems, bell-and-spigot pipe is widely used because of the flexibility that can be had at the joints in laying the pipe in trenches. The standard types of bell-and-spigot fittings are shown in Fig. 12. For exposed and supported cast-iron pipe in pump stations and treatment plants, flanged joints are commonly used, with a few bell-and-spigot joints strategically located to facilitate making connections. American standard flanged fittings for steam, as shown in Fig. 13, are

[1] For further discussion of pipe lines see Sec. 10.

Table 7.—Bell-and-spigot Cast-iron Pipe Data

Nominal inside diameter of pipe, in.	A.W.W.A. class B. 200-ft head or 86-psi working pressure			A.W.W.A. class C. 300-ft head or 130-psi working pressure			Class 150. 346-ft head or 150-psi working pressure					Approx. lb lead per joint 2 in. deep	Approx. lb leadite per joint 2½ in. deep	Approx. lb yarn per joint
	Pipe thickness, in.	Weight with bell, lb/ft		Pipe thickness, in.	Weight with bell, lb/ft		Pipe thickness, in.	Weight with bell, lb/ft						
		12-ft length	16-ft length		12-ft length	16-ft length		12-ft length	16-ft length	18-ft length	20-ft length			
4	0.45	21.7	21.2	0.48	23.3	22.8	0.34	16.4	16.1	15.9	.....	7.50	2.50	0.21
6	0.48	33.3	32.5	0.51	35.8	35.0	0.37	26.3	25.7	25.5	.....	10.25	3.40	0.31
8	0.51	47.5	46.6	0.56	52.1	50.9	0.42	39.4	38.6	38.3	.....	13.25	4.38	0.44
10	0.57	63.8	62.5	0.62	70.8	69.4	0.47	53.3	52.2	51.8	.....	16.00	5.33	0.53
12	0.62	82.1	80.6	0.68	91.7	90.0	0.50	67.4	66.1	65.6	.....	19.00	6.25	0.61
14	0.66	102.5	....	0.74	116.7	....	0.55	88.5	86.9	86.3	.....	22.00	7.50	0.81
16	0.70	125.0	....	0.80	143.8	....	0.60	110.2	108.1	107.4	106.9	30.00	10.00	0.94
18	0.75	150.0	....	0.87	175.0	....	0.65	132.8	130.4	129.6	128.9	33.80	10.95	1.00
20	0.80	175.0	....	0.92	208.3	....	0.68	155.0	152.0	151.0	150.2	37.00	12.50	1.25
24	0.89	233.3	....	1.04	279.2	....	0.76	206.8	202.9	201.6	200.5	44.00	15.00	1.50
30	1.03	333.3	....	1.20	400.0	....	0.89	.....	290.2	.....	.....	54.25	18.70	2.06
36	1.15	454.2	....	1.36	545.8	....	1.01	.....	395.6	.....	.....	64.75	23.40	3.00
42	1.28	591.7	....	1.54	716.7	....	....	.....	.....	.....	.....	75.25	30.00	3.62
48	1.42	750.0	....	1.71	908.3	....	....	.....	.....	.....	.....	85.50	37.50	4.37

usually employed; but where longer radius bends are desired, some fittings with radii and laying lengths in accordance with A.W.W.A. standards may be obtained. For dimensions and weights of fittings, see "Cast Iron Pipe Handbook" or manufacturers' catalogues.

Cast-iron pipe fittings with some of the special types of joints described below may be had from the manufacturers. Small-diameter cast-iron pipe in sizes 1¼, 1½, 2, 3 in., and larger may be had from several manufacturers with several different

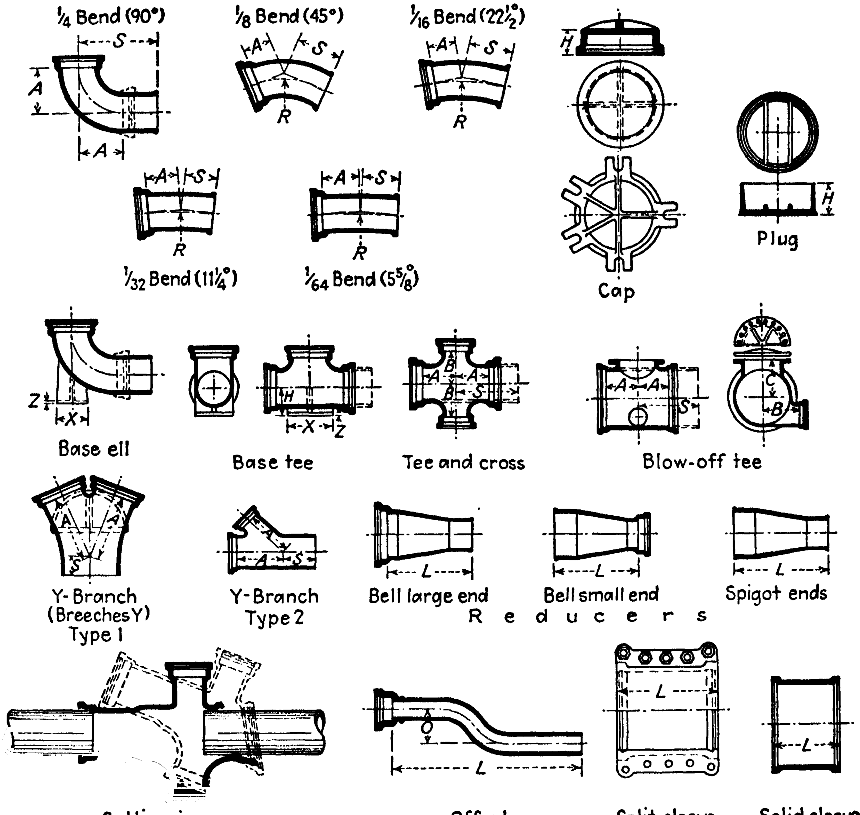


Fig. 12.—A.W.W.A. standard cast-iron bell-and-spigot fittings. (*The Cast Iron Pipe Research Association.*)

types of joints. Small pipes are useful in filter plants and pumping stations and for service pipes.

Joints for Cast-iron Pipes. A number of types of joints for cast-iron pipe are shown in Fig. 14.

The most common type of joint[1] for water pipe laid in trenches is the bell-and-spigot joint made with cast lead and yarn. The yarn is usually made from jute or hemp (oakum) and is calked in the annular space between the spigot and bell to hold the spigot concentrically in the bell and to prevent the lead from running into the

[1] See Flinn, Weston, and Bogert, "Waterworks Handbook," McGraw-Hill Book Company, Inc., 1927.

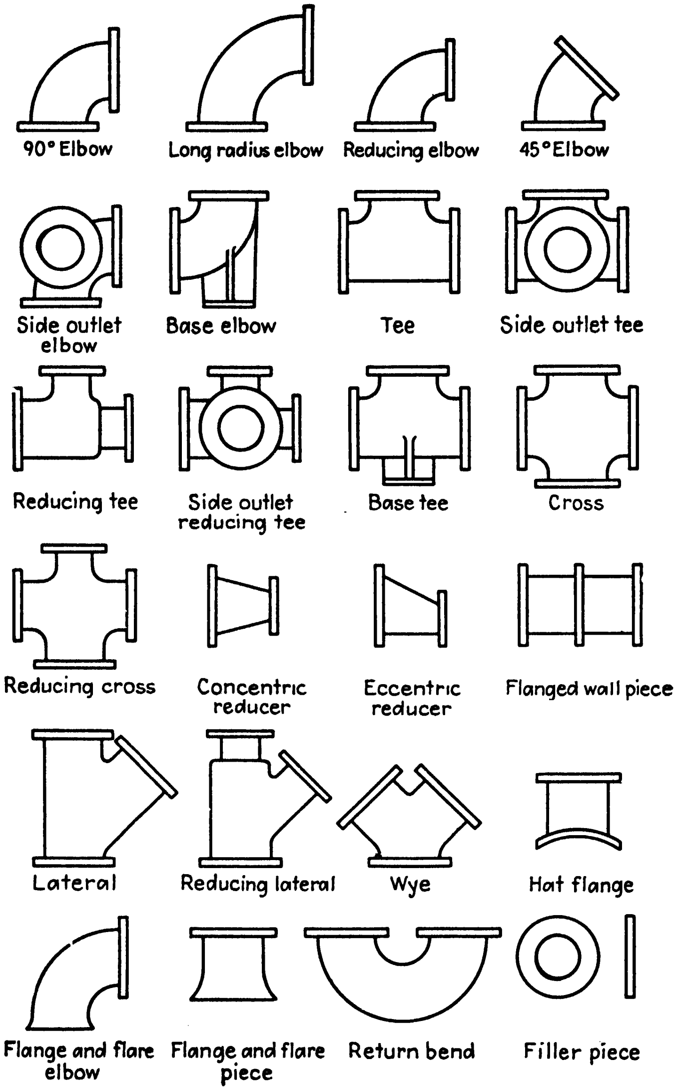


FIG. 13.—American standard flanged fittings for cast-iron pipe. (*The Cast Iron Pipe Research Association.*)

FIG. 14.—Typical joints for cast-iron pipe. (*A*) A.W.W.A. standard bell-and-spigot pipe. (*B*) American standard flanged joint. (*C*) Dresser- or sleeve-type coupling for plain-end pipe. (*D*) U.S. joint, 3- to 12-in. pipe (*U.S. Pipe and Foundry Company*). (*E*) Molox ball joint for subaqueous pipe lines, 4- to 24-in. pipe (*American Cast Iron Pipe Company*. (*F*) Thomas joint, 3- to 12-in. pipe (*Warren Foundry & Pipe Corporation*). (*G*) Victaulic joint (*American Cast Iron Pipe Company*). (*H*) Doublex simplex joint, 2- to 36-in. pipe (*American Cast Iron Pipe Company*). (*I*) Simplex joint, 2- to 6-in. pipe (*American Cast Iron Pipe Company*). (*J*) Precalked joint, 1¼- to 12-in. pipe (*McWane Cast Iron Pipe Company*). (*K*) Threaded joint for small cast-iron pipe (*American Cast Iron Pipe Company*). (*L*) C-N Mechanical joint, 3- to 24-in. pipe (*James B. Clow & Sons*).

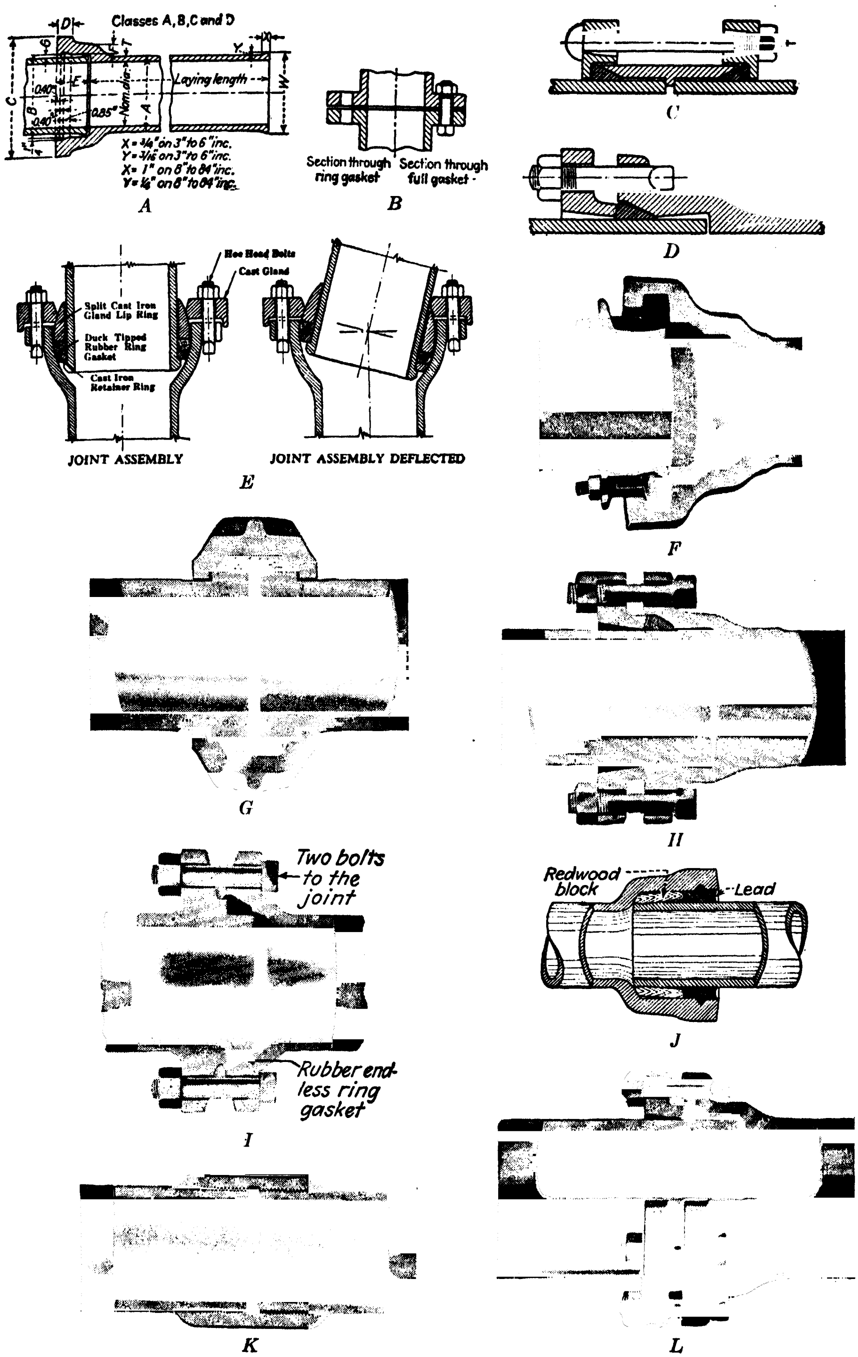


FIG. 14.—See facing page for legend.

pipes. After the joint is yarned to leave a depth of 2 to 2½ in. for the lead, a lead runner of asbestos is placed around the pipe and snugly against the face of the bell, being clamped at the top of the pipe in such a way as to form an opening for pouring the molten lead. A pouring gate is built up with clay above the runner, and the molten lead (melting point about 620F) is poured from a ladle into the joint. The lead hardens quickly as it cools, and the runner is then removed and the joint calked to make it tight.

A number of substitutes[1] for molten lead are available, some of which have advantages over the lead. Lead wool made of shredded lead in the form of a loose rope may be calked into the joint without heating. It is more expensive and more difficult to calk than cast lead, but it may be used under water if necessary. Molten lead alloys containing small amounts of tin and antimony are sometimes used for high

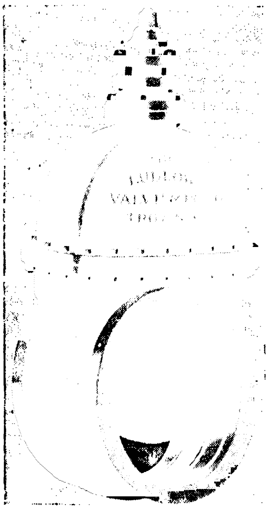


FIG. 15a.—Inside-screw type cast-iron gate valve. (*Ludlow.*)

FIG. 15b.—Outside-screw-and-yoke type cast-iron gate valve with spur gears and by-pass. (*Rensselaer.*)

pressures where lead is too ductile. Among the best known substitutes for lead are Leadite, composed of iron, sulphur, slag, and other substances; Hydro-tite; and Mineralead. These compounds are all much lighter than lead, are cheaper, and melt with less heat (about 240F). No calking is required since the cool material is hard and tightens when moist. These compounds are nonconductors and therefore prevent the passage of as much electric current along the pipe line as could pass with lead joints. Neat Portland cement mortar, also a nonconductor, has been used successfully as a jointing material. Many of the special joints shown in Fig. 14 are of the mechanical type. Among the advantages of some of these joints are ease of assembly; flexibility under vertical loads, vibrations, and settlement; suitability for longitudinal expansion of pipe; and resistance to the flow of electric currents due to the use of nonconducting gaskets. An added advantage of the sleeve-type joint is that it permits the use of short-pieces of pipe without waste. With the precalked-type

[1] *Ibid.*

joint, lead and packing are furnished already in place in the bell, sufficient clearance being left for the spigot to be inserted, after which the lead is calked tight.

Valves and Hydrants. Gate valves for distribution mains are usually of the inside-screw or nonrising-stem type illustrated in Fig. 15 (*a* and *c*). The outside screw and yolk or rising-stem type valve, illustrated in Fig. 15*b*, is advantageous in that the position of the stem indicates how wide the valve is open; but this type cannot ordinarily be used on mains because of the depth of trench required for the rising stem. Valves for underground mains are usually provided with bell ends; but spigot ends, flanges, and special types of joints are available.

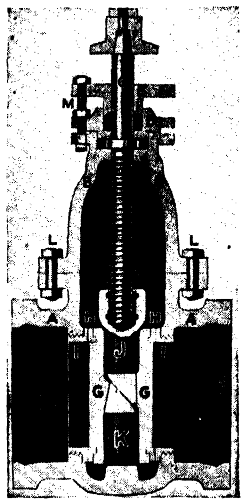


FIG. 15*c*.—Details of inside-screw-type gate valve. (*Ludlow.*) (*A*) Case; (*B*) cover or bonnet; (*C*) stem or spindle; (*D*) packing plate or stuffing box; (*E*) stuffing-box gland or follower; (*F*) wrench nut; (*GG*) gates; (*H*) gate rings; (*I*) case rings; (*J*) top wedge; (*K*) bottom wedge; (*L*) throat flange bolts; (*M*) stuffing-box or follower bolts.

The standard specifications[1] for gate valves of the A.W.W.A. and the N.E.W.W.A. embrace hand-operated, inside-screw, iron-body, bronze-mounted gate valves of both the solid-wedge and double-disk (either parallel seat or inclined seat) type, ranging in size from 3 to 48 in., for ordinary water service in approximately level setting under operating pressures not exceeding 150 psi. Valves are required to withstand an internal test pressure of 300 psi and to operate satisfactorily with 150-psi pressure on one side of the gate. The diameter of the waterway must be not less than the pipe diameter. In valves 3 in. in size and smaller, gates are of solid bronze. In larger valves, disks may be of cast iron with bronze rings. Valve seat rings, thrust bearings, packing glands, gear spindles, wedging devices, guides, rollers and tracks, and indicator mechanisms are made of bronze or are bushed or faced with bronze. Valve stems, stem collars, and nuts are made of manganese bronze having a tensile strength of not less than 60,000 psi. Bell-end valves are made with sockets for class C pipe. Valves 16 in. in size and larger, designed to lie on their sides with horizontal stems, are required to have tracks and rollers to carry the weight of the gate as it moves into the bonnet. Stuffing boxes are packed with graphited hydraulic packing made of flax or properly lubricated braided asbestos. Wrench nuts are of cast iron, 2 in. square at the base, and provided with an arrow to mark the direction of turn for opening the valve. Gears for larger valves may be of cast iron or of steel if enclosed in an oiltight cast-iron gear case. All ferrous parts of the valve, except finished or bearing surfaces, are required to have two coats of pipe dip or varnish on the interior and three coats outside.

Large valves are usually provided with gears to facilitate hand operation. Vertical-stem valves are provided with spur gears (Fig. 15*b*) and horizontal-stem valves with bevel gears (Fig. 15*d* and *e*). Such valves may also be provided with by-passes (Fig. 15*b*) to facilitate opening. Smaller valves are usually provided with valve boxes (Fig. 15*f*), and for such valves it is necessary to remove the box and excavate down to the valve in order to repack the stuffing box. It is advisable to install large valves (16 in. and larger) in vaults or manholes in order to facilitate repacking and repairs.

Gate valves are sometimes used on lines between low-pressure and high-pressure districts, and in order to obviate the effects of dead ends in the pipe on both sides of

[1] *Jour. A.W.W.A.*, **31**, 502, 1939.

the gate they are sometimes kept partly open. The gates thus act as throttling valves. Ordinary gate valves are unsatisfactory when used for this purpose owing to the chatter of the disks and their deflection downstream which causes vibrations and wear on the seats. Square-bottom valves designed especially for this purpose are now available.

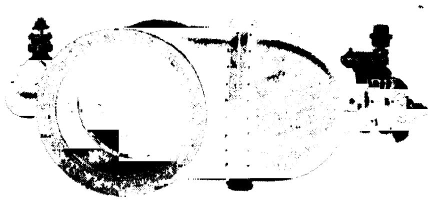


FIG. 15d.—Inside-screw-type cast-iron gate valve with horizontal stem, cast-iron bevel gears and by-pass. (*Rensselaer.*)

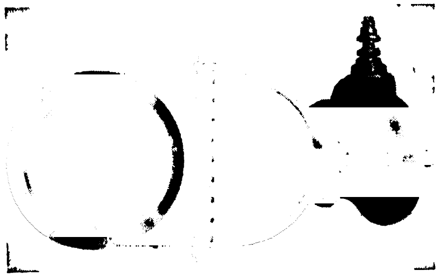


FIG. 15e. Inside-screw-type cast-iron gate valve with horizontal stem and oil-encased steel bevel gears. (*Ludlow.*)

When a distribution system receives its supply by gravity from a distant reservoir at a much higher elevation or through pipes from a high-pressure district, pressure-reducing valves are required in order to limit the pressure in the system at times of minimum draft. Automatic pressure-reducing valves which maintain the pressure constant on the downstream side of the valve are available for this purpose. Figure 16 illustrates a typical pressure-reducing valve, shown in the closed position. The pilot control valve is adjusted by hand for any desired pressure on the delivery side of the main valve. Whenever the delivery pressure lowers below the adjusted value, the spring against the pilot diaphragm causes the pilot valve to open and permit water above piston *B* to escape through *M* and *N* to the delivery side. The valve thus opens owing to the higher pressure under piston *B*. While the valve is open, water flows through the main valve and also through port *L* and needle valve *S* into the chamber above *B*. When the flow through the main valve becomes sufficient to raise the delivery pressure to the adjusted value, the pilot valve closes and the piston is held in position at the proper opening.

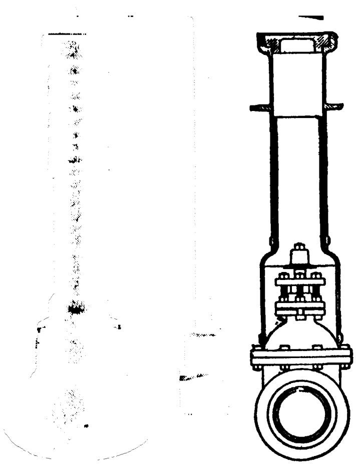


FIG. 15f.—Extension valve boxes and valve wrench. (*Kennedy.*) (*a*) Outside view of screw type; (*b*) valve wrench; (*c*) sectional view of sliding type.

Altitude-control valves are used on the inlet pipes to distribution reservoirs and elevated tanks to shut off the inflow and prevent overflow when the water level gets too high. Figure 17 shows a typical altitude valve in the closed position. When the valve is open, water may flow in either direction. During inflow when the reservoir water level reaches the value sufficient to overcome the adjustment of the spring *W* by exerting pressure above diaphragm *R*, the pilot exhaust valve *I* is closed and pilot valve *H* is opened. Water is thus permitted to flow through *L* and *M* into

the chamber above B and close the valve. In the type of valve shown in the figure, the valve will not open until the reservoir water level is lowered. Hence for removing

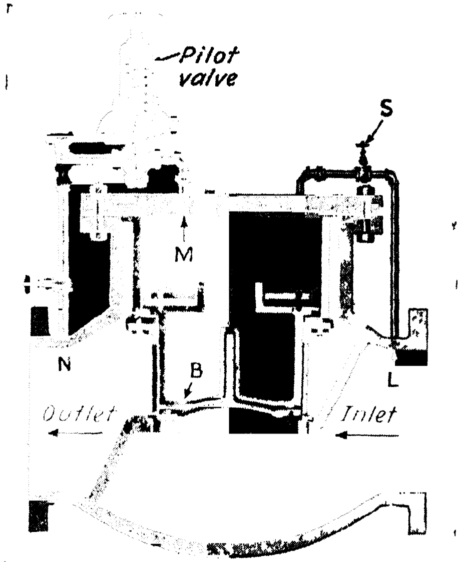


FIG. 16.—Automatic pressure-reducing valve. (*Golden-Anderson*)

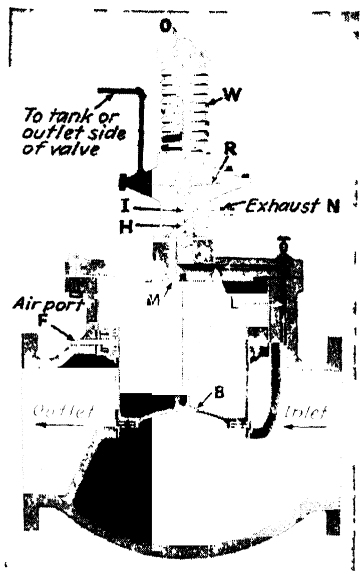


FIG. 17.—Single-acting altitude-control valve. (*Golden-Anderson.*)

storage water for distribution when the altitude valve is closed, a separate discharge line from the reservoir or a by-pass around the altitude valve is required, as shown in Fig 18. Other styles of valves are available, some of which are double acting and will open when the pressure on the inlet side of the valve lowers below the pressure corresponding to the reservoir level. Other valves are designed to close when a high-pressure fire pump is discharging into the system regardless of the reservoir water level.

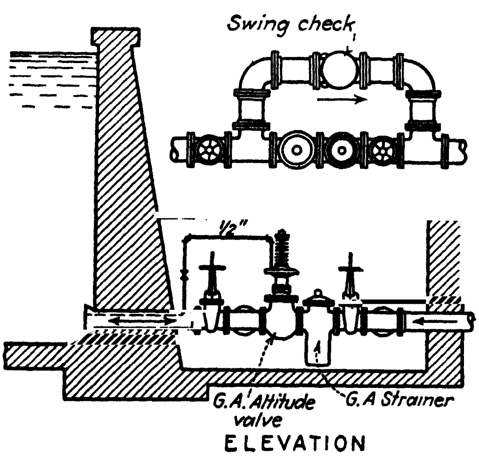


FIG. 18.—Installation of single-acting altitude valve. (*Golden-Anderson.*)

Pressure-reducing and altitude valves are particularly susceptible to trouble from freezing in cold weather because of the presence of the small-size control water pipes and passages in which water is often still. For this reason, such valves should be adequately housed where they are readily accessible, and provisions should be made for heating the interior of the housing in extremely cold weather.

Check valves, which permit flow through a pipe in only one direction, have their principal use in the suction and discharge lines of pumps. They are sometimes used

in distribution systems, however, one use being in connection with altitude valves as shown in Fig. 18. One important use in distribution systems is for the prevention of

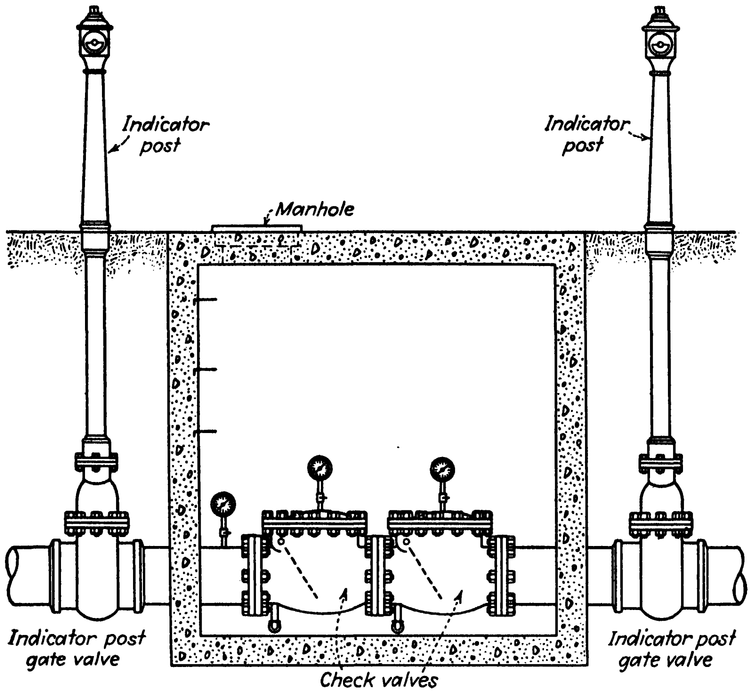


Fig. 19.—Fire-service connection using two Factory Mutual check valves. (*Ludlow.*)

backflow of a separate contaminated industrial fire supply into the municipal distribution system. Any such cross-connection between an impure and a potable water is a potential health hazard, because of the possibility that the valves may not seat tightly. An installation consisting of two swing check valves in series (Fig. 19) is recommended by the Associated Factory Mutual Laboratories and is accepted by some health departments. Figure 20 illustrates a typical horizontal swing or flap check valve. There is considerable friction loss through this type of check valve. In cases where it is necessary to minimize the friction loss, automatic power-operated check valves of various types which provide openings the full area of the pipe may be had. Motor- or hydraulic-operated gate valves may be used, but the recently developed revolving cone valve is superior. Cone valves may also be used as gate, altitude, and pressure-reducing valves with suitable controls.

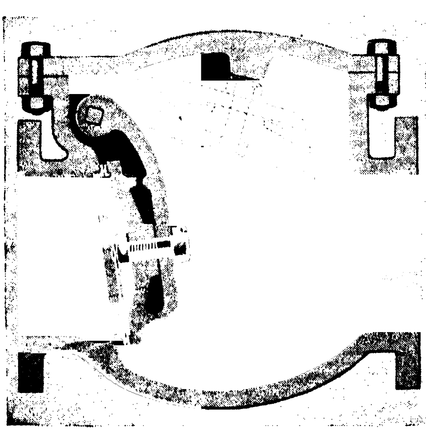


Fig. 20.—Typical horizontal swing check valve. (*Ludlow.*)

Tilting-disk check valves are also much superior to swing checks with regard to head loss and to chatter.

Fire hydrants are made in three general types: (1) the post hydrant with a vertical barrel extending 2 or 3 ft above the ground surface; (2) the flush hydrant in which the top of the barrel and the nozzle are underground in a box whose cast-iron cover is flush with the ground surface; (3) and the wall hydrant which is set back in the wall

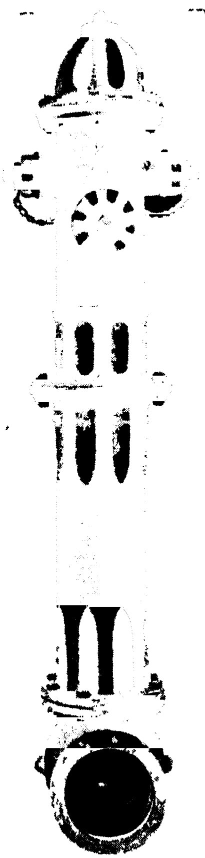


FIG. 21a.—Compression-type post fire hydrant. (*Mueller-Columbian.*)

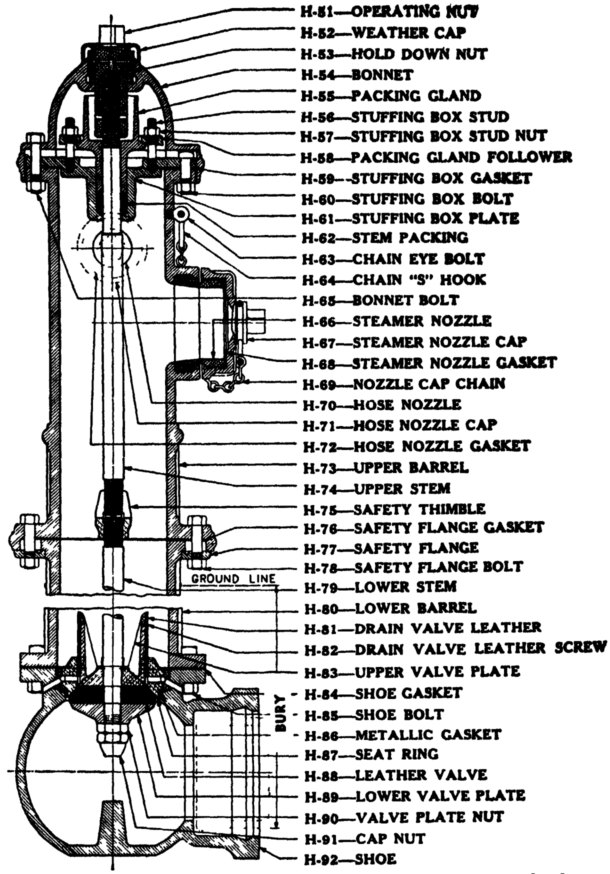


FIG. 21b.—Details of compression-type post fire hydrant. (*Mueller-Columbian.*)

of a building. Post hydrants are usually just in back of the curb line, and because of this exposed position they are frequently damaged by motor vehicles. The other two types of hydrants are not thus subject to damage, but they are not as satisfactory as post hydrants because of their limited capacity and because they are more difficult to find quickly. Flush hydrants are not satisfactory in northern climates because of the difficulty of keeping the covers clear of snow.

The standard specifications[1] (1940) of the A.W.W.A. and the N.E.W.W.A. are for post hydrants of the compression-valve (Fig. 21) and gate-valve (Fig. 22) type. The

[1] *Jour. A.W.W.A.*, **32**, August, 1940.

design should permit the ready removal of the valve without excavation and should be such that the valve will remain closed in case of damage to the top of the barrel. Hydrants are classed as single-hose, two-hose, and two-hose and pumper, according to the arrangement of hose and pumper nozzles. The size of the hydrant is designated by the diameter of the valve opening, which should be at least 4 in. for two-hose, 5 in. for three-hose, and 6 in. for four-hose hydrants. The length of the hydrant is defined as the vertical distance from the ground surface to the bottom of the connecting pipe. Friction losses should be under 1 psi for each 250-gpm fire stream. When the barrel is made in sections, the flanges or connection should be at least 2 in. above the ground surface. Positive operating drain valves are required to drain the hydrant completely and quickly when the main valve is closed in order to prevent freezing and to close tightly when the main valve is opened. The hydrant top should be designed to prevent interference with operation due to freezing, and provision should be made for convenient lubrication. Barrels, hydrant heads, valve gates, and nozzle caps may be made of cast iron. Outlet nozzles, valve seats or seat rings, drain valves, stuffing-box glands and gland-bolt nuts, and either the operating stem or the operating nut should be made of bronze or other noncorrodible material. The main valve should be faced with rubber, leather, or balata or, in case of slide-gate type valves, with a bronze ring. All iron parts inside and out, except the outside surface above the ground which should be painted, are required to have a hot bituminous dip coat or two bituminous paint coats. The outside surface above the ground should be painted a color[1] to distinguish between private and public hydrants, and the hydrant tops and nozzle caps should be given a proper color to indicate the hydrant capacity.

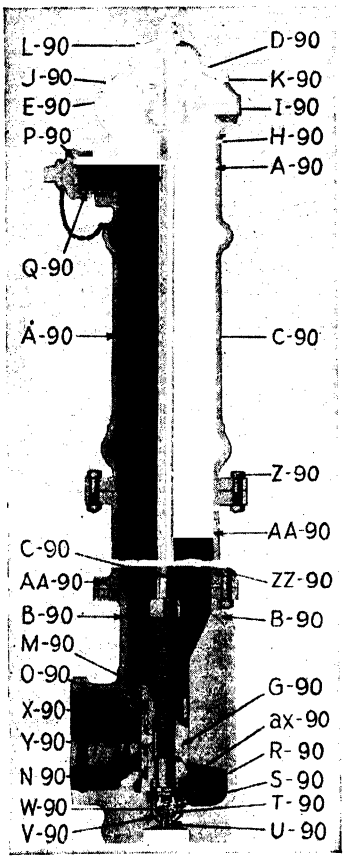


FIG. 22.—Slide-gate type post fire hydrant. (*Ludlow.*)

A recent improvement in post-hydrant construction is the provision of a safety joint above the ground surface which permits the top of the hydrant to be knocked off by motor vehicles with a minimum of damage (see Fig. 21). In installing a hydrant, it is desirable that the drain valve be below the frost line and be provided with a reliable outlet to ensure proper emptying of the barrel in cold weather. A usual provision is to surround the bottom of the barrel with gravel, but this is inadequate if the ground-water table is above the drain valve or if the earth surrounding the gravel is impervious. A connection to a sewer provides positive drainage. It is good practice to provide hydrants with gate valves on the supply pipes in order to make repairs to a hydrant without the necessity of shutting off a section of main. In order to prevent strains on hydrant barrels when the surrounding ground heaves owing to frost

[1] *Jour. A.W.W.A.*, **29**, 449, 1937.

action, the barrel should be surrounded with gravel to the ground surface and flanges placed above the ground as provided by the standard specifications of the A.W.W.A. and the N.E.W.W.A. Some manufacturers provide sliding frost cases of cast iron for protection against heaving ground.

Both hydrants and gate valves in distribution systems are so infrequently used that they are subject to jamming due to corrosion and injury from other causes which may make them useless when needed. Complete inspections of all valves and hydrants should be made yearly, during which each valve and hydrant is opened and closed, lubricated, repacked, and repaired if necessary. Such inspections frequently result in the discovery of closed gate valves which should be open.

Corrosion and Electrolysis.[1] Corrosion may be broadly defined as the chemical action of certain external agencies on metals which causes their deterioration or destruction. Metals tend to revert to more stable compounds, of which the metal ores, as found in nature, are familiar examples. Corrosion is of great economic importance in water distribution, for the deterioration of pipe lines and their loss of capacity with age is due primarily to corrosion and its products, scale and rust tubercles. It is also of great importance in water treatment, for the protection of the piping against corrosion and the prevention of red-water troubles is now recognized as a major function of water treatment.

A number of theories have been proposed to explain the mechanism of corrosion, one of which, the electrochemical theory, is now generally accepted as being the most satisfactory. This theory is presented briefly below in terms of the corrosion of iron, lead, copper, and zinc, the principal metals used for water distribution.

Water is corrosive to a solid metal when it tends to dissolve the metal as positive ions or to furnish negative ions to react with the metal at the interface. Corrosion proceeds by a transfer of negative electrons at *anodic* areas from the water to the metal. These electrons flow through the metal to *cathodic* areas where they are given up to constituents in the water. This flow of electrons constitutes a flow of electric current, and the circuit is completed through the water by the motion of ions between the two electrodes of the cell. In order that the current may flow, there must be an electrochemical or *half-cell* reaction at the anode and an equivalent half-cell reaction at the cathode, as shown in Fig. 23. The two reactions which take place in a particular case are those which produce the greatest potential between the electrodes. The actual potential is determined by the difference, $\Delta E°$, of the *standard oxidation potentials* of the half-cell reactions, and the concentrations of the ions and dissolved substances which take part in the reactions.

In order to determine what reaction will take place at the anode for a particular case, it is necessary to examine all possible anodic reactions in pairs to determine which is the most anodic. Similarly to determine the cathodic reaction, all possible cathodic reactions must be examined in pairs to find the most cathodic. One reaction is anodic to another when, at 25C,

$$\frac{Q_1^{\frac{1}{n_1}}}{Q_2^{\frac{1}{n_2}}} \text{ is less than } \frac{\Delta E_{25}°}{10^{0.05914}} \tag{24}$$

where the subscript 1 refers to the anodic reaction, the subscript 2 to the cathodic reaction, Q is the ratio of the product of the activities of the reaction products to the product of the activities of the reactants, and n is the number of electrons transferred.

[1] SPELLER, F. N., "Corrosion—Causes and Prevention," 3d ed., McGraw-Hill Book Company, Inc., 1951. See also CAMP, T. R., Corrosiveness of Water to Metals, *Jour. N. E. W. W. A.*, **60,** 1946, pp. 188 and 282.

The principal reactions in the corrosion of iron, lead, copper, and zinc in fresh water are shown in Table 8. Solid reaction products are underlined. The cathodic reactions proceed in the direction opposite to that of the arrows.

In the corrosion of iron, reaction 4 prevails at pH values below about 9 and ferrous iron enters solution at the anode. At higher pH values, OH^- and CO_3^{--} are plated out at the anode forming solid coatings which retard the rate of corrosion. In the presence of dissolved oxygen, the principal cathodic reaction in the corrosion of iron in unchlorinated water is reaction 9 resulting in the formation of ferric iron rust. Water is not formed by reaction 8 unless there is a negligible amount of iron in solution, and

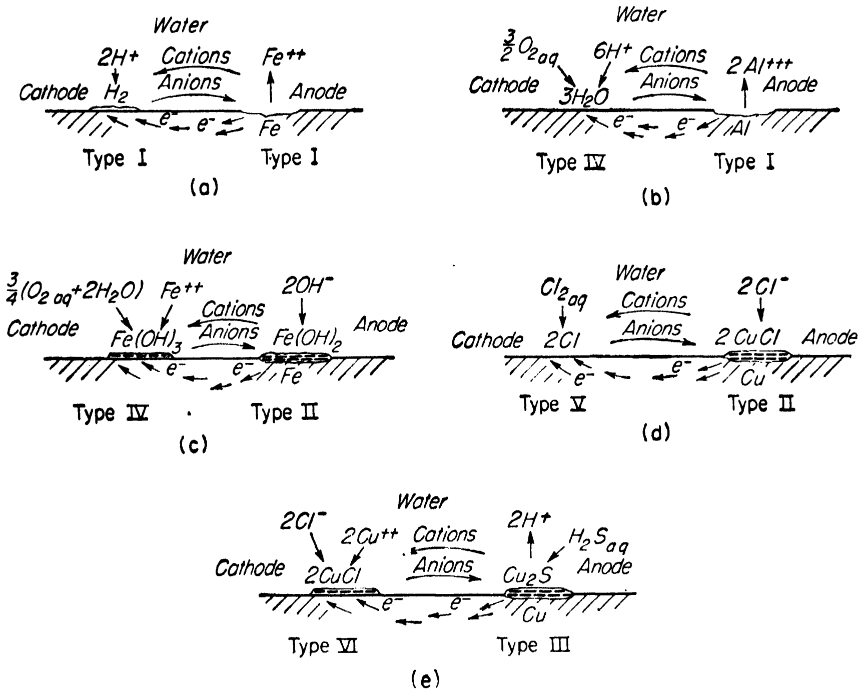


FIG. 23.—Electrochemical action.

H_2 gas is not plated out at the cathode except at low pH values (pH 3 or less). In chlorinated water, chlorine replaces oxygen in similar cathodic reactions.

In the corrosion of lead in fresh water, reaction 16 prevails at pH values less than 8, reaction 14 at pH values from 8 to 10 when the water contains carbonates in sufficient concentration, and reaction 13 at pH values above 10. The cathodic reactions with lead are similar to those with iron, except that a soluble lead oxide is produced at pH values below about 6.5.

In the corrosion of copper in fresh water, it will be noted that ammonia has a deleterious effect at the anode. It will also be noted that the voltages for the anodic reactions are so low that reaction 5, the plating out of hydrogen at the cathode, cannot take place. Also, because of this fact the rate of corrosion of copper is much less than for the other metals. In the absence of ammonia, reaction 20 prevails at the anode at pH values above about 5.5, and reactions 23 and 24 at lower pH values. The presence of ammonia may be deleterious at pH values below 8 or 9.

Brass is an alloy of copper and zinc, the relative amounts ranging from 60 per cent

TABLE 8.—CORROSION OF SOME METALS IN FRESH WATER

Principal Reactions	$E_{25°}$, Volts
IRON	
Anodic:	
(1) $2OH^- + \underline{Fe} \rightarrow \underline{Fe(OH)_2} + 2e^-$	+0.877
(2) $CO_3^{--} + \underline{Fe} \rightarrow \underline{FeCO_3} + 2e^-$	+0.755
(3) $HPO_4^{--} + \underline{Fe} \rightarrow \underline{FeHPO_4} + 2e^-$	?
(4) $\underline{Fe} \rightarrow Fe^{++} + 2e^-$	+0.440
Cathodic:	
(5) $H_{2gas} \rightarrow 2H^+ + 2e^-$	+0.000
(6) $2CO_3^{--} + H_2O \rightarrow 2HCO_3^- + \frac{1}{2}O_{2aq} + 2e^-$	−0.66
(7) $CO_3^{--} + H_2O \rightarrow H_2CO_3 + \frac{1}{2}O_{2aq} + 2e^-$	−0.779
(8) $H_2O \rightarrow \frac{1}{2}O_{2aq} + 2H^+ + 2e^-$	−1.273
(9) $\underline{Fe(OH)_3} \rightarrow \frac{3}{4}O_{2aq} + Fe^{++} + \frac{3}{2}H_2O + 2e^-$	−1.386
Cathodic in chlorinated water:	
(10) $Cl^- + 2OH^- \rightarrow OCl^- + H_2O + 2e^-$	−0.878
(11) $Cl^- + H_2O \rightarrow HOCl + H^+ + 2e^-$	−1.49
(12) $3Cl^- + \underline{Fe(OH)_3} \rightarrow 3HOCl + Fe^{++} + 5e^-$	−1.58
LEAD	
Anodic:	
(13) $3OH^- + \underline{Pb} \rightarrow HPbO_2^- + H_2O + 2e^-$	+0.54
(14) $CO_3^{--} + \underline{Pb} \rightarrow \underline{PbCO_3} + 2e^-$	+0.506
(15) $HPO_4^{--} + \underline{Pb} \rightarrow \underline{PbHPO_4} + 2e^-$	+0.463
(16) $\underline{Pb} \rightarrow Pb^{++} + 2e^-$	+0.126
Cathodic:	
(5) $H_{2gas} \rightarrow 2H^+ + 2e^-$	+0.000
(6) $2CO_3^{--} + H_2O \rightarrow 2HCO_3^- + \frac{1}{2}O_{2aq} + 2e^-$	−0.66
(7) $CO_3^{--} + H_2O \rightarrow H_2CO_3 + \frac{1}{2}O_{2aq} + 2e^-$	−0.779
(17) $HPbO_2^- \rightarrow O_{2aq} + Pb^{++} + H^+ + 2e^-$	−0.857
(18) $\underline{PbO_2} \rightarrow O_{2aq} + Pb^{++} + 2e^-$	−1.086
(8) $\underline{H_2O} \rightarrow \frac{1}{2}O_{2aq} + 2H^+ + 2e^-$	−1.273
Cathodic in chlorinated water:	
(10) $Cl^- + 2OH^- \rightarrow OCl^- + H_2O + 2e^-$	−0.878
(11) $Cl^- + H_2O \rightarrow HOCl + H^+ + 2e^-$	−1.49
(19) $2Cl^- + 2H^+ + \underline{PbO_2} \rightarrow 2HOCl + Pb^{++} + 2e^-$	−1.53
COPPER	
Anodic:	
(20) $2OH^- + \underline{2Cu} \rightarrow \underline{Cu_2O} + H_2O + 2e^-$	+0.361
(21) $2NH_{3aq} + \underline{Cu} \rightarrow Cu(NH_3)_2^+ + e^-$	+0.11
(22) $4NH_{3aq} + \underline{Cu} \rightarrow Cu(NH_3)_4^{++} + 2e^-$	+0.05
(23) $\underline{Cu} \rightarrow Cu^{++} + 2e^-$	−0.345
(24) $\underline{Cu} \rightarrow Cu^+ + e^-$	−0.552
Cathodic:	
(6) $2CO_3^{--} + H_2O \rightarrow 2HCO_3^- + \frac{1}{2}O_{2aq} + 2e^-$	−0.66
(7) $CO_3^{--} + H_2O \rightarrow H_2CO_3 + \frac{1}{2}O_{2aq} + 2e^-$	−0.779
(25) $\underline{Cu(OH)_2} \rightarrow \frac{1}{2}O_{2aq} + Cu^{++} + H_2O + 2e^-$	−1.01
(8) $\underline{H_2O} \rightarrow \frac{1}{2}O_{2aq} + 2H^+ + 2e^-$	−1.273
Cathodic in chlorinated water:	
(10) $Cl^- + 2OH^- \rightarrow OCl^- + H_2O + 2e^-$	−0.878
(26) $2Cl^- + \underline{Cu(OH)_2} \rightarrow 2HOCl + Cu^{++} + 4e^-$	−1.365
(11) $Cl^- + H_2O \rightarrow HOCl + H^+ + 2e^-$	−1.49
ZINC	
Anodic:	
(27) $2OH^- + \underline{Zn} \rightarrow \underline{Zn(OH)_2} + 2e^-$	+1.42
(28) $4OH^- + \underline{Zn} \rightarrow ZnO_2^{--} + 2H_2O + 2e^-$	+1.216
(29) $4NH_{3aq} + \underline{Zn} \rightarrow Zn(NH_3)_4^{++} + 2e^-$	+1.03
(30) $\underline{Zn} \rightarrow Zn^{++} + 2e^-$	+0.762

copper and 40 per cent zinc in yellow brass to more than 90 per cent copper in red brass. As has been shown above, the copper is not readily corrosive in natural waters. The corrosion of brass usually takes place by dezincification, or zinc solution, which appears to be greatest in soft waters of low pH value and with high zinc content in the brass. The principal anodic reactions for zinc corrosion are much higher in potential than those for copper. Hence in brass the anodic areas are on the zinc and the cathodic areas are on the copper. At pH values below 6 or 7, zinc enters solution at the anode as Zn^{++} or ZnO_2^{--}. At higher pH values, reaction 27 prevails, and $Zn(OH)_2$ is formed at the anode. Ammonia is relatively unimportant for zinc except for high concentrations, but it does affect the copper in brass.

A similarity may be noted in the corrosion of all the metals considered above. At low pH values, the metals enter solution at the anode as positive ions, and at high pH values solid precipitates are formed at the anode. It is evident, therefore, that corrosion may be retarded by anodic protection through a rise in pH value to plate out hydroxide, carbonate, or phosphate at the anodic. The degree of protection furnished depends upon the effectiveness of the coating produced. Ferrous hydroxide formed on iron will probably oxidize in the presence of dissolved oxygen to form ferric hydroxide or ordinary iron rust which is known to be a relatively poor coating. For all the metals, the most important cathodic reactions depend upon the presence of dissolved oxygen which enters into the reactions. In natural waters, the corrosion rate is almost directly proportional to the concentration of dissolved oxygen.

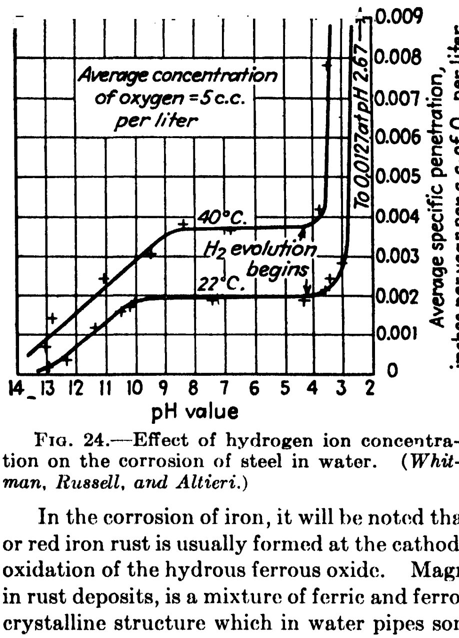


FIG. 24.—Effect of hydrogen ion concentration on the corrosion of steel in water. (*Whitman, Russell, and Altieri.*)

In the corrosion of iron, it will be noted that ferric hydroxide (hydrous ferric oxide) or red iron rust is usually formed at the cathode, and it may be formed at the anode by oxidation of the hydrous ferrous oxide. Magnetic iron oxide, Fe_3O_4, sometimes found in rust deposits, is a mixture of ferric and ferrous oxides. Iron rust is a loosely formed crystalline structure which in water pipes sometimes contains organic deposits all of which are easily scoured from the metal surface during heavy drafts to produce red water. If permitted to grow unrestricted, rust may be built up into tubercles or blisters which restrict the area of the pipe and reduce its capacity. One of the principal objects of corrosion abatement therefore, is to inhibit the formation of iron rust.

The influence of hydrogen-ion concentration[1] on the corrosion of steel in water free from salts or impurities which form protective coatings is shown in Fig. 24. It is evident from the figure that the corrosion of iron in water may be divided into three zones of varying pH value. According to Speller, the principal factors that influence the corrosion of iron in water in the three zones may be arranged in the order of their importance as shown in Table 9.

The relative corrosiveness of various metals and alloys under conditions prevailing in water-works practice is shown in a general way by Table 10,[2] with the more resistant

[1] WHITMAN, RUSSELL, and ALTIERI, *Ind. Eng. Chem.*, **16**, 665–670, 1924. Also see SPELLER, *op. cit.*
[2] From MCKAY and WORTHINGTON, "Corrosion Resistance of Metals and Alloys," Reinhold Publishing Corporation, 1936, and other sources.

TABLE 9.—RELATIVE INFLUENCE OF IMPORTANT FACTORS ON CORROSION OF STEEL IN WATER

	Alkaline zone (pH greater than 10) corrosion slow	Neutral zone (pH 4.3 to 10) corrosion medium	Acid zone (pH less than 4.3) corrosion rapid
1	Protective coatings	Oxygen concentration	pH value
2	Oxygen concentration	Protective coatings	Hydrogen overvoltage
3	Composition of metal	pH value	Composition of metal
4	pH value	Composition of metal	Oxygen concentration
5	Hydrogen overvoltage	Hydrogen overvoltage	Metal-ion concentration
6	Metal-ion concentration	Metal-ion concentration	Protective coatings

TABLE 10.—RELATIVE CORROSION RESISTANCE OF METALS AND ALLOYS

In fresh water				In air
Acid solutions		Neutral	Alkaline solutions	
No oxidation	With oxidation			
Silicon iron	Silicon iron	Copper Nickel Bronze Stainless steel Aluminum (below pH 9)	Nickel Stainless steel Iron (above pH 10)	Copper Bronze Aluminum Stainless steel Tin Brass Lead
Stainless steel	Stainless steel	Zinc (pH 8–9)	Copper	Silicon iron
Aluminum	Aluminum	Tin	Bronze	Zinc
Tin	Copper Nickel Bronze Brass	Brass	Brass	Nickel
Copper Nickel	Lead	Lead (pH 6–9, not good with soft waters)	Tin	Iron
Bronze	Tin	Silicon iron	Zinc (above pH 11)	
Brass	Iron	Zinc (pH 7)	Lead (above pH 10)	
Iron	Zinc	Iron (pH 4.3–10)	Aluminum (above pH 9)	
Lead			Silicon iron	
Zinc				

metals placed at the top. The metals that are underlined are too corrosive for practical use. Bracketed metals are about equally resistant. Bronze is a copper-tin alloy. Cast bronze contains about 85 per cent copper, 5 to 10 per cent tin, and the remainder zinc and sometimes lead. Silicon-manganese bronze, which is quite strong, contains 96 per cent copper, 3 per cent silicon, and 1 per cent manganese. Brass is a copper-zinc alloy containing 60 to about 90 per cent copper. Red brass with an 85–15 ratio is comparatively immune from dezincification. Manganese bronze is a

60–40 brass modified in casting form with small amounts of iron, manganese, aluminum, and nickel for strength and hardness.

Extensive experiments by the U.S. Bureau of Standards[1] on the corrosion of various types of steel, cast iron, and wrought iron in 22 of the more corrosive soils in the United States indicate that no one of the materials is superior to another (with the exception of cast iron with 14.25 per cent silicon). Some soils are more corrosive than others. Cinder fills are particularly corrosive to metal pipes.

Crenothrix[2] and several other forms of bacteria found in natural waters are capable of assimilating dissolved iron and subsequently eliminating it as ferrous oxide. They are not dependent upon iron for existence, but they thrive best in slightly acid water containing more than 2 ppm of iron in solution. These organisms apparently do not attach metallic iron and therefore have little effect upon the corrosion rate; but their presence does result in increased rate of growth of rust tubercles owing to the precipitated iron.

When any metal, such as iron, is placed in contact with a more cathodic material, such as copper, the corrosion potential is greater than for a single metal. A galvanic cell is thus set up, and a form of corrosion known as *electrolysis* is produced. With steel in contact with brass, the corrosion rate of steel is approximately doubled and that of brass greatly reduced. If iron is in contact with a more anodic metal such as zinc, the zinc tends to dissolve. Zinc coatings (galvanizing) on iron thus protect iron from corrosion until the coating is destroyed. Although zinc tends to dissolve faster than iron, protective coatings are produced by zinc in neutral waters which are more effective in retarding the corrosion of zinc than the analogous compounds are in retarding iron corrosion. In the acid range, however, and in the presence of free CO_2, zinc galvanizing is not a very effective protection.

Another form of electrolysis known as *stray-current electrolysis* is best exemplified by the corrosion of iron pipes laid in the soil near electric railway tracks. Direct current escapes from the rails, or is returned through both the rails and the soil, and flows from soil to pipe where the potential is in the proper direction and from pipe to soil where the potential is reversed. Where the positive current leaves the pipes, the iron goes into solution and pitting of the pipe results. With alternating current, the corrosive effect is usually less than 1 per cent of that with direct current because of the counteracting effect of the reverse in potential. The effects of stray-current electrolysis[3] on piping may be mitigated by (1) better bonding of rails, (2) track insulation, (3) reinforcement of rail conductivity, (4) increasing the number of power substations, (5) interconnection of tracks, (6) insulated negative feeders, (7) three-wire system, (8) reversed polarity trolley system, (9) double-contact conductor system, (10) a-c system, (11) safe location of pipes with respect to rails, (12) insulation of pipes and cables, (13) insulating joints in pipes, and other methods.

The grounding of electric circuits in buildings on water pipes for the safety of persons using the electrical equipment is widely practiced. No deleterious effects seem to result from this practice provided the connections do not result in the continuous or intermittent flow of stray currents through the pipes under normal operating conditions.

There are two general methods for minimizing or preventing the corrosion of metals in contact with water or moisture: (1) the protection of the metal itself by the application of a protective lining or covering, and (2) the treatment of the water to remove the agents contributing to corrosion or to form precipitates that act as protective films. For metal tanks, pipes, and structures in contact with the air, water, or soil,

[1] LOGAN and TAYLOR, *U.S. Bur. Standards, Jour. Research*, **12**, 119–145, 1934.
[2] SPELLER, *op. cit.*
[3] *Ibid.*, pp. 601–609.

protective coatings are used. They may be in the form of paints, bituminous coatings, enamels, alloys, zinc coatings, rubber linings, cement linings, and concrete encasement. The relative merits of these coatings are fully discussed by Speller.[1] The corrective treatment of water is discussed in Sec. 21.

Cathodic protection[2] is now being used for the interior wetted surface of steel water tanks. In this process, a direct current is passed continuously from anodes immersed in the water to the tank plates as cathodes.

Rust tubercles in iron pipe lines may be partly removed by pulling cleaners through the pipes or by water-driven turbine cleaners. A portion or all of the pipe line to be cleaned must be taken out of service during the cleaning operation. Pipe lines have been restored to almost their original capacity by cleaning, but corrosion proceeds more rapidly after cleaning owing to the fact that the surface of the metal is exposed unevenly. Loose rust, growths, and sediment in pipes may be partly removed by flushing through fire hydrants or blowoffs.

Pipe Linings and Protective Coatings. The A.W.W.A. specifications (1908) for bell-and-spigot cast-iron pipe and fittings require a coating inside and out of coal-tar pitch varnish. Castings are heated to 300F and then dipped for at least 5 min into vats containing the pitch and a small amount of oil also at about 300F. A coating of about 1/32 in. is thus produced for a single dip. Asphalt is sometimes used in place of coal-tar pitch for dip coatings. Coal-tar pitch coatings are more durable than asphalt coatings for underground water pipes after the pipes are in service, but they are more subject to damage in shipping and handling the pipes because the temperature range between the brittle and softening points of coal-tar pitch usually does not exceed 45F, whereas asphalts have a range of 120 to 130F. Nearly all bell-and-spigot water pipe is furnished with dip coatings unless more effective coatings are specified. Flanged pipe is commonly furnished without coating.

Two types of interior linings for cast-iron pipe are being widely used for corrosion protection: *viz.*, special bituminous linings with inert fillers and cement linings. A well-known example of the former type is Bitumastic Enamel,[3] which consists of a coal-tar pitch base and finely divided mineral filler. When the enamel is applied at the foundry, the pipe is first coated by the regular dip method and after cooling is placed in a horizontal turning rig and revolved at high speed. A trough carrying the hot enamel is then inserted into the pipe and turned over, the coating being thus deposited on the pipe surface to a thickness of about 3/32 in. The susceptibility of this coal-tar pitch coating to temperature changes is somewhat modified to suit particular climatic conditions by the use of a fluxing oil. For lining pipes in the field with Bitumastic, a portable rolling rig may be used; for reconditioning old pipes in place, the coating may be brushed on if the pipe is 30 in. or larger in size. In both cases, the pipe is first cleaned thoroughly, dried, and then primed by brushing on a cold bituminous solution. Fittings are lined by hand brushing. The temperature of molten lead used for joints should be kept less than 800F to prevent injury to Bitumastic lining.

The American Standards Association standard specifications (1939) for cement lining for cast-iron pipe call for minimum thicknesses of 1/8 in. for pipes 4 to 12 in. in diameter, 3/16 in. for pipes 14 to 24 in. in size, and 1/4 in. for pipes 30 to 48 in. in size. A satisfactory mortar may be obtained with 1 part Portland cement to 1 part sand by volume. Pipe to be lined with cement should not be precoated on the inside with tar or asphalt and should be cleaned thoroughly. New pipe is cement lined at the foundry by the centrifugal process, the method being similar to that used in applying

[1] SPELLER, *op. cit.*
[2] HAMILTON, *Water Works and Sewerage*, November, 1939, p. 433.
[3] Wailes Dove-Hermiston Corp., New York. See STUART, *Jour. A.W.W.A.*, **25**, 1431, 1933.

Bitumastic lining. Fittings are lined by hand brushing. When water is first applied to cement-lined pipe, some free lime and other materials are leached out of the cement and cause hardness and alkalinity in the water. This effect usually lasts for only a few days but may continue if the water is soft and corrosive and result in the slow disintegration of the lining. Smaller pipes may be given hot tar dip coatings after the cement lining has set, but for larger pipes the expansion due to the heat may result in breaking the bond between pipe and cement. Bituminous paint may be used with larger pipes.

Cement lining of small pipes in place (up to 16 in.) may be accomplished by the Tate process,[1] in which mortar dumped into a section of the pipe is shaped into lining by means of a mandrel which is pulled through the pipe. Pipes larger than 16 in. may be lined in place with cement by special machines. Pipes 3 to 24 in. in diameter may be lined in place after cleaning with a bituminous lining applied electrolytically (Eric process).

Other Pipe Materials. Steel is widely used for pipe lines larger than 30 in., particularly for supply lines and other lines in distribution systems that are not too frequently interconnected with smaller distributors. Both riveted and welded pipe are used. Because of the thinness of the plate used, it is particularly important to protect steel pipe against corrosion with interior lining and covering. Galvanized-steel and wrought-iron pipes with screw joints are widely used for small distributors (less than 4 in.) upon which there are no hydrants. Galvanized pipe stands up well until the zinc coating is perforated, after which corrosion is very rapid.

Reinforced-concrete pipe made by centrifugal methods or by casting in forms is sometimes used for low heads. A new type of reinforced-concrete pressure pipe is now being widely used in supply lines for medium pressures and larger sizes in competition with cast-iron and steel. This pipe consists of a thin steel cylinder lined with concrete on the inside and tension-wrapped on the outside with high-tensile-strength wire which is protected from corrosion with a mortar coating. Reinforced-concrete pipe should be given a bituminous coating on the interior surface to retard the leaching of free lime from the cement. Woodstave pipe is used for both supply lines and distribution mains, but its use in distribution systems is diminishing. The steel bands on the outside of wood-stave pipe are particularly subject to soil corrosion and must be adequately protected. Taps to pipe made of the foregoing materials for service connections are not so readily made as with cast-iron pipe, and the taps are subject to more trouble from leakage. Service connections to small galvanized pipe are made with tees.

Asbestos-cement pipe made from a mixture of asbestos fiber and Portland cement in the approximate proportions of 15 and 85 per cent by weight, respectively, is proving satisfactory for distribution systems. The American product, Transite pressure pipe[2] and Century[3] pipe are made in four classes corresponding to those used for centrifugal cast-iron pipe and in sizes from 2 to 36 in. The pipes are 13 ft long for sizes above 4½ in. and are made with plain ends. Figure 25 shows the type of coupling used. The thickness of the pipe is 1.5 to 3 times that of the corresponding class of centrifugal cast-iron pipe. Cast-iron fittings and gate valves are used, but, because of the extra thickness of asbestos cement pipe, special bells are required for use with the larger sizes of class 150 and heavier pipe.

Asbestos-cement pipe is lighter than cast iron, class 150 asbestos cement weighing 60 to 85 per cent of the weight of the corresponding class and size of cast-iron pipe (the percentage increasing with pipe size). The material is a nonconductor of electricity

[1] WIGGIN, *Jour. A.W.W.A.*, **27**, 1073, 1935.
[2] Johns-Manville Corporation, New York.
[3] Keasbey & Mattison Company, Ambler, Pa.

and is not subject to tuberculation but may collect iron and manganese oxides from waters heavily charged with iron or manganese. The material can be cut with a saw and other woodworking tools. Drilling and tapping for house services are readily accomplished, and the connections are relatively free from excessive leakage. Tar coating of asbestos-cement pipe is desirable to retard the leaching of free lime from the cement.

Capacity of Mains. The capacity of water mains is generally expressed in the United States in terms of the Hazen-Williams coefficient C. The 1935 Report[1] of the Committee on Pipe Line Friction Coefficients of the N.E.W.W.A. contains a comprehensive summary of existing information on this subject.

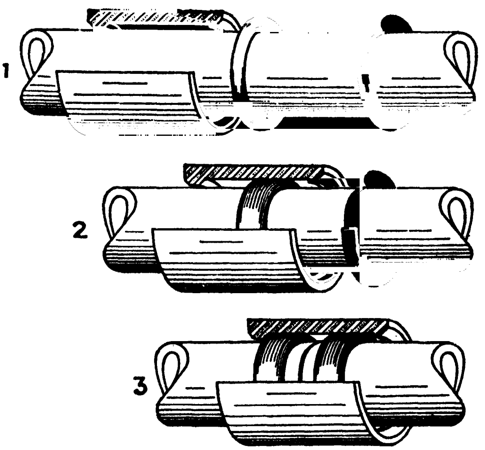


FIG. 25.—Simplex joint for Transite pressure pipe. (*Johns-Manville.*) Assembly of Simplex coupling: (1) starting position; (2) second position—one rubber ring compressed; (3) final position—both rubber rings compressed.

The average capacity of tar-coated cast-iron pipes of all sizes and their loss of capacity with age are shown in a general way in Fig. 26, taken from the Report. The effect of the pH value of the water upon the loss of capacity of tar-coated pipe is indicated by Table 11. The value of C adopted for the design of new tar-coated cast-iron pipe lines is 135 for mains 16 in. and larger and 125 for smaller mains which includes an allowance for friction due to tees, valves, bends, etc.

For cement lining applied centrifugally to cast-iron pipe 4 to 24 in. in diameter, the data[1] indicate an average value of C of 134 based on nominal diameter and 150 based on actual net diameter. Service tests indicate very little loss of capacity with age. The data[2] on new Bitumastic enamel, centrifugally applied, indicate values of C from 145 to 160 for supply lines 16 in. and larger and from 140 to 150 for distribution mains less than 16 in. in diameter. Bitumastic lining if properly applied is effective in sustaining hydraulic capacity, but experience indicates that it adheres better to steel than to cast-iron pipe, and for satisfactory adherence the metal surface must be thoroughly

TABLE 11[1]

pH Value of Water	Average Capacity Loss in Tar coated Cast-iron Pipe in 30 Years, per cent
8.0	30
7.5	35
7.0	45
6.5	60
6.0	85

[1] *Jour. N.E.W.W.A.*, **49**, 235, 1935.

clean. Tests on Transite[3] and other asbestos-cement pipe indicate values of C from 145 to 165. For design purposes, a value of C of 140 is recommended for Transite

[1] *Jour. N.E.W.W.A.*, **49**, 235, 1935.
[2] *Idem.*
[3] MCGINNIS, *Jour. A.W.W.A.*, **26**, 596, 1934.

pressure pipe. The capacity of asbestos-cement pipe is expected to remain constant, for no deterioration was observed in some lines which had been in service for 16 years.

Measurements of head losses due to 6- and 12-in. bends, tees, and crosses in distribution systems by Schoder and Vanderlip[1] indicate that the additional loss in excess of that due to straight pipe is less than the velocity head for a single fitting even for the worst case of flow division. The average loss for 90-deg bends is about 0.1 the velocity head for A.W.W.A. standard long-turn bends and about 0.27 for short-turn bends. The average loss for crosses and tees with deviated flow is about 0.51 times the entry velocity head for A.W.W.A. standard long-turn fillets and about 0.57 for short-turn fillets. An investigation by Ricketts[2] of the head losses between two points

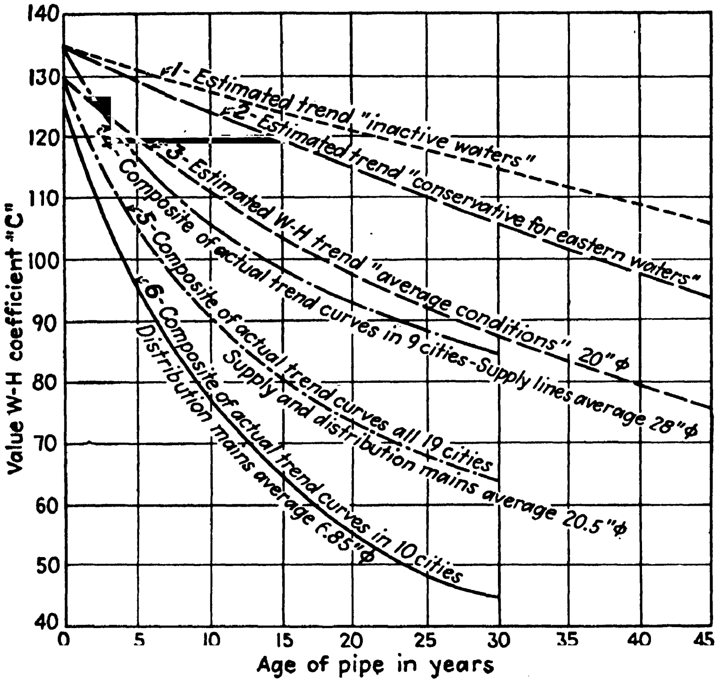


FIG. 26.—Summary of estimated and actual trends of age-coefficient relations for tar-coated cast-iron pipe. Curves 1 and 2 are based on data for steel pipe (*U.S. Dept. Agr., Bull.* 150, 1930). Curve 3 is based on William-Hazen tables for average soft unfiltered river water. Curves 4, 5 and 6 are composites of actual trends for tar-coated cast-iron pipe.

in a typical distribution system for both average and fire flows, in which the Schoder-Vanderlip head-loss values for tees and crosses was used, indicated that the loss due to fittings was about 1.3 per cent of the total loss and was about the same for both long and short types. A saving of 3 to 10 per cent of the cost of fittings was indicated by the use of short instead of standard A.W.W.A. tees and crosses. From a study by Wiggin[2] of the economics of bends, the long-turn bends were shown to be more economical because of the reduced friction loss.

Customers' Services. Service taps ½ to 2 in. in size may be made to mains under pressure by means of corporation tapping machines (Fig. 27*a*). With such a machine held tight against the main, the pipe is drilled and tapped with a special tool (Fig. 27*b*). The tool is then withdrawn above the flap valve, which is closed to prevent the

[1] *Cornell Univ. Eng. Exp. Station, Bull.* 20, September, 1935.
[2] *Idem.*

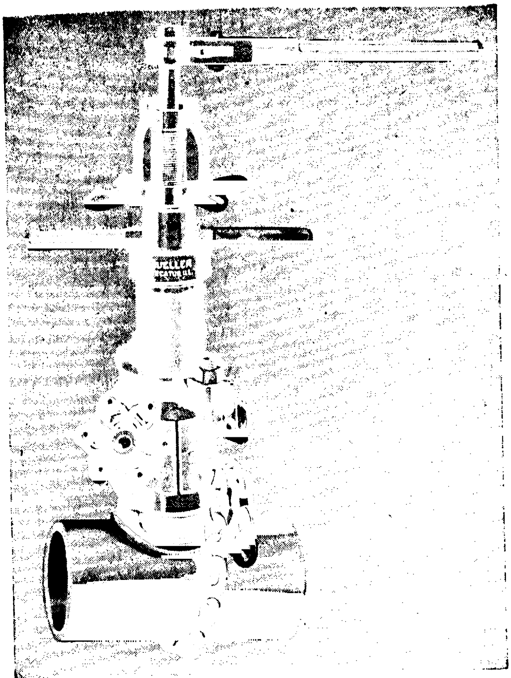


FIG. 27a.—Corporation tapping machine. (*Mueller.*)

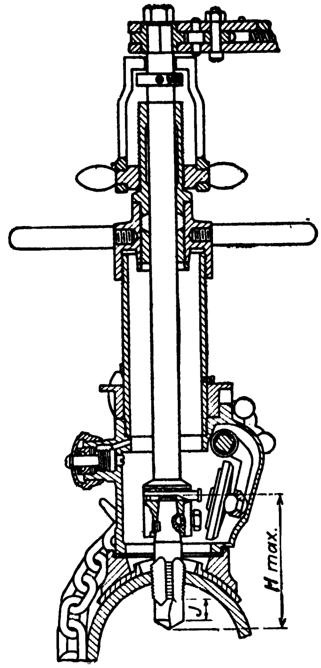


FIG. 27b.—Corporation tapping machine with tapping tool in use. (*Mueller.*)

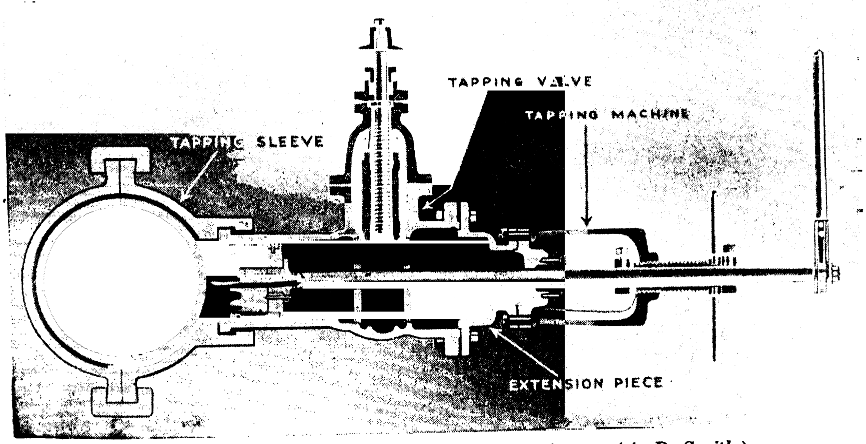


FIG. 28.—Tapping machine for large connections. (*A. P. Smith.*)

escape of water, after which the tool is removed from the cylinder by unscrewing the cap. A corporation cock is then substituted for the tool in the boring bar, which is inserted in the cylinder and the cap is screwed on. The flap valve is opened and the corporation stop is screwed into the pipe, after which the machine is removed.

Taps 2 to 8 in. in size may be made to mains under pressure with the tapping machine shown in Fig. 28. This machine operates through a tapping valve which is held to the pipe permanently by means of a tapping sleeve, the service pipe being attached to the flanged end of the valve. Similar machines are available for making

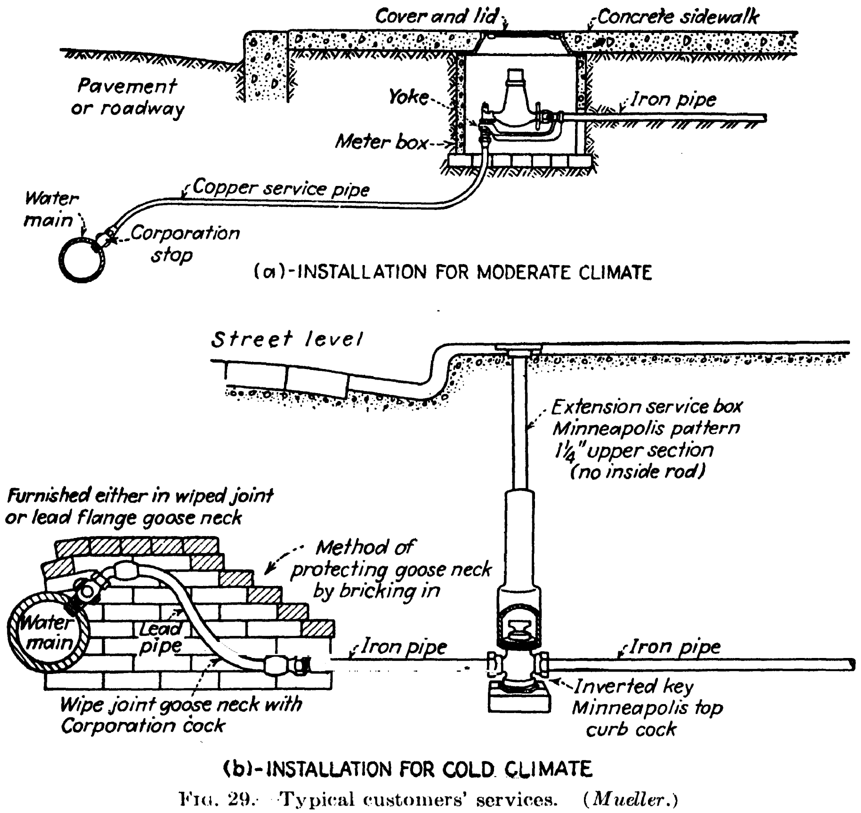


FIG. 29.—Typical customers' services. (*Mueller*.)

larger taps up to 42 in., the larger machines being power operated. Most service taps are made with corporation tapping machines. The larger taps are useful for large consumers, for fire-protection services, or for connecting new mains or hydrants into the system.

Small service lines consist of corporation cocks with or without lead goosenecks, the service pipe to the curb at which point an accessible curb cock is usually placed, the service pipe into the building, the meter, and a stop-waste valve. The meter is preferably placed at the curb (Fig. 29*a*), except in very cold climates or where cellars are available. Goosenecks are for the purpose of providing flexibility to relieve strains due to unequal settlement. They are omitted with lead pipe and sometimes with copper tubing. Services should be deep enough to avoid freezing in winter.

Small service pipes, ⅝ to 1¼ in., may be made of copper tubing, lead, galvanized wrought iron, galvanized steel, and iron-pipe-size brass or copper, the preference[1] in

[1] *Jour. A.W.W.A.*, **23**, 1435, 1931.

America being in the older named. For sizes 1¼ in. and larger, cast iron is also available, with and without lining. Service pipes may be laid by excavation for the full length of the pipe, by boring under the pavement, and by jacking or driving the pipe under the pavement. Lead and copper tubing cannot be jacked. According to Pracy,[1] the relative costs of services installed in San Francisco under pavements where driving was used for iron-size pipes was as follows for several different materials in the order of increasing cost: (1) galvanized steel, (2) I.P.S. brass, (3) I.P.S. copper, (4) copper tubing, and (5) AA lead. Where no pavement was encountered, the relative costs were as follows: (1) copper tubing, (2) galvanized steel, (3) I.P.S. brass, (4) I.P.S. copper, and (5) AA lead. The type of material best suited for a given case depends not only on first cost but also on the corrosive qualities of the water and the soil.

The size of services should be determined from the customers' demand, the available pressure, and the friction losses, but should preferably be not less than ¾ in.

TABLE 12.—PRESSURE LOSSES IN NEW SERVICES[1]
Values of k in $p = kQ^2$
p = pressure loss, psi for Q gpm

Size, in.	Corp. cocks	100 ft of service pipe				Curb cocks	Meter yokes		Compression stop-waste valves
		Copper	Lead	G.W. iron	Cast iron		Straight	Ram's-horn	
⅝	0.0166	......	0.40	....	.....	......	0.011	0.036	0.031
¾	0.0095	0.15	0.16	0.11	.....	0.0082	.....	0.013	0.020
1	0.0029	0.04	0.045	0.03	.....	0.0010	.....	.....	0.013
1¼	......	0.0085	0.0085	.....	0.01				
1½	......	0.0035	0.0035	.....	0.004				
2	......	0.0008	0.0008	.....	0.001				

[1] Compiled from data from Niemeyer and Bruhn, *Jour. A.W.W.A.*, **24**, 631, 1932, and other sources.

Corporation cocks and meters[2] should preferably be the same size as the service, except that meters must also be selected for accurate registration at low flows and this provision may call for a smaller meter. Friction losses in new services may be estimated by means of Table 12.

Meters. Cold-water service meters serve two main purposes, to give a basis for charging for water used and to restrain waste. For the second purpose, it is highly desirable that all services be metered, both to restrain all customers from wasting water and to furnish a basis for detecting waste due to leakage from mains by balancing meter readings against total inflow. Service meters register continuously the total flow that passes through. The important characteristics of meters are accuracy and sensitiveness, durability, low pressure loss, cost, and ease and economy of maintenance.

Meters are classified as displacement, current or velocity, compound and fire-service meters. Displacement meters are of the piston, rotary, and nutating-disk types and displace a fixed quantity of water with each stroke or revolution. They are made in sizes from ⅝ to 6 in. Displacement meters of the disk type (Fig. 30) are almost universally used on supply lines to dwellings. Current meters, operated by the flow of water through a propeller or water wheel, are made in sizes from 1½

[1] *Jour. A.W.W.A.*, **24**, 1819, 1932; and **25**, 444, 1933.
[2] *Jour. A.W.W.A.*, **23**, 1435, 1931.

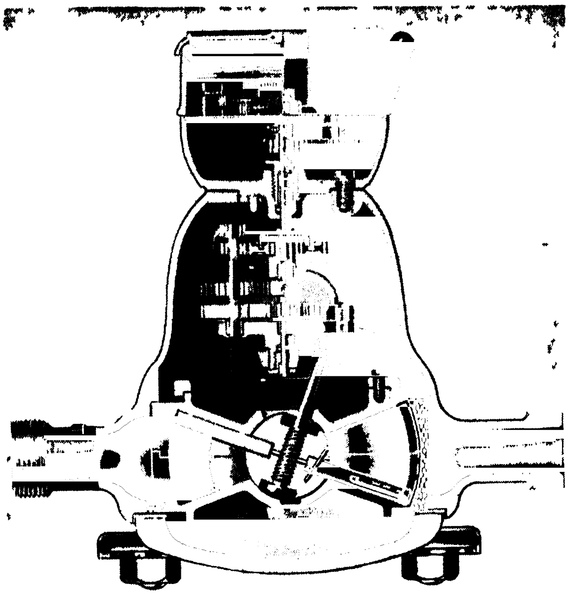


FIG. 30 —Disk meter with open gear train and frostproof housing (*A P Smith*)

to 20 in. When a low friction loss is required, current meters of the proportional type may be used In this type, only a portion of the water passes through the propeller, which is on a by-pass, the remainder flowing through the main waterway, which contains a friction ring between the by-pass connections Current meters are not sensitive to small flows. Compound meters (Fig 32) consist of a combination of a main-line meter of the current or displacement type for measuring large flows and a small by-pass meter of the displacement type for measuring small flows, together with an automatic valve mechanism for diverting the small flows through the by-pass meter This valve remains closed for low flows. Compound meters are made in sizes from $1\frac{1}{2}$ to 12 in Fire-service meters (Fig. 33) are compound meters having the main-line meter of the proportional type. They are the type required by the Fire Underwriters on private sprinkler and fire-hydrant connections and are made in sizes from 3 to 12 in.

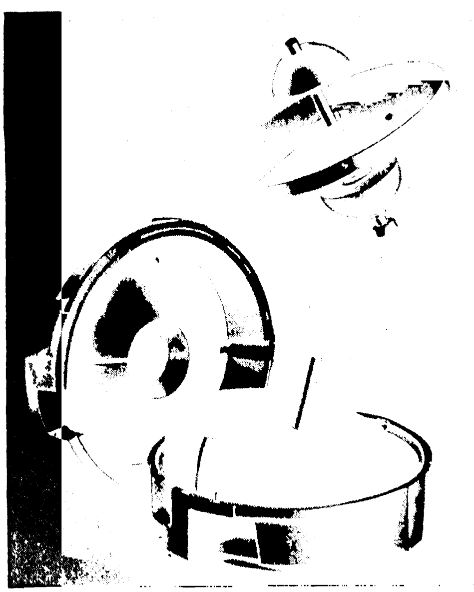


FIG. 31 —Details of measuring chamber and nutating disk. (*A P Smith*)

In the disk-type meter, a disk (Figs. 30 and 31) moves in a circular chamber owing to the passage of water through the chamber on both sides of the disk. The motion of the disk is described as nutating or similar to that of a spinning top in dying except that the disk as a whole does not revolve. It is kept from revolving by a vertical

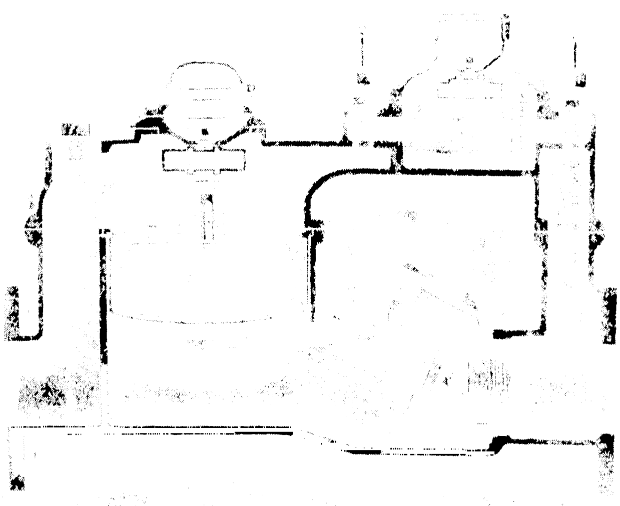


Fig. 32 Compound meter. Disk meter above lever valve, current meter at left. (*Hersey.*)

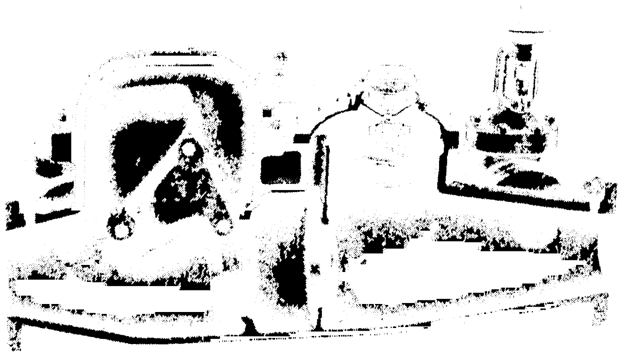


Fig. 33. Fire-service meter. Disk meter on by-pass, proportional meter and automatic valve on main line. (*Hersey.*)

diaphragm fastened to the chamber which passes through a slot in the disk. Water enters the chamber on one side of the diaphragm and passes around the disk ball and out on the other side of the diaphragm. Disks are made of hard rubber, usually in three pieces which are held together by a threaded spindle. The top of the spindle, revolving around a pivot, drives the gear train (which may be either open or of the oil-enclosed type), which in turn drives the register in the top of the meter Two

types of meter housings are furnished, split casing and frostproof. The bottom of a frostproof housing consists of a separate plate so bolted on that should the contents of the meter freeze the bottom will fail, with a minimum of damage to the working parts of the meter.

The specifications[1] for cold-water service meters of the A.W.W.A. and N.E.W.W.A. (1946) require meters of all types to be designed for an operating pressure of 150 psi. Bronze or nonferrous material is required for measuring chambers and cages of all meters and for outer cases of disk meters 2 in. and smaller. Larger meters may have cases of cast iron with suitable protective coating. Gear trains and strainers are to be of nonferrous metal and measuring disks and wheels of vulcanized rubber. Registration of new meters is required to be accurate within the normal test flow limits given in Table 13 to 1.5 per cent for disk meters and to 3 per cent for other types. The registered flow at minimum test flow must be not less than 95 per cent of the actual flow for all types. The pressure loss at the upper normal test flow limit should not exceed 15 psi for disk meters 1 in. and less in size, 20 psi for larger disk meters, current and compound meters, and 4 psi for fire-service meters.

Table 13.—Test Flow Limits for Cold-water Service Meters
(In gallons per minute)

Size	Disk meters		Current meters		Compound meters		Fire-service meters	
	Normal test flow limits	Minimum test flow	Normal test flow limits	Minimum test flow	Normal test flow limits	Minimum test flow	Normal test flow limits	Minimum test flow
5/8	1 to 20	1/4						
3/4	2 to 30	1/2						
1	3 to 50	3/4						
1½	5 to 100	1½	12 to 100	7	2 to 100	1/2		
2	8 to 160	2	16 to 160	10	2 to 160	1/2		
3	16 to 300	4	24 to 350	15	4 to 320	1	8 to 400	2
4	28 to 500	7	40 to 600	20	6 to 500	1½	8 to 700	2
6	48 to 1,000	12	80 to 1,400	30	10 to 1,000	3	16 to 1,600	4
8	....	....	144 to 2,500	50	16 to 1,600	4	28 to 2,800	7
10	....	....	224 to 3,800	75	32 to 2,300	8	48 to 4,400	12
12	....	....	320 to 5,800	100	32 to 3,100	14	48 to 6,400	12
16			400 to 11,500	150				

The sensitiveness and accuracy of meters at low flows are very important. Most well-made modern disk meters will retain their accuracy fairly well within the normal test flow limits for 15 to 20 years, but their sensitiveness to low flows falls off with age. Even new meters which conform to the preceding specifications for accuracy will fail to account for about 2 to 3 per cent of the total water passed because of under-registration at low flows. Periodic testing of meters and repair of those which need it are essential to minimize loss of revenue. The water works associations[2] recommend testing at intervals not exceeding 5 years for small meters and more frequently for meters larger than 1 in.

Tests of 824 5/8-in. meters of the open-gear type which had been in continuous service in Indianapolis[3] for periods up to about 20 years showed that the oldest group conformed to about 2 per cent accuracy within the normal test-flow limits. At 0.17 gpm, however, the registration varied from about 80 per cent of the total flow

[1] *Jour. A.W.W.A.*, December, 1941, April, 1946, and February, 1947.
[2] See "Water Works Practice," A.W.W.A., 1929, pp. 700, 707.
[3] Niemeyer, Lebo, and Horstman, *Jour. A.W.W.A.*, **26**, 819, 1934.

for the 5-year old group to only 41 per cent for the 20-year old group. The calculated unrecorded flow at the time these meters were removed was estimated at 5.4 per cent for the 5-year old group to 17.3 per cent for the oldest group. Comparative tests indicated the superiority of oil-enclosed gear trains over open gear trains in sustained sensitiveness to low flows. The Indianapolis system is fully metered, 96.9 per cent of the meters (in 1933) being ⅝-in. disk meters. It was estimated that about 6 per cent of the unaccounted-for water was due to underregistration of meters.

At Hartford,[1] where (in 1932) 87 per cent of the meters are ⅝- and ½-in. disk type and account for about 53 per cent of the water revenue, studies of the rate of flow to a number of one-, two-, and three-family houses indicated that 27 to 50 per cent of the total water was used at rates less than 1 gpm in houses with tank-type water closets. At rates less than 2 gpm, 38 to 78 per cent was used. In houses with flushometer closets, 10 to 33 per cent was used at rates less than 1 gpm and 20 to 55 per cent at rates less than 2 gpm. In tests of the accuracy of the meters in these houses both before and after repairing them, it was found that about 7 per cent of the water was unrecorded before repair and about 2.3 per cent after repair.

DISTRIBUTING RESERVOIRS

Classification and Purpose. Distributing reservoirs are used for storing water within or contiguous to the distribution area. Such storage may be designated as *elevated storage* if it serves to control the pressure table and *ground storage* if the water must be pumped into the mains. Distributing reservoirs are of two general types: (1) surface reservoirs which have little or no elevation above the ground and which are usually constructed of earth or masonry or a combination of earth and masonry; and (2) elevated reservoirs built entirely above the ground such as standpipes and elevated tanks which are usually of steel, reinforced concrete, or wood. Many surface reservoirs are built on hills and thus comprise elevated storage. Reservoirs are said to be "floating on the system" when the water enters and leaves by the same pipe.

Distribution reservoirs serve a variety of purposes as described below.

With regard to water quantity:

1. *Fire Storage.* The immediate availability of large quantities of water within the system for fire-fighting safeguards the community and results in lower fire insurance rates. Elevated storage is a more effective protection and results in lower insurance rates than ground storage.

2. *Storage for Fluctuating Demand.* Reservoirs are filling when the rate of pumping or filtration exceeds the demand rate and are emptying when the reverse occurs. This action permits pumps and treatment plants to operate at constant rates throughout any one day and thereby allows the use of pumps and treatment plants of less capacity. Filtered water reservoirs come within this classification if the discharge from them fluctuates with the demand.

3. *Emergency Storage.* The storage of sufficient water within the system gives protection against the failure of a supply conduit or intake delivering water to the system from a distant source.

With regard to pressures:

4. *Equalizing Pressures in Distribution System.* Reduction of the amount of fluctuation in pressure at points in the system due to fluctuating demand results in improved service to consumers and better pressures to fire hydrants.

5. *Raising Pressures at Remote Points.* Location of elevated storage near points distant from pumping stations or main supply results in improved pressures during periods of peak demand. The same improvement may be accomplished with ground storage and booster pumps.

[1] GRISWOLD and GENTNER, *Jour. N.E.W.W.A.*, **46**, 288, 1932.

6. *Equalizing Heads on Pumps.* Location of elevated storage near pumping plants results in more uniform pumping heads and permits the selection of pumps for and their operation at highest efficiency.

Distributing reservoirs are built with and without covers. In order to prevent the contamination of the water from dust, fumes, bird droppings, algae growth, and other causes, it is highly desirable that distribution reservoirs be covered. This precaution seems particularly appropriate for waters that have already been purified at considerable expense. Elevated tanks and standpipes are usually covered. Many small surface reservoirs are provided with concrete or wood covers, but many large surface reservoirs built in natural depressions at comparatively small expense are too large to be covered economically.

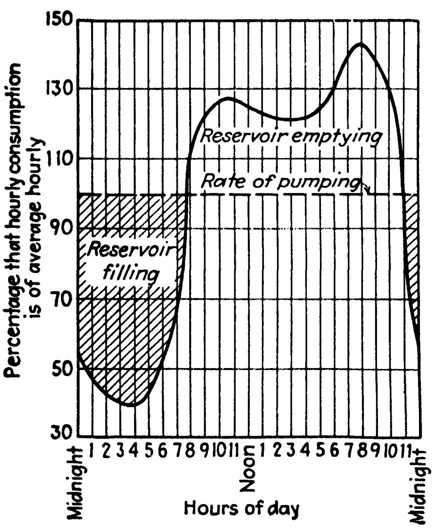


FIG. 34.—Storage for fluctuating demand. Constant pumping rate throughout the 24 hr.

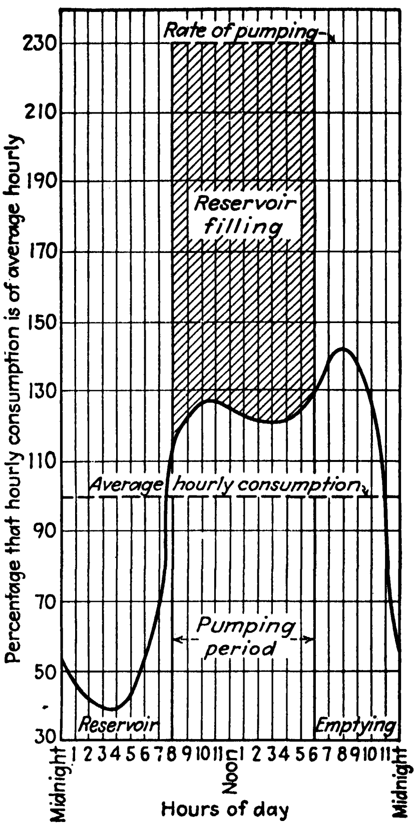


FIG. 35.—Storage for fluctuating demand. One-shift pumping at constant rate from 8 A.M. to 6 P.M.

Capacity and Location. The capacity required for distribution reservoirs is determined from their functions, 1, 2, and 3, above. The fire storage may be estimated from the population in accordance with the Underwriters' requirements (page 850, Sec. 19). The storage required for fluctuating demand should be determined from the fluctuations on the maximum day and the proposed period of pumping or filtration. Wherever possible, filter plants should be operated throughout the 24 hr in order to secure uniformity of results, but since this requires several shifts it is not economically feasible in small towns.

The storage required for hourly demand fluctuations is the maximum cumulative difference between the amount of water pumped or filtered and the water used. It may be computed or determined graphically from either a mass or rate curve. For example, in the rate curve illustrated by Fig. 34, the shaded area corresponds to the

storage required for a constant 24-hr rate of pumping. In this case, the storage is 17.5 per cent of the daily demand. Either area between the two rate curves, *i.e.*, above or below the pumping rate curve, may be used. Figure 35 shows the same demand curve and the storage required for one pumping shift from 8 A.M. to 6 P.M., the storage being 48 per cent of the daily demand for this case.

The emergency storage should be determined from the estimated time required to repair the supply works that have been damaged and to place them back into service. The amount will vary greatly with the length and size of the conduit and the means by which and the extent to which the works are subject to damage. Such storage may vary from a few hours' supply to several months' supply.

The total amount of distribution storage required may be estimated from a reasonable combination of the three classes of storage, *viz.*, fire, fluctuating demand, and emergency. A major fire may readily occur on a day of large demand, but it is quite unlikely that emergency storage will be required at the same time. If the conditions are such that the required emergency storage is very large in comparison to the sum of the other two classes, the latter may be neglected safely.

The location of storage may be determined by the function of the reservoirs, the available sites, or both. Storage for the control of pressures should be elevated, and the location of reservoirs for this function should be within or near the regions where pressure improvement is desired. If there are hills in the proper location, surface reservoirs may be used, otherwise elevated tanks are required. Elevated storage is more effective if distributed strategically among a number of reservoirs, but the cost usually is greater than for a single reservoir. When a number of reservoirs are used, the capacity of each should be determined by the demand of the district it is intended to serve.

Surface Reservoirs. Surface reservoirs are usually constructed partly by excavation and partly by building up of embankment. The most economical sites are usually those which require a minimum of excavation and embankment, such as the Eden Park Reservoir site at Cincinnati, which is a natural valley with a dam across it, or the Loudonville Reservoir at Albany, which consists of three basins built in natural depressions of glacial origin. An economical design for a given site is usually one in which the volume of excavated material suitable for embankment balances the volume of embankment. The bottom of a reservoir should be on firm virgin soil or in cut. The sides of a reservoir may be of earth or of masonry walls usually surrounded by embankment. In the former case, as much of the sides as feasible should be in cut. Slopes of cuts depend upon the material but should generally be not steeper than 1 to 1½. Sides constructed of earth embankment should be well compacted to minimize settlement which is particularly important if the basin is to be lined. A satisfactory method is to place the fill in 6- to 12-in. layers, each wetted and compacted by rolling. Interior slopes of embankments should preferably be 1 to 2 or flatter. Embankment placed around structurally independent masonry walls does not require careful compacting, for settlement is of less importance. For more detail on methods of construction of earth embankment, see Sec. 5.

If it is economically feasible, surface reservoirs should be lined to protect the quality of the water and to prevent leakage or inflow of ground water. Reinforced concrete is invariably used for lining in new construction. Bottom linings consist of reinforced-concrete slabs 1½ to 12 in. thick, usually provided with contraction joints to care for shrinkage and expansion of the concrete. If the side walls of the basin are of earth, the interior slopes may be lined in the same manner; but particular care should be taken with the joints on embankment slopes. Figure 36[1] shows some typical joints used with concrete basin linings. Such joints should permit of free

[1] SMITH, C. A., *Water Works and Sewerage*, February, 1931, p. 37.

movement of the slabs without excessive leakage. Figure 37[1] shows the type of joint used in basin C of the Loudonville Reservoir where prevention of leakage was of paramount importance. In this basin, slabs are 22½ by 50 ft and 8 in. thick, reinforced near the center. In order to minimize leakage, lining in some basins is

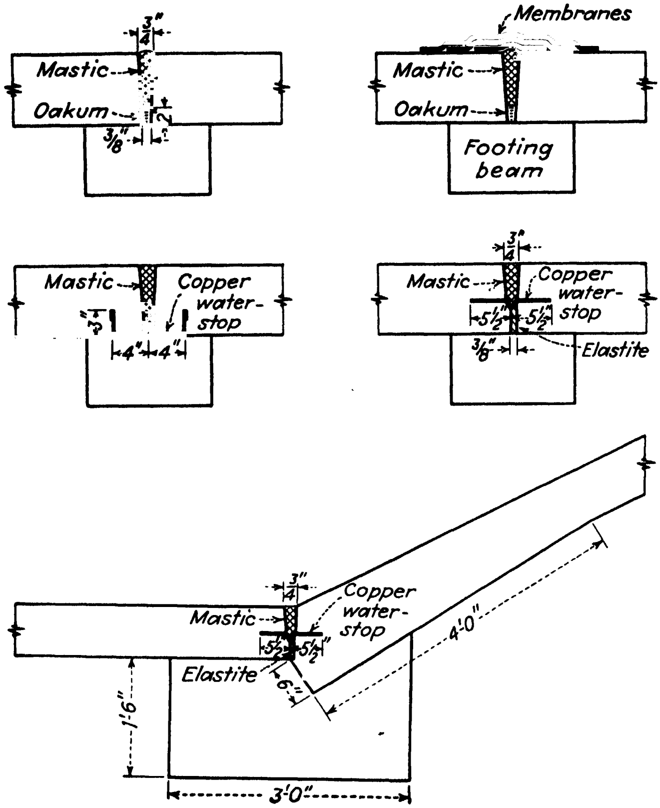


FIG. 36.—Typical joints for concrete linings of surface reservoirs. (*C. A. Smith.*)

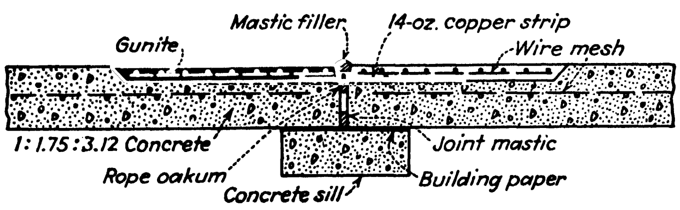


FIG. 37.—Leak-resistant joint for concrete lining.

composed of two or more layers of concrete with roofing felt in between. Linings should be placed upon firm or well-compacted soil or gravel, so that it will not be necessary for the slabs to take bending loads. Reinforcement, usually of wire mesh, 0.2 to 0.4 per cent of the cross-sectional area of the concrete is used to reduce shrinkage cracks. In some cases, the entire lining is composed of gunite 1½ to 3 in. thick placed around wire-mesh reinforcement.

[1] SMITH, B. L., W.P.A. Publication, 1937.

Concrete side walls may be designed as cantilever retaining walls, counterforted walls, vertical slabs supported top and bottom when a concrete roof is used, and as ring-tension cylindrical walls for small circular reservoirs. Figure 38[1] is an example of a circular covered reservoir in which the wall is designed as a vertical cantilever retaining wall. In ring-tension reservoirs, sufficient horizontal steel is placed in the circular wall to take the entire water load in tension. Since the concrete cannot take tension, it tends to crack and is thus subject to leakage. In some cases, as in the Hewett system, the steel is prestressed after the concrete has set, a sufficient amount of compression being thrown into the concrete so that it will remain in compression when the water load is applied. There is some cantilever action[2] in ring-tension

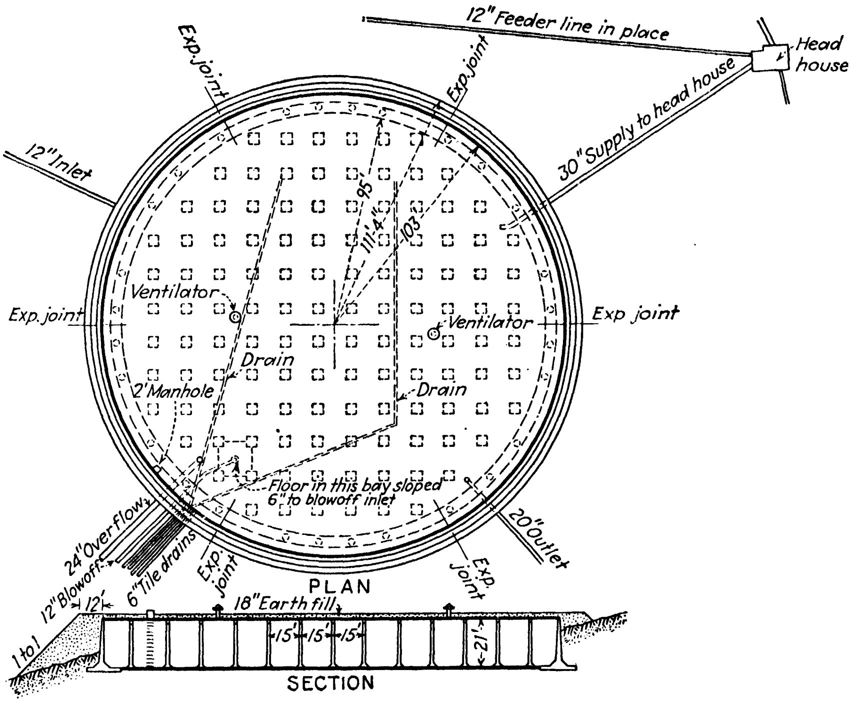


FIG. 38.—Equalizing reservoir at Provo, Utah. (*Ullrich.*)

walls because of the restraint at the bottom. In some cases where embankment is placed against concrete reservoir walls, the passive earth pressure is relied upon to assist the wall in carrying the water load. This practice may be unsafe owing to the possibility of shrinkage of the embankment away from the wall or of its subsequent removal for placing pipe lines or for other causes. Walls should be designed strong enough to withstand full water pressure without supporting embankment and full embankment pressure with the reservoir empty.

Provisions should be made to drain off water that leaks through linings and to prevent the ground-water table from rising above the reservoir bottom; otherwise the lining may be ruptured by the upward water pressure when the basin is emptied. To accomplish this purpose, open drains are usually placed under the lining at the bottom of side slopes and at intervals under the bottom lining, preferably near and

[1] ULLRICH, *Eng. News-Record*, **109**, 63, 1932.
[2] See CRIST, *Jour. A.W.W.A.*, **26**, 39, 1934.

parallel to joints. Drain pipes should be surrounded by gravel or crushed stone, and in some cases it is desirable to place the side slope lining upon a layer of well-compacted gravel.

Surface-reservoir covers are now generally made of reinforced concrete except that wood roofs are sometimes used where funds are scarce. Flat-slab construction such as is shown in Fig. 38 is widely used and is economical. It is desirable that concrete roofs be covered with a layer of earth for the protection of the concrete and to equalize the temperature of the water in the basin. It is also desirable that reservoirs be constructed in two or more parts so that repairs may be made to one part without taking the entire reservoir out of service. Inlet and outlet pipes should be so arranged that the water circulates through the reservoir, particularly if the basin is large.

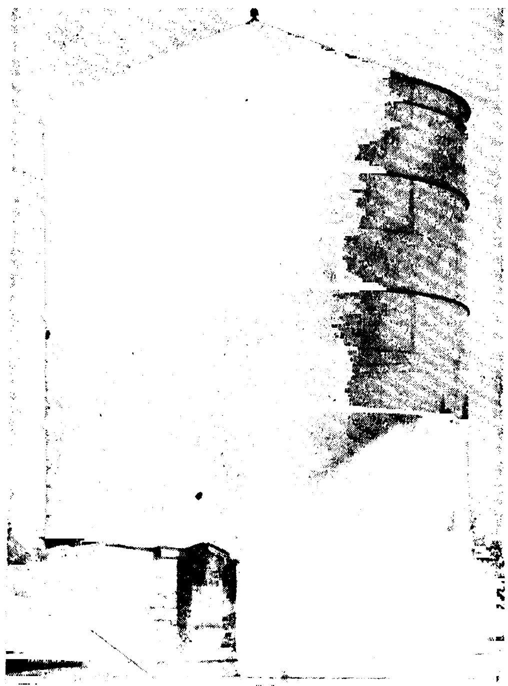


FIG. 39.—Standpipe at Burlington, Iowa. (*Pittsburgh-Des Moines Steel Company.*)

The proper depth for a reservoir depends upon its function, the site, and the cost. Reservoirs for pressure control should be shallow in order to minimize fluctuations in pressure. According to Turneaure and Russell,[1] the economic depth of a surface reservoir based upon first cost varies approximately as the fourth root of the capacity. In practice, depths vary from 10 to about 35 ft, usually increasing with the capacity. According to C. A. Smith,[2] the cost per million gallons for embankment concrete-lined reservoirs with wood roofs ranges from about $15,000 for capacities from ½ to 1 million gal to about $5,500 for capacities from 10 to 15 million gal. Reservoirs with reinforced-concrete walls and roofs cost one and one-half to three times these values.

Standpipes and Elevated Tanks. Circular reservoirs built above the ground surface to heights exceeding 25 or 30 ft are called standpipes (Fig. 39). They are built of either steel or reinforced concrete, steel being more generally used because

[1] "Public Water Supplies," John Wiley & Sons, Inc., 1940, p. 619.
[2] SMITH, C. A., *op. cit.*

of the difficulty of securing watertightness in concrete shells with relatively high heads. The water in the bottom of a standpipe used for elevated storage is usually not available as storage due to the fact that distribution-system pressure requirements limit the allowable fluctuations in water level to 25 or 30 ft. The bottom part of the standpipe shell therefore serves only to support the upper useful portion of the standpipe; and standpipes become uneconomical when their height is such that the tower

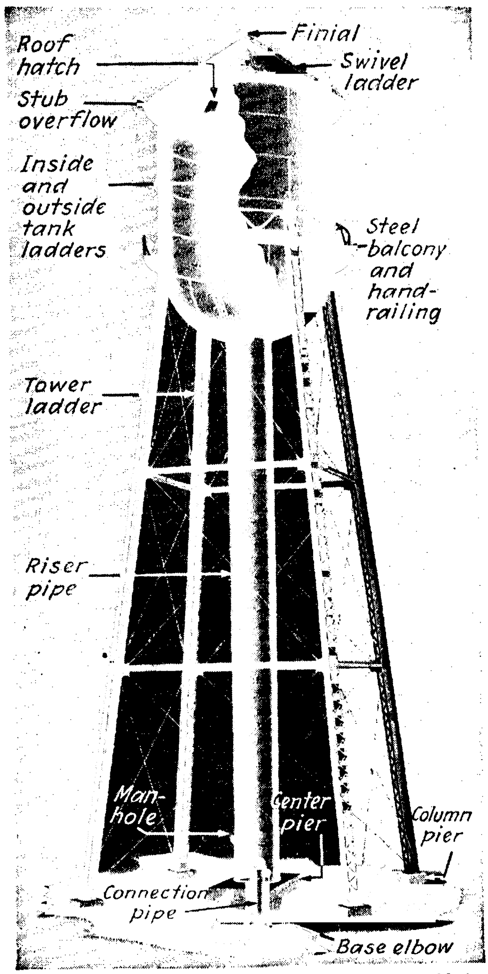


Fig. 40.—Component parts of elevated tank. (*Pittsburgh-Des Moines Steel Company.*)

structure of an elevated tank becomes cheaper than the supporting part of the standpipe shell.

Steel is the most satisfactory material for elevated tanks. The component parts of an ordinary steel tank and tower are shown in Fig. 40. Elevated tanks are widely used for elevated storage with capacities up to 2,000,000 gal. Recent developments in design are illustrated in Fig. 41.[1] Type I has a hemispherical bottom, a cylindrical

[1] Compiled largely from private communication of H. C. Boardman, Chicago Bridge and Iron Works, 1936. See also Young, *Jour. A.W.W.A.*, **26**, 1046, 1934.

shell, a cone roof, and a small riser pipe protected by a frost case. The bottom is suspended and connected by an expansion joint to the riser pipe, the water load being carried entirely by the tower. Figure 40 is the same type tank except that a large-diameter riser is used to avoid freezing, the riser being riveted to the bottom assists the tower to carry the water load. This type is limited to a capacity of about 60,000 gal for a 25-ft range of head. Type II has a hemiellipsoidal bottom, and type III has a hemiellipsoidal bottom and a dome roof partly used as storage space. Dome

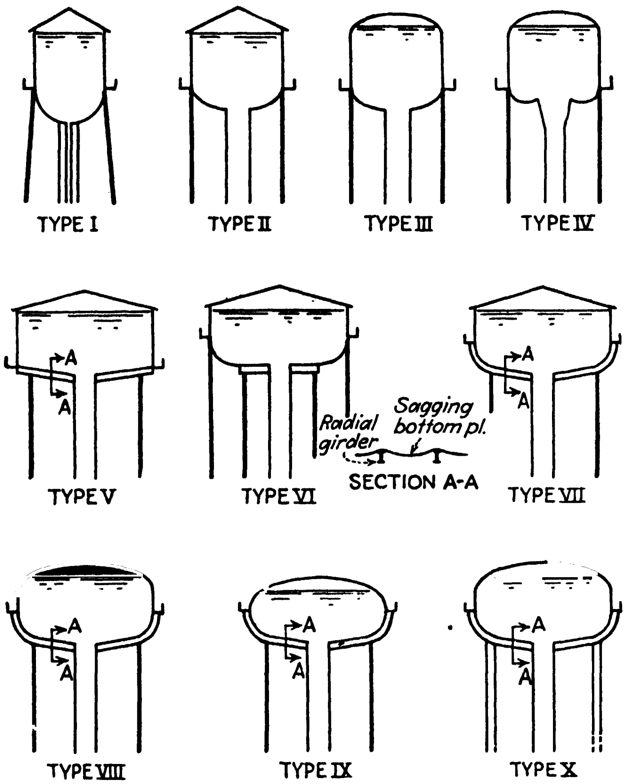


Fig. 41.—Types of elevated tanks.

roofs of large diameter are self-supporting, whereas large cone roofs are not. Types II and III are limited to a capacity of about 200,000 gal for a 25-ft drawing depth. Type IV has a toroidal bottom suspended between the shell and the riser which carries a large part of the water load. In this type, a more constant and more easily calculated distribution of the water load between the tower and the riser is obtained. For large capacities and small range of head, radial girders are used as in types V to X. Type V has radial girders, supported by tower columns and riser pipe, and a radial cone bottom. Type VI has radial girders, supported by tower columns and riser, which in turn support a flat bottom on a grillage, and a toroidal bottom suspended between the shell and the flat bottom. Type VII is the same as type V except that the bottom turns up to make a tangent connection to the cylindrical shell. Type VIII is the same as type VII except that it has a dome roof partly used for storage. Type IX is the same as type VIII except that the cylindrical shell is omitted. In type X,

tubular columns have been used instead of rolled sections. The improved appearance due to the use of curved plates and tubular columns may be noted from Fig. 42. Figure 43 is an illustration of a tank of type VI.

The welding of tanks and standpipes instead of riveting is growing in favor. It is common practice with some builders to weld the risers, girders, bottoms, and roofs of all tanks with radial girder bottoms and to rivet only the cylindrical part of the shell and the tower connections.

The Standard Specifications[1] of the A.W.W.A. for riveted-steel standpipes and elevated tanks give approved methods of design for ordinary loads, materials, and working stresses, ordinary loading conditions, and other details. Many elevated

FIG. 42.—500,000-gal elevated tank at Decatur, Ga. (*Chicago Bridge and Iron Works.*)

tanks in regions subject to earthquakes have collapsed. According to Ruge,[2] the commonly used statical method of design against earthquakes cannot be applied safely to elevated tank structures. A new type of tower is suggested embodying increased flexibility by means of springs in the diagonals. The Specifications for Painting Standpipes of the N.E.W.W.A.[3] require that mill scale and rust be removed from the steel after erection, preferably by sandblasting, and that three coats of paint be applied inside and out. Several formulas of suitable paints are given. Both linseed oil and bituminous paints may be used. Aluminum paint is widely used for the outside of elevated tanks and standpipes.

Appurtenances and Regulating Devices. Distributing reservoirs are usually provided with automatic altitude valves which shut when the water level reaches a predetermined maximum. In case of failure of the altitude valve, an overflow should be provided which leads to a sewer or other suitable outlet. The capacity of the overflow should be equal to the maximum rate of inflow to the reservoir. Suitable

[1] *Jour. A.W.W.A.*, **27**, 1606, 1935.
[2] *Trans. A. S. C. E.*, **64**, 889, 1938.
[3] *Jour. N.E.W.W.A.*, **44**, 553, 1930.

valves should be provided on inlet and drain pipes so that reservoirs and tanks may be emptied for repairs or painting.

Many types of water-level indicators are available for reservoirs and tanks. The simplest type for elevated tanks consists of a float attached by wire or chain over a pulley to a telltale on the outside. Float gages are very accurate, except in very cold weather when ice on the water surface may interfere with their operation. To avoid this difficulty, the float may be surrounded with heating coils. Another type of water-level indicator not subject to sheet-ice troubles is the pneumatic gage

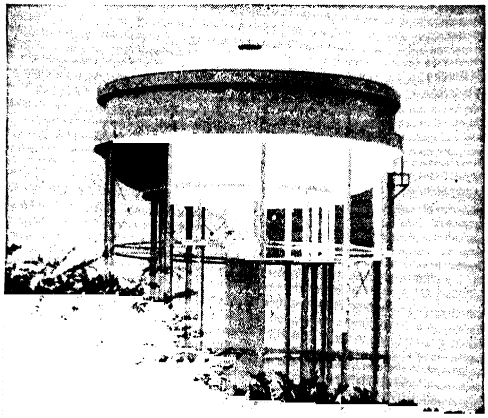


FIG. 43.—750,000-gal toroidal bottom tank at Dubuque, Iowa. (*Pittsburgh-Des Moines Steel Company.*)

consisting of a closed diaphragm box or bellows containing air which is immersed below the water surface to a fixed position and which transmits the water level in terms of air pressure through copper tubing to the gage. Both float- and pneumatic-operated gages may be equipped for remote recording, so that pumping-station operators may follow the performance of the reservoir. In some cases, telephone company wires have been used for transmitting signals[1] indicating the liquid level in a reservoir. It is sometimes desirable that altitude valves be wired for remote control[1] so that they may be operated either automatically or from the pump station. Remote recording of the position of the altitude valve in automatic operation is also helpful to pump-station operators.

[1] RUPARD, *Jour. A.W W.A.*, **24**, 1005, 1932.

SECTION 21

WATER TREATMENT

BY THOMAS R. CAMP

The treatment of water to improve its sanitary quality is called water purification. Purification consists primarily of the removal or destruction of bacteria and the removal of turbidity and color. It is accomplished by sedimentation, filtration, and disinfection; with or without pretreatment of the water by chemical coagulation. A complete plant for this purpose is known as a purification or filtration plant or, more broadly, a treatment plant. In modern treatment plants, many other processes not related to sanitation are applied to the improvement of water quality to meet the exacting requirements of the consumers. These processes include corrective treatment to prevent corrosion, removal of iron and manganese, removal of odors, and softening.

QUALITY OF WATER

Composition. The quality of natural water depends upon its content of impurities. Water itself is an associated liquid consisting of hydrol H_2O, dihydrol $(H_2O)_2$, trihydrol $(H_2O)_3$, and hydrogen and hydroxyl ions H^+ and OH^-. The impurities in water occur in three progressively finer states of subdivision, suspended, colloidal, and dissolved, which are of significance in that they influence the methods required for the removal of the impurities. The total amount of solid impurities in a water is obtained by the total solids[1] test, in which a sample of unfiltered water is evaporated and the residue weighed. The result is expressed in parts per million by weight, and it includes suspended, colloidal, and dissolved solids.

A suspension is a dispersion of solid particles that are large enough to be removed by filtration or settling. Such particles are macroscopic and contribute turbidity to the water. The concentration of suspended matter in water is measured by its turbidity[1] or by the suspended-solids[1] analysis. The turbidity of a water is its capacity for absorbing or scattering light and is measured by the concentration of fine silica in ppm which produces an equivalent effect. The suspended-solids content of a water is the concentration in ppm by weight of solid matter removed from the water by filtration through a Gooch crucible or filter paper. There is no definite relation between suspended solids and turbidity inasmuch as the latter is influenced by the size and character of suspended particles as well as their concentration by weight. The ratio of the suspended solids to the turbidity, called the *coefficient of fineness*, is a measure of the size of particles causing turbidity, the particle size increasing with the coefficient of fineness. The suspended-solids determination is widely used for concentrated suspensions such as sewage but is difficult to apply to relatively clear water and therefore is not commonly used in routine water analysis.

A colloid, or sol,[2] is a finely divided dispersion of one material called the *dispersed phase* in another called the *dispersion medium*. An aqueous suspensoid colloid is a

[1] For analytical methods, see "Standard Methods for the Examination of Water and Sewage," American Public Health Association, 1946.

[2] See KRUYT and VAN KLOOSTER, "Colloids," John Wiley & Sons, Inc., 1930; and WEISER, "Inorganic Colloid Chemistry," John Wiley & Sons, Inc., 1935.

water sol of solid particles that are too small to be removed effectively by ordinary filters and which are so small that they exhibit Brownian motion (*i.e.*, they diffuse) and that the electric charges on their surfaces are large enough in comparison with their mass to cause the particles to repel one another when they move within the sphere of action of each other's charges. The electric charge is due to the presence of adsorbed ions on the surface of the solid, and the sign of the charge is determined by the material of the particle and the pH value of the liquid. Neutral or acid materials such as silica, glass, and most organic particles tend to acquire negative charges in neutral water; whereas basic materials such as the metallic oxides Al_2O_3 and Fe_2O_3 tend to be positively charged. Many colloidal particles also adsorb water, and when the amount of adsorbed water is large as compared with the solid matter in the particle the colloid is called an *emulsoid.* Most of the properties of colloidal particles are due to their size. The upper limit of the size of colloidal particles varies widely with the character of the particle but is approximately 1μ ($1\mu = 1$ micron $= 0.001$ mm). The lower limit of size is approximately that of single molecules of the substance, or about $1\mu\mu$ ($1\mu\mu = 1$ millimicron $= 0.001\mu$).

Colloidal particles cannot be seen with the naked eye except with the aid of a Tyndall cone of light. In ordinary light, a suspensoid appears clear but is usually colored. Most of the color of water is due to the presence of colloidal particles, but some colloids such as silica are colorless. There is no convenient means in routine laboratory technique for measuring the amount of colloidal matter in water, but the color[1] test is an indication of the concentration of certain types of colloidal matter. The color of a water is the amount of platinum in platinum-cobalt color standards expressed in ppm required to match the strength of color of the water. In order to remove colloidal particles from water, they must first be combined into larger particles by coagulation, after which settling and filtration are effective. Some color may also be removed by adsorbents.

A solution is a molecular or ionic dispersion. Solids, liquids, and gases are dissolved in natural waters. Some substances, particularly organic compounds, remain in solution largely as molecular dispersions. Other substances, the strong electrolytes, ionize completely when they are dissolved in natural water. Practically all inorganic rocks or salts found in true solution in natural waters are fully ionized in the concentrations in which they normally occur. The concentration of total dissolved solids,[2] usually expressed in ppm, is obtained by weighing the residue after evaporation of the water from a filtered sample. The determination will include colloidal matter if present. Substances in true solution may be removed from water in a variety of ways. Some dissolved solids may be removed by adding a chemical that reacts with the soluble substance to form a precipitate, the precipitate being subsequently coagulated and removed by sedimentation and filtration. Some dissolved substances may be removed by an exchange process with zeolites and some by adsorption on activated carbon or other adsorbents. Still another method is aeration for the liberation of gases, volatile, and odoriferous substances and for the precipitation of iron.

Most of the substances that occur in natural waters are shown in Table 1. All these substances do not occur in a single water, and their concentration varies widely for different waters. Some substances of sanitary significance occurring in water because of artificial contamination, such as phenols and cyanides, are not specifically included in the table. Other substances, such as the secretions of certain microorganisms which cause odors but which occur in such minute amounts that they cannot be detected by chemical analysis, are not shown. Their presence and con-

[1] "Standard Methods for the Examination of Water and Sewage," *op. cit.*

[2] *Idem.*

TABLE 1.—SUBSTANCES OCCURRING IN NATURAL WATERS

Substance	Suspended	Colloidal	Dissolved		
			Not ionized	Positive ions	Negative ions
Of mineral origin	Clay Sand Other inorganic soils	Clay Silica, SiO_2 Iron Oxide, Fe_2O_3 Alumina, Al_2O_3 Manganic oxide MnO_2		Calcium, Ca^{++} (40) Magnesium, Mg^{++} (24.3) Sodium, Na^{+} (23) Potassium, K^{+} (39.1) Iron, Fe^{++} (55.8) Manganese, Mn^{++} (54.9) Hydrogen, H^{+} (1)	Bicarbonate, HCO_3^{-} (61) Sulphate, SO_4^{--} (96) Chloride, Cl^{-} (35.5) Nitrate, NO_3^{-} (62) Carbonate, CO_3^{--} (60) Hydroxyl, OH^{-} (17) Silicate $HSiO_3^{-}$ (77.1) Borate, $H_2BO_3^{-}$ (60.8) Phosphate, HPO_4^{--} (96) $H_2PO_4^{-}$ (97) Iodide, I^{-} (126.9) Fluoride, F^{-} (19)
Of organic origin	Organic soil (topsoil) Decomposing organic wastes	Vegetable coloring matter Organic wastes	Vegetable coloring matter Organic wastes Ammonia, NH_4OH Carbonic acid, H_2CO_3 Other organic acids	Ammonium, NH_4^{+} Hydrogen, H^{+} Other organic acids	Nitrate, NO_3^{-} Nitrite, NO_2^{-} Hydroxyl, OH^{-} Bicarbonate, HCO_3^{-}
Gases			Free carbon dioxide, CO_2 Oxygen, O_2 Nitrogen, N_2 Hydrogen, H_2 Hydrogen sulphide, H_2S Methane, CH_4 Sulphur dioxide, SO_2 Ammonia, NH_3 Odorivectors		
Living organisms	Fish life Algae, diatoms Minute animals	Bacteria, viruses Algae, diatoms Minute animals			

Figures in parentheses after the ions are the ionic weights.

centration is indicated by odor[1] measurements. Several other substances of some sanitary significance, such as lead, copper, zinc, and chlorine, Cl_2, which enter water because of treatment or from the pipes of the distribution system, are not shown since they do not usually occur in natural waters.

For methods of identification and measurement of concentration of the various impurities in water, see "Standard Methods for the Examination of Water and Sewage."[1] Living microorganisms are identified and counted in accordance with the microscopic and bacteriological examinations. Chemical substances are determined from the sanitary chemical and mineral examinations. A mineral analysis is preferably stated in terms of the concentration of the ions. The sum of the milliequivalents of basic radicles (positive ions) equals the sum of the milliequivalents of acid radicles (negative ions) in an accurate analysis. The milliequivalent of an ion is equal to its concentration in ppm divided by its combining weight. The combining weight is the molecular or ionic weight divided by the valence. The valence of an ion is indicated by the number of charges. The molal concentration of a substance is its concentration in ppm divided by its molecular or ionic weight and multiplied by 10^{-3}.

Ionization and pH Value. Water ionizes in accordance with the following reaction:

$$H_2O \rightleftarrows H^+ + OH^- \tag{1}$$

The reaction goes to the right very slightly, only one out of every 555 million H_2O molecules being dissociated in pure water. The law of mass action applied to the dissociation of water results in the following

$$\frac{[H^+][OH^-]}{[H_2O]} = K \tag{2}$$

This equation means that the product of the molal concentrations of the ions divided by the molal concentration of the undissociated water is a constant for a given temperature. K is known as an equilibrium constant, or, since reaction (1) is a dissociation, K is also known as a dissociation or ionization constant. Since the denominator is so large, its value is unaffected by changes in the concentration of the ions. Hence

$$[H^+][OH^-] \doteq K_w \tag{3}$$

K_w is called the *ionic product* of water, and its value is $10^{-14.0}$ at 25C. The value[2] of K_w decreases with the temperature in such a manner that the numerical value of the exponent of 10 is increased by about 0.18 for each 5°C decrease in temperature. It is common practice to assume that K_w is 10^{-14}, in which case both the hydrogen-ion concentration $[H^+]$ and the hydroxyl-ion concentration $[OH^-]$ are equal to 10^{-7} in pure water.

A substance that dissolves in water to yield H^+ or to add to itself OH^- is called an *acid*. Both these reactions, examples of which are shown below, increase the $[H^+]$ and decrease the $[OH^-]$.

$$HCl \rightleftarrows H^+ + Cl^- \tag{4}$$

$$CO_2 + OH^- \rightleftarrows HCO_3^- \tag{5}$$

NOTE: Reaction (5) may also be written $CO_2 + H_2O \rightleftarrows H_2CO_3 \rightleftarrows H^+ + HCO_3^-$.

Conversely, a substance that dissolves in water to yield OH^- or to add to itself H^+ is called a *base*, or *alkali*. Typical reactions are as follows:

[1] A.P.H.A., 1946.

[2] A.P.H.A., 1946. See LANDOLT-BÖRNSTEIN, "Physikalischchemische Tabellen," Springer, Berlin, 1923.

$$NaOH \rightleftarrows Na^+ + OH^- \quad (6)$$
$$Na_2CO_3 \leftrightarrows 2Na^+ + CO_3^{--} \quad (7)$$

and
$$CO_3^{--} + H^+ \rightleftarrows HCO_3^-$$

A substance that ionizes in a manner which does not affect the $[H^+]$ is called a *salt*. An example of such a reaction is

$$NaCl \rightleftarrows Na^+ + Cl^- \quad (8)$$

It follows that a solution is acid when $[H^+]$ is greater than 10^{-7} and basic when $[H^+]$ is less than 10^{-7}. For convenience, the exponent of 10 without sign, *i.e.*, the logarithm of the reciprocal of the hydrogen-ion concentration, is used to express the acidity or alkalinity of a solution. This figure is known as the *pH value*. A solution with a pH of 7.0 is neutral. Solutions with a pH value less than 7 are acid and greater than 7 are alkaline. Most natural waters have pH values between 6 and 8.

The pH value of dilute solutions of strong acids and bases in pure water may be computed from the amount of the material in solution, as strong electrolytes ionize completely in dilute solutions. For example, a $0.001N$ solution of HCl or any other strong acid in pure water has a $[H^+]$ of 10^{-3} and the pH is 3. The pH value of solutions of weak acids and bases in pure water cannot be computed directly from the amount of material in solution, for the compounds are not fully ionized. In this case, it is necessary to take account of the ionization constants of the material and water. If a strong acid is added to water containing HCO_3^-, the change in $[H^+]$ is not proportional to the amount of acid added. As the acid is added, the bicarbonate ionizes in accordance with reaction (5) and the OH^- liberated reacts with the H^+ from the acid to form water. A similar phenomenon occurs when a strong base is added to water containing HCO_3^-. In this case, as the OH^- is increased by the base, the bicarbonate ionizes as follows:

$$HCO_3^- \rightleftarrows H^+ + CO_3^{--} \quad (9)$$

The H^+ liberated in reaction (9) combines with the OH^- of the base to form water. This phenomenon is called *buffer action*, and a buffer solution is one in which the change in $[H^+]$ upon the addition of an acid or base is less than would be expected from the ionizing characteristics of the substance added. The buffer acts as a reservoir of acidity or alkalinity to oppose changes in the pH value. Weak acids or bases exhibit buffer action. The pH value of a buffered solution may be computed by the simultaneous solution of equations involving the concentrations of all the substances in the water, including the ions, and the ionization constants.

The pH of a solution may be determined by test[1] electrometrically or colorimetrically.

Alkalinity and Hardness. The alkalinity[1] of a water represents its content of OH^- or of other ions that combine with H^+ upon the addition of acid. The most important of these other ions is HCO_3^- which is usually present in considerable quantity. In alkaline waters, normal carbonate, CO_3^{--}, is a source of alkalinity since it forms HCO_3^{--} upon the addition of acid. Borates, silicates, and phosphates also cause alkalinity, but they are usually not present in natural waters in appreciable quantities. Alkalinity is measured by titration with acid. The amount of acid required to bring the water to pH 4 expressed in ppm of equivalent $CaCO_3$ is called the *total*, or *methyl orange, alkalinity*. If the water is alkaline, the amount of acid required to bring the pH down to 8 expressed in ppm of equivalent $CaCO_3$ is called the *phenolphthalein alkalinity*. Methyl orange and phenolphthalein are indicators that have significant color changes at about pH 4 and pH 8, respectively.

[1] "Standard Methods for the Examination of Water and Sewage," *op. cit.*

Alkalinity is commonly differentiated into three kinds: hydroxide or caustic alkalinity, carbonate alkalinity, and bicarbonate alkalinity. Caustic alkalinity is caused by OH^-, and it is negligible ($[OH^-] = 0.34$ ppm or 1 ppm as $CaCO_3$ at pH 9.3) at pH values less than 9.3. The relation between the carbonate and bicarbonate alkalinity and the relation between the bicarbonate and the free CO_2 are functions of the pH value. The mass-action relation for Eq. (9) is

$$\frac{[H^+][CO_3^{--}]}{[HCO_3^-]} = 4.69 \times 10^{-11} \text{ or } 10^{-10.32} \text{ at 25C} \tag{10}$$

and for Eq. (5) with the OH^- expressed in terms of H^+ it is

$$\frac{[H^+][HCO_3^-]}{[CO_2]} = 4.31 \times 10^{-7} \text{ or } 10^{-6.37} \text{ at 25C} \tag{11}$$

The numerical value of the exponent of 10 in the preceding ionization constants[1] is increased by about 0.01 for each degree centigrade decrease in temperature and

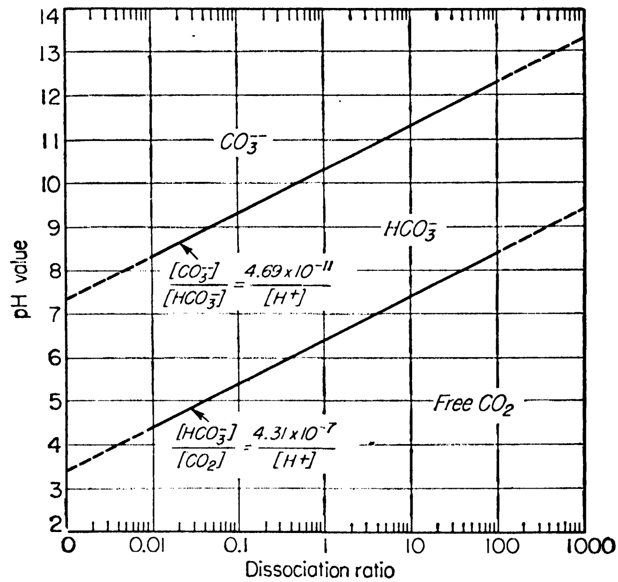


FIG. 1.—Bicarbonate ionization at 25C.

appears to be decreased slightly by increase in concentration of total dissolved solids. Langelier notes a decrease of about 0.01 in the exponent of the constant of Eq. (10) for each 100 ppm of total solids.

The significance of these equations is shown graphically on Fig. 1. The ratio of the CO_3^{--} alkalinity to the HCO_3^- alkalinity is twice the corresponding dissociation ratio, as shown on the figure, and the ratio of the HCO_3^- alkalinity to the free CO_2 expressed at $CaCO_3$ is one-half the corresponding dissociation ratio. It will be noted from the figure that, in titrating an alkaline water with acid, at pH 8 practically all the CO_3^{--} has been converted to HCO_3^- and at pH 4 practically all the HCO_3^- has been converted to free CO_2. A water must be caustic with a pH above 12 in order to convert practically all the HCO_3^- to CO_3^{--}.

[1] See LANDOLT-BÖRNSTEIN, *op. cit.* Also MACINNES and BELCHER, *Jour. Am. Chem. Soc.*, **55**, 2630, 1933, and LANGELIER, *Jour. A.W.W.A.*, **28**, 1500, 1936.

The hardness[1] of a water represents its content of metals which form precipitates under the normal conditions of use of the water. These include all the metals of Table 1 except Na^+ and K^+ whose salts are all soluble. Most of the hardness of a water is due to the presence of Ca^{++} and Mg^{++}, for these elements occur in substantial amounts. Hardness, like alkalinity, is expressed in ppm as $CaCO_3$. It is objectionable principally because of soap waste and boiler scale. The hardness metals form insoluble precipitates with soap, and a lather cannot be obtained until all the hardness has been so precipitated. The precipitated hard soap, moreover, may form stains upon fabrics being laundered, particularly if Fe and Mn are present. Boiler scale is obtained by the precipitation of the metals as salts (principally carbonates, sulphates, chlorides, and nitrates) owing to their increased concentration upon the evaporation of the water. If the hardness is less than 100 ppm, a water is generally considered soft, but for efficient boiler use and for certain industrial processing purposes waters of zero hardness are desirable. The hardness of many waters of the Middle West in the United States exceeds 500 ppm, whereas the hardness of many waters on the Eastern seaboard is less than 50 ppm. It has been demonstrated that the saving[2] in soap for home use alone accomplished by the softening of water with a hardness exceeding about 150 ppm is sufficient to pay for the cost of softening.

Sanitary Significance of Impurities. The impurities of greatest sanitary significance in water to be used for drinking purposes are the pathogenic bacteria and other pathogenic microorganisms. The most serious water-borne diseases are cholera in the Old World and typhoid fever in America, but other important human diseases, such as dysentery and diarrhea, are known to be water-borne, and still others may be. The isolation of the causative organism of these diseases from water is impractical in routine water examination. Because these diseases are of intestinal origin and the source of the germs in water is human excreta, the presence of sewage in water is evidence of the possibility of the presence of infectious organisms. The presence of ammonia and nitrites, and of chlorides in abnormal amounts, is tentative but not conclusive evidence of sewage pollution. The presence of the colon bacillus (*B. coli*, or *E. coli*) whose normal habitat is the alimentary canal of man and other mammals is the best evidence of sewage pollution.

The 1946 U.S. Public Health Service Drinking Water Standards[3] for water used for drinking and culinary purposes on interstate carriers, which standards have been adopted by many public health authorities and are widely used, require a *most probable number* (M.P.N.) of coliform bacteria not exceeding 1 per 100 ml of water for all samples collected in any one month. Other requirements of the U.S.P.H.S. Standards are as follows:

Physical characteristics:

1. Turbidity shall not exceed 10 ppm (silica scale).
2. Color shall not exceed 20 ppm (standard cobalt scale).
3. There should be no objectionable taste or odor.

Chemical characteristics:

1. Lead, Pb, shall not exceed 0.1 ppm.
2. Fluoride, F, shall not exceed 1.5 ppm.
3. Arsenic, As, shall not exceed 0.05 ppm.
4. Selenium, Se, shall not exceed 0.05 ppm.
5. Hexavalent chromium, Cr^{++++++}, shall not exceed 0.05 ppm.
6. Salts of barium, Ba, hexavalent chromium, Cr^{++++++}, heavy metal glucosides, or other substances with deleterious physiological effects shall not be added for water treatment purposes.
7. Copper, Cu, should not exceed 3.0 ppm.
8. Iron, Fe, and manganese, Mn, together should not exceed 0.3 ppm.
9. Magnesium, Mg, should not exceed 125 ppm.

[1] "Standard Methods for the Examination of Water and Sewage," *op. cit.*
[2] See HOOVER, "Water Supply and Treatment," National Lime Assoc., 1943.
[3] *Jour. A.W.W.A.* March, 1946, p. 361.

10. Zinc, Zn, should not exceed 15 ppm.
11. Chloride, Cl, should not exceed 250 ppm.
12. Sulfate, SO_4, should not exceed 250 ppm.
13. Phenolic compounds should not exceed 0.001 ppm in terms of phenol.
14. Total solids should not exceed 500 ppm, but may be permitted up to 1,000 ppm.
15. For chemically treated waters, the pH should not be greater than about 10.6, the normal carbonate (CO_3^{--}) alkalinity should not exceed 120 ppm as $CaCO_3$, and the total alkalinity should not exceed the hardness by more than 35 ppm as $CaCO_3$.

Many of the requirements of the U.S.P.H.S. Standards have no health significance although they are of aesthetic importance. The presence of too much Pb may result in lead poisoning. Lead is not present in natural waters but may enter the water by solution from lead services and plumbing systems if the water is corrosive to lead. Arsenic, selenium, and hexavalent chromium are all toxic, and their concentrations must be limited. The toxicity[1] of copper to man has been the subject of much discussion, but it now appears that Cu is not injurious up to concentrations of about 20 ppm. The taste of water becomes disagreeable when the Cu content reaches about 5 ppm. Zinc[2] appears to be safe in drinking water up to concentrations of about 40 ppm, but at that concentration it will impart a milky appearance and an astringent taste to the water. Too much $MgSO_4$[3] (Epsom salt) and Na_2SO_4 (Glauber's salt) in water produce laxative effects, NaCl and $NaNO_3$ tend to produce thirst, and carbonates and hydroxides tend to neutralize the acid of the stomach. Iron is beneficial to the health, but it is objectionable because of red water and stains. Manganese, is more objectionable than Fe because of stains and because of its interference with the orthotolidine test for residual chlorine. Its concentration should be limited to less than 0.1 ppm.

The presence of fluoride[4] in concentrations exceeding 0.5 ppm may result in mild endemic dental fluorosis (mottled enamel) in children, although about 1.5 or more ppm of F is required for severe cases. It has recently been found that fluoride in the drinking water is accompanied by low incidence of dental caries (tooth decay) in children, and many health officials are now advocating the addition of fluorides to drinking water up to about 1.0 ppm in regions where caries is prevalent. Because simple goiter has been shown to be due to a deficiency of iodine[5] in the thyroid gland, many attempts have been made to supply the deficiency in regions where this condition is endemic by adding NaI to the water supplies. Studies of the relationship of the incidence of goiter to the I content of drinking waters have given conflicting results, however, and it is probable that I in organic combination in foods where it is also more concentrated is more readily assimilated than I in water. Disinfection by means of ionic and colloidal silver[6] is growing for swimming-pool waters, but this method should not be used for drinking water until more is known about the effects upon the human body of Ag thus ingested.

TYPE AND CAPACITY OF PLANT

Slow and Rapid Sand Filtration. Slow sand filters, also known as English-type filters, are beds of sand 30 to 40 in. deep in concrete basins, each about 1 acre in area. They were first used about 1830. Slow sand filters are usually preceded by plain settling basins with no chemical coagulation. The water passes downward through the sand at rates of 2 to 8 mgd/acre. Filter runs are from 2 to 8 weeks, and in order to clean the sand the beds are usually taken out of service. Cleaning is accomplished

[1] See SCHNEIDER *Jour. N.E.W.W.A.*, **44**, 485, 1930.
[2] See ANDERSON, REINHARD, and HAMMEL, *Jour. A.W.W.A.*, **26**, 49, 1934.
[3] See POLLARD, *Jour. A.W.W.A.*, **28**, 1038, 1936.
[4] DEAN and ELVOVE *Am. Jour. Pub. Health*, June, 1936, p. 567.
[5] WESTON, *Am. Jour. Pub. Health*, July, 1931, p. 715.
[6] See JUST and SZNIOLIS, *Jour. A.W.W.A.*, **28**, 492, 1936, and GIBBARD, *Am. Jour. Pub. Health*, February, 1937, p. 112.

by the removal of the top layer of sand to a depth of about an inch or by washing it in place by means of special machines. Slow sand filtration is adapted to waters low in color and with turbidities less than about 30 ppm. Because this type of plant is not suitable for many treatment processes now required, its use for new plants has been superseded by the rapid or mechanical type of filter plant which first came into use about 1890.

An essential feature of rapid filtration is the use of chemical coagulants applied to the water preceding filtration and usually preceding settling. Because of this feature, colloidal impurities are flocculated and effectively removed by the filters at rates of filtration from 125 to 180 mgd/acre (125 mgd/acre is about 2 gpm/sq ft). Very much less bed area is required than for slow filters, and the length of filter runs is correspondingly reduced, ranging from about 3 hr to a week. Cleaning is accom-

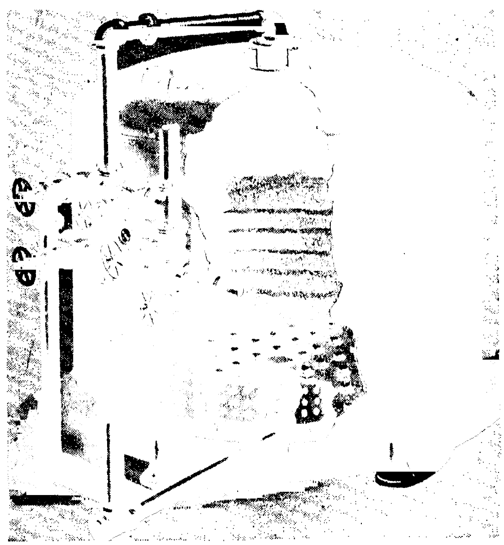


FIG. 2.—Phantom view of pressure filter. (*Courtesy of Graver Tank & Manufacturing Company, Inc.*)

plished by upward flow of clean water through the bed at rates sufficient to suspend the sand, about eight times the rate of filtration. The washing process results in stratification of the sand according to grain size with the small grains at the top. The underdrainage system is designed especially for the washing function, which is accomplished in about 5 min. The depth of sand commonly used varies from 24 to 30 in.

Both slow and rapid filters for municipal use are usually of the gravity type; *i.e.*, the filters are open at the top with the water level 3 to 6 ft over the sand. Rapid filters are also constructed in enclosed steel tanks (see Fig. 2) so that the water may be pumped direct to the distribution system through the filter. Such filters, called *pressure filters*, are extensively used in industry. They are not so satisfactory as gravity filters, because they are not accessible for inspection of the filtering material or washing efficiency during operation.

Types of Rapid Filter Plants. Rapid filter plants are employed both for purification and for softening by the lime-soda process. The essential features are chemical coagulation (including the softening reactions of the lime-soda process) accomplished in mixing basins, sedimentation accomplished in settling basins, and filtration. Other processes may also be employed such as disinfection, carbonation (addition of CO_2),

aeration, zeolite softening, and activated carbon treatment, some of which require separate treatment units. The number, size, and arrangement of the units depend upon the character of the treatment process as a whole and the plant size.

Figures 3, 4, and 5 are typical flow diagrams suggested by Baylis,[1] showing the place of application of chemicals and the treatment units used in both purification and softening plants. All the chemicals shown may not be required for any single plant. The heavy lines show the usual or preferred points of application of chemicals, but some plants may find it desirable to apply chemicals continuously at points

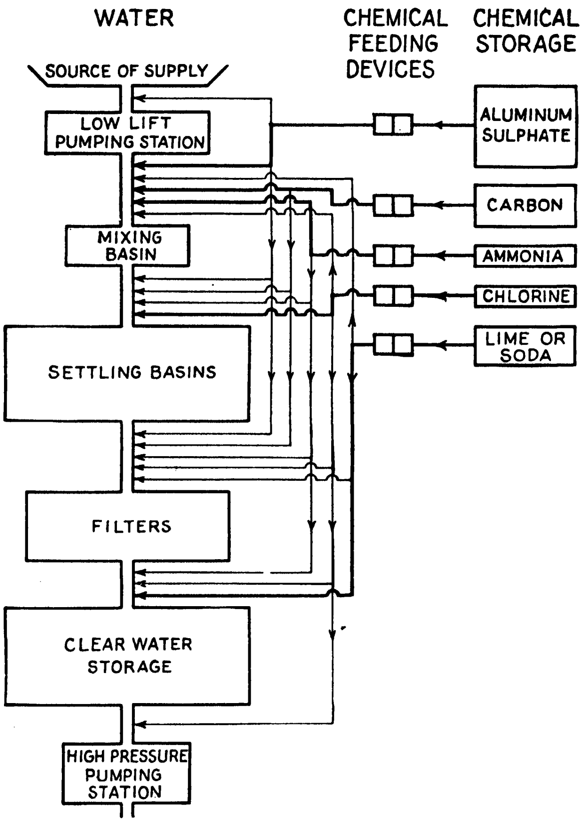


FIG. 3.—Flow diagram for alum treatment.

indicated by the lighter lines. Purification plants with low bacterial loads not troubled with odors and employing chlorine for disinfection only usually apply it after filtration. This practice is called *postchlorination.* Figures 3 and 4 represent simple purification plants requiring no softening. If chlorinated copperas, ferric chloride, or ferric sulphate is used for the coagulant, the diagram will be similar to Fig. 3. Figure 6[2] is a diagrammatic profile through a purification plant. Activated carbon, when needed, is usually dosed into the water in powdered form and then removed by settling and filtration; but in some plants continuously troubled with odors, granulated carbon contact beds are used following filtration. Additional

[1] BAYLIS, *Water Works and Sewerage,* February, 1934, p. 65.
[2] BESOZZI, *Water Works and Sewerage,* June, 1937, p. 194.

units similar to filters are required for granular carbon. Two-stage mixing and clarification as illustrated in Fig. 5 for softening plants is a new development for greater efficiency and flexibility which is not in use in many lime-softening plants. Double sedimentation or double coagulation and settling is sometimes required for purification plants treating highly polluted or very turbid water. Softening plants for clear, colorless water such as well water may consist entirely of zeolite units, which are constructed similar to rapid filters. Some lime-softening plants employ zeolite units following filtration to remove part of the hardness.

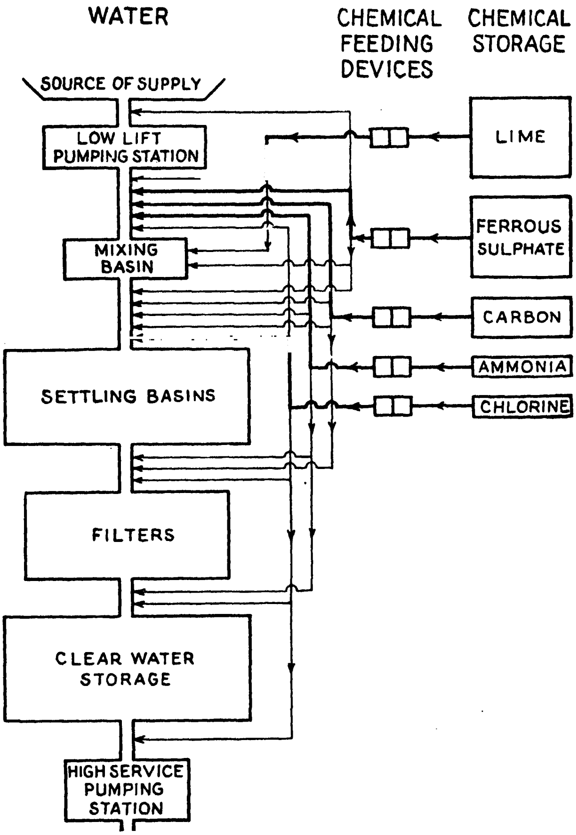


FIG. 4.—Flow diagram for iron and lime treatment.

Figure 8 illustrates an arrangement for the units of an ordinary purification plant as suggested by Hansen[1] for compactness and flexibility. Each settling basin in Fig. 8 is designed with an 180-deg turn in the direction of flow so that both inlets and outlets are in the filter building. Sludge-removal equipment is not ordinarily employed with this type of basin. A lime-soda softening plant usually requires a more complicated layout. Figure 9[2] is the plan of a lime-softening plant with two-stage mixing and clarification and zeolite beds following the filters. The settling tanks are equipped with sludge removal equipment, which is usually desirable for softening plants because

[1] *Eng. News-Record*, **114**, 634, 1935.
[2] ROBERTS, Ohio Conference on Water Purification, *11th Ann. Rept.*, 1931, p. 52.

of the large amount of sludge produced. Both carbonation and recarbonation are practiced at this plant. The plant is so arranged that any portion of the lime-softened water may be passed through the zeolite beds.

Plant Capacity. It is good practice to operate filters and basins continuously and at constant rates during any one day, except for the smallest plants where the expense of more than one shift is prohibitive. Hourly fluctuations in demand are cared for by

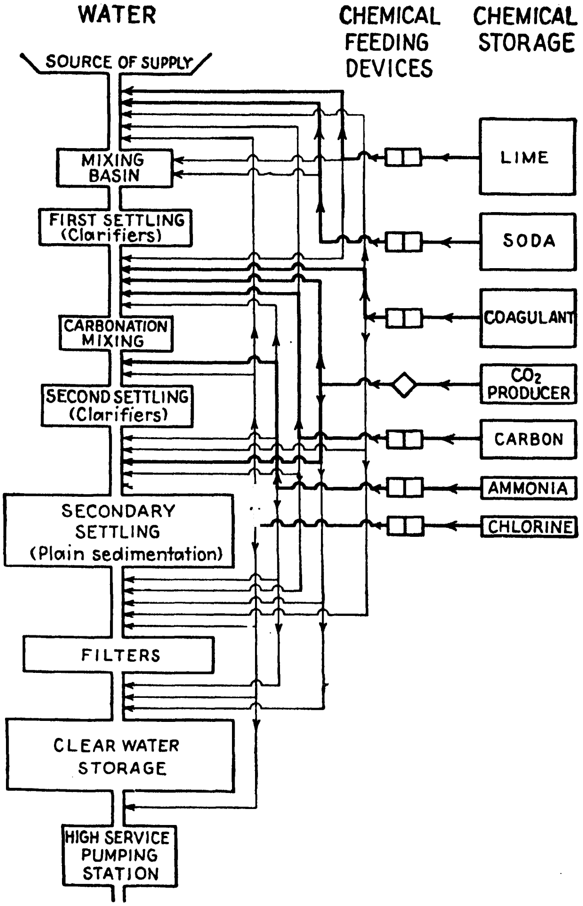


FIG. 5.—Flow diagram for lime-soda softening.

elevated and filtered water storage. The capacity of a plant is therefore determined by the maximum daily demand at the end of the design period (*E* of Table 3, Sec. 19). The nominal capacity should be such that the plant is not overloaded by more than 25 to 50 per cent on the day of maximum demand at the end of the design period. The design period for units that are readily duplicated, such as mixing basins, settling tanks, and filters, is usually limited to only about 10 years, but the design period selected for the pump rooms, office, laboratory, and chemical rooms may be 25 to 50 years.

The number of filter units and basins in parallel is established from the minimum daily demand (*B* of Table 3, Sec. 19) and economic considerations. There should be

not less than two units of any type in order that one may be taken out of operation without disturbing the process Filter units are built with capacities up to about 5 mgd Each basin usually serves two or more filters Where feasible, it is desirable to arrange the plant so that flow may be divided throughout the plant This makes it possible to treat the two portions of the flow differently and to compare the results on the same raw water

Plant Requirements for Purification. The bacterial removal of ordinary rapid sand filter plants without chlorination is usually 97 to 99 per cent in terms of the M P N of coliform organisms This implies an average raw water coliform content of not more than 50 to 100 per 100 ml if the plant is to produce an effluent conforming with the U S P H S Standard Based upon extensive studies by the U.S. Public

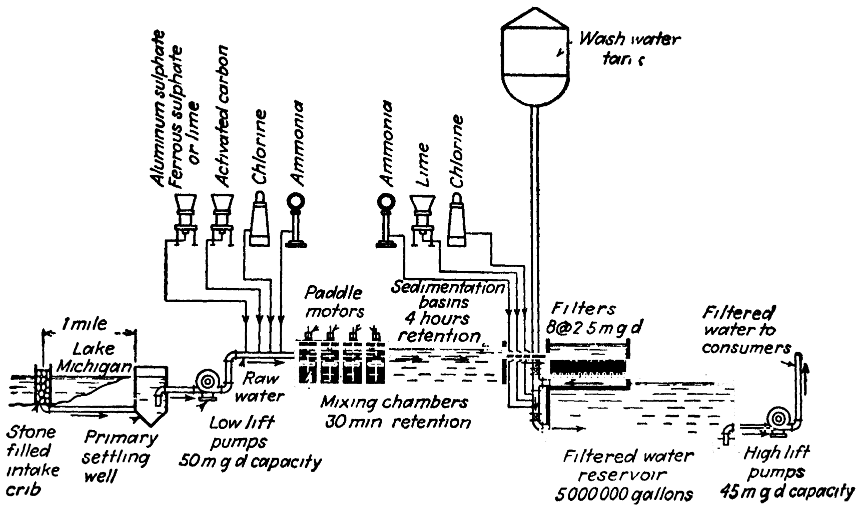


Fig 6 Purification process at Hammond Ind

Health Service[1] the limiting raw water content of coliform bacteria for various types of plants in order that they may produce water conforming to the U S P H S Standard is as follows

Type of plant	Coliform bacteria per 100 ml of raw water	
	Average	5 per cent of time
Simple chlorination	50	400
Rapid sand filtration with postchlorination	5 000	20 000
Rapid sand filtration with pre and postchlorination	20 000	

Chlorination as an adjunct to filtration for bacterial removal from polluted waters is of such importance that it is almost universally used in ordinary purification plants With chlorination, bacterial efficiency of the filters is of less importance, and their effectiveness, along with coagulation and settling, in the removal of turbidity and color

[1] Streeter *Jour A W W A* **27,** 1110 1935

FIG. 7.—Three Rivers softening and filtration plant, Ft. Wayne, Ind. (*Courtesy of Shoecraft, Drury and McNamee, Consulting Engineers.*)

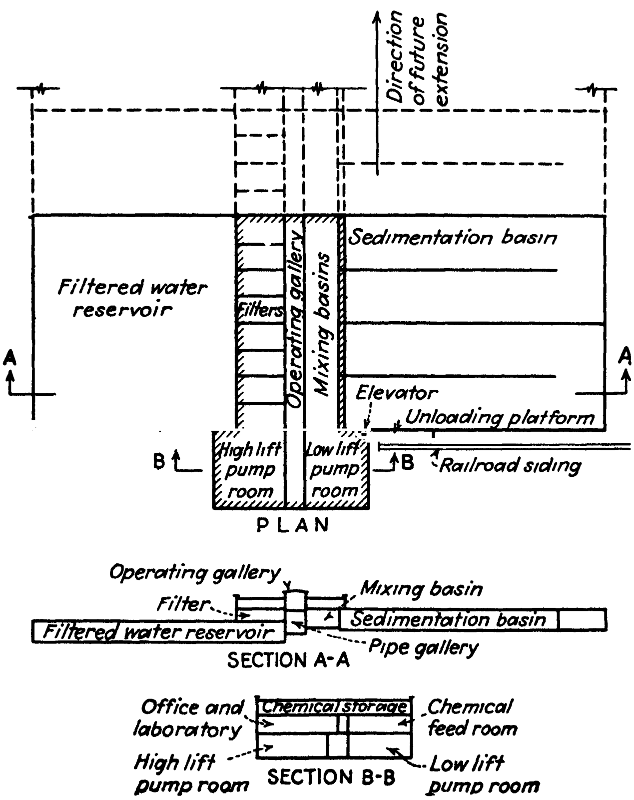


FIG. 8.—A compact type of general layout.

can be promoted. Modern rapid filter plants should produce effluents with an average turbidity of less than 0.5 ppm. In order to accomplish this, the pretreatment must be such that the water going to the filters has a turbidity of not more than 10 to 20 ppm. The overflow rates usually used in the settling basins of ordinary purification plants such as illustrated in Fig. 8 for comparatively clear waters range from 600 to 1,500 gpd per sq ft of floor area at the nominal plant capacity. For turbid waters, the required overflow rate may be as low as 100 gpd per sq ft. The overflow rate in gallons per day per square foot equals 180 times the depth in feet divided by the settling period in hours.

According to Fleming,[1] the treatment of waters with turbidities in excess of 1,000 ppm, such as that of the Mississippi River, requires double coagulation and sedimenta-

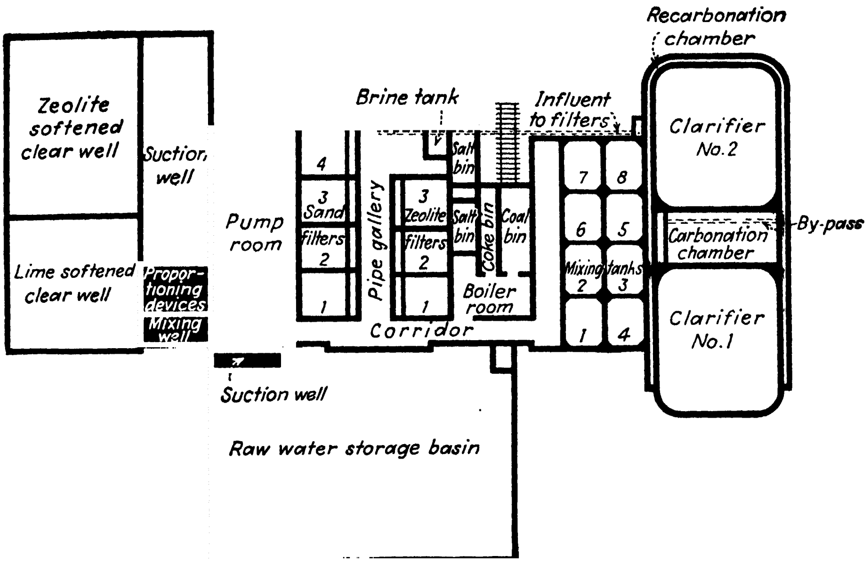


FIG. 9.—Plan of 4-mgd water-softening plant at Findlay, Ohio. (*H. P. Jones Co. and C. P. Hoover, Engineers.*)

tion with 8 to 12 and 4 to 6 hr for the first and second settling periods, respectively. If the turbidity is 1,200 to 2,000 with the fineness coefficient greater than 1.1, or is greater than 2,000 with the fineness coefficient greater than 0.7, the double coagulation-sedimentation process should be preceded by 3 to 5 hr of plain settling. A considerable amount of fine sand is present when the fineness coefficient exceeds 0.7, and plain settling to reduce the turbidity to 1,200 or less results in the saving of chemicals. Grit chambers are effective if the fineness coefficient exceeds 1.1. For turbidities in excess of 2,000 with the fineness coefficient less than 0.7, triple coagulation and settling is required with settling periods of 10 to 12, 4 to 6, and 4 to 6 hr, respectively.

SEDIMENTATION

Settling Velocities of Individual Particles. When a particle is permitted to move without interference through a fluid owing to the difference between its density and that of the fluid, the settling or rising velocity with respect to the fluid quickly becomes constant, the resistance becoming equal to the weight of the particle in the fluid. The

[1] *Jour. A.W.W.A.*, **22**, 1559, 1930.

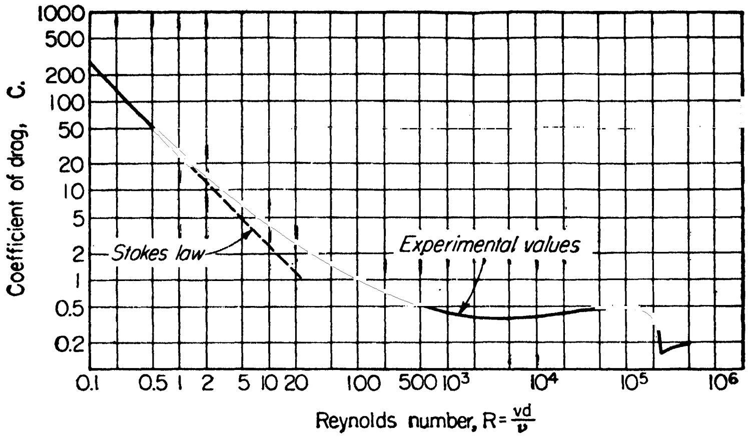


FIG. 10.—Drag coefficients of spheres in fluids as a function of Reynolds number. (*After Schiller.*)

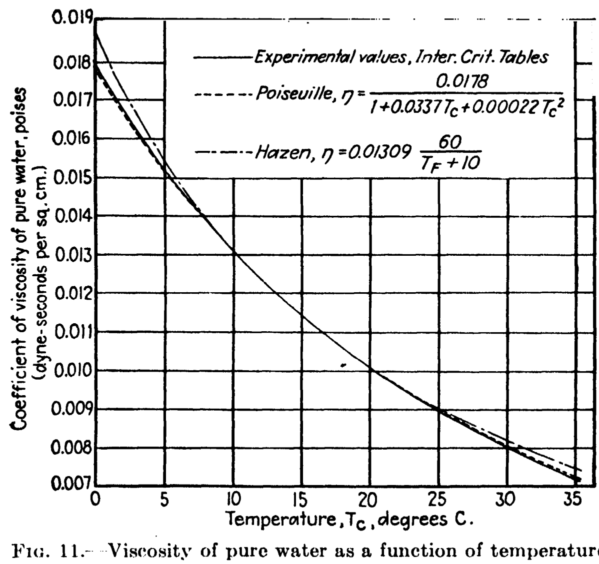


FIG. 11.—Viscosity of pure water as a function of temperature.

general equation for the settling velocity[1] of a sphere, obtained by equating Newton's value of the resistance, or drag, to the weight of the sphere in the fluid, is as follows:

$$v = \sqrt{\frac{4}{3}\frac{g}{C}\frac{\rho_1 - \rho}{\rho}d} \tag{12}$$

in which v = settling velocity.

g = acceleration of gravity.

C = drag coefficient.

ρ = density of the fluid.

ρ_1 = density of the particle.

d = diameter of the particle.

[1] See CAMP, Sedimentation and the Design of Settling Tanks, *Trans. A. S. C. E.*, 1946, p. 895.

Experimental values of the drag coefficient of spheres in terms of Reynolds number are shown in Fig. 10.[1] For Reynolds numbers up to about 0.5 to 1, the drag is due entirely to viscosity, and for larger Reynolds numbers, eddying resistance comes into play.

The viscous drag,[2] as developed by Stokes, may be equated to the weight of the sphere to obtain Stokes' law, which is given below:

$$v = \frac{1}{18}\frac{g}{\eta}(\rho_1 - \rho)d^2 \tag{13}$$

in which η is the absolute viscosity of the fluid.

Values of η in poises for pure water at various temperatures are given in Fig. 11. The presence of dissolved solids in concentrations up to 1,000 ppm will generally affect

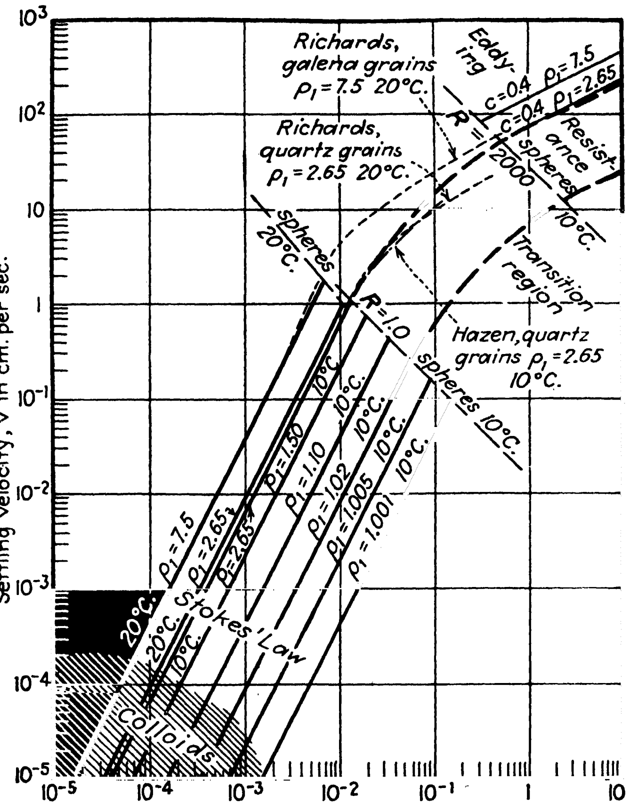


FIG. 12.—Settling velocities of particles in still water.

the viscosity by less than 0.5 per cent. Figure 12 shows the settling velocities of particles in still water. The heavy lines are for values computed for spheres from Eqs. (12) and (13) with drag coefficients from Fig. 10. Sand grains and heavy floc particles settle in the transition region, but most of the particles significant in forecasting removal by settling in water-treatment plants settle well within the Stokes'

[1] CAMP, *Sewage Works Jour.*, September, 1936, p. 742.
[2] CAMP, *Trans. A. S. C. E.*, 1946, p. 895.

law region. Particles with irregular shapes settle somewhat more slowly than spheres of equivalent volume.

When the volumetric concentration of suspended particles exceeds about 1 per cent, settling is appreciably hindered and the settling velocities are reduced by 10 per cent or more.

Clarification Theory.[1] **Ideal Basin and Discrete Particles.** The following simplifying assumptions are made to develop the theory for the ideal rectangular basin:

1. The direction of flow is horizontal, and the velocity is the same in all parts of the settling zone. Hence each particle of water is assumed to remain in the settling zone for a period equal to the retention period, which is the volume of the settling zone divided by the rate of discharge.

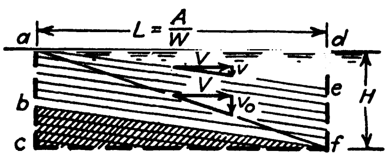


Fig. 13.

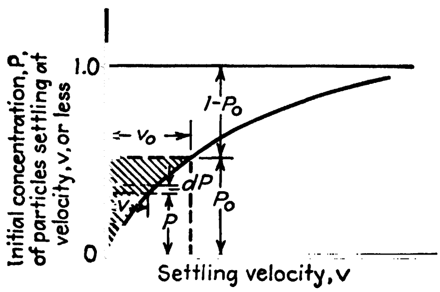


Fig. 14.—Typical settling-velocity analysis curve of suspension of discrete particles.

2. The concentration of suspended particles of each size is the same at all points in the vertical plane perpendicular to the direction of flow at the inlet of the settling zone.
3. All suspended particles maintain their shape, size, and individuality during settling and settle without interference. Hence each particle is assumed to settle at constant velocity for a given temperature.
4. A particle is removed when it strikes the bottom of the settling zone.

The settling path of a particle is determined by the vector sum of its settling velocity and the velocity of the liquid. Hence, in the ideal basin, particles having settling velocities equal to or greater than v_0, as shown in Fig. 13, will all settle out; and the removal of particles with any settling velocity v less than v_0 will be

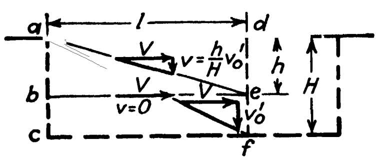


Fig. 15.

$$r = \frac{v}{v_0} = \frac{Av}{Q} \tag{14}$$

in which r is the removal ratio, and v_0 the overflow rate, or the discharge per unit of tank surface area $= Q/A$.

If the settling-velocity analysis of the original suspension is represented by a curve, such as Fig. 14, the total removal ratio for the entire suspension is

$$r = 1 - P_0 + \frac{1}{v_0}\int_0^{P_0} v\,dP \tag{15}$$

The value of the last term in Eq. (15) is the average vertical distance from the curve (Fig. 14) to the horizontal line for $P = P_0$. The concentration of suspended matter at any point in the basin, such as at e (Fig. 15) is

$$x = \int_0^{P_1} dP = P_1 \tag{16}$$

where P_1 is the value of P from Fig. 14 for $v = \frac{h}{H} v_0'$ from Fig. 15.

The above theory, Eqs. (14), (15), and (16), also applies to ideal radial-flow basins

[1] Camp, *Trans. A. S. C. E.*, 1946, p. 895.

in which the velocity is horizontal in a radial direction from the center and varies inversely as the distance from the center.

As a consequence of this theory, the following conclusions may be drawn with respect to settling of discrete particles in an ideal basin:

1 For any given discharge, the removal is a function of the surface area and independent of the depth of the basin; or the removal is a function of the overflow rate and not the retention period.

2. The concentration of suspended matter at any cross section in the settling zone increases with the depth below the surface; and decreases with the proximity of the cross section to the outlet end of the basin.

The settling-velocity analysis curve of a suspension of discrete particles may be determined experimentally by quiescent settling in a container such as Fig. 16. At various time intervals after the start of the test, samples are withdrawn from a given point, such as D, and the concentration is determined for each sample. The concentration may be measured in terms of suspended solids, turbidity, iron, alumina, color, or any other quantity that is reduced by settling. The corresponding settling velocity is obtained by dividing the depth of water above the sampling point by the period of settling. Extreme care must be exercised to maintain a constant temperature and to reduce eddy and convection currents to a minimum. If the test is satisfactory and no flocculation of the particles is taking place, the same analysis curve should be produced by samples taken from any depth. On the other hand, if the velocity-analysis curve from deep samples lies below the analysis curve for shallow samples, coagulation is taking place and the velocities obtained are meaningless. If the particles coalesce, the preceding theory is not applicable.

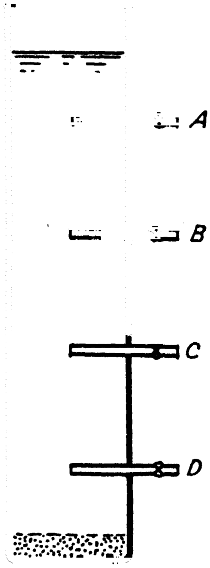


FIG. 16.

Clarification Theory.[1] **Ideal Basin and Coalescing Particles.** An analysis of a suspension which includes the effect of flocculation and is satisfactory for forecasting settling-basin removal may be made by determining concentrations in samples from each sampling point of the container (Fig. 16) at the end of various time intervals. The effects of turbulent mixing[2] may be approximated by means of a mixing grid moving up and down in the sample. Table 2 shows a typical analysis of this type. The depth of the container should be as great as that of the settling basin for best results, and the capacity of the container should be large enough so that the depth is not appreciably changed by the withdrawal of samples.

TABLE 2

Sampling point	Concentration ratio					
A	1.0	0.82	0.66	0.47	0.28	0.21
B	1.0	0.83	0.69	0.50	0.33	0.25
C	1.0	0.85	0.72	0.54	0.38	0.30
D	1.0	0.87	0.75	0.59	0.43	0.36
Time, min	0	20	40	80	160	300

[1] CAMP, *loc. cit.*

[2] CAMP and STEIN, Velocity Gradients and Internal Work in Fluid Motion, *Jour. Boston Soc. Civil Engrs.*, October, 1943, p. 219.

The removal may be estimated for any settling period by subtracting from unity the average of the concentration ratios. For example, the removal ratio for a basin the same depth as the container for a 160-min retention, from Table 2, is 0.645. For a basin half the depth of the container, the removal is 0.695 for the same period and 0.515 for half the period. It may be seen, therefore, that the removal for this suspension is not independent of the depth and is influenced by both overflow rate and retention period.

The particles in most of the suspensions dealt with in water-treatment plants flocculate during settling. This is notably true of particles of floc formed by chemical coagulation, but it has also been observed with clay suspensions.

Velocity and Basin Dimensions.[1] The velocity in an actual rectangular basin is not uniform over the cross section even though uniform dispersion at the inlet end and uniform collection at the outlet end of the settling zone is effected. Because of the drag on the walls and floor the velocity at these boundaries is zero, and it is greater than the average at some points out from the boundaries. The velocity distribution over the cross section of most settling basins is not stable, moreover, owing to the disturbing influences of masses of water of varying density. This variation in density, which is due to temperature differences and differences in concentration of solids and entrained gases, though slight is nevertheless sufficient to cause vertical movements of water masses, dead spaces, and reversals in the direction of flow. As a result of these disturbing influences, the probable time of passage of all the particles in a given volume of water will be less than the retention period, and the volume will disperse itself while passing through the basin so that the time interval between the passing of the first and last particles at the basin outlet will be much greater than at the inlet. This phenomenon is called *short-circuiting.* If two or more succeeding volumes of water passing through the basin at the same rate of flow require markedly different times for passage, the basin lacks stability of flow.

A particular type of short-circuiting common to many settling basins is caused by *density currents.* If the incoming suspension is heavier than the basin contents, and the basin velocity is insufficient to cause mixing, the heavy suspension will flow along the bottom as a density current. Similarly, light suspensions will flow along the surface. Since the incoming suspension contains more solids than the clarified water in the basin and is therefore likely to be heavier, it is better to introduce it near the bottom and to make the basin shallow.

Short-circuiting may be measured by inserting a charge of dye or other substance into the basin influent and observing its concentration in the effluent after various time intervals as the slug of water containing the charge passes out of the basin. Figure 17 shows a typical analysis of short-circuiting through a model settling basin. In the ideal basin, both the minimum time T_i and the probable flowing through time T_f would be equal to the retention period T. The flow conditions are best in actual basins in which these ratios approach nearest to unity. In making measurements of short-circuiting, it is very important to select a dye or chemical solution whose density is so near that of the suspension in the basin that the flow pattern will not be modified by the introduction of the dye.

Instability of flow in a basin is marked by the inability to reproduce the short-circuiting curve, such as Fig. 17, in succeeding runs at the same discharge. Short-circuiting is least and stability is greatest in basins with the highest ratio of inertia to gravity forces. This ratio is expressed by Froude's number $F = V^2/Rg$, where V is the mean basin velocity, R the hydraulic radius of the basin cross section, and g the gravity constant. High values of Froude's number imply high velocities and long shallow basins.

[1] *Idem.*

Since, from the clarification theory with discrete particles in an ideal basin, removal is independent of depth for a given rate of discharge, it is evident that the most economical basin will be that with the least practicable depth. Camp[1] has shown that this is also true for actual basins, even with flocculation and turbulence during settling, and that the least practicable depth is that for impending scour of sludge at the peak rate of flow. For a particular type of sludge deposit, scour depends primarily upon the magnitude of the basin velocity. Little is known about the limiting velocities for various kinds of sludges beyond which scour will obtain, but experience indicates that the limiting velocities are considerably greater than those actually used in practice. Langelier[2] reports unusually good removal of turbidity and color from an alum-coagulated water by settling in a tunnel of the supply line. The velocity was 12 fpm, the retention period was 11 hr, and the value of F was 6×10^{-4}.

Reductions in short-circuiting and improvements in settling efficiency of existing basins have been reported due to the judicious use of baffles in the basins normal to the

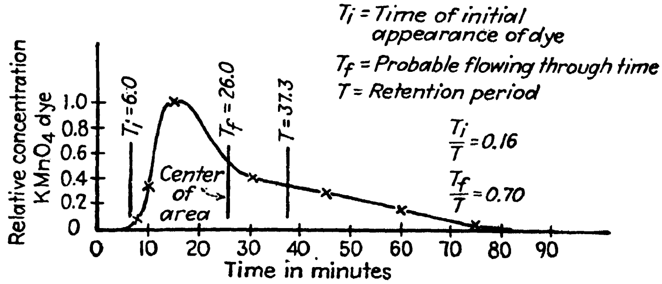


FIG. 17.

direction of flow. This procedure has the effect of increasing F, but it also may introduce dead spaces and eddy currents and cause disturbance to the deposited solids. A more satisfactory procedure for existing basins is the use of longitudinal round-the-end baffles or the use of trays. Eddy currents and dead spaces are produced at the bends, but their effect is not so harmful as with cross baffling.

The design of new settling basins should be based upon the overflow rates required to obtain the desired removal of suspended matter at the expected rates of discharge. The basin floor area will thus be established. The required floor area may be provided in single-story basins or in multiple-story basins, the choice depending largely upon the amount of sludge to be produced and the methods available for sludge removal. Multi-story basins will generally prove to be the more economical to construct, since the required floor area may be provided in the least basin volume the greater the number of trays. The depth of a single-story basin to the top of the sludge blanket or the clear depth of a single pass of a multi-story basin from ceiling to the top of the sludge, should be the minimum consistent with no scour for best economy except as limited by the method of removal of the sludge.

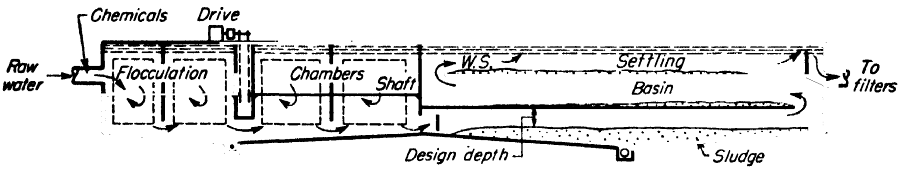
For the usual type of water purification plant, the amount of sludge produced is usually not sufficient to warrant the cost of mechanical equipment for continuous sludge removal. Sludge is removed intermittently at intervals of several weeks by withdrawing a basin from service, draining it, and flushing the walls and floors by hand. In this type of basin, the clear height from floor to ceiling should be about 6.5 ft so as to provide headroom for workmen during cleaning operations.

Figure 18 illustrates in longitudinal section two multi-story basins of this type,

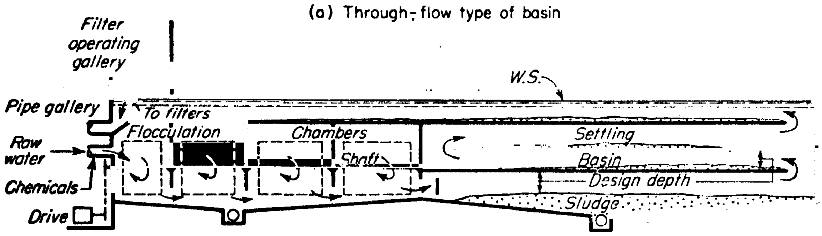
[1] *Idem.*
[2] *Jour. A.W.W.A.*, **22**, 1484, 1930.

each in series with flocculation chambers. Figure 18*a* shows a through-flow type of installation where the settling basin is between the flocculation chambers and the filter building. Figure 18*b* shows a return-flow type of installation with part of the flocculation chambers within the filter building. In both cases the design depth of trays is premised upon the amount of sludge to be accumulated between cleanings.

It will be noted that Figure 18*b* is similar functionally to the layout for single-story basins illustrated in Fig. 8, both layouts permitting the application of chemicals within the filter building. The return-flow for the single-story basin requires a horizontal round-the-end bend which occupies a considerable portion of the basin area, whereas the returns in the multi-story basin are made vertically with much less attendant loss in effective floor area. The spaces occupied by inlet and outlet zones and by return bends should not be counted as effective floor area for settling purposes because of excessive turbulence and upward components of velocity within these zones.



(a) Through-flow type of basin



(b) Return-flow type of basin

FIG. 18.

Experiments indicate that, with well-designed inlets and outlets, both the inlet and the outlet zone will extend out into the basin for a distance about equal to the basin depth. Since the floor area in these zones is ineffective for settling, it is obvious that the length of a basin should be great as compared to its depth. Since single-story radial-flow circular tanks and single-story square tanks are quite inefficient for this reason, the design of the inlets and outlets for such tanks is a critical problem. In order to effect good distribution at the inlet of such a tank and good flow distribution over the cross section at the outlet end, with a minimum length for both inlet and outlet zones, it is essential to use orifice walls to the full depth and to have a relatively high velocity through the orifices. Unfortunately, such a wall is detrimental at the inlet end since the high velocity through the orifices may destroy floc.

The problem of inlet and outlet design is much simplified if the length of the basin is made so great as compared to its depth that the inlet and outlet zones are a negligible part of the gross length. For example, the effective settling area in a basin with a 20 to 1 ratio of length to depth is about 90 per cent of the total floor area. Orifice walls are of no benefit in such a tank.

Where the amount of sludge produced is sufficient to warrant the use of mechanical sludge collectors, the basin depth will be controlled primarily by the depth required for the equipment which is available. Most of the commercially available sludge collector equipment is designed for single-story basins and is not readily adapted to

multi-story tanks. Straight-line collectors for rectangular tanks with scraper blade motion parallel to the direction of flow require tank depths of 5 ft or more in order to accommodate the sprockets. Headroom for workmen is not important where mechanical equipment is used, and smaller depths are desirable for multi-story tanks if satisfactory collectors can be devised.

Settling basins should be covered in order to minimize convection and eddy currents due to temperature changes and wind action and to obviate ice difficulties. The covering of basins is becoming common practice in design unless the basins are so large that the cost is too great. Occasionally natural sites are available, as at the Providence purification plant, which permit the construction of very large settling

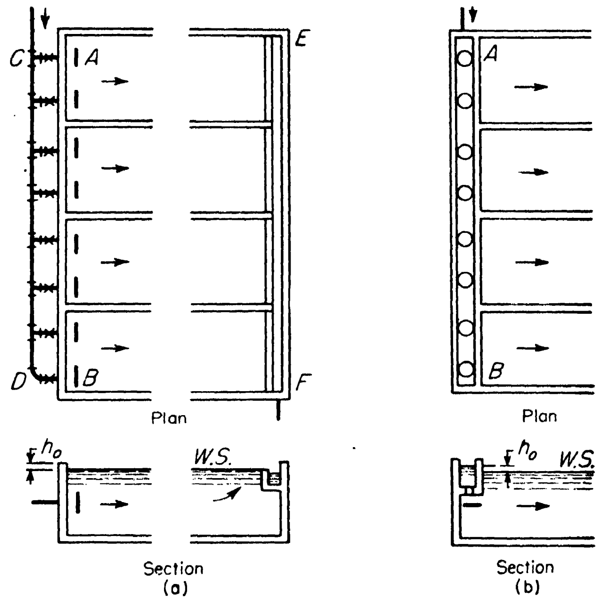


FIG. 19.

basins with much more capacity than is needed at a minimum of cost. Under such conditions, short-circuiting is of little importance and roofs are not needed.

Inlet and Outlet Devices. The purpose of properly designed inlets and outlets is to distribute the suspension uniformly among the basins and uniformly over the cross section of each settling basin at the inlet end and to collect the effluent uniformly at the outlet end. Properly designed inlets and outlets assist in the reduction of short-circuiting and are very important for short basins with low velocities. The velocity in the basin must be reduced to less than 1 per cent of the velocity in the influent conduit in many cases.

The suspension may be distributed across the width of the basins by bringing it in through several pipes at intervals across the width, as shown in Fig. 19*a*, or by bringing it in through a single conduit to a transverse influent flume, as shown in Fig. 19*b*, which in turn distributes the suspension across the basin width through orifices or sluice gates. A baffle should be placed in front of each inlet pipe or orifice in order to help dissipate the kinetic energy of the jet. An equal division of flow to all the basins is most readily approached if the water level is the same in all, which may be accomplished by proper design of outlets.

A uniform distribution of flow through inlet pipes or orifices, all of the same size, may be approached by making the head loss at the inlet pipes or orifices large as compared with the maximum difference in energy head available at the inlets. The maximum difference in available energy head in Fig. 19 will be between inlets A and B owing to the change in energy head through the pipe CD or the flume. The head loss at one of the two inlets considered is $h_o = kq^2$, where q is the discharge through the inlet. If h_f is the difference in energy head available at A and B and m is the ratio of the rates of discharge at the two inlets, the head loss at the other inlet is $h_o - h_f = k(mq)^2$. Then

$$\frac{h_o}{h_o - h_f} = \frac{1}{m^2} \tag{17}$$

This equation may be used to compute the required inlet head loss h_o for any desired variation in discharge between the inlets. For example, if it is desired that the discharge at A vary by not more than 10 per cent from the discharge at B, $m \gtreqless 0.9$ and $h_o \gtreqless 5.3\ h_f$. The value of h_f may be estimated from friction losses and velocity-head changes. The size of orifices or gates may be determined by introducing the required value of h_o in the orifice formula with the proper discharge coefficient. The proper size of inlet pipes may be determined by making the velocity head in the pipes equal to the required value of h_o. The design should be based upon the peak flow and should be checked for minimum-flow conditions.

The velocity in inlet pipes varies directly as the discharge. A lower range of variation in velocity may be obtained with flumes inasmuch as the depth varies with the discharge. The velocity in the inlet conduits should be sufficiently high to prevent settling in the conduits and not high enough to cause destruction of floc in coagulated waters. Velocities between 0.4 and 3 fps are usually satisfactory with coagulated water, but fragile floc may require a lower maximum.

For settling basins treating water with very fragile floc, high velocities at inlets and through dispersion walls are not permissible. Very low velocities in the inlets and less satisfactory distribution may be required. Permissible velocities for any particular floc can be determined only by test. The best solution for fragile floc and also the simplest from the standpoint of inlet design is the use of flocculation tanks just preceding each settling tank as shown in Fig. 18. With this design the inlets are transferred to the inlet end of the flocculation tanks where the velocity may be made as high as is required for good distribution of flow among parallel units. The wall openings between flocculation tanks and settling basins may be made quite large, the size being limited only by the desirability of minimizing short-circuiting.

The effluent from settling tanks may be collected uniformly across the width of each basin, and the basin water levels may be kept the same by means of freely discharging weirs at the same level across the width of each basin at the effluent end. Such weirs discharge into effluent flumes, such as EF (Fig. 19a), which, hydraulically, are lateral spillway channels. In some plants, the effluent weirs are submerged in operation in order to eliminate the high velocity of a freely discharging weir and thus prevent the destruction of small floc particles going to the filters. Since the surface profile in lateral spillway channels drops in the direction of flow, the use of submerged weirs promotes short-circuiting toward the downstream end of the flume. Short-circuiting due to this cause is of little consequence in long shallow basins, and in short deep tanks it may be partly prevented by the use of an effluent orifice wall. In vertical planes perpendicular to the effluent weir, the direction of flow is radial approaching the weir, but there is a tendency for most of the water to short-circuit toward the water surface. Effluent orifice walls eliminate this tendency.

The drawdown of the water-surface curve, the effect of friction being excluded, in

a lateral spillway channel of rectangular cross section and with a level invert (see Fig. 20) may be estimated by means of the following equation:[1]

$$H = \sqrt{h^2 + \frac{2q^2x^2}{gb^2h}} \tag{18}$$

where H = depth at the upstream end.
h = depth at distance x.
q = discharge per unit length of weir.
g = gravity constant.
b = width of the channel.

This equation results from the integration for the special case of the general differential equation developed by Hinds[2] from the momentum theory. It is based upon the assumption of hydrostatic-pressure distribution, which is nearly correct in treatment-plant applications. Equation (18) is correct for parallel-wall flumes of other than rectangular cross section provided the invert is level; and the effects of sloping invert and friction may also be taken into account with additional terms. Experimental studies indicate that the friction loss in the flume may be estimated with values of Darcy's f varying from about 0.03 to 0.12, depending upon the turbulence in the channel, and that friction will account for 6 to 16 per cent of the water-surface drawdown.

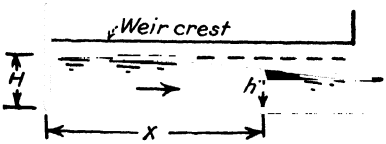


FIG. 20.

Sludge Removal. For ordinary purification plants treating comparatively clear water, the settling basins may be designed without mechanical equipment for sludge removal. Such basins are usually designed with bottoms sloping gently to one or more sludge drain pipes (see Fig. 18); the sludge is removed by taking a basin out of service, draining it, and hosing the sludge to the drains. The period of accumulation of sludge between cleanings varies from a few weeks to several months. According to Baylis,[3] it is economical to use mechanical removal when the volume of the sediment is over 0.1 per cent of the volume of the water. It is probably not economy in most plants when the volume is less than 0.02 per cent. Baylis' 0.1 per cent corresponds with 200 retention periods, or 25 days at 3-hr retention, before the sludge accumulates to 0.2 the basin depth, after which the sludge should probably be removed. The moisture content of the deposited sludge varies from about 90 per cent for very turbid waters and for some softening plants to more than 99.5 per cent for ordinary coagulation of clear waters. Hence 0.1 per cent of the volume may correspond with a suspended solids content in the treated water of about 20 to 100 ppm.

Two general types of mechanical sludge-removal equipment are in common use in water-treatment plants: the straight-line type of equipment suitable for rectangular tanks and rotary equipment suitable for square tanks and for radial-flow circular or square tanks. In the first type, the sludge is scraped in a straight line to sludge hoppers at one end of the basin, usually the inlet end, from which it is removed through a sludge pipe. In the second type, the sludge is scraped radially to the center of the basin where it is removed.

The velocity of the sludge-removal equipment should not be sufficient to throw the sludge back into suspension. Lower velocities are required for flocculent sludge of high moisture content than for granular solids. On the other hand, the velocity of the mechanism must be high enough to remove the sludge as fast as it deposits.

[1] CAMP, *Trans. A. S. C. E.*, **105**, 606, 1940.
[2] HINDS, *Trans. A. S. C. E.*, **89**, 885, 1926.
[3] *Water Works*, **67**, 38, 1928.

This consideration is of little importance for most water plants, but may become of consequence with long narrow basins and for very turbid water. Sludge-removal mechanism may be operated intermittently with waters of low suspended content, but continuous operation is necessary for highly turbid waters and for lime-softening plants.

COAGULATION

Purpose of Coagulants. Coagulants are added to water to assist in the removal of finely divided or colloidal impurities that require agglomeration before they can be effectively removed by settling and filtration. Coagulation in its strictest sense means the agglomeration or flocculation of smaller particles to form larger ones. In water treatment, the term is generally used to include all the processes that take place from the addition of the chemicals to the formation of the floc.

The most important of these processes are the formation of colloidal precipitates, the neutralization of charges on the colloidal particles, the flocculation of the particles by Brownian motion followed by mechanical stirring, and the adsorption by the floc of impurities in the water. In most cases, the floc is formed from the precipitation of the chemicals added to the water by interreactions or by reaction with soluble constituents of the water. In some cases, however, the floc is produced by coagulation of colloidal particles, such as vegetable coloring matter, already present in the water. A part of the lime-softening process consists of coagulation, inasmuch as the precipitates formed in the removal of hardness must be flocculated before they can be removed from the water.

Chemistry of Coagulation. The most commonly used coagulent is filter alum [sulphate of alumina, $Al_2(SO_4)_3 \cdot 14H_2O$] which is available commercially in both lump and powdered form. Another chemical in common use is copperas (ferrous sulphate, $FeSO_4 \cdot 7H_2O$), also available in solid form; and coagulants growing in popularity are ferric chloride, $FeCl_3$, available as an amorphous solid or in concentrated aqueous solution; ferric sulphate, $Fe_2(SO_4)_3$, available in solid form; and chlorinated copperas made at the plant by combining Cl_2 with copperas.

The reactions are much the same with all the coagulants, with some variations that will be pointed out below. In the case of sulphate of alumina, for example, the first reaction upon the addition of the chemical to the water is its solution, as follows, which with the small concentrations used goes to completion to the right:

$$\underline{Al_2(SO_4)_3 \cdot 14H_2O} \rightleftarrows 2Al^{+++} + 3SO_4^{--} + 14H_2O \tag{19}$$

The Al^{+++} ions combine with OH^-, which is liberated from the alkalinity of the water in accordance with reaction (5), as follows:

$$2Al^{+++} + 6OH^- \rightleftarrows \underline{Al_2O_3 \cdot xH_2O} \tag{20}$$

The extent to which this reaction goes to the right to form the hydrous aluminum oxide precipitate depends upon the available alkalinity and the equilibrium constant for reaction (20). It will be noted that three OH^- ions are required for each Al^{+++} ion. If sufficient alkalinity is not available, it must be added to precipitate the Al as hydrous oxide. For this purpose, hydrated lime, $Ca(OH)_2$, is usually added, or, to avoid increase in hardness, soda ash (sodium carbonate, Na_2CO_3), or lye (sodium hydroxide, NaOH) may be used. Ferric coagulants produce hydrous ferric oxide, $Fe_2O_3 \cdot xH_2O$, as a precipitate in reactions analogous to (20). Ferrous sulphate cannot be used without lime or alkali, as will be explained below.

When a relatively insoluble compound, such as Al_2O_3 or Fe_2O_3, is in equilibrium with a solution of its ions, the mass action expression may be written by omitting the solid term. The equilibrium constant is called a *solubility product.* The solubility

product for reaction (20) is

$$[Al^{+++}][OH^-]^3 = 1.9 \times 10^{-33} \text{ at } 25C^1 \tag{21}$$

and for Fe_2O_3 it is

$$[Fe^{+++}][OH^-]^3 = 4 \times 10^{-38} \text{ at } 25C^1 \tag{22}$$

The value of the solubility product for these equations is influenced by both temperature and concentration of dissolved solids.

In the presence of caustic alkalinity, Al_2O_3 dissolves as an aluminate as follows:

$$\underline{Al_2O_3} + 2OH^- \rightleftarrows 2AlO_2^- + H_2O \tag{23}$$

The solubility product for this reaction is

$$[AlO_2^-][H^+] = 4 \times 10^{-13} \text{ at } 25C^1 \tag{24}$$

Reaction (23) is peculiar to Al. It does not take place with the iron salts.

When copperas is dissolved in water, Fe^{++} and SO_4^{--} ions are liberated in a reaction analogous to (19), and the ferrous ions react with the OH^- ions in the solution to produce the hydrous ferrous oxide, $FeO \cdot xH_2O$. The solubility product for hydrous ferrous oxide is

$$[Fe^{++}][OH^-]^2 = 1.65 \times 10^{-15} \text{ at } 25C^1 \tag{25}$$

which indicates that ferrous iron oxide does not begin to precipitate until the pH exceeds 8.5. For this reason lime is always used in conjunction with copperas, and enough lime is used to raise the pH to 8.5 to 10.0. The process is sometimes called the *iron and lime* process. Ferrous oxide is unstable in the presence of dissolved oxygen and is rapidly oxidized to hydrous ferric oxide as follows:

$$\underline{4FeO} + O_2 \rightleftarrows \underline{2Fe_2O_3} \tag{26}$$

Aeration is helpful in this process as a first step before lime is added in order to furnish dissolved oxygen and to save lime by the liberation of free CO_2.

The significance of the foregoing solubility products is indicated in Fig. 21 which shows the approximate solubility of aluminum and iron oxide floc at various pH values. The best precipitate is obtained at points of least solubility, which for alum is pH 5.0 to 7.5 and for ferric coagulants for all pH values above about 4.0, the solubility decreasing greatly as the pH is increased. The least solubility for ferrous iron is at pH values above 9.5.

Copperas may be used successfully as a coagulant at low pH values if the ferrous iron is first oxidized to ferric by the introduction of chlorine into a copperas solution to form chlorinated copperas as follows:

$$6FeSO_4 \cdot 7H_2O + 3Cl_2 \rightleftarrows 6Fe^{+++} + 6SO_4^{--} + 6Cl^- + 7H_2O \tag{27}$$

This reaction requires about 1 lb of chlorine to 8 of copperas. Prechlorination of alum-treated waters has been found to assist in the removal of color, probably because it oxidizes ferrous iron present in the water. Organic-bound iron in waters of high color has been effectively oxidized and precipitated by $KMnO_4$ at a pH of 8.8 to 9.8.

The hydrous aluminum or ferric oxide when first formed is a positively charged colloid in the acid region up to a pH of about 6 to 7.5, adsorbing H^+ and probably some Al^{+++} or Fe^{+++} from solution. In order to flocculate a colloid, a sufficient concentration of flocculating ions of opposite charge must be present. The flocculating value of an ion is measured by the concentration required to completely floccu-

[1] LATIMER, "Oxidation Potentials," App. III, Prentice-Hall, Inc., 1938.

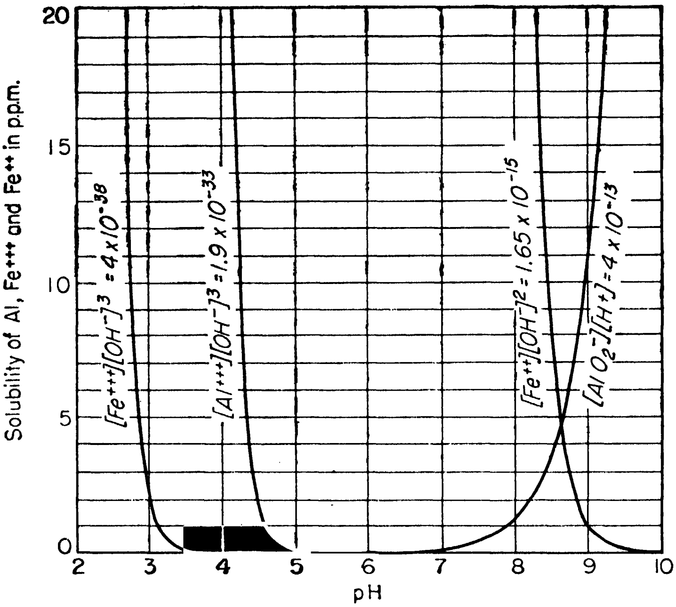


FIG. 21.

late a sol in a given time. The flocculating value[1] increases enormously with the valence of the ion, the relative flocculating values of trivalent, bivalent, and monovalent ions being roughly in the order of 500 to 7 to 1. In the case of coagulation with alum or ferric sulphate in the acid region, bivalent SO_4^{--} is automatically present to flocculate the hydrous oxides effectively. If $FeCl_3$ is used, however, flocculation is not so effective because only the monovalent Cl^- is present. Recent investigators[2] have shown that additional negative ions extend the range of pH in the acid region over which rapid flocculation of alum and ferric salts takes place (see Fig. 22) and that, as expected, SO_4^{--} is much more effective than Cl^-. These investigators have also shown that the colloidal oxides of Fe and Al are negatively charged in the alkaline region, for additional positive ions promote more rapid flocculation. This effect is not appreciable in the case of alum floc because of reaction (23); but it is quite evident for ferric oxide floc, and, as expected, Ca^{++} is much more effective than Na^+.

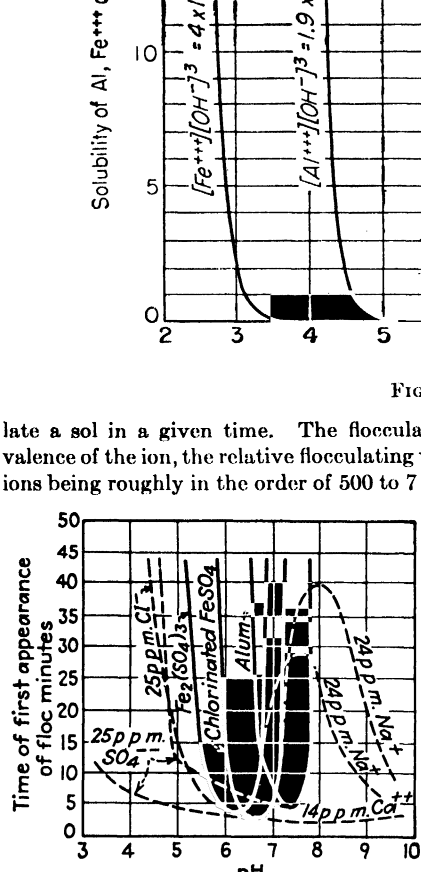


FIG. 22.—Time of flocculation as a function of pH value. (Concentration of Al and Fe equivalent to about 2 gpg of alum. Temperature not reported.)

The curves in Fig. 22 are for coagulation in distilled water containing only the added salts. SO_4^{--} and Cl^- were added as sodium salts and Na^+ and Ca^{++} as chlorides.

[1] KRUYT and VAN KLOOSTER, *op. cit.*; WEISER, *op. cit.*

[2] PETERSON and BARTOW; BLACK, RICE and BARTOW; BARTOW, BLACK and SANSBURY: *Ind. Eng. Chem.*, January, 1928, p. 51; July, 1933, p. 311; and August, 1933, p. 898.

The presence of other salts and the concentration of dissolved solids may be expected to affect the position of the curves. The position of the curves is also shifted by changes in temperature, and the shift is greatest for smaller doses.

In the case of highly colored waters, best removal of color may be obtained by flocculating at low pH values, 4 or less, in which case very little oxide is formed and Al^{+++} or Fe^{+++} becomes an effective flocculating agent for the negatively charged color particles. Double coagulation is often required with an alkali added in the second stage in order to remove the residual Al or Fe as hydrous oxide.

The amount of coagulant required varies from about 0.3 gpg (1 grain per U.S. gallon, gpg = 17.1 ppm) for clear waters to more than 6 gpg for some waters of high turbidity. The proper coagulant to use for a given water, the amount required, and the optimum pH value for effective flocculation can best be determined by jar tests on a laboratory stirring device (Fig. 23).[1] Flocculation is initiated by the Brownian motion of the colloidal particles which brings them in contact with one

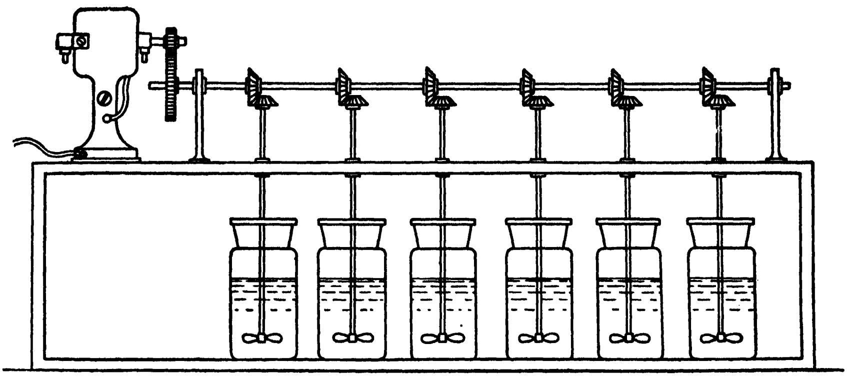


FIG. 23.—Laboratory stirring device.

another. When the particles become too large to be influenced by Brownian motion, they are still too small for effective settling and filtration. Hence, in order to continue the flocculation process, mechanical stirring is required. The stirring must be sufficiently violent to prevent settling but not violent enough to destroy the floc particles after they are formed. The laboratory stirring device performs in the laboratory the function of the mixing chamber or flocculator, and for comparable results the velocity gradients and temperature should be the same in the jars as in the flocculation chambers.

In coagulation of very clear waters, it is sometimes difficult to produce a good floc because of the absence of nuclei in the water about which the precipitates may collect. Finely divided clay in small amounts (1 to 7 gpg) has been successfully used in some cases to promote floc formation and hasten settling.

Feeding and Solution of Chemicals. Chemicals are proportioned to the water by dry feeding or by solution feeding. In the former method, the dry granulated chemical is fed volumetrically or gravimetrically from a dry-feed machine of which there are several types commercially available. Alum, ferric sulphate, and powdered activated carbon can be proportioned without difficulty with dry feeders. Hydrated lime and soda ash may be fed successfully if sufficient agitation is provided in the hoppers to prevent arching. It is difficult to feed copperas dry because of its tendency to cake. Dry feeders for dusty chemicals such as hydrated lime should be wholly

[1] HOOVER, *op. cit.*

enclosed. Feed machines are most conveniently fed from storage bins above. Figure 24[1] illustrates such storage bins for a lime-softening plant.

The chemicals are usually raised to the storage rooms or bins by means of elevators, but where large quantitites of chemicals are used, pneumatic conveyors such as illustrated in Fig. 24 are economical. Dry feeders usually dose the chemicals into a small stream of water which is discharged through a pipe or hose into the water at the desired point.

In solution feeding, the solid chemical is first dissolved in tanks provided for that purpose to produce a solution of known strength which is then proportioned to the water with a pump or through an orifice. Solutions of alum, copperas, and soda ash are usually prepared with a strength of less than 6 per cent. Hydrated lime, which is too insoluble to be prepared as a clear solution in small quantities of water, is prepared as milk of lime with about the same strength. Quicklime, CaO, must be slaked before it can be used. Slaking is usually done with warm water in iron tanks. Solid ferric sulphate is usually proportioned dry but should be mixed for not less than 20 min with water heated to above 25C to produce a 35 per cent colloidal solution. Ferric chloride is usually fed as a 2 to 3 per cent solution. Liquid ferric chloride, which is shipped in drums or tank cars as a 38 to 44 per cent solution, is diluted to 2 to 3 per cent for feeding.

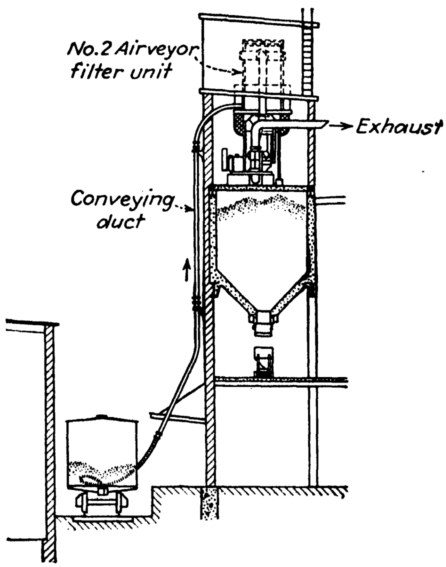


FIG. 24.—Pneumatic conveyor. (*Fuller Co.*)

The time required to dissolve solid chemicals is directly proportional to the size of the particles in dilute solutions. The rate of solution of alum, ferric sulphate, and ferric chloride increases two- to three-fold for each 10C increase in temperature and appears to be independent of the concentration of the solute up to about 8 per cent. Alum and ferric sulphate require about 10 min for each 1.0 mm of size of particle to dissolve at 10C in dilute solutions. Ferric chloride dissolves at nearly a hundred times this rate, but lime dissolves at very much slower rates than alum. Solutions should be kept stirred during preparation in order to procure uniform concentrations and to speed up the process.

Solutions of alum and iron salts are quite corrosive. Glazed tile, hard rubber, and lead-lined pipe are satisfactory for conveying the solutions. Solution tanks should be lined with rubber, acidproof brick, or other acidproof lining. Iron pipe may be used for lime and soda solutions, but with milk of lime, flexible rubber hose is better in order that deposits may be readily removed. Milk of lime, when cold, may be stored in concrete tanks. Soda ash, however, should be dissolved and stored in iron tanks, since it attacks concrete.

Mixing and Flocculation. When the chemicals are added to the water, they should be quickly and uniformly dispersed throughout in order to hasten the chemical reactions. The chemical reactions after the chemicals are dissolved are almost instan-

[1] Public Works Magazine, "Manual of Water Works Equipment and Materials," 1936.

taneous, but the formation of the floc is a time-consuming process. The principal function of a mixing chamber or flocculator is to form the floc, and its capacity is determined from the time required for this function. Another function of mixing, however, is the dispersion of the chemicals in the water. This function is automatically performed by all mixing chambers, but dispersion may be hastened by a violent initial mixing. Dispersion, which is particularly important when the chemicals are not wholly dissolved, may be accomplished almost instantaneously by means of the low-lift pumps, aerators, the hydraulic jump, chutes, falls, or flash mixers. The value of a violent initial mix has not been conclusively demonstrated for water treatment, but Rudolfs[1] has shown a decided improvement in clarification of sewage by chemical coagulation when flash mixing precedes flocculation.

The agglomeration of small dispersed particles to form larger ones is brought about first by true diffusion or Brownian motion and thereafter by the relative motion of the suspension usually by turbulent mixing. It has been shown by Camp, Root, and Bhoota[2] that the Brownian motion phase of coagulation is completed in a few seconds and is therefore of negligible importance in fixing tank dimensions as compared to the turbulent mixing phase.

Camp and Stein[3] have shown that the speed of flocculation at a point in a moving suspension is directly proportional to the concentration of suspended particles and to the space rate of change of velocity or *velocity gradient* at the point. For efficient coagulation, therefore, the motion of the suspension should be great enough to prevent settling of the floc particles and to produce velocity gradients of sufficient magnitude to promote rapid coalescence. The velocity gradients should not be great enough, however, to shear apart floc particles already formed. For best results, flocculation should be carried out in several stages in a series of tanks with the velocity gradients progressively decreased as the floc particles grow in size. This procedure was first developed by Langelier and is known as the Langelier process.

The velocity gradients throughout a flocculation tank vary considerably, but the speed of flocculation may be taken as proportional to the *mean velocity gradient, G,* which is given by the relation:[3]

$$G = \sqrt{\frac{W}{\eta}} \tag{28}$$

in which W is the work done on the water by shear per unit of volume per unit of time and η is the absolute viscosity.

The value of W for tanks equipped with mechanical mixers may be computed from the power input to the shaft with suitable deductions for friction losses in shaft bearings and stuffing boxes. For tanks in which mixing is produced by an inlet jet, W may be computed from the kinetic energy of the jet. For baffled mixing channels, W may be computed from the discharge and head loss as follows:

$$W = \frac{Qg\rho h_f}{AL} \tag{29}$$

where Q = discharge,
g = gravity constant,
ρ = mass density,
h_f = head loss,
A and L = cross-sectional area and length of channel.

[1] *Sewage Works Jour.*, July, 1936, p. 547.
[2] *Jour. A.W.W.A.*, **32**, 1913, 1940.
[3] CAMP and STEIN, *loc. cit.*

The values of the mean velocity gradient, G, used in existing flocculation basins[1] for water purification plants throughout the United States vary from about 20 to 75 fps per ft (sec^{-1}). These figures represent average values for each plant. Where the Langelier process is used, it is probable that the initial value of G may exceed 100 and the final value may be less than 10 sec^{-1} in some cases. It is probable that the optimum values for G vary widely for different types of floc, but little is known about the optimum values.

The effect of temperature on the velocity gradient, G, and hence on the speed of flocculation, is felt through the factor $\sqrt{h_f/\nu}$ for mixing channels and the factor $\sqrt{C/\nu}$ for mechanical flocculating devices, where ν is the kinematic viscosity and C is the drag coefficient of the paddles. Since the flow is almost fully turbulent in mixing channels, h_f changes very little with the temperature. Since the value of ν for water is about doubled for a temperature change from 80F to the freezing point, the speed of flocculation in mixing channels should be reduced about 30 per cent by this change in temperature. For mechanical mixers, however, the value of the drag coefficient C decreases with increases in Reynolds number, and the effect of temperature on the speed of flocculation may therefore be considerably less.

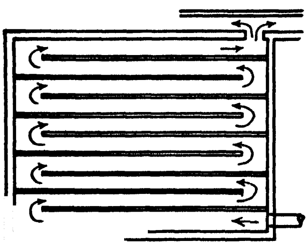


FIG. 25.—Plan of round-the-end baffled mixing basin.

Since the speed of flocculation varies directly with the magnitude of the mean velocity gradient G, it follows that satisfactory coagulation should be produced with particular values of the product GT where T is the flocculation period. If G and T are expressed in seconds, GT is a dimensionless number. The values of GT for American flocculation basins[1] of all types range from 23,000 to 210,000, and correspond to floc periods ranging from 10 to 100 min. The velocities used for channel mixing chambers range from about 0.3 to 3 fps, and the same range of velocities is generally employed for paddle-tip speeds in mechanical mixers.

Two general types of mixing basins are in common use: baffled basins and tanks with mechanical stirring devices or tangential flow. Baffled basins are of two types:

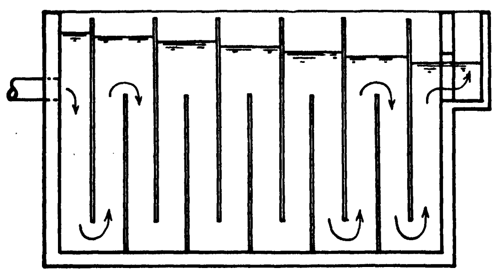


FIG. 26.—Section through over-and-under baffled mixing chamber.

with round-the-end baffles as illustrated in Fig. 25 and with over-and-under baffles as illustrated in Fig. 26. One of the important considerations in the use of baffled mixing chambers is the head loss, which ranges from about 0.5 to nearly 3 ft, depending upon the number of 180-deg bends. With round-the-end baffles, the head loss per bend is approximately 3.5 times the velocity head in the adjacent channels. With over-and-under baffles, the head loss per bend does not vary directly with the channel velocity head, because the cross section area above the over baffles increases with the discharge

[1] Water Treatment Plant Design *A. S. C. E. Manual* 19, 1940, p. 29.

which is not true of the area under the under baffles. Relatively long channels and few bends are desirable to conserve head and decrease short-circuiting. For this purpose, round-the-end baffles are better. Because of economic limitations to the basin depth, over-and-under baffles are suitable only for small plants. On the other hand, round-the-end basins are not suitable for small plants. The channel cross section should have a minimum width of about 15 in. for economy in construction, and the depth should be relatively great as compared with the head loss in order that the velocity may not vary too much along the channel. For economy, the depth should generally exceed 6 ft.

Mixing basins with mechanical agitators are usually made with two or more basins in series, as shown in Fig. 18 for horizontal shafts and in Fig. 27 for vertical shafts. In some plants, the mechanical stirrers have been omitted, and sufficient velocity is provided at inlets to impart a whirling motion to the basin contents. The great advantage of mechanical mixers is the provision for varying the velocity of stirring independently of the discharge. This advantage is lost in basins without stirring devices. In them, the velocity is proportional to the discharge as it is with baffled basins. Another advantage in basins of this type, with or without stirrers, is the negligible head loss.

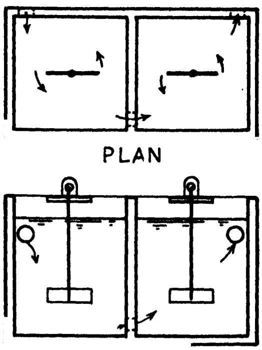


FIG. 27.—Mixing chambers with mechanical agitation.

A serious drawback to mechanical and tangential flow tanks is short-circuiting. Hydraulic studies at the Massachusetts Institute of Technology[1] on miniature mixing basins indicate that the minimum time is almost zero and the probable flowing-through time may range from about 0.50 to 0.90 of the retention period with an average of about 0.75 for a mechanical mixer similar to Fig. 27; whereas in a baffled basin of the round-the-end type the minimum time was 0.75 and the probable flowing-through time ratio was 0.94 of the retention period. The practice of using two or more mechanical mixing basins in series, as is required with the Langelier process, also results in reduced short-circuiting. The probable flowing-through time, for two basins in series should be about 0.87 and for three basins about 0.92 of the retention period.

FILTRATION

Operation of Rapid Filters. The essential features of a rapid sand filter are illustrated in Figs. 28 and 29.[2] Each filter is equipped with four valves: influent, effluent, wash, and waste. Sometimes a connection is provided from the manifold to the drain and equipped with a filter-to-waste valve. The effluent pipe is provided with a rate controller which keeps the discharge constant.

During filtration, the influent and effluent valves are open and the others are closed. The water enters from the settling tanks, passes up through and over the wash gutters and thence downward through the sand and into the underdrainage system and out through the effluent valve. The water level over the filters is controlled by regulating the rate of inflow, sometimes by means of a butterfly valve.

When a filter needs to be washed, it is taken out of service by first closing the influent valve. When the water has drained down below the level of the gutters, the effluent valve is closed and the drain is opened. The wash-water valve is then opened slowly, which permits filtered water to enter the manifold from the wash-water

[1] BAGNULO and KNIGHT, bachelor's thesis, M.I.T., 1936.
[2] HOOVER, *op. cit.*

pump or tank and pass up through the bed over the gutter weirs and out through the gutters and drain. When the rate of wash reaches a maximum after about 1½ min, most of the sand is in suspension and the bed is expanded 20 to 60 per cent above its normal depth. This rate of wash is maintained until the water above the sand begins to clear, usually 2 to 4 min, and then the wash valve is slowly closed. To place the bed back into service, the drain is closed and the influent and effluent valves are reopened. When the filter is provided with a filter-to-waste connection, the

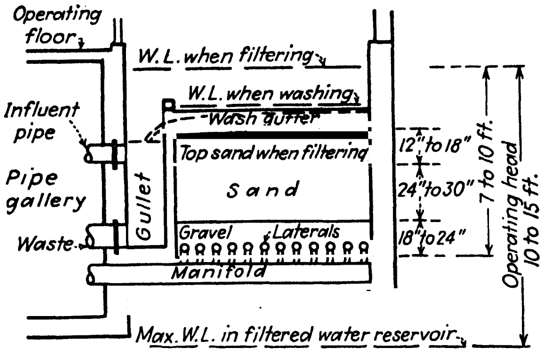


FIG. 28.—Diagrammatic section through a rapid sand filter.

filtered water is sometimes discharged to the drain for a few minutes after washing. This practice, which is seldom necessary, is to prevent sediment that may possibly be near the bottom of the sand from passing into the filtered water reservoir.

The permissible length of a filter run between washes is fixed by the head lost through the bed or the character of the filter effluent. Usually 7 to 10 ft of the operating head is available for loss through the sand and rate controller. When a clean

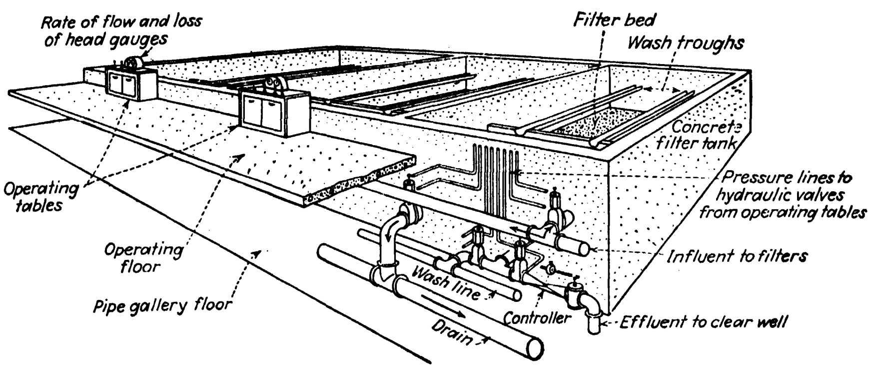


FIG 29.—Battery of three rapid sand filters. (*Hoover.*)

bed is put back into service, the head loss through the sand is about 1 ft, the remainder of the 7 to 10 ft being taken by the controller which is nearly closed. As the bed clogs and the head loss through it builds up, the controller opens so that the total loss through bed and controller remains constant until the controller is wide open. At this point, the rate will change; hence washing becomes necessary before this point is reached. In many cases, however, a filter will begin to pass abnormal turbidity before all the available head is used up.

The length of filter run is an important factor in the economy of filtration. If filter runs are very short, the capacity of the plant is reduced because of the time filters are out of service for cleaning. Moreover, the portion of filtered water required for washing, which is subsequently wasted, becomes abnormally high and results in higher costs of plant operation. Generally, the water required for washing should be less than 2 per cent of the total water filtered. The length of filter runs is determined principally by the efficiency of pretreatment, the rate of filtration, and the character of the sand.

Hydraulics of Flow through Sand. Viscous flow of clean water through clean sand and other loose granular media, which has been studied by many investigators appears to be represented very well by an equation developed rationally by Kozeny[1]

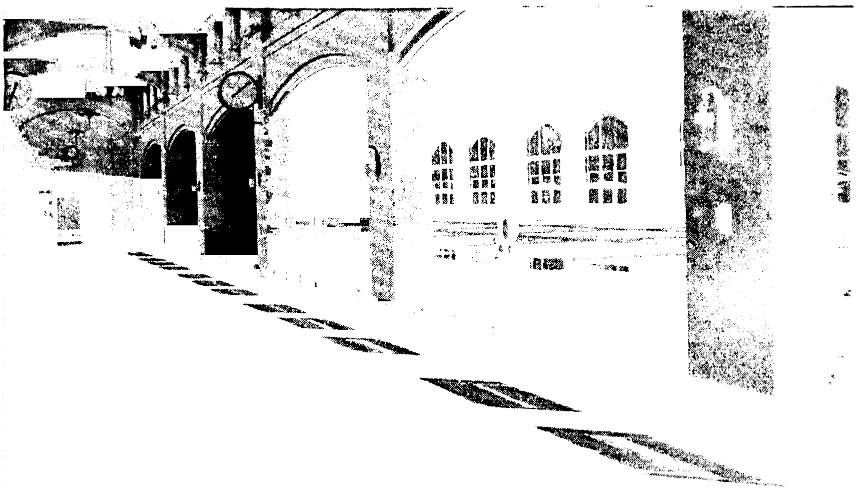


FIG. 30.—Operating floor of water treatment plant at Saginaw, Mich.

from Poiseuille's law of flow through capillary tubes. The same equation was later developed independently by Fair and Hatch.[2] This equation states that the hydraulic slope through homogeneously packed sand of uniform grain size is

$$S = \frac{h}{l} = \frac{\beta\eta}{g\rho}\frac{(1 - p)^2}{p^3}\frac{s^2}{d^2}v \tag{30}$$

in which h = head loss through sand bed thickness, l.
p = porosity ratio.
s = a shape factor for the sand, the constant obtained for a given shape by dividing the surface area by the volume (6 for spheres).
d = diameter of the grains.
v = rate of filtration as a velocity.
β = a constant.

η, g, and ρ are the same as for Eqs. (12) and (13). It will be noted that the value of the permeability coefficient k of Eq. (2), Sec. 19, is, from Eq. (30),

$$k = \frac{g\rho}{\beta\eta}\frac{p^3}{(1 - p)^2}\frac{d^2}{s^2} \tag{31}$$

[1] KOZENY, J., *Wasserkraft und Wasserwirtschaft*, **22**, 67, 1927. See also DONAT, *Wasserkraft und Wasserwirtschaft*, **24**, 228, 1929.
[2] *Jour. A.W.W.A.*, **25**, 1551, 1933.

For a homogeneously packed sand bed of varying grain size but constant shape factor, the Fair-Hatch equation is

$$\frac{h}{l} = \frac{\beta\eta}{g\rho}\frac{(1-p)^2}{p^3}vs^2\left(\int_{P=0}^{P=1}\frac{dP}{d}\right)^2 \tag{32}$$

in which P is the ratio of the weight of the grains of size d or less to the total weight of the sand. For the evaluation of the term under the integral, P must be determined as a function of d from the sieve-analysis curve (such as Fig. 31).

For a bed of varying grain size, constant shape factor, uniform porosity, and stratified according to grain size, the hydraulic slope dh/dl at any depth l at which the grain size is d is given by Eq. (30). The total head loss to any depth l less than the bed thickness L (since $l = PL$) is

$$h = \frac{\beta\eta}{g\rho}\frac{(1-p)^2}{p^3}vs^2L\int_{P=0}^{P=P}\frac{dP}{d^2} \tag{33}$$

Equation (33) is approximately correct for a clean rapid filter that has been thoroughly backwashed, but stratification of grain size is not perfect, and the porosity and shape factor are probably not the same from top to bottom.

If the sand size in the sieve-analysis curve is stated in terms of the size of sieve openings (curve B, Fig. 31), the value of β[1,2] is approximately 5 in any consistent system of units. If the sand size is determined by Hazen's[3] count and weigh method, the value of β is approximately 6. In this method, a number of grains that barely pass a sieve are counted and weighed to determine the size of separation of the sieve. The size of separation is the diameter of the sphere whose volume equals that of the average of all the grains weighed. The porosity of sands varies from about 0.34 to 0.48; and that of a single sand varies over more than half this range, according to the degree of compactness. The value of the porosity must be accurately known or the errors in the use of the preceding equations will be great. According to Fair and Hatch,[1] the value of the shape factor s varies from 6 for spheres to about 7.7 for angular grains. It is more practical to evaluate βs^2 as an over-all characteristic of a sand than to obtain separate values for β and the shape factor.

The size of a sand can be given completely only in terms of a sieve-analysis curve. Hazen attempted to define a sand in terms of a single size that would give an equivalent head loss and the same filtration characteristics for a slow sand filter. This size, called the *effective size*, is, according to Hazen, the 10 per cent size. The uniformity of grain size is measured roughly by Hazen's uniformity coefficient, the ratio of the 60 per cent size to the 10 per cent size. These terms are widely used for comparing sands; and for rapid filters, the 10 per cent size ranges from about 0.35 to 0.60 mm and the uniformity coefficient ranges from about 1.2 to 1.7. It will be noted that the size which produces an equivalent head loss for the sand whose analysis is given in Fig. 31, curve A, is 0.61 mm for a homogeneous bed such as would be found in slow sand filters [Eq. (32)] and is 0.60 mm for a stratified bed of a rapid filter [Eq. (33)]. For this particular sand, the true effective size as regards head loss is nearly the 50 per cent size.

The relation of Reynolds number to friction factor for spherical grains as given by Eq. (30) is shown in Fig. 32. Based upon experiments by several investigators,[4] the effect of eddying resistance begins to be felt at Reynolds numbers exceeding about

[1] FAIR and HATCH, *idem*.
[2] PHILLIPPI, bachelor's thesis, M.I.T., 1938.
[3] *Mass. Dept. Health, Ann. Rept.*, 1892, p. 544.
[4] HICKOX, *Trans. Am. Geophys. Union*, 1934, Part II, p. 567; BAKHMETEFF and FEODOROFF, *Jour. Applied Mech.*, A.S.M.E., September, 1937, p. A97.

12. The position of the curves in Fig. 32 for higher values of R was estimated approximately from the experimental data of these investigators. In rapid filtration, the value of R ranges from about 0.4 to 3.0 for clean filters and is less for dirty filters. The value of R corresponding to the maximum rate of wash in use is about 30, but

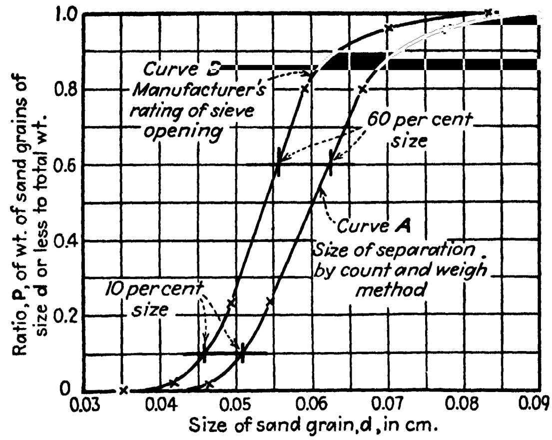


FIG. 31.—Sieve-analysis curves of typical filter sand.

the corresponding head loss cannot be determined from Fig. 32. During washing, the sand grains are in suspension and in motion. The value of f appears to be about the same under these conditions as for still sand at lower values of R, the corresponding value of β being about 4 on the basis of sieve sizes and about 5 on the basis of count and weigh sizes, according to the experiments of Fair and Hatch and of Eliassen.[1]

Mechanism of Filtration. Suspended matter is removed from water by sand filters through the adhesion of small particles to the surfaces of the grains and by the straining of large particles in the pore constrictions. The small particles are brought into contact with the surfaces of the grains by the convergence of the streamlines upstream from each constriction in the flow paths. In slow sand filters, removal by adhesion and straining is expedited to some extent by flocculation of suspended particles within the pores; but experiments by Stein[2] indicate that the velocity gradients (being of the order of 200 sec^{-1}) are too high in rapid sand filters to permit much flocculation.

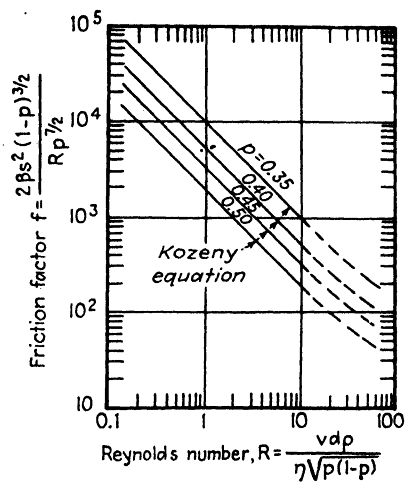


FIG. 32.—Kozeny equation for rounded sand grains ($\beta s^2 = 240$).

As the filter passageways become reduced in cross-sectional area owing to the accumulation of adhering material, the velocities in the pores and hence the shear will increase. When the shear forces become large enough, the rate of adhesion is decreased and suspended matter will be carried more deeply into the filter.

[1] ELIASSEN, doctor's thesis, M.I.T., 1935.
[2] STEIN, doctor's thesis, M.I.T., 1940.

If the floc particles are larger than the pore constrictions, they will deposit right at the top of the bed and filter runs will be very short; if very much smaller, they will penetrate deeply into the bed and filter runs will be longer. As the top pores become clogged, some of the suspended particles will settle on top of the bed above these pores and thus will be removed by sedimentation. The mud blanket thus formed, called the *schmutzdecke*, was once thought to be essential to proper filter performance. It is now known, however, that it bears no relation to the removal effected and is a positive detriment to rapid filters as it is the main source of formation of mud balls during the washing process. Many filters perform efficiently without a mud blanket.

A certain ripening process of a filter after it is first put into service, during which the sand grains become partly coated, is known to improve the efficiency of the filter. This improvement is probably due to an increase in effective surface area of the grains. Slow sand filters are known to be poor removers of negative color colloids. Rapid-filter sand becomes coated with alumina or ferric oxide which is adsorbed from the floc. This adsorption process is so slow, however, as to be a negligible factor in the removal accomplished in a single filter run.

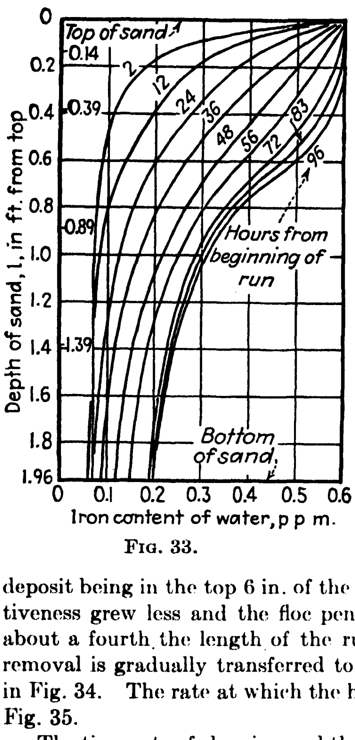


Fig. 33.

The mechanism of filtration is illustrated well by the data from a filter run taken by Eliassen[1] during studies made with an experimental rapid filter at the Providence, R. I., water-treatment plant. The finer sand grains were screened out to ensure penetration of all the floc into the bed, and the performance of the filter as regards removal and length of run before too much floc was passed was correspondingly impaired. The average size of iron floc particles in water samples taken from the filter varied from about 11μ above the filter to about 8μ at a depth of 0.89 ft. The size of the pore constrictions was about ten to twenty times the floc size. The sand size for this bed is shown in Fig. 31. The penetration of floc into the bed is illustrated in Fig. 33. It will be noted that at the start of the run about 90 per cent removal was accomplished, nearly all the deposit being in the top 6 in. of the bed. As the top sand became clogged, its effectiveness grew less and the floc penetrated deeper, the total removal decreasing after about a fourth the length of the run. During the clogging process, the burden of removal is gradually transferred to sand deeper in the bed, which is illustrated better in Fig. 34. The rate at which the head loss built up during this run is illustrated in Fig. 35.

The time rate of clogging and the depth of penetration of suspended matter depend upon the sand size and the quantity and size of suspended matter in the applied water. The time rate of clogging as measured by lost head dh/dt appears from many observations to be approximately constant or to increase slightly with time, if the size and amount of suspended matter applied to the filter, the rate of filtration, and the temperature of the water remain constant during a run. The time rate of clogging for a given water, according to experiments by Baylis[2] and others,[3] varies inversely with

[1] *Op. cit.*
[2] *Water Works and Sewerage*, October, 1934, p. 352.
[3] *Proc. A. S. C. E.*, December, 1936, p. 1543.

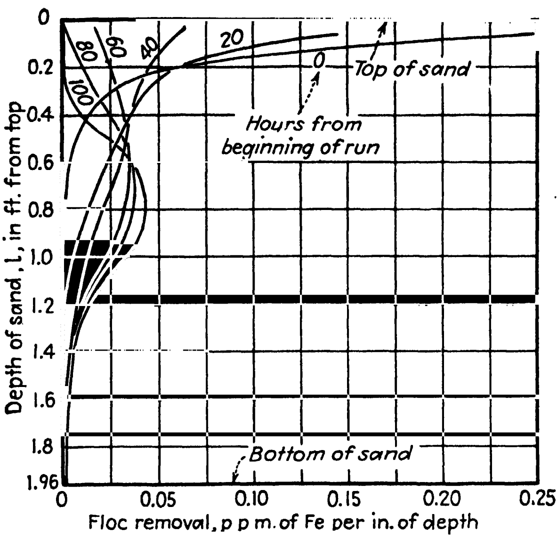


FIG. 34.

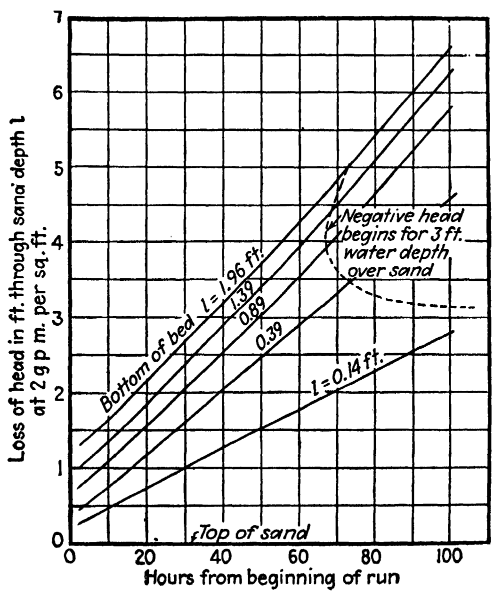


FIG. 35.

some power between 1 and 2.15 of the sand size; and according to Stein[1] varies inversely with the square of the sand size.

Numerous experiments under the author's direction appear to indicate that *dh/dt* is about directly proportional to the volume of suspended matter in the water applied to the filter, other factors remaining constant. Inasmuch as the head loss

[1] *Op. cit.*

and the rate of application of suspended matter to a filter are both directly proportional to the rate of filtration, it might be expected that *dh/dt* should vary as the square of the rate of filtration. This is not the case, however, for the solids penetrate deeper into the bed with increasing velocities. Experiments indicate that *dh/dt* increases approximately with some power of the rate of filtration less than 1.

The head at any point in a filter during filtration is equal to the static head at the point less the lost head in the sand layer above the point. When the lost head exceeds the static head, a partial vacuum called a *negative head* is produced at the point. Negative heads are produced in nearly all filters near the end of a run when the clogging is great. For example, if a 3-ft water depth is assumed over the sand in the filter run illustrated by Fig. 35, negative heads will begin to develop for the depths and times marked by the dotted line. A partial vacuum is not very harmful, but if it becomes too high it will result in the evolution of dissolved air from the water which may cause air-binding of the sand until the bed is washed. Negative heads may be prevented in design by making the static head greater than the proposed maximum head loss through the bed. This procedure calls for a water depth over the sand generally exceeding 8 ft, and the resulting filter structure will in most cases be costly.

The pressure corresponding to the head loss in the top layer of a filter, which becomes very high because of clogging, tends to compact the sand and results in shrinkage. Shrinkage is greater for high porosities and for dirty sand. The path of flow adjacent to the filter walls is less tortuous and the head loss correspondingly less than through the bed proper. Therefore the head at any depth is greater at the wall than at a small distance from it. This difference in head results in flow inward from the wall and a corresponding shrinkage of the sand away from the wall. As the shrinkage cracks open, short-circuiting of more water and sediment to these cracks results. Mud sometimes penetrates through the sand because of shrinkage, and caking of the bed near the wall results. Shrinkage cracks also occur in the interior of some sand beds owing to inequalities in flow and sand shrinkage. No adequate remedy has been found for this trouble, but shrinkage can be minimized by using coarse sand and washing it clean.

Clean silica sand is the most widely used filtering material, but other materials, including anthracite coal and magnetite, are used to some extent. The filtration process itself depends only upon the size and shape of grains, and any durable material is satisfactory. The washing process, however, is also influenced by the density of the material as shown in the following article.

Washing of Rapid Filters. The effective washing of a rapid filter requires (1) a sufficient velocity of water (or air) past each grain to dislodge the sediment, (2) a sufficient expansion of the bed to permit the sediment to pass upward between the grains, (3) a sufficient wash rate to carry the sediment upward above the bed, and (4) a length of wash great enough to allow the sediment to pass out of the filter.

Early rapid filters were washed at rates of about 18-in. vertical rise per minute (12-in. rise per minute corresponds to a filtration rate of 7.5 gpm/sq ft) when water alone was used and at lesser water rates when air or mechanical raking of the surface of the beds was also used. Air is usually distributed to the underside of the sand through the water underdrainage system or through a separate perforated-pipe system just above the underdrainage system. The difficulty of distributing air uniformly, the extra cost of using air, and the danger of air-binding have led to the general abandonment of the air-wash method in the United States in favor of higher velocity of wash with water. It is general practice to use rates of wash from 24 to 42 in., depending upon the size and density of the filtering material and the temperature.

In order to wash a bed properly, all the grains to a depth at which there is any appreciable deposit of floc should be suspended, and a sufficient rate of wash should be

available to suspend the entire bed when necessary. When the grains are suspended, the head loss is the head corresponding to the weight of the material in water, and the maximum head loss is reached when all the grains are suspended. Since the weight is constant, the head loss is independent of the temperature and the amount of expansion; and since the head loss is a function of the velocity past the grains, the cleansing action of the water is not increased with increasing rates of wash after the bed is in suspension.

By following the method of Fair and Hatch[1] and equating the friction loss through the thickness dl_e of the expanded bed to the weight of the sand in water, it is found that

$$dh_e = dl_e \frac{\rho_1 - \rho}{\rho} (1 - p_e) \tag{34}$$

in which dh_e = head loss through the expanded layer of depth dl_e.
dl_e = expanded depth of the layer whose original depth is dl.
p_e = expanded porosity.
ρ_1 = density of the grains of the filtering material.

By equating the value of dh_e/dl_e from Eq. (34) to the corresponding value from Eq. (30), the following equation results for the expanded bed:

$$v_w = \frac{g(\rho_1 - \rho)}{\beta \eta s^2} \frac{p_e^3}{1 - p_e} d^2 \tag{35}$$

in which v_w is the wash rate as a velocity.

The wash rate at which expansion starts may be computed from (35) for values of $p_e = p$. The amount of expansion of the layer dl is

$$e = \frac{dl_e}{dl} - 1 = \frac{p_e - p}{1 - p_e} \tag{36}$$

and the total expansion is

$$E = \int \frac{p_e - p}{1 - p_e} \tag{37}$$

The value of p_e for Eq. (36) may be determined from (35) for any wash rate.

A study of the experiments of Hulbert and Herring[2] on the washing of rapid filters indicates that the expansion may be computed approximately from Eqs. (35) and (36) for most sands by using the 30 per cent count and weigh size and a value of β of 5.

As shown by Eqs. (35) and (36), the rate of wash for a given expansion, and therefore the cleansing efficiency, varies directly as the weight of the material in water. Light materials such as coal require large grain sizes for efficient cleansing. The amount of expansion required to get all the sand in suspension depends upon the size and grading of the material. Expansions used in practice vary from about 30 to 50 per cent. The freeboard between the top of the sand during filtration and the bottom of the wash troughs (see Fig. 28) should be great enough for the maximum expansion required. In many sand filters, the surface sand is too small to be washed effectively by suspension alone. Baylis[3] recommends a supplementary surface wash of 2 to 8 gpm/sq ft applied through perforated pipes immediately above the sand surface at a pressure of about 10 lb in the pipes in order to break up the clogged masses at the surface before the bed is fully expanded. Mechanical raking is also of value.

Hydraulic Design of Filters. The sizes of pipes and conduits in the pipe gallery should be kept small for economy, but in general the velocities at the nominal capacity of the plant and at the maximum wash rate should not exceed the following:

[1] *Jour. A.W.W.A.*, **25**, 1551, 1933.
[2] *Jour. A.W.W.A.*, **21**, 1445, 1929.
[3] *Water Works and Sewerage*, January, 1935, p. 20.

Pipe	Fps
Filter influent	2
Filter effluent	5
Wash water	12
Waste water	8
Filter to waste	15

For fragile floc, limiting influent velocities should be estimated from limiting velocity gradients which will not rupture the floc. Local considerations, such as inadequate head, may require lower velocities than given above for the other lines.

The principal function of a filter bottom or underdrainage system is to distribute the wash water uniformly to the underside of the sand bed. The most commonly used type of filter bottom is one composed of perforated pipes surrounded by gravel as illustrated in Fig. 28. The perforations are usually circular orifices drilled in the underside of the laterals at uniform intervals preferably not greater than 12 in. The velocity of the wash water issuing from the orifices is largely dissipated against the floor and in the gravel around the laterals.

The function of the gravel is to support the sand and to spread the wash water over the covering area of each orifice before it reaches the sand and thus to prevent jetting through the sand. This cannot usually be accomplished with less than 18 in. of gravel, which should be graded from about 10 mesh at the top to about 1½ in. at the bottom. The grading should be such that the particles of gravel are large enough at all depths to remain undisturbed by the velocity of the wash water past them, and it should be such that particles above cannot fall through the interstices of the lower gravel. According to Baylis,[1] the depth to any size gravel from the underside of the sand may be determined by the following empirical formula:

$$M = 12 \log d \tag{38}$$

where M is the depth in inches, and d the gravel size in millimeters. The head lost through graded gravel during washing, according to Dixon,[2] is approximately 0.1 ft for each 12 in. depth at a wash rate of 12 in./min and is in direct proportion for other depths and rates in common use. The maximum possible loss is equivalent to the weight of the gravel in water. Most of the head is lost through the finer material at the top which is the effective agent for the final distribution of the water to the sand.

A uniform distribution of the wash water to the orifices may be approximated by making the orifice head loss great as compared with the maximum difference in energy head available for flow through the orifices. Equation (17) may be used to determine the required orifice head loss. It is good practice to make the orifice ratio (ratio of orifice area to total bed area) 0.3 to 0.5 per cent, which, for a 36-in. wash rate and an orifice discharge coefficient of 0.75, corresponds to an orifice velocity of 16.7 to 10 fps and a head loss of 7.7 to 2.8 ft. The velocity head at the entrance to each lateral should generally be less than one-fifth the orifice head loss for good distribution. The velocity should preferably be less than 6 fps for the laterals and 8 fps for the manifolds.

Inasmuch as the fine gravel supporting the filter sand is the effective agent for distribution of the wash water to the sand and since the coarse gravel surrounding the orifices functions as an equalizing chamber, uniform distribution of wash water to the orifices is not so important as is commonly supposed.

The use of umbrella-type strainer heads fastened into the tops of the laterals was formerly widespread but has now been largely abandoned because of the superiority and economy of perforated pipes. In order to ensure uniform discharge coefficients for all the orifices, they may be fitted with brass or bronze bushings. Laterals

[1] *Jour. N.E.W.W.A.*, **51**, 1, 1937.
[2] *Water Works and Sewerage*, April, 1935, p. 103.

and manifolds are generally made of cast-iron pipe which should be lined with cement or bitumastic to retard corrosion. Numerous other types of filter bottoms have been developed. One of the most satisfactory is the Wheeler bottom illustrated in Fig. 36[1] which requires less gravel and has the advantage of a low head loss. The head loss through a Wheeler bottom, according to tests by Barbour,[2] is approximated by the following formula:

$$h = 0.004v_w^2 \tag{39}$$

where h is the head loss in feet, and v_w the wash rate in inches per minute. The wash water is delivered to the underside of a Wheeler bottom through concrete channels or from an equalizing chamber below the Wheeler slab, in which latter case the slab is supported above the filter floor on columns.

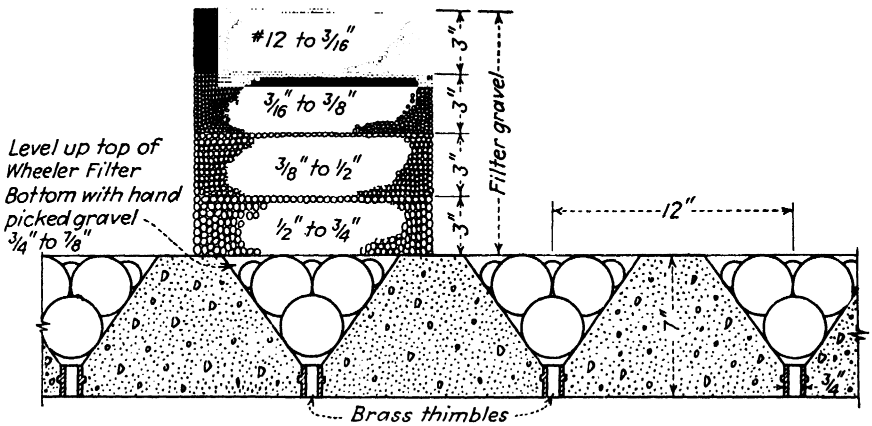


FIG. 36.—Wheeler filter bottom.

All filter bottoms which require graded gravel to support the sand depend upon the top layer of loose fine gravel to effect uniform distribution of wash water to the underside of the sand bed. In order to function properly, the top layer of fine gravel must be carefully sized and placed and must not be displaced by the washing process. Unfortunately, some sediment will always pass through the sand during filtration, and some of this will accumulate in the pores of the fine gravel so as to increase the velocity through this gravel during washing until ultimately the velocity is sufficient to lift the gravel and mix it with the sand. This is a defect inherent with the use of loose gravel, and it requires the periodic removal and replacement of the sand and gravel at intervals of 2 to 10 years.

Another type of filter bottom which is coming into more general use is the porous-plate bottom first developed by Camp[3] in which the filtering material is supported directly upon a false bottom of porous plates. The plates are constructed of granular material cemented rigidly together, and they are supported above the floor as shown in Fig. 37 to produce an equalizing chamber under the plates. The porous plates perform the same function as the top layer of gravel in other types of bottoms, but the grains in the plates are not subject to displacement by the washing process. The pores of the plates gradually clog with sediment, and the eventual replacement of the plates or periodic cleaning with acid is indicated. Experience indicates that porous

[1] DIXON, *idem.*
[2] *Trans. A. S. C. E.*, **80**, 1415, 1916.
[3] CAMP, *Jour. N.E.W.W.A.*, **49**, 1, 1935.

bottoms may be operated up to 10 years or more without cleaning, and that cleaning may be accomplished without removal of the sand. The clogging of the plates is a self-equalizing process so that wash-water distribution is not impaired with partially clogged plates.

Wash-water gutters with level bottoms may be designed by means of Eq. (18), due account being taken of the additional drawdown of the water surface due to friction. Usually, gutters discharge freely into the gullets, in which case the depth h at the discharge end may be assumed at the hydrostatic critical depth without serious error. Equation (18) then becomes

$$H = \sqrt{3}\,h_c \tag{40}$$

in which h_c = the hydrostatic critical depth, $= \sqrt[3]{\frac{q^2x^2}{gb^2}}$.

In practice, gutters are spaced with clear distances between the weirs of 3 to 8 ft. The spacing is not important if the sand is not permitted to expand above the gutter bottoms. Gutters are made of steel, cast iron, and reinforced concrete. Metal gutters should be adequately coated to reduce corrosion. Gullets may also be designed by means of Eq. (18) except that the inflow is at intervals and not continuous along the length of the gullet. Gullets should be wide enough for the access of workmen.

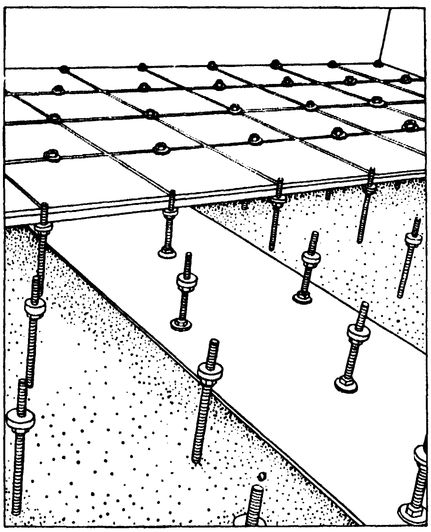


FIG. 37.—Porous-plate filter bottom. (*Courtesy of Walker Process Equipment Inc.*)

Most rapid filter plants are equipped with elevated tanks or standpipes for the wash water. Tanks should have capacities about 50 per cent in excess of the amount of water required for a single wash unless the number of filters is so great that two must be washed at the same time. For small plants, wash-water pumps, in duplicate, with capacities sufficient to refill the tanks between washes should be provided. In large plants where washing is almost continuous, the capacity of the pumps must be so great that water may be supplied direct from pumps to filter and the tanks may be dispensed with.

The vertical height between the weir of the wash troughs and the bottom of the wash-water tank should be in excess of the sum of the following heads for the maximum wash rate:

1. Loss of head through expanded sand
2. Loss of head through gravel
3. Loss of head through filter bottom
4. Loss of head through wash-water piping

The drawing depth in the tank should be small for economy in pumping.

Filter Accessories. The gate valves for modern rapid filters are usually power operated, most frequently by means of hydraulic cylinders. Since these valves are operated very frequently, they should be well constructed and bronze fitted through-

out. The valves are controlled from operating tables, each filter being equipped with a table as shown in Figs. 29 and 30. Loss of head and rate of flow gages are also mounted on the operating tables, and in order to have continuous records of the performance of each filter, these gages should be recording.

The discharge line from each filter should be equipped with a rate controller as shown in Fig. 29, and for large plants master rate controllers are sometimes desirable. All rate controllers consist of two essential parts, a measuring device and a valve actuated by the measuring device. Venturi tubes are commonly used as the measuring device and are superior to orifices because of the lower head loss and more constant discharge coefficient. Figure 38 illustrates one type of rate controller.

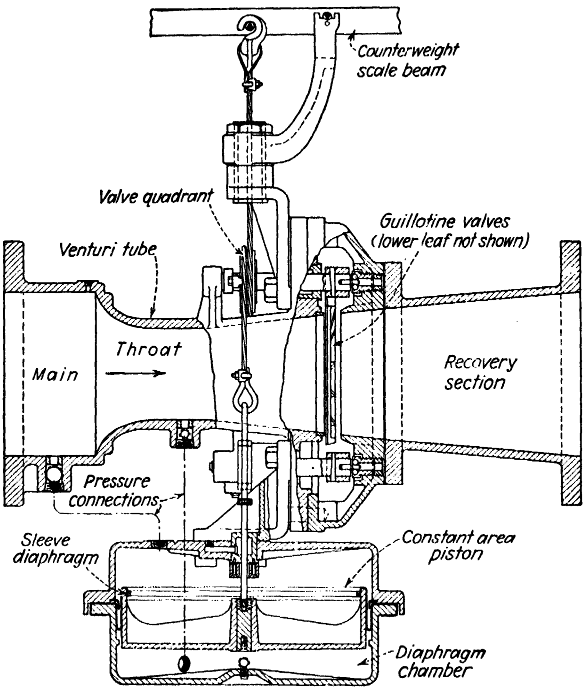


FIG. 38.—Typical rate controller. (*Courtesy of Simplex Valve and Meter Company.*)

Many other accessories such as sampling devices, sand-expansion indicators, and automatic-control equipment for wash-water pumps are desirable for modern plants.

SOFTENING

Lime-soda Process. The purpose of adding lime in softening is to convert free CO_2 and HCO_3^- to normal CO_3^{--} which reacts with Ca^{++} to form the precipitate $CaCO_3$ and to add sufficient OH^- to remove Mg^{++} as the insoluble $Mg(OH)_2$. Ca^{++} and Mg^{++} will be removed to the extent that there is CO_3^{--} and OH^- available to form the precipitates. If the water contains SO_4^{--}, NO_3^-, or Cl^-, the so-called incrustants that contribute to noncarbonate hardness, the conversion of CO_2 and HCO_3^- to CO_3^{--} does not furnish sufficient CO_3^{--} to remove the Ca^{++}. Additional CO_3^{--} must therefore be furnished, and the usual method is to add soda ash, Na_2CO_3.

The reactions with lime are as follows:

$$Ca(OH)_2 \rightleftarrows Ca^{++} + 2OH^- \tag{41}$$
$$CO_2 + OH^- \rightleftarrows HCO_3^- \tag{42}$$
$$HCO_3^- + OH^- \rightleftarrows CO_3^{--} + H_2O \tag{43}$$
$$Ca^{++} + CO_3^{--} \rightleftarrows \underline{CaCO_3} \tag{44}$$
$$Mg^{++} + 2OH^- \rightleftarrows \underline{Mg(OH)_2} \tag{45}$$

Reaction (41) may go to completion, but the others do not. The ionization constants for reactions (42) and (43) are given, respectively, by Eqs. (11) and (10). The solubility product for reaction (44) is

$$[Ca^{++}][CO_3^{--}] = 10^{-8.32} \text{ at } 25C \tag{46}$$

The numerical value[1] of the exponent of 10 in the foregoing solubility product increases about 0.01 for each degree centigrade increase in temperature and appears to decrease with increase in dissolved solids by an amount approximately equal to 0.02 times the square root of the concentration of dissolved solids in ppm. The solubility product[2] for reaction (45) is

$$[Mg^{++}][OH^-]^2 = 5.5 \times 10^{-12} \text{ at } 25C \tag{47}$$

The amount of chemicals required is usually estimated on the assumption that reactions (41) to (45) go to completion. Sufficient lime is added in the ordinary process to change all the free CO_2 and HCO_3^- to CO_3^{--}, and sufficient soda is added to balance the incrustants with carbonates. In order to remove the Mg^{++}, sufficient additional lime is required to produce $Mg(OH)_2$ and to leave an excess caustic alkalinity of 25 to 50 ppm. This process is called the *excess-lime* process, and it should be followed after precipitation of the Mg^{++} by recarbonation with CO_2 gas to neutralize the causticity and to precipitate the excess Ca^{++} added. Carbonation is used in many lime-softening plants just before the water reaches the filters in order to minimize the incrustation of the sand with $CaCO_3$.

The residual hardness and the analysis of the water after softening may be estimated by the solution of simultaneous equations involving concentrations and equilibrium constants. The results of such a computation are based upon the assumptions that the reactions have reached equilibrium and that all precipitates have been removed from the water. The $CaCO_3$ precipitate is difficult to flocculate and hence may not all be removed by settling and filtration. $Mg(OH)_2$ floc is similar to alum floc and settles readily, although more slowly than $CaCO_3$.

The computations for the amounts of chemicals required and for the residual hardness are illustrated in the following example for a water having a hardness of 221.6 ppm.

When the reactions have reached equilibrium, the Ca, Mg, CO_3, and OH appear both as solid and as ions. The $[CO_3^{--}]$, $[HCO_3^-]$, and $[OH^-]$ are also modified by the ionization reactions. The concentration of all constituents is determined by the solution of the simultaneous equations shown in Table 6.

The results obtained by the solution of the equations in Table 6 are shown in Table 7. The residual hardness is 26.27 ppm, nearly all due to the Ca^{++} in the excess lime, and the caustic alkalinity is 42.1 ppm which corresponds to a pH value of 10.92.

If the free CO_2 content of the raw water is high, it is frequently economical to remove a portion of it by aeration before the chemicals are added. Spray nozzles are most effective for aeration and will sometimes accomplish the removal of as much

[1] LANGELIER, *loc. cit.*
[2] LATIMER, *loc. cit.*

TABLE 3.—ANALYSIS OF RAW WATER

Ion	ppm	Mol. wt	ppm as $CaCO_3$	
			Positive	Negative
Ca^{++}	59	40	147.6	
Mg^{++}	18	24.3	74.0	
Na^{+}	10	23	21.7	
HCO_3^-	222	61	.....	182.0
CO_3^{--}	0.6	60	.....	1.0
SO_4^{--}	38.6	96	.....	40.2
Cl^-	12	35.5	.....	16.9
NO_3^-	4	62	.....	3.2
			243.3	243.3
Free CO_2	8	44	18.2 ppm or $CaCO_3$	
pH	7.82			

Chemicals required:

Lime, for CO_2 18.2 ppm as $CaCO_3$
HCO_3^- 182.0 ppm as $CaCO_3$
Mg^{++} 74.0 ppm as $CaCO_3$
Excess 40.0 ppm as $CaCO_3$
Total = 314.2 ppm as $CaCO_3$
or 233 ppm as pure $Ca(OH)_2$

Soda ash, for SO_4^{--} 40.2 ppm as $CaCO_3$
Cl^- 16.9 ppm as $CaCO_3$
NO_3^- 3.2 ppm as $CaCO_3$
Total = 60.3 ppm as $CaCO_3$
or 64 ppm as pure Na_2CO_3

TABLE 4.—BALANCE OF RADICALS AFTER ADDITION OF CHEMICALS

	ppm as $CaCO_3$							
	Positive			Negative				
	Ca	Mg	Na	SO_4	$Cl + NO_3$	CO_3	HCO_3	OH
Original	147.6	74.0	21.7	40.2	20.1	1.0	182.0	
Added	314.2	....	60.3	....	....	60.3	.....	314.2
From CO_2	.....	....	....	....	....	+18.2	.....	−18.2
Total	461.8	74.0	82.0	40.2	20.1	79.5	182.0	296.0
		= 617.8				= 617.8		

TABLE 5.—TOTAL MOLAL CONCENTRATION OF RADICALS

$[Ca] = 4.618 \times 10^{-3}$	$[Cl + NO_3] = 0.402 \times 10^{-3}$
$[Mg] = 0.74 \times 10^{-3}$	$[CO_3] = 0.795 \times 10^{-3}$
$[Na] = 1.64 \times 10^{-3}$	$[HCO_3] = 3.64 \times 10^{-3}$
$[SO_4] = 0.402 \times 10^{-3}$	$[OH] = 5.92 \times 10^{-3}$

as 75 per cent of the free CO_2. As the free CO_2 gas is liberated, the HCO_3^- breaks down in accordance with reaction (42) to furnish OH^- and thus raise the pH value. Since some of the OH^- reacts with H^+ to form water, the alkalinity is very slightly reduced by aeration.

Recarbonation is required after settling in order to eliminate the caustic alkalinity added in the excess lime. The reactions that take place upon the addition of CO_2 are shown in Eqs. (42) and (43). If sufficient CO_2 is added to barely remove the

TABLE 6

Unknowns	Equations	
(1) $[Ca^{++}]$	$[H^+][OH^-] = k_w$	(1)
(2) $[OH^-]$	$\dfrac{[H^+][CO_3^{--}]}{[HCO_3^-]} = K_1$	(2)
(3) $[CO_3^{--}]$	$[Ca^{++}][CO_3^{--}] = K_2$	(3)
(4) $[HCO_3^-]$	$[Mg^{++}][OH^-]^2 = K_3$	(4)
(5) $[H^+]$	$[Ca^{++}] + [Ca_s] = A = 4.618 \times 10^{-3}$	(5)
(6) $[Mg^{++}]$	$[Mg^{++}] + [Mg_s] = B = 0.74 \times 10^{-3}$	(6)
(7) $[Ca_s]$	$[Ca_s] = [CO_{3s}]$	(7)
(8) $[Mg_s]$	$2[Mg_s] = [OH_s]$	(8)
(9) $[CO_{3s}]$	$[CO_3^{--}] + [HCO_3^-] + [CO_{3s}] = C = (3.64 + 0.795)10^{-3}$	(9)
(10) $[OH_s]$	$[OH^-] + [OH_s] - [HCO_3^-] = D = (5.92 - 3.64)10^{-3}$	(10)

OH^-, much of the remaining Ca^{++} will be precipitated and the water will be further softened. If this is desired, carbonation should be followed by secondary flocculation with a coagulant or returned sludge and settling before the water goes to the filters; this process may be followed by secondary carbonation just prior to filtration to minimize afterprecipitation of $CaCO_3$ on the sand grains. The Ca^{++} may also be precipitated by secondary treatment with Na_2CO_3, but the causticity will remain and must be removed later by recarbonation. The final carbonation may be so adjusted

TABLE 7.—BALANCE OF IONS AFTER COMPLETION OF REACTIONS

	ppm as $CaCO_3$							
	Positive			Negative				
	Ca^{++}	Mg^{++}	Na^+	SO_4^{--}	$Cl^- + NO_3^-$	CO_3^{--}	HCO_3^-	OH^-
Ions	26.2	0.07	82.0	40.2	20.1	5.31	0.7	42.1
Total	108.27			108.41				

as to constitute corrective treatment of the water for protection against corrosion. If carbonation is carried far enough to throw the residual Ca back into solution, incrustation of filter sand will be lessened but further corrective treatment after filtration with soda or lime will usually be required. A small dose of sodium hexametaphosphate[1] may be used instead of secondary carbonation to prevent incrustation of the filter sand.

[1] RICE and HATCH, *Jour. A.W.W.A.*, **31**, 1171, 1939.

CO_2 gas for carbonation may be produced by the following types of plants: (1) gas burning, (2) combined gas and coke burning, (3) oil burning, (4) combined oil and coke burning, (5) coke burning, (6) utilization of stack gas, and (7) generation and use of producer gas. A carbonation plant consists principally of (1) a combustion chamber so designed as to permit proper admixture of air and fuel, (2) a washer or scrubber where the products of combustion are cleaned and cooled, (3) a drier for removing the water that may come over with the gas, (4) a compressor or blower for forcing the gas into the carbonation chamber, (5) a gas flow meter, and (6) a carbonation chamber in which the gas is diffused into the water.

For small treatment plants, gas- or oil-burning carbonation plants are most satisfactory because of the ease with which the rate of combustion may be controlled. Fuel oil yields about 2.3 lb of CO_2 per pound; kerosene about 3.1 lb of CO_2 per pound; and gas 82 to 115 lb of CO_2 per 1,000 cu ft of gas. For large softening plants, gas-producer carbonators as illustrated in Fig. 39[1] are economical. Gas is produced from

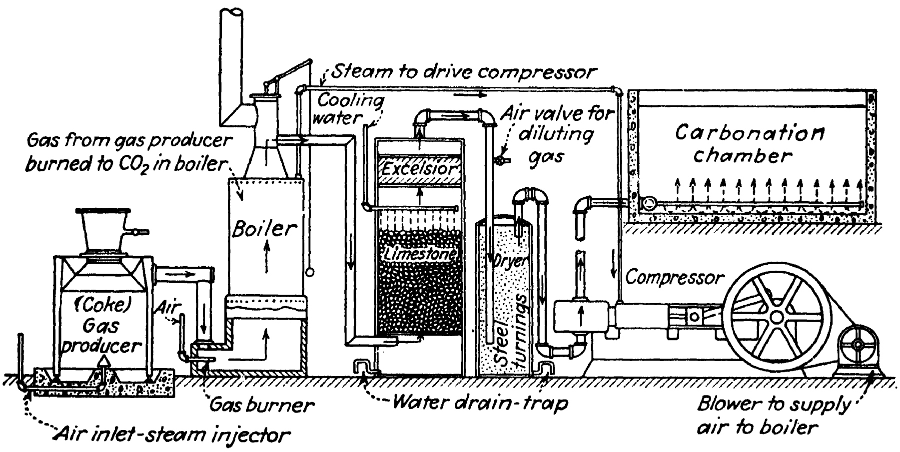


FIG. 39. Gas-producer carbonation plant.

coke and burned under a boiler, the steam from the boiler being used to drive the compressor or for other power or heating purposes. Coke produces about 3 lb of CO_2 per pound of coke. The burned gas from this type of plant contains up to 19 per cent CO_2; whereas, from the ordinary burning of coke and other types of carbonators, the gas contains only about 10 to 12 per cent CO_2. One cubic foot of CO_2 weighs 0.1145 lb at atmospheric pressure and 70F.

The gas from the compressor is diffused into the water through perforated pipes or diffuser plates in the bottom of the carbonation chamber. The depth of the chamber may be made the same as the adjacent basins for structural convenience. The retention period for the water in the chamber is of no significance, and the size may be made just sufficient to accommodate the diffuser system. Perforated-pipe diffusers may be designed according to the same principles used for filter underdrains. Brass or copper pipe should be used.

Zeolite Process. Zeolites for water softening are insoluble compounds consisting generally of silicates of Al and Fe or synthetic resins containing Na which, when in contact with hard water, is exchanged for the positive ions in the water. The zeolite or base-exchange process consists of passing the hard water through beds of granular zeolite similar to rapid filters during which the Na of the zeolite is exchanged for Ca

[1] HOOVER, "Water Supply and Treatment," National Lime Assoc., 1934, p. 93.

and Mg and other basic ions in the water. The reaction may be represented as follows:

$$NaZ + Ca^{++} \rightleftarrows CaZ + Na^{+} \tag{48}$$

Water of zero hardness is produced, and the zeolite units are operated until they begin to pass hard water. They are then taken out of service, washed, and the zeolite is regenerated with a 5 to 10 per cent solution of NaCl, after which the beds are rinsed and placed back into service. In the regeneration process, reaction (48) is reversed.

Two types of mineral zeolites[1] are available: treated natural zeolites, or greensand, and synthetic zeolites the first example of which was Permutit. All mineral zeolites have a specific gravity between 2 and 2.4, but the grains of artificial zeolites are more porous than greensand and thus have a smaller apparent specific gravity. Greensands weigh about 100 lb/cu ft moist and contain less than 10 per cent water by weight; whereas, artificial zeolites weigh 55 to 70 lb/cu ft and contain up to 50 per cent of water. The grain size usually used for greensand is slightly larger than the sand size for rapid filters, and the grain size for synthetic zeolite is still larger. The softening capacity of a zeolite is the number of grains of hardness removed per cubic foot up to the point where zero hardness is no longer produced. The softening capacity of greensands of about ½ mm grain size is about 2,500 to 3,000 grains/cu ft. The softening capacity increases with the porosity of zeolite grains up to more than 10,000 grains/cu ft. According to Hoover,[2] the softening capacity of zeolites increases with the amount of salt used in regeneration up to a maximum, the economical amount of salt usually being less than the maximum and between 0.3 and 0.5 lb per 1,000 grains of hardness removed. Softening capacity is somewhat less for excessively hard water or for excessive amounts of Na or Mg.

Synthetic resinous or carbonaceous zeolites (organic cation exchangers) weigh about 30 lb/cu ft dry and have exchange capacities up to about 15,000 grains of hardness per cubic foot. Resinous zeolites require for regeneration about 0.4 to 0.5 lb of salt per 1,000 grains of hardness removed.

Organic cation exchangers may be used on the hydrogen cycle instead of the sodium cycle, and thus exchange H^+ for all basic ions in the water including Na^+. When used on the hydrogen cycle, H_2SO_4 is usually used for regeneration with a requirement of about 0.5 lb of 66Bé sulphuric acid per 1,000 grains of base (as $CaCO_3$) removed. The effluent from hydrogen zeolite is strongly acid and requires further treatment either for neutralization or demineralization. If the effluent from the cation exchanger is passed through an anion exchanger, the acids will be absorbed and a demineralized water will be produced. CO_2 is removed by aeration. Anion exchange resins are regenerated with strong solutions of hydrated lime or soda ash.

Zeolite softeners are built in units similar to both pressure- and open-type rapid filters. In addition to the piping and valves provided for rapid filters, zeolite units are provided with salt-solution inlets and outlets. The brine may be introduced by a perforated-pipe system at the surface of the bed, in which case the brine flows downward, or through the underdrainage system, in which case the brine flows upward through the bed. Because of the smaller specific gravity of zeolites, the wash rates are smaller than for sand filters. A wash rate of 6 to 8 gpm/sq ft will ordinarily accomplish the desired expansion of the bed. Zeolite beds are made 30 to 75 in. deep and are operated as either downflow or upflow units at rates up to 4 to 8 gpm/sq ft. Wash-water requirements range up to 5 to 10 per cent for synthetic zeolites and up to 20 to 25 per cent for greensands, the requirements for a single wash and rinse being about 100 gal/sq ft.

[1] See APPLEBAUM, *Jour. A.W.W.A.*, **13**, 213, 1925.
[2] HOOVER, *op. cit.*, p. 98.

Brine is prepared in a salt storage bin, usually made of concrete with duplicate compartments, as a saturated solution containing about 25 per cent salt. The bottom of the bins is usually made hopper shaped and provided with a screen and about a foot of graded gravel to support the salt. The salt is covered with water which dissolves it, the solution being withdrawn from below to a collecting reservoir of sufficient capacity to regenerate one softening unit. From the collecting reservoir, the brine is withdrawn, diluted to the proper strength, and applied to the unit. The brine solution is passed slowly and continuously through greensand, but it is customary to hold the brine in synthetic zeolite beds 5 to 15 min.

It is usually not economical to operate zeolite units as filters, as the sediment that deposits on the grains reduces the softening capacity of the material. Zeolite units may be used alone with clear well waters not too high in iron, but they should follow filters with turbid waters. Zeolite units will remove Fe and Mn in true solution by base exchange, but with dissolved O_2 present some of the iron nearly always takes the form of Fe_2O_3 and as such coats the zeolite grains and reduces their softening capacity. Mineral zeolite disintegrates in acid waters and in waters having appreciable free CO_2 content, but this is not true of resinous zeolites.

Although zeolite units produce water of zero hardness, the total dissolved solids in the softened water is nevertheless greater than in the raw water and the water is high in Na. In contrast with the lime-soda process, which may produce 200 cu ft or more of sludge per million gallons of water, there is no sludge-disposal problem with the zeolite process. As regards softening economy,[1] lime costs only about half as much as salt for removing bicarbonate hardness, whereas soda ash costs about 25 per cent more than salt for removing noncarbonate hardness. Hence zeolite is sometimes used following lime-softening to remove noncarbonate hardness.

Iron and Manganese Removal.[2] Iron and manganese in hard water are effectively removed by the lime-softening process. Fe precipitates as ferrous oxide or carbonate which is rapidly oxidized to Fe_2O_3 [reaction (26)] by the dissolved O_2. Mn precipitates by a similar reaction as white hydrous manganous oxide, $MnO{\cdot}H_2O$, which is similarly oxidized quite rapidly to hydrous manganic oxide, $Mn_2O_3{\cdot}XH_2O$, and hydrous manganese dioxide, $MnO_2{\cdot}XH_2O$, brownish-black precipitates.

In purification plants treating waters that do not require softening, Fe and Mn may be removed by the addition of sufficient lime or other alkali to raise the pH value to about 9.6 for Fe^{++} and 9.9 for Mn^{++}. The solubility product for hydrous ferrous oxide is given by Eq. (25) and for hydrous manganous oxide is as follows:

$$[Mn^{++}][OH^-]^2 = 7.1 \times 10^{-15} \text{ at } 25C^{3} \tag{49}$$

The solubilities of ferrous iron and manganous manganese corresponding to these solubility product constants are shown in Fig. 40, which also shows the solubilities of Fe^{+++} and Mn^{+++}. The solubility product for hydrous ferric oxide is given by Eq. (22) and for hydrous manganic oxide is as follows:

$$[Mn^{+++}][OH^-]^3 = 2.5 \times 10^{-47} \text{ at } 25C^{4} \tag{50}$$

Figure 40 shows that if the Fe^{++} and Mn^{++} are oxidized by chlorination or other means to Fe^{+++} and Mn^{+++}, iron and manganese may both be precipitated effectively at any pH value above 1.0 for Mn and above 4.0 for Fe.

The addition of lime in many cases is undesirable because of the increase in hardness, particularly with waters used entirely for industrial purposes. It is also undesirable if the only fault with the water is its Fe or Mn content. In such cases, so-called

[1] BEHRMAN, *Jour. A.W.W.A.*, **26**, 618, 1934.
[2] ZAPFFE, *Jour. A.W.W.A.*, **25**, 655, 1933; and WESTON, *Jour. N.E.W.W.A.* **50**, 231, 1936.
[3] LATIMER, *loc. cit.*
[4] Computed from data given by Latimer, *loc. cit.*

deferrization or demanganization plants may be constructed. Such plants involve the use of aeration and contact beds of coke or crushed stone followed by settling for about 1 hr and filtration. Aeration is for the double purpose of raising the pH value by CO_2 reduction and of increasing the dissolved O_2 content. The surfaces of the contact material after usage collect deposits of MnO_2 and sometimes iron and manganese bacteria. MnO_2 acts as a catalyst to promote the precipitation of manganous and ferrous oxides at lower pH values than are required when chemicals are used. The iron and manganese bacteria, of which there are some six varieties, have the power of adsorbing dissolved Fe and Mn on their sheaths where the metals are precipitated owing to the alkaline reaction of the sheaths. MnO_2 ore, pyrolusite, has been used as contact medium. It possesses the advantage of high catalytic power.

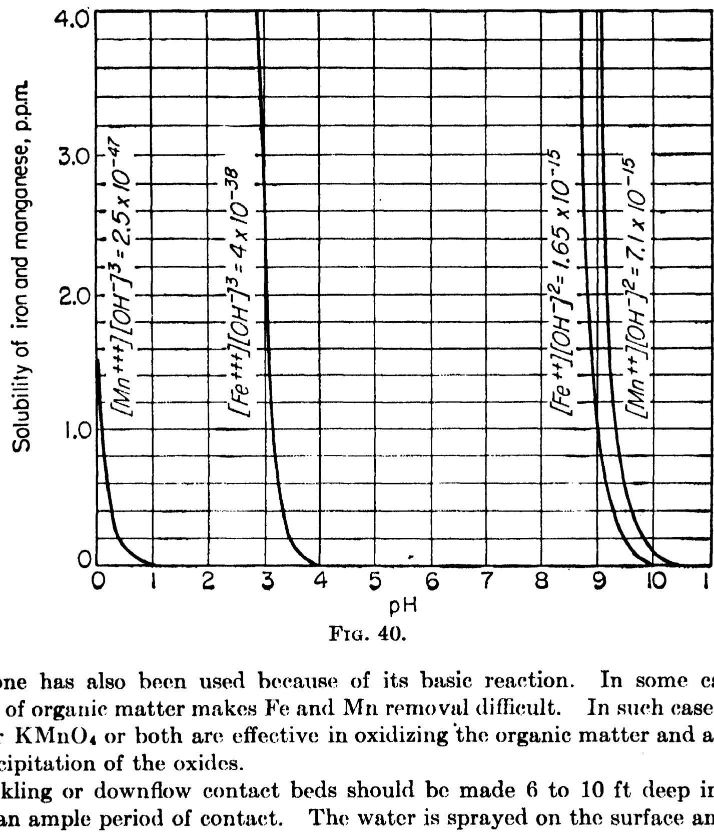


FIG. 40.

Limestone has also been used because of its basic reaction. In some cases, high content of organic matter makes Fe and Mn removal difficult. In such cases, chlorination or $KMnO_4$ or both are effective in oxidizing the organic matter and assisting in the precipitation of the oxides.

Trickling or downflow contact beds should be made 6 to 10 ft deep in order to ensure an ample period of contact. The water is sprayed on the surface and allowed to trickle downward at rates of 50 to 80 mgd/acre. Upflow contact beds are more effective than downflow beds because of the longer period of contact, provided the water is previously aerated. Such beds are apt to unload and are more easily cleaned than tricklers. Aeration is best effected by spray nozzles or superimposed trays filled with coke. Filters for deferrization plants may be either rapid or slow sand filters. Slow filters are preferable because of the longer period of contact and may be operated at rates up to 10 mgd/acre. If coagulants are used, rapid filters are essential.

Manganese zeolite, sodium zeolite treated with manganous chloride and subsequently with sodium permanganate, is effective in the removal of Fe and Mn from water. The removal is due to the precipitation of ferrous and manganous oxides

through the catalytic action of the MnO_2 on the zeolite grains and the subsequent oxidation of the precipitates by the higher oxides of manganese on the grains. The precipitates are removed by filtration in the zeolite bed. When the oxidizing power of the zeolite is exhausted, the bed is regenerated with sodium permanganate. Precipitates are removed from the bed by backwashing.

CORRECTIVE TREATMENT OF CORROSIVE WATERS

Two general methods of attack are available for reducing the corrosiveness of water: (1) the removal of the dissolved oxygen and (2) the adjustment of the pH value.

Dissolved-oxygen Removal. Two methods of removal of dissolved oxygen on a small scale have been used in industrial plants for some time. One of these methods, called *deactivation*, consists of heating the water to reduce the solubility of O_2 and then passing it over steel scrap which unites with the oxygen to form rust. In the other method, called *deaeration*, the water is heated to just below boiling to free the O_2 and is then allowed to trickle over a series of copper trays to liberate the gas. Both of these methods require heat and are expensive.

Powell and Bacon[1] describe a method in which deaeration is accomplished by vacuum. The water is introduced into the top of an enclosed steel tower and allowed to trickle downward over bundles of wood laths, while a vacuum of about 28.5 in. is maintained inside the tower by means of a vacuum pump taking suction at the top of the tower. A 95 per cent removal of O_2 is obtained by means of the vacuum. The remaining dissolved oxygen is removed after the water flows from the bottom of the tower by means of a dose of sodium sulphite, a reducing agent.

Oxygen removal is more expensive generally than adjustment of the pH with alkali and is used only where additional chemicals, particularly Ca^{++}, are objectionable in the water.

Adjustment of the pH Value. The chemical adjustment of water to reduce corrosiveness is for the double purpose of raising the pH value and of stabilizing protective coatings on the interior surfaces of metal pipes. As has been shown in Sec. 20, the metals tend to enter solution as metallic ions at low pH values but are protected by the plating out of OH^- and CO_3^{--} at the anode in the alkaline range. A pH of 9.5 to 10.0 is required to ensure plating out of OH^- and CO_3^{--} on iron.

The most commonly used method of chemical treatment for corrosion abatement in iron pipes is to lay down a coating of $CaCO_3$ on the interior surfaces of the pipes. Except for very soft waters, this may be done at pH values lower than 9.5 to 10.0. If a water is undersaturated with $CaCO_3$, it will dissolve $CaCO_3$ coatings and iron from the surface of iron pipes to which it is exposed. If supersaturated, $CaCO_3$ will be precipitated upon the surface of the pipe and thus form a protective coating. If the water is just saturated, coatings of $CaCO_3$ will be stable and will protect the iron against corrosion. Two methods are available for determining whether a water is in equilibrium with $CaCO_3$: (1) by chemical test using powdered $CaCO_3$ and (2) by computation from the analysis of the water.

The chemical test as described by Baylis[2] requires several days for completion, in order that the water may come to equilibrium with the powdered $CaCO_3$ with which it is placed in contact. If after the contact, the pH and alkalinity of the water are unchanged, the water is in equilibrium with $CaCO_3$. If the pH and alkalinity are increased, the water is corrosive to $CaCO_3$, and if decreased, the water is supersaturated with $CaCO_3$. Baylis presents a graph showing the relation of pH to alkalinity where a water is in equilibrium with $CaCO_3$. Since the concentrations of Ca^{++}

[1] *Water Works and Sewerage*, April, 1937, p. 109.
[2] BAYLIS, *Jour. A.W.W.A.*, **27**, 220, 1935.

and total dissolved salts are influencing factors, as has been shown by Langelier,[1] the graph is not the same for all waters even at the same temperature.

Langelier has developed a method for computing the equilibrium values of a water from the analysis and equilibrium constants. If the water is just saturated with $CaCO_3$, Eq. (46) holds, and the solution of this equation simultaneously with Eq. (10) gives the following.

$$[H^+]_s = \frac{K_1}{K_2}[HCO_3^-][Ca^{++}] \tag{51}$$

in which $[H^+]_s$ is the hydrogen-ion concentration at saturation and K_1 and K_2 the equilibrium constants of Eqs. (10) and (46), respectively. $[H^+]_s$ is computed by means of this equation from the values of $[HCO_3^-]$ and $[Ca^{++}]$ as determined by chemical analysis. The influence of temperature and total solids makes itself felt in the values of K_1 and K_2. If the computed value of $[H^+]_s$ is less than the $[H^+]$ as determined by analysis (pH_s greater than pH), the water is corrosive to $CaCO_3$; and if $[H^+]_s$ is greater than $[H^+]$ (pH_s less than pH), the water is supersaturated with $CaCO_3$. pH_s may vary from about 7.5 for hard waters to 10 for very soft waters.

The chemical adjustment of a corrosive water is usually done by adding lime, or by adding soda ash or caustic soda if it is desired not to increase the hardness. Adjustment may be made with CO_2 if the water is oversaturated with $CaCO_3$. Aeration and contact treatment through beds of marble or limestone are also used for corrosive waters. Equation (51) cannot be used to compute the amount of treatment or chemicals required. These are determined by trial and error, the equation being used to check the results. The addition of lime and contact treatment with marble, it will be noted, change all three variables in Eq. (51). Soda ash, caustic soda, and CO_2 affect two of the variables. Aeration affects only the $[H^+]$, the influence on alkalinity being very slight. Aeration alone is usually not sufficient treatment for a corrosive water, for only about 75 per cent of the free CO_2 can be removed; but it may suffice, according to Cox,[2] if the alkalinity exceeds about 100 ppm. Contact treatment with limestone or marble, on the other hand, is not economical if the alkalinity exceeds about 50 ppm and the free CO_2 about 30 ppm. The hardness is increased too much by the treatment, and the contact period is too long. For waters amenable to treatment by the contact method, Cox recommends stone about 2 mm in size and contact periods of 10 to 20 min. Beds should be equipped for washing.

When corrective treatment is started with a corrosive water, the water should be slightly overtreated in order to lay down a $CaCO_3$ coating on the interior surface of pipes. If water is slightly supersaturated, several hours are required for precipitation of the excess $CaCO_3$ and distant pipes will thus be coated. A coating of $\frac{1}{64}$ to $\frac{1}{32}$ in. is satisfactory and does not reduce the carrying capacity of pipes greatly. When a satisfactory coating is produced, the water should be brought to equilibrium with $CaCO_3$ and maintained there. When repairs and extensions are made to the distribution system, the coatings in the uncovered pipes should be examined where possible as a guide to corrective treatment. Experience indicates that it is very difficult to produce a satisfactory protective coating uniformly throughout a distribution system, and that flushometer screens and small meters may be clogged in the attempt.

Sodium hexametaphosphate (Calgon) is being widely used to prevent the formation of *red water* where considerable iron is present in a water. Its action in this connection is as a peptizing agent to prevent the growth of iron-rust crystals. It is also effective following lime softening to prevent *afterprecipitation* of $CaCO_3$. It is

[1] LANGELIER, *Jour. A.W.W.A.*, **28**, 1550, 1936.
[2] COX, *Jour. A.W.W.A.*, **25**, 1505, 1933.

claimed that this chemical is also effective in corrosion abatement by inhibiting the solution of iron, but conflicting results have been obtained in practice. It is probable, as indicated in Sec. 20, that anodic protection may be obtained by the plating out of phosphate ions if the pH is high enough.

DISINFECTION

Methods. Disinfection may be effected by the use of (1) chemical disinfectants, (2) ultraviolet light, and (3) heat. The application of heat is a very satisfactory method of disinfection on a small scale for laboratory or emergency purposes, but it is too expensive for plant-scale disinfection. Complete sterilization may be obtained by boiling water 30 min, and disinfection (*i.e.*, destruction of pathogenic bacteria) may be obtained by heating to 60C for 15 min or by boiling for a shorter period. Chemical disinfectants are the more generally used in treatment plants. The following chemicals have found practical application: (1) liquid chlorine, (2) hypochlorites, (3) chloramine, (4) excess lime, (5) ozone, (6) silver, (7) permanganate, (8) bromine, (9) iodine, and (10) chlorine dioxide. Some of these chemicals are strong oxidizing agents. The first three chemicals are the most widely used and will be considered together.

Chlorine and Chloramines. Bleaching powder, or chlorinated lime, has been widely used as a source of chlorine; but it has been largely displaced by gaseous Cl_2 owing to the perfection of chlorinators for proportioning gaseous chlorine. Bleach contains 25 to 37 per cent available chlorine. Two commercial hypochlorites, H.T.H. and Perchloron, which contain 65 to 75 per cent true calcium hypochlorite, $Ca(ClO)_2$, are in use in the United States. Chlorinated lime and hypochlorites dissolve in water to form hypochlorus acid, HClO. Chlorine also reacts with water to produce hypochlorus acid as follows:

$$Cl_2 + H_2O \rightleftarrows HClO + H^+ + Cl^- \tag{52}$$

The reaction goes almost to completion to the right in dilute solutions but is said to require several hours for equilibrium.

Hypochlorus acid ionizes as follows:

$$HClO \rightleftarrows H^+ + ClO^- \tag{53}$$

The dissociation constant is

$$\frac{[H^+][ClO^-]}{[HClO]} = 3.7 \times 10^{-8} \text{ at } 25C \tag{54}$$

Equation (54) indicates that a chlorine solution is about 96 per cent HClO at pH 6 and less than 3 per cent HClO at pH 9.

Chloramine for water treatment is produced by combining ammonia with chlorine in the water. Anhydrous gaseous ammonia, NH_3, is usually used and is applied by means of ammoniators similar in type to chlorinators. Some plants use sulphate of ammonia, which is fed by dry-feed machines. The chemical reactions of NH_3 with Cl_2 in water treatment are varied and not well understood. The most important reactions[1] are as follows:

$$NH_4^+ + HClO \rightleftarrows \underset{\text{Monochloramine}}{NH_2Cl} + H^+ + H_2O \tag{55}$$

$$NH_2Cl + HClO \rightleftarrows \underset{\text{Dichloramine}}{NHCl_2} + H_2O \tag{56}$$

$$NHCl_2 + HClO \rightleftarrows \underset{\text{Nitrogen trichloride}}{NCl_3} + H_2O \tag{57}$$

$$2NH_2Cl + H^+ \rightleftarrows NH_4^+ + NHCl_2 \tag{58}$$

[1] FAIR, *Eng. News-Record*, July 29, 1948, p. 60.

Monochloramine forms most quickly at pH 8.3 and less rapidly at higher and lower pH values. Monochloramine is formed at pH values above 7.5, dichloramine at pH values from 5 to 6.5, and nitrogen trichloride at lower pH values. The relative proportions of mono- and dichloramine depend upon both pH and NH_4^+ concentration, and may be computed from the equilibrium constant for reaction (58) which follows:

$$\frac{(NH_4^+)(NHCl_2)}{(H^+)(NH_2Cl)} \quad 6.7 \times 10^5 \text{ at } 25C \tag{59}$$

The manner in which chlorine and chlorine compounds effect their killing action upon bacteria has received much study, but little has been learned of the mechanism. Disinfection was once thought to be due to the liberation of nascent oxygen from HClO, but this theory has lost support owing to the fact that chloramines contain no oxygen. In the case of chlorine and hypochlorites, it appears that undissociated HClO is the important factor[1] in disinfection inasmuch as the speed of germicidal action is markedly increased at low pH values. In the case of chloramines, the speed of disinfection appears to be less than that of chlorine and hypochlorites and to be affected to a much smaller extent by changes of pH value. The chloramine molecules themselves appear to be the effective germicides, and it appears that dichloramine is slightly more effective than monochloramine. The presence of positive chlorine atoms in both HClO and the chloramines seems to be related to the germicidal efficacy. Oxidation potential has been suggested as a measure of germicidal efficiency, but it has been found to be unrelated in the case of some germicides.

The speed of germicidal action is almost directly proportional to the concentration of residual chlorine, other factors remaining the same. Germicidal speed also increases markedly with increase in temperature. The germicidal efficacy of chlorine and chloramine varies with different bacteria and even with different strains of the same bacterium. *B. typhosus* and *B. coli* are generally killed at about the same rate, but some strains[2] of *B. typhosus* have been found to be more resistant than *B. coli*. Bacterial spores are killed very slowly by disinfectants.

It is general practice in chlorination to maintain a residual of 0.1 to 0.2 ppm chlorine 10 to 20 min after chlorine application. With chloramine treatment, a residual twice as high is usually required for the same germicidal efficiency, and a reaction period of 20 min or longer before making the residual chlorine test[3] is desirable. The residual to be used for a given water should be selected as a result of tests upon the bacterial death rate, made preferably with *B. coli*. A chlorine dose higher than the residual is required in order to satisfy the chlorine demand of the water. Organic and other foreign matter in the water combines with the chlorine and makes it ineffective as a germicide, and the amount of chlorine required to combine with this matter is called the *chlorine demand*. The chlorine demand can be determined by test only. For chlorinating clear waters, the chlorine dose ranges from about 0.1 ppm up to about 0.6 ppm. For turbid waters, as in the case of prechlorination prior to coagulation in a filtration plant, the required dose may exceed 1.0 ppm. The rate at which the chlorine demand is satisfied varies with the temperature and the kind of organic matter, but the reaction is frequently completed in less than 5 min. Chlorine residuals are sometimes dissipated in the distribution system where chlorine or hypochlorites are used, but chloramine residuals are very persistent.

If rapid disinfection is required, as is the case with water systems in which the

[1] See CHARLTON and LEVINE, *Iowa Eng. Exp. Sta., Bull.* 132, 1937; MALLMAN, *Jour. A.W.W.A.*, **24**, 1054, 1932; FAIR, MORRIS, and CHANG, *Jour. N. E. W. W. A.*, **61**, 285, 1947.

[2] *Public Health Reports*, Oct. 2, 1936, p. 1367.

[3] See "Standard Methods for the Examination of Water and Sewage," A.P.H.A., 1946, for test methods.

consumers are in close proximity to the point of chlorination, chlorine or hypochlorites should be used. A contact period between the chlorine and water of as little as 10 min may be sufficient to procure safe disinfection with HClO. With the ammonia-chlorine process, however, longer periods are required up to several hours, depending upon the temperature, the pH value, the amount of bacterial contamination, the chlorine dose, the ratio of NH_3 to Cl_2, and the order of application of the chemicals.

There are two purposes in the use of ammonia[1] with chlorine: (1) to prevent odors produced by overchlorination or the combination of the Cl_2 with substances in the water and (2) to create a persistent chlorine residual that is effective in preventing aftergrowths of bacteria in the distribution system. If the ammonia is used to prevent odors, it is more effective if applied before the chlorine. The chlorine should follow within 10 min or some of the NH_3 may be lost. Passage of ammoniated water through filters prior to the application of chlorine sometimes results in the absorption of much of the NH_3 by the filter sand. If rapid germicidal action together with a persistent residual is desired, it may sometimes be accomplished by applying the

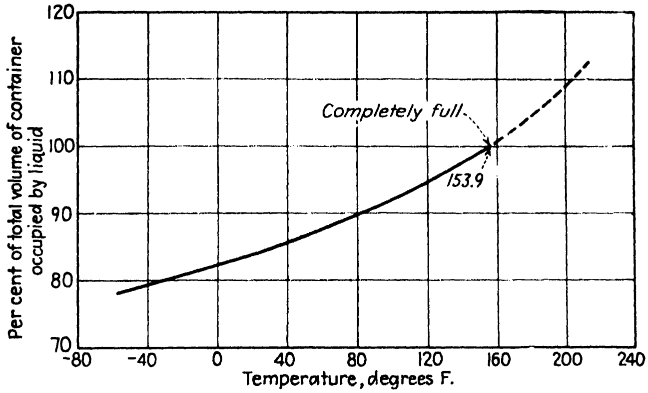


FIG. 41.—Volume-temperature relations of liquid chlorine in closed containers.

chlorine first followed in a few minutes by the ammonia. This process also results in a saving of ammonia, but it may result in odors with some waters. The approximate ratio of ammonia to chlorine required for the formation of monochloramine is 1:4 and for dichloramine is 1:8, as indicated by Eqs. (55) and (56). In practice, the average ratio used is about 1:3, a slight excess of NH_3 being found desirable. In some cases, however, ratios as low as 1:12 have proved adequate. The ratio required in any case depends principally upon the original ammonia content of the water, the order of application of chlorine and ammonia, and the odor problem.

Hypochlorites are prepared as solutions with a strength of about 1 per cent available Cl_2 and then fed through orifice boxes into the water. Solutions should be protected from light and if stored for more than two weeks before use should be stabilized with alkali in order to prevent loss of Cl_2. Commerical feed apparatus is available for both manual and automatic control of the dose.

Liquid chlorine[2] is commercially available in the United States in closed drums of 100 lb, 150 lb, and 1 ton capacity and in 30-ton tank cars. The weight of liquid chlorines varies inversely with the temperature, being 91.8 lb/cu ft at 32F and 78 lb at 153.9F. When shipped under Interstate Commerce Commission regulations, the

[1] See ENSLOW, *Jour. N.E.W.W.A.*, **48**, 6, 1934; and GRIFFIN, *Am. Jour. Pub. Health*, March, 1937.
[2] HEDGEPETH, *Jour. A.W.W.A.*, **26**, 1602, 1934.

volume-temperature relations in the containers are as shown in Fig. 41.[1] It is obvious from the figure that containers should not be heated to above 153.9F, nor should they be filled at lower temperatures with more liquid than is indicated by the curve. The expansion of the liquid due to increase in temperature will in either case be dangerous. The vapor pressure of liquid chlorine for various temperatures is shown in Fig. 42,[1] from which it may be seen that the gage pressure of a container is about

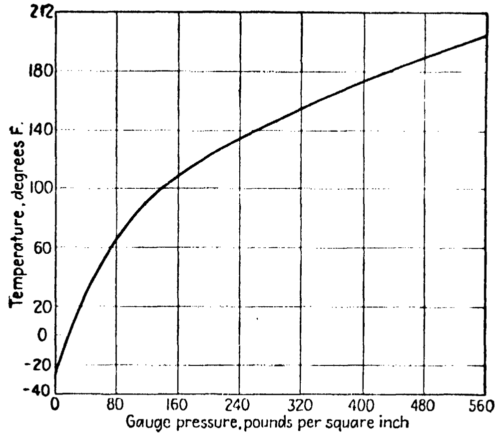


FIG. 42.—Vapor pressure of liquid chlorine.

85 psi at room temperature and about 39 psi at 0C. When the pressure on the cylinder is released, the liquid boils away as a gas.

Liquid chlorine is withdrawn from the container as a gas and fed through a chlorinator into the water. Several types and makes of chlorinators are available, of which some feed the chlorine as a water solution and others feed the gas directly to the water. The solubility of Cl_2 in water as affected by the temperature is shown in Fig. 43.[1]

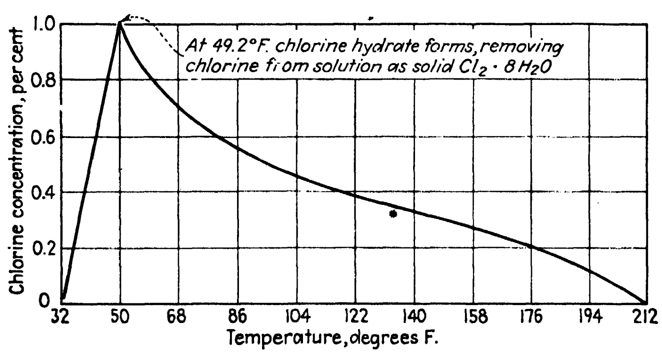


FIG. 43.—Solubility of chlorine in water.

Below 49.2F, solid chlorine hydrate is formed. It is therefore desirable that chlorine solutions be kept at temperatures above 50F, which may require heated rooms in cold weather for the apparatus. When a heated room is not available, the chlorine should be fed as a dry gas in cold weather.

In the presence of moisture, chlorine gas is exceedingly corrosive. It seriously

[1] *Ibid.*

affects the linings of the throat, nose, and lungs, and if inhaled in sufficiently large quantities will produce death. The dry gas or liquid may be conveyed from the cylinders to the chlorinator by high-pressure iron or copper pipe, but wet gas and chlorine solutions should be conveyed in pipes made of earthenware, glass, rubber, silver, or special alloy. Rubber should not be used with liquid chlorine. Chlorine is about 2.5 times as heavy as air, and it therefore sinks to low places when it escapes owing to accidents or leaks. Chlorinator rooms should be provided with forced-draft ventilation exhausting at the floor, and the exhaust pipe should discharge at sufficient height to secure adequate diffusion of the gas before it reaches the ground. Sprinkler systems are an added precaution, for the water will convert some of the gas on the floor to hydrate and will absorb some of it. All chlorinating plants should be equipped with gas masks and other approved safety devices.

Ammoniators are designed similar to chlorinators, except that different materials are used for the conduits in order to avoid corrosion. Ammonia gas[1] is also an irritant to the nose and throat and in the presence of moisture is corrosive to copper. Steel pipe and valves may be used for conduits. Ammonia is only about half as heavy as air and hence rises. Where ammonia is used, ventilation exhausts should be at both floor and ceiling. Anhydrous ammonia is sold commercially in steel drums as a liquid under pressure. At room temperature, the gage pressure is about 115 psi and at 0C about 50 psi. Precaution should be taken never to mix chlorine and ammonia gases, or concentrated solutions of the gases, before they are applied to the water, for under certain conditions they may combine to form the highly explosive nitrogen trichloride. Sprays of water are effective in removing gaseous ammonia from a room since it is highly soluble. Chlorine and ammonia apparatus should be housed in a special room provided for that purpose, and in large plants the apparatus and containers are preferably housed in separate rooms.

Experience indicates that a maximum continuous rate of evaporation may be maintained for liquid chlorine of about 35 lb per day from 100- and 150-lb cylinders and 400 lb per day from ton containers. Ammonia may be evaporated at a rate of about 30 lb per day from 100- and 150-lb cylinders.

In dosing chlorine and ammonia into water, it is important that the solutions be well mixed with the water to ensure uniform diffusion. Violent mixing is desirable and if obtained will usually result in a saving of chemicals.

A recent development in chlorination is the use of the *breakpoint* method[2] in which sufficient chlorine, usually several parts per million, is applied to react with all the ammonia in the water and produce a residual of *free* available chlorine. It is claimed that with most waters high residuals of free chlorine may be obtained by this method with no trouble from chlorinous odors and with increased bactericidal efficiency.

Chlorine dioxide[3] has recently been adapted to use for disinfection and odor control. Chlorine dioxide, ClO_2, is a powerful oxidizing agent produced by the reaction of chlorine with a solution of sodium chlorite, $NaClO_2$. It has been found effective for taste and odor control with some waters, and it gives promise of being an effective germicide at pH values up to and above 10.

Other Disinfectants. Mallman[4] showed that germicidal action begins with *B. coli* when sufficient alkali is present for a pH of 11 and that sterilization is complete in less than 1 min at pH 13. According to Hoover,[5] in softening the Columbus, Ohio, water, when enough lime is added to precipitate Mg, *B. coli* and *B. typhosus* are killed in 48 hr, provided the water does not have a large organic content. Disinfection is

[1] Hedgepeth and Bingham, *16th Ann. Rept.*, Ohio Conference on Water Purification, 1936.
[2] Griffin and Chamberlin, *Am. Jour. Pub. Health*, **35**, 199, 1945.
[3] Aston and Synan, *Jour. N. E. W. W. A.*, **62**, 80, 1948.
[4] Mallman, *loc. cit.*
[5] Hoover, *Jour. A.W.W.A.*, **23**, 1196, 1931.

complete in 5 to 24 hr when an excess of ½ to 1 gpg of lime is added. Disinfection by excess-lime treatment is practical only in lime-softening plants.

Ozone,[1] O_3 is a powerful oxidizing agent and a very effective germicide. It is usually produced by the electrolysis of air at 7,000 to 20,000 volts. At 25C, about 10 g of ozone are produced per cubic meter of air. For disinfection, a dose of 1 to 3 ppm of ozone is required. The water should be clear and low in organic matter in order to avoid waste of ozone due to oxidation of organic matter. Ozonization of the water is usually accomplished by allowing the O_3 gas to pass upward through a bed of coke or gravel through which the water is trickling downward. As ozonized water is very corrosive, it must be aerated before it is passed into the distribution system. Ozone is effective in odor control because it oxidizes odor-producing organic compounds. Ozonization has been extensively used in Europe, but it has been found costly in the United States.

Ultraviolet light rays with a wave length of 0.3μ or less are effective germicides in clear water. The light is usually produced by mercury-vapor quartz lamps, but carbon arcs are also applicable. According to Perkins and Welch,[2] ultraviolet rays are effective to a depth of 20 in. in clear colorless water, and the killing action is instantaneous on all save certain resistant forms of bacteria. The resistant forms comprise less than 10 per cent of the bacteria. The usual method of application of ultraviolet light is to pass the clarified water in thin layers much less than 20 in. past a series of lamps, but, according to Perkins and Welch, this practice is not necessary since the resistant forms will survive the whole series. Thoroughly clarified and colorless water is essential for the success of this method. The advantages of the method are (1) no change in the chemistry of the water and (2) no possibility of an overdose. The method is more expensive than the use of chlorine or chloramines.

Ionized silver[3] in concentrations of 0.05 to 0.3 ppm effects sterilization of water after 2 to 8 hr. The silver may be added to the water as a salt such as silver nitrate, as colloidal silver, by means of silver electrodes, or the water may be placed in contact with or passed through silver-coated sand or porcelain. For very high bacterial counts, the required silver concentration is proportional to the count. With low counts, water treated with silver retains its germicidal property until the ionized silver is lost by adsorption on the surfaces of the container and suspended matter. The germicidal effect has been found by Shapiro and Hale[4] to persist for 24 hr with little loss of strength but to be substantially gone in 6 days. The silver appears to be ineffective in salt water and weakly effective in the presence of free ammonia. The rate of disinfection is increased with the temperature. The silver process is sometimes called the *katadyn* process after a type of silver developed by Krauze; the germicidal effect is known as an *oligodynamic* property, a term originated by Nägeli. No odors are produced by the process. It is very much more expensive than chlorination.

Algaecides. Plankton which grow in open reservoirs are troublesome owing to the production of odors and because they cause rapid clogging of filters. The most widely used method of destruction of plankton is by the use of copper sulphate distributed evenly over the water surface. The chemical is customarily distributed by dragging a sack of the crystals systematically through the water by boat until the desired dose is obtained. Treatment is required only at times when the algae growth is heavy enough to cause trouble. The killing action of dissolved copper is similar to that of silver. A list of the names of common troublesome microorganisms together with characteristic odors produced and the required copper sulphate dose to kill each

[1] See HORWOOD, "Sanitation of Water Supplies," Thomas Publishing Company, 1932.

[2] *Jour. A.W.W.A.*, **22**, 959, 1930.

[3] See JUST and SZNIOLIS, *Jour. A.W.W.A.*, **28**, 492, 1936; and GIBBARD, *Am. Jour. Pub. Health*, September, 1933, p. 910.

[4] *Jour. N.E.W.W.A.*, **51**, 113, 1937.

is given in "Water Works Practice."[1] The required dose ranges from 0.05 to 10 ppm, depending upon the organism. Methods of identification and counting plankton are given in "The Microscopy of Drinking Water"[2] and "Standard Methods for the Examination of Water and Sewage."[3] If copper sulphate is applied in sufficient concentration, it may result in the destruction of fish life. The safe dose ranges from about 0.14 ppm for trout to 2.1 ppm for black bass.

Chlorine is also used as an algaecide, both in liquid form and as hypochlorite. It is usually applied at treatment plants, but portable chlorinators are available for distributing chlorine over the surface of reservoirs. The dose required varies from about 0.3 to 3 ppm. If the organisms are causing odors, chlorine may accentuate them at the time of application.

TASTES AND ODORS

Sources and Measurement. The term *tastes and odors*[4] as applied to drinking water usually refers to cases where only odors are present. Only four true taste sensations are recognized: (1) sour or acid, associated with hydrogen ions; (2) sweet, found in sugars and associated with hydroxyl; (3) salty, produced by chlorides, nitrates, and sulphates; and (4) bitter, associated with alkaloids. Most other so-called tastes are due to odors that reach the olfactory organs at the time the material is tasted.

The sources of odors in water may be classified[5] as follows:

1. Essential oils from plankton
2. Decaying vegetation
3. Sulphur gases in ground water
4. Chlorine
5. Phenolic wastes from
    - Dye plants
    - Phenol-manufacturing plants
    - Coke-oven plants
    - Gas plants
    - Wood-distillation plants
    - Tar and oil refineries
6. Nonphenolic wastes from
    - Paper mills
    - Tanneries
    - Beet-sugar mills
    - Canneries
    - Starch plants
    - Sewage-treatment plants
7. Chlorine combined with substances listed in 2, 5, and 6.

Odor-producing substances are volatile, and it is the gaseous molecules that produce the sensation of smell in the olfactory region of the nose. The concentration in solution which is required to produce an odor is so extremely small for many odor-producing substances that relatively few of them have been identified chemically. The only means of measuring the concentration of such substances is through the strength of the odor produced, and the human nose is the measuring instrument. Consequently, the measurement of odors is not yet on a very satisfactory basis. The smallest amount of a substance required to produce an odor, usually measured in terms of its concentration in air, is called the *threshold value* of the odor. An odor in water is measured by sniffing the air in contact with the water when the air and water have come to equilibrium with reference to the odor-producing substance. The threshold point may be reached by diluting either the water with odor-free water or the

[1] A.W.W.A., 1929, p. 168.
[2] WHIPPLE, FAIR, and WHIPPLE, John Wiley & Sons, Inc., 1927.
[3] A.P.H.A., 1946.
[4] See FAIR, *Jour. N.E.W.W.A.*, **47**, 248, 1933.
[5] See BUSWELL, "Chemistry of Water and Sewage Treatment," The Chemical Catalog Company, 1928, p. 195.

air with odor-free air until the odor is just discernible. The ratio of the volume of the diluted sample to the undiluted sample is called the *threshold number*,[1] or *odor intensity*.

Methods of Control. The following methods[2] of treatment for control of odors have found practical application:

1. Ammonia, chlorine, chlorine dioxide
2. Powdered activated carbon
3. Prechlorination
4. Aeration
5. Copper sulphate treatment of reservoirs
6. Superchlorination followed by dechlorination
7. Potassium permanganate
8. Granular activated carbon
9. Bleaching clay
10. Chlorine for hydrogen sulphide
11. Ozone

The first five methods are widely used. The best method for any particular case depends upon the circumstances and sometimes can be determined only by trial. Many of the processes have treatment value in other respects, and they therefore cannot be evaluated solely on the basis of odor control. Ammonia-chlorine treatment, for example, has considerable value in the long-sustained chlorine residual; and all the methods involving the use of chlorine, ozone, and permanganate may be primarily for disinfection purposes. Aeration has value in iron removal, softening and corrosion control due to carbon dioxide removal. For odor removal, aeration is effective in the case of H_2S and for very volatile odorous substances from certain of the microorganisms. The destruction of plankton by copper sulphate is of value in increasing filter runs as well as for odor control.

Prechlorination may be used when chlorine successfully oxidizes the odor-producing compound or destroys the causative microorganisms and is not itself required in sufficient concentration to produce an odor. When the usual dose of chlorine combines with odor-producing substances to accentuate the odor, it has sometimes been found that an overdose of chlorine will destroy the odor provided the residual chlorine is later removed by an antichlor. The earliest dechlorinating methods involved the use of reducing agents, among which are sulphur dioxide gas, sodium thiosulphate, sodium sulphate, sodium bisulphate, and sodium metabisulphate. The reaction of Cl_2 with all these agents produces Cl^- ions. Activated carbon may also be used for dechlorination, the reaction producing CO_2 and Cl^-.

Activated carbon effectively destroys most odors by adsorbing the compounds from solution onto the surface of the solid carbon. The adsorption process is not well understood, and the mechanism probably varies with the character of the compound adsorbed. Adsorption is a surface phenomenon, and the rate of adsorption is a function of the amount of unsaturated surface area on the adsorbent. Activated carbon is prepared from both vegetable and animal (bone) chars which are very porous and hence have immense surface areas. The carbons most widely used in water treatment are prepared from lignite, a paper-mill wood pulp, and birch and maple wood. The process of activation consists of heating the char in steam or air to a temperature somewhat less than 600C. Activation apparently results in the volatilization of adsorbed compounds, such as hydrocarbons, leaving the primary carbon surface free.

Carbon is widely used in the powdered form and is dosed into the water by means of either dry or solution feed machines. It may be applied with the coagulating

[1] For methods see "Standard Methods for the Examination of Water and Sewage," A.P.H. A., 1946; SPAULDING, *Am. Jour. Pub. Health*, 1931, p. 1038; FAIR, *Jour. N.E.W.W.A.*, **47**, 248, 1933; FAIR and WELLS, *Jour. A.W.W.A.*, **26**, 1670, 1934; and BAYLIS and GULLANS, *Jour. A.W.W.A.*, **28**, 507, 1936.

[2] For extended discussion, see BAYLIS, J. R., "Elimination of Taste and Odor in Water," McGraw-Hill Book Company, Inc., 1935.

chemicals into the mixing chamber, during flocculation or just prior to filtration. In the first case, much of the carbon is removed from the water by settling, but in the latter case all must be removed by the filters and filter runs may be shortened. On the other hand, a larger dose is required if the carbon is fed with the coagulants because some of the carbon becomes coated with suspended matter. The amount of powdered carbon required varies from about 2 ppm up to 50 or 60 ppm, depending upon the strength and type of odor. Powdered carbon is wasted after use with the sludge of the settling tanks and with waste wash water.

A more economical use of carbon may be obtained with granular carbon beds, but additional units similar to rapid filters are required at considerable expense. The additional capital cost is not usually justified unless odors persist throughout the year. Powdered-carbon treatment is more economical for seasonal odor troubles. Carbon beds are made 2 to 4 ft deep and are operated at rates of 1 to 6 or more gpm/sq ft. The required rate of application of water depends upon the concentration and type of odor-producing substances to be adsorbed. Carbon units are made either upflow or downflow, the former being limited to rates of about 4 gpm/sq ft in order to prevent suspension of the material. Carbon beds are usually used after filtration, but the beds nevertheless require washing occasionally and should be designed for washing. Relatively low wash rates are required for expansion, owing to the lightness of the material. The activity of granular carbon is gradually exhausted in service because of adsorption of many substances in addition to odor-producing compounds. The life of granular carbon may be several years, in which case it may be more economical to replace the carbon with new material than to reactivate it.

SECTION 22

SEWERAGE

By Samuel A. Greeley and William E. Stanley

This section covers the hydraulic aspects of the design of sewers. Other factors also must be considered by the engineer in determining the best plan for a specific project. For example, an early decision must be reached as between a *combined* system of sewers, carrying both sewage and storm water, and a *separate* systems of sewers in which sewage and storm water are carried in separate conduits, designated as sanitary sewers and storm sewers, respectively.[1] Engineering practice and the regulatory codes of state departments of health tend toward separate sewers, although in some cases, local considerations require combined sewers. A factor favoring separate sewers is the lesser cost of sewage treatment. Conclusions as to the type of system to be adopted are properly based upon the extent and character of existing sewers, the geographical location, the relative elevations of sewers and waterways, the extent of sewage treatment, and the relative costs both present and future.

1. QUANTITY OF DOMESTIC, INDUSTRIAL, AND COMMERCIAL SEWAGE

Domestic sewage comprises the soiled water of a community and a limited amount of infiltration. It is important, therefore, to determine the tributary population, the portion of the water consumption that finds its way into the sewers, and the amounts of commercial and industrial wastes and of infiltration.

Design Period. Sewers should be designed with a capacity sufficient to provide for some future development, the extent of which depends upon local considerations and the character and use of the sewers.

Generally, lateral and submain sewers are designed for the anticipated ultimate development of the area, whereas the capacity of main sewers, outfalls, and intercepting sewers is based upon estimates of conditions 40 or 50 years in the future. Another procedure, sometimes useful, is to consider what capacity the present population might reasonably undertake to provide for future growth. Some typical data (Table 1) from intercepting sewer-design experience indicate ratios of future to present population ranging from 1.3 to 3.8 with an average of about 2.5. Similar data for sewage-treatment works are given in Table 2.

Area Development. Sewer design should include a consideration of the probable expansion of the community in area, and a reasonable amount of capacity should be provided in the main sewer lines to take sewage from the likely extensions of sewered areas. Table 3 shows the extensions of area for which provisions were made in a number of illustrative comprehensive sewerage projects.

The areas within a city may be classified with regard to their effect upon sewage quantities as follows:

1. Residential
   a. Light residential, single houses
   b. Heavy residential, multiple dwellings and apartment houses
2. Commercial
3. Industrial
4. Public use: parks, playgrounds, cemeteries, etc.

[1] A glossary of terms for use in water and sewage engineering entitled "Glossary, Water and Sewage Control Engineering," was published in March, 1949, by a Joint Committee representing A.P.H.A., A.S.C.E., A.W.W.A., and F.S.W.A.

Zoning regulations aid materially in anticipating future developments, but they are sometimes modified, at the expense of the single-home residential areas, when there is a need for more extensive commercial, industrial, or apartment house districts.

Some indication of the relative proportions of urban areas likely to be devoted to various uses is furnished by the typical data in Table 4.

TABLE 1.—POPULATION INCREASES FOR WHICH TRUNK-LINE SEWER CAPACITY HAS BEEN PROVIDED AT VARIOUS PLACES

City	Population		Ratio population basis of design to population at time of design
	At time of design	Design basis	
Boston, Mass.:			
North Metropolitan District	741,000	836,000	1.13
South Metropolitan District	844,000	1,000,000	1.19
Milwaukee, Wis	414,000	862,000	2.08
Buffalo, N. Y	600,000	1,100,000	1.83
Toledo, Ohio, East Side	43,180	164,270	3.80
Toledo, Ohio, West Side	211,520	617,630	2.93
Louisville, Ky	105,103	274,850	2.62
District of Columbia	500,000	850,000	1.70
Philadelphia, Pa., Northeastern	230,000	740,000	3.22
Philadelphia, Pa., Southwestern	368,000	651,000	1.77
Cincinnati, Ohio, Mill Creek	138,738	308,664	2.22
Cincinnati, Ohio, Ohio River	238,794	303,826	1.27
Springfield, Ill	62,000	193,000	3.12
Oklahoma City, Okla	118,000	380,000	3.22
Rockford, Ill	85,000	210,000	2.47
Urbana-Champaign, Ill	35,000	70,000	2.00
Peoria, Ill	100,000	200,000	2.00
Decatur, Ill	43,818	120,000	2.74
Elgin, Ill	28,260	70,000	2.48
Evansville, Ind	105,000	225,000	2.14
Eau Claire, Wis	30,000	70,000	2.33
Oshkosh, Wis	40,000	70,000	1.75
Winona, Minn	21,000	45,000	2.14
Appleton, Wis	27,000	58,000	2.15
Maximum	....	....	3.80
Minimum	....	....	1.13
Average	....	....	2.26
Proposed for Madison, Wis	100,000	185,000	1.85

Several bases have been used in the past to forecast the future growth of cities as follows:

1. *A uniform percentage rate of increase* per decade for future growth based upon the rate of growth between recent census periods.
2. *An arithmetical increase* in population per decade or per year.
3. *A graphical extension* of the curve of past growth into the future.
4. *A graphical comparison* with the growth of other similar but larger cities after the date at which their population was the same as that of the city under consideration.
5. *A decreasing percentage rate of increase* for each decade in the future. This is particularly applicable to larger cities.

The method of graphical comparison has been, perhaps, the most suitable for general use, but it is frequently advisable to compare the results obtained by several procedures. Local conditions that may have been responsible for peculiar changes

in past growth or that are likely to affect future growth in any unusual way should be given careful study. Consideration should also be given to the relative growth of other near-by cities and to the current trend of birth and mortality rates. Owing to the difficulty of predicting the future population with any great degree of accuracy, it is sometimes useful to represent the probable future growth by two limiting lines on the population curve, as illustrated in Fig. 2.

TABLE 2.—POPULATION INCREASES FOR WHICH SEWAGE-TREATMENT PLANTS HAVE BEEN DESIGNED AT VARIOUS PLACES

City	Population		Ratio population basis of design to population at time of design
	At time of design	Design basis	
Boston Mass.:			
North Metropolitan District	741,000	790,000	1.07
South Metropolitan District	844,000	980,000	1.16
Milwaukee, Wis	414,000	588,750	1.42
Indianapolis, Ind	350,000	500,000	1.43
Rochester, N. Y	250,000	438,000	1.75
District of Columbia	500,000	650,000	1.30
Buffalo, N. Y	600,000	750,000	1.25
Decatur, Ill	43,818	60,000	1.40
Urbana-Champaign, Ill	35,000	45,000	1.29
Elgin, Ill	28,500	37,000	1.32
Oklahoma City, Okla	111,600	136,000	1.48
Springfield, Ill	58,300	90,000	1.55
Holland, Mich	15,000	22,500	1.50
Worcester, Mass	200,000	275,000	1.38
East Orange, N. J	86,000	133,000	1.55
Schenectady, N. Y	94,000	120,000	1.28
Albany, N. Y	101,000	150,000	1.49
Fitchburg, Mass	42,000	55,000	1.31
Rockford, Ill	95,000	140,000	1.48
Medford, Ore	11,000	17,000	1.54
Winona, Minn	21,000	30,000	1.43
Appleton, Wis	27,000	32,500	1.20
Neenah-Menasha, Wis	20,000	25,000	1.25
Leominster, Mass	21,000	25,000	1.19
Eau Claire, Wis	30,000	35,000	1.17
Maximum	....	....	1.75
Minimum	....	....	1.07
Average	....	....	1.37
Proposed for Madison, Wis	100,000	134,500	1.35

Metropolitan Areas. Estimates of the future development of large metropolitan areas or of smaller communities in close proximity to large cities may be made by relating their growth to the predicted future growth of the principal city (Fig. 3). Average population densities for a number of large cities are given in Table 6.

Population Distribution. As a basis for estimating sewage quantities for various sewers, the city should be subdivided into sewer districts and subdistricts, with boundaries determined largely by topography and by artificial developments of the city, and the present and estimated future populations should be distributed into the several districts. Distribution of the present population may be estimated by obtaining the number of residences in the several districts from fire insurance maps, airplane

TABLE 3.—EXTENSION OF CITY AREA FOR COMPREHENSIVE SEWERAGE DEVELOPMENTS
(Areas in Acres)

City	Year of study	Area considered in sewerage-project study		
		Presently incorporated	With future extensions	Ratio
Bloomington, Ind.	1930	2,250	4,941	2.2
Buffalo, N. Y.	1936	21,370	31,225	1.5
Cincinnati, Ohio	1913	44,800	89,600	2.0
Dallas, Tex	1930	28,160	114,340	4.1
Decatur, Ill.	1924	4,885	9,650	2.0
Detroit, Mich.	1915	30,200	74,500	2.5
Milwaukee, Wis.	1910	16,400	37,790	2.3
Minneapolis-St. Paul, Minn.	1928	72,000[1]	134,200[1]	1.9
Rockford, Ill.	1927	7,574	28,600	3.8
Sheboygan, Wis.	1929	3,286	12,621	3.9
Springfield, Ill.	1924	7,544	12,472	1.7
Toledo, Ohio	1919	15,945	47,300	3.0

[1] Includes streets, alleys, and school grounds, but does not include parks, playgrounds, or cemeteries.

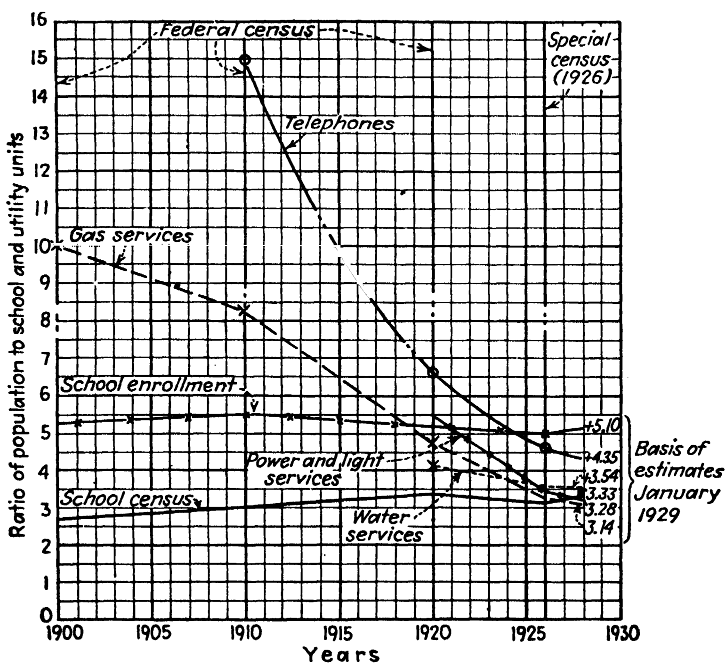


FIG. 1.—Ratio of population to school and utility units, Sheboygan, Wis.

TABLE 4.—PERCENTAGE OF CITY AREAS ALLOTTED TO VARIOUS USES

City	Year	Percentage of total area			
		Residential	Industrial	Commercial	Parks
Decatur, Ill.	1924	71.8	8.8	5.9	13.5
Cincinnati,[1] Ohio	1950	79.6	12.2	4.9	3.3
Louisville,[2] Ky	1960	83.8	5.1	1.8	9.3
Milwaukee, Wis	1930	87.8	4.0	0.3	7.9
Toledo,[3] Ohio	1918	84.2	10.5	...	5.3
Buffalo, N. Y	1935	81.0	10.1	3.5	5.4

[1] Estimated for Sewer districts 40 to 80.
[2] Estimated for Beargrass Intercepting Sewer District.
[3] Ten Mile Creek District.

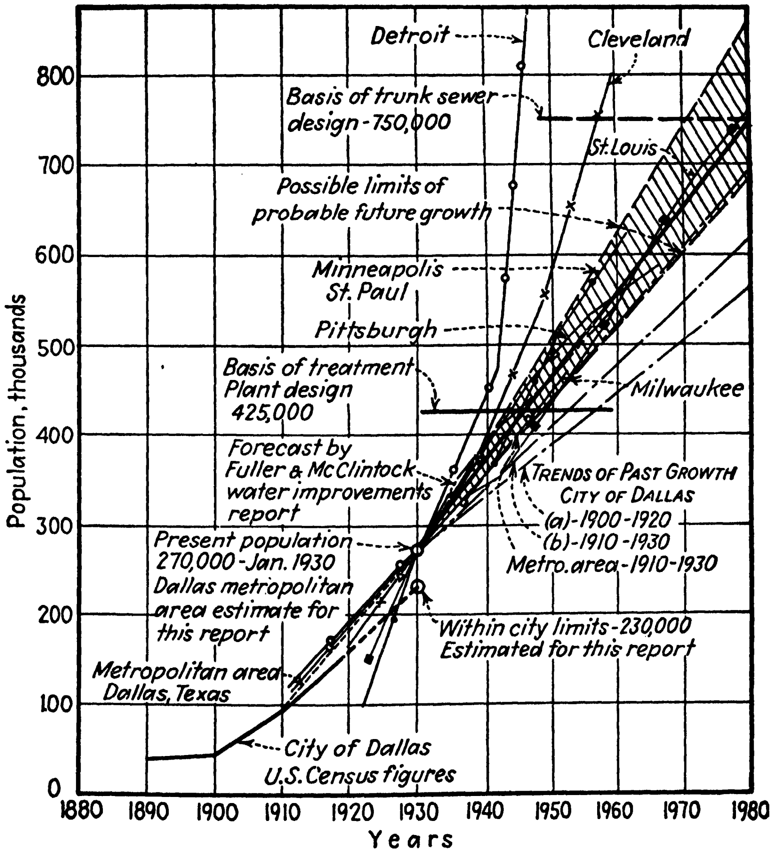


FIG. 2.—Estimated population, Dallas, Tex.

NOTE: United States Census of 260,475 for Apr. 1, 1930, included some area annexed during March and April.

TABLE 5.—VARIOUS ESTIMATES OF PRESENT POPULATION OF SHEBOYGAN, WIS., MADE IN 1929

Basis	Population
1. School census, 11,788 at 3.33	39,200
2. School enrollment, 7,443 at 5.10	37,900
3. Water services, 11,173 at 3.54	39,600
4. Gas services, 12,872 at 3.14	40,400
5. Power and light services, 11,797 at 3.28	38,700
6. Telephones, 9,015 at 4.35	39,300
7. Chamber of Commerce estimate (1928)	42,408
8. Telephone Company commercial survey (1924 forecast based on past growth) (1930)	37,400
Estimated	40,000
U.S. Census, Apr. 30, 1930	39,251

maps, actual field counts of houses, by the registration of voters by precincts or by recent census counts by enumeration districts.

Based upon a study of such factors as present population densities, type of development, zoning regulations, and any indicated migration of population from one district to another, the future density can be estimated for each district. The population of various types of districts tends to approach maximum densities (Fig. 4), approximating 15 to 20 persons per acre for light residential (single-residence) areas, 55 per

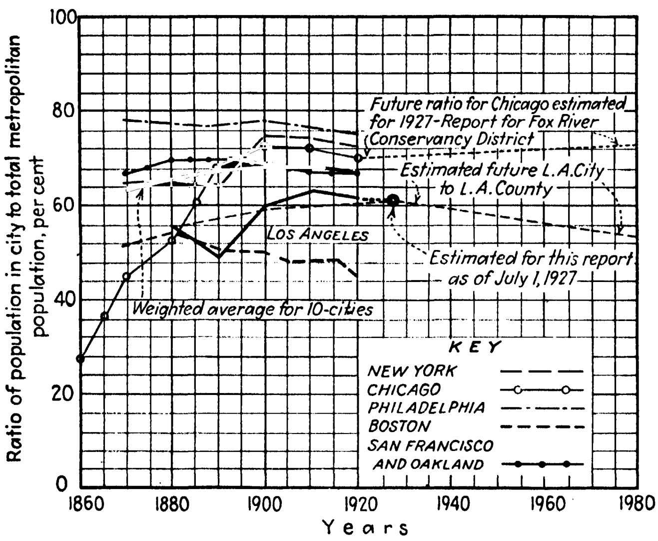


FIG. 3.—Population data, ratio of city to metropolitan population. Los Angeles County, California, January, 1928.

acre for multiple-dwelling and small apartment house districts, and ranging from 10 to 30 per acre for commercial and industrial districts (Fig. 4).

Capacity Factors. The distribution of future population into sewer districts is less certain than the population estimate for the entire city. This uncertainty is less (1) for larger districts than for smaller districts, and (2) for more densely populated districts than for sparsely developed districts. Some allowance should be provided in the computed capacity of main and submain sewers to cover such uncertainties of population distribution. In practice, this may be done by multiplying the estimated future population for a given area by a capacity factor, ranging from 1.0 to 2.0

TABLE 6.—POPULATION DENSITY COMPARISON[1]

Municipality	Population	Area, acres	Population density, persons per acre
New York, N. Y.	6,930,446	191,360	36.2
Chicago, Ill.	3,376,438	129,216	26.1
Philadelphia, Pa.	1,950,961	81,920	24.7
Detroit, Mich.	1,568,662	88,256	17.8
Cleveland, Ohio	900,429	45,286	19.9
St. Louis, Mo.	821,960	39,040	21.0
Baltimore, Md.	804,874	50,381	16.0
Boston, Mass.	781,188	28,096	27.8
San Francisco, Calif.	634,394	26,880	23.6
Milwaukee, Wis.	578,249	26,329	21.9
Buffalo, N. Y.	573,076	24,896	23.0
Washington, D. C.	486,869	39,680	12.3
Cincinnati, Ohio	451,160	45,702	9.9
Indianapolis, Ind.	364,161	34,656	10.5

[1] From U.S. Census, 1930.

(Fig. 5), to obtain a figure that may be designated as the basis of design population, which is then multiplied by the selected per capita flow to obtain the rate of sewage flow for which capacity should be provided.

Designation of Sewage-flow Rates. Unit domestic sewage quantities are frequently stated in terms of gallons per capita per 24 hr (gpcpd). Thus it is convenient to state total flow rates in terms of million gallons per 24 hr (mgd). However, it is rather a general practice to state capacities of pumps in gallons per minute (gpm) and to express storm sewage flows in cubic feet per second (cfs). These terms may be converted from one to another by the following factors:

To change from	Multiply by	Or divide by
gpm to mgd	0.00144	694
gpm to cfs	0.00223	449
mgd to gpm	694	0.00144
mgd to cfs	1.55	0.646
cfs to gpm	449	0.00223
cfs to mgd	0.646	1.55

Computation of Sewage Quantities. Estimates of probable future sewage flows may be based upon one or more of the following items:

1. Water consumption from public and private sources with a deduction for water that does not reach the sewers and an addition for probable infiltration
2. Short-term records of sewer gagings
3. Long-term records of sewer gagings
4. Arbitrarily selected per-capita sewage-flow rates based upon experience elsewhere
5. The tributary area and a unit per-acre allowance for the sewage flow based upon experience elsewhere

The preceding order possibly represents the frequency of use, *i.e.*, the first method is perhaps the most commonly used. Long-term gagings give the most reliable results but are frequently not available.

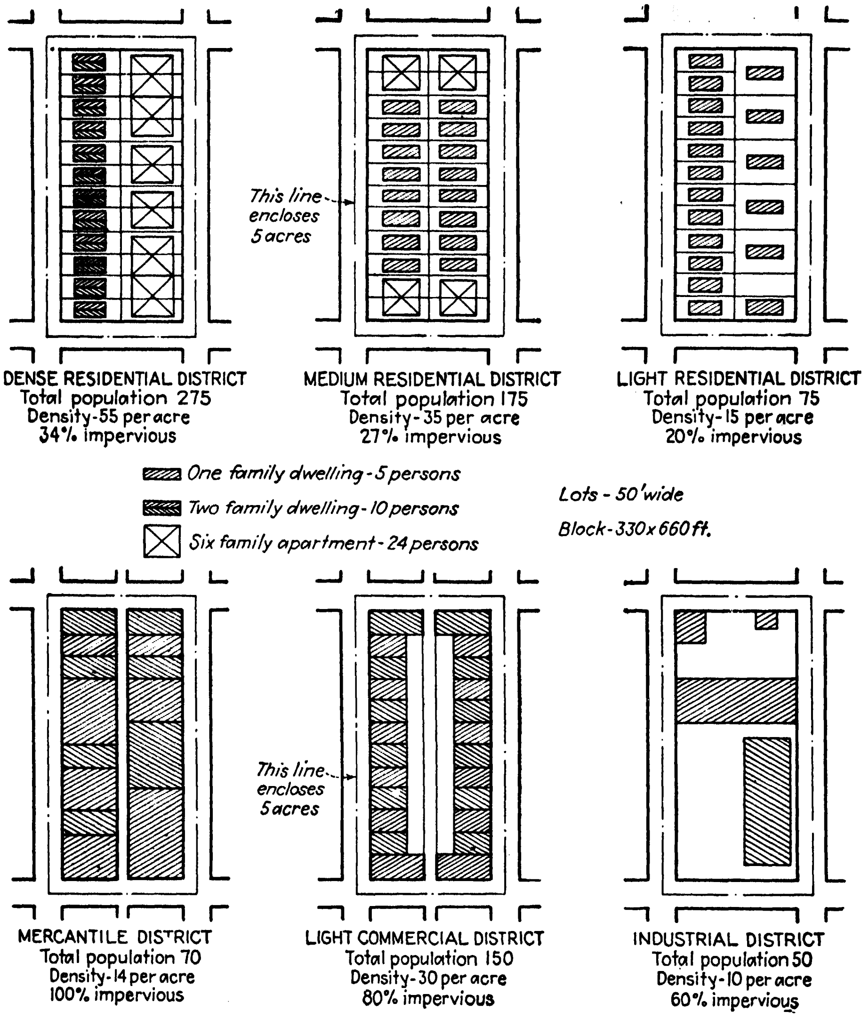


FIG. 4.—Typical blocks for different kinds of zones showing percentages of impervious surfaces and assumed population per acre contributing domestic sewage.

TABLE 7.—RATIO OF SEWAGE FLOW TO WATER PUMPAGE IN VARIOUS CITIES

Place	Year	Average sewage flow, percentage of average water pumpage
Austin, Minn.	Jan., 1937	111
Brockton, Mass.	1926	94
Metro. Distr. Com., Mass. North Metro. Sewerage Distr.	1926	123
Milwaukee, Wis.	1934	95
Providence, R. I.	1926	182
Quincy, Mass.	1926	80
Worcester, Mass.	1926	144

In estimating future sewage flows, the available records of water or sewage quantities should be related to the actual present population to which the quantities apply in order to obtain per capita quantities for use with the future population. Sewers must be designed for maximum or peak rates of flow which usually are obtained most conveniently by applying suitable ratio factors to the average unit rates.

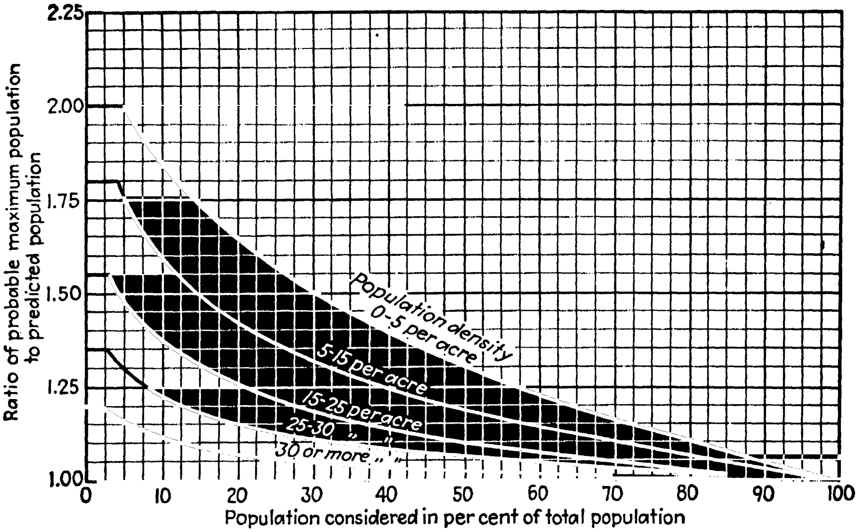


FIG. 5.—Capacity factors for unequal population development.

Typical computations of the relative quantities of water consumption and sewage flow are as follows:

Milwaukee Sewerage Commission (1911):		Gpcpd
Total water pumpage		105
Quantities not reaching sewers:		
Steam railroads	5	
Manufacturing and mechanical purposes	5	
Street sprinkling	5	
Lawn sprinkling	2½	
Consumers not connected to sewers	7½	
Leakage from mains and services	15	
Total	40	
Proportion not reaching sewers		38%
Bloomington, Ind. (1929):		
Total water pumpage	2.46 mgd	
Probable amount reaching sewers	1.87 mgd	
Proportion not reaching sewers		24%

In localities having private as well as public water supplies or where infiltration of ground water into the sewers is large, the total sewage flow may exceed the total water consumption. Table 7 indicates the lack of uniformity as regards the relation of the average sewage flow to the average water pumpage for various cities.

In a study of water-supply quantities as related to sewage flows, consideration should be given to the seasonal, daily, and hourly variations as well as the yearly average rate in gpcpd.

The yearly average per capita water-consumption rates differ widely for various cities (see Table 8). The seasonal, daily, and hourly variations in water-consumption rates for a typical city are illustrated by Figs. 6, 7, and 8.

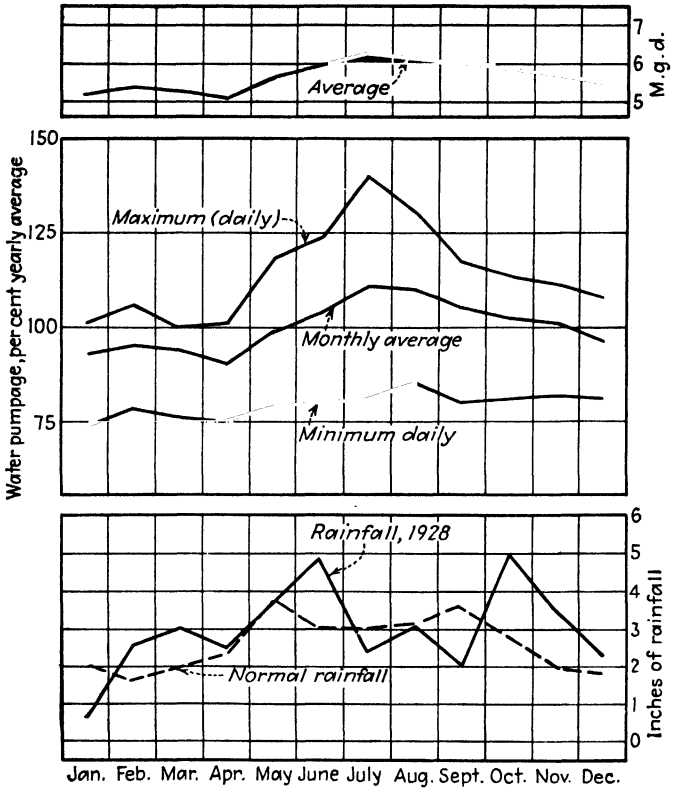


Fig. 6.—Monthly water pumpage rates, Sheboygan, Wis., 1928.

A record of water-consumption data from 67 cities in Massachusetts ranging in population from 2,000 to 146,000 gives the following data:

Item	Data from 67 cities		
	Avg of all	Max	Min
Yearly average daily consumption per capita	63	168	22
Maximum rates, per cent of yearly averages:			
Max monthly	128	217	102
Max weekly	147	272	106
Max daily	198	368	119

The hourly and daily variations of the flows in sewers are so affected by infiltration and other factors that frequently there is little relation between the sewage flow and the water consumption. This is illustrated by a series of careful gagings of sewers at Springfield, Ill. (Fig. 9).

Sewer Gagings and Sewage-flow Records. In the absence of the more reliable long-term records of measured sewage flows, it is sometimes desirable to secure approxi-

TABLE 8.—AVERAGE YEARLY WATER-CONSUMPTION DATA

City	Period	Water supply, gpcpd
Appleton, Wis.	1926–1930	77
Baltimore, Md.	1932	102
Chicago, Ill.	1931	268
Kaukauna, Wis.	1926–1930	35
Louisville, Ky.	1933	123
Menasha, Wis.	1926–1930	179
Milwaukee, Wis.	1934	126
Neenah, Wis.	1927–1930	71

mate flow data from short-term gagings. These gagings may be made by installing weirs in selected manholes and making measurements at intervals of 1 hr or less over a period of a few days. Less accurate methods make use of floats or dye to indicate the time of travel between manholes, the flow being computed from the velocity thus obtained and the depth of flow found by careful measurement.

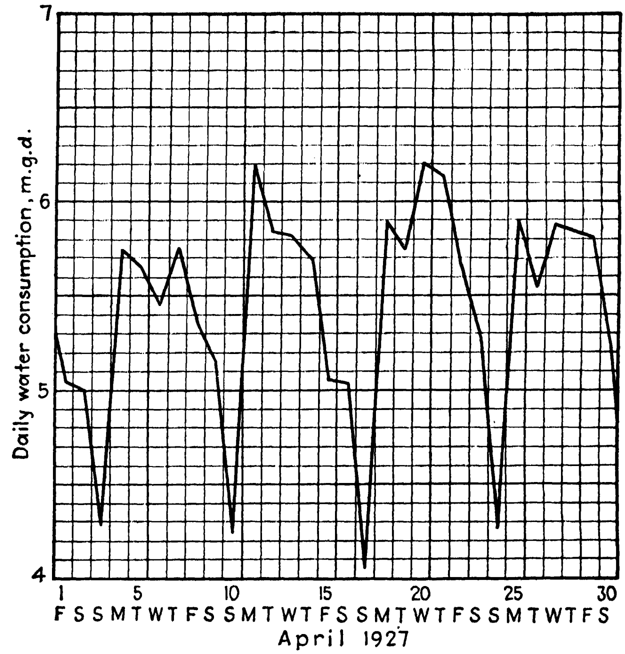


FIG. 7.—Daily variations in water pumpage, Sheboygan, Wis.

Such short-term records must be used judiciously and with due regard to groundwater elevations and rainfall and runoff conditions just preceding and during the period of measurement. Frequently the flows during a wet season are as much as twice the dry-weather flow, as indicated by the following typical results of sewer gagings.

City	Ratio Wet-season to Dry-season Sewage Flows
Bloomington, Ill.	2.0
Decatur, Ill.	1.5
Oklahoma City, Okla.	1.7
Toledo, Ohio	1.4
Urbana-Champaign, Ill.	2.0

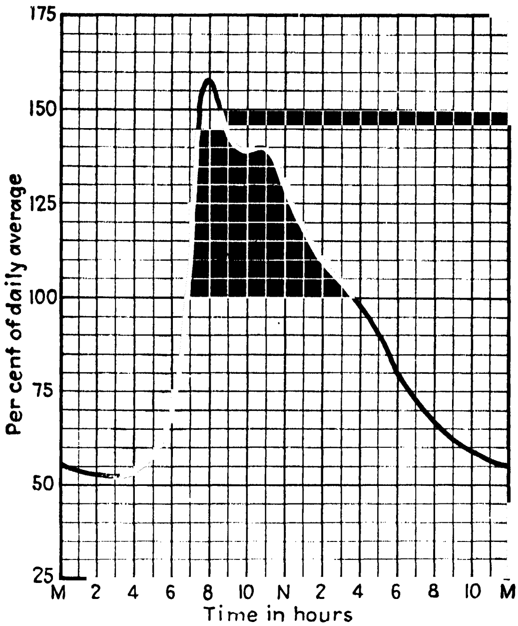


FIG. 8.—Hourly variations of water pumpage, Sheboygan, Wisconsin.

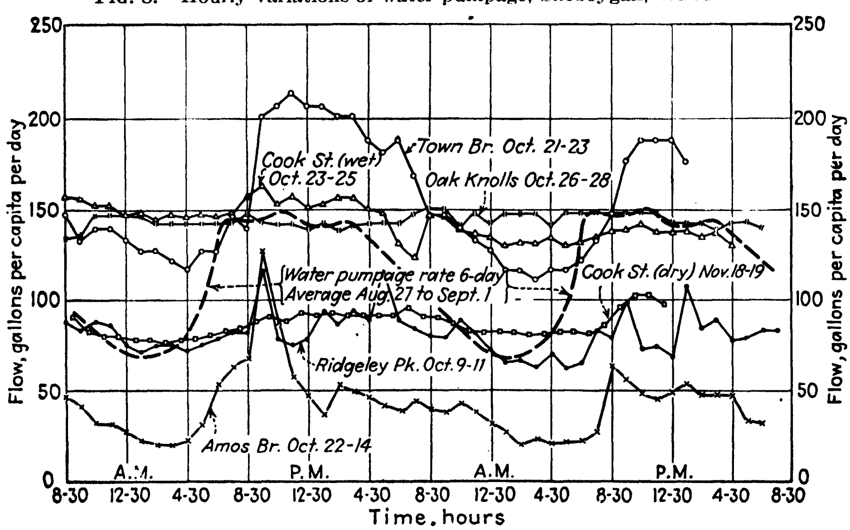


FIG. 9.—Sewage gagings, Springfield, Ill., December, 1923.

TABLE 9.—ESTIMATED QUANTITIES OF DOMESTIC SEWAGE IN VARIOUS CITIES

City	Tributary population	Yearly avg, gpcpd
Los Angeles, Calif.	1,060,000	79
Long Beach, Calif.	124,300	64
Pasadena, Calif.	83,000	65
Orange County, Calif.	60,500	55
Houston, Tex.	100,000	68
Fort Worth, Tex.	161,000	65
Toledo, Ohio	254,700	52
Milwaukee, Wis.	414,000	50
Lincoln, Neb.	65,000	61
Fitchburg, Mass.	37,320	75
Utica, N. Y.	100,000	47
Urbana-Champaign, Ill.	20,500	57
Pontiac, Mich.	38,200	75
Brockton, Mass.	24,800	55
Cincinnati, Ohio	32,693	85

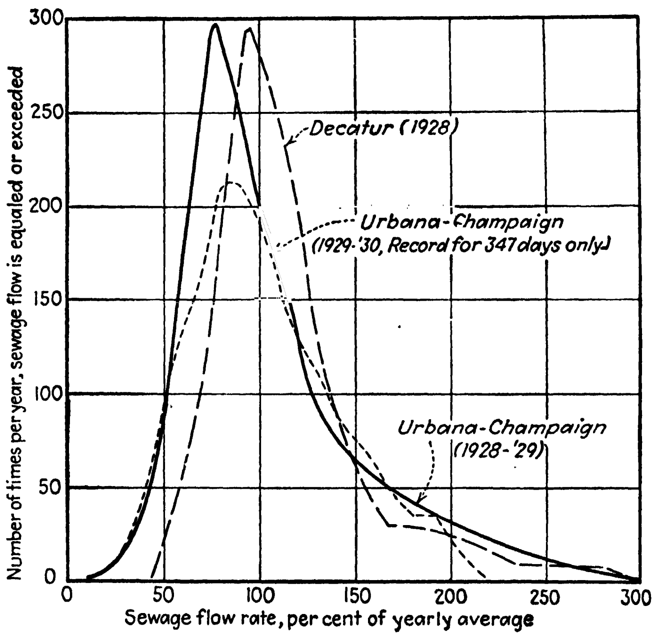


FIG. 10.—Occurrence of various rates of sewage flow, Urbana-Champaign Sanitary District.

NOTE: Yearly average 1928–1929 = 3.22 mgd. Yearly average 1929–1930 = 3.90 mgd. Each occurrence includes the entire period during which the sewage flow remains equal to or greater than the given rate.

Tables 9 and 10 show some data on average domestic sewage-flow rates for various cities. In general, the yearly average sewage flows range from 100 to 135 gpcpd for larger cities to as low as 50 to 80 gpcpd for smaller cities or for the less congested residential areas in larger cities. Figure 10, based upon a study of sewage flows at Urbana-Champaign and Decatur, Ill., indicates the number of occurrences each year of various sewage-flow rates in terms of the yearly average; whereas in Fig. 11 data are

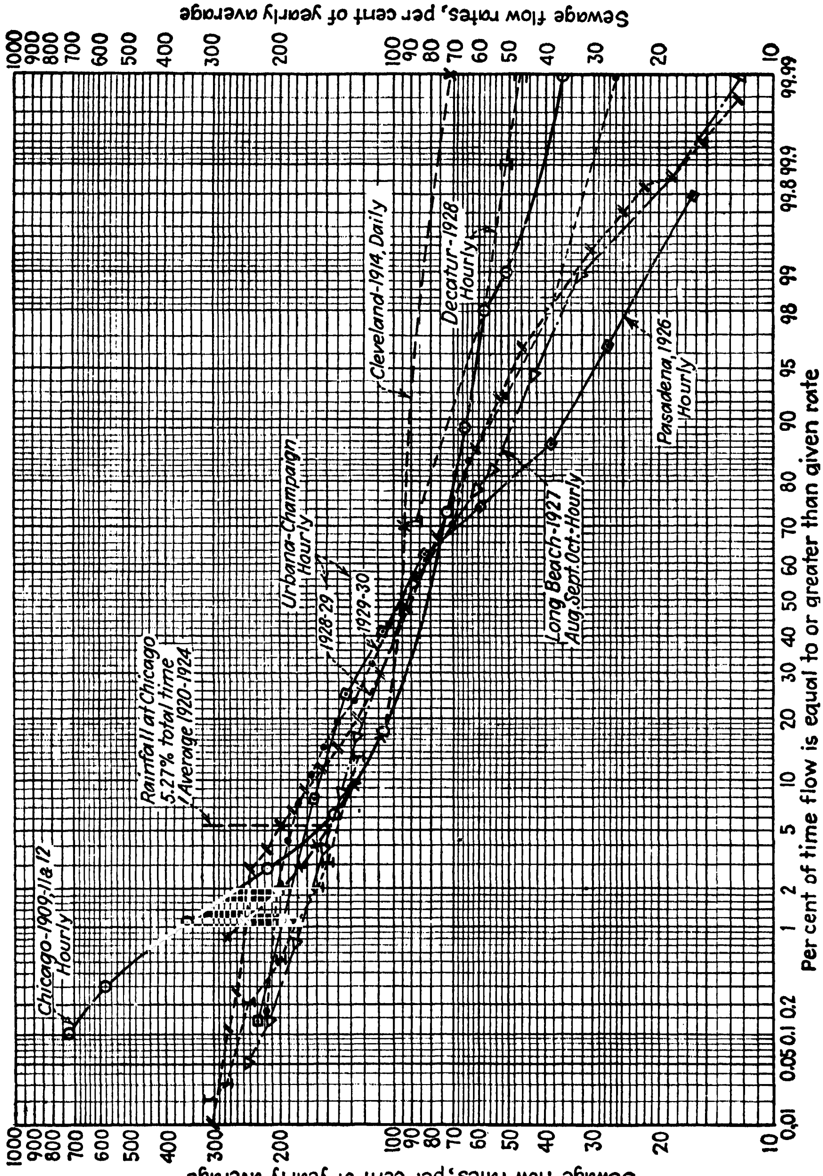


FIG. 11.—Probable variations of flow in sewers.

given on the proportion of time for various sewage-flow rates, based on hourly records, in terms of the yearly average daily rate, *i.e.*, flow-duration curves.

Data from extensive sewer gagings at Minneapolis-St. Paul, Minn., relating to the hourly variations in sewage flows are given in Fig. 12 for domestic or residential areas, industrial areas, and a composite curve for all areas. It will be noted that the general effect of industrial sewage and infiltration is to reduce the hourly variations somewhat.

The hourly variations of domestic sewage flow in terms of the yearly average daily rates are indicated by Fig. 13 for flows in the Pasadena main outlet sewer. These data include very little industrial sewage, but reflect the effect of occasional rains,

TABLE 10.—ACTUAL YEARLY AVERAGE SEWAGE FLOW RATES IN VARIOUS CITIES

City	Tributary population	Avg sewage flow	
		mgd	gpcpd
Alliance, Ohio	20,000	2.62	131
Antigo, Wis	5,600	0.65	116
Baltimore, Md	714,032	63.49	88
Columbus, Ohio	299,000	29.97	100
Decatur, Ill	50,000	10.82	204
Indianapolis, Ind	324,000	50.0	154
Milwaukee, Wis	600,000	86.0	143
Pasadena, Calif	85,000	8.37	68
Rochester, N. Y. (Irondequoit)	240,000	33.0	138
Rockford, Ill	57,100	9.09	159
Sioux Falls, S. D	28,000	5.1	182
Syracuse, N. Y	150,000	27.88	186
Worcester, Mass	185,000	21.8	118

although, in general, the normal amount of ground-water infiltration into the Pasadena sewers is quite small because of the low rainfall.

Maximum or Peak Rates of Flow. Long-term records of sewage flow, available for many cities, furnish helpful information regarding the relation of maximum or peak sewage-flow rates to the yearly average daily rates.

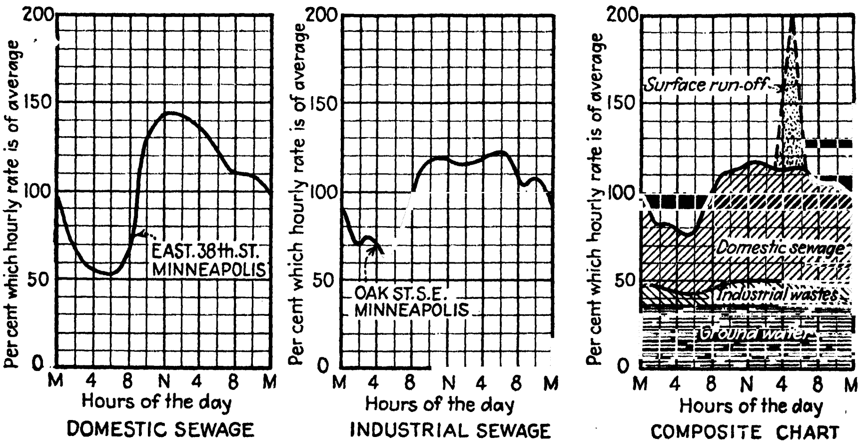


FIG. 12.—Hourly variations in sewage flows.

NOTE: Data based on sewer gagings by Metropolitan Drainage Commission, Minneapolis-St. Paul, Second Annual Report, 1928.

Figure 11, based upon sewage-flow records, is helpful in evaluating the proportion of time for various rates of sewage flow. Thus the curve for hourly readings at Pasadena indicates the following:

Ratio of maximum rate to yearly average	Proportion of time flow occurs	
	Per cent of time	Hours per year
2.3	0.14	12
2.0	0.8	70
1.8	3.9	342
1.5	13.8	1209

It seems reasonable to expect higher maximum rates of flow, as compared with average rates, for smaller areas or smaller numbers of people. This principle is reflected in many of the empirical procedures that have been proposed for arriving at

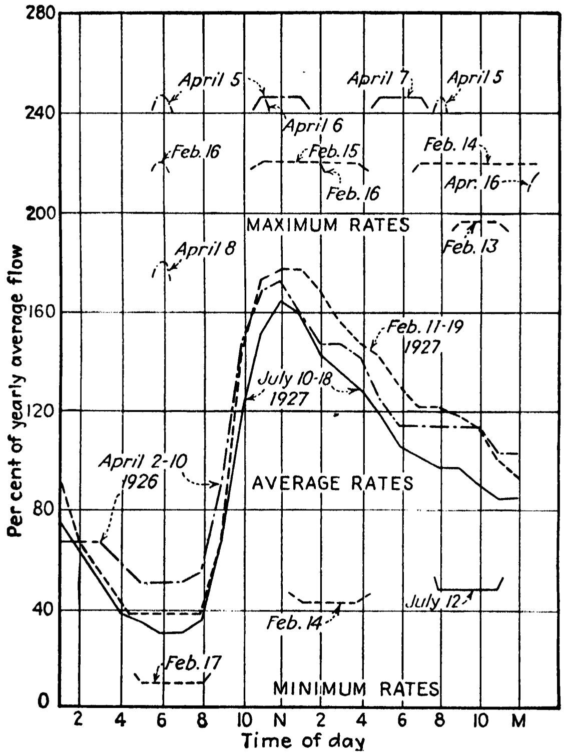


FIG. 13. Hourly variation in sewage flow, Pasadena, Calif.

peak rates of flow for sewer design. Two of these are (1) the formula given by Babbitt,[1] for use between the limits of 1,000 and 1,000,000 population, and (2) the relation offered by Harmon,[2] as follows:

[1] "Sewerage and Sewage Treatment," 6th ed., p. 87, John Wiley & Sons, Inc., 1947.
[2] *Eng. News-Record*, **80**, 1235, 1918.

$$(1)\ M = \frac{5}{P^{1/6}} \qquad (2)\ M = 1 + \frac{14}{4 + P^{1/2}}$$

in which P is the tributary population in thousands, and M the ratio of peak rate to average rate.

These formulas relate to the design capacity to be provided for domestic sewage flows. The capacity for fluctuations in other sewages may be less owing to the leveling influence of ground-water infiltration and of industrial and commercial sewages.

The Maryland State Department of Health in 1914 proposed that the basis of design of sanitary sewers should be related to the average sewage flow according to a certain chart, from which the following ratios have been computed. Similar charts are in use by the Washington Suburban Sanitary District and in New York City.

Sewage flows, mgd		Ratio
Average sewage flow	Basis of sewer design	
0.2	0.75	3.7
0.5	1.8	3.6
1.0	3.3	3.3
2.0	5.6	2.8
4.0	10.0	2.5
6.0	13.8	2.3
10.0	21.0	2.1
16.0	32.0	2.0[1]

[1] This factor applies to all flows above 16 mgd.

On the basis of 100 gpcpd for the yearly average daily domestic sewage flow, these three procedures yield the following comparative results:

Population	Average domestic sewage flow, mgd	Ratios of maximum to average sewage flow rates as computed by various methods		
		$M = \frac{5}{P^{1/6}}$	$M = 1 + \frac{14}{4 + P^{1/2}}$	Maryland State Dept. of Health diagram (1914)
10,000	1.0	3.1	3.0	3.3
25,000	2.5	2.6	2.6	2.7
50,000	5.0	2.3	2.3	2.4
100,000	10.0	2.0	2.0	2.1
200,000	20.0	1.7	1.8	2.0
500,000	50.0	1.4	1.5	2.0
1,000,000	100.0	1.3	1.4	2.0

The following tabulation shows the ratio of the peak hourly to the average flow, as indicated by flow records, for several cities.

City	Population (approx.)	Ratio of peak hourly to yearly average flow	
		Equaled or exceeded 0.1 % of time	Equaled or exceeded 1 % of time
Long Beach, Calif.	125,000	2.2	1.8
Los Angeles, Calif.	1,060,000	1.8	1.5
Pasadena, Calif.	70,000	2.4	2.0
Urbana-Champaign, Ill.	40,000	2.8	2.5

On the basis of a number of gagings at Buffalo, where the sewage flow in combined sewers is intercepted for treatment, ratios of peak flows to the average flow at the treatment plant have been estimated as follows:

Period	Per cent of time per year	Ratio of flow to yearly average
Peak rate	.....	4.44
4-hr maximum	0.046	3.80
24-hr maximum	0.274	3.00
2-day maximum	0.55	2.50
3-day maximum	0.82	2.00
Maximum month	8.2	1.33
Minimum	.....	0.50

Quantity of Industrial and Commercial Sewage. Careful consideration should be given to the quantities of industrial and commercial sewage for which sewer capacity should be provided. In smaller communities with no large volumes of industrial wastes, the allowance for such wastes can generally be included in the per capita flow of domestic sewage; but in the larger cities having numerous industries and commercial establishments, the quantities of these wastes are preferably estimated as separate items and added to the estimated domestic sewage quantity. The proportion of the city area used for industrial and commercial purposes is usually small (see Table 4), and the probable future extension of these areas should be carefully estimated.

A usual practice is to estimate future flows of industrial and commercial sewage in terms of gallons per acre per day based upon a study of the quantities discharged from presently developed areas and comparative data from other cities. Special consideration should be given to existing industries that produce exceptionally large quantities of wastes. Table 11 gives illustrative data on the volumes of wastes produced in various types of industries. Sewer-capacity allowances for industrial and commercial areas based on careful analyses of existing conditions are given in Table 12 for several cities.

TABLE 11.—SEWAGE QUANTITIES PRODUCED BY VARIOUS INDUSTRIAL AREAS[1]

Type of Industry	Industrial Wastes, gal/acre/day
Large machinery manufacturers	20,300
Miscellaneous machine shops, metal workers and foundries	11,000
Breweries	137,500
Packing houses	62,000

[1] Sewerage Commission of the City of Milwaukee 18th *Ann. Rept.*

TABLE 12.—COMMERCIAL AND INDUSTRIAL SEWAGE FLOWS AT DATES OF ULTIMATE ESTIMATE—BASIS OF SEWER DESIGN

Municipality	Population	Combined commercial and industrial acreage	Combined commercial and industrial wastes		
			Total million gal/day	Gal/acre/day	Gpcpd
Minneapolis-St. Paul, Minn., exclusive of South St. Paul and Newport	1,415,000	14,884	Max 104.9	7,050	74.
			Avg 74.9	5,020	53.
Entire metropolitan area	1,450,000	16,099	Max 139.9	8,700	96.5
			Avg 99.9	6,200	69.
Milwaukee, Wis	862,000	4,609	Max 88.4	19,100	102.
			Avg 50.2	10,900	58.
Detroit, Mich	1,500,000	5,500	Max 92.6	16,850	62.
			Avg 61.5	11,180	41.
Toledo, Ohio	781,900	4,976	Max 74.9	15,050	96.
			Avg 45.7	9,185	58.
Buffalo, N. Y	800,000	4,633	Max 99.1	21,400	124.
Cincinnati, Ohio	720,000	8,903	Max 102.5	11,500	142.
Louisville (Beargrass Creek), Ky	230,000	688	Max 16.9	24,600	73.5
Fitchburg, Mass	87,200	1,256	Max 10.0	8,000	115.
			Avg 5.57	4,440	64.

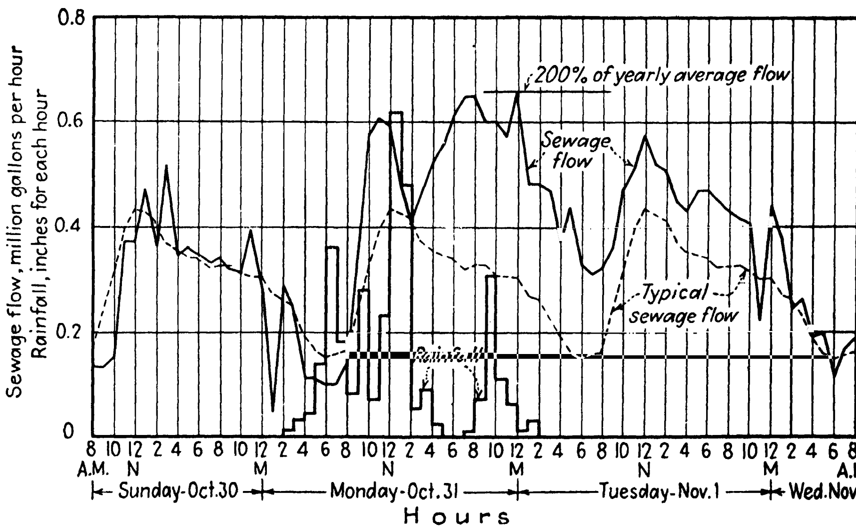


FIG. 14.—Effect of rain of Oct. 31, 1927, on the hourly sewage flows at Long Beach, Los Angeles County, Calif.

Infiltration. The flow in sewers is increased by surface water and ground water from two sources: (1) the entrance of runoff from rains directly through inlets such as downspouts, street inlets, and other openings; (2) the entrance of water from the ground into the sewers through leaky joints and other structural defects.

In spite of the fact that most modern plumbing codes and sewer regulations properly prohibit the discharge of rain water or other surface water into sanitary sewers,

it is sometimes necessary to provide capacity for a limited amount of surface runoff. Figures 14 and 15 show the influence of rains on domestic sewage flows in two specific cases.

The infiltration of ground water into sanitary sewers depends upon four major factors:

1. The level of the ground water with reference to the sewer.
2. The character of the subsoil. Sand and gravel will permit more water to leak into a sewer than will clay.
3. The water tightness of joints and the provisions to prevent cracking of the sewer pipes.
4. The character of the construction of the house connections. This usually relates to the extent and care with which the construction of the house connections is supervised.

Reduction of the amount of surface water entering existing sewers is sometimes possible, but it is difficult and costly to reduce the ground-water infiltration after a sewer system and the house connections have been built. Therefore, every reasonable effort should be made during the period of construction to assure watertight sewers.

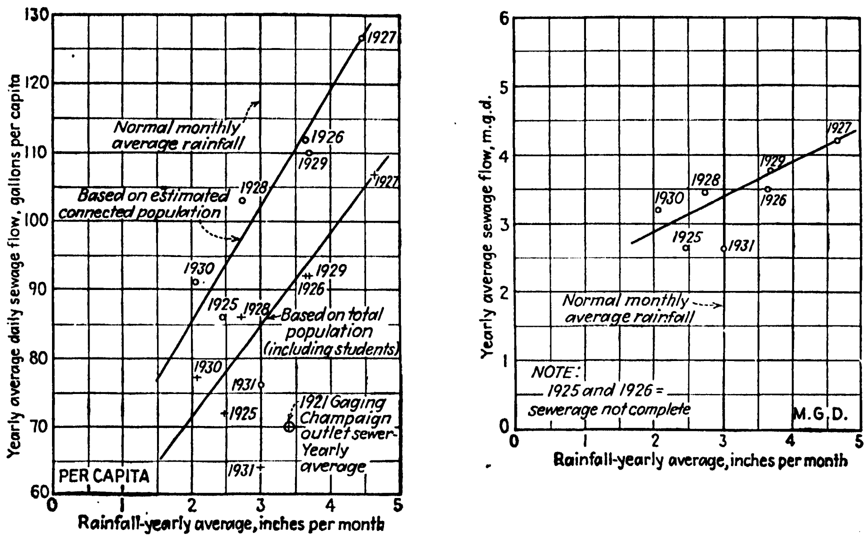


FIG. 15.—Relation of rainfall to yearly average sewage flows, Urbana-Champaign Sanitary District, May, 1932.

In some Western cities, the elevation of ground water is raised several feet during the irrigation season and the infiltration is much increased as indicated by the following rates of sewage flow at Yakima, Wash., in 1939:

Season			
Nonirrigation		Irrigation	
Month	Gpcpd	Month	Gpcpd
January	181		
February	189		
March	205		
April	315		
		May	448
		June	567
		July	595
		August	667
		September	571
		October	436
November	238		
December	215		

Several units of measurement of infiltration have been suggested, such as gallons per 24 hr per capita, per acre, per foot of joint, per square yard of interior surface, and per mile of sewer. The rather extensive available data on measured leakage into sewers shows quite a wide range in amounts even for new sewers built under careful supervision. Accordingly, there appears no justification for any refinements of the units of measurement. The units of gallons per 24 hr per mile of sewer and per acre of sewered area are easily applied and are the most commonly used.

The approximate lengths of sewer lines per acre of sewered area provide a basis for changing the infiltration rate from one to the other of the latter two terms. Some data on the length of sanitary sewers per acre of sewered area are given in Table 13.

The following data are typical of measured quantities of infiltration and may be useful in determining reasonable allowances for design purposes:

1. Newly constructed sewer lines:

Municipality	Ground-water Infiltration, Gallons per Mile of Sewer per 24 Hr
Danville, Ill	14,500
Decatur, Ill	22,100
Harrodsburg, Ky	26,400
Kaukauna, Wis	23,000
Peoria Heights, Ill	3,147
Springfield, Ill	27,200

2. Entire sewerage system including house connections:

Municipality	Ground-water Infiltration, Gallons per Mile of Sewer per 24 Hr
Boston, Mass	40,000
East Orange, N. J	22,400
Joint Trunk Sewer, N. J	25,000
Newark, N. J	115,752

TABLE 13.—SEWER LENGTHS PER ACRE AND PER CAPITA FOR VARIOUS CITIES

City	Population (total)	Area, acres	Length of sanitary sewers, ft	
			Per capita	Per acre
Minneapolis, Minn	415,460[1]	21,000[1]	7.4	147[3]
St. Paul, Minn	261,585[1]	19,300[1]	10.8	145[3]
Dallas, Tex	217,000	22,075	9.7	95[3]
Brockton, Mass	70,600	13,678	10.4[2]	54[2]
Fitchburg, Mass	42,513	17,584	9.0[2]	22[2]
Forth Worth, Tex	165,000	26,406	13.4[2]	84[2]
Grand Rapids, Mich	145,000	13,830	9.8	103
Milwaukee, Wis	484,595	17,250	6.4	178
Rockford, Ill	85,000	7,574	18.1[2]	202[2]
Washington, D. C	486,936	39,680	8.6	105
Sheboygan, Wis	38,907	3,286	11.2[2]	133[2]
				146[3]

[1] Provided with combined sewers, 1927 (2d *Ann. Rept.* Metropolitan Sewerage Community, 1928).
[2] Total length of sanitary and storm sewers.
[3] Related to sewered area only.

The infiltration provided for in the design of a number of well-considered sewerage projects, helpful as a guide to judgment, is as follows:

Municipality	Ground-water infiltration per 24 hr, gal per mile of sewer	Gpad[1]
Louisville, Ky	50,000	1,400
Milwaukee, Wis	36,000	1,000
Minneapolis–St. Paul, Minn	97,000	2,700

[1] Gallons per acre per day based on 146 ft of sewer per acre (Table 13.)

2. QUANTITY OF STORM SEWAGE

The design of storm sewers or combined sewers involves a decision as to the degree of protection to be provided against property damage, nuisance, and inconvenience from surcharged sewers. It is not economically feasible to construct sewers of suffi-

TABLE 14.—COST OF SEWERS PER FRONT FOOT AS ESTIMATED FOR THE PROPOSED NORTH SIDE SEWER DISTRICT DECATUR, ILL. (1924)

Projects	Cost of sewers per foot of assessed frontage (1924 cost data)			Approx. capacity of sewers, cfs/acre	
	Main sewers	Laterals	Total	400 acres	100 acres
Combined sewers:					
Rainfall frequency					
10 years	$2.12	$2.88	$5.00	0.73	1.06
3 years	1.78	2.61	4.39	0.42	0.76
1 year	1.67	2.37	4.04	0.38	0.57
Storm sewers:					
Rainfall frequency					
3 years	1.78	2.45	4.23		
1 year	1.67	2.22	3.89		
4 times per year	1.30	1.79	3.09	0.19	0.28
Sanitary sewers:	0.26	1.21	1.47	Capacity 300 gal per capita and 19 people per acre, sewers flowing full	
Separate sewers:					
(Storm and sanitary)					
Rainfall frequency					
3 years	2.04	3.66	5.70		
1 year	1.93	3.43	5.36		
4 times per year	1.56	3.00	4.56		

cient size to take the runoff from the extreme storms likely to occur at infrequent intervals. Surcharging of combined sewers is more objectionable than surcharging of storm sewers, because of the nuisances and health hazards that result from the flooding of basements and the overflowing of domestic sewage. Thus, the quantity of storm water for which sewer capacity should be provided is a balance among the first cost and the capitalized damage to private property, the hazard to health, and the curtailed convenience to the public.[1] These factors, therefore, should be given consideration as well as the technical hydraulic factors.

Basis of Storm-sewer Design. General. An exact determination of the permissible frequency of surcharging is impossible in present practice, and the conclusion as to

[1] BERNARD, M. M., *Trans. A. S. C. E.*, **96**, 1150, 1932.

the basis of design depends, finally, upon the judgment and experience of the designing engineer. Data with reference to sewer costs, sewer capacities for storm water, and the results of operating experience in a number of cities are helpful guides to judgment.

A study[1] of new and relief sewers at Decatur in 1924 for a residential area, including consideration of sewer assessments (costs per foot of assessed frontage) and sewer capacities, resulted in the data in Table 14 for a proposed sewer district of about 525 acres, approximately 9,000 ft long by 2,500 ft wide. Comparative data from a number of cities are given in Tables 15 and 16. The Decatur estimates indicated that

TABLE 15.—SEWER CAPACITIES AND ASSESSMENTS
Summary of Approximate Data for Various Cities (1924)

City	Kind of sewer system	Sewer capacity cfs/acre		Cost of equivalent combined sewer service per front foot			
		100 acres	500 acres	As reported	On basis of 200 ft frontage per acre	On basis of 250 ft frontage per acre	Trend (1924)
Louisville	C	1.46	1.16	....	$6.72	$5.37	$5.50
St. Louis	C	2.55	2.44	....	6.24	4.99	5.50
Chicago	C	0.26	0.16	....	....	.....	4.50
Indianapolis	C	0.60[1]	0.43	....	.....	.....	3.00
Buffalo	C	0.82	....	8.38	.....	.....	8.38
Milwaukee	S	0.80	0.65	4.00[2]	.....	.....	5.50
Detroit	C	....	....	....	3.24[3]	2.59[3]	5.50
Syracuse	C	0.50	0.75	3.00[4]			
New Bedford	C	1.0	1.0	2.00[4]			
Los Angeles	S	....	....	4.00[2]	.....	.....	5.50
Average	..	....	....	....	.....	.....	5.42

C = combined
S = separate
[1] Based on $C = 0.4$ and slope of 2/1,000
[2] Storm sewers only.
[3] Laterals only.
[4] Assessment fixed by law.
NOTE: These cost data may be related to present cost levels by *Engineering News-Record* or other indices.

a storm sewer for a 3-year storm must have a capacity 2.2 to 2.7 times as great and would cost 1.43 times as much to build as one for a storm frequency of four times per year.

In computations of storm-water quantities, engineers have used two general procedures: (1) the empirical-formula method, and (2) the rational method.

In either of these procedures, the computed quantity of storm water is a function of the area to be drained, in acres, as definitely determined by field surveys or by scaling from a relatively accurate map; the rainfall intensity, which results from an analysis of rainfall records and storm frequencies; and the maximum rate or coefficient of runoff. The latter factor depends upon the surface slope and the estimated condition of the drainage area with reference to the proportion of the rainfall that will run off.

[1] GREELEY, SAMUEL A., Some Notes on Relation between the Capacity of Combined and Storm Sewers and the Frontage Assessment, *Jour. Western Soc. Eng.*, **30**, No. 1, 13–24, 1925.

The Empirical Formula Method. Some engineers, in the past, whose experience has been large, have sought to express their judgment in formulas by which their experience gained in one or more localities might be applied by themselves or others to other localities.

These formulas are no longer generally used, although they may be useful in large places where the results in practice of the application of a formula are known and the sewer department has become used to expressing its judgment through the formula.

TABLE 16.—COST RATIOS OF STORM SEWERS FOR VARIOUS STORM FREQUENCIES

City	Date	Storm frequency	Cost per acre (estimated)	Ratio to cost for 4 times per year storm
Decatur, Ill.[1] (525 acres)	1924	3 year	$780	1.43
		1 year	706	1.30
		4 times per year	546	1.00
Oshkosh, Wis.[1]	1929	2 year	438	1.46
		1 year	400	1.33
		3 times per year	325	1.08
		4 times per year	300	1.00

[1] Reports by Greeley and Hansen on new and relief sewers.

The Rational Method. The widely used rational method relates the runoff directly to the tributary area and the rainfall intensity and to an estimated proportion of the rainfall reaching the sewer as direct runoff.

This procedure may be represented by the formula

$$Q = ciA$$

in which Q = storm water runoff, or the quantity of storm water entering the sewer, cfs.

c = a coefficient representing the ratio of runoff to rainfall, commonly called the runoff coefficient.

i = rainfall intensity, cfs/acre, or (close enough for all practical computations) the rate of rainfall, in./hr. (1.00 in./hr = 1.008 cfs/acre).

A = tributary drainage area, acres.

The application of the rational method requires the exercise of sound judgment in the selection of the coefficient c and the rainfall intensity i. The area A can be more definitely determined.

In practice, the *first step* is a tentative arrangement of the sewer lines on a plan of the district for which the sewers are to be computed, a division of the district into subdistricts tributary to the several concentration points along the proposed sewer lines, and the successive summation of the tributary areas above the various concentration points.

The *second step* is the selection of the rainfall frequency for which capacity is to be provided and the determination of the rainfall intensity of this frequency for a storm duration equal to the time of concentration. The *time of concentration* includes two factors: the *inlet time*, or the time required for rain falling on the most remote point of the tributary area to flow across the ground surface, along pavement gutters to the street inlet, and through the inlet into the sewer; and the *time of travel* within the sewer from the uppermost inlet to the concentration point under consideration.

The *third step* is a determination of the proportion of the rainfall which finds its way directly into the sewer as runoff, both expressed as rates.

Rainfall Data. The usual procedure in studying rainfall data for storm-sewer design is to obtain the records of all excessive rainstorms for as many years as they are available. The greatest average rate of rainfall, in inches per hour for periods of 5, 10, 15 min and up, may be determined throughout the duration of each storm. The intensities are then tabulated in order of magnitude for each duration of time, and

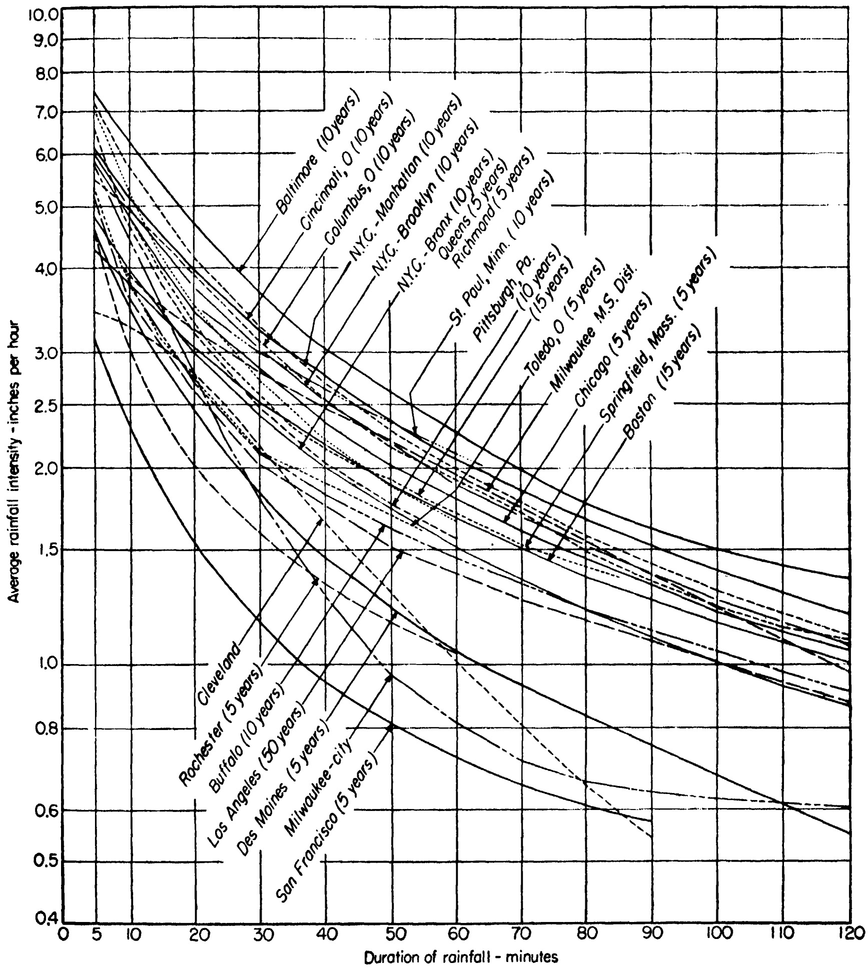


FIG. 16.—Rainfall curves used for sewer design.

the frequency in years for each intensity is determined by dividing the number of years of record by the number of occurrences. By plotting these data and drawing smooth curves through the plotted points, a series of rainfall-intensity curves is obtained, representing various frequencies of storm occurrence.

The rainfall curves used for storm-sewer design in a number of cities are compared graphically in Fig. 16.

Some investigators have combined the records of several rain-gaging stations assuming, for example, that the storm records of two stations for 10 years at each

station are equivalent to a 20-year record at one station. There is some doubt as to the correctness of this procedure.

Figure 17*A* gives the results of an investigation of the intensities and durations of rainstorms in Chicago, as recorded at the U.S. Weather Bureau rain gage on the U.S. Court House, for the years 1900 to 1923.[1] Figure 17*B*, based upon a later investigation[2] of the records of 13 recording rain gages within the Chicago area, equivalent to 330 station-years, gives similar though somewhat higher (6 to 20 per cent) results than Fig. 17*A*, which is based upon a shorter record of only one station. The semi-hyperbolic plotting used in Figure 17*B* yields straight-line results. About 1944 the Chicago Bureau of Sewers adopted the Eltinge-Towne formula for 5-year storm, $i = 90\ (t^{0.9} + 11)$, as the basis for combined sewer design.

Curves for other cities may be developed from an analysis of the available records of local recording rainfall gages.

Many efforts have been made to represent the time-intensity relation for rainfall by a mathematical formula. Many of these follow the form devised by Talbot in 1891 of $i = A/(t + b)$, in which i is the inches per hour, t the storm duration in minutes, and A and b are constants. The study by Schafmayer[2] seems to indicate that this hyperbolic type of formula is as good as any for showing the relation between intensity and duration for storms not exceeding 120 min. Factors A and b for the curves in Fig. 17*B*, as computed by Schafmayer are as follows:

Storm frequency, years	Factors in $i = \frac{A}{t+b}$	
	A	b
Schafmayer (1937) for Chicago area		
2	102	16
5	138	19
10	166	21
(15)[1]	(182)	(21)
20	193	22
(25)[1]	(203)	(22)

[1] Interpolated from Schafmayer's data, Table 5, *Proc. A. S. C. E.*, February, 1937.

Similar factors proposed by certain other authorities[3] are as follows:

Authority	Year	A	b	Application
Talbot	1891	360	30	Max storms in Eastern U.S.
Talbot	1891	105	15	Ordinary storms in Eastern U.S.
Dorr	1892	150	30	Basis of design, Boston
Kuichling	1905	120	20	Basis of design, Boston
deBruyn-Kops	1908	191	19	Max storms, Savannah
deBruyn-Kops	1908	163	27	Once in 2 years, Savannah
deBruyn-Kops	1908	141	27	Once a year, Savannah
Hill	1907	120	15	Chicago
Hendrick	1911	300	25	Max storms for Baltimore
Hendrick	1911	105	10	Basis of design, Baltimore
Allen[1]	....	200	20	25-year curve for Central Park, N. Y.

[1] BABBITT, "Sewerage and Sewage Treatment," 3d ed., John Wiley & Sons, Inc., 1928.
[1] GREELEY, SAMUEL A., *Jour. West. Soc. Eng.*, 1925.
[2] SCHAFMAYER, A. J., Rainfall Intensities and Frequencies, *Proc. A. S. C. E.*, February, 1937.
[3] Compiled by Meyer, A. F., "Elements of Hydrology," 2d ed., 1928.

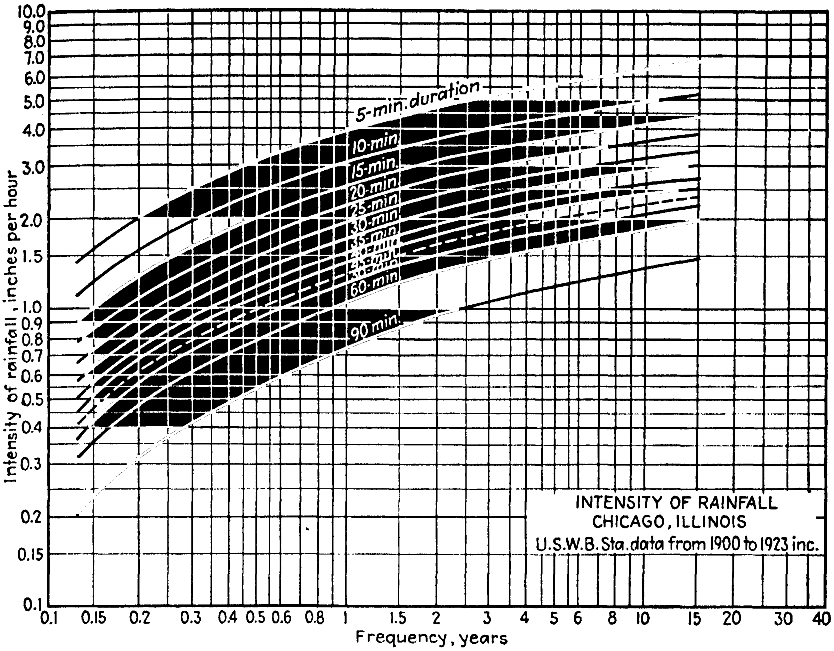


FIG. 17A.—Intensity of rainfall, Chicago, Ill. U.S. Weather Bureau Station data, 1900 to 1923.

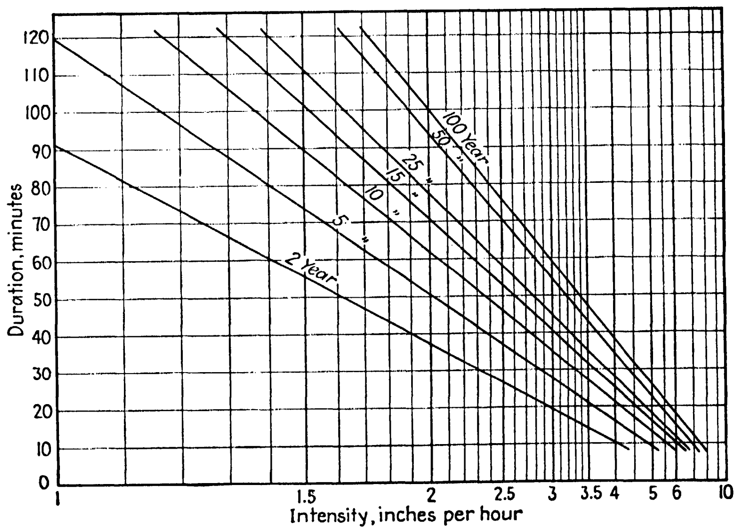


FIG. 17B.—Intensity and duration of rainfall, Chicago area.

TABLE 17.—MEYER'S CONSTANTS IN RAINFALL FORMULA $i = \frac{A}{t + b}$

Regional group	Constants	Storm frequency, years						
		1	2	5	10	25	50	100
1	A	145	180	220	276	355	450	600
	b	23	24.5	27	32	40	50	65
2	A	100	131	171	214	252	289	325
	b	18	21	23.5	26	28	30	32
3	A	72	96	122	150	181	216	256
	b	13	16	18	19.5	21	23	25
4	A	60	84	108	132	160	186	210
	b	15	16	17.5	19	20	21	22
5	A	60	75	90	105	126	152	180
	b	13	13	13	13	14	16	18

TABLE 18.—LIST OF RAINFALL STATIONS USED BY MEYER AS BASIS FOR HIS FORMULAS

Cities in various regional-group numbers				
1	2	3	4	5
Galveston New Orleans Jacksonville	New York Philadelphia Washington Norfolk Raleigh Savannah Atlanta Little Rock Forth Worth Abilene Bentonville St. Louis Kansas City Lincoln Des Moines	Boston Albany Pittsburgh Elkins Asheville Knoxville Memphis Cairo Indianapolis Cincinnati Cleveland Detroit Grand Haven Chicago Madison St. Paul Moorhead Yankton Dodge	Duluth Escanaba Buffalo Rochester	Denver Bismarck

Some authorities consider that an exponential formula such as $i = A/t^b$, or $i = A/\sqrt{t + b}$, may fit the rainfall data better. However, neither the accuracy of the available data nor the reliability of the application in storm-sewer design warrants much complexity in the mathematical formulas representing the relation between duration time and average rainfall intensity.

The formulas proposed by Meyer[1] may be found useful in the absence of any better

[1] MEYER, *op. cit.*, pp. 196–200.

analysis of a long-term record of local data. These are in the form $i = A/(t + b)$, and he gives values of A and b (Table 17) based upon the rainfall records of various groups of localities (Table 18).

Rainfall Distribution. The meager available data on rainfall distribution indicates that the average rainfall intensity reduces as the size of the area increases, the reduction being greatest for storms of short duration. The data indicate, however, that for the small areas, usually included in storm-sewer design, the distribution factor may be omitted or provided for in the selection of the runoff coefficient.

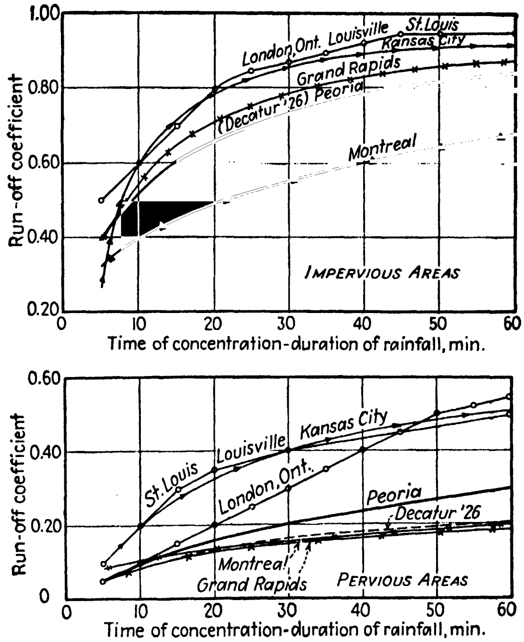


FIG. 18.—Runoff coefficients for pervious and impervious areas, Peoria, Ill.

Inlet Time. The inlet time is affected materially by the character and slope of the ground surface, and its selection has a considerable influence upon the rainfall-intensity rate. The inlet time may be as short as 5 min for closely spaced street inlets of ample size on areas highly developed or with steep slopes, or 10 to 15 min on flatter slopes with greater inlet spacings. In certain cases, an inlet time up to 20 or 30 min may be considered for relatively flat residential areas with inlets spaced at relatively long distances and where some surface ponding is not objectionable. Practice in several cities is reported to be as follows:

City	Inlet Time, min
Buffalo	5–10[1]
Cincinnati	8–10
Cleveland	8
Detroit	15
District of Columbia	5–12
Milwaukee	5–10
New York City—Manhattan	4
St. Louis	20[2]

[1] Minimum time of concentration used in design, 15 min.
[2] Minimum rainfall time including inlet time.

Decreasing the size, and hence the capacity, of street inlets or spacing them at greater distances, thus keeping the water on the street for a few minutes longer, has the effect of reducing the maximum rate of storm-water runoff for which sewer capacity need be provided. This procedure can be used best in relatively flat areas. Such surface ponding explains the adequacy of many sewers that are apparently too small.

Runoff Coefficient. Pondage in depressions, evaporation, and absorption, and other factors reduce the runoff, so that not all of the rainfall, even on impervious surfaces, reaches the sewer inlets. The longer the storm duration, the larger may be the percentage of rainfall runoff.

The proportion of pervious and impervious areas probably affects the runoff as much as any other factor. In designing storm sewers, a practical procedure includes a reasonable estimate of the probable future percentage of impervious area. The typical curves in Fig. 18 represent the basis for selecting runoff coefficients used for certain projects in various cities.

Estimates of the relative proportion of pervious and impervious areas can be made by considering the present development of a number of typical areas in the more completely developed sections of the city for which sewers are to be designed and comparing these with the anticipated future development of the districts to be sewered.

Certain illustrative data on the percentage of impervious surfaces, used in connection with various storm-sewer projects, are as follows:

Project and Type of Area	Impervious Surfaces, %
1. Peoria, Ill.,[1] 1926	
Mercantile	70
Commercial	40–50
Industrial	35
Residential, persons per acre:	
5–15	20
15–22	25
22–32	30
32–40	35
40–50	40
50–60	45
2. Oskhosh, Wis.,[1] 1929 (40,000 population)	
Highly developed commercial areas	50
Fairly well developed residential areas	25
Parks and undeveloped areas	5–10
3. Decatur, Ill.[2]	
Mercantile	70
Industrial	60
Commercial	50
Residential	25
(density 16–30 per acre)	
Parks and undeveloped	10
4. Analyses of small typical areas for several American cities[3]	
Louisville, Ky., residential	32–56
Springfield, Mass., residential	49
Cincinnati, Ohio	
Residential, persons per acre:	
20	35
55	55
135	84
Commercial	100
Detroit, Mich., residential	28–50
Quincy, Mass., residential	31–42
Schenectady, N. Y., residential	35–42

[1] GREELEY and HANSEN, Engineering reports on new and relief sewers.

[2] GREELEY, SAMUEL A., *Jour. Western Soc. Eng.*, **30**, of No. 1, 13–24, 1925.

[3] METCALF and EDDY, "American Sewerage Practice," vol. 1, 2d ed. p. 286, McGraw-Hill Book Company, Inc., 1928.

To apply these coefficients, the city or sewer district under consideration is divided into zones representing the anticipated future extent of impervious area and a tabulation is made showing the pervious and impervious areas tributary to each concentration point, from which an average coefficient of runoff may be computed corresponding to the time of concentration.

These computations may be simplified for use in a study of several proposed sewer districts by constructing a diagram, such as Fig. 19, to give the computed runoff in cfs per acre for various times of concentration and proportions of impervious area.

Another method, sometimes used, includes a fixed coefficient of runoff based upon the anticipated future development of the area. This, in effect, assumes that the uncertainties in forecasting area development and the selection of rainfall intensities make unnecessary the refinements in the foregoing procedure. Table 19 gives the runoff coefficients used in a number of cities.

Arbitrary Unit-rate Method Related to Existing Sewer Capacities. A study of the capacities of existing sewers in a given city, related to the location and frequency of objectionable surcharge, is a helpful guide to judgment as to the proper capacity for new or relief storm sewers in that locality.

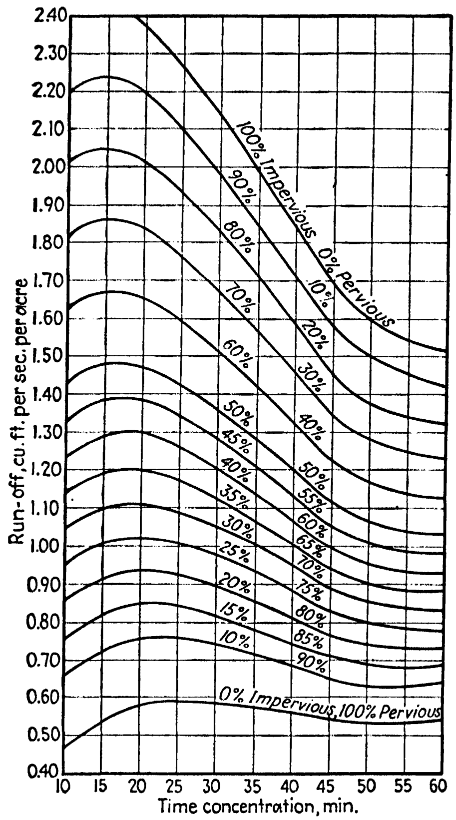


FIG. 19.—Runoff from sewered areas computed for rainfall intensities exceeded once in 10 years, Peoria, Ill.

Care must be taken to study the capacity of each existing sewer for its full length in and below the area that has experienced surcharging troubles. This can best be done by computing the sewer capacity at various points, based upon the slope and size, and plotting the results in cfs per acre with the area as the independent ordinate. Frequently the critical capacity of the sewer will be found at a considerable distance below the section where surcharge has been experienced but where no reports of difficulties have been received because the elevation of the sewer is below basement levels.

Typical data are given by Fig. 20 from a study of capacities of existing sewers together with the estimated storm-water runoff by several computations. Application of this procedure will indicate certain per acre runoff quantities which should be considered the minimum quantities for designing new or relief storm sewers in a particular city. Some typical results of this procedure are given in Table 20.

Summary Statement. The rational method represents the most practical present basis for estimating storm-water runoff quantities when used with mature judgment based upon experience in relating the capacity of storm sewers to troubles from sur-

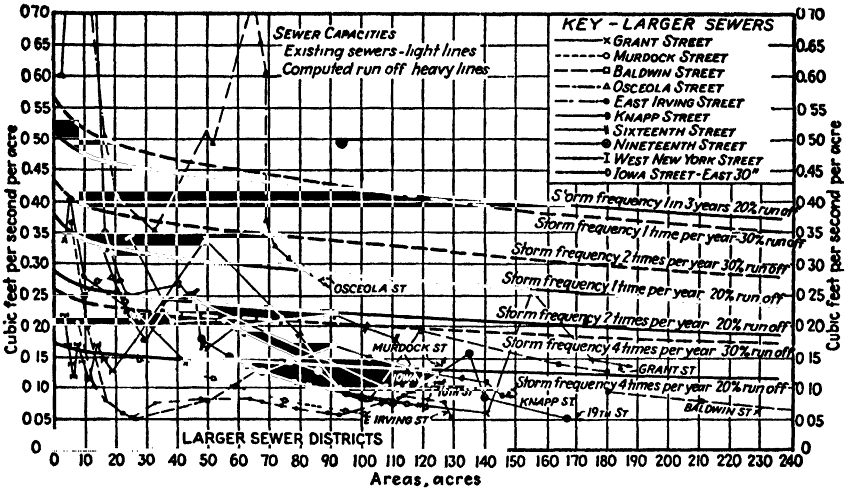


Fig. 20. Sewer capacities, Oshkosh, Wis.

Note. Existing sewers = light lines, computed runoff = heavy lines

TABLE 19—RUNOFF COEFFICIENTS USED IN VARIOUS CITIES

City	Type of area	Runoff coefficients
Buffalo, N Y	Residential	0 48 0 58
	Residential (apartments)	0 60–0 65
	Commercial	0 60–0 70
	Industrial	0 55–0 60
Cincinnati, Ohio	Suburban (large lots)	0 30
	Residential	0 35–0 40
	Residential (apartments)	0 50–0 60
	Tenements and industrial	0 70
	Commercial	0 80–0 85
Cleveland, Ohio	Residential	Varying 0 60–1 00
	Downtown areas	1 00
Detroit, Mich .	Impervious	Varying 0 50–0 95
	Pervious clay	Varying 0 10–0 70
	Pervious sand	Varying 0 01–0 55
District of Columbia		0 60 0 85
Louisville, Ky	Impervious	Varying 0 60 0 95
	Pervious	Varying 0 20–0 70
Milwaukee, Wis	Residential	0 40–0 50
	Local business	0 65
	Commercial and industrial	0 80
	Special cases	0 90
New York City—Manhattan	Central Manhattan	0 75
	More open sections	0 60
	Parks	0 25
	Streets	1 00
Rochester, N. Y............. .	Residential	0 25 0 40
	Commercial	0 50 0 85
	Industrial	0 60
St. Louis, Mo.................	Impervious	Varying 0 50–0 95
	Pervious	Varying 0 22–0 60

charge or actual measurement of storm-water flows from various types of sewered areas. It is not an easy method for an inexperienced engineer to use. Recent investigations, by a number of engineers who have given much study to this problem, indicate the possibility of developing such computation procedures along somewhat new lines. However, these have not been sufficiently developed for routine use. Typical discussions of such methods of computing runoff are given in the following references:

1. HORTON, ROBERT E., *Pub.* 101, February, 1935.
2. SHERMAN, LEROY K., *Eng. News-Record,* **108,** 501, 1932.
3. HORNER and FLYNT, *Trans. A. S. C. E.*, **101,** 140, 1936.
4. BERNARD, M. M., *Trans. A. S. C. E.*, **100,** 347, 1935.

TABLE 20.—ILLUSTRATIVE MINIMUM STORM WATER RUNOFF QUANTITIES FOR PROPOSED SEWER DESIGN, VARIOUS PROJECTS

Project	c, per cent	Storm frequency	Proposed minimum runoff rates, cfs per acre		
			100 acres	200 acres	400 acres
Decatur, Ill.[1] (population 48,500, 1924):					
Combined sewers	..	Once in 10 years	1.06	....	0.73
Separate sewers	..	Once in 3 years	0.76	....	0.42
Oshkosh, Wis.[2] (population 40,000, 1928):					
New sewers (separate)	20	4 times per year	0.13	0.12	
Combined sewers (new)	20	2 times per year	0.22	0.20	
Relief sewers (combined)	30	2 times per year	0.33	0.29	
Sheboygan, Wis.[2] (population 40,000, 1929):					
New sewers (separate)	30	2 times per year	0.41	0.36	0.30
New sewers (combined)	30	1 time per year	0.55	0.48	0.38
Relief sewers (combined)	40	1 time per year	0.73	0.65	0.51
Chicago, Ill. (1950), new sewers (combined):					
Type 3 area (population 10 per acre plus 1,000 sq ft of roof)	..	Once in 5 years	0.72	0.65	0.57
Type 5 area (population 30 per acre plus 1,000 sq ft of roof)	..	Once in 5 years	0.97	0.89	0.78

[1] GREELEY, S. A., *Jour. Western Soc. Eng.*, **30,** No. 1, 13–24, 1925.
[2] GREELEY and HANSEN, Engineering reports on new and relief sewers.

3. HYDRAULICS OF SEWERS

The laws of hydraulics are applied to sewers for three general purposes:

1. To determine the proper size and slope in the design of a new sewer to carry certain anticipated sewage quantities.
2. To determine the capacity of an existing sewer in relation to anticipated sewage quantities which may require relief capacity to prevent surcharge.
3. To determine the flow in a sewer as a means for measuring sewage quantities.

The practical limitations in the accuracy with which the sewage quantities may be anticipated (as indicated in the foregoing sections of this chapter) make unnecessary any unusual refinements in the application of the laws of hydraulics to sewers in the first two cases. Moreover, the impracticability of an accurate determination of the physical factors, such as roughness and variations in stream section, which control the hydraulic resistance to flow, makes any extreme refinements unnecessary in the third case.

Sewage is composed of about 99.9 per cent water and about 0.1 per cent pollutional matter, partly suspended and partly dissolved. Industrial wastes may be, at times, somewhat more concentrated, and storm water may include considerable grit and other debris from street washings. With reference to their hydraulic capacity to carry sewage, industrial wastes, and storm water, sewers are treated in hydraulic computaions as though they were conduits carrying clear water, with the following exceptions:

1. Velocities of flow must be maintained sufficiently high to prevent deposits of solids.
2. Sewers are usually designed to flow partly full, and the sewer invert generally is approximately parallel to the hydraulic gradient of the flowing sewage. Thus, the sewer is an open channel with the depth of the flowing stream varying with the quantity of sewage.
3. Openings or connections cut into the sewers at frequent intervals, together with variation in alignment of the short pipe sections, cause a reduction in the carrying capacity of the sewer.
4. There may be some reduction in the carrying capacity of sewers, with the passage of time, owing to deposits or adhesions of matter from the sewage, or to deterioration of the interior of the sewer surface as a result of chemical reactions of sewage substances and the sewer material.

The first two items relate to the minimum slopes at which sewers should be constructed, and the last two items relate to the selection of the roughness coefficients, or friction factors, which are to be used in the hydraulic formulas for computing the sizes of the sewers.

Ordinarily, sewers are considered as open channels in the selection of hydraulic formulas, except for pressure lines such as depressed sewers or pumping-station discharge lines.

Hydraulic Formulas. Various hydraulic formulas are given in Appendix A for flow in both open channels and in pipes under pressure. The Kutter formula has been used most generally in sewage-flow computations for open-channel conditions, and the Hazen and Williams formula has found wide application in connection with pressure pipes.

In the use of Kutter's formula, the selection of the proper value of the roughness coefficient n is most important. The Swiss engineers, Ganguillet and Kutter, originally suggested values of n as shown in Table 21. A more extensive list of n values, prepared by Robert E. Horton[1] in 1916, is given in Table 22.

TABLE 21.—VALUES OF n RECOMMENDED BY GANGUILLET AND KUTTER[1]

	n
1. Channels lined with carefully planed boards or with smooth cement	0.010
2. Channels lined with common boards	0.012
3. Channels lined with ashlar or with neatly jointed brickwork	0.013
4. Channels in rubble masonry	0.017
5. Channels in earth, brooks, and rivers	0.025
6. Streams with detritus or aquatic plants	0.030

[1] METCALF and EDDY, "American Sewerage Practice," vol. 1, 2d ed., p. 80, McGraw-Hill Book Company, Inc., 1928.

The results of a considerable number of actual measurements of flows in sewers are available showing values of n ranging from 0.011 to 0.016 for sewers in reasonably clean condition and up to 0.020 for sewers in poor alignment or with deposits in the bottom.

Metcalf and Eddy[2] suggested the following values of n for the design of sewers "to be maintained in reasonably good operating condition."

For pipe sewers 24 in. or less in size	0.015
For concrete sewers of large section and best work	0.012
For concrete sewers under good ordinary conditions of work	0.013
For brick sewers lined with vitrified or reasonably smooth hard-burned brick and laid with great care, with close joints	0.014
For brick sewers under ordinary conditions	0.015
For brick sewers, rough work	0.017–0.020

[1] *Eng. News*, **75**, 373, 1916.
[2] "American Sewerage Practice," vol. 1, 2d ed., p. 90, McGraw-Hill Book Company, Inc., 1928.

TABLE 22.—R. E. HORTON'S VALUES OF n; TO BE USED WITH KUTTER'S FORMULA

Surface	Best	Good	Fair	Bad
Uncoated cast-iron pipe	0.012	0.013	0.014	0.015
Coated cast-iron pipe	0.011	0.012[1]	0.013[1]	
Commercial wrought-iron pipe, black	0.012	0.013	0.014	0.015
Commercial wrought-iron pipe, galvanized	0.013	0.014	0.015	0.017
Smooth brass and glass pipe	0.009	0.010	0.011	0.013
Smooth lockbar and welded OD pipe	0.010	0.011[1]	0.013[1]	
Riveted and spiral steel pipe	0.013	0.015[1]	0.017[1]	
Vitrified sewer pipe	0.010 0.011	0.013[1]	0.015	0.017
Common clay drainage tile	0.011	0.012[1]	0.014[1]	0.017
Glazed brickwork	0.011	0.012	0.013[1]	0.015
Brick in cement mortar, brick sewers	0.012	0.013	0.015[1]	0.017
Neat cement surfaces	0.010	0.011	0.012	0.013
Cement-mortar surfaces	0.011	0.012	0.013[1]	0.015
Concrete pipe	0.012	0.013	0.015[1]	0.016
Wood-stave pipe	0.010	0.011	0.012	0.013
Plank flumes:				
Planed	0.010	0.012[1]	0.013	0.014
Unplaned	0.011	0.013[1]	0.014	0.015
With battens	0.012	0.015[1]	0.016	
Concrete-lined channels	0.012	0.014[1]	0.016[1]	0.018
Cement-rubble surface	0.017	0.020	0.025	0.030
Dry rubble surface	0.025	0.030	0.033	0.035
Dressed ashlar surface	0.013	0.014	0.015	0.017
Semicircular metal flumes, smooth	0.011	0.012	0.013	0.015
Semicircular metal flumes, corrugated	0.0225	0.025	0.0275	0.030
Canals and ditches:				
Earth, straight and uniform	0.017	0.020	0.0225[1]	0.025
Rock cuts, smooth and uniform	0.025	0.030	0.033[1]	0.035
Rock cuts, jagged and irregular	0.035	0.040	0.045	
Winding sluggish canals	0.0225	0.025[1]	0.0275	0.030
Dredged earth channels	0.025	0.0275[1]	0.030	0.033
Canals with rough stony beds, weeds on earth banks	0.025	0.030	0.035[1]	0.040
Earth bottom, rubble sides	0.028	0.030[1]	0.033[1]	0.035
Natural stream channels:				
1. Clean, straight bank, full stage, no rifts or deep pools	0.025	0.0275	0.030	0.033
2. Same as (1), but some weeds and stones	0.030	0.033	0.035	0.040
3. Winding, some pools and shoals, clean	0.033	0.035	0.040	0.045
4. Same as (3), lower stages, more ineffective slope and sections	0.040	0.045	0.050	0.055
5. Same as (3), some weeds and stones	0.035	0.040	0.045	0.050
6. Same as (4), stony sections	0.045	0.050	0.055	0.060
7. Sluggish river reaches, rather weedy or with very deep pools	0.050	0.060	0.070	0.080
8. Very weedy reaches	0.075	0.100	0.125	0.150

[1] Values commonly used in designing (according to Horton).

It has been the practice of the authors to base preliminary designs of sewerage projects for sewers 8 to 108 in. in size upon $n = 0.015$ to cover all hydraulic losses and then in the preparation of final design drawings to use $n = 0.013$ for straight sewer sections with additional fall provided for hydraulic losses caused by manholes, curves, junctions, and other factors.

Computation Diagrams. Diagrams for the general solution of Kutter's formula and the Hazen and Williams formula are given in Appendix A.

Figures 21 and 22 are diagrams originally prepared by John H. Gregory for

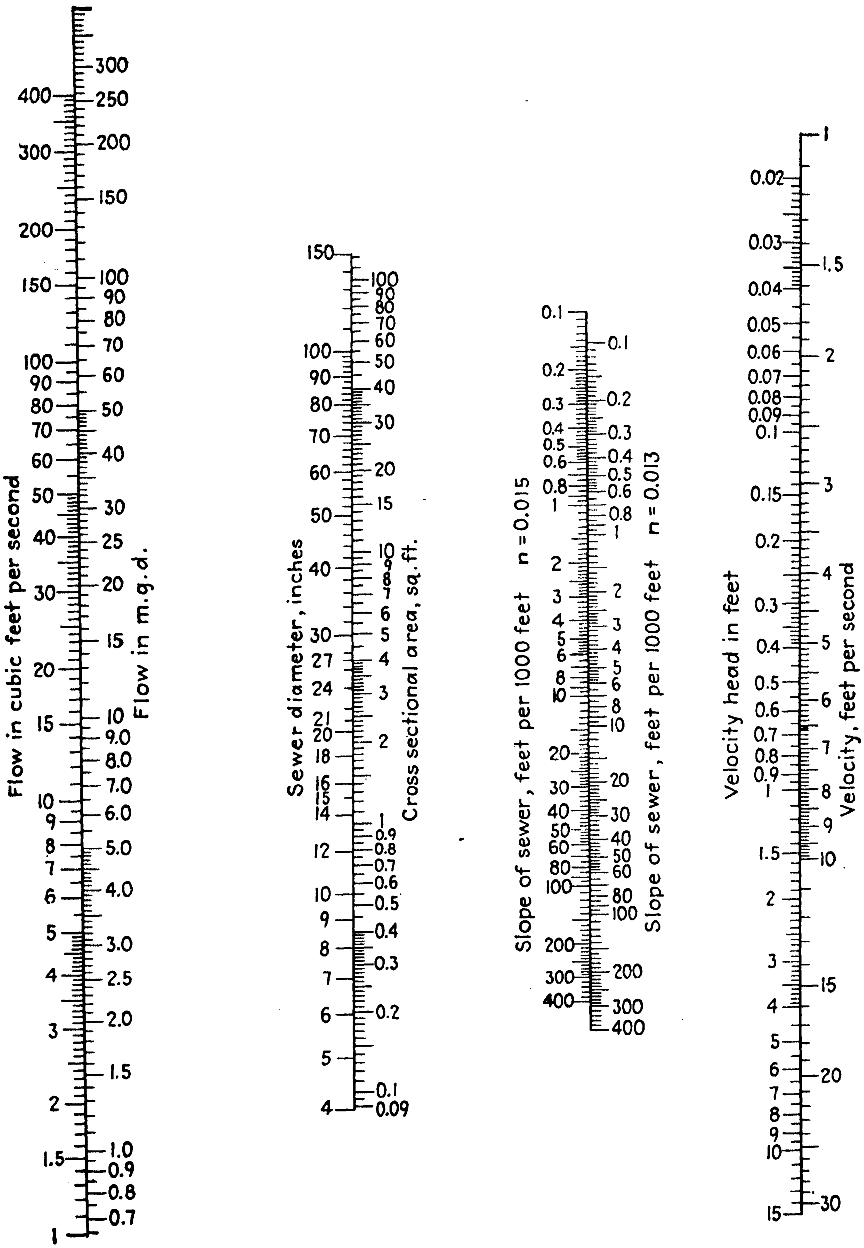


FIG. 23.—Flow in sewers.

NOTE: Full capacity for $n = 0.013$ and $n = 0.015$ (accurate to within $\pm$ 3 per cent).

n = 0.013 and 0.015. These Gregory diagrams cover a wide range of all the factors involved in sewer design and have been found very useful for circular sewers ranging in size from 4 to 144 in. (except that interpolations must be made for present-day commercial sizes of 15 and 21 in.). As an illustration of their use, suppose that it is desired to select the size of pipe, laid on a slope of 1.2 ft per 1,000 ft, that will be required to carry a flow of 5.0 mgd when running full, on the assumption that n = 0.013. Enter the right-hand side of the diagram marked Fig. 21 with the known flow, proceed horizontally to the left to the intersection with the vertical line representing the slope. This point lies just below the line sloping upward to the right which represents 24-in. pipe, the required size in this case. By interpolation between the lines sloping upward to the left, it is further found that the velocity of flow will be about 2.5 fps.

A somewhat more convenient type of diagram to use is the nomogram, shown as Fig. 23, which permits the use of a straight edge to show the relation of the several factors that determine sewer capacity for n = 0.013 or 0.015. The results obtained from this diagram are not so close as those from the Gregory diagram, but, in general, are as close as the basic data warrant.

These diagrams will meet the usual needs in computing sewer capacities for circular sections. Figures 24 and 25 relate to semielliptical and horseshoe sewer sections (as used at Buffalo, N. Y.), whereas general problems in open channel flow may be solved by means of the Scobey diagram (Appendix A, Fig. 2).

Figure 26 shows the relation of the factors of slope and velocity for a given sewer with roughness coefficients ranging from 0.010 to 0.025 as compared with the slope and velocity for n = 0.015.

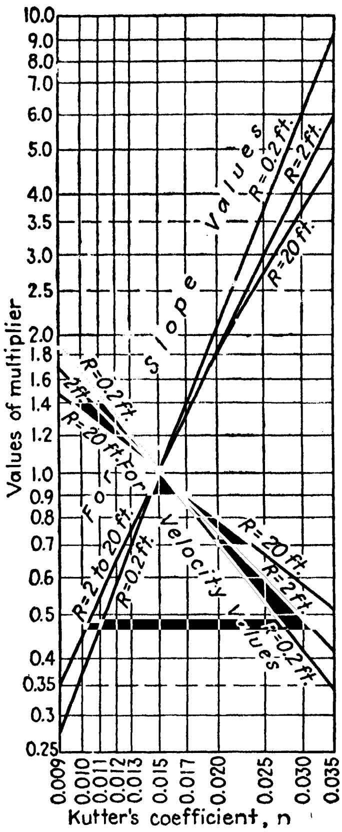


FIG. 26.—Relation between Kutter's n and corresponding slopes and velocities.

Velocities and Minimum Grades. Sewers should operate with velocities of flow sufficient to prevent excessive deposits of the solid materials suspended in the liquids, otherwise objectionable clogging may result. The controlling velocity is that near the bottom of the stream flowing in the sewer and is considerably less than the average velocity of flow. In closed conduits flowing full, the maximum velocity is usually at the center of the pipe, with the mean velocity at the upper one-third depth and equal in value to about 85 per cent of the maximum velocity. The bottom velocity may be about one-half the maximum, or about 60 per cent of the mean velocity.

In open channels or sewers flowing partly full, the maximum velocity is usually in the upper one-third of the stream section, with the mean velocity about 80 per cent of the maximum, and the bottom velocity possibly 50 per cent of the mean velocity.

Experience has indicated that average velocities, when flowing full, of 2.0 fps for sanitary sewers and 2.5 fps for storm-water sewers will prevent objectionable deposits,

whereas frequent troubles will result from lower velocities. However, relatively high construction costs sometimes warrant designing sewers with flatter slopes and lower velocities at the expense of somewhat more frequent operating troubles.

Some general idea of the possible effects of low velocities may be obtained by considering the data on the lifting and transporting capacity of water at various velocities, as given in Table 23.

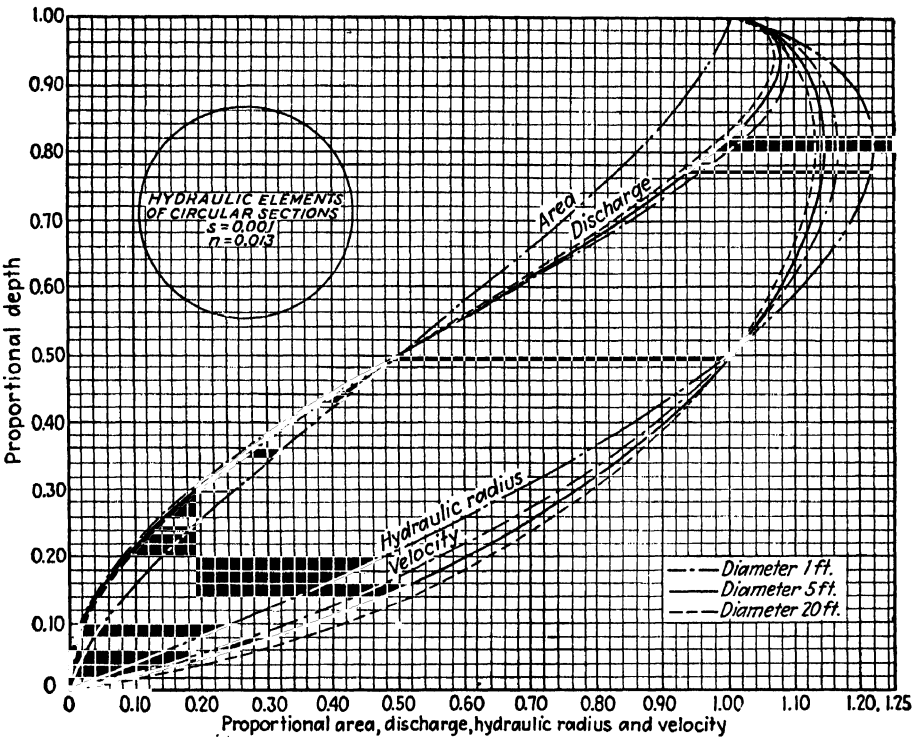


FIG. 27.

In general, the minimum grades for the design of sewers are controlled by various State Board of Health codes setting forth minimum desirable grades based upon general experience. The sewer grades normally considered as the desirable minimum and the flattest grades that should be considered in any well-designed sewer system are indicated by Table 24.

Maximum Velocities. Maximum velocities in sewers are important mainly because of the possibilities of excessive erosion on sewer inverts and occasionally because of the piling up of water or sewage at the lower end of a high-velocity section due to the

TABLE 23.—CURRENTS NECESSARY TO MOVE SOLIDS[1]

Kind of Material	Velocity Required to Move Material on Bottom, fps.
Fine clay and silt	0.25
Fine sand	0.50
Pebbles ½ in. in diameter	1.0
Pebbles 1 in. in diameter	2.0

[1] Metropolitan Sewerage Commission, New York.

Table 24.—Minimum Sewer Grades
(n = 0.013)

Size	Ordinary min grades		Extreme min grades[1]	
	Slope, ft/1,000	Velocity when full, fps	Slope, ft/1,000	Velocity when full, fps
8	3.0	1.7	2.0	1.4
10	2.2	1.8	1.5	1.5
12	1.8	1.8	1.2	1.5
15	1.4	1.9	0.85	1.5
18	1.1	1.9	0.65	1.5
21	0.96	2.0	0.60	1.6
24	0.8	2.0	0.51	1.6
27	0.7	2.0	0.44	1.6
30	0.6	2.0	0.38	1.6
33	0.5	2.0	0.34	1.6
36	0.44	2.0	0.30	1.6
39	0.42	2.0	0.28	1.6
42	0.36	2.0	0.24	1.6
45	0.34	2.0	0.22	1.6
48	0.32	2.0	0.20	1.6

[1] These grades should be used only when they will make pumping or excessive excavation unnecessary and then with extraordinarily careful construction and operating precautions.

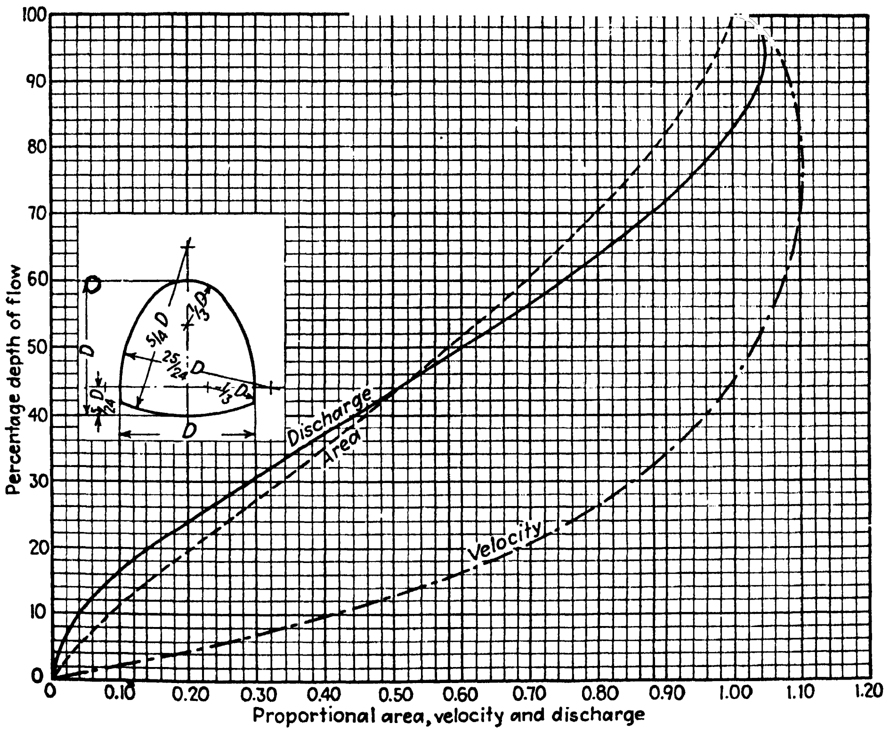


Fig. 28.—Proportional velocity and discharge for semi-elliptical sewers flowing partly full. n = 0.013.

Note: Figured from Kutter's formula on grade of 0.0005; 0.05 per cent. O = 10′ – 0″.

abrupt reduction in velocity. Furthermore, the computations of sewer capacities become less reliable for velocities exceeding about 10 fps.

It is feasible to construct sewers with dense concrete or vitrified-clay interior surfaces and free from depressions or variations in the interior surface so that velocities of flow may be safely used considerably higher than formerly. Special construction may be occasionally required to reduce the maximum velocities directly at the bottom of the steep section of a sewer, such as a series of drops or a specially designed hydraulic-jump chamber.

Sewers Partly Filled. Frequently it is necessary to know the capacity for a given depth of flow, the depth of flow for a given quantity of flow less than full sewer capacity, or the velocity of flow for a sewer flowing partly filled. These may be determined for circular sewers by reference to Fig. 27, and for semi-elliptical and horseshoe sections of the proportions represented on Figs. 24 and 25 by reference to Figs. 28 and 29. Figure 27 shows, for example, that a 5 ft diameter sewer flowing 0.7 full will have a discharge of 84 per cent and a velocity of 112.5 per cent of the discharge and velocity obtaining when the section is flowing at full depth.

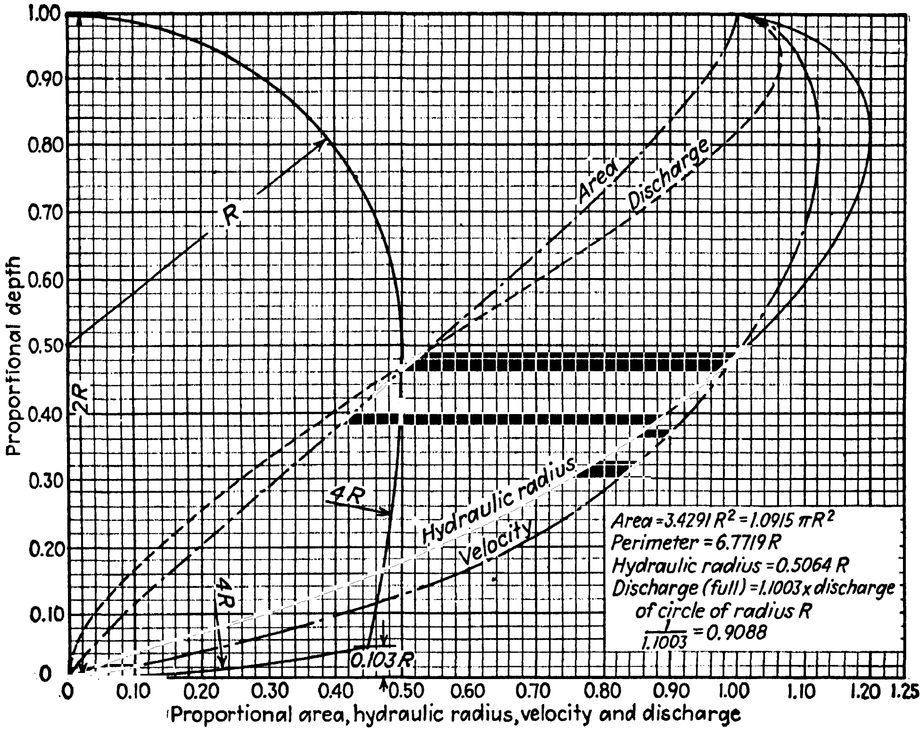


FIG. 29.—Hydraulic elements of horseshoe section; with S = 0.0004 and Kutter's n = 0.013.

Sewer Sections Other than Circular. Factors for determining sewer sizes and capacities for 30 different types of sewer sections have been published by Robert S. Beard.[1] Certain of these sections are shown in Fig. 30, and various constants are given in Table 25. The factor X for a particular section multiplied by the required capacity gives the capacity of a circular sewer having the same velocity. The factor Y is the

[1] *Eng. Record,* **72,** 608–610, 1915.

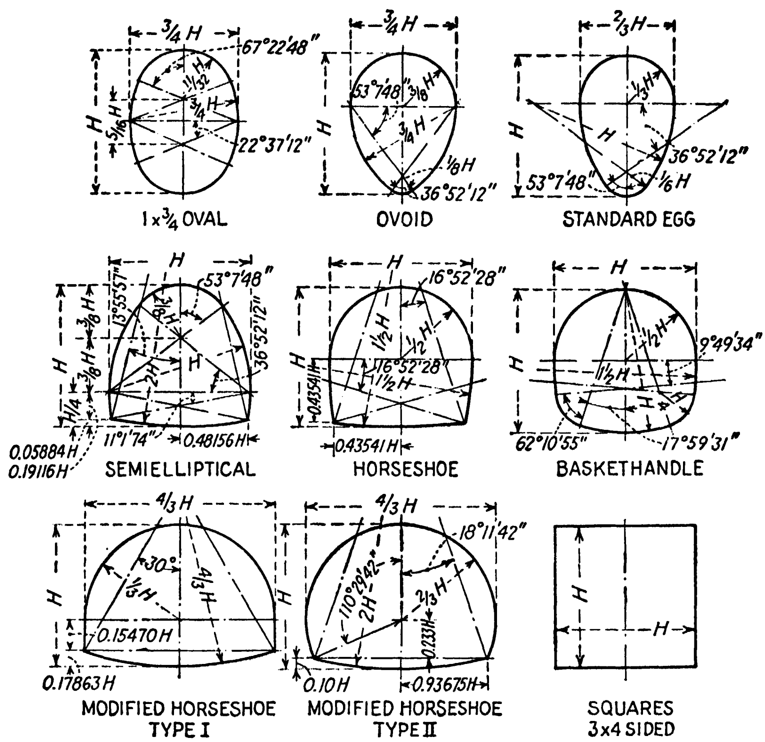


FIG. 30.—Typical sewer sections.

ratio of the height of the particular section to the diameter of a circular section having the same hydraulic radius.

Head Losses Other than Friction. At various places in a sewer system, there may be head losses other than friction losses. These may be due to changes in the velocity

TABLE 25.—HYDRAULIC FACTORS FOR SEWER SECTIONS OTHER THAN CIRCULAR

Type of section	Relative to circular section		Hydraulic elements, ratio to height		
	Capacity X	Height Y	Area	Wetted perimeter	Hydraulic radius
1 × ¾ oval	0.96857	1.15783	$0.60488H^2$	$2.80138H$	$0.21592H$
Ovoid	0.95280	1.20782	$0.56505H^2$	$2.72991H$	$0.20698H$
Standard egg	0.91808	1.29456	$0.51046H^2$	$2.64330H$	$0.19311H$
Semi-elliptical	0.91603	1.02687	$0.81311H^2$	$3.33984H$	$0.24346H$
Horseshoe	0.95554	0.98499	$0.84719H^2$	$3.33789H$	$0.25381H$
Basket handle	0.98535	0.97922	$0.83126H^2$	$3.25597H$	$0.25530H$
Modified horseshoe type I	0.92717	0.89166	$1.06544H^2$	$3.80006H$	$0.28037H$
Modified horseshoe type II	0.92522	0.88392	$1.08646H^2$	$3.84140H$	$0.28383H$
Square (4 sides wet)	0.78540	1.00000	$1.00000H^2$	$4.00000H$	$0.25000H$
Square (3 sides wet)	1.39626	0.75000	$1.00000H^2$	$3.00000H$	$0.33333H$

of flow or to the hydraulic resistance due to turbulence or eddies produced by obstructions or changes in the sewer section. The more common locations of special head losses include manholes, curves, change in sewer size, junctions of two or more sewers, and connections.

Some engineers include the more common head losses by the use of a larger value of Kutter's coefficient n. Other engineers use a lower value of n corresponding to the straight channel friction losses and add a special allowance for other losses.

There are few available data on actual measurements of special losses in sewers. Therefore, the allowances for these losses must be somewhat arbitrary and often relate to similar losses in closed conduits. The following allowances are considered reasonable for practical design purposes:

Item	Loss of Head Allowance
Manholes on sewers smaller than 72 in. flowing full	About 0.08 ft
Right-angle curves:	
Radius = diameter	$0.5 \frac{V^2}{2g}$
Radius = 2 to 8 times diameter	$0.25 \frac{V^2}{2g}$
Junctions of two or more sewers	Subject to special study
Changes in sewer size	$0.8\ (D_1 - D_2)$ft[1]
Sudden reduction in sewer section	$0.5 \frac{v^2}{2g}$ (vel. in smaller section)
Entrance losses	$0.5 \frac{v^2}{2g}$

[1] D_1 and D_2 in feet, also add manhole loss of 0.08± feet and record invert elevation of pipe to nearest 0.01 ft.

Nonuniform Flow. Flow in an open channel is considered uniform when the depth, cross-sectional area, and other elements of flow are substantially constant from section to section along the channel. The foregoing discussion of sewer hydraulics relates to uniform flow, which is the usual condition in sewers.

Whenever the depth and other features of flow such as the velocity, the cross-sectional area, and slope vary from section to section, the flow is nonuniform or varied. Steady nonuniform flow exists when a constant quantity of water flows with variable cross sections, slopes, and velocities. Under these conditions, the surface of the water is not parallel with the sewer invert.

The more common examples of nonuniform flow in sewers include:

1. The *drawdown curve*, which occurs near the free outlet end of a sewer in which the velocity increases toward the outlet.
2. The *backwater curve* caused by an obstruction in the sewer such as a dam or by discharge into a body of water whose surface is above the normal level of flow in the sewer. A flattened grade in the sewer also will produce a backwater curve. In such cases, the velocity and surface slope decrease and the depth increases toward the obstruction or flattened grade.
3. The *hydraulic jump* which is the rise in the surface of a moving stream of sewage likely to occur when sewage moving at high velocity in a relatively shallow stream strikes a stream having a substantial depth and generally moving at a lower velocity.

The first two types of nonuniform flow are the more common, and of them the backwater curve is perhaps the more important in that it frequently is the cause of surcharging in sewers. The hydraulic jump is not common, but it may be quite important under certain conditions.

A profile of the drawdown or backwater curve can be computed by the application of the general formula for steady nonuniform flow in open channels

$$Q = A_m C \sqrt{R_m s}$$

in which A_m and R_m are the average values of area and hydraulic radius for the two ends of the section of channel or conduit being studied. Consideration must be given to the shape and slope of the conduit, the quantity of flow, the coefficient of roughness, and the depth of flow at some known point.

In the solution of any specific problem, use is made of the following procedure:

1. The depth of flow at the outlet is computed.
2. The depth of flow at a section at some selected distance upstream is assumed.
3. The loss of head between these two sections is computed.
4. The distance between the two sections is related to the difference in total head (depth of flow plus velocity head) at each section. If the computed factors check the assumed factors, they become known.
5. The computation is repeated for the next section until sufficient points have become known to establish the drawdown or backwater curve.

The procedure is general and is applicable to all conditions of steady nonuniform flow. It is limited only by the accuracy of the assumptions on which it is based.

4. CAPACITY DESIGN

The capacity design of any system of sewers involves three major steps:

1. The preparation of a general arrangement plan showing the sewer lines and the area tributary to each.
2. The determination of the anticipated sewage quantities for which capacity is to be provided.
3. The determination of proper sewer sizes and grades to carry the anticipated sewage quantities.

Procedures for computing sewage quantities have been given in the foregoing sections, and the application of hydraulics to sewer design has been discussed. In the following paragraphs, the practical application of sewage quantities and sewer hydraulics to the computation of sewer sizes and grades will be discussed.

Four general types of sewer systems, each including trunk sewers, submains, and laterals, will be considered as follows:

1. Sanitary sewers
2. Storm-water sewers
3. Combined sewers
4. Intercepting sewers

Some consideration pertinent to the relative capacities of the several types of sewers are as follows:

1. Sewers required to carry off the domestic sewage and industrial wastes should be provided with ample capacity at any cost, as they concern the public health.
2. There is more leeway in determining the economical or desirable capacity of storm sewers as surcharging of these would back relatively clean storm water onto the streets. Thus, there is a factor of convenience as distinct from a factor of public health.
3. Combined sewers should have larger capacities than separate storm sewers because of the public health hazards from basement flooding.
4. The capacity provided in storm or combined sewers to carry off storm water is in the nature of an insurance against inconvenience resulting from flooded streets or flooded basements. Thus, the relationship between cost and capacity is pertinent.

Maps and Profiles. An early step in sewer design requires an accurate map of the district to be sewered, on which should be given street lines, lot lines, waterways, and contours or surface elevations to outline the ground slopes and to permit the preparation of preliminary profiles. Such a map should be drawn to a reasonably large scale (around 200 to 400 ft to 1 in.).

Sewer Design. An arrangement of proposed sewers should be made with the sewers located to follow the natural slopes in the shortest line toward the outlet of the district. Several arrangements may sometimes be compared in a preliminary way to determine the most economical plan.

When the sewer arrangement is fixed, the subareas tributary to each lateral separate sewer or to each storm-water inlet should then be outlined and computed. Subarea boundaries must be located in anticipation of the probable future street and

lot improvements within the sewer district. Sewerage for each lot must be anticipated in the layout of separate and combined sewers. Subdistrict areas may be determined from the map by scaling or by a planimeter.

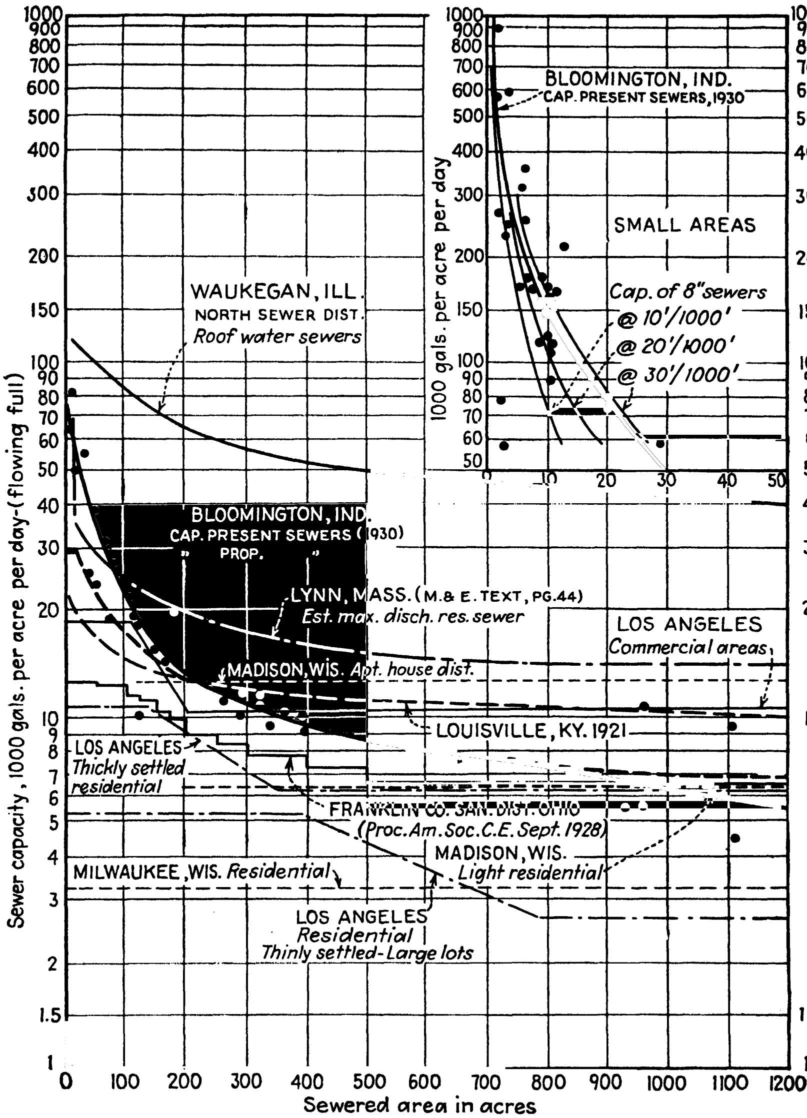


FIG. 31.—Basis of design. Separate sewers, relation of capacity to area.

Finally, the sewer sizes may be computed by one of the several procedures outlined in the following paragraphs. Good engineering practice includes 8- and 12-in. pipe as the minimum sizes for sanitary and storm-sewer systems, respectively. Smaller sizes are more likely to become obstructed by debris, tree roots, and the like, and are also more difficult to clean.

In selecting the proper sewer sizes, it is helpful to determine, first, a general slope corresponding to the ground-surface slope. The final sewer invert slopes will be somewhat flatter over a long line of sewers than the ground surface, as part of the available fall must be used for the invert drops required at changes in sewer size to keep the 0.8 points of the two sewers at the same elevation. The minimum depth of the sewer invert should, in general, be not less than 5 ft, or sufficient to provide a minimum cover of 2 to 3 ft, for storm sewers and 7 or 8 ft in depth for sanitary or combined sewers. The depth for storm sewers is governed by the minimum required cover, and the depth for sanitary or combined sewers is determined in many instances by the necessity of providing drainage for basement plumbing fixtures.

Computation Procedure for Sanitary Sewers. A procedure for sanitary sewer design which has been found useful is the adoption of a curve representing the quantity per acre for which capacity is to be provided (Fig. 31). Such a curve is based upon the estimated population densities, the per capita sewage flows, the allowances for ground water, and the like, as illustrated by the computation procedure shown in Table 26. Obviously, any single curve will apply only to one set of conditions.

TABLE 26.—ILLUSTRATIVE COMPUTATIONS TO DETERMINE UNIT PER ACRE CAPACITY FOR SEWERS

Total population, 40,000

Item	Quantities for various areas, acres					
	10	50	100	300	500	1000
Estimated future population per acre	20	20	20	20	20	20
Factor for uncertain development (see Fig. 5)	1.6	1.6	1.5	1.3	1.2	1.1
Basis of design population per acre	32	32	30	26	24	22
Estimated average daily per capita sewage flow	100	100	100	100	100	100
Factor for peak flows, $M = 1 + \dfrac{14}{4 + \sqrt{p}}$	4.1	3.6	3.4	3.1	2.9	2.6
Per capita peak flows	410	360	340	310	290	260
Total daily sanitary sewage flows, gal/acre	13,100	11,500	10,200	8,100	7,000	5,700
Infiltration at 1,000 gal/acre	1,000	1,000	1,000	1,000	1,000	1,000
Total sewer capacity, gal/acre	14,100	12,500	11,200	9,100	8,000	6,700

The quantities so computed can be compared with capacities at other places by referring to Fig. 31. In certain cities, unit sanitary sewer capacities considerably lower than those shown in the illustrative example have been found satisfactory as a basis of design. Among these are the following:

Madison, Wis. The original system was designed in 1885. Data from the city engineer indicate that the original design was substantially satisfactory, some relief sewers being required only in the commercial and more congested districts. All the sewage is pumped two or more times, and determined efforts are made to prevent downspout and surface-water connections and to reduce infiltration. The design of

new sewers in 1930 was based upon 100 gpcpd, with population densities of 25 per acre for light residential areas and 50 per acre for apartment districts, plus 25 per cent for infiltration and allowance in the design for 100 per cent overload. Thus, these two bases for the sewer flowing full become equal to

Area	Gallons per Acre Daily
Light residential	6,250
Apartment	12,500

Recent sewer designs (1949) for areas of anticipated heavy future development have been checked as to capacity by the engineers of the Madison Metropolitan Sewerage District, on the following basis:

Population to be served	30 per acre
Maximum hourly rate	300 gpcpd
Maximum rate of sewage flow	9,000 gal/acre/day
Infiltration per acre	2,000 gal/acre/day
Maximum rate of flow in sewer	11,000 gal/acre/day

Milwaukee, Wis. The basis of design used by the city engineers for sanitary sewers has been as follows:

Description of Area	Quantity of Sewage, Gal per Acre per 24 Hr
1. Residential	12,000
2. Commercial	60,500
3. Industrial:[1]	
5 acres or less	242,000
10 acres or less	129,200
50 acres or less	32,300
100 acres or less	24,000
500 acres or less	11,000

[1] Within the 1916 limits of city use, double these rates.

Los Angeles, Calif. The basis of design for sanitary sewers has been as follows:

Description of area	Sewage quantities per acre per 24 hr		Sewer capacities flowing full, gal per acre	
	Cfs	Gal	15 in. and smaller[1]	18 in. and larger[2]
Residential-single-family zone[3]	0.004	2,580	5,160	2,800
Residential-unlimited zone	0.020	12,920	25,840	14,000
Commercial zones	0.015	9,690	19,380	10,500
Industrial-light industries	0.021	13,570	27,100	14,700

[1] Sewers 15 in. or smaller designed to flow one-half full (0.5 full capacity).
[2] Sewers 18 in. or larger designed to flow three-quarters full (0.92 full capacity).
[3] Two, three, and four-family zones have unit capacities 2, 3, and 4 times single-family zone.

Computations for determining the required sewer sizes and slopes may be simplified by using a form similar to that shown in Table 27. The rate per acre, obtained from the curve previously adopted as a basis for the design, is entered in column 6 and is multiplied by the total area (column 5) to obtain the total flow (column 7) for which capacity is to be provided. Column 14 is the sum of the losses due to bends, manholes, junctions, and special structures.

TABLE 27.—FORM FOR SANITARY-SEWER DESIGN COMPUTATIONS

Sewer location			Tributary area, acres		Max rate of sewage flow		Design				Profile				
Street	From man-hole No.	To man-hole No.	Incre-ment	Total	Rate per acre, gpd	Total mgd	Diam-eter, in.	Slope, ft/1,000	Capacity when full, mgd	Vel. when full, fps	Length, ft	Fall, ft	Other losses, ft	Invert elev. upper end	Invert elev. lower end
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Third..............	10	11	40.0	40.0	13,000	0.52	8	5.3	0.52	2.3	325	1.72	0.00	95.33	93.61
Third..............	11	12	15.0	55.0	12,500	0.69	8	9.5	0.69	3.1	400	3.60	0.08	93.53	89.93
Chestnut...........	12	13	12.4	67.4	12,000	0.81	10	3.8	0.82	2.3	350	1.33	0.21	89.72	88.39

TABLE 28.—FORM FOR STORM-SEWER DESIGN COMPUTATIONS

Sewer location			Tributary area, acres		Run-off coeffi-cient	Time of flow, min		Rain-fall inten-sity, in./hr	Runoff, cfs		Design				Profile				
Street	From man-hole No.	To man-hole No.	Incre-ment	Total		To upper end	In sec-tion		Rate per acre	Total	Diam-eter, in.	Slope, ft/1,000	Capac-ity, cfs	Vel., fps	Length, ft	Fall, ft	Other losses, ft	Invert elev. upper end	Invert elev. lower end
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Oak..........	1	2	7.20	7.20	0.47	15.0	1.7	3.2	1.50	10.8	21	5.0	11.1	4.6	470	2.35	0.00	45.00	42.65
Chestnut.....	2	3	6.40	13.60	0.50	16.7	1.0	3.0	1.50	20.4	24	10.0	22.8	7.3	430	4.30	0.28	42.37	38.07
Maple........	3	4	6.68	20.28	0.51	17.17	1.0	2.9	1.48	30.0	30	7.0	34.5	7.0	430	3.01	0.48	37.59	34.58

Roof-water Sewers. In those cities where the discharge of roof water into sanitary sewers is permitted or occurs pending the construction of storm sewers, additional capacity should be provided for some roof water. At Waukegan, Ill., where a count in several sewer districts showed that 48 to 92 per cent of the houses had downspout connections, capacity was provided for roof water from 90 per cent of existing houses, computed for a 15-min storm duration and a runoff of 80 per cent. The computation of sewer sizes and slopes is, in general, similar to that for combined sewers.

Computation Procedure for Storm Sewers. Application of the rational method, which is discussed in Art. 2 of this section, to the selection of sewer sizes and slopes may be simplified by the use of a form similar to that shown in Table 28. Values of the runoff coefficient relating to the total area tributary at a particular point are entered in column 6. The rainfall intensity (column 9), corresponding to the time of concentration (column 7), is obtained from a previously adopted rainfall time-intensity curve. The rate of runoff per acre (column 10) is the product of the runoff coefficient (column 6) and the rainfall intensity (column 9), whereas the total runoff for which capacity is required is the product of the total area (column 5) and the rate per acre.

If the sewer capacity is to be based upon the arbitrary method, in which a per acre runoff is selected for future conditions from experience with existing sewers, as discussed in Art. 2, the rate per acre may be taken directly from a curve similar to that shown in Fig. 20, prepared especially for the city under consideration. In this case, columns 6, 7, 8, and 9 (of Table 28) are not needed. This procedure is useful in the design of new and relief sewers covering substantial portions of a city already provided with storm water or combined sewers. Since this procedure does not include consideration of the time of concentration, care must be taken in the selection of the runoff curve to give due consideration to differences in surface slopes and the characteristics of individual sewer districts.

Computation Procedure for Combined Sewers. Sizes and slopes of combined sewers are computed in the same manner as for storm sewers, with the exception that the estimated maximum rate of sewage flow must be added to the storm-water runoff to determine the required sewer capacity.

Intercepting Sewers. Intercepting sewers are generally used in connection with sewage-treatment projects and serve as the collection system to take such part of the flow from existing sewers as may be necessary to remove objectionable waterway pollution. Intercepting sewers may be classed as follows:

1. Sewers to take the flow of existing sanitary sewers
2. Sewers to take the dry-weather flow and a first portion of the storm-water flow from combined sewers

The first class of intercepting sewer is essentially a collecting sewer, directly connected to the existing sewers and designed to take the full flow at the peak rates that may reasonably be expected within the design period.

Intercepting sewers of the second class are more common, but the basis of their design is less easily determined because of the rapid and large increases in the rate of flow in the tributary sewer systems during rainstorms. The major problem is to determine how much of the storm-water runoff should be diverted into the intercepting sewer before the sewage, diluted with rainwater, is permitted to overflow through the old outlets into the watercourse.

Determination of the quantity of storm water to be admitted to the intercepting sewer requires consideration of the following principal factors:

1. The pollutional characteristics of the sewage to be intercepted
2. The quantity of sewage in relation to the minimum flow in the waterway receiving the overflow
3. The use to be made of the watercourse or body of water
4. The extent, frequency, duration, and intensity of rainfall and the resulting overflow
5. Considerations of cost

The design of large intercepting sewers must be based upon an extended investigation including both the capacity of the waterway to take pollution without creating objectionable conditions and experience elsewhere in actual operation of similar intercepting sewer systems.

Some data on the capacities of intercepting sewers at various places are summarized in Table 29. Generally, intercepting sewer capacities have been 350 to 500 gpcpd for the future population. However, in certain instances a much larger capacity has been considered desirable, as at Springfield, Ill., where the southeast intercepting sewer was designed for a capacity of twenty-five times the dry-weather flow, or about 2,200 gpcpd, as it was anticipated at the time of the design that the stream into which the overflow would discharge would flow into the future source of water supply.

The intercepting sewers for the Minneapolis-Saint Paul sewage-disposal project were designed with capacities for the following sewage quantities:

1. Domestic sewage estimated at 65 gpcpd
2. Infiltration computed on the basis of 800 gal/acre/day, plus special allotments up to a total of 3,000 gal/acre daily in certain districts
3. Commercial and industrial quantities estimated on the basis of minimum rates of 300 gal/acre daily, plus additional quantities for certain areas to provide for present or anticipated special wet industries.
4. Concentration or capacity factor applied to the sum of the average domestic sewage flow and allowances for commercial and industrial wastes, which factor ranged for various areas from 1.5 to 2.0, averaging for all districts about 1.75.
5. Storm water computed on the basis of rainfall intensities equal to 0.04 in./hr, with runoff coefficients ranging from 0.2 to 0.8, depending upon the size and characteristics of the existing sewer districts, provision for storm water being included only for areas presently served by combined sewers.

TABLE 29.—COMPARATIVE DATA ON INTERCEPTING SEWER CAPACITIES

	Average dry-weather sewage flow, gpcpd	Interceptor capacity at date of ultimate design	
		Gpcpd	Gal/acre daily
Minneapolis-St. Paul, Minn.			
Metropolitan Drainage Commission:			
Interceptor *A*	140[1]	389	6,400
Interceptor *B*	197[1]	529	6,840
Interceptor *C*	142[1]	422	4,600
Interceptor *D*	158[1]	472	6,500
Interceptor D_1	156[1]	434	6,500
Rochester, N. Y.	120	420	12,200
Detroit, Mich.	155	329	6,620
Syracuse, N. Y.	187	314	11,800
Milwaukee, Wis.	150	364	8,370
Louisville, Ky., Bear Grass Creek	146	412	12,100
Kansas City, Mo., Blue River	100	378	10,100
Decatur, Ill., at treatment plant	...	457	6,700
Springfield, Ill., at treatment plant	...	397	3,300
Cincinnati, Ohio	175	366	10,980
Des Plaines, Ill.	125	250	5,000
Cleveland, Ohio	135	405	8,100
Albany, N. Y.	200	450	7,200
Fitchburg, Mass.	...	445	4,450

[1] Total average domestic sewage and commercial and industrial wastes plus one-half quantities added for wet-season flows.

5. STRUCTURAL DESIGN RELATED TO HYDRAULICS

The structural design of sewers has a bearing on their hydraulic characteristics, particularly with reference to the types of sewer sections, the materials of construction, and the care with which the construction work is done. Also, the effective capacity of sewers may be materially reduced by partial failure of the sewer conduit or by excessive infiltration because of defective construction work or improper design as regards structural strength to carry the trench loadings.

Materials of Construction. The materials of construction relate to hydraulic considerations in two ways: (1) the variations in carrying capacity due to frictional resistance because of interior surface roughness or changes in alignment, and (2) the commercial sizes and shapes available for the sewer construction.

Some of the materials used in the construction of sewers are the following:

1. Vitrified clay pipe
2. Precast concrete pipe (both plain and reinforced)
3. Monolithic concrete (built in place, both plain and reinforced)
4. Vitrified-clay segment block
5. Brick
6. Cast-iron pipe
7. Steel pipe (or some iron alloy)
8. Wood pipe
9. Asbestos-cement pipe

At present, most sewer construction involves one of the first three kinds of materials. Segment block and brick have been used extensively, but not so much in recent years. Cast-iron pipe is often used in pressure lines and sometimes for river crossings, railway crossings, and at special locations where high structural strength and fewer joints are desirable. Wooden pipes are occasionally used in certain sections of the country. Asbestos-cement pipe has recently come into use.

Vitrified-clay pipe is commonly manufactured in sizes up to 36 in., and occasionally up to 42 in. Plain concrete pipe is made up to 24 in. in diameter and reinforced concrete pipe in sizes of 24 up to 108 in. and occasionally up to 154 in. Standard dimensions for vitrified-clay and precast concrete sewer pipes may be found in the Standards of the American Society of Testing Materials. The standard commercial sizes of cast-iron pipe vary in inside diameter by 2-in. intervals from 2 to 24 in. and then by 6-in. intervals above 24 in., with 84 in. being the largest pipe commonly manufactured. Sizes of steel and iron pipes are frequently designated by their outside diameters so that the thickness of the metal must be taken into consideration in computations of carrying capacity.

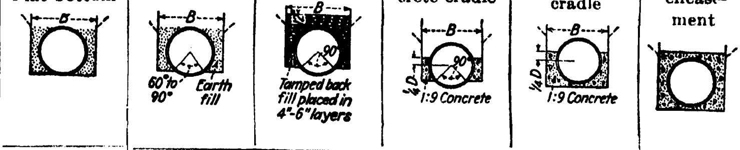
Owing to the short lengths of vitrified-clay and precast concrete pipes, and the consequent large number of joints, it is necessary to watch carefully the construction to minimize (1) the surface roughness at the joint due to lack of uniformity in dimensions or placement, (2) the deviations in alignment, and (3) the imperfections in the joint which permit infiltration.

As indicated in Art. 3, page 1042, the same coefficient n may be used in Kutter's formula for either vitrified-clay pipe or precast concrete pipe. Some concrete pipe may have relatively rough surfaces, and in some cases slight corrugations may be noted. On the other hand, some vitrified-clay pipe is distorted in shape or may have surface or dimensional imperfections. Any of these several conditions tends to reduce the carrying capacity of sewers.

Present construction methods permit obtaining smooth, dense, well-aligned surfaces in monolithic concrete sewers so that a higher carrying capacity can often be obtained than with short precast pipes. Thus, a value of $n = 0.012$ may sometimes be used in design instead of $n = 0.013$. However, some deterioration of the interior

surface may occasionally result from a chemical reaction of the sewage, or of the products of sewage decomposition, usually hydrogen sulphide, with the cement. Therefore, a possible reduction in sewer capacity should be kept in mind, particularly in long sewer lines or where the water supply contains a considerable amount of sulphates.

TABLE 30[1]



Internal diameter of pipe sewer, in.	B Recommended maximum width of trench ⅘D + 8"	Flat bottom		Ordinary laying		First-class laying		Partial concrete cradle		Concrete cradle		Concrete encasement
				Recommended depth of trench to invert of pipe								
		Sand	Clay	Sand or loam	Clay	Sand or loam	Clay	Sand or loam	Clay	Sand or loam	Clay	
12	2'0"	See notes	Not recommended	10'9"	7'0"	17'0"	9'0"	24'0"	11'0"	24'0"	24'0"	For very shallow sewers that are subject to shock loads, or For sewers laid in cuts exceeding the maximum recommended for concrete cradle
15	2'0"			9'9"	7'3"	13'9"	8'6"	20'0"	10'6"	24'0"	21'0"	
18	2'8"			8'6"	7'0"	10'9"	8'3"	14'6"	9'6"	24'0"	16'6"	
21	3'0"			9'0"	7'6"	11'3"	8'9"	14'0"	10'3"	24'0"	16'6"	
24	3'4"			9'6"	8'0"	12'3"	9'3"	15'0"	11'0"	24'0"	17'0"	
27	3'8"			9'3"	8'0"	11'0"	9'6"	14'0"	11'0"	24'0"	16'3"	
30	4'0"			9'6"	8'3"	11'3"	9'6"	13'9"	11'0"	24'0"	16'3"	
33	4'4"			9'6"	8'3"	11'3"	9'9"	13'3"	11'0"	21'6"	16'3"	
36	4'8"			9'9"	8'9"	11'6"	10'0"	13'9"	11'6"	21'0"	16'3"	

[1] American Society of Municipal Engineers, Report of Committee on Sewer Specifications, October, 1931.

Supporting strength of pipe used allows factor of safety of 1.5.

Concrete cradles or encasement may be required, where bad subsoil conditions prevail, in cuts less than shown.

Brick sewers and sewers built of vitrified-clay segment block may not be built with as smooth and true surfaces as can be obtained with concrete, and somewhat different values of n might be used ($n = 0.014$, see page 1043). However, in practice this difference is often properly disregarded, as the future condition of the surface is difficult to predict.

Cast-iron and steel pipes in short sections are frequently designed with the same Kutter's coefficient n as the remainder of the sewer. However, the standard diameters of these pipes differ so that the selection of sizes for short sewer lengths often is determined by factors other than the hydraulic computations. If these pipes are to flow

full, the Hazen and Williams tables or slide rule can be used to compute flows by taking a roughness coefficient of 100.

Sewer Sections. The circular sewer section will, in general, be found the most economical for smaller sewers, owing in part to the available standard sizes, and is the most commonly used section for sizes up to 7 or 8 ft in diameter. The egg-shaped section, formerly used with great frequency, requires a greater invert depth in relation to the elevation of the hydraulic grade line than many other sections.

Larger sewers of reinforced concrete are frequently of horseshoe or semi-elliptical section, although in some cases other special sections have been used. The choice of the sewer cross section is governed largely by relative construction costs, modified to some extent by possible operating difficulties where small flows, and hence low velocities, may be expected.

Earth Loadings and Strength of Sewer Sections. Loads on pipes in ditches may be obtained by the use of the formulas and diagrams given in Sec. 20.

Since the width of the ditch is such an important factor in the computations for earth loading, a narrow trench is desirable. However, to provide adequate working space, construction specifications often prescribe a minimum width or breadth B of trench, which has been based upon one of the two following formulas:

(1) B = pipe diameter plus 12 in.

(2) B = $\frac{4}{3}$ pipe diameter plus 8 in

The strength of sewer conduits in place depends in large measure upon the type of foundation provided under the sewer. Table 30 shows maximum trench widths and trench depths as recommended by a committee of the American Society of Municipal Engineers[1] for various foundations.

6. SEWER APPURTENANCES AND SPECIAL STRUCTURES

The more common sewer appurtenances include manholes, junction chambers, street inlets, catch basins, and flush tanks. The special structures used occasionally for specific purposes include inverted siphons (depressed sewers), sewage regulating devices, overflows, and tide gates.

Manholes are constructed to provide ready access to sewers for inspection, cleaning, and repairs. Where one sewer joins another with a difference in invert elevation of 2 to 3 ft or more, a drop manhole (Fig. 32) is usually constructed to permit the sewage to fall vertically through a pipe placed just outside the manhole barrel and to enter the manhole near the bottom with a minimum of disturbance to flow and inconvenience to workmen who must enter the manhole.

To provide for the head losses resulting from the disturbance to flow caused by manholes, it is good practice to allow an additional elevation drop for each manhole, ranging from 1 in. where there is no angle in the sewer line to as much as 3 in. where the angle of divergence is great.

Manholes should generally be provided on smaller sewers at changes in alignment or grade, at sewer junctions, and at such intermediate points that the distance between manholes will not exceed 200 to 300 ft. For sewers larger than about 42 in., the manholes may be placed farther apart, but they are usually included at each sewer junction and near each angle in alignment of 15 deg or greater.

Junction chambers for small sewers usually are manholes, but special structures are frequently included for the junction of large sewers (48 in. and larger). The major hydraulic problem is the prevention of deposits and head losses incident to the checking of a high velocity in one sewer by a lower velocity in the other. In so far as practicable, the velocities of flow in both sewers at a junction should be the same. To prevent backing up of flow in one sewer due to a higher elevation in a larger sewer, it

[1] *Proc. Am. Soc. Munic. Eng.*, **37**, 43, 1931–1932.

is usually desirable to bring the sewers together so that the 0.8 depths of both sewers will be at the same elevation. The resulting difference in invert elevations is sometimes avoided by giving the smaller sewer a steep slope for the last few feet.

As long as reasonable care is exercised to avoid sudden enlargements or contractions and changes of direction are accomplished by the use of smooth, easy curves, the hydraulic features of junction structures generally present no very complex problem for the usual velocities of flow. However, when the velocities are as much as 6 or 8 fps, the junction structure should be proportioned with great care so that the two flowing streams will blend together without abrupt changes in the elevation of the water surface.

Street inlets, used to permit entrance of storm water from the street, should usually be located upstream of the crosswalk at each street intersection and at low points between streets.

The time of entrance, and hence the required capacity of the storm-water sewer, may be controlled to a limited extent in flat areas by restricting the capacity of the inlet or by spacing the inlets at greater intervals; but on steep grades, the inlets must be designed to catch the storm water as it flows along the gutter and thus prevent objectionable accumulation of the water on lower property.

It is frequently necessary, particularly on steep slopes, to depress the gutter at the inlet, but such depressions should be carefully designed to avoid traffic hazards. The maximum height of the inlet opening is limited by the curb height and the permissible depth of depression. Experience in many places indicates that heights of 4 to 6 in. are desirable. On steep grades, the inlet openings may be made longer than elsewhere to provide sufficient inlet capacity.

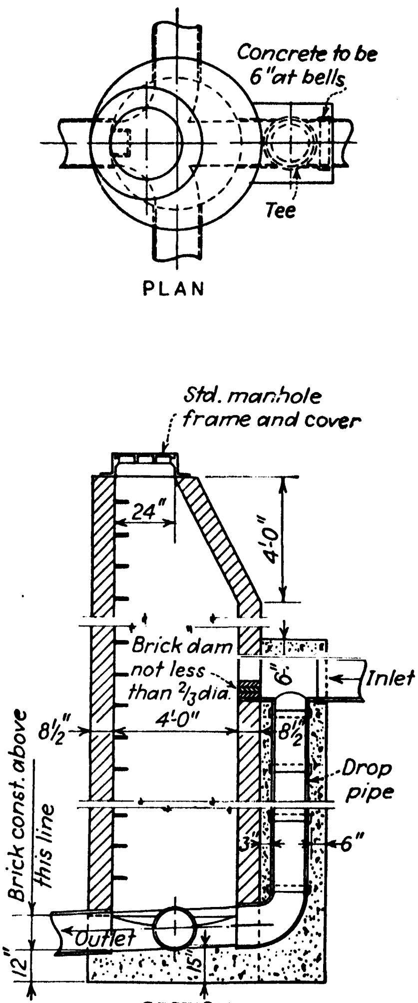


FIG. 32.—Sanitary District of Bloom Township, Chicago Heights, Ill., sewers plan and profile. Station 62 + 81.0 to station 79 + 01.0.

NOTE: Standard manhole the same except drop omitted.

Formerly, bar gratings were provided on the vertical openings of side inlets to prevent entrance of materials of sufficient size to clog sewers. Practice today varies, but with more paved streets, there is a tendency to use clear openings with no bars. Data on inlet capacities are few, and quite generally the type and number of inlets are determined by local experience. It is not feasible to check capacities closely by experimental installations as partial clogging of the inlets by debris is frequent. Some data from tests by Horner at St.

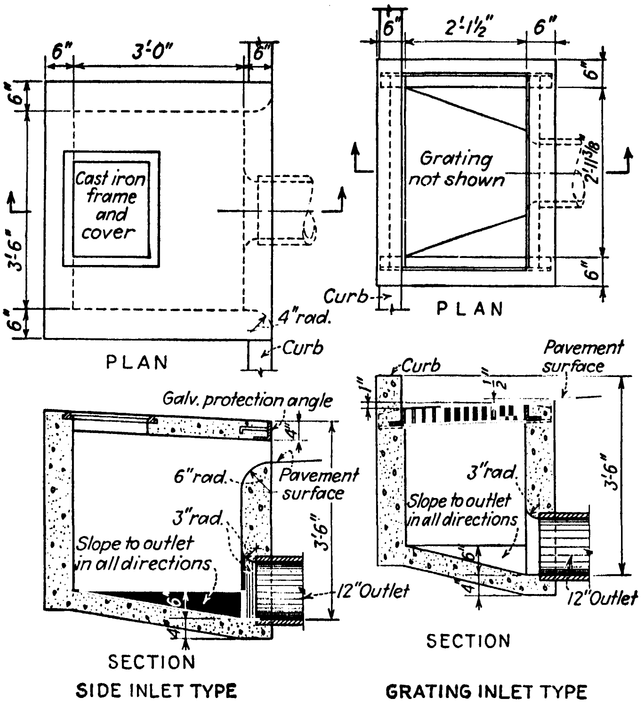


FIG. 33.—Street inlets.

Louis, Mo.,[1] on a side inlet opening 6 in. by 4 ft 6 in. are shown in Table 31. Data from studies at Los Angeles, Calif.,[2] are given in Tables 32 and 33. These data are based on specific designs of the local depressions, and changes in these designs are said to affect materially the inlet capacities.

TABLE 31.—APPROXIMATE INLET CAPACITIES
(Tests by W. W. Horner of St. Louis, Mo.)[1]

Street grade, %	Inlet capacity for various depressions, cfs			
	0	2 in.	4-in.	6 in.
1.0	0.08	0.42	1.20	2.30
2.0	0.05	0.34	1.01	1.96
3.0	0.04	0.27	0.84	1.58
4.0	0.02	0.19	0.67	1.22
5.0	0.00	0.12	0.50	0.85

Data are for a side inlet 4 ft 6 in. long, 6 in. high.
[1] *Munic. County Eng.*, **57**, 147, 1919.

Examples of street-inlet designs are shown by Fig. 33. The designs vary considerably in different cities.

[1] *Munic. County Eng.*, **57**, 147, 1919.
[2] *Eng. News-Record*, **107**, 54–56, 1931.

Catch basins are used to retain solids washed off the street and prevent them from reaching the sewer. Often a design of new sewers may be worked out with economy by using steeper grades so that catch basins may be omitted with desirable improvements in sewer operation and maintenance.

There are few hydraulic considerations in connection with catch basins other than their effect on the time of entrance of storm water or on the prevention of deposits in sewers. A typical catch-basin design is shown in Fig. 34.

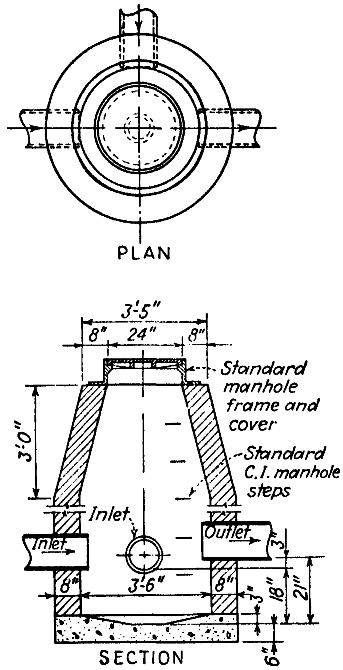


FIG. 34.—Standard catch basin.

Flush tanks are used to produce periodically velocities of flow sufficient to clear deposits from flat sewers or the little used upper ends of lateral lines.

Automatic flushing by the use of flush tanks and dosing siphons is particularly useful with sanitary sewers where sewer grades must be kept very flat. Many cities find hand flushing less expensive, and flush tanks are being used less frequently, except in the unusually flat communities.

Inverted Siphons (Depressed Sewers). Sewers normally are constructed to grade and operate with the hydraulic gradient approximately parallel with the sewer invert. Occasionally it is necessary to construct a sewer over or, more frequently, under some obstacle such as a stream, another conduit, a subway, or some other structure. In this case, a siphon may be used. A true siphon carries the sewage flow above the hydraulic gradient. This type is used very rarely.

An inverted siphon, or depressed sewer, is a sewer constructed below the hydraulic gradient so that the sewer flows under pressure. Usually an inverted siphon includes one or more conduits, an inlet chamber, and an outlet chamber. The inlet chamber includes controlling devices to direct the sewage flow into the several conduits so as to develop self-cleansing velocities in them.

TABLE 32.—CAPACITY OF STANDARD GRATING INLETS

Street Grade, %	Capacity of Inlet, cfs
0	3.70
1	3.55
2	3.38
3	3.20
4	3.02
5	2.85
6	2.69
7	2.53
8	2.38
9	2.25
10	2.12
12	1.88
14	1.71
16	1.57
18	1.47
20	1.40

Data are for an inlet 25½ by 36 in. over-all in plan, with ½-in. bars and 1-in. clear spaces. Bars placed parallel to curb. Two-inch local depression in pavement. Capacity figures are for no overflow past inlet. With a slight overflow the rated capacity may be increased 50%.

TABLE 33.—CAPACITY OF SIDE INLETS

Street grade, %	Length of inlet, ft	Capacity of inlet, cfs, for given depth of local depression			
		0	3.5 in.	6 in.	8 in.
0.5	3.5	0.5	1.8	2.7	3.5
	7.0	1.8	3.4	4.6	5.6
	10.0	3.0	4.7	6.0	7.0
	14.0	4.6	6.4	7.7	8.8
1.0	3.5	0.5	1.7	2.7	3.4
	7.0	2.0	3.6	4.8	5.8
	10.0	3.2	4.9	6.2	7.3
	14.0	4.9	6.8	8.2	9.3
2.0	3.5	0.4	1.5	2.8	4.0
	7.0	1.9	3.0	4.8	6.3
	10.0	3.1	4.4	6.1	7.7
	14.0	4.8	6.2	8.1	9.6
4.0	3.5	0.2	1.3	2.3	3.3
	7.0	1.6	2.5	4.0	5.3
	10.0	2.7	3.6	5.2	6.4
	14.0	4.1	5.1	6.6	7.9
6.0	3.5	0.0	0.9	1.8	2.7
	7.0	0.8	1.7	3.0	4.3
	10.0	1.6	2.4	3.8	5.3
	14.0	2.6	3.4	4.7	6.8

Capacity of inlet not affected by increase in height of opening above a minimum of about 4 in.

TABLE 34.—DATA ON TYPICAL INSTALLATIONS OF INVERTED SIPHONS

Place	Capacity, mgd	Length, ft	Pipe diameter, in.	Diff. between invert elev., ft	Max. vel. fps
Appleton, Wis.	17.4	950	1—15	3.71	4.8
			1—30		
Los Angeles, Calif.	36	800	1—30	0.4	4.6
			1—36		
Charlevoix, Mich.	1.55	210	2—8	4.30	3.4
Carlisle, Pa.	0.92	51.5	1—12	0.95	1.8
Urbana, Ill.	5.8	42	1—12		
			1—14	0.44	2.7
			1—16		
Boston, Mass., Chelsea Creek	88	890	1—60	3.87	6.9
	320	1000+	2—60	....	12.6
Buffalo, N. Y.:					
Black Rock Harbor	570	545	2—96	1.58	8.8
Buffalo River	90	210	1—36	3.36	5.2
			1—48 × 77 Section with 23.85 sq ft area		

The hydraulics of an inverted siphon relate to the velocities required to prevent the depressed sections from filling up with deposits. Either the velocities during minimum flows must be sufficient to prevent deposits, or the velocities during maximum flows must be sufficient to scour out any deposits that may form during periods of low flow. A considerably higher velocity may be required to scour out deposits than is necessary to prevent their formation.

Experience in the design and operation of inverted siphons has indicated that velocities for the estimated near future minimum dry-weather flows of short duration

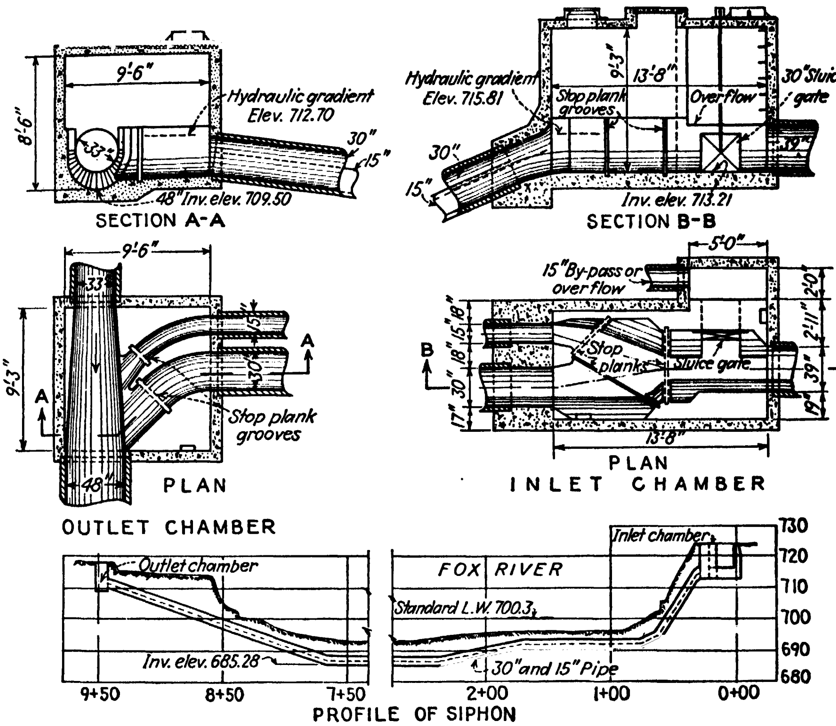


FIG. 35.—Profile of crossing, plan of inlet chamber, and plan of outlet chamber.

should not be less than 1.0 to 1.5 fps and that velocities during normal flows should not be less than 2 to 3 fps for sanitary sewers and 3 to 4 fps for combined sewers.

Where sufficient elevation difference may be obtained between the two ends of the depressed sewer section without undue cost and the sewage quantities are sufficient to create the proper velocities, a single pipe line may be used. However, it is usually uneconomical to use the required head to create velocity; and, as the sewage flows vary over quite a range, the more economical design to obtain suitable velocities may include two or more pipes of different sizes. Some typical data for siphons, which have been in successful operation for a number of years, are given in Table 34.

The hydraulic design of an inverted siphon requires the determination of the following and their interrelationship:

1. The present and future maximum and minimum rates of sewage flows to be taken through the depressed sewer.

2. The difference of elevation in feet which is available or may be obtainable across the depressed sewer within reasonable limitations as to construction and operating costs.

3. The head losses across the siphon for various sewer sizes with one, two, or three depressed conduits.
4. The minimum and maximum velocities of flow for several combinations of sewer sizes. (If there is more than one pipe, the minimum flows may be taken by the smaller pipe, in which case the other pipe or pipes will have a minimum velocity of zero.)
5. The pipe materials to be used in construction. This may modify slightly the conduit sizes as he standard commercial sizes vary for different materials.

The total head available to force sewage through the siphon at desirable velocities is the difference in water-level elevations in the two sewers on either side of the siphon.

The total loss of head in a siphon includes the head lost at entrance and exit, at bends, and changes in cross section plus the frictional resistance along the length of the depresssed conduit between the inlet and outlet chambers.

One typical design of an inverted siphon for a river crossing at Appleton, Wis., is shown by Fig. 35. The losses through the river crossing controlled the elevations and gradient of the main south side intercepting sewer for some 2,500 ft downstream of the outlet chamber and also the elevation of the sewage-treatment plant. Available elevation differentials were limited. Accordingly, it was important that the losses across the siphon be held to a minimum, and several computations were made to determine the proper conduit sizes. The sizes finally selected were one 15-in. and one 30-in. concrete pipe (with an alternate design including one 16-in. and one 30-in. cast-iron pipe for alternate bids which proved to be higher in cost of construction). The following computations for this particular case illustrate the procedure:

COMPUTATION FOR INVERTED SIPHON FOR RIVER CROSSING AT APPLETON, WIS.

Size of sewer above siphon	39	in.
Size of sewer below siphon	48	in.
Length of siphon	950	ft
Flow through siphon:		
Maximum	17.4	mgd
Minimum (50 gpcpd)	1.25	mgd

Try using a 15- and a 30-in. pipe. A 30-in. pipe will carry 6.3 times as much as a 15-in. pipe for the same loss of head.

Maximum flow:

Flow in 15 in. $= \frac{17.4}{7.3} = 2.4$ mgd

Flow in 30 in. $= 17.4 - 2.4 = 15$ mgd

15 in. at 2.4 mgd; loss $= 3.2/1{,}000$; vel. $= 3.0$ fps; vel. head $= 0.14$ ft

30 in. at 15 mgd; loss $= 3.2/1{,}000$; vel. $= 4.7$ fps; vel. head $= 0.35$ ft

Inv. elev. of 48-ft sewer	709.50
Depth of flow in 48 in. (0.8 point)	3.20
Elev. of W.S. in 48-in. sewer	712.70
Friction loss 0.95 × 3.2	3.04
Entrance loss (0.5 velocity head)	0.07
Elev. of W.S. in 39-in. sewer	715.81
Depth of flow in 39-in. sewer (0.8) point)	2.60
Inv. elev. of 39-in. sewer	713.21
Hydraulic loss	3.11 ft
Drop in sewer inverts	3.71 ft

Minimum flow:

15 in. at 1.25 mgd; vel. = 1.5 fps

Intercepting Devices. Intercepting devices are used principally to regulate the quantity of storm water and sewage that is allowed to flow into intercepting sewers from a system of combined sewers before overflow to a near-by watercourse. Also an intercepting device may be used to regulate the overflow from a combined sewer of inadequate capacity into a relief sewer.

The hydraulics of any intercepting device involve (1) the quantity to be diverted, and (2) the determination of the proper size and setting of the controlling device so

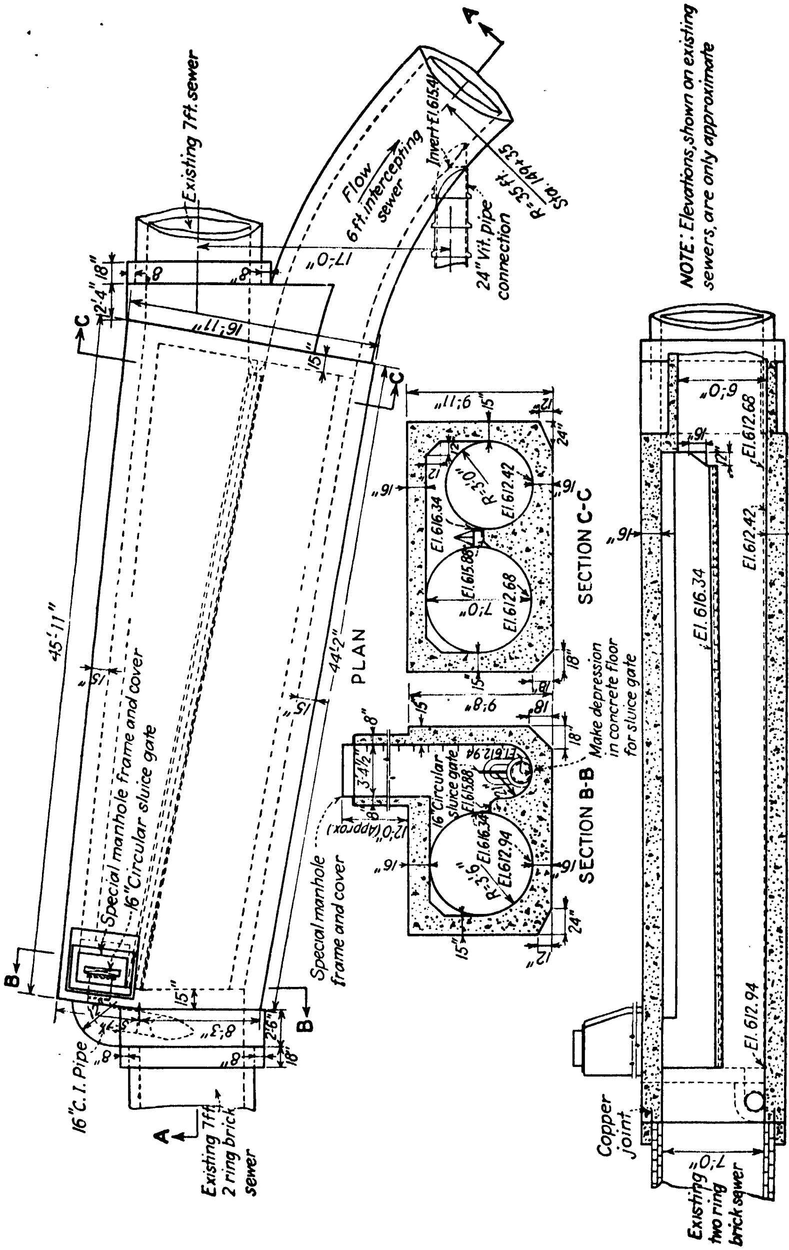


FIG. 36.—Sanitary District of Decatur, Ill., intercepting chamber. Contract 3, Broadway interceptor.

that the desired quantity will be diverted with a minimum variation in the intercepted quantity due to changes in the sewage elevation in the combined sewer. The quantity to be diverted will be determined from a consideration of the factors mentioned in Art. 3, Sec. 23.

The design of the intercepting devices should include some provision for adjustment by raising or lowering weirs, enlarging or reducing orifices, and the like, so that the quantities intercepted may be readily modified as operating experience may indicate to be desirable.

Some of the more commonly used types of intercepting devices are as follows:

1. Overflow weirs
2. Leaping weirs
3. Orifices
4. Float-controlled mechanical regulators
5. Tipping-vane mechanical regulators
6. Siphon spillways

The *overflow weir* usually comprises a structure built into the line of an existing sewer, including a low weir, usually placed diagonally across (sometimes directly across) the old sewer, and an outlet through which the intercepted flow is diverted into the intercepting sewer by the weir acting as a dam. The excess storm water overflows the weir and continues along the existing sewer. This type of intercepting device is particularly useful where the available head is limited. Since the head on the outlet to the intercepting sewer, and hence the rate of diversion, increases as the depth of flow over the weir becomes greater, the length of the weir governs to a large extent the amount diverted in excess of the desired quantity.

The hydraulic computations involve the following items:

1. The rate of sewage flow at which overflow is to begin.
2. The capacity of the existing sewer when flowing full.
3. The proportional depth and the true depth above the sewer invert for the sewage flow to be diverted (see Fig. 27).
4. The probable increase in diverted flows above the desired quantity and the likely effect on the intercepting sewer capacity for various alternative lengths of overflow weir.

The length and elevation of the weir are selected on the basis of these computations and a consideration of construction costs. Occasionally the importance of closely regulating the quantity of diverted flow will warrant constructing an overflow weir of considerable length, such as at the Broadway overflow chamber at Decatur, Ill. (see Fig. 36), but the expense of such long structures is not always warranted. The hydraulics of this type of design has been considered on a theoretical basis by various hydraulicians, and several formulas for practical application are given in some detail by Metcalf and Eddy.[1]

For intercepting chambers on small sewers, it has often been feasible to build the diversion weirs in manholes (see Fig. 37). Sometimes the connecting sewer extending from the intercepting chamber to the intercepting sewer can be used as part of the control to prevent interception of quantities greatly in excess of the desired diversion. In other cases, additional control has been provided by using a sluice gate as illustrated in Fig. 37, an orifice plate, or stop logs to throttle the discharge into the intercepting sewer.

The *leaping weir* comprises an opening in the invert of the sewer of such dimensions as to permit the flow that is to be intercepted to fall through, the increase in velocity and depth at times of storm causing the storm water to pass over or leap the opening and continue along the sewer to the storm-water outlets. Figure 38 shows a typical design with adjustable plates to permit some modifications in the quantity of intercepted flow.

This type of intercepting device is particularly useful for removing sewage from

[1] "American Sewerage Practice," vol. 1, 2d ed., p. 631, McGraw-Hill Book Company, Inc., 1928.

existing sewers with steep grades and where some difference of elevation is available between the invert of the combined sewer and the connection to the intercepting sewer.

The hydraulic computations involve the following steps:

1. The determination of the size and grade of the existing sewer at and directly above the point where the flow is to be intercepted
2. Computation of the quantity of flow to be intercepted
3. Determination of the depth of flow and mean velocity in the existing sewer for the quantity to be intercepted
4. Computation of the width of the weir opening and the form of the weir plate

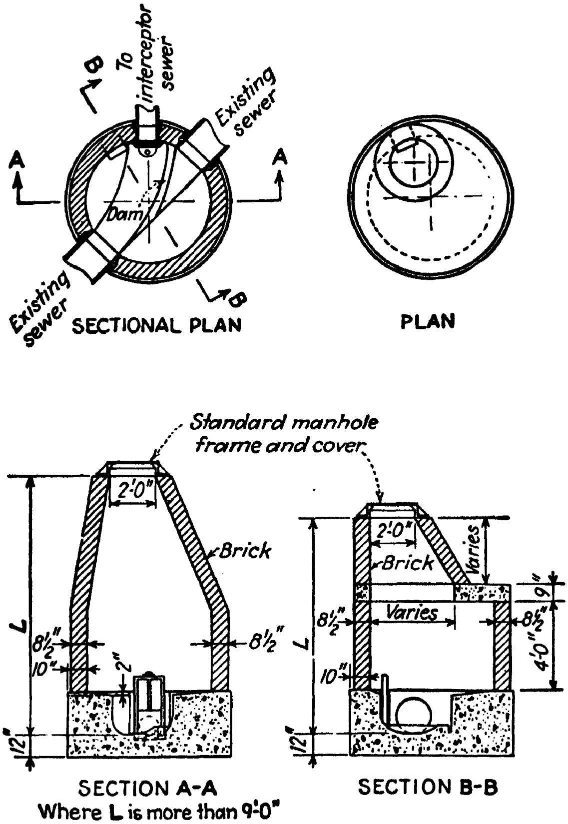


FIG. 37.—Standard intercepting chambers.

The weir opening will depend upon the depth of flow and the surface velocity at any point across the stream flowing in the sewer. The theory for this has been developed by McClenahan[1] substantially as follows (see Fig. 39):

Let d = depth of flow, ft, at center of sewer (computed from Fig. 27 by proportional quantity data).
d' = depth, ft, at any distance X from center line.
R = radius of the sewer, ft.
L = distance across weir opening at center, ft.
Lx = distance across weir opening at any distance X from center, ft.
θ = angle whose sine is X/R.
V = effective surface velocity, fps.

[1] *Ill. Soc. Eng., Ann. Rept.*, 1922 (prize paper).

$$d' = d - R \text{ vers } \theta$$

$$L = V\sqrt{\frac{2d}{g}} \qquad (1)$$

$$Lx = V\sqrt{\frac{2d}{g} - \frac{2R}{g}\text{ vers }\theta} \qquad (2)$$

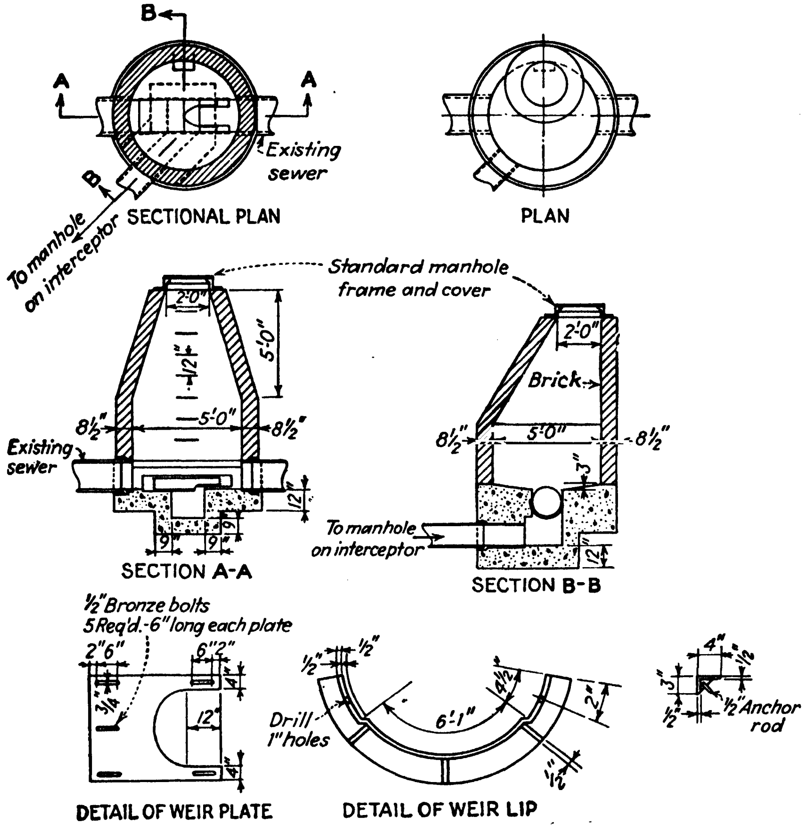


FIG. 38.—Leaping-weir intercepting chamber.

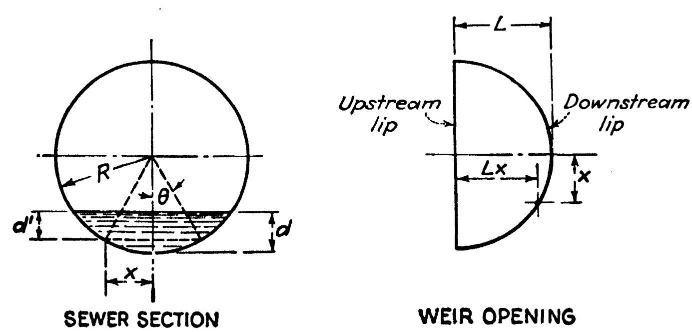


FIG. 39.—McClenahan's formula for leaping weirs. Illustration of symbols.

(1) This formula applies for distance across weir opening at center of sewer only.
(2) This formula applies for distance across weir opening at any distance X from center of sewer.

In a given sewer, the effective surface velocity may be somewhat greater than the mean velocity and may vary across the width of the stream. However, the uncertainties as to the exact flows in the sewers and the amounts to be diverted are sufficient to warrant ignoring the refinements relating to the variations in velocity across the section of the flowing stream.

The usual practice is to construct the weir as a movable plate and to provide for adjustments of about 50 per cent of the L distance in each direction. Thus, at the Bond Street intercepting chamber in Springfield, Ill., the L distance was computed to be 7 in. and the weir plate was designed to provide an opening adjustable from 5 to 11 in.

To determine the dimensions for cutting the weir plate before it is bent into a circular form, the Lx distances may be taken at a distance X' from the center line, where

$$X' = \frac{\pi}{180} R\theta$$

In larger intercepting chambers, it is desirable to cut the weir plates to the computed forms, but in many cases in small sewers a simple circular cut will be sufficiently accurate.

In many cases, it may be desirable to so design these intercepting chambers that backwater from the connecting sewer will offset the tendency to discharge larger quantities with larger flows in the combined sewer.

The *orifice control* includes an orifice set horizontally (and occasionally vertically) somewhat below the flow line of the combined sewer. A low dam in the combined sewer diverts the normal flow to the orifice. The area a of the orifice will depend upon the quantity to be intercepted and upon the distance that the orifice is placed below the crest of the diversion dam and may be computed from the orifice formula

$$q = Ca\sqrt{2gh}$$

with C taken at 0.60 and in which h is the head. Experience in operation has indicated that orifices less than about 6 in. in diameter should not be used as they require excessive attention to prevent clogging. Figure 40 shows a typical installation.

The hydraulic computations include the following steps:

1. Computation of the quantity of flow to be intercepted
2. Determination of orifice sizes required for various heads
3. Determination of the dam heights required to provide the needed head
4. Comparison of dam heights with the normal depth of flow in the existing sewer, or with the backwater conditions to determine how high the dam may be without serious reduction either in velocity for the flow to be diverted, or in the capacity of the combined sewer to discharge the maximum flow.
5. Computation of the maximum capacity of the orifice when the combined sewer is flowing full, consideration being given to the throttling effect of the connecting sewer and the effect of the sewage elevation in the main interceptor

The weir should be of such height that the velocity in the combined sewer at times of dry-weather flow is not less than 1.5 to 2.0 fps. The loss due to the obstruction caused by the dam will depend upon the characteristics of any particular installation and must be based largely upon the judgment of the designer. For practical purposes the loss may be considered as the difference between the velocity head at the restricted section and in the full waterway area. The permissible loss is the difference in depth of flow required for the total flow in the combined sewer and that required for the total flow less the maximum amount to be diverted.

Closer regulation of this type of controlling device may be obtained by permitting backwater from the connecting sewer or the main intercepting sewer to reduce the

effective head on the orifice and thus counteract the increase in head due to increased sewage depth in the combined sewer above the orifice. Detailed computations should be made to determine the probable effect of these factors, and, if the connecting sewer is quite long, consideration should be given to the costs of various sizes of sewer lines as related to the size of the orifice. Smaller orifices will permit closer regulation than

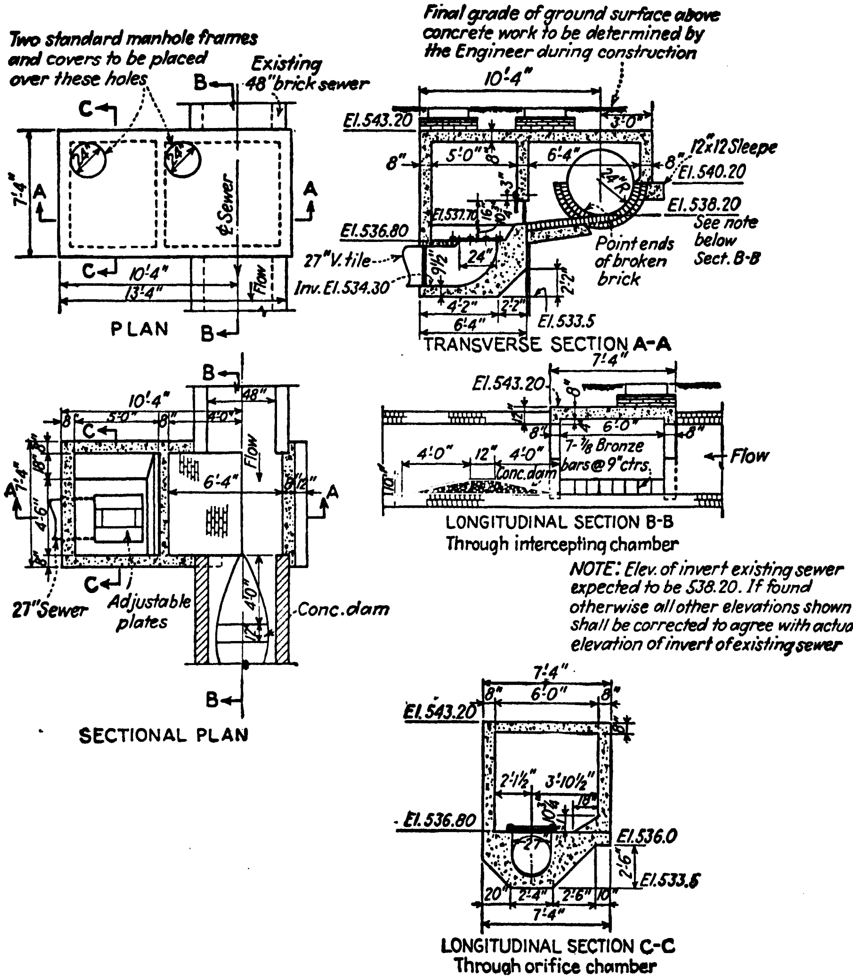


FIG. 40.—Springfield Sanitary District, west side intercepting sewer. Division E, north section. Ridely Park intercepting chamber.

larger ones but will use a greater proportion of the total available head and hence will require larger connecting sewers.

Under certain conditions, the orifice might be omitted and a sewer size selected which will act as the regulator. With long connecting sewers, the variation in discharge due to fluctuations in sewage level in the combined sewer will be small. This method was successfully used in the Sanger Street and South Street intercepting chambers in Peoria, Ill. The principal disadvantage of this method is that the quan-

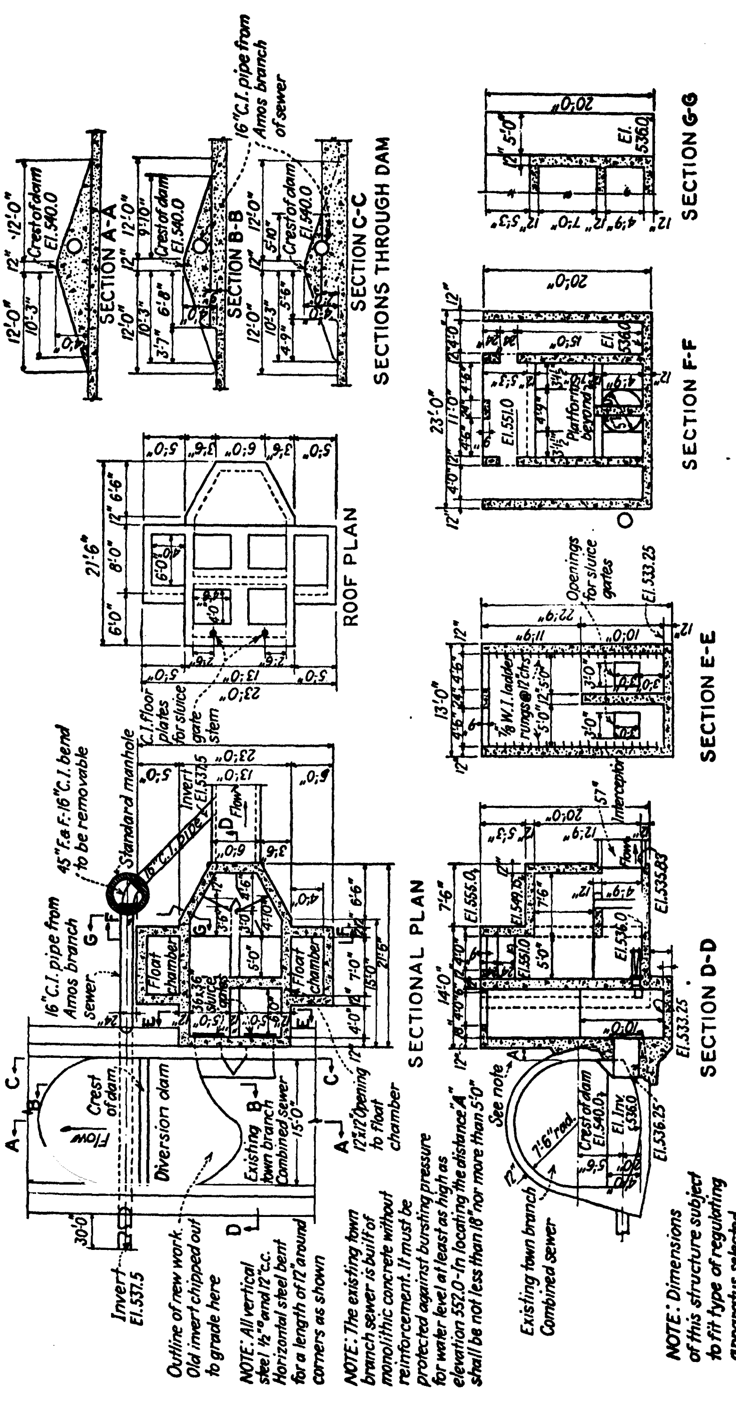


FIG. 41.—Springfield Sanitary District, west side intercepting sewer. Division F, tunnel section. Town branch connection chamber.

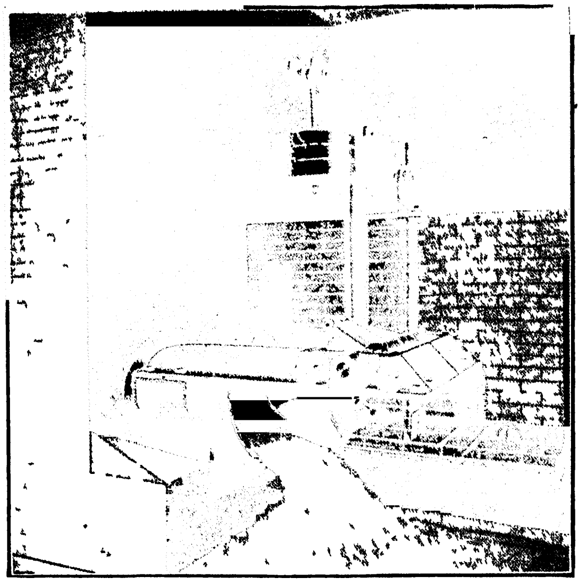


Fig 42. -McNulty toggle-joint regulatoı

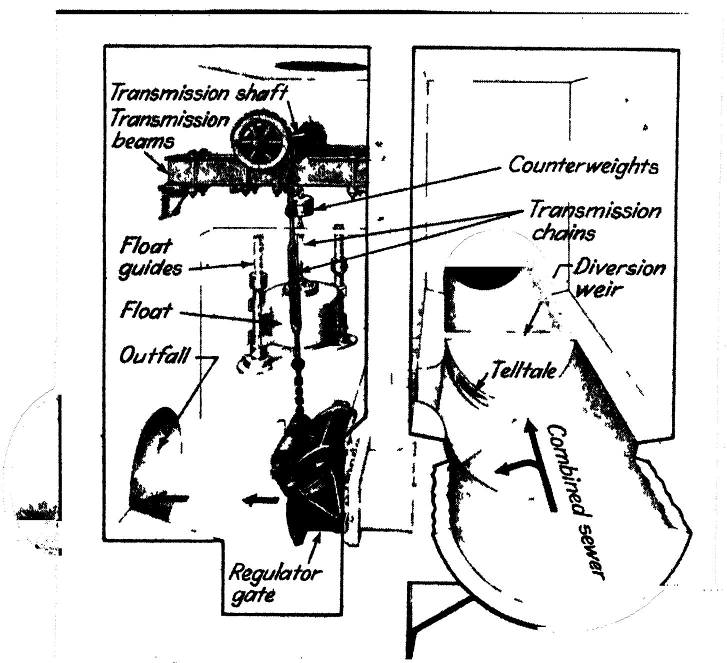


Fig. 43.—Brown & Brown automatic sewage regulator.

TABLE 35.—CAPACITY OF SHEAR-GATE TYPE FLOAT-CONTROLLED REGULATORS

Head, ft	Capacities: Discharge, cfs, for various orifice sizes, in.																	
	5″×6″	5″× 7½″	5″× 9¼″	7½″× 7¾″	7½″× 9¾″	7½″× 12⅜″	7½″× 15⅜″	12″× 12″	12″× 15″	12″× 18¾″	12″× 23⅜″	16″× 21⅜″	16″× 27½″	16″× 34¼″	21″× 34¼″	21″× 40¾″	28″× 38¼″	28″× 47⅞″	28″× 59¾″
0.1	0.53	0.63	0.79	0.99	1.22	1.54	1.92	2.40	3.00	3.75	4.69	5.86	7.32	9.16	11.45	14.29	17.89	22.38	27.98
0.2	0.75	0.89	1.12	1.40	1.74	2.18	2.73	3.41	4.26	5.32	6.66	8.33	10.40	12.99	16.24	20.30	25.38	31.76	39.70
0.3	0.92	1.11	1.36	1.71	2.13	2.67	3.34	4.17	5.21	6.51	8.14	10.17	12.71	15.89	19.84	24.83	31.04	38.84	48.55
0.4	1.06	1.26	1.58	1.97	2.45	3.07	3.86	4.82	6.02	7.52	9.41	11.76	14.70	18.36	22.94	28.68	35.84	44.85	56.07
0.5	1.18	1.41	1.76	2.21	2.75	3.45	4.31	5.39	6.73	8.41	10.52	13.15	16.42	20.54	25.76	32.08	40.89	50.16	62.71
0.6	1.30	1.54	1.93	2.42	3.01	3.78	4.72	5.90	7.37	9.22	11.52	14.40	17.98	22.48	28.10	35.12	43.90	54.93	68.68
0.7	1.40	1.67	2.09	2.61	3.25	4.08	5.10	6.37	7.96	9.96	12.45	15.54	19.43	24.30	30.36	37.95	47.44	59.30	74.21
0.8	1.50	1.78	2.23	2.89	3.47	4.36	5.45	6.81	8.51	10.64	13.30	16.63	20.77	25.95	32.44	40.55	50.69	63.40	79.30
0.9	1.59	1.89	2.37	2.96	3.69	4.63	5.78	7.23	9.03	11.30	14.12	17.64	22.04	27.55	34.44	43.04	53.80	67.32	84.16
1.0	1.68	2.00	2.49	3.12	3.90	4.88	6.10	7.62	9.52	11.90	14.88	18.60	23.24	29.03	36.29	45.36	56.70	70.88	88.70
0.1	1.76	2.09	2.62	3.27	4.07	5.11	6.40	7.99	9.98	12.48	15.60	19.51	24.35	30.44	38.05	47.56	59.46	74.40	93.01
0.2	1.84	2.19	2.72	3.40	4.26	5.34	6.68	8.35	10.43	13.04	16.30	20.39	25.49	31.74	39.64	49.71	62.14	77.79	97.21
0.3	1.91	2.28	2.84	3.56	4.43	5.55	6.94	8.68	10.85	13.56	16.95	21.18	26.47	33.08	41.36	51.70	64.62	80.81	101.08
0.4	1.98	2.36	2.95	3.70	4.60	5.77	7.22	9.02	11.26	14.08	17.60	22.01	27.51	34.37	42.94	53.68	67.09	83.96	104.96
0.5	2.05	2.44	3.05	3.83	4.76	5.97	7.46	9.33	11.66	14.57	18.22	22.77	28.44	35.55	44.44	55.54	69.43	86.88	108.60
0.6	2.12	2.52	3.15	3.93	4.92	6.17	7.71	9.64	12.05	15.06	18.82	23.52	29.40	36.73	45.92	57.40	71.76	89.70	112.26
0.7	2.19	2.60	3.25	4.07	5.07	6.37	7.95	9.94	12.42	15.54	19.40	24.25	30.32	37.87	47.33	59.16	73.95	92.44	115.68
0.8	2.24	2.68	3.35	4.19	5.21	6.54	8.17	10.22	12.78	15.97	19.96	24.94	31.17	38.94	48.68	60.86	76.07	95.20	119.01
0.9	2.30	2.74	3.44	4.32	5.36	6.73	8.41	10.51	13.14	16.42	20.51	25.64	32.06	40.04	50.05	62.56	78.20	97.85	122.32
2.0	2.37	2.82	3.53	4.41	5.49	6.89	8.62	10.77	13.46	16.82	21.03	26.26	32.85	41.06	51.32	64.14	80.17	100.20	125.42
3.0	2.90	3.46	4.32	5.40	6.74	8.46	10.57	13.20	16.50	20.63	25.78	32.25	40.26	50.32	62.89	78.62	98.27	122.67	153.73
4.0	3.35	4.00	4.99	6.24	7.78	9.76	12.20	15.25	19.06	23.89	29.78	37.24	46.56	58.10	72.63	90.78	113.47	141.99	177.51
5.0	3.74	4.65	5.58	6.97	8.69	10.92	13.63	17.04	21.30	26.62	33.28	41.62	51.98	64.94	81.20	101.47	126.84	158.72	198.84
6.0	4.11	4.89	6.11	7.64	9.52	11.95	14.94	18.68	23.35	29.17	36.74	45.65	56.96	71.17	88.96	111.20	139.00	173.93	217.44

Capacity values computed by equation $Q = cAv$. Coefficient of discharge $C = 0.95$. This factor is confirmed by exhaustive tests at Ohio State University. These tests are reproduced fully by Brown & Brown, Inc., *Bull.* 55, 1935.

tity intercepted is fixed and cannot be altered to meet changing future conditions. The quantity intercepted in this way should, therefore, be a relatively small part of the total flow.

Mechanical regulators include a float-controlled gate on the opening between the combined and intercepting sewers. The float may be actuated by the water level in either the combined sewer or the intercepting sewer and may be so arranged that the gate will begin to close when the flow into the intercepting sewer has reached the desired rate. As the depth of flow increases, the gate opening becomes progressively smaller, in this manner counteracting the effect of the increased head.

This type of equipment is particularly useful where there is considerable head variation on the diversion outlet, such as will frequently occur when the diversion is from a large sewer. It is also useful in certain instances when flood water may back up through the overflow outlet.

A typical illustration of mechanical regulators is shown in Fig. 41 for the Town Branch connection at Springfield, Ill., in which two 24- by 36-in. float-controlled gates were installed.

The hydraulic computations include the following steps:

1. Determination of the amount of flow to be diverted from the combined sewer
2. Determination of the allowable variation in head within which the regulator must be closed
3. Selection of the regulator size that will have the required capacity at the available head

Two types of float-controlled regulators have been used in a sufficient number of installations to demonstrate their suitability for such service. These are a toggle-joint type of regulator, as manufactured by the McNulty Engineering Company of Boston (Fig. 42), and a shear-gate type, as manufactured by Brown & Brown, Inc., of Lima, Ohio (Fig. 43). The essential difference in these is the method of closing the gate.

The approximate size of gate opening required for the available head may be determined by the short-tube discharge formula, $Q = Ca\sqrt{2gh}$, with C taken at 0.95, which is approximately correct for wide open gates. As the gates start to close, the coefficient of discharge reduces and it is necessary to use capacity curves which may be obtained from the manufacturers. The capacities for certain standard sizes of regulator gates are given in Table 35.

The head available to produce the desired flow through the regulator will be determined by the allowable height of a diversion dam and the allowable drop from the combined sewer to the intercepting sewer. Some consideration must be given to the relative construction costs of various sizes of regulating devices as compared with possible modifications in the size, grade, and depth of the intercepting sewer. It is desirable to so design the regulator that no large flows in excess of the desired quantity will pass into the interceptor. In certain cases (such as the Town Branch interceptor at Springfield, Ill., Fig. 41), it may be advisable to close the gate entirely after a certain dilution is reached. In the Springfield installation, for example, diversion of 50 cfs was desired, but there was little need to divert 50 cfs when the flow in the combined sewer approached the full capacity of 2,860 cfs. Accordingly, the regulator was arranged to close completely when the water level in the combined sewer rose to a depth of 8 ft, representing a flow of about 1,800 cfs.

7. SEWAGE-PUMPING STATIONS

Sewage-pumping stations may have pumping units with either horizontal or vertical shafts. The pumps may be submerged in the sewage in a wet well or may be set in a dry well with suction pipes extending into a wet well or into an enlarged section of the incoming sewer. Electric motor-driven centrifugal pumping units are used extensively for sewage-pumping service. A typical cross section of a sewage-pumping

station is shown by Fig. 44. In smaller stations, the discharge lines often include check valves instead of the gooseneck siphons shown.

The hydraulic design of the pumping station requires the determination of the following major items:

1. The number and capacity of pumping units
2. The head against which the pumps must operate
3. The pump-performance characteristics and the combination of pumping units to give the most economical installation

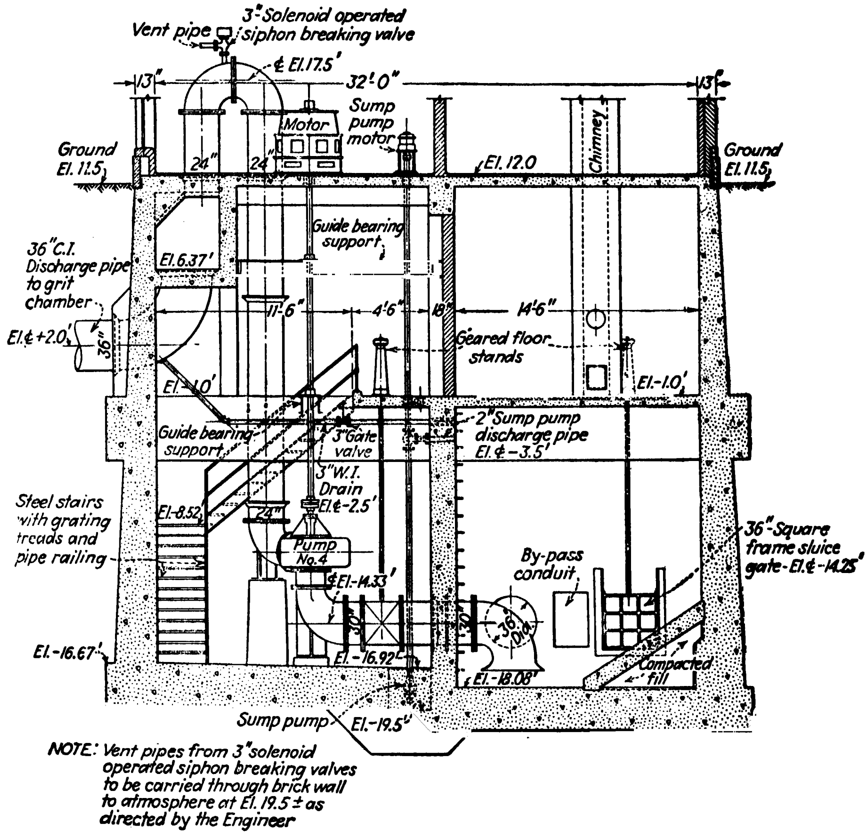


FIG. 44.—Cross section of typical sewage-pumping station.

The head against which the pumping units must operate for any given rate of pumpage is determined by the following items:

1. Static head, which is the difference in sewage levels on the suction and discharge sides of the pumping units
2. Hydraulic losses, which include the following:
   a. Friction losses through straight pipe sections (including any discharge force main)
   b. Velocity head losses at entrance and exit
   c. Losses due to turbulence of flow through valves, bends, reducers, and other special fittings.

The usual type of sewage-pumping station includes some storage volume in a suction sump or wet well to reduce the effect of differences between the rate of sewage flow and the rate of pumping. This results in a fluctuating sewage elevation in the wet well, which is usually limited by an automatic float switch so set as to start or stop

the various pumping units as the sewage elevation rises or falls beyond certain limits. The sewage elevation in the wet well, therefore, may not be constant for any particular rate of pumping or sewage flow. The sewage level in the outlet conduit may also vary with the rate of pumping. With such an arrangement, the static head is the difference between the sewage elevation in the wet well and in the outlet conduit and will usually vary with the sewage flow or pumpage rate.

Generally, some type of bar screen is placed at the end of the incoming sewer to protect the pumps from large solid materials likely to cause clogging. To allow for loss of head through the screen, the maximum sewage level in the wet well should be 6 to 12 in. lower than the maximum permissible level in the incoming sewer.

The losses through sections of straight pipe can be determined by the Hazen and Williams formula or by hydraulic tables. Usually a friction coefficient of 100 may be assumed with sufficient accuracy. To keep the losses within an economical range, the velocity in the pump discharge pipes should usually be about 8 fps. The suction pipes are commonly made larger than the discharge pipes to reduce the suction lift and to minimize the entrainment of air which may adversely affect pump operation. A suction velocity of about 4 fps has usually been found satisfactory.

Entrance losses of about one-half the velocity head and exit losses about equal to the velocity head are reasonable to assume. By carefully designing the piping, it may be possible to reduce these losses somewhat. Thus, a bellmouth entrance to the suction pipe will give lower losses and the discharge pipe may be arranged to conserve a part of the discharge velocity head.

The losses through bends, valves, and special fittings may be computed either by assuming the losses in terms of the velocity heads in the adjacent pipe, or by considering that the special items are each equivalent to a certain additional length of straight pipe. The velocity-head method is less confusing and is easy of application.

The following velocity-head coefficients have been used in pumping-station designs with satisfactory results:

Type of fitting	Coefficient K in $hf = K\frac{V^2}{2g}$	Authority
Gate valve (wide open)	0.19	*Univ. Wis. Bull.* 252
Check valve	3.0 -10.0	
Standard 90-deg bend (4 to 18 in.)	0.25- 0.40	"Handbook of Hydraulics," King
Long-radius 90-deg bend (4 to 18 in.)	0.20- 0.35	"Handbook of Hydraulics," King
Standard 45-deg bend (4 to 18 in.)	0.20- 0.30	"Handbook of Hydraulics," King
Standard reducers	0.04[1]	"Hydraulics," Daugherty

[1] Based on velocity head at smaller section.

It will be noted that the check-valve loss is the largest and the most uncertain individual loss. Some data on measured check-valve losses are given in Table 36.

Check valves are also undesirable as they sometimes clog and may be quite noisy. Sometimes the hammering, due to the rapid closing of the check valve, sets up undesirable vibrations in the piping. For these several reasons, the usual type of check valve may be replaced by an automatic cone type of valve. Automatic check valves are relatively expensive and require considerable operating attention because of their complex construction. In another type of design which has been successfully used in many medium-sized and larger pumping stations, the discharge pipe is extended above the sewage level in the outlet conduit and is then turned down so as to be sealed below the sewage surface. Thus, during the pumping period, the gooseneck acts as a

TABLE 36.—CHECK-VALVE LOSSES
(Loss in terms of velocity head in pipe)

Size of check valve, in.	Velocity in pipe, fps	Velocity head $V^2/2g$	Estimated loss through check valve, ft	K
12	2.84	0.125	0.2	1.6
12	5.67	0.518	0.5	1.0
14	5.8	0.53	4.6	8.7
14	6.1	0.58	2.8	4.9
16	5.0	0.39	3.0	7.7
16	3.2	0.16	0.3	1.9
18	2.52	0.099	0.8	8.1
18	5.05	0.40	1.5	3.8

Data derived from tests in pumping stations at Whiting, Ind., and Urbana, Ill.

TABLE 37.—TYPICAL COMPUTATION OF HEAD-DISCHARGE CURVE, WALNUT STREET PUMPING STATION, SPRINGFIELD, ILL.
(Computations based upon anticipated pump-performance curves*)

Pump in operation	Station discharge, g.p.m.‡	Sewage elevation†			Static head, ft		Hydr. losses, ft‡	Total head, ft‡	
		In 57-in. sewer	In wet well						
			H.W.L.	L.W.L.	H.W.L.	L.W.L.		H.W.L.	L.W.L.
No. 1 alone.....	2,450	534.4	520.9	.....	13.5	....	4.1	17.6	
	1,900	534.2	.....	515.5	....	18.7	2.5	....	21.2
No. 2 alone.....	3,550	534.8	520.9	.....	13.9	....	5.3	19.2	
	2,900	534.6	.....	515.5	....	19.1	3.5	....	22.6
No. 3 alone.....	6,200	535.5	522.3	.....	13.2	....	7.3	20.5	
	5,250	535.2	.....	516.5	....	18.7	5.2	....	23.9
No. 4 alone.....	9,050	536.2	522.3	.....	13.9	....	8.8	22.7	
	7,600	535.9	.....	516.5	....	19.4	6.2	....	25.6
No. 1 and 2....	6,000	535.4	521.6	.....	13.8	....	5.5	19.3	
	4,870	535.1	.....	516.0	....	19.1	3.6	....	22.7
No. 1, 2, and 3..	11,380	536.8	522.3	.....	14.5	....	7.8	22.3	
	8,450	536.1	.....	516.5	....	19.6	6.2	....	25.8
No. 1, 2, and 4..	12,710	537.4	522.3	.....	15.1	....	8.9	24.0	
	11,300	536.9	.....	516.5	....	20.4	7.1	....	27.5
No. 3 and 4....	13,750	537.8	523.0	.....	14.8	....	11.1	25.9	
	12,150	537.2	.....	517.0	....	20.2	8.4	....	28.6

* Performance curves of pumping units finally installed compared closely with assumed curves.
† Sewage levels determined as follows:
a. In 57-in. discharge sewer by proportional depth.
b. In wet well: H.W.L. Level by permissible backing up into 39-in. inlet sewer with 0.55-ft loss through screen and 0.7-ft steps between starting of pumps.
L.W.L. Level by lowest drawdown before sucking air into pumps.
‡ Station discharge, hydraulic losses, and total heads vary somewhat with the different pumps actually in operation, but not sufficiently to affect reliability of the method. Losses include friction, velocity head, and Venturi meter losses.

true siphon and the static head is measured to the sewage surface in the discharge conduit. To prevent the sewage being back-siphoned out of the conduit with the stopping of the pump, an electrically controlled siphon-breaking valve is provided at the top of the gooseneck. In some larger installations, mechanically or electrically operated butterfly or pivot valves are used.

TABLE 38.—SEWAGE-PUMPING STATIONS
Ratio of Installed Pump Capacity to Incoming Sewer Capacity

Station	Year	Capacity, mgd		Ratio, installed pump capacity to sewer capacity
		Sewer to P. S.	Installed pumps	
East Chicago, Ind.:				
Magoun Ave	1921	37.2	75.0	2.02
Alder St. (present)	1925	118.0	145.0	1.23
Alder St. (future)	....	118.0	195.0	1.65
North Shore San. Dist.:				
Deerfield	1922	0.40	0.58	1.45
Highwood	....	0.80	0.86	1.08
Urbana-Champaign	1924	8.25	10.5	1.27
Elgin, Ill.:				
P. S. No. 1	1924	32.0	32.0	1.00
P. S. No. 2	1924	17.0	21.0	1.24
P. S. No. 3	1924	8.0	10.5	1.30
Holland, Mich. (future)	1929	10.0	9.5[1]	0.95
Springfield, Ill.:				
Cook St	1925	13.80	18.0	1.30
Walnut St	1926	16.25	27.0	1.66
Rockford, Ill. (future)	1929	6.10	7.0	1.15
Hinsdale, Ill	1929	16.0	24.0[2]	1.50
Hinsdale, Ill. (future)	....	16.0	32.0[3]	2.00
Centralia, Ill	1928	2.4	3.0	1.25
Jacksonville, Ill	1928	2.5	4.5	1.80
Highland Park:				
Intermediate	1929	9.0	17.0	1.89
Treatment plant	1929	14.0	27.0	1.93
Peoria, Ill	1929	90.0	102.0	1.13
Lima, Ohio:				
Findlay Road	1921	2.04	2.9	1.42
Waukegan	1935	32.6	36.0	1.10
Neenah-Menasha	1936	35.0	36.0	1.03
Buffalo	1937	570.0	720.0	1.26

[1] Pump capacity for average head.
[2] Present, 8 mgd to plant and 16 mgd low-head by-pass.
[3] Future, 16 mgd to plant and 16 mgd low-head by-pass.

The head-discharge conditions will not be the same for all pumping units within a given station, but for most situations it is sufficiently accurate to assume that the head conditions will vary with the total station pumpage. Some preliminary assumptions are usually necessary as to the number of pumping units and their respective capacities in order to compute the hydraulic losses. On the basis of these computations, preliminary head-discharge curves may be prepared representing the anticipated performance of the pumping station. These preliminary head-discharge conditions should then be compared with head-discharge curves for pumps likely to be available,

after which revisions may be made if necessary in the computations. Table 37 and Fig. 45 are computations and station curves for the Walnut Street station at Springfield, Ill.

The total rated capacity of the several pumping units within a pumping station usually should be somewhat more than the capacity of the incoming sewer. Some

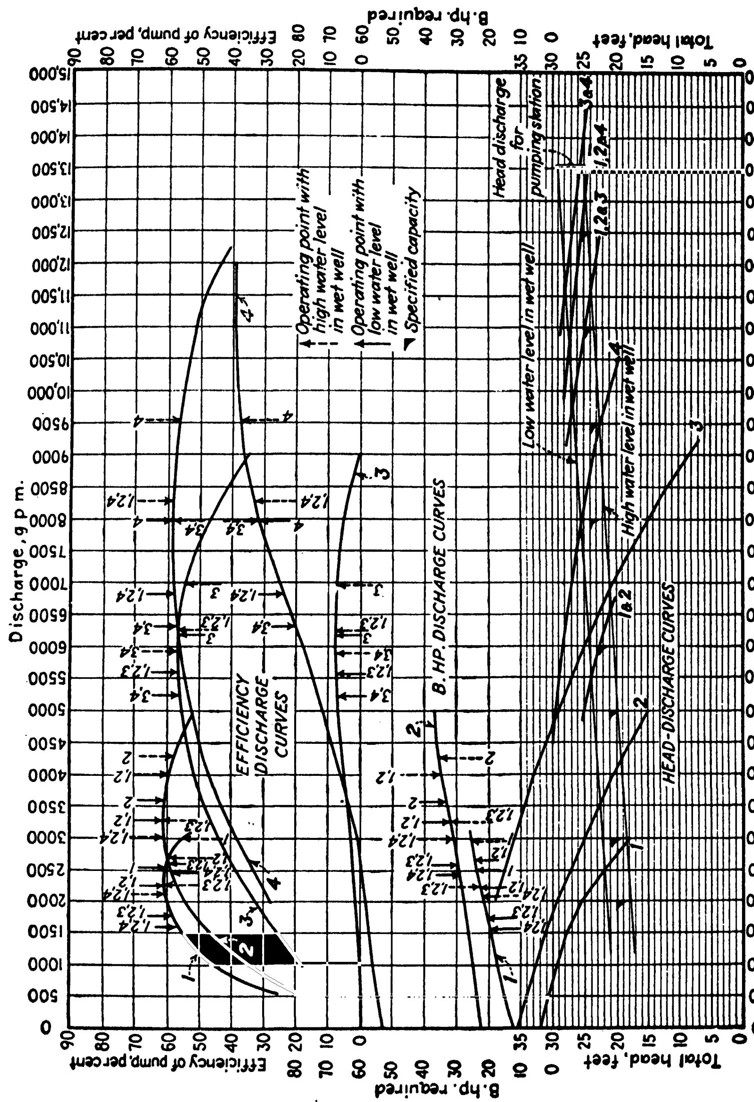


FIG. 45.—Performance of pumping units. The Springfield Sanitary District, Walnut Street sewage-pumping station.

data on the ratio of total rated capacity of pumps installed to the capacity of the sewer leading to the station are given in Table 38.

The capacity and number of the individual pumping units will depend upon the head-discharge performance curves for the pumps as compared with that of the pump-

ing station (Fig. 45). Where the station head-discharge curve is relatively flat, two or more of the pumps may be operated economically in combination and fewer pumps will be necessary to cover the required pumping range. Should the station head-discharge curve be rather steep (as will be the case where relatively long force mains are included), then each pump must operate alone and a larger number of pumping units will be required and hence a larger ratio of installed pump capacity compared to the incoming sewer capacity.

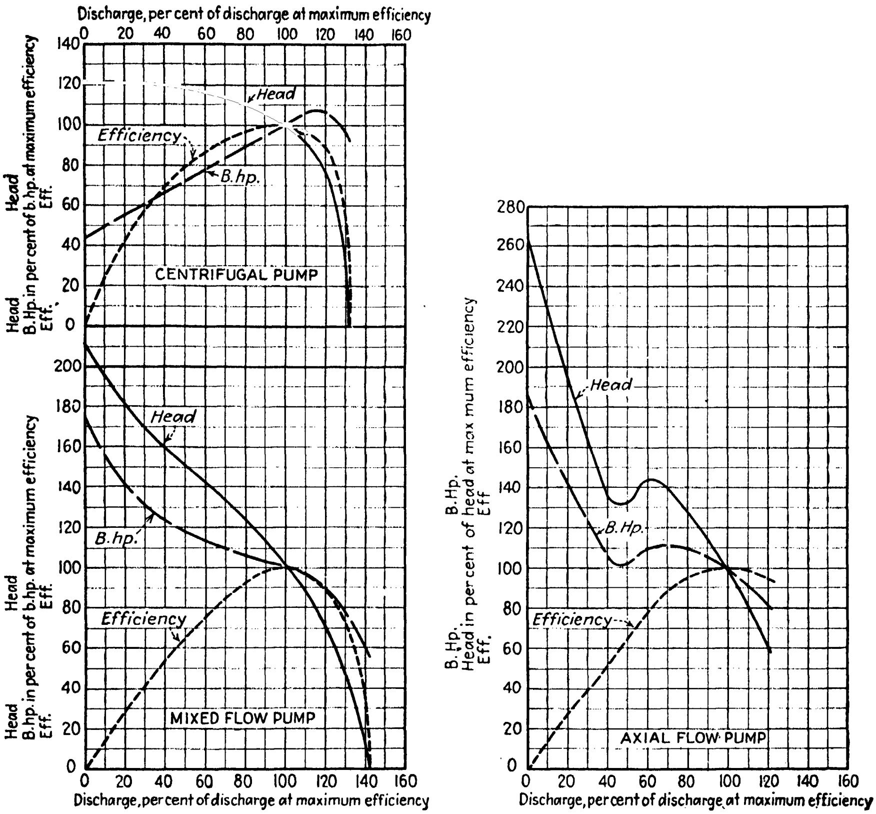


Fig. 46.

The head-discharge characteristics of each pumping unit will depend upon the type of the pump. The usual type of sewage pump includes an impeller which produces pressure at the discharge side by centrifugal action of the rotating impeller and outward discharge at right angles to the pump shaft. Sometimes an axial-flow type pump has been used, and again a mixed-flow or screw-type pump has better fitted the desirable characteristics. Typical distinguishing characteristics of these several types of curves are graphically illustrated by Fig. 46.

Each pump may have a separate discharge pipe connected directly into the upper end of a gravity-flow conduit, or the several discharge lines may be brought together into a single pipe. The former arrangement is preferable as there is less interference of one pump with another. If two or more pumps discharge through the same discharge line, the discharge velocities should be somewhat lower. In either of these

cases the sewage elevation in the outlet conduit will vary with the rate of flow and it will be necessary to determine the depth of flow by the relation between the pumpage rate and the full conduit capacity (see Fig. 27 for a circular conduit).

Frequently, the discharge from the pumping station will be through a long force main. In this case, two factors must be given consideration, which may, at times, be somewhat opposing. On the one hand, the pipe must be small enough so that unduly low velocities will not result with the minimum pumpage rates in order to avoid

TABLE 39.—TYPICAL COMPUTATION TO DETERMINE SIZE OF FORCE MAIN

Sewage quantities:		
1. Minimum 1930		0.47 mgd
2. Dry-weather flow (Dwf) 1930		0.82 mgd
3. Yearly average flow		1.10 mgd
Total average monthly pumpage		34 mg
4. Maximum rate, 1940		6.6 mgd
5. Maximum rate, 1970		13.2 mgd
6. Maximum pump capacity, say		7.0 mgd
7. Force main maximum capacity		13.2 mgd
Force main:		
Length		21,000 ft
Static lift (wet well to end of force main)	11 ft	
Loss through P. sta (assumed constant)	4	
Total lift	15 ft	

Diam. F.M. (in.)	Pump capacity for V. of 1.0 fps		V. in F.M. fps		% time min. pump. operating		Friction per 1,000 ft			Losses, ft total			Total head, ft.		
	Mgd	Gpm	At max pump.	At 1970 max	Min	Dwf	Min pump.	Max pump.	1970 max	Min pump.	Max pump.	1970 max	Min pump.	Max pump.	F.M. capacity
20	1.4	972	4.96	9.3	0.333	0.58	0.33	6.4	21	7.0	134	440	22.0	149	455
24	2.0	1389	3.45	6.5	0.235	0.41	0.26	2.63	8.5	5.5	55	178	20.5	70	193
30	3.0	2083	2.20	4.2	0.157	0.27	0.18	0.89	2.9	3.8	18.7	61	18.8	33.7	76
36	4.4	3056	1.53	2.9	0.107	0.19	0.17	0.37	1.2	3.6	7.8	25	18.6	22.8	40

Head considerations indicate force main size should be 24 or 30 in., depending on relative costs (see below).

COMPARATIVE COST OF FORCE MAINS

Item	Construction cost	Fixed charge at 7½%	Power cost	Total annual cost
24-in. force main, pipe line only	$133,000	$10,000	$4,040	$14,040
30-in. force main, pipe line only	145,000	10,900	3,700	14,600

Use 30-in. force main.

objectionable deposits. On the other hand, there should be an economic balance between construction costs and operating costs, which may require somewhat larger pipes. Table 39 shows a typical illustration of the computations made to determine the size of a force main.

It is desirable that some reasonably accurate record be kept of the pumpage and also, where feasible, of the heads against which the pumps operate. These records make possible routine checking of the efficiencies of the several pumps and the proper control of operation, so that the greatest over-all efficiency may be obtained

from the station. The discharge of a pumping station is frequently measured by a weir or a Venturi meter. More recently the Venturi flume has been used (see Sec. 23, page 1103) as a means of flow measurement. Any sewage measuring device requires some head for operation which must be determined (see Sec. 23, page 1108) and included in the computations of heads for the given station. A measuring device located on the incoming sewer may in some cases give more accurate results, but often it is difficult to so locate it.

SECTION 23

SEWAGE-TREATMENT HYDRAULICS

BY SAMUEL A. GREELEY AND WILLIAM E. STANLEY

1. THE SEWAGE-DISPOSAL PROBLEM

Liquid wastes (sewage) from communities and industries usually flow by gravity or are pumped into waterways for disposal. This practice is reasonable so long as the pollution introduced by the sewage does not cause a danger to public health, a nuisance, or damage to property.

Sewage treatment constitutes the correction factor between the quantity of pollutional material in sewage and the capacity of the waterway to assimilate pollutional material without causing objectionable conditions. The pollutional matter is in part inorganic, in part organic of animal or vegetable origin, and in part living organisms, largely bacteria; it comprises particles in suspension and particles in solution.

Nature provides for the conversion in waterways of the organic matter in sewage into mineral matter. Oxygen is an important element in this conversion and is supplied from the water. The water absorbs oxygen from the air through its surface, the amount increasing as the surface becomes turbulent, and also receives it from tributary streams. If the supply of oxygen is sufficient, so that the oxygen demand of the organic matter does not eventually exceed or nearly exceed it, digestion and assimilation proceed normally.

Other pollution factors which determine the need for sewage treatment include the visual nuisance of objectionable floating matter; deposits of suspended solids which form sludge banks; poisonous or discolored wastes from industries; and especially the bacterial pollution which is dangerous to water supplies, bathing and recreation, shellfish, and the public health.

The objectives of sewage treatment are the removal of the suspended solids, the conversion of the putrefactive dissolved solids into inocuous substances, and the reduction of the bacteria.

The sewage-treatment problem comprises three main parts: (1), a determination of the extent or degree of treatment necessary and the most economical or advantageous treatment process; (2) computations of plant capacities and hydraulics; (3) the design of the structures. The factors involved in the first item are extensive and complicated and are beyond the scope of this discussion.

The commonly used sewage-treatment methods and the structures involved are only briefly described in the following paragraphs to clarify the hydraulic computations necessary in the design of the various treatment-plant elements.

2. SEWAGE QUANTITIES AND CHARACTERISTICS

Quantities of sewage and rates of sewage flow are given in Sec. 22. The ratio of the maximum to the average yearly rate of flow to be passed through each element of the treatment plant largely determines the plant capacities and hydraulic losses. The following ratios represent reasonable practice:

Type of Treatment	Ratios of Maximum Hydraulic Capacity* to Average Rate of Sewage Flow
Plain sedimentation tank	2.5–4.0
Chemical treatment	2.5
Activated-sludge treatment plants	
Screens, grit chambers, preliminary settling tanks	2.5–3.0 or more
Aeration units and final tanks	1.5
Trickling filter plants	2.5

* Hydraulic capacity provided in pipes, channels, and openings to take high rates of flow.

The maximum rates of sewage flow permitted to pass through a treatment plant may be controlled by providing overflows or by-passes, and these hydraulic losses are separate from those of the treatment plant. The minimum rates of sewage flow will be determined by the actual flows from the sewers and cannot be avoided. Adequate velocities or other means must be provided to prevent objectionable deposits in conduits at the minimum rates of flow.

Sewage Characteristics. Sewage may be considered as soiled water, the water comprising over 99.9 per cent of the total volume. The pollutional material comprises larger floating matters which are usually removed by screening, heavier inert material called grit which is removed by a grit chamber, and finer particles in suspension or solution. A portion of such suspended and dissolved matter may be removed by plain sedimentation and by chemical or biological treatment.

The two most important characteristics of sewage are the suspended solids and the biochemical oxygen demand. Typical quantities of these characteristics are as follows:

Item	Pounds per Capita Daily
Suspended solids (s.s)	
Separate sewers	0.17 (1/6)
Combined sewers	0.33 (1/3)
Five-day Biochemical Oxygen Demand (B.O.D.)	
Residential areas only with separate sewers	0.12 (1/8)
Residential cities including ordinary commercial establishments with separate sewers	0.17 (1/6)
Same, with combined sewers	0.25 (1/4)
Manufacturing city, with no large special industrial wastes	0.33 (1/3)
Industrial wastes (frequently quite high)	0.33–0.50 (1/2)

3. SEWAGE-TREATMENT PROCESSES

As sewage-treatment hydraulics depend upon the type, arrangement, and location of the treatment plant, a brief description of the more commonly used types is pertinent.

Sewage-treatment processes have resulted from years of operating experience and laboratory research. Developments have been rapid during recent years, and additional modifications may be expected.

Dilution. This comprises the mixing and dispersing of sewage into a body of water in which natural subsidence causes deposition of the heavier solids and in which living organisms, in the presence of dissolved oxygen, cause the conversion of putrescible organic matter into mineral matter and stable compounds. When the quantity of pollutional matter is too great, the self-purification capacity of the body of water is overloaded and the natural processes are disturbed and inhibited.

The sewage-treatment methods in common use to supplement the natural purification processes include the removal of solids in suspension by screens, grit chambers, and sedimentation tanks and the removal or conversion of the organic material in solution by biological oxidation or chemical precipitation and sometimes filtration which leaves a residue of pollutional material to be finally reduced by natural purification processes.

Bar Screens. The relatively large, coarse suspended solids such as fruit skins, rags, sticks, pieces of paper, and wood are removed by passing the sewage through racks or screens made of parallel iron bars spaced perhaps ½ in. or more apart (Figs. 1 and 2). The quantity of material thus removed is relatively small but would be troublesome in the succeeding treatment elements. This material may be disposed of by burial, incineration, or grinding into finely divided particles and returning to the sewage. Comminutors are frequently used instead of bar screens (Fig. 14).

Grit Chambers. Considerable sandy or granular material designated as grit gets into the sewage from surface runoff during storms, especially with combined

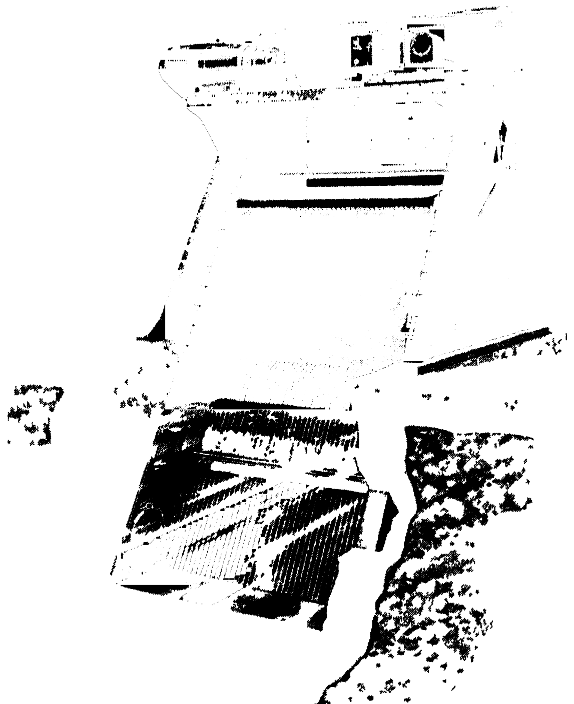


FIG. 1.—Chain Belt mechanically cleaned bar screen.

sewers. This causes trouble in various plant elements by settling out and forming hard compact masses difficult to handle. Grit is removed by passing the sewage through long narrow channels (Fig. 3) in which the velocity is reduced to about 1 fps, which has been found sufficiently slow to permit this material to settle out with a minimum of organic material. During rains, the quantity of grit may be ten or more times the annual average. It is difficult to design a grit chamber so as not to deposit some organic matter, particularly during periods of low flow. It is therefore advisable in many cases to provide methods for washing the grit through the use of mechanical equipment or otherwise (Fig. 3). Grit is generally disposed of by dumping.

Fine Screens. Fine screens in the form of revolving disks, drums, or traveling bands (Figs. 4, 4a, and 4b) with openings $\frac{1}{32}$ to $\frac{1}{8}$ in. in width and possibly 2 in. in length are sometimes used to remove a larger portion of the coarser particles in suspen-

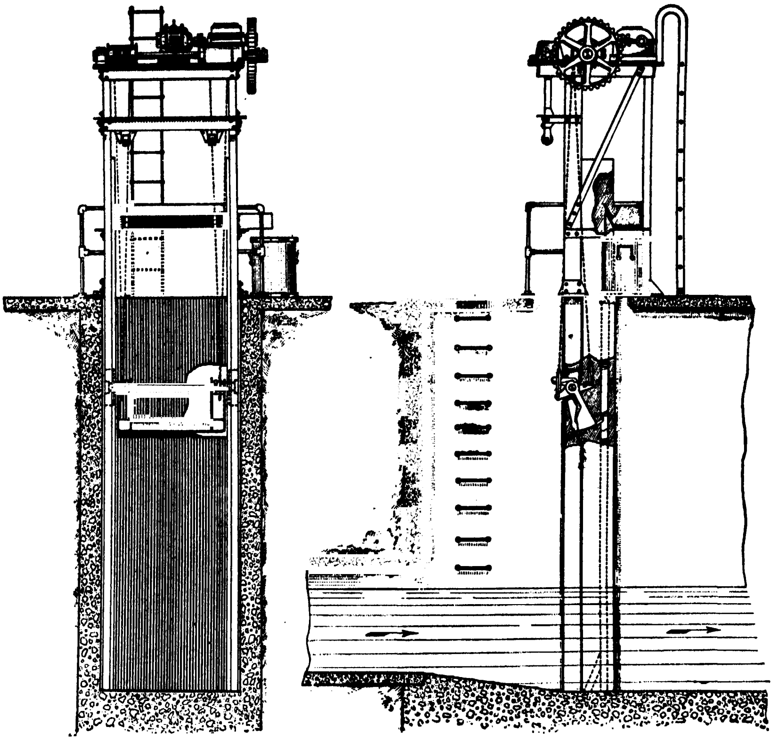


Fig. 2.—Link Belt straight-line mechanically cleaned bar screen.

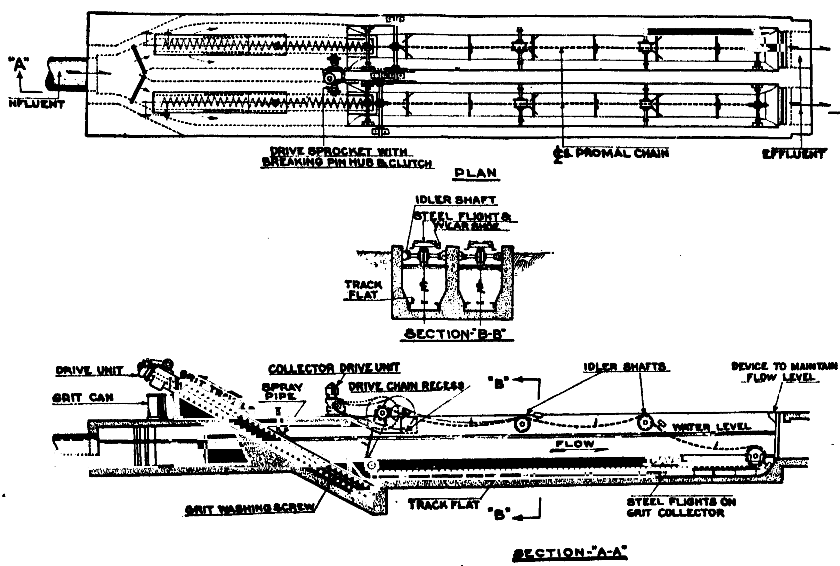
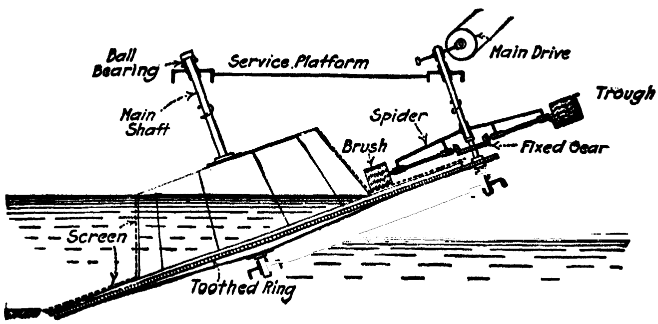
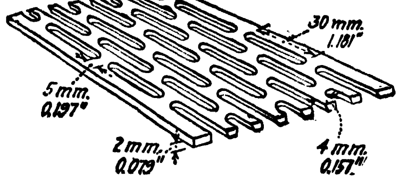


Fig. 3.—Link Belt grit collector and washer.

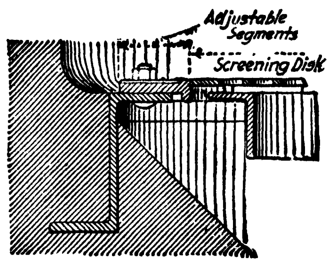
sion. These solids or screenings are removed from the screen by hand or by mechanical means and disposed of by burial or incineration. Such fine screens do not remove so high a percentage of suspended solids as is removed by plain sedimentation, and their use is, therefore, limited.



General arrangement.

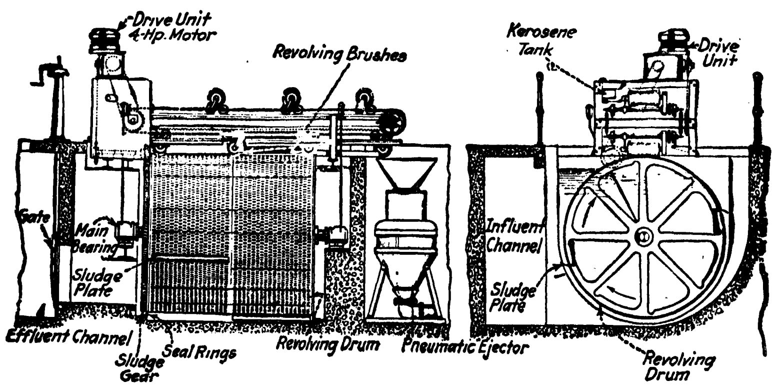


Screening plate.



Detail of seal.

FIG. 4.—Riensch-Wurl screen. (*From Metcalf and Eddy.*)



Side Elevation Section

FIG. 4A.—Link Belt Tark screen. (*From Metcalf and Eddy.*)

Grease Removal. Oil and grease in the sewage tend to rise to the surface and are removed by skimming either in especially constructed tanks or from the surface of the sedimentation tanks. A somewhat greater proportion of grease and oil may be removed by agitating the sewage for a short period by compressed air or paddles with

or without the application of a small quantity of chlorine. Oil and grease skimmings are disposed of by burial, incineration, or digestion with the sewage sludge.

Plain Sedimentation. The first major treatment of sewage following the foregoing preliminary steps is usually the settling out of suspended solids in a sedimentation tank by holding the sewage in the tank for a period of ½ to 3 hr or more, which is sufficient to allow about 40 to 65 per cent, and occasionally more, of the finely suspended solids to settle to the bottom to be removed by mechanical collectors as

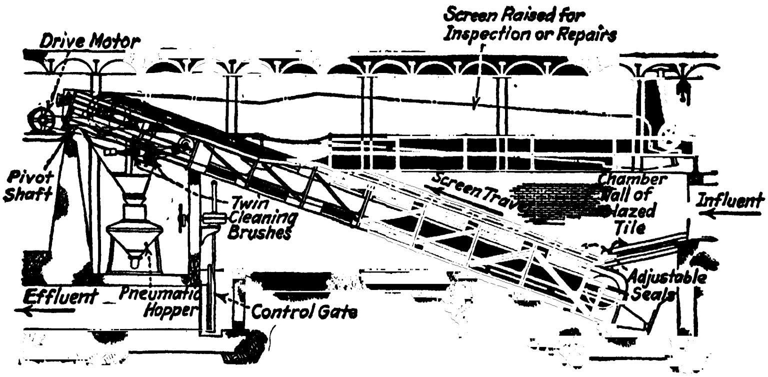


Fig. 4B.—Chain Belt Rex screen. (*From Metcalf and Eddy.*)

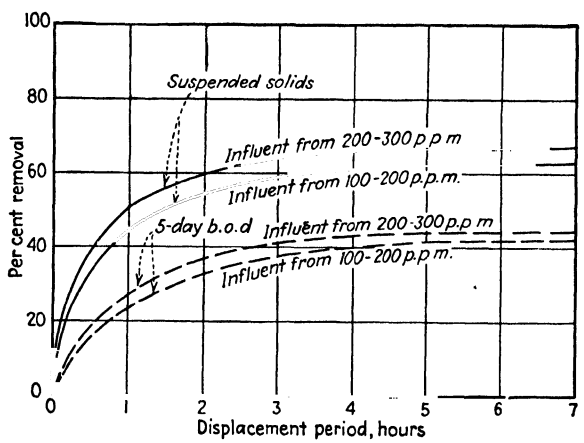


Fig. 5.—Per cent removal of suspended solids and 5-day biochemical oxygen demand. (*After G. J. Schroepfer, Sewage Works Journal, March, 1933.*)

sewage sludge. The exact proportion of solids removed depends upon many factors. Figure 5 indicates the approximate relationship of displacement period and concentration of suspended solids upon the percentage of removal. This step may reduce the total oxygen demand about 33 per cent. In some projects, plain sedimentation comprises the entire treatment, whereas in other projects it is a preliminary step to more complete treatment.

Chemical Treatment. More complete removal of the suspended solids may be accomplished by adding to the sewage certain chemicals such as lime, iron or aluminum salts, or combinations of such chemicals. This is an intermediate step between plain

sedimentation and so-called complete treatment by biological oxidation. The chemicals should be introduced into and mixed with the sewage and given a period of 20 min or more for chemical reaction to form a flocculant precipitate which, when introduced into the sedimentation tank, tends to carry down a considerable proportion of the finely divided suspended solids and some of the colloidal matter. Under favorable conditions of operation and with sufficient quantities of chemicals, this procedure may remove 75 per cent or more of the suspended solids and 65 per cent or more of the biochemical oxygen demand of the sewage. This is one of the older methods of sewage treatment which has come back into use during recent years due to improved equipment for handling and applying chemicals and for the removal of sludge, together with the availability of relatively cheap chemicals.

More Complete Methods of Treatment. Several methods for the greater reduction of pollutional material in sewage depend on the biological oxidation of the putrescible organic matter, in suspension and in solution, by living organisms. Among these more complete methods of sewage treatment are the following:

- Broad irrigation or sewage farming
- Sewage filters
  - Intermittent sand
  - Trickling
  - Contact
- Activated sludge
  - Diffused-air aeration
  - Mechanical aeration

One of the oldest methods of sewage treatment is broad irrigation or the spreading of the sewage, usually after plain sedimentation, over large areas of farm land, a method not used extensively in this country.

Sewage Filters. A development of broad irrigation called *intermittent sand filtration* comprises the intermittent application of the effluent from settling tanks over natural or artificial beds of sand. Each dose is oxidized during its passage through the sand. Between doses, the sand recovers its capacity to treat another dose. As large areas are required, the usefulness of this method is limited.

Since about 1890, the so-called *trickling* or *sprinkling filter* has been in use in which the sewage effluent is sprayed over a bed of stone (or other coarse material) 6 to 10 ft deep through which it trickles down to underdrains placed on the floor of the filter. Bacteria and other organisms grow on the stone surfaces and oxidize the putrescible matter in the sewage as it passes down through the filter. This type of plant has been frequently used.

In recent years *high-rate trickling filters* have come into common use, with sewage applied at substantially higher unit rates than for the earlier filters, now generally called *low-rate trickling filters.* Recirculation of filter effluent, settled or unsettled, is used to dilute a strong sewage or to maintain a sufficient flow rate to keep distributor equipment moving, usually an application rate more than 10 million gal/acre/day.

Activated Sludge. This biological treatment method has been in general use since about 1920. When a screened or settled sewage is agitated in the presence of oxygen, a sludge floc is formed from sewage particles on which many bacteria and small living organisms develop and the sludge floc becomes active in the absorbing and oxidizing of organic matter. Hence, it is called *activated sludge.* When in good condition, this sludge and its load of microscopic life settles rapidly and takes with it nearly all the suspended solids and much of the solids in the colloidal state. The quantity of sludge floc in the sewage is maintained by returning into the sewage an amount of the settled sludge equal in volume to 10 to 25 per cent or more of the sewage flow. The combined sewage and returned sludge, called *mixed liquor,* flows into the aeration tank, which has sufficient volume to provide a relatively long dis-

placement period, generally 4.5 to 6 hr for domestic sewage up to 10 or 12 hr for strong industrial wastes. This mixture is agitated and mixed with air or paddles, during which time the flocculation, absorption, and oxidation of the suspended and colloidal matter, and some of the matter in solution, takes place. The mixture of sewage and sludge passes from the aeration tank into a final settling tank where the activated sludge floc settles out. There is then left a clear liquid with very little organic matter to flow into the waterway.

Final Sedimentation. The effluent from trickling filters and aeration tanks contains a large amount of solids in suspension so that to complete the sewage treatment a final sedimentation tank (Fig. 6) is included to provide sedimentation of these solids.

Effluent Filters. Efforts have been made to remove more of the suspended solids than will settle out by passing the effluent of sedimentation tanks through an effluent

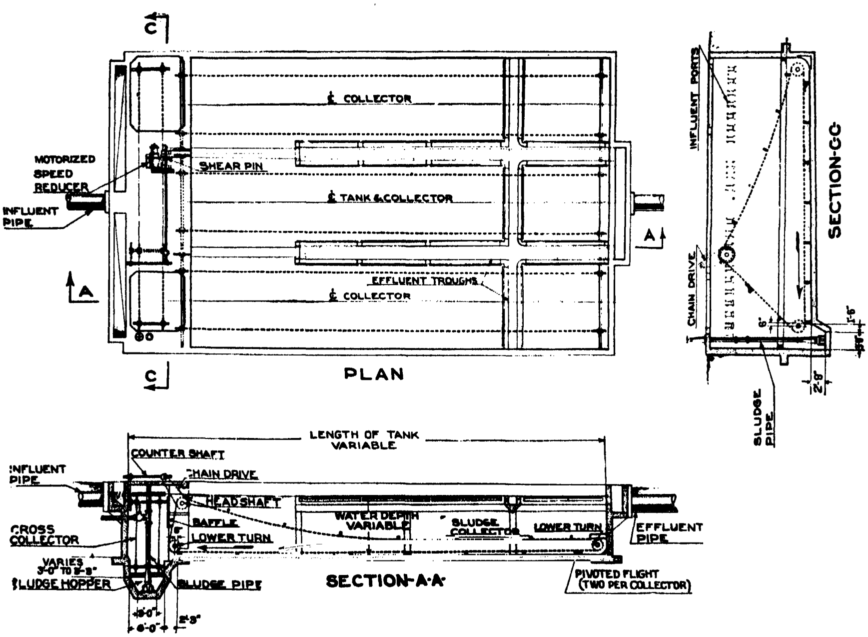


Fig. 6.—Link Belt final sedimentation tank.

filter. One type comprises a thin layer of magnetite sand in a shallow tray with a perforated bottom placed along the effluent weir (Fig. 7) or in a separate structure. The sewage effluent flows up or down through the sand layer, and the suspended solid particles are caught by the sand layer. From time to time, the sand is cleaned by lifting it with an electromagnet and passing sewage effluent through it.

Disinfection. The several methods of sewage treatment do not completely remove or destroy pathogenic and other bacteria. Where the effluent of the treatment plant goes into a waterway being used for water supply, bathing, or shellfish, a greater reduction in bacteria is desirable and is obtained by disinfecting the effluent with chlorine. This method is quite common practice, not overly expensive, and can remove 98 to 99 per cent of the bacteria. A special tank is sometimes included to provide a contact period of 15 to 30 min.

Miscellaneous Methods. Many special methods of sewage treatment have been proposed from time to time. Some years ago, the so-called *direct oxidation process*

was tried, using lime and electricity. Another method called the *miles acid process* was tried out many years ago but has not been used in later years. The Guggenheim process, designated as the *biochemical process*, has been used in a few places. The settled sewage effluent is treated with an iron salt combined with returned acti-

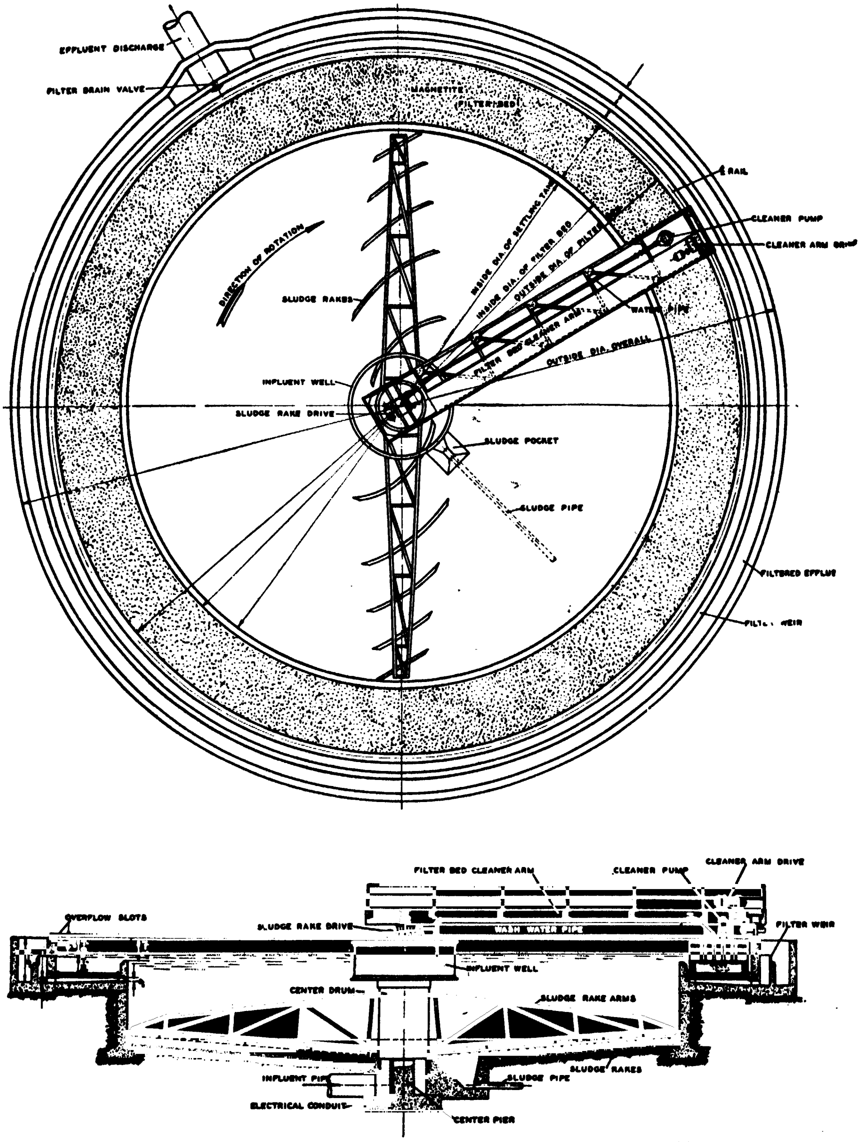


FIG. 7.—Filtration Equipment Corporation magnetite filter.

vated sludge, aerated, and agitated for a short period and then settled. Other processes have been proposed, many of them involving some form of chemical treatment. The use of high-rate (high-capacity) trickling filters has brought added emphasis to plant hydraulics.

4. TYPICAL SEWAGE-TREATMENT ARRANGEMENTS

Plant Elements. The several structures that make up a sewage-treatment plant are as follows:

I. Sewage-treatment elements
  1. Incoming sewer
  2. Preparatory devices
    a. Bar screens (or comminutor) to remove (or reduce) coarse material
    b. Grit chambers,[1] to remove grit
    c. Grease basins,[1] to remove oil and grease and other floating material
    d. Flocculation tanks
  3. Sewage measuring weir, meter, or flume
  4. Preliminary sedimentation tanks,[2] to remove suspended particles
  5. Biological treatment (oxidation)
    a. Trickling filters (or sand filters)
    b. Aeration tanks
  6. Final sedimentation tanks
  7. Effluent filters[1] to remove finely divided suspended solids
  8. Conduits to conduct sewage from element to element
  9. Outfall sewer

II. Sludge-disposal elements
  1. Sludge from sedimentation tank
    a. Sludge collectors
    b. Sludge piping
    c. Sludge pumps
    d. Sludge (or cake) conveyors
  2. Sludge treatment
    a. Digestion tanks
    b. Lagooning
    c. Drying beds
    d. Conditioning tanks
    e. Dewatering filters
  3. Sludge disposal
    a. Dumping (wet or dried)
    b. Fertilizing land
    c. Incineration

[1] Sometimes not needed.
[2] Often the final and main treatment element.

Certain of the preceding elements for the treatment of sewage or sludge are alternative items used only in case the others are not.

Typical Plant Arrangement. The arrangement of plant elements varies a great deal depending upon the type of plant and the topography and shape of the site. Usually a number of tentative arrangement plans are considered and that one selected which will provide operation most economically and conveniently and also leave space and head for future enlargements.

Flow Diagrams. Typical flow diagrams are illustrated by Figs. 8, 9, 10, and 11 for plain sedimentation, chemical treatment, trickling filter (low rate), and activated-sludge methods of sewage treatment, including provisions for sludge treatment and disposal. These also indicate several alternate possibilities.

5. HYDRAULIC LOSSES. GENERAL

Sewage flowing through the various treatment-plant elements requires a difference in elevation of the sewage level between the entrance of the plant and the outlet to overcome the various hydraulic losses. These head differences will vary with the rate of flow, and the successful operation of sewage-treatment works depends in a large measure on the skill with which the hydraulic losses have been determined.

Types of Losses. Head losses at various points through the treatment works may be classified, generally, into the following major divisions:

1. Friction losses through conduits
2. Velocity-head losses
3. Heads required for discharge over weirs, through orifices and other controlling and measuring devices
4. Water-level drops at various points such as a free fall over a weir
5. Head allowance for future extensions
6. Head allowance for high water in the receiving waterway

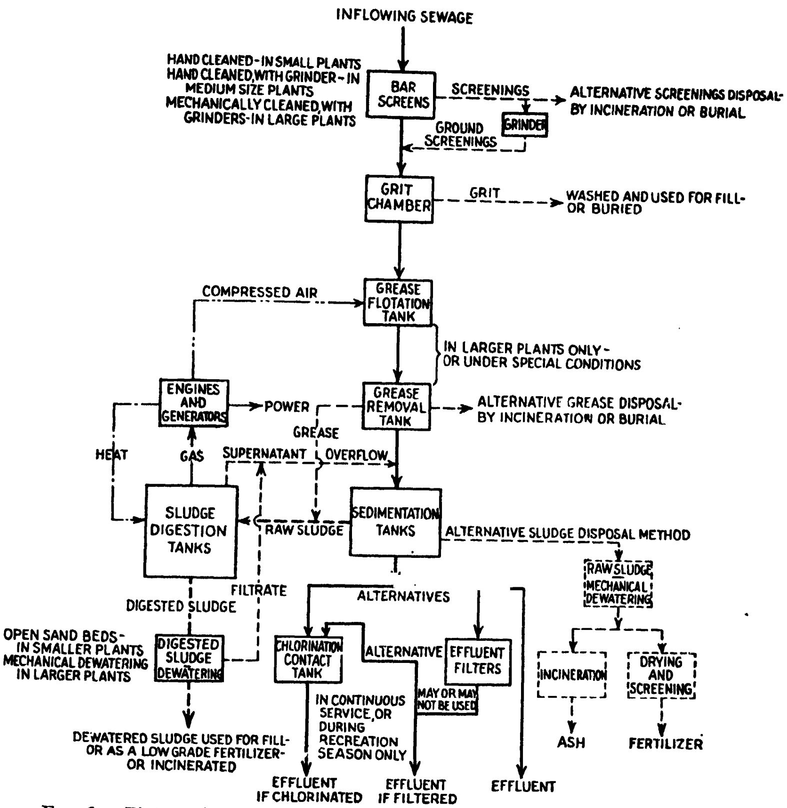


Fig. 8.—Plain sedimentation sewage-treatment plant, typical flow diagram.

The extent and variation in the several hydraulic losses may be affected by a number of factors as follows:

1. Variations in the sewage flow from minimum to maximum rates, as the greatest hydraulic loss occurs at the maximum rate of flow for which treatment is to be provided. This is an important factor.
2. The type and effectiveness of the devices employed to measure and control the distribution of sewage through various plant elements. Thus, close control of distribution and accurate measurements of quantities may require larger hydraulic losses than would be necessary with more approximate methods.
3. The tendency for deposition of sewage solids, particularly in advance of the sedimentation tanks. Thus, the use of higher velocities or agitation by air, or other methods to prevent deposits in conduit. may result in greater hydraulic losses than would be necessary with clear water or well-settled sewage.
4. The size and character of openings and the arrangements of flow channels. Thus, small openings and abrupt changes in direction of flow increase velocity-head losses.

The operation of sewage-treatment plants is influenced somewhat by the hydraulic design, and vice versa. Thus, in sedimentation and aeration tanks, it is advisable to

maintain the sewage level practically constant. The distribution of sewage equally into each of several tanks and the delivery of the sewage quietly and gently into the cross section determines the effective use of the displacement period.

It is good practice in the design of sewage-treatment plants to anticipate future enlargements of various elements. Head allowances should be included at various points to provide for such enlargements. Often a plant extension of 50 to 100 per cent of the first capacity is reasonable to anticipate.

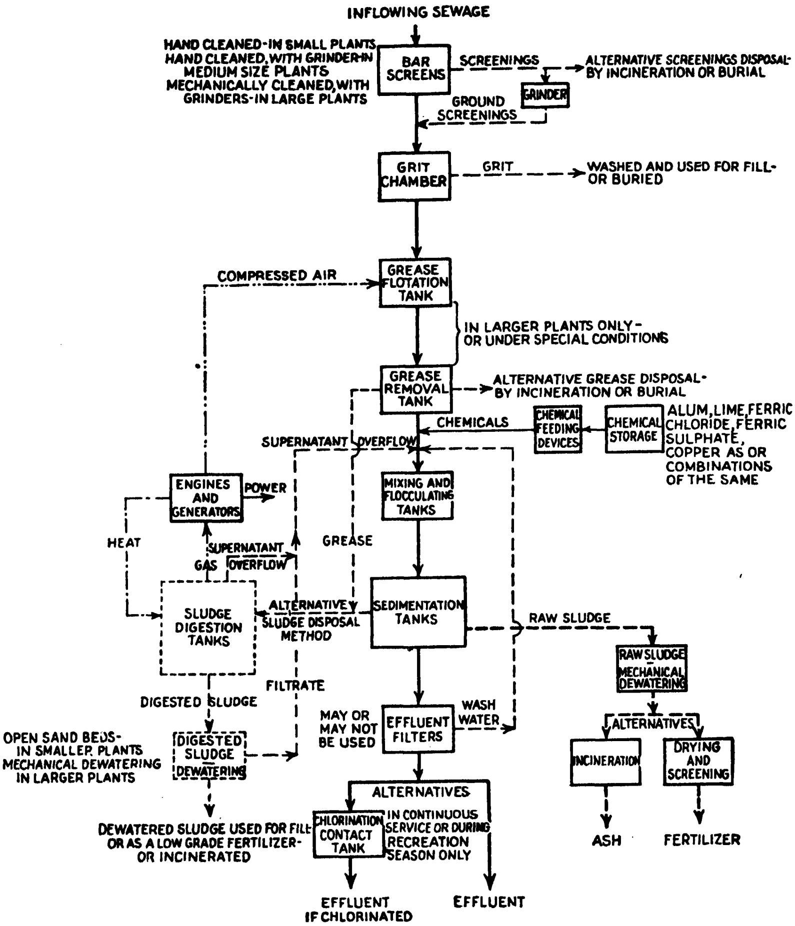


FIG. 9.—Chemical sewage-treatment plant, typical flow diagram.

The head available for the operation of a treatment plant is the difference in elevation between some elevation of the sewage in the inlet sewer and some elevation of high water in the receiving body of water. The frequency of occurrence of these controlling elevations is important (Fig. 12). Some less effective hydraulic operating control is permissible at infrequent intervals, if the cost of avoiding this is great and the resulting small pollution is not objectionable. Thus, in some cases, a higher inlet elevation may be had by backing up the sewage in the inlet sewer, if the resulting lower velocity

is infrequent and not great, say somewhat above 1 fps. The high water in the outlet for the computation of the hydraulic gradient may be somewhat below the maximum and may be taken generally as that exceeded only 1 per cent or occasionally some 10 per cent of the time.

During periods of normal water level, there will be some excess head available over and above that required to overcome the hydraulic losses through the plant.

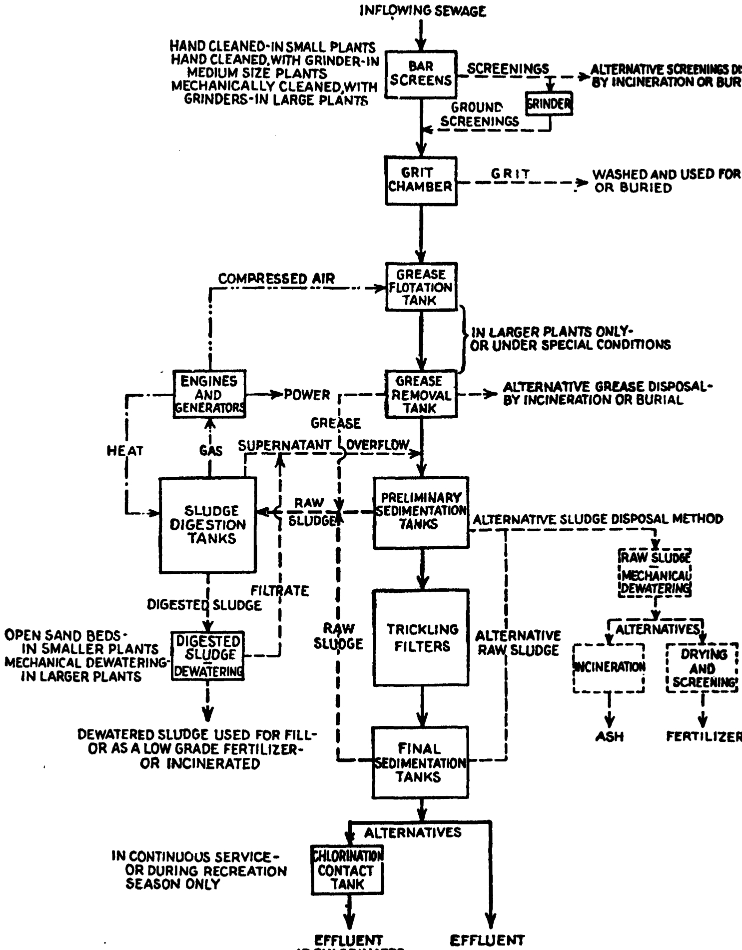


FIG. 10.—Trickling filter sewage-treatment plant, typical flow diagram (low rate).

6. HYDRAULIC COMPUTATIONS FOR MAJOR ELEMENTS OF SEWAGE-TREATMENT PLANTS

General. The detailed hydraulic computations at various points in a sewage-treatment plant are described in the following paragraphs. In an actual problem, the hydraulic computations might start with the high water level of the river or other body of water into which the plant effluent is to be discharged and extend up through

the outlet sewer and the plant in reverse direction to the flow of the sewage, particularly if the plant hydraulics are determined before the incoming sewer is designed.

The usual treatment-plant hydraulics allow only the small variation of the sewage level in sedimentation or aeration tanks required by the outlet weirs. A minimum variation of sewage level is desirable for the best operation of sedimentation and aeration tanks and in the high water level in dosing tanks for trickling filters.

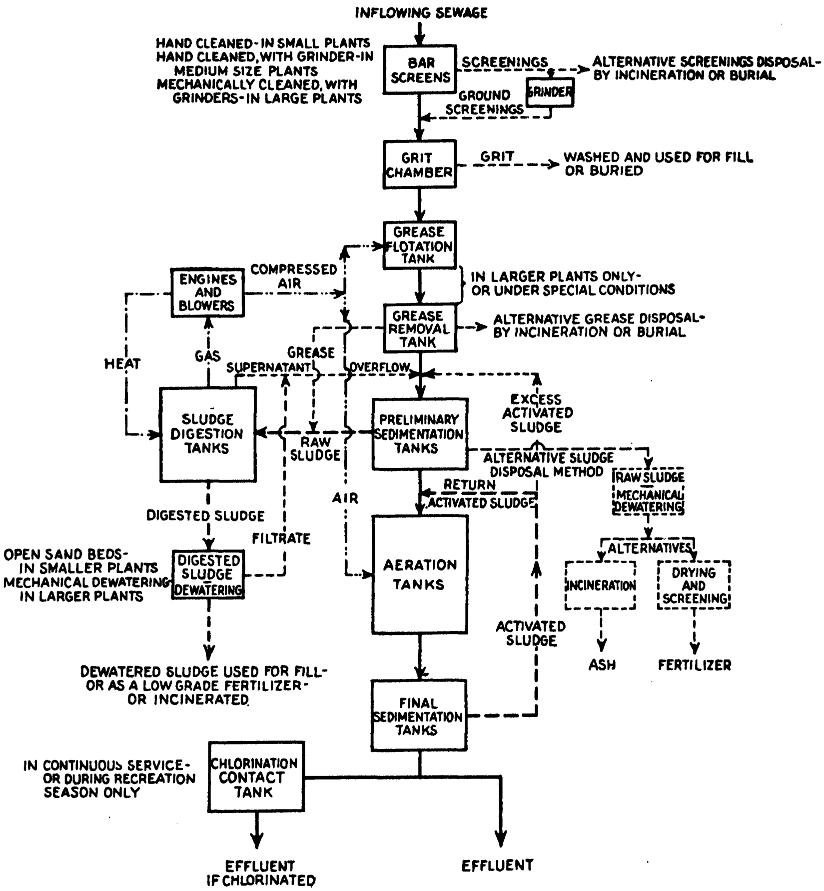


FIG. 11.—Activated sludge sewage-treatment plant, typical flow diagram.

To determine the proper elevations for various plant elements, the hydraulic gradient through the plant should be computed for the maximum and minimum as well as the average rate of flow.

Entrance Losses. At the entrance to a sewage-treatment plant, the following two types of head losses may occur:

1. Loss due to the difference between the high sewage level in the incoming sewer and the high sewage level in the treatment plant
2. Friction and velocity head losses, due to control gates and changes in conduit direction or section

The considerable variation in sewage level in the incoming sewer between maximum and minimum flows should be used as much as possible to overcome hydraulic

losses at tne higher rates of flow in the various plant elements, such as the bar screens, grit chambers, measuring weir or meter, and conduits.

If a pumping station is required in conjunction with the sewage-treatment works, some part of the variation in sewage level in the incoming sewer may be utilized to reduce the head against which the pumps must operate.

The friction and velocity head losses at the entrance to a sewage-treatment plant usually are not large, depending upon the type and design of control gates and the necessary changes in direction of flow.

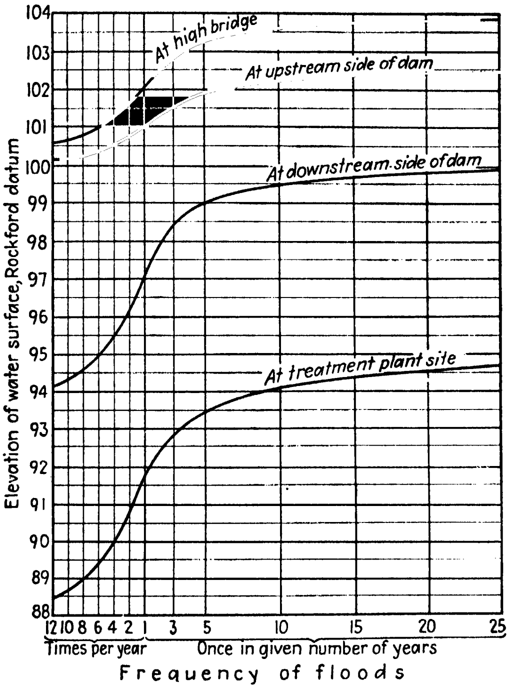


FIG. 12.—Frequency curves of flood heights, Rock River, Rockford Sanitary District.

Losses through inlet gates may be computed by the velocity-head formula $h_v = k(V^2/2g)$ with the following values of k as reasonable for practical design purposes:

Type of Entrance	Coefficient k
Sluice gate	
As a submerged port in a 12-in. wall	0.8
As a contraction in a conduit	0.5
Width equal to full conduit width and without top submergence	0.2

Conduits. Conduits may flow as open channels or as pressure pipes. They are generally circular but may have other shapes. Aerated channels are usually designed to flow as open channels.

Head losses through conduits include:

1. Friction resistance to flow
2. Velocity-head losses due to
   a. Entrance disturbances
   b. Sudden enlargements
   c. Gradual enlargements
   d. Sudden contractions
   e. Obstructions
   f. Bends

The head requirements for flow in circular channels may be computed as indicated for sewers in Sec. 22. Uncertainties due to velocity disturbances usually make close refinements in the selection of the roughness coefficient n unnecessary. The Hazen-Williams formula and diagrams (Appendix A) or tables based upon this formula are often used for flow in pipes under pressure.

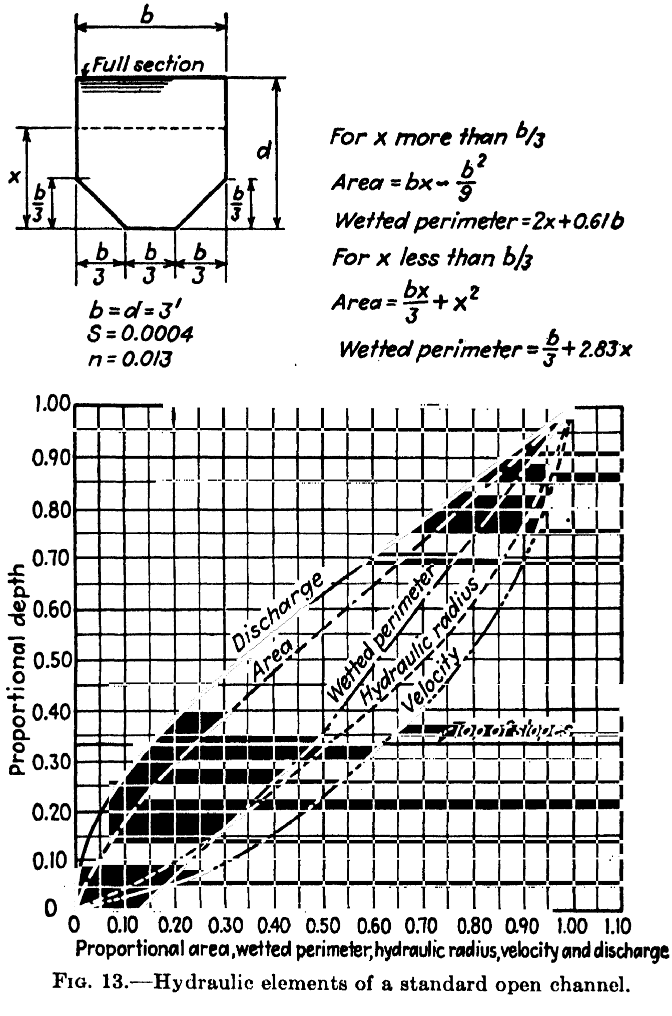


FIG. 13.—Hydraulic elements of a standard open channel.

The head requirements of open channels of rectangular, trapezoidal, or other form can be readily determined by means of Scobey's diagram for the solution of Kutter's formula (Appendix A) after computing the hydraulic radius (R equals area of section divided by the wetted perimeter) from the dimensions of the channel section. Figure 13 gives the hydraulic elements of a rectangular channel with the bottom formed by 45-deg-angle fillets in the corners for one-third the width.

The head losses in conduits other than friction may be determined with sufficient accuracy by considering them in terms of the velocity head. Detailed analyses of

these head losses have been made by King,[1] from which the following data have been taken:

1. Entrance losses

   Inward projecting, $h_1 = 0.78 \dfrac{V^2}{2g}$

   Sharp corners, $h_1 = 0.5 \dfrac{V^2}{2g}$

   Slightly rounded, $h_1 = 0.23 \dfrac{V^2}{2g}$

   Bellmouth, $h_2 = 0.04 \dfrac{V^2}{2g}$

   See also Table 1.
2. Sudden enlargements (Table 2)
3. Gradual enlargements

   $h_1 = K \dfrac{V^2}{2g}$. (See Table 3 for values of K)
4. Sudden Contractions (Table 4)
5. Obstructions, such as valves partly open (see Table 5)
6. Bends, tees, and Y branches

   For losses in 90-deg bends (see Table 6)

   For 45-deg bends, use three-fourths of the losses for 90-deg bends

   For 22.5-deg bends, use one-half of the losses for 90-deg bends

   For tees, use the losses for a 90-deg bend with zero radius

   For Y branches use three-fourths of the losses for a tee

Aerated Conduits. Frequently, diffused air is blown up through the sewage in conduits to prevent deposits of solids with low velocities or high solids content. Two types of aeration have been used, the ridge-and-furrow type and the spiral-flow type.

There is meager data on the effect of air agitation on the head losses in conduits. Some tests at Milwaukee were reported by D. W. Townsend[2] from which it was concluded that Kutter's coefficient n might range as follows:

Velocity in Aerated Channel	Value of n in Kutter's Formula[1]
1.3	0.034
1.0	0.039
0.9	0.043

[1] Scaled from Fig. 5, *Trans. A. S. C. E.*, **100**, 562, 1935.

TABLE 1.—LOSS OF HEAD, H_0, AT ENTRANCE TO PIPES

Condition at entrance	Velocity, fps														
	2	3	4	5	6	7	8	10	12	14	16	18	20	25	30
Inward projecting	.05	.11	.19	.30	.44	.59	.78	1.21	1.75	2.38	3.10	3.93	4.85	7.58	10.91
Sharp-cornered	.03	.07	.12	.19	.28	.38	.50	.78	1.12	1.52	1.99	2.52	3.11	4.86	7.00
Slightly rounded	.01	.03	.06	.09	.13	.13	.23	.36	.51	.70	.92	1.16	1.43	2.24	3.22
Bellmouth	.00	.01	.01	.02	.02	.08	.04	.06	.09	.12	.16	.20	.25	.39	.50

[1] King, H. W., "Handbook of Hydraulics," 3d ed., pp. 190–193, McGraw-Hill Book Company, Inc., 1939.

[2] *Trans. A. S. C. E.* **100**, 558, 1935.

Table 2.—Loss of Head (H_2) Due to Sudden Enlargement in Pipes

$\frac{d_2}{d_1}$ = ratio of diameter of larger pipe to diameter of smaller pipe

v = velocity in smaller pipe

$\frac{d_2}{d_1}$	Velocity, v, fps												
	2	3	4	5	6	7	8	10	12	15	20	30	40
1.2	0.01	0.01	0.02	0.04	0.06	0.07	0.10	0.14	0.21	0.32	0.55	1.20	2.08
1.4	0.02	0.04	0.06	0.10	0.14	0.18	0.23	0.36	0.51	0.78	1.36	2.96	5.14
1.6	0.02	0.05	0.09	0.14	0.20	0.28	0.36	0.55	0.78	1.19	2.07	4.50	7.82
1.8	0.03	0.07	0.12	0.18	0.26	0.35	0.45	0.70	0.99	1.52	2.64	5.74	9.97
2.0	0.04	0.08	0.14	0.22	0.31	0.41	0.53	0.81	1.16	1.77	3.08	6.71	11.65
2.5	0.05	0.10	0.17	0.27	0.38	0.51	0.66	1.01	1.44	2.20	3.83	8.34	14.48
3.0	0.05	0.11	0.19	0.30	0.42	0.57	0.74	1.13	1.60	2.46	4.27	9.29	16.14
4.0	0.06	0.12	0.22	0.33	0.47	0.63	0.82	1.25	1.78	2.76	4.73	10.30	17.90
5.0	0.06	0.13	0.23	0.35	0.49	0.66	0.85	1.31	1.86	2.85	4.95	10.79	18.73
10.0	0.06	0.14	0.24	0.37	0.52	0.70	0.91	1.39	1.97	2.96	5.25	11.44	19.87
∞	0.06	0.14	0.24	0.37	0.53	0.71	0.92	1.42	2.01	3.09	5.36	11.66	20.26

Table 3.—Values of K_2 for Determining Loss of Head Due to Gradual Enlargements in Pipes from the Formula $H_2 = K_2 \frac{v^2}{2g}$

$\frac{d_2}{d_1}$ = ratio of diameter of larger pipe to diameter of smaller pipe

Angle of cone is twice the angle between the axis of the cone and its side

$\frac{d_2}{d_1}$	Angle of cone													
	2°	4°	6°	8°	10°	15°	20°	25°	30°	35°	40°	45°	50°	60°
1.1	0.01	0.01	0.01	0.02	0.03	0.05	0.10	0.13	0.16	0.18	0.19	0.20	0.21	0.23
1.2	0.02	0.02	0.02	0.03	0.04	0.09	0.16	0.21	0.25	0.29	0.31	0.33	0.35	0.37
1.4	0.02	0.03	0.03	0.04	0.06	0.12	0.23	0.30	0.36	0.41	0.44	0.47	0.50	0.53
1.6	0.03	0.03	0.04	0.05	0.07	0.14	0.26	0.35	0.42	0.47	0.51	0.54	0.57	0.61
1.8	0.03	0.04	0.04	0.05	0.07	0.15	0.28	0.37	0.44	0.50	0.54	0.58	0.61	0.65
2.0	0.03	0.04	0.04	0.05	0.07	0.16	0.29	0.38	0.46	0.52	0.56	0.60	0.63	0.68
2.5	0.03	0.04	0.04	0.05	0.08	0.16	0.30	0.39	0.48	0.54	0.58	0.62	0.65	0.70
3.0	0.03	0.04	0.04	0.05	0.08	0.16	0.31	0.40	0.48	0.55	0.59	0.63	0.66	0.71
∞	0.03	0.04	0.05	0.06	0.08	0.16	0.31	0.40	0.49	0.56	0.60	0.64	0.67	0.72

Bar Screens. Hydraulic computations for the design of bar screens include the determination of the screen area and the head loss to be allowed for the operation of the screen. Bar screens may include iron bars 2 to 3 in. wide and ¼ to ⅜ in. thick spaced ½ to 2 in. apart. The more common spacing is ½ to ¾ in. for mechanically cleaned screens and 1 to 1½ in. for manually cleaned units.

The required area of submerged screen surface may be determined by assuming a velocity of flow through the openings (when clean) of 2.0 fps for average sewage flows

TABLE 4.—LOSS OF HEAD (H_3) DUE TO SUDDEN CONTRACTION

$\frac{d_2}{d_1}$ = ratio of larger to smaller diameter

v = velocity in smaller pipe

$\frac{d_2}{d_1}$	Velocity, v, fps												
	2	3	4	5	6	7	8	10	12	15	20	30	40
1.1	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.06	0.09	0.15	0.29	0.75	1.49
1.2	0.00	0.01	0.02	0.03	0.04	0.06	0.07	0.12	0.18	0.28	0.54	1.38	2.74
1.4	0.01	0.02	0.04	0.07	0.10	0.13	0.17	0.27	0.40	0.65	1.14	2.68	4.98
1.6	0.02	0.04	0.06	0.10	0.14	0.20	0.26	0.40	0.67	0.89	1.56	3.44	5.97
1.8	0.02	0.05	0.08	0.13	0.19	0.25	0.33	0.51	0.73	1.12	1.92	4.05	6.72
2.0	0.02	0.05	0.09	0.14	0.21	0.28	0.36	0.55	0.79	1.19	2.06	4.28	7.09
2.2	0.02	0.06	0.10	0.15	0.22	0.30	0.38	0.59	0.84	1.28	2.20	4.56	7.41
2.5	0.03	0.06	0.10	0.16	0.23	0.31	0.40	0.62	0.88	1.34	2.30	4.76	7.71
3.0	0.03	0.06	0.11	0.17	0.24	0.32	0.42	0.65	0.92	1.40	2.41	4.98	8.11
4.0	0.03	0.06	0.12	0.18	0.25	0.34	0.44	0.69	0.97	1.48	2.53	5.24	8.48
5.0	0.03	0.07	0.12	0.18	0.26	0.35	0.46	0.70	1.00	1.52	2.60	5.36	8.67
10.0	0.03	0.07	0.12	0.19	0.27	0.36	0.47	0.72	1.02	1.56	2.68	5.56	9.06
∞	0.03	0.07	0.12	0.19	0.27	0.36	0.47	0.72	1.03	1.58	2.71	5.68	9.36

TABLE 5.—LOSS OF HEAD (H_4) DUE TO VALVES OR OBSTRUCTIONS IN PIPES

$\frac{A}{A_0}$ = ratio of area of pipe to area of opening in obstruction

v = velocity of water in the pipe

$\frac{A}{A_0}$	Velocity, v, fps												
	1	2	3	4	5	6	7	8	10	12	15	20	30
1.05	0.00	0.01	0.01	0.03	0.04	0.06	0.08	0.10	0.15	0.23	0.36	0.61	1.37
1.1	0.00	0.01	0.03	0.05	0.07	0.11	0.15	0.19	0.30	0.43	0.67	1.20	2.70
1.2	0.01	0.03	0.06	0.10	0.16	0.24	0.32	0.42	0.65	0.94	1.47	2.61	5.88
1.4	0.01	0.06	0.13	0.24	0.37	0.54	0.73	0.95	1.49	2.14	3.35	5.95	13.35
1.6	0.02	0.10	0.22	0.38	0.60	0.86	1.17	1.53	2.39	3.44	5.38	9.56	21.52
1.8	0.03	0.13	0.30	0.54	0.84	1.22	1.66	2.16	3.38	4.87	7.64	13.21	30.42
2.0	0.04	0.17	0.38	0.67	1.05	1.51	2.06	2.69	4.20	6.05	9.46	16.82	37.84
2.2	0.05	0.20	0.46	0.81	1.27	1.83	2.49	3.26	5.09	7.33	11.45	20.35	45.78
2.5	0.06	0.25	0.56	1.00	1.56	2.24	3.05	3.98	6.23	8.97	14.01	24.91	56.05
3.0	0.08	0.31	0.71	1.26	1.97	2.83	3.85	5.03	7.87	11.33	17.70	31.47	70.80
4.0	0.10	0.42	0.94	1.68	2.62	3.78	5.14	6.71	10.49	15.10	23.60	41.95	94.40
5.0	0.12	0.50	1.12	1.99	3.11	4.48	6.10	7.97	12.46	17.94	28.02	49.82	112.10
6.0	0.15	0.58	1.31	2.33	3.64	5.25	7.14	9.33	14.58	20.99	32.79	58.30	131.18
7.0	0.16	0.65	1.46	2.59	4.05	5.83	7.93	10.36	16.19	23.31	36.41	64.74	145.67
8.0	0.18	0.72	1.59	2.82	4.41	6.35	8.64	11.28	17.63	25.39	39.66	70.51	158.65
9.0	0.19	0.78	1.74	3.10	4.84	6.97	9.49	12.39	19.37	27.89	43.57	77.47	174.31
10.0	0.21	0.84	1.89	3.36	5.26	7.57	10.30	13.45	21.02	30.27	47.30	84.09	189.20

TABLE 6.—LOSS OF HEAD, H_b, IN FEET, DUE TO 90-DEG BENDS

R ft	Velocity, v, fps												
	2	3	4	5	6	7	8	10	12	15	20	30	40
0.0	0.06	0.16	0.31	0.50	0.76	1.08	1.45	2.40	3.62	5.98	11.42	28.44	54.32
0.25	0.03	0.07	0.14	0.22	0.34	0.48	0.65	1.08	1.61	2.66	5.08	12.64	24.14
0.50	0.02	0.05	0.09	0.15	0.23	0.32	0.43	0.71	1.07	1.77	3.38	8.43	16.97
1.	0.01	0.03	0.06	0.10	0.16	0.22	0.30	0.49	0.74	1.22	2.33	5.79	11.07
2.	0.01	0.03	0.06	0.09	0.14	0.19	0.26	0.43	0.65	1.08	2.06	5.12	9.78
3.	0.01	0.03	0.05	0.09	0.14	0.19	0.26	0.43	0.64	1.06	2.02	5.03	9.62
4.	0.01	0.03	0.05	0.09	0.13	0.19	0.25	0.42	0.63	1.05	2.00	4.97	9.50
5.	0.01	0.03	0.05	0.09	0.13	0.19	0.25	0.41	0.63	1.03	1.97	4.91	9.38
6.	0.01	0.03	0.05	0.09	0.13	0.18	0.25	0.41	0.62	1.02	1.95	4.85	9.26
7.	0.01	0.03	0.06	0.09	0.14	0.19	0.26	0.43	0.65	1.07	2.05	5.10	9.74
8.	0.01	0.03	0.06	0.10	0.15	0.22	0.29	0.48	0.73	1.20	2.29	5.71	10.91
10.	0.02	0.04	0.08	0.13	0.19	0.27	0.36	0.60	0.90	1.48	2.83	7.06	13.48
15.	0.02	0.06	0.11	0.18	0.27	0.38	0.52	0.85	1.28	2.12	4.04	10.07	19.23
20.	0.03	0.07	0.14	0.22	0.34	0.48	0.64	1.06	1.60	2.64	5.05	12.57	24.02
25.	0.03	0.08	0.15	0.25	0.37	0.52	0.71	1.17	1.76	2.91	5.55	13.82	26.40

and 3.0 fps for maximum rates of sewage flow. The larger of the two areas thus obtained is the controlling area, but it may be increased for small installations to provide a minimum working width of 18 in. for manually cleaned units and 2 to 3 ft for mechanically cleaned units. In general, the effective screen area should be about twice the cross sectional area of the incoming sewer.

Head losses through bar screens will vary, depending upon the amount of coarse material in the sewage and the frequency of cleaning. A computation may be made of the head loss for a clean screen by the formula

$$h_1 = \frac{V^2 - v^2}{2g} \times \frac{1}{0.7}$$

or

$$h_1 = \frac{0.5V^2}{2g} + \frac{V^2 - v^2}{2g}$$

in which V and v represent, respectively, the velocity between the bars and in the approach channel and h_1 is the head loss due to a clean bar screen.

In practice for large installations, the screen may be kept clean by continuous operation of the cleaning mechanism, but for the more usual size of installation, the cleaning mechanism may operate intermittently, in which case some arbitrary allowance is usually included to permit some backing up, due to clogging of the screen, and a high water overflow is provided. A reasonable amount for this allowance is about 3 to 6 in.

Fine Screens. Screens with openings less than ¼ in. in width are sometimes used. These may be the disk (Rienschwurl), the drum (Dorr Company or Tark), or the band (Rex) type. The loss of head through the disk type has been estimated[1] by the orifice formula with a coefficient of discharge of 0.4 which results in the following:

$$h = 9.7 \frac{(Q)^2}{(s)}$$

[1] METCALF and EDDY, "American Sewerage Practice," vol. III, 3d ed., p. 266, McGraw-Hill Book Company, Inc., 1935.

where Q = quantity of sewage, cfs.
s = area of submerged openings, sq ft.
h = head loss, ft.

The hydraulic losses in fine screens are different for each type and are determined by test. Usually, records of such tests are furnished by the manufacturer.

Comminutors. The comminutor is a slotted drum rotating in the sewage channel, which serves to reduce coarse material in lieu of removal by a bar screen (Fig. 14). Sewage flows through the slots, out of the bottom of the drum, and then into the downstream channel. Cutters on the drum pass through a cutting comb to shear all

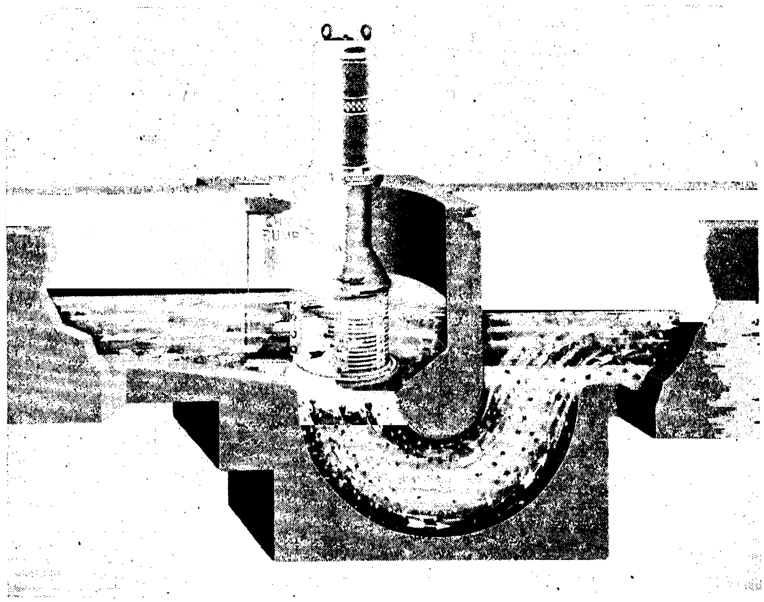


FIG. 14.—Chicago Pump Company comminutor.

material that is engaged by the cutters. This material is sheared to small pieces that pass through the $\frac{3}{16}$-, $\frac{1}{4}$-, or $\frac{3}{8}$-in. slots.

Hydraulic considerations include conduit design to maintain sufficient velocity to hold coarse material in suspension and sufficient velocity through the slots of the comminutor to carry coarse material to the face of the drum where it can be shredded by the cutters.

The head loss for a comminutor has been considered as the differential in sewage levels upstream and downstream of the comminutor. The manufacturer has furnished data on head losses and capacities as follows:

Size, in.	Range of capacity, mgd		Range of head loss, in.	Hp of motor furnished	Speed of drum, rpm
	Minimum present flow	Maximum future flow			
7	0.0	0.35	2– 7	¼	56
10*A*	0.3	1.1	4–10	½	48
15*A*	0.4	2.3	4–10	¾	37.5
25*M*	1.0	6.0	4–10	1½	23
25*A*	1.0	11.0	4–15	1½	23
36*A*	1.5	24.0	4–15	2	15

Grit Chambers. The hydraulic computations for grit chambers include the design of the velocity-control device, such as a proportional weir, and the head losses required for operation.

One important hydraulic factor in grit chamber design is the control of the depth of flow so as to maintain a velocity of about 1.0 fps for all variations in rates of flow.

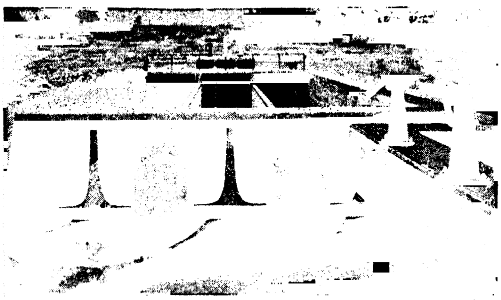


FIG. 15.—Outlet end of grit chamber, Springfield, Ill. (*From Metcalf and Eddy.*)

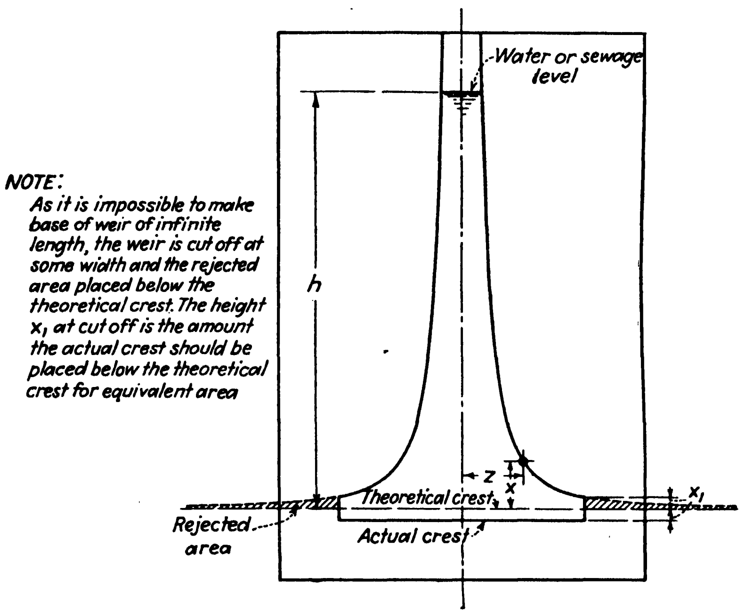


FIG. 16.—Rettger proportional-flow weir.

This is often done by providing a special weir at the outlet of the grit chambers (Fig. 15), so designed as to proportion the depth of flow through the channels to the rate of flow and thus maintain a fairly uniform velocity of flow. The Rettger proportional-flow weir (Fig. 16) has been used for this purpose in the design of grit chambers.

The discharge for this weir may be computed by the formula

$$Q = C\tfrac{1}{2}\sqrt{2g}\,b\pi h$$

where Q = discharge, cfs.
h = depth of flow above the weir crest.
b = weir constant.
C = coefficient of discharge.

Experimental values of C are given in Table 7. For all practical computations, a value of 0.62 may be used. The weir constant b may be computed after maximum values of Q and h are selected. The dimensions of the curves for the sides of the weir opening may be determined by the formula

$$z = \frac{b}{2\sqrt{x}}$$

where x equals the vertical distance above the theoretical crest of the weir, and z the distance from the center line of the opening to the side of the opening.

TABLE 7.—DISCHARGE COEFFICIENTS FOR PROPORTIONAL WEIRS
(Based on Cornell Tests, 1915[1])

Head on weir, ft	Discharge coefficient		
	Experimental results		Use for design computations
	Weir A	Weir B	
0.2	0.656	0.650	**0.65**
0.4	0.628	0.628	**0.63**
0.6	0.617	0.621	**0.62**
0.8	0.610	0.620	**0.62**
1.0	0.606	0.620	**0.62**
1.5	0.607	0.622	**0.62**
2.0	0.608	0.623	**0.62**
2.5	0.610	0.624	**0.62**
3.0	0.611	0.627	**0.62**

[1] *Eng. News*, **74**, p. 1018, 1915.

Head losses in the operation of grit chambers include the losses through the control gates and the inlet and outlet channels. These losses may be computed as indicated for conduit losses. Where velocity control is by means of proportional weirs, there will be a substantial head differential on the two sides of the weir. Sometimes the velocity control is produced by head variations through the measuring device, such as a Venturi flume, following the grit chamber. In some cases, the loss of head through the grit chamber can be coordinated with the depth of flow in the incoming sewer, as at Decatur (Fig. 17).

Sewage Meters. Some head is required for the main sewage meter often located directly following the grit chamber. A weir, Venturi meter, or Venturi flume may be used. The proportional weir for velocity control in the grit chamber has been used.

Head losses for a weir include the discharge head obtained by an appropriate weir formula plus 2 or 3 in. free fall.

Venturi-meter losses depend upon the meter design and the ratio of the inlet to the throat velocity. Builders Iron Foundry suggest the following losses in terms of the velocity head at the throat of the meter:

Meter	Head loss, ft
Long tube	$0.108 \frac{V^2}{2g}$
Short tube	
Throat to inlet ratio	
$\frac{4}{12}$	$0.27 \frac{V^2}{2g}$
$\frac{6}{12}$	$0.18 \frac{V^2}{2g}$
$\frac{7}{12}$	$0.143 \frac{V^2}{2g}$
$\frac{8}{12}$	$0.14 \frac{V^2}{2g}$
$\frac{9}{12}$	$0.135 \frac{V^2}{2g}$

The Simplex Valve & Meter Company suggests the following formula for the computation of the losses in a Venturi meter: $H_L = K \times V_t^2$. In this formula, H_L is the

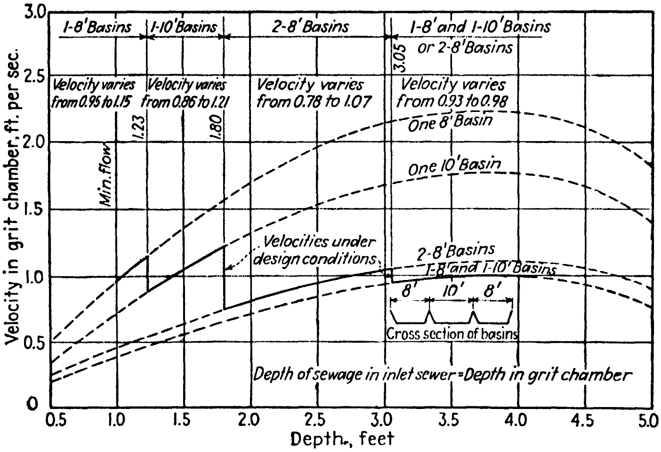


FIG. 17.—Velocities in grit chamber, Decatur, Ill.

head loss in feet and V_t is the velocity in the throat of the Venturi tube, and the following values of K are suggested for several sizes of the main in which the meter tube is placed:

Size Main Diameter Tube, in.	K
6	0.0021
10	0.00196
16	0.00190
20	0.00185
24	0.00180
30	0.00175
36	0.00172
42	0.00170
48	0.00169
60	0.00166

These coefficients apply to a concentric long pattern having an upstream angle of 10.5 deg. and a downstream angle of 2.5 deg. For eccentric or flat invert designs of Venturi tubes, Simplex Valve & Meter Company suggests the following values of K:

For short eccentric tubes, $K = 0.0044$
For long eccentric tubes, $K = 0.002$

The discharge of a Venturi meter is determined by the differential between the inlet and throat pressure heads (as explained in Appendix A). An approximation of the head loss is 15 per cent of the head differential.

The Parshall flume, an improved type of Venturi flume, was developed for measurement of water for agricultural irrigation. It is a reasonably accurate water-measuring device, although not so accurate as a Venturi meter, and it has the advantage of relatively low cost. The Parshall flume has been used as a measuring device in many sewage-treatment plants where low cost was an important consideration. It has proven a satisfactory measuring device and also very useful for controlling velocities in grit chambers.

Venturi meters and Parshall flumes, when provided with appropriate devices, may be used also to control the rate of application of chemicals to sewage in direct proportion to the rate of sewage flow.

Sedimentation Tanks. Head losses for sedimentation tanks include the following items:

1. Control-chamber losses
2. Losses due to distribution devices
3. Conduit losses
4. Velocity head losses at inlets
5. Head on outlet weir and free fall below weir.

The control chamber often comprises a small chamber with outlets closed by sluice gates. The head losses are a portion of the velocity head on entrance to the chamber through a sluice gate, additional proportions of the velocity head if changes in direction of flow are necessary, and further losses through outlet openings.

In any project comprising two or more tank units, the flow of sewage should be equally divided between the several tanks. Sometimes measuring devices are included, such as Venturi tubes, pitot tubes, or weirs, by which the flow to each tank is actually measured and the division controlled with considerable accuracy. The considerable hydraulic losses through these devices can be computed from the data given elsewhere.

The conduit losses may be computed by methods already described. Thus, the losses in the inlet conduit might include about one-half the velocity head with a sharp-edged entrance ($0.5\ V^2/2g$), plus the friction loss along the length of the conduit, plus the full velocity head on discharge into the sedimentation tank.

The head required for the outlet weir will depend upon the flow per unit length of weir and may be computed by the Francis formula

$$Q = 3.33H^{3/2}$$

Thus, at Buffalo, for a maximum flow of 570 mgd (882 cfs) four round sedimentation tanks were included, each having a diameter of 160 ft and 500 lin ft of effluent weir. This amounts to 0.441 cfs per foot of effluent weir and requires, according to the preceding formula, a head of 0.26 ft. The present tendency is to hold the flow over a unit length of weir down to about 15,000 gal per day per ft of weir.

In addition to the head over the weir, some free fall should be allowed, so as not to submerge the weir. A reasonable amount of the free fall is 0.2 to 0.5 ft, depending in part on the total head available.

The computed hydraulic losses for a number of sedimentation tanks are given in Table 8.

Trickling Filters. Hydraulic computations for trickling filters usually involve consideration of the following major factors:

1. Area, depth, and shape of the filter
2. Head requirements to provide satisfactory operation

Trickling filters are either rectangular with fixed nozzles for sewage distribution or round with rotary distributors and range in depth from 3 to 10 ft, a general average being about 6 ft. The volume of sewage per unit of filter area by which to determine the filter size may range from 2.0 up to 30.0 mgd/acre. The greater number of older filters were designed for a rate of application of 2.0 to 3.0 mgd/acre. Many recently constructed trickling filters have been designed for higher rates.

The head losses of trickling filters are caused by three main items, as follows:

1. Hydraulic losses in the distribution system and dosing tank
2. Depth of filter bed
3. Hydraulic losses in the collection and underdrainage system and filter floor

TABLE 8.—TYPICAL HYDRAULIC LOSSES FOR SEDIMENTATION TANKS

Item	Buffalo, N.Y., 570 mgd	Appleton, Wis., 30 mgd	Danville, Ill., 15 mgd	Eau Claire, Wis., 15 mgd
Inlet chamber	0.13	0.10		0.42
Inlet conduit	1.04	0.15	0.55	
Riser pipe[1]	0.59			1.12
Fall across tank				
Head on weir	0.25	0.56	0.07	0.30
Free fall below weir	1.48	−0.56[3]		
Collecting conduit	0.14	0.14	0.29	0.70
Outlet connection[2]	0.49	0.15		
Total	4.12	0.54	0.91	2.54

[1] For round center feed tanks.
[2] Connection from individual tank to main outlet conduit.
[3] Submerged weir.

Distribution System and Dosing Tank. The intermittent but uniform distribution of settled sewage over the surface of a filter of fixed-nozzle type is usually accom-

TABLE 9.—HEAD REQUIREMENTS FOR SPRAY NOZZLES—TYPICAL PLANT

Plant	Size, acres	Actual heads, ft			Total head, ft	
		Depth of filter	Distribution system[1]	Underdrainage[2]	Actual	Adjusted to filter 6 ft deep
Akron, Ohio	14.0	10.0	9.23	1.77	21.00	17.00
Sioux Falls, S. D	3.77	8.5	8.95	1.25	18.70	16.20
Fort Worth, Tex	3.18	8.2	7.80	2.05	18.05	15.85
Decatur, Ill	3.0	6.0	6.55	0.93	13.48	13.48
Bloomington, Ill	2.5	8.0	8.83	2.98	19.81	17.81
Urbana, Ill	1.6	10.0	8.36	2.28	20.64	16.64
Elgin, Ill	1.5	8.0	7.79	2.48	18.27	16.27
Downers Grove, Ill	0.25	8.0	8.00	1.00	17.00	15.00
Fargo, N. D	1.6	8.0	4.74	2.18	14.92	12.92
Average (omitting Fargo, N.D.)	.....	....	....	....	.....	16.03

[1] High water level in dosing tank to stone surface.
[2] Above water level in final settling tanks.

plished by a siphon placed in a dosing tank. The siphon discharges intermittently as the dosing tank fills and empties. The head requirements include the following losses:

1. Dosing tank } (Not required for high-rate filters)
2. Siphon }
3. Friction through connecting pipe
4. Rotary distributor
5. Friction and velocity head through distribution pipes (fixed or rotating)
6. Discharge head on nozzles or orifices
7. Height of nozzles or orifices above surface of filter

Typical head losses for trickling filters with fixed spray nozzles are given in Table 9. The head requirements with rotary distributors are usually less than for fixed spray nozzles (Table 10).

TABLE 10.—HEAD REQUIREMENTS FOR ROTARY DISTRIBUTORS—TYPICAL PLANTS

Plant	Filter units		Head required, ft
	Number	Diameter	Distribution system[1]
Clinton, Ill.	1	140	2.5
Dayton, Ohio	20	165	4.83
Fort Dodge, Iowa	4	151	3.5
Portage, Wis.	1	140	2.25
Plymouth, Wis.	1	140	3.0
Highwood, Ill.	1	125	3.0
Hot Springs, Ark.	1	108	2.75
Marion, Iowa	1	100	2.00
Richmond, Ky.	1	134	3.00
Average	..	...	2.98[2]

[1] High water level in dosing tank to stone surface, including the distance from stone surface to the center of the rotating arms.
[2] Allowing 2.0 ft for underdrainage plus 6 ft of media gives an average total head of 11.0 ft.

The hydraulic computations required to determine the foregoing losses are as follows:

1. *Dosing Tank.* The dosing-tank loss is the drop in average level during the time required to bring the distribution system into full operation. This depends upon the area of the dosing tank which, in turn, depends upon the dosing cycle. Operating experience indicates a minimum dosing time for fixed nozzle distribution of about 3 min. The dosing loss is computed from the formula suggested by Pacific Flush Tank Company

$$DTL\ (\text{in.}) = \frac{58.4d^4}{A \times \text{gpm}}$$

where A = area of the dosing tank, sq ft.
d = diameter of the siphon, in.
gpm = rate of discharge of the rotary distributor at maximum head.
58.4 = an assumed constant, determined by test.

Usually, records of such tests are furnished by the manufacturer.

2. *Siphon.* The siphon loss will depend upon the size and design of the siphon. For the frequently used trapless siphon, the loss is obtained by the following formula suggested by Pacific Flush Tank Company:

$$h_s = 2.78\,\frac{V^2}{2g}$$

where h_s equals siphon loss in feet and V equals the velocity in feet per second through the siphon.

3. *Connecting Pipe.* The loss through the connecting pipe from the siphon to the vertical shaft of the distributor at the center of the filter is a pipe friction loss and may be so computed, a coefficient n in Kutter's formula of about 0.015 being used.

4. *Rotary Distributor.* This loss includes the losses in the center column, the distribution arms, and the orifices and depends upon design factors, such as the special construction required to control the rate of flow into the rotating arms, the number of arms, and the design of discharge nozzles or orifices.

The center column loss can be obtained by actual tests, usually by the manufacturers. The general magnitude of these losses in terms of the equivalent additional length of connecting pipe is indicated by data from two manufacturers, as follows:

Diameter of connecting pipe, in.	Equivalent additional length of connecting pipe in feet to give same loss as the center column loss			
	Company *A*	Company *B* For different arm sizes, in.		
		4	5	6
6	15	9		
10	40	92	42	22
14	50	463	203	106

The losses in the distributor arm include friction and velocity-head losses. The friction losses are computed as for straight pipe, short lengths being used for each computation to approximate the decrease of flow from orifice to orifice. Experience indicates velocities in the arms not over 4 fps for best operation. The velocity head loss depends upon the recovery of the velocity head, which varies from about 80 per cent for arms with uniform diameter to 0 per cent for arms tapered to maintain a uniform velocity through the length of the arm. The centrifugal force of rotation also adds a pressure head which partly offsets the friction and velocity-head losses. The head due to rotation is computed by the following formula:

$$h_c = 0.00051D^2N^2$$

where h_c is the head in inches, due to rotation, D the diameter of the distributor in feet, and N the rate of rotation in revolutions per minute which varies from 2.0 or more rpm for smaller to 0.5 or less for larger distributors.

The head required for the discharge nozzles or orifices of the rotary distributor may be computed by the formula

$$Q = Ca\sqrt{2gh}$$

where Q = discharge, cfs.
a = area, sq ft.
h = head, ft, for each orifice or the average head for a small number of orifices.
c = coefficient of discharge depending upon the design of the orifice, ranging from about 0.6 to 0.8.

5. *Height of Arms above Filter.* The arm-clearance loss, *i.e.*, the distance from the surface of the filter to the center line of the distributor arms, depends upon the diameter of the filter and the details of the distributor. It is a static loss or clearance of approximately 9 in. per 100 ft of filter diameter with a minimum of about 6 in.

The foregoing procedure will permit an estimate of the head losses in the design of a rotary distributor. In practice, it is usual to state the maximum, average, and

minimum rates of sewage flow, the filter size, and the total head available from the high water level in the dosing tank to the surface of the filter and then to require proposals from manufacturers for distributors to operate within the capacity and head limitations.

Fixed-nozzle Distribution. The hydraulic computations for the design of a fixed-nozzle distribution system comprise (1) the determination of head losses through dosing tank, siphon, and distribution piping; (2) the size, number, and spacing of nozzles; (3) the size and shape of dosing tank; and (4) the slope of the filter to compensate for the difference in the head on the far and near nozzles.

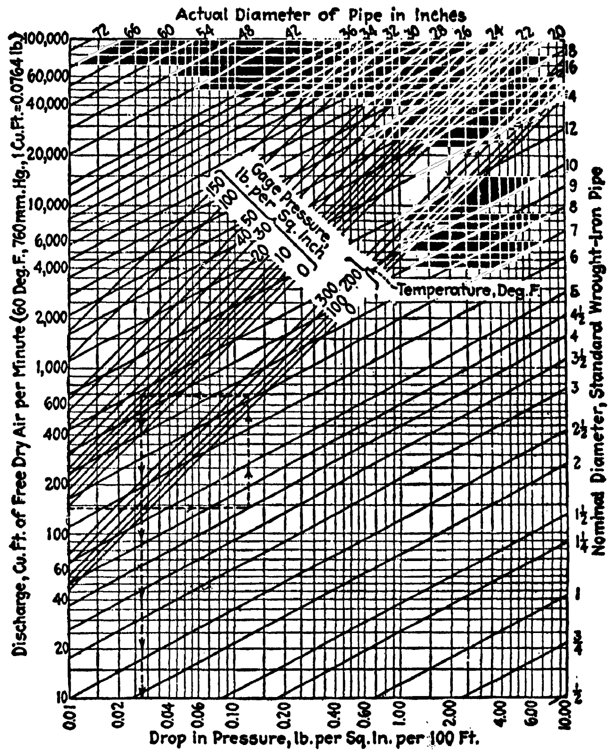


FIG. 18.—Flow of air in circular pipes based on Fritzsche formula. (*From Metcalf and Eddy.*)

With the total discharge of the nozzles known, the several head losses for the dosing tank, siphon, and distribution piping can be computed. However, the nozzle discharge depends upon the net head on the nozzle which, in turn, depends upon the head losses in the dosing tank, siphon, and distribution piping.

Accordingly, a usual procedure is to assume, for a first trial computation, a net head on the nozzles of about 75 per cent of the total head available between the high water level in the dosing tank and the nozzle elevation.

Generally, two or more repetitions of the computation are necessary before finding the size of dosing tank and siphon, the pipe sizes and the nozzle spacing to secure, from the available head, for the most uniform distribution of sewage.

There are a number of factors affecting these hydraulic computations which may be briefly stated, as follows:

1. ***Rates of Sewage Application.*** These determine the quantity of sewage to be handled per unit of time. Thus an average application rate of 2.2 million gal/acre/day might have a maximum rate of 0.06 gpm/sq ft.
2. *The Number, Size, and Spacing of Nozzles.* The area covered by a nozzle spray is a function of the size, the design, and the head on the nozzle. Nozzles must be spaced so that the overlapping of sprays will be such as to give uniform distribution.
3. *The Size of Dosing Tank and Siphon.* The size of the dosing tank and siphon must be selected. In accordance with the empirical formula for dosing tank losses (page 1111), a siphon larger than the nozzle capacities increases the dosing-tank losses unnecessarily; and if the siphon is too small, compared with the number of nozzles, the siphon loss will increase.
4. *The Size and Shape of the Dosing Tank.* These must be selected, so that equal quantities of sewage per square foot of filter surface will result for different sewage levels in the dosing tank.
5. *Slope of the Filter.* This compensates for differences in head losses to the far and near nozzles.

Reliable hydraulic formulas and coefficients for siphons and nozzles, based upon experimental tests, have been published by the Pacific Flush Tank Company in their Catalogue 30, to which the reader is referred. This catalogue also includes tables based on the test data which facilitate solving the hydraulic problems described above. These data relate to unit rates of sewage application of 2.0 to 3.5 million gal/acre/day. Some adjustments would be required for higher rates of sewage application.

Aeration Tanks. Inlet, outlet, and conduit capacities and head losses are computed as described for sedimentation tanks. Conduits carrying mixed liquor and aeration-tank effluents are usually aerated to prevent the depositing of solids. The relatively small losses through the aeration tank can be neglected. Head variations on the effluent weirs are computed by the Francis weir formula, as described for sedimentation tanks.

The diffused-air type of aeration tank involves hydraulic computations of the capacity and the head losses of air piping and diffuser plates. The main object is to obtain uniform air diffusion throughout the length of the aeration tank. Reasonably liberal pipe sizes should be used, so that velocities do not exceed 2,000 to 3,000 fpm.

Air piping losses are computed by the Fritzsche formula (Fig. 18), which in English units may be written as follows:

$$P = \frac{1.268(t + 460)Q^{1.852}L^*}{10^6(p + 14.7)d^{4.973}}$$

where P = drop in pressure, psi per foot of piping.
p = mean gage pressure, psi.
t = mean air temperature, F.
d = pipe diameter, in.
L = length of pipe, ft.
Q = cubic feet of free air per minute at 60F and 760 mm pressure.

Bushee and Zack have reported[1] the results of experiments by the Chicago Sanitary District on air-pressure losses through piping, meters, valves, and diffuser-plate connections.

Measurements with 4-in. steel pipe gave losses practically the same as computed by the Fritzsche formula, whereas the measured losses for 10- and 24-in. Brown and Sharpe cast-iron pipe were about 25 per cent greater than the computed losses.

Measurements of air-pressure losses through Venturi meters gave results summarized as follows:

* McMillan, *Eng. News-Record*, **91**, 178, 1923.
[1] *Eng. News-Record*, **93**, 823, 1924.

Size of meter, in.	Compressed-air factors		Pressure losses	
	Pressure,[1] psi	Temperature, F	Per cent of meter differential head	Psi per 1-ft differential
10 × 5	21.5	86	18	0.078
5 × 2	22.0	80	22	0.095
4 × 2	18.7	86	21	0.091
3 × 3/4	21.5	78	24	0.104

[1] Absolute.

Bushee and Zack state that meters may be obtained to keep the total loss to 0.1 psi.

Tests of air pressure losses through a 10-in. check valve with an aluminum flap gave results as follows:

Rate of free air, cu ft/min	Loss of pressure, psi	
	Check valve and elbow	Check valve alone[1]
500	0.042	0.04
1,000	0.066	0.06
1,500	0.074	0.06
2,000	0.078	0.06

[1] Assuming loss through elbow equal to 30 diameters of straight pipe.

Air-pressure losses through elbows and bends in pipe lines in terms of equivalent length of pipe have been given as follows:[1]

Diameter, in.	Equivalent length of pipe, ft	
	Elbow	Bend
1	1.5	0.23
2	4.9	0.74
3	9.4	1.41
4	14.5	2.2
6	25.9	3.9
8	38.0	5.7
10	50.7	7.6
12	63.7	9.6
14	76.7	11.5
16	90.1	13.5
18	104	15.5
20	117	17.5
24	144	21.6

[1] METCALF and EDDY, "American Sewerage Practice," vol. III, 3d ed., p. 653, McGraw-Hill Book Company, Inc., 1935.

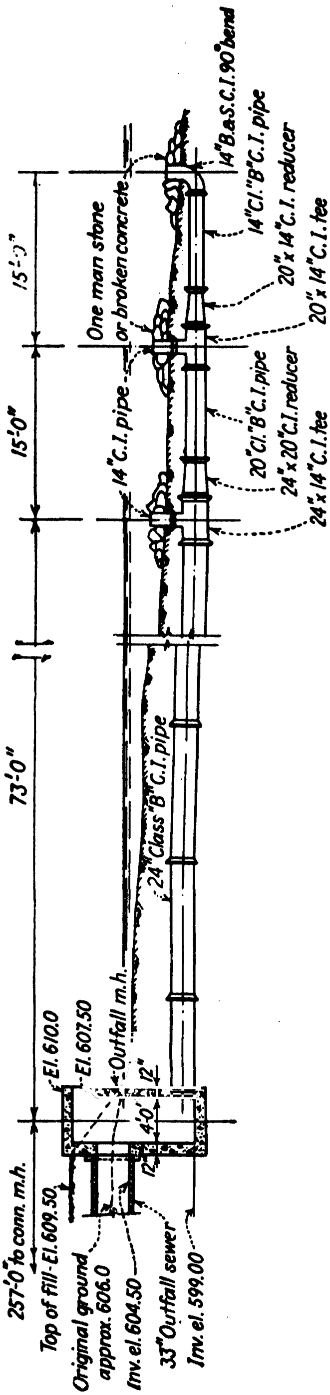


FIG. 19.—Sewage-treatment plant, Kaukauna, Wis.

Air-pressure losses through globe valves, tees, and elbows in terms of equivalent length of straight pipe have been given as follows:[1]

Nominal pipe diameter	Additional length of straight pipe, ft	
	Globe valves	Tees and elbows
1	2	2
1½	4	3
2	7	5
2½	10	7
3	13	9
3½	16	11
4	20	13
5	28	19
6	36	24
7	44	30
8	53	35
10	70	47
12	88	59
15	115	77
18	143	96
20	162	108
22	181	120
24	200	134

Outfall Losses. Hydraulic losses for sewer outfalls from treatment plants depend upon the distance to the waterway, the design of the entrance to the outfall sewer and of the outlet chamber, and whether or not there is a submerged discharge.

The friction losses in the outfall sewer leading to the waterway are computed as described in Sec. 22. except that there is no limit on the minimum velocity, as the settleable solids in the sewage have been removed. Entrance and outlet chamber losses are computed as a portion (generally 0.5 to 1.0) of the velocity head.

Usually a free fall at the plant outlet is provided equal to the rise in river or lake level above the normal water level at which the plant is designed to operate. A portion of this fall may be used to provide head to operate an overflow weir. A gate included in the outlet structure may be opened at high water levels to compensate for submergence of the overflow weir.

A common practice with partial treatment is to extend the outfall sewer to a single outlet or to a series of submerged outlets, so as to

[1] HARRIS, E. G., "Compressed Air," 2d ed., p. 170, McGraw-Hill Book Company, Inc.

distribute the effluent into the water. Figure 19 illustrates such a design. The head losses include 0.5 $V^2/2g$ at the entrance to the outfall, the pipe friction losses through the straight pipe sections, the velocity head in the pipe line, and the nozzle head required to force the discharge out through the openings, which may be computed by the formula

$$h_1 = \frac{1}{C^2} \times \frac{V_0^2}{2g}$$

where h_1 is the discharge head which is considered all lost, and V_0 the velocity through the openings. For a short-tube type of opening, the discharge coefficient C may be taken at 0.82, and the head loss is 1½ times the velocity head ($h_1 = 1.5\ V_0^2/2g$). If the opening is an orifice cut in the pipe, C may be taken at 0.6, and the head loss will be $h_1 = 2.8\ V_0^2/2g$.

The height of the overflow weir above normal water level will provide the head to force the effluent through the submerged pipe and outlets. Thus, a greater rate of flow can be discharged through the submerged outlets during low water levels than during high water levels.

Sludge Handling. In a general way, the quantities and characteristics of sludge for domestic sewage are as follows:

Treatment process	Gallons per million gallons of sewage	Approximate moisture, %
Plain sedimentation:		
Raw	2,500	95
Digested	1,200	94
Chemical treatment	3,400	95
Trickling filters:		
Final tank	400	94
Activated sludge:		
Final tank	150,000	98.5
Excess through preliminary		
Sedimentation tank	1,700	96
Digested	700	94

Sewage sludge does not constitute a homogenous fluid, and the more common hydraulic laws can be applied only approximately. The character of sludge varies from time to time as to its moisture content and the kind of solids. Present data do not permit close computations of the hydraulics in handling sludge, and empirical methods based upon experience are generally used.

Sludge-pipe sizes are selected somewhat arbitrarily. The short lines of piping used about a treatment plant range in size from 6 in. for smaller plants to 10 in. for larger plants with 8-in. pipe very often used. Long sludge lines have ranged in size from 5 to 14 in., as shown in the table at the top of page 1118.[1]

The pipe friction losses for sludge may range from 1.5 to 4.0 times the losses for water. At times, sludge lines may become partly clogged by grease or other matter.

Accordingly, sludge-pumping equipment should be sturdily built and have sufficient motor power to overcome these occasional high heads.

[1] Data taken from Metcalf and Eddy, *op. cit.*, Table 131, p. 707.

Place	Sludge pipe line		Discharge head, ft
	Length, ft	Diameter, in.	
Chicago, Ill.	74,000	14	180
Baltimore, Md.	2,252	12	50
Birmingham, England	21,000	12	116
Bradford, England	21,800	8	139
Syracuse, N. Y.	12,000	5	462

7. TOTAL HEAD REQUIRED AND TYPICAL HYDRAULIC PROFILES

It will be seen from the foregoing that some of the hydraulic losses in sewage-treatment plants are readily computed, whereas others depend on special allowances made by the engineer to provide for satisfactory operation, future additions, and the like. Therefore, the computations should be related to and checked by actual plant-operating experience. The following paragraphs illustrate the actual total head in

TABLE 11.—OVER-ALL HEAD ALLOWANCES FOR VARIOUS TYPES OF SEWAGE-TREATMENT PLANTS

Plant	Rated plant capacity, mgd		Loss of head through plant, ft[1]	
	Average flow	Maximum flow	At average flow	At maximum flow
Plain sedimentation:				
Rockford, Ill.	17.3	50.0	1.25	3.07
Oklahoma City, Okla., South Side	......	30.0	.....	3.45
Buffalo, N. Y.	150.0	570.0	4.34	6.22
Chemical treatment:				
Danville, Ill.	......	15.0	.....	3.01
Appleton, Wis.	10.0	30.0	2.80	5.34
Minneapolis-St. Paul	134.0	268.0	3.29	3.64
Trickling filters:[2]				
Hinsdale, Ill.	......	8.0	.....	23.67
Elgin, Ill.	......	10.3	.....	22.00
Oklahoma City, Okla., North Side[3]	1.45	2.7	17.65	18.04
Activated sludge:				
Springfield, Ill.	......	33.75[4]	.....	3.42
Peoria, Ill.	22.0	50.0[5]	2.50	2.88
Milwaukee, Wis.	......	231.0	.....	4.25
Chicago, North Side	175.0	370.0	3.06	5.30

[1] Generally the difference in elevation of the hydraulic gradient above the screens and the upper end of the outfall sewer.

[2] Depth of filter stone: Hinsdale, 7.5 ft; Elgin, 8 ft; Oklahoma City, 8 ft.

[3] Head allowance made for future construction of trickling filters.

[4] Maximum flow through aeration and final sedimentation tanks 22.5 mgd.

[5] Maximum flow through aeration and final sedimentation tanks 33.0 mgd.

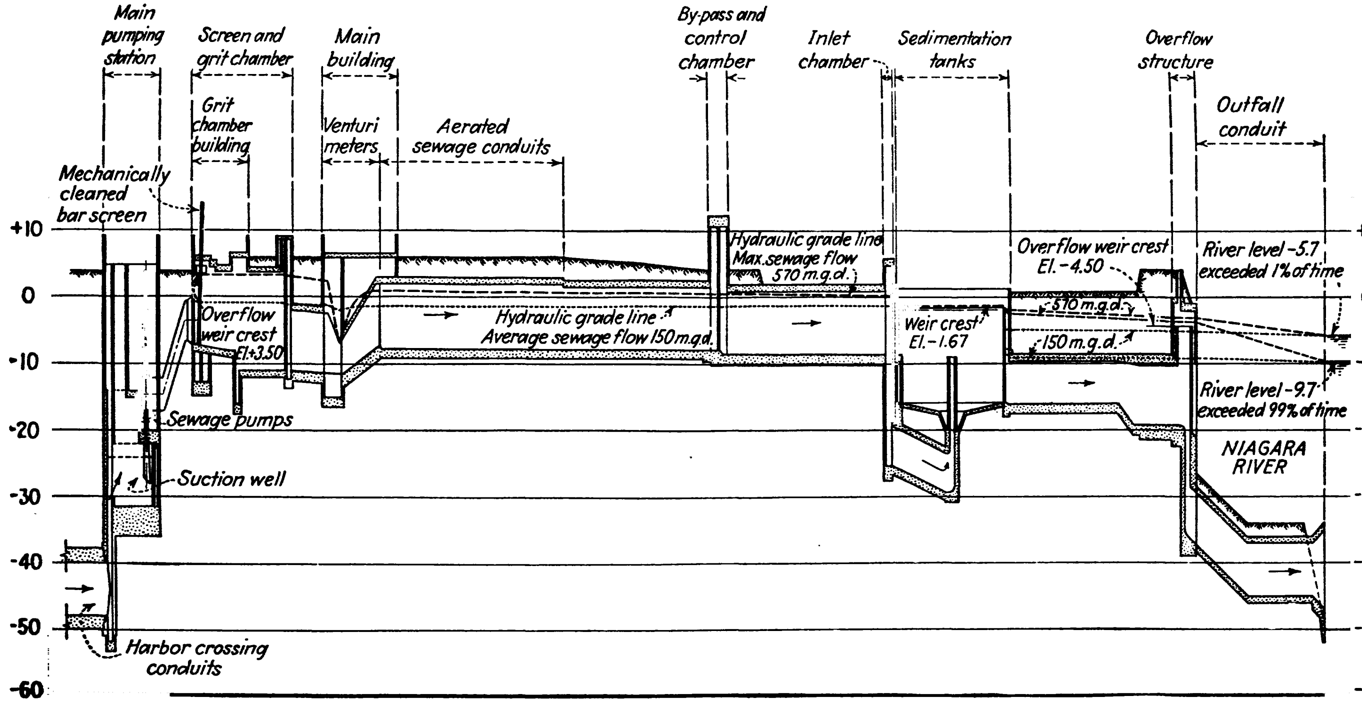


FIG. 20.—Sewage-treatment plant, hydraulic profile. Buffalo Sewage Authority, Buffalo, N. Y.

NOTES:

Buffalo City datum is 575.89 ft. above mean tide at New York (U.S. Engineers 1903 level adjustment).
With river level at −5.7 and sewage flow of 570 mgd, 330 mgd will discharge through the submerged outlet and 240 mgd through the overflow weir.
With river level at −9.7 and sewage flow of 570 mgd, 500 mgd will discharge through the submerged outlet and 70 mgd through the overflow weir.
With river level at −5.7 sewage flows up to 225 mgd will be discharged through the submerged outlet without overflow.
With river level at −9.7 sewage flows up to 470 mgd will be discharged through the submerged outlet without overflow.

sewage-treatment plants with typical hydraulic losses. The total head available is the difference in elevation between the high level of the sewage in the inlet sewer and the high level of the water surface in the receiving waterway. Where this difference does not provide sufficient head for the operation of the treatment plant, pumping is required. Hydraulic profiles are computed to determine the head required for the operation of the treatment plant and generally are the difference in elevation of the hydraulic gradient above the coarse screens at the inlet to the treatment plant and below the last structure of the plant. These over-all head requirements will vary for different types of sewage-treatment plants and for different plants of the same type.

Over-all Head Allowances (Table 11). Sewage treatment by plain sedimentation requires the lowest over-all head. The greatest losses in this type of treatment occur through the preparatory elements (screens, grit basin, etc.) and the conduits. If grit chambers are not required (as with sanitary sewers), the over-all head will be reduced by some 1.0 to 1.5 ft, depending upon the plant size.

Chemical treatment plants usually include a flocculation tank and some extra length of conduit and require some head in addition to that for plain sedimentation.

An effluent filter may be included, either constructed around the sedimentation tank or as a separate structure. Effluent filters may be operated on a 3-in. normal and about 6-in. maximum head. To provide a factor of safety, 8 or 9 in. are sometimes allowed. A separate structure would require additional head for extra conduits.

Activated-sludge treatment plants will require additional head allowances for conduits, aeration tanks, and final sedimentation tanks.

Trickling-filter treatment plants usually require more head than other types, as head is required to distribute properly the sewage over the filter, to provide for the depth of the filter and for the underdrainage system.

In some cases, a contact tank is included for chlorination, in which case moderate additional losses will occur.

A hydraulic profile illustrating the head requirements for a typical plain sedimentation plant is given in Fig. 20.

SECTION 24

HYDRAULIC MODELS

By George H. Hickox

1. DEFINITION OF A MODEL

A model may be defined as a system by whose operation the characteristics of other similar systems may be predicted. This definition is general and applies to other than hydraulic models. A model is not necessarily smaller than the system to which it is to be compared. Actually, it may even be larger. Hydraulic models, however, are generally smaller than their prototypes; in fact, the chief difficulty experienced is in making them large enough.

This definition of a model implies nothing as to its appearance, although it is generally thought of as a small-scale reproduction of the prototype. Hydraulic models usually bear a recognizable resemblance to their prototypes, although they are frequently considerably distorted. There is undoubtedly a psychological advantage in building models that look like their prototypes. Laymen are much more favorably impressed by a good appearance than by a technical description of the reasons for differences in appearance between prototype and model.

2. LAWS GOVERNING MODELS

In general, the laws governing the relationship of model to prototype are derived from the laws that govern the action of the phenomena under investigation. Occasionally the law governing action in the prototype is unknown or very poorly known. In such cases, models can be used only qualitatively.

The operation of, and results obtained from, hydraulic models may usually be transferred to the prototype by the use of model laws which may be developed from principles of dynamic similarity. The relationships most commonly used may be expressed in the form of dimensionless groups which by their numerical value characterize the type of flow under consideration. The derivation of the model laws assumes that for equal values of the dimensionless characteristics the corresponding flow patterns in model and prototype are similar. This assumption has been verified to a sufficient degree by experimental results. The dimensionless groups most commonly used in hydraulic experimentation are designated as Froude's number, Reynolds' number, and Weber's number. They are derived from a consideration of the forces of gravity, viscosity, and surface tension, respectively, in conjunction with the resisting force of inertia. For derivation of these model laws, the interested reader is referred to the publications of such writers as Rouse,[1] Chick,[2] and Dodge and Thompson.[3]

Froude's law, which may be written as

$$Fr = \frac{V^2}{Lg}$$

[1] Rouse, Hunter, "Fluid Mechanics for Hydraulic Engineers," McGraw-Hill Book Company, Inc., Chap. 1, 1938.

[2] Chick, Alton C., Dimensional Analysis and the Principle of Similitude as Applied to Hydraulic Experiments with Models. Hydraulic Laboratory Practice, A.S.M.E., 1929, pp. 775–827.

[3] Dodge and Thompson, "Fluid Mechanics," McGraw-Hill Book Company, Inc., Chap. 15, 1937.

expresses the condition for similarity of gravity and inertia forces; *i.e.*, when Froude's number is the same for both model and prototype, the ratios of gravity forces to inertia forces are the same, and the paths of flow are similar. This simple application of Froude's law supposes that neither viscosity nor surface tension have any appreciable influence on the phenomena under investigation. Froude's number is used as the basis of design and interpretation of models in which friction forces are negligible, such as spillways and other structures in which there is a rapid change in the elevation of the water surface in a short distance.

Reynolds' number may be expressed as

$$Re = \frac{VL\rho}{\mu}$$

and states the conditions under which the ratios of viscous forces to inertia forces are the same both in model and prototype. Equality of Reynolds' number for both model and prototype indicates that similar paths of flow occur when viscosity and inertia are the governing forces. Gravity and surface tension are neglected in the use of this criterion. Reynolds' number is chiefly applicable to closed systems of flow, such as pipes or conduits where there is no free water surface.

Weber's number is commonly written as

$$We = \frac{V^2L\rho}{\sigma}$$

It expresses equality of the ratios of surface tension to inertia forces in model and prototype and is useful in certain studies of surface waves, formation of drops and air bubbles, entrainment of air in flowing water, and other related phenomena. The use of Weber's number as a criterion supposes that the forces of gravity and viscosity are negligible.

In these paragraphs, the symbols have the following meanings:

V = a characteristic velocity of the system; it may be the mean, surface, or maximum velocity.
L = a characteristic linear dimension, as diameter or depth.
g = acceleration of gravity.
ρ = density of the fluid.
μ = viscosity of the fluid.
σ = surface tension of the fluid.

All these symbols must be expressed in consistent units in order that the combinations will be dimensionless.

There are other dimensionless ratios that have significance in other fields, but none of practical importance in the use of hydraulic models for river and structure problems. Dimensionless criteria involving elasticity are valuable in the study of water hammer.

The design of models is not always so simple that the application of one or the other of these moduli is sufficient. Frequently two of them must be utilized, and in this process the judgment of the engineer is required. The difficulty encountered is illustrated by considering the necessary variation of velocity with length in order to ensure similarity by each of the three criteria. If all other quantities remain the same, as is the case when the model fluid is water, Froude's number requires that $V \propto L^{1/2}$; Reynolds' number, $V \propto L^{-1}$; and Weber's number, $V \propto L^{-1/2}$. No two of these requirements can be satisfied simultaneously unless the model scale is 1:1.

3. TYPES OF MODELS

Model structures may be divided into two types: models of structures and models of rivers. These types are distinguished by the behavior of the water surface. In models of structures, there is usually a rapid change in the elevation of the water surface and a corresponding dependence on Froude's number for similarity. In

models of river channels, the change in water-surface elevation is very gradual, being governed chiefly by friction. In such models, similarity is governed by Reynolds number or by the laws of friction in river channels. Both types sometimes occur in one model, such as a sudden constriction in a river model which may be caused by bridge piers or cofferdam structures. In such cases, it is necessary to satisfy both Froude's law and a friction law at the same time, or if this is not possible, the model must be designed so that one feature or the other is unimportant.

1. Models of Structures. In the design of models of hydraulic structures, consideration must be given to (1) the results desired, (2) the space available for construction of the model, (3) the water supply, and (4) the cost. In general, the model should be built as large as possible in order to obtain the most accurate results. Space limitations frequently dictate the size, and in many cases it is further limited by the amount of water or the funds available. The fundamental relationships for models of structures based on Froude's law are

$$
\begin{array}{ll}
\text{Length scale} & = L_r \\
\text{Velocity scale} & = V_r = L_r^{1/2} \\
\text{Area scale} & = A_r = L_r^2 \\
\text{Discharge scale} & = Q_r = L_r^{5/2} \\
\text{Time scale} & = T_r = L_r^{1/2} \\
\text{Volume scale} & = B_r = L_r^3 \\
\text{Acceleration scale} & = A_r = 1
\end{array}
$$

The choice of a length scale fixes all the others. Once the length scale is selected, the size of the model and the water supply are determined. If the water supply is limited, this fixes the discharge ratio and thus the length ratio and size.

a. Spillway Discharge Coefficients. In determining discharge coefficients of spillways, it is assumed that the model faithfully represents the action of the prototype. The discharge ratio

$$Q_r = L_r^{5/2}$$

assumes that the flow patterns in model and prototype are similar and that the discharge coefficients for the model and the prototype are the same. Verification of this is still meager. Where quantitative comparisons of model and prototype are available, however, they indicate that the results obtained by a model operating under a head of at least 0.3 ft are reliable within 2 or 3 per cent. Friction in very small models is not negligible, and methods of correction have not yet been developed.

(1) *Plain Spillway without Piers or Gates.* A spillway of this type is usually so long that the effect of end contractions need not be considered. It is customary not to model the entire length of the spillway but to make a model of a short section, suppressing the end contractions. The length of crest should be at least equal to the head of the water over it in order to minimize the effect of the channel walls. Suppose that the head on the prototype spillway is 10 ft and the laboratory water supply is 5 cfs. It is desired to build the largest model that will utilize this flow. By assuming a maximum discharge coefficient of 4.1, which is seldom exceeded, the prototype discharge is 130 cfs per foot of length. For a crest length of 1 ft and a discharge of 5 cfs, the head on the model will be 1.14 ft. The ratio of the heads in model and prototype is approximately 1:9. The head is slightly greater than the length, and the scale should be reduced slightly. At a scale ratio of 1:10, the head on the model will be 1 ft, and the discharge will be 4.1 cfs per foot of length, allowing a crest length of 1.25 ft. The size of the model will then be governed by space limitations. If the dam is 40 ft high, the model height will be 4 ft. The addition of 1 ft for the depth over the crest would give a total height of 5 ft not including the necessary freeboard.

The correct reproduction of approach conditions is important in the determination of discharge coefficients, and they should be simulated as nearly as possible. This

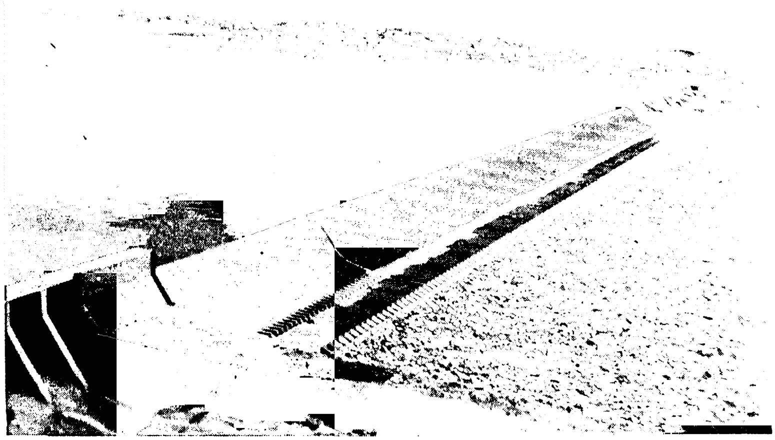


FIG. 1.—1:40 scale model of Imperial Dam, All-American Canal project. (*Courtesy of Bureau of Reclamation.*)

FIG. 2.—Study of effect of angularity of approach on discharge coefficients. (*Courtesy of Dr. K. C. Reynolds, Department of Civil and Sanitary Engineering, M.I.T.*)

will frequently result in a smaller scale model than would otherwise be built, owing to space limitations. If the spillway crest is submerged, care should be taken to reproduce the get-away conditions, as these also influence the discharge.

The effect of submergence should be carefully investigated if the tail water rises appreciably above the spillway crest. It has been found that published values of submergence coefficients are applicable only to the crest on which they were measured. and in many cases only to one particular discharge. This is illustrated by results reported by Soucek.[1]

[1] SOUCEK, EDWARD, Meter Measurements of Discharge, University Dam, *Trans. A. S. C. E.*, **109**, 86–99, 1944.

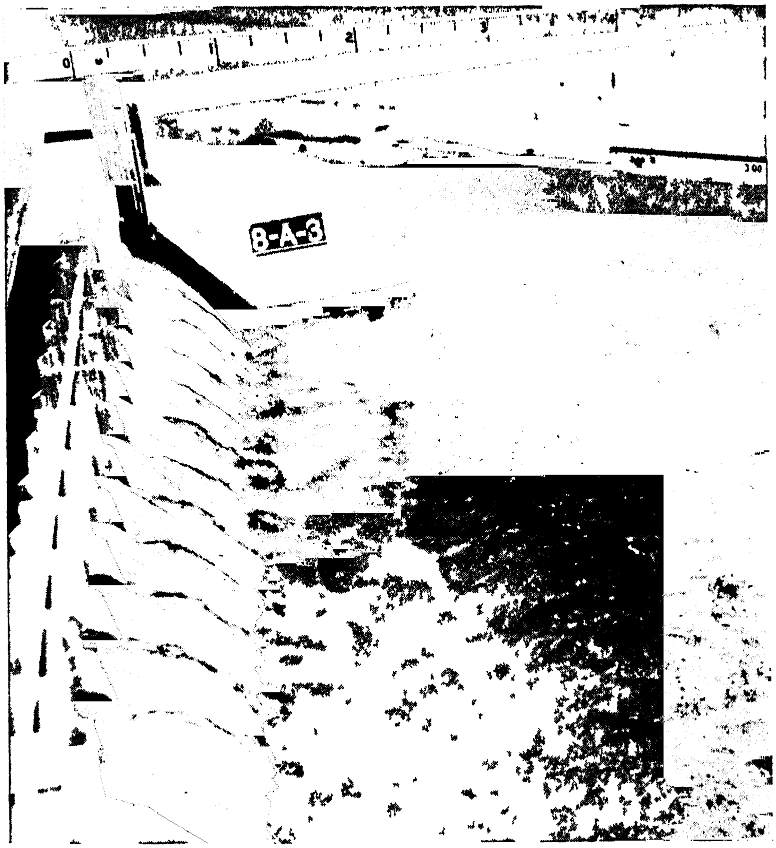


FIG. 3.—1:70 scale model of Possum Kingdom Dam spillway. Discharge 250,000 cfs (*Courtesy of U.S. Waterways Experiment Station.*)

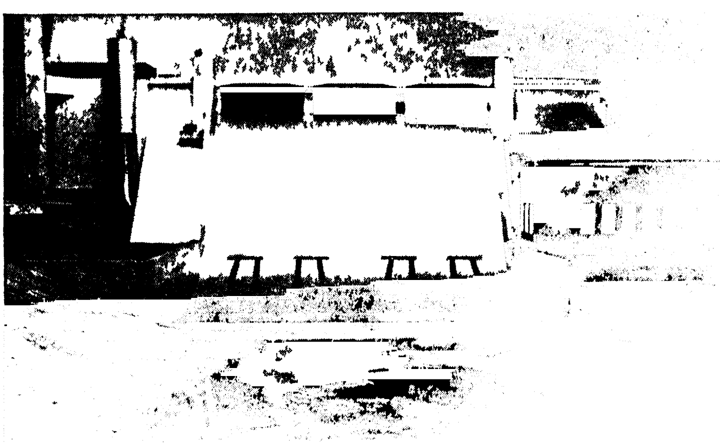


FIG. 4.—1:72 scale model of Norris Dam, T.V.A. (*Courtesy of Bureau of Reclamation.*)

Figure 1 illustrates a spillway without piers, and Fig. 2 shows a model arranged for studying the effect of angularity of approach on the discharge coefficient.

(2) *Spillways with Piers.* Many spillways are divided into short sections by piers on the crest. These piers may support a bridge over the structure, or gates. When building a model of such a structure, it is necessary to use an integral number of sections or bays. The design of the model is similar to the spillway without piers except

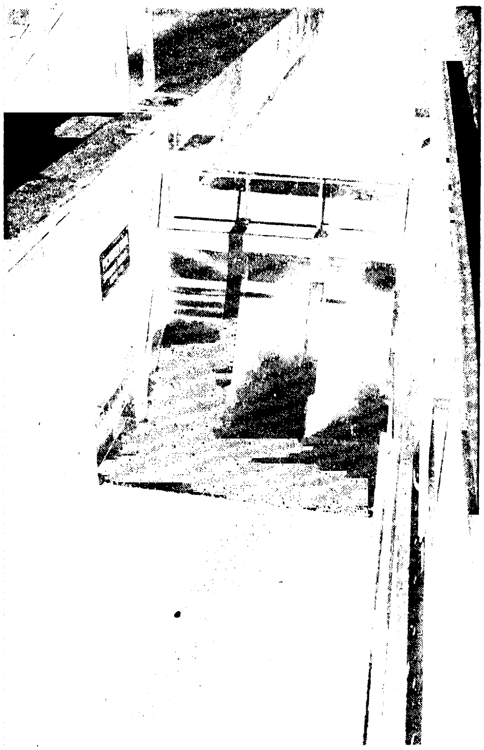


FIG. 5.—1:25 scale model of Pickwick Landing spillway, T.V.A. Three bays, showing crest gates. (*Courtesy of Tennessee Valley Authority.*)

that the length of the model crest is a multiple of the length of one bay instead of an arbitrary length. The model should be wide enough so that the effect of end contractions may be observed, and provision should be made for eliminating the contractions for a portion of the tests.

Figures 3 and 4 illustrate typical models of spillways with piers on the crests.

When there are spillway crest gates between piers, the discharge through any bay is influenced by the discharge of the adjacent bays. This is illustrated by the results

TABLE 1.—EFFECT OF OPERATION OF ADJACENT GATES ON SPILLWAY DISCHARGE COEFFICIENTS

Method of Operation	Coefficient
One spillway operating alone	3.71
Middle of three open spillways	3.78
Middle of seven open spillways	3.90
End gate of two open spillways	3.74
End gate of five open spillways	3.72

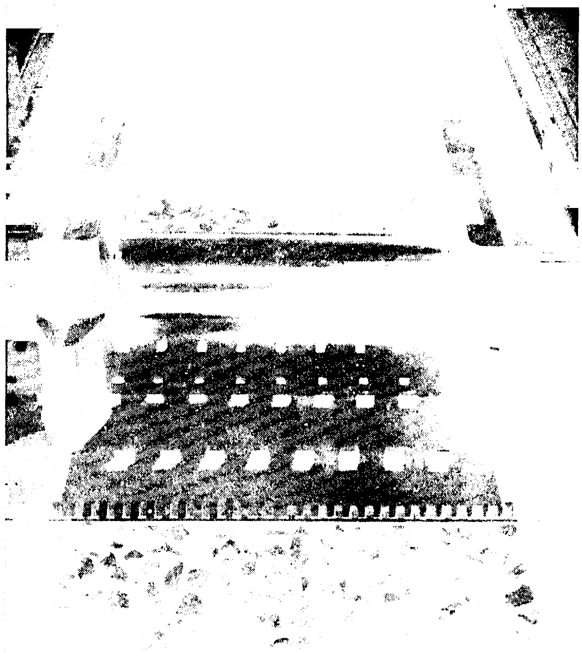


FIG. 6.—Model of roller gates on Dam 5, Upper Mississippi River. Gate in normal position. (*Courtesy of St. Paul office, U.S. Army Engineers.*)

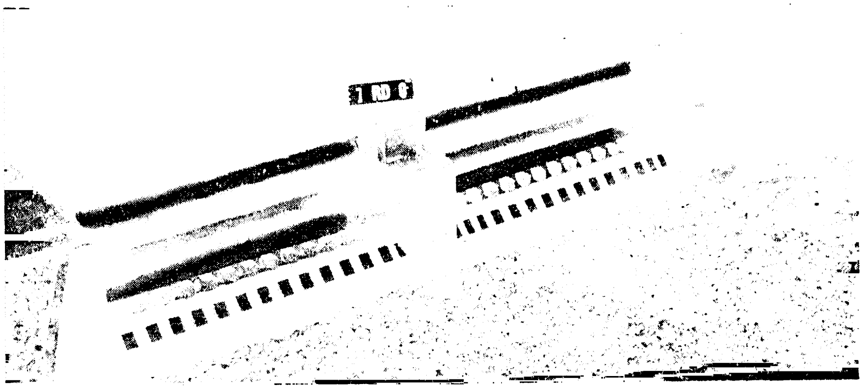


FIG. 7.—1:48 scale model of Roza Diversion Dam, Yakima Project, Washington. (*Courtesy of Bureau of Reclamation.*)

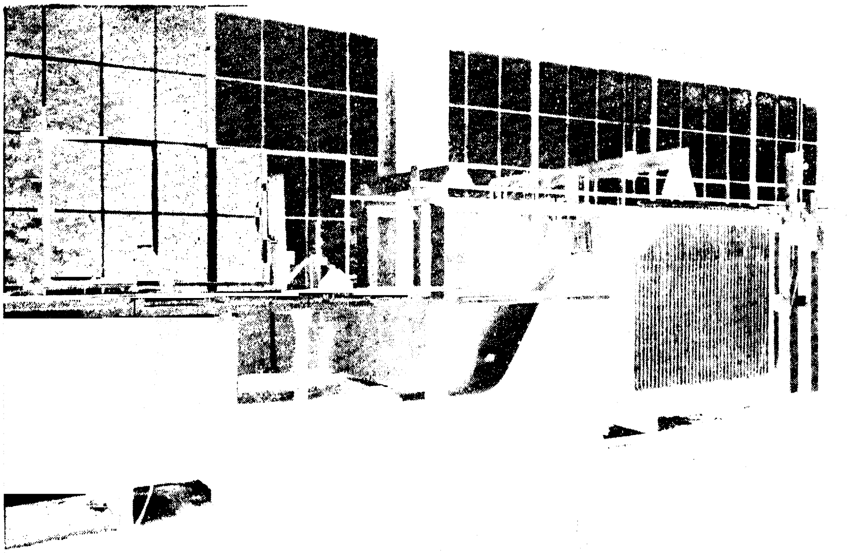


FIG. 8.—1:48 Scale model of Mahoning Dam. (*Courtesy of Prof. Geo. E. Barnes, Case School of Applied Science.*)

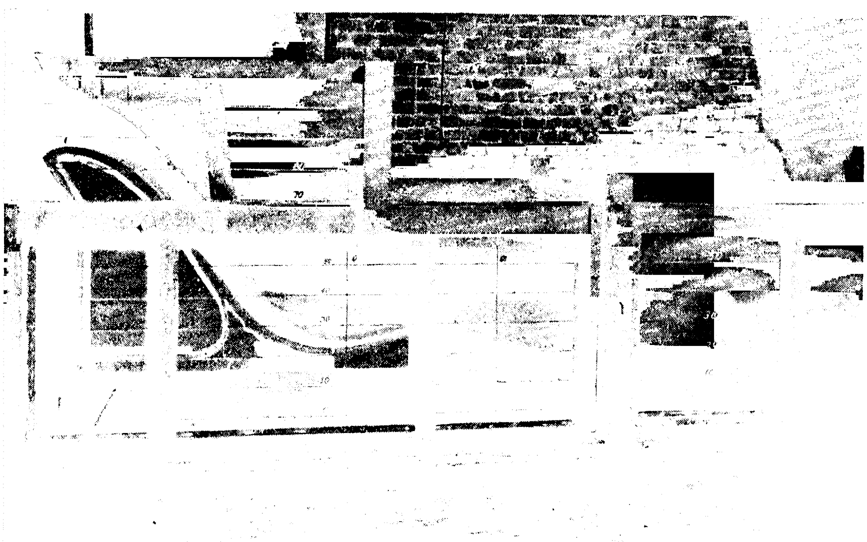


FIG. 9.—Model of spillway and stilling basin, Conowingo Dam. (*Courtesy of Prof. C. M. Allen, Worcester Polytechnic Institute.*)

obtained by Nagler and Davis[1] in measurements of the discharge over Keokuk Dam. They found the results in Table 1.

Model tests were made on three bays at a scale of 1:11. The coefficient obtained for all three bays was 3.82 and for the center bay alone was 3.69. This is good agreement.

b. Gate Discharge Coefficients. The design of a model to determine gate discharge coefficients is similar to that of a model for spillway discharge coefficients. It may

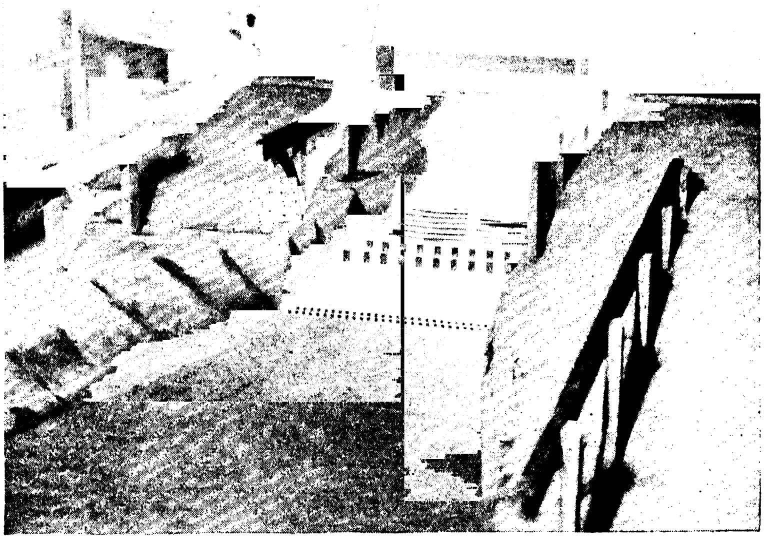


FIG. 10.—1:40 scale model of stilling basin for Mohawk Dam. (*Courtesy of Prof. Geo. E. Barnes, Case School of Applied Science.*)

be assumed that discharge coefficients for the model and prototype are equal and that the equation

$$Q = CAH^{1/2}$$

is applicable where discharge occurs beneath the gate. In the case of a taintor gate or a leaf gate which is lowered so that discharge occurs over it, the procedure is identical with that for a spillway with piers. Figures 5 to 7 illustrate typical models with spillway gates.

c. Stilling-basin Characteristics. An important phase of model testing is determining the effect of spillway discharge on the river bed below a dam. It is not necessary to simulate approach conditions as accurately as for discharge coefficient measurements, but the model should have a channel that will simulate conditions below the dam. Preliminary tests of erosion tendencies may be made on a large scale with a sectional model, but where gates are provided on the crest, and it is possible

[1] NAGLER and DAVIS, Experiments on Discharge over Spillways and Models, Keokuk Dam, *Trans. A. S. C. E.*, **94**, 777–820, 1930.

for the distribution of discharge to vary, tests of the entire structure should be made, and the river channel below should be reproduced in detail.

The type of apron that will be most effective in reducing energy may be determined approximately by the methods outlined in Sec. 7. The effect of the overfalling water on the erodible material at the end of the apron in the prototype cannot be determined precisely. In many cases, the bed is rock and its erosion characteristics are unknown. If the bed is gravel, it will erode easily. One laboratory expedient is the use of an erodible river bed composed of material that is fine enough to be moved readily, yet that will not be carried away by the normal velocities that exist

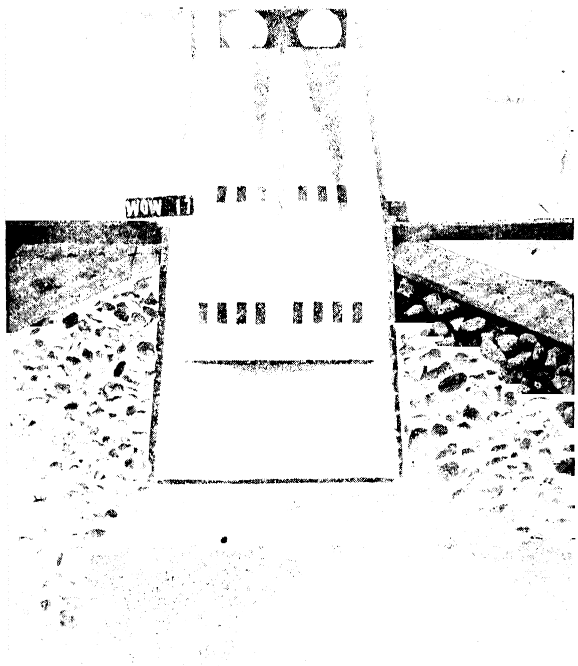


FIG. 11.—1:20 scale model of outlet works for Wickiup Dam, Deschutes Project, Oregon. (*Courtesy of Bureau of Reclamation.*)

below the spillway. It is important that the material be carefully screened so that its range of size is limited. If a well-graded material is used, the fines wash away first. In order to reproduce tests, it is necessary that the fines be collected and remixed with the coarse particles. This is practically impossible, and comparative results are very difficult to obtain with such material.

Sometimes it is possible to simulate the rock bed and banks of a stream in a qualitative manner by forming them in concrete so weak that it will be eroded by the appropriate model velocities. This is particularly true when the banks are steep and cannot be made to hold their slopes with sand or gravel. A reasonably satisfactory material has been produced by the use of "lumnite" cement and sand. Portland cement cannot be used because of its characteristic increase of strength with age.

It is not possible to predict the precise amount of erosion that may be expected below a spillway. This does not restrict the use of models in design, however, because it is not usually desired that erosion shall occur. A properly constructed model is

capable of predicting tendencies toward erosion or deposition of material, and it is entirely feasible so to construct the stilling basin that the action of the water leaving it will tend to deposit material at the toe of the apron, rather than to remove it. An apron designed in such a way should never fail by undermining, provided, of course, that there is no tendency toward degradation of the channel, as in the case of the Prairie du Sac Dam.[1]

A great many models have been tested to determine the best means of dissipating energy and preventing erosion. The conditions are different at each site, and a number of devices have been developed. Figures 8 to 12 are typical examples.

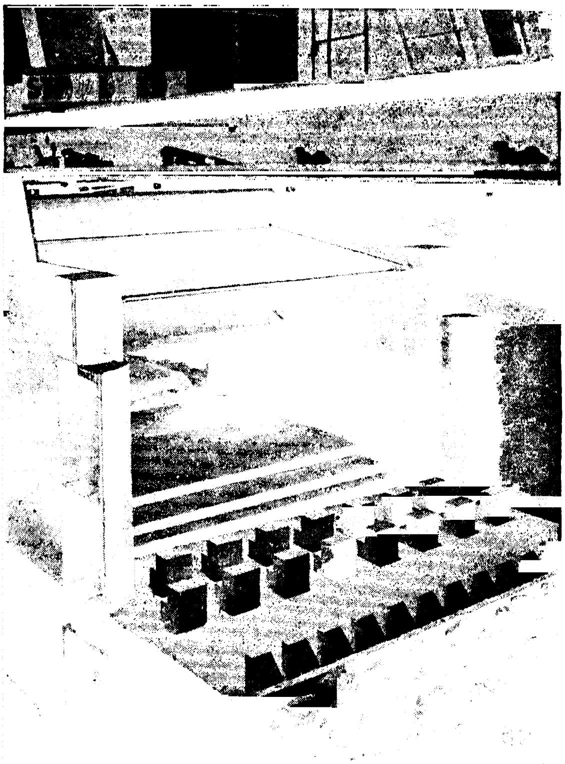


FIG. 12.—Details of sill and apron for Tainter gate section. Dam 5, Upper Mississippi River. (*Courtesy of St. Paul Office, U.S. Army Engineers.*)

Figure 13 illustrates a model of the Almanor spillway tested by the Pacific Gas and Electric Company. This model was built and tested subsequent to the construction of the prototype to determine the character and amount of work required at the lower end of the spillway chute in order to prevent erosion of the bed and banks of the river channel to an extent that might endanger the safety of the dam. An unusual feature of the model was that in order to fit it into the available ground space at the testing laboratory it was built left-hand or reversed. The prototype structure curves downward to the right instead of to the left as indicated in the illustration of the model.

[1] WARD, C. N., and HUNT, HENRY J., Correction of Tailwater Erosion at Prairie du Sac Dam, *Trans. A. S. C. E.*, **112**, 1173–1200, 1947.

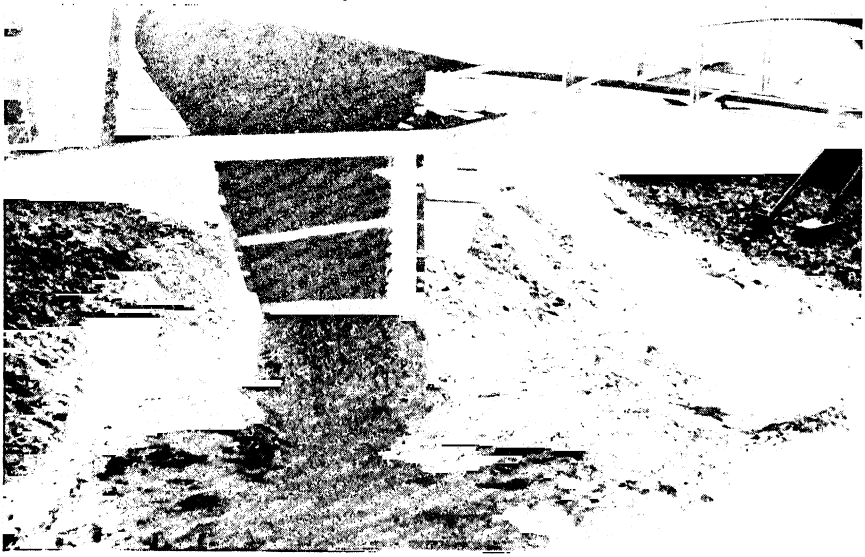


FIG. 13.—Reversed model of the Almanor spillway. (*Courtesy of Pacific Gas and Electric Company.*)

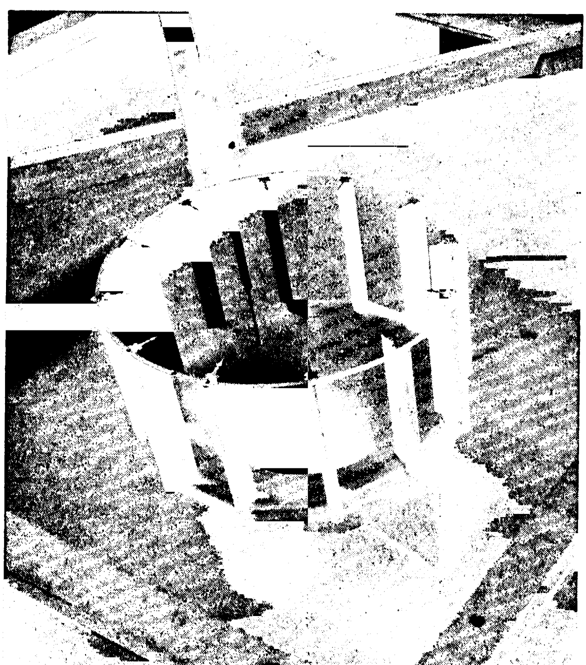


FIG. 14.—1:40 scale model of shaft spillway, Keystone Dam, Nebraska. (*Courtesy of Prof. Geo. E. Barnes, Case School of Applied Science.*)

d. Special Spillway Structures. Models are well adapted to the investigation of the performance of special spillway structures, such as side-channel spillways, morning-glory spillways, and siphon spillways. Hinds[1] has stated the theory of operation of the side-channel spillway. His discussion is limited to motion parallel to the axis of the channel and does not predict the occurrence or behavior of the eddies having a longitudinal axis which invariably exist. A fine study of a side-channel spillway has been made by the Bureau of Reclamation.[2]

Morning-glory spillways are another type of structure in which model studies are very useful (See Fig. 14). Spiral flow in the bell has been found to be eliminated by using the profile of the under side of the nappe of a circular sharp-crested weir, as defined by the experiments of Camp and Howe.[3] Choking of the spillway and irregular flow in the lower portion of the vertical shaft and horizontal tunnel may be prevented and improved by model tests. The operation of morning-glory or shaft spillways is usually complicated by the entrainment of air.

The discharge characteristics of siphon spillways may also be investigated by models, although Stevens[4] has shown that it is possible to predict the operation of a siphon spillway with reasonable accuracy by analytical methods. Care should be taken in working with models of siphon spillways, or structures where negative pressures occur, to see that the model pressures when transferred to the prototype do not go below absolute zero. It is feasible, for example, in a model built to a scale of 1:15, to develop a negative pressure of 3 ft of water which would correspond to minus 45 ft in the prototype. Before this point is reached, the model ceases to represent prototype conditions.

Both morning-glory and siphon spillways introduce a certain amount of friction that makes the application of Froude's law uncertain. For this reason, the hydraulic model should be used chiefly to observe the action at certain specific locations, rather than to predict the discharge of the structure as a whole.

e. Cavitation Studies. Cavitation is a frequent source of trouble in flow systems without a free water surface. It occurs where the average pressure is low (at or below atmospheric) and where the pressure is still further reduced by irregularities of surface that cause sharp local curvature of flow. The presence of cavitation is usually indicated by a sharp snapping or crackling noise. The damage caused by it may be considerable. A good account of the causes of cavitation and examples of its occurrence is given in a Symposium[5] presented by a sub-committee of the Committee on Hydraulic Research of the American Society of Civil Engineers. The possibility of cavitation may be studied by operating the model under prototype conditions and measuring pressures immediately below any irregularities or sharp bends. If these pressures when transferred to the prototype indicate pressures at or near absolute zero, the existence of cavitation at that point should be suspected. The remedy is to increase the pressure, either locally by removing near-by obstructions or smoothing out an angular entrance, or generally by constricting the conduit somewhere downstream. A general increase of pressure at a section may also be secured by lowering the conduit at that section.

Cavitation at sluice entrances has been studied by enclosing the entire model system and evacuating it until absolute pressures in the model bear the same relationship to absolute pressures in the prototype as do the linear dimensions. Cavitation

[1] HINDS, JULIAN, Side Channel Spillways, *Trans. A. S. C. E.*, **89**, 881–927, 1926.

[2] U.S. Bureau of Reclamation, Boulder Canyon Project Report, Part VI, Hydraulic Investigations, Book 1, 1938. Model studies of spillways.

[3] CAMP, CECIL S., and HOWE, J. W., Tests of Circular Weirs, *Civil Eng.*, **9**, No. 4, April, 1939, pp. 247–248.

[4] STEVENS, J. C., On the Behavior of Siphons, *Trans. A. S. C. E.*, **99**, 986–1005, 1934.

[5] VENNARD, JOHN K., HARROLD, JOHN C., WARNOCK, JACOB E., and HICKOX, GEORGE H., Cavitation in Hydraulic Structures, *Trans. A. S. C. E.*, **112**, 1–124, 1947.

is thus actually produced in the model. Damage to a structure can be simulated by building it of a suitably weak material. This method should be used with caution because the vapor pressure of water is the same both in the model and in the prototype It would be possible, if the model scale were small enough, to operate the entire model at pressures less than the vapor pressure of water, thus indicating cavitation in regions that were actually at atmospheric pressure or above.

The model relationships follow Froude's law until cavitation is indicated, when the model no longer represents the prototype. Figure 15 illustrates a model of a sluice inlet tested for cavitation conditions by the Tennessee Valley Authority.

f. Emergency-gate Loads. Gates designed to operate in flowing water under considerable heads are subjected to loads parallel to their direction of motion. Water passing under the gate reduces the pressure on its lower face, whereas the upper surfaces, being in a region of low velocity, are under full pressure. The load due to this differential pressure may be much greater than that of the gate itself. Both

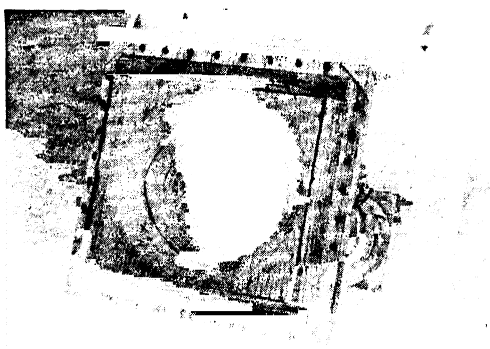


FIG. 15.—1:15 scale model of sluice inlet, Hiwassee Dam, T.V.A. Note piezometers along crown of inlet where pressures are lowest and cavitation might occur. (*Courtesy of Tennessee Valley Authority.*)

the design of the gate and the capacity of the hoist are directly affected. The load may be reduced by increasing the area of flow beneath the gate and by reducing the effective horizontal area of the gate in the region of high velocity.

Satisfactory determination of the hydraulic load can be made only by means of a model. The model must simulate the prototype in all respects that influence the flow including both approach and get-away conditions. The load may be determined by weighing. It is important to minimize mechanical friction of the gate in the model. Figure 16 illustrates a model of the emergency gate for the Hiwassee Dam sluices.

g. Lock-filling Systems. Another type of hydraulic structure that is governed largely by Froude's law is the operation of filling systems for navigation locks. Successful operation of a lock-filling system requires a minimum disturbance of boats moored in the lock chamber. The hydraulic design of the model offers no particular difficulties. The determination of stresses in the hawsers mooring the boat is another matter. Krey[1] has developed a scheme of measuring hawser stresses, and this has been modified and improved on in this country by Prof. C. M. Allen at Worcester Polytechnic Institute and by the T.V.A. Figures 17 and 18 illustrate a typical installation of the device used to record the motion of the boat during the period of lock filling as developed by the T.V.A.

[1] KREY, H., Neuere Versuche fur Schiffschleusen, *Zentralblatt der Bauverwaltung*, Nos. 45 and 47, 1914.

If the filling system is composed of a culvert through the lock walls with ports into the chamber, friction undoubtedly plays at least a minor part in governing the operation of the system. However, if the lock is filled from the upper end by short culverts either around or through the upper miter sill or by discharge through the upper gates such as sector or Tainter gates, friction may be neglected entirely.

h. Tower Intake Structures. Tower intakes are frequently used for discharging water from a reservoir when it is desirable to draw water from various elevations, as for water supply or other reasons. No different principles are involved in the design of the model, but care must be taken in applying Froude's law if the outlet tunnel is so long that appreciable friction develops.

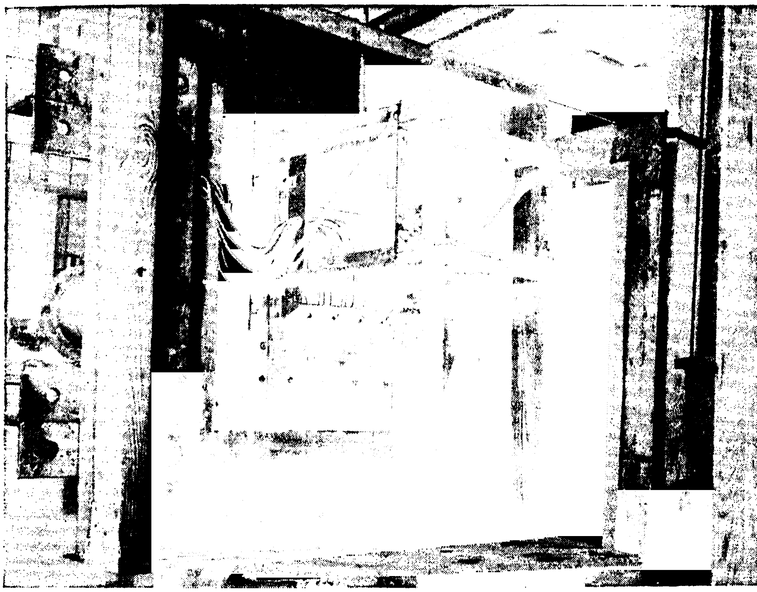


FIG. 16.—1:15 scale model of emergency sluice gate for Hiwassee Dam, T.V.A. Piezometers were used for measuring pressures in addition to weighing the total load. (*Courtesy of Tennessee Valley Authority.*)

i. Fish Ladders. The design of fish ladders by means of model tests is a comparatively recent development, and very little literature on the subject is available. Considerable work has been done by the Hydraulic Laboratory of the State University of Iowa under the direction of Prof. E. W. Lane, and also by the U.S. Army engineers in the design of Bonneville Dam.

j. Drops and Chutes. In irrigation or soil-erosion control work, it is often necessary to lower the level of a canal or other stream of water. This is accomplished by checks, drops, or chutes. The prevention of erosion at the foot of such structures is the same problem met with in designing stilling basins for dam spillways, but on a much smaller scale. Little attention has been paid to these structures because of their size, and considerable difficulty has been experienced with erosion at the lower level. The design can be improved by hydraulic-model tests. The University of Wisconsin,[1] the Bureau of Reclamation, and the Soil Conservation Service[2] have

[1] KESSLER L. H., Experimental Investigation of the Hydraulics of Drop Inlets and Spillways for Erosion Control Structures, *Univ. Wis., Eng. Exp. Sta. Series, Bull.* 80, 1934.

[2] MORRIS, B. T., and JOHNSON, D. C., Hydraulic Design of Drop Structures for Gully Control, *Trans. A. S. C. E.*, 108, 887–913, 1943.

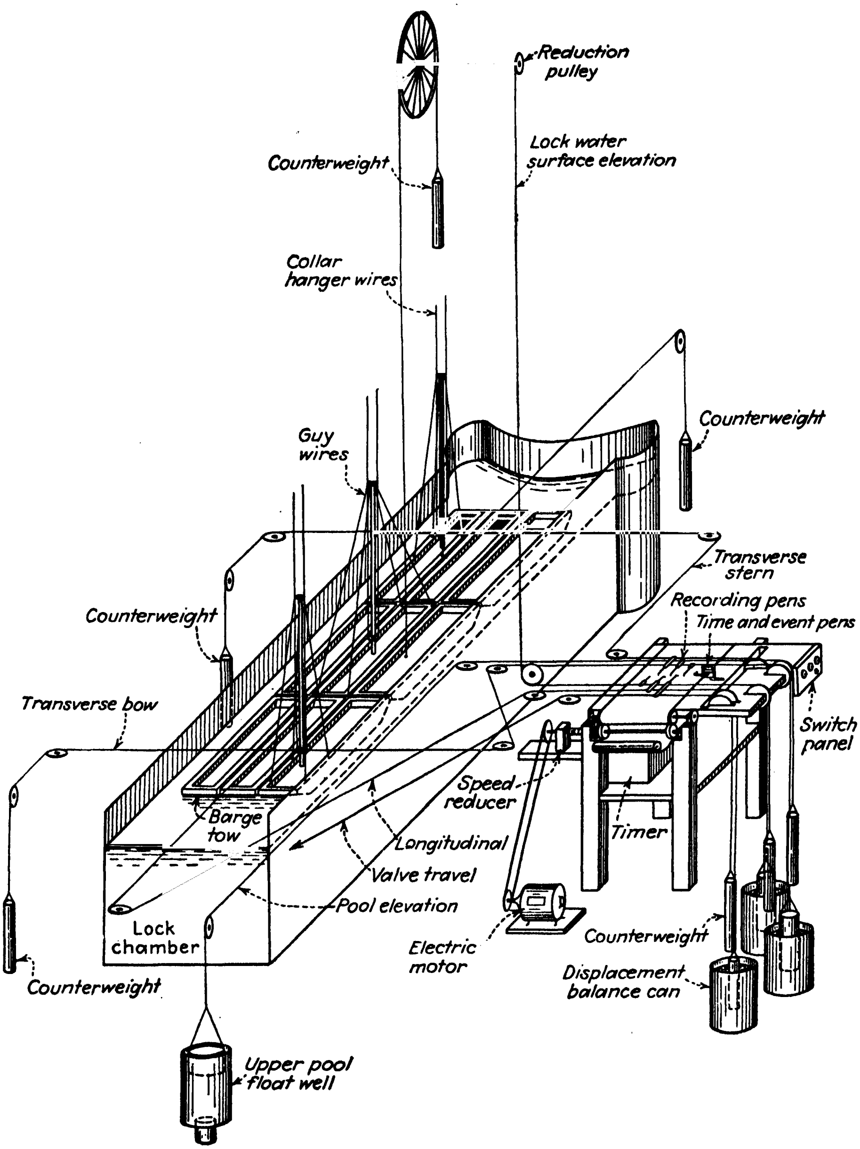


FIG. 17.—Diagrammatic sketch of device for recording movement of a barge tow in a lock. (*Courtesy of Tennessee Valley Authority.*)

attacked the problem. Figure 19 is a model of a check drop developed by the Bureau of Reclamation. Chutes may be of such length that friction is an important factor. In such cases, Froude's law is no longer applicable.

2. Structures in Which Friction Is Important. The types of models described in the foregoing section have been those in which friction played a relatively unimportant part. Some structures, however, are materially affected by friction losses and some of these will be considered briefly.

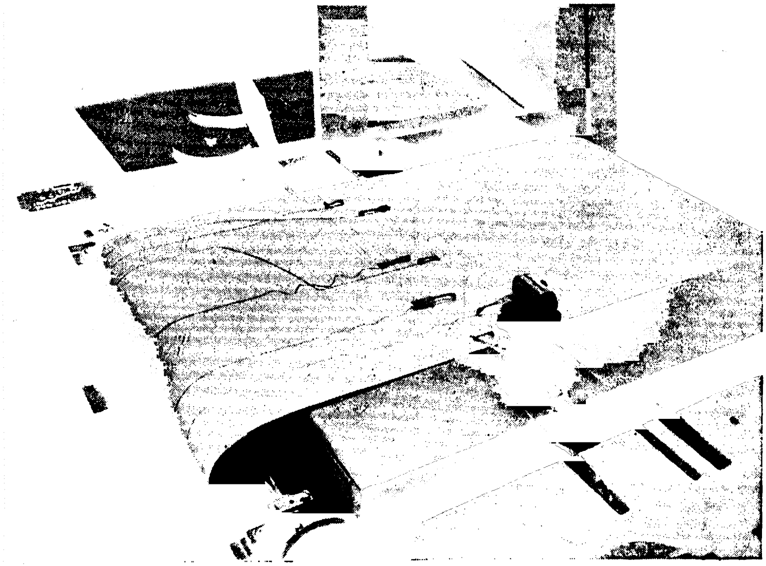


FIG. 18.—Detailed view of recorder. (*Courtesy of Tennessee Valley Authority.*)

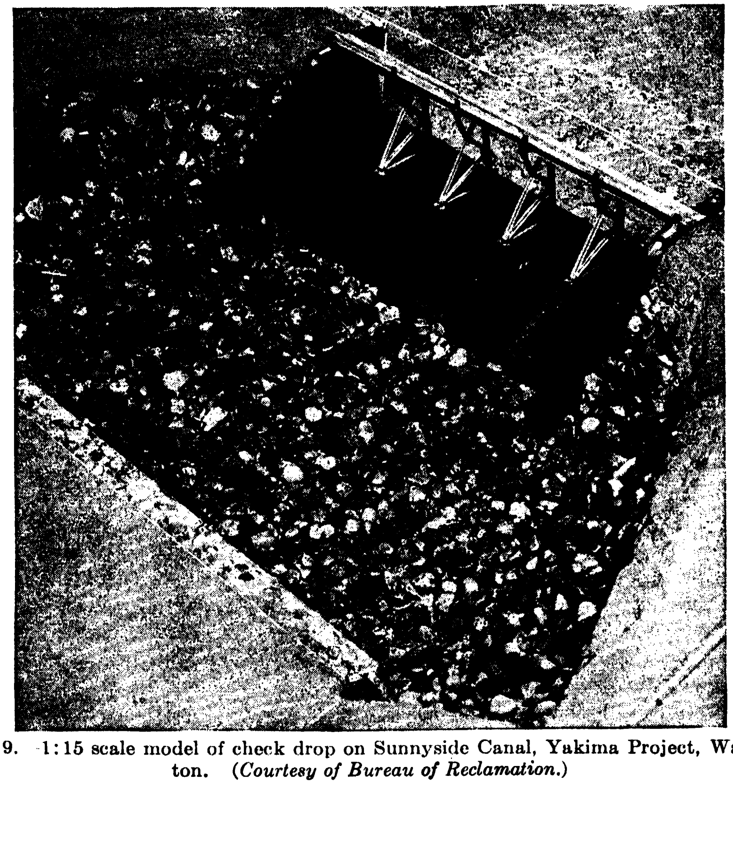


FIG. 19. 1:15 scale model of check drop on Sunnyside Canal, Yakima Project, Washington. (*Courtesy of Bureau of Reclamation.*)

a. Lock-Filling Systems. Lock-filling systems composed of culverts in the lock walls and lateral ports are subject to friction losses, although the effects are generally neglected without serious error.

b. Spillways Discharging through Tunnels. Figure 20 illustrates the tunnels of the Keystone Dam spillway. The two long conduits are supplied by a tower intake and a morning-glory spillway, respectively. The performance of these structures is governed by Froude's law. From the intake structures, the water flows through the

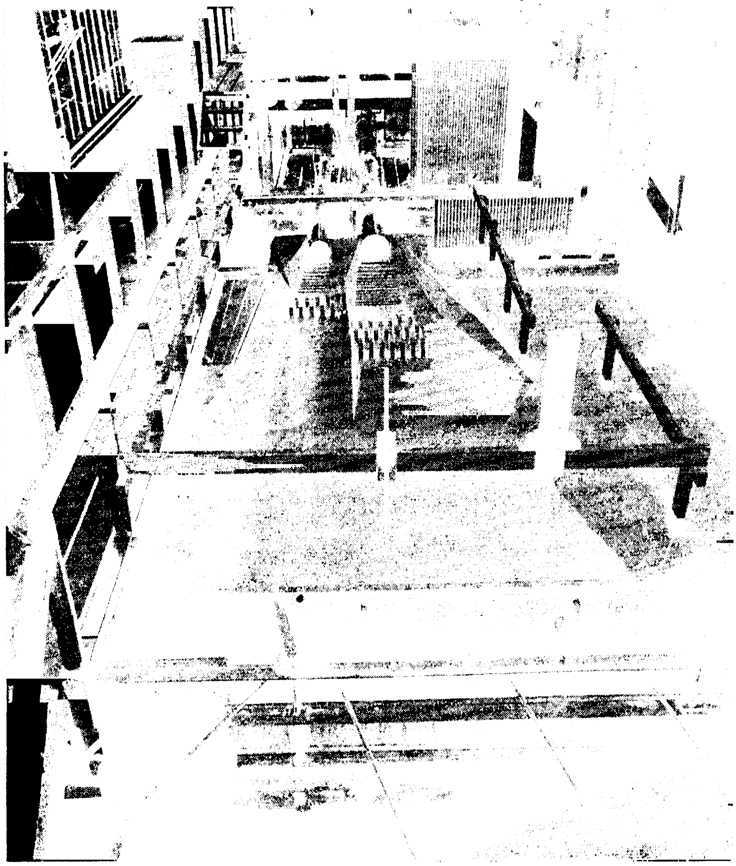


FIG. 20.—1:40 scale model of spillway and outlet works, Keystone Dam, Nebraska. Note the manometer banks for determining friction losses and pressures in the conduits. (*Courtesy of Prof. G. E. Barnes, Case School of Applied Science.*)

conduits, where its action is governed by friction, to the stilling basins below, where Froude's law again holds. If the friction slope in the conduit is greater for the model than for the prototype, as is usually the case, the discharge of the system is reduced and the performance of the intake and stilling basins for given headwater and tailwater elevations is not correct. Pressures in the conduit are not correctly represented.

The remedy in such a situation is a careful study of friction losses in the conduit and a comparison with similar friction losses to be expected in the prototype structure. The conduit can then be omitted from quantitative consideration except as it affects the back pressure on the intakes or the discharge into the stilling basin, and these

elements may be tested independently. In his report on the tests on these structures, Prof. Barnes[1] avoids the difficulty by the use of an approximate correction to the discharge obtained by Froude's law. The ratio of model roughness to prototype roughness that will give similarity by Froude's law is $L_r^{1/6}$. If the actual roughness of model and prototype be n_m and n_p, respectively, the discharge that will give similar gradients in the conduit for model and prototype is obtained by multiplying that obtained by Froude's law by

$$\frac{n_m}{n_p L_r^{1/6}}$$

c. Surge-Tank Models. Surge tanks are generally used only on tunnels that are so long that friction losses are material. In such models it is desirable to satisfy similarity requirements with regard to both inertia and friction. Since it is not usually possible to reproduce the friction losses to scale in a geometrically similar model, it becomes necessary to adjust the dimensions of the model tunnel to an "equivalent" tunnel having the desired friction and inertia characteristics. The details are too lengthy for inclusion but the procedure is indicated in "Applied Fluid Mechanics."[2] The dimensions of the surge tank proper are reproduced to scale since friction in the tank itself can be neglected.

Other types of models in which both gravity and friction forces are important will occur. No fixed rule for handling all situations can be stated. The treatment will depend on the specific conditions and the ingenuity of the model designer.

3. Models of River Channels. *a. General Considerations.* The design of models of river channels presents more difficulty than the design of models governed by Froude's law alone. The flow of water in rivers is governed largely by friction. The change in water surface is very gradual so that gravity forces play a relatively minor part, except at marked changes in the cross section where the flow becomes decidedly nonuniform.

The source of many of the troubles encountered with river models is the size of the area to be investigated. Space limitations are usually such that when a scale has been selected that will permit an undistorted model to be placed in the space available, the depths of flow are so small that laminar flow may occur. The laws governing laminar flow are considerably different from those governing turbulent flow, and since turbulent flow almost always occurs in nature, it must also exist in the model. The use of an undistorted model usually results in water-surface slopes so flat that differences in elevation cannot be measured satisfactorily. These objections can be overcome by distorting the model. Distortion may be accomplished in several ways. The slope may be increased by tilting the model, by arbitrarily changing the discharge scale, by using different horizontal and vertical scales, or by changing the roughness.

The scale relationships for river models are usually based on Manning's friction formula. On this basis, the ratio of velocities is

$$V_r = \frac{D_r^{2/3}}{L_r^{1/2} n_r}$$

in which the ratio of the slopes is

$$S_r = \frac{D_r}{L_r}$$

[1] BARNES, G. E., Hydraulic Model Studies for the Keystone Dam on the North Platte River, Report to the Central Nebraska Public Power and Irrigation District, Hastings, Neb., 1936. Privately published by G. E. Barnes.

[2] O'BRIEN, M. P., and HICKOX, G. H., "Applied Fluid Mechanics," McGraw-Hill Book Company, Inc., 1937.

The discharge ratio is

$$Q_r = \frac{L_r^{1/2} D_r^{13/6}}{n_r}$$

In these equations, V, Q, D, L, and n represent velocity, discharge, depth, length, and roughness, respectively. The subscripts m and p refer to model and prototype, and the subscript r denotes the ratio of model to prototype.

In certain cases where both gravity and friction forces must be satisfied simultaneously, the necessary condition may be found by equating the velocity ratios for the two conditions.

$$L_r^{1/2} = \frac{D_r^{7/6}}{L_r^{1/2} n_r}$$

from which

$$n_r = \frac{D_r^{7/6}}{L_r}$$

For undistorted models, this reduces to

$$n_r = L_r^{1/6}$$

The dimensions of the model can be determined to a first approximation by the use of the foregoing relationships. In addition, the flow must be turbulent. It has been established by measurements made at the U.S. Waterways Experiment Station that flow will be turbulent if

$$\frac{VR}{\nu} \cong\!\!> 4{,}000$$

where V = mean velocity, fps.
R = hydraulic radius, ft.
ν = kinematic viscosity, ft squared per sec.

For water at 70F, the kinematic viscosity is approximately 0.00001. For this value of ν, turbulent flow in the model is assured if $VR \gtreqqless 0.04$. A product of VR less than 0.04 should be considered sufficient reason for enlarging the model, or if this is not possible, of interpreting the results with extreme caution. Values of $VR < 0.02$ should not be used under any conditions as laminar flow is almost certain to occur.

b. Movable-bed Models. A type of river model often requiring investigation is one that involves movement of the bed material. This is the most difficult type of model to design and operate, and also the one that gives the least satisfactory results.

The laws governing the movement of bed material in a river channel are very imperfectly known. The movement of bed load has been considered to depend on the tractive force exerted on the channel bed by the flowing water. If the component of the weight of the water parallel to the bed is equated to the resistance offered by the channel and if it is assumed that the resistance is uniform over the wetted perimeter, the tractive force may be expressed as

$$T = wRS$$

where T = tractive force, lb per sq ft.
w = unit weight of water, lb per cu ft.
R = hydraulic radius, ft.
S = slope of the water surface.

The critical tractive force or the force at which movement of sand particles begins has been found by several experimenters to be proportional to the diameter of the sand grains, or

$$T_c = KD$$

where T_c = critical tractive force, psf.

D = diameter of the sand particle, ft.

K = a constant that varies from 0.2 to 0.4.

Tractive forces in models are extremely small, and it is difficult to obtain the necessary force to cause sand movement without very steep water-surface slopes when sand is used as the bed material. Attempts have been made to reduce the necessary tractive force by using a much lighter bed material. Substances that have been

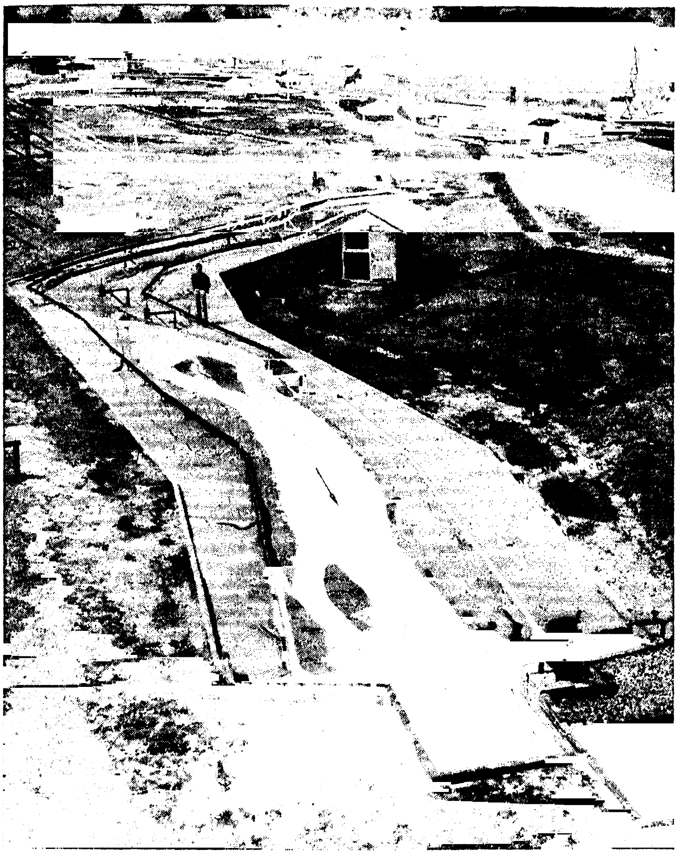


FIG. 21.—Movable-bed model of the Pryors Island reach of the Ohio River. Scales: horizontal, 1:600; vertical, 1:150. (*Courtesy of U.S. Waterways Experiment Station.*)

used in movable-bed models for this purpose are lignite, haydite, and cornmeal. The effect of these lighter materials is to lessen the need for extreme distortion of the model and to decrease the time necessary to accomplish completion of the experiment.

The dimensions of the model may be determined from the foregoing equations, due consideration being given to space available, water supply, tractive force, and turbulence. Frequently, the material used for the movable bed is not of the proper size to satisfy the requirements of friction in obtaining similarity. As a result of this, and of the lack of knowledge of laws governing transportation of material, movable-bed models are built as the result of experience and the model ratios are

determined experimentally. The U.S. Waterways Experiment Station has probably developed the technique of movable-bed models further than any other agency. At this laboratory, the model scales are determined by repeated experiments with varying rates of discharge, slope distortion, and times, until the model will reproduce a set of conditions known to exist at two specified times in the prototype. The set of conditions producing satisfactory correspondence is chosen to give the model scales. The process of determining these scales is called *verification*. Subsequent operation of the model is based on the assumption that these model scales will apply equally well after the desired modifications have been made. Since the changes generally consist of cutoffs or dikes and a resulting nonuniformity of flow whereas the

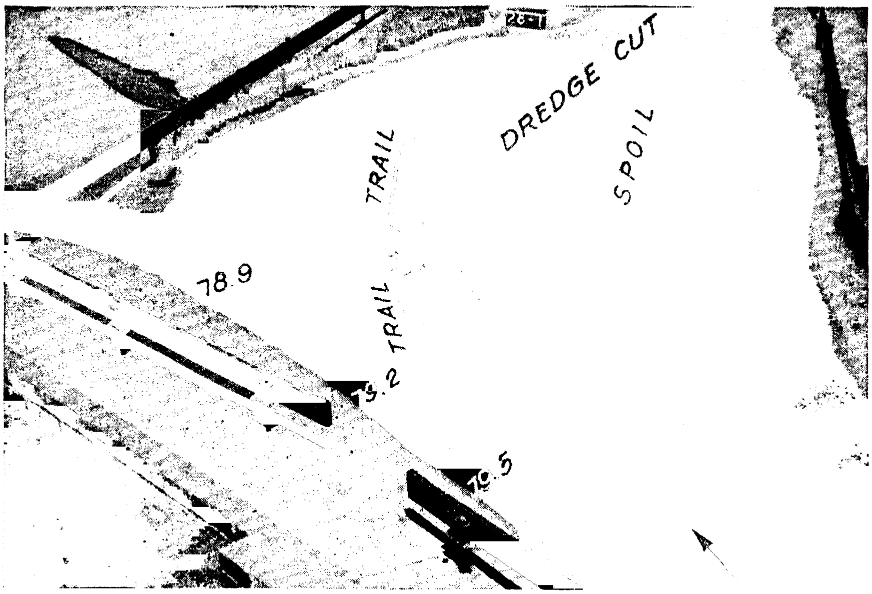


FIG. 22.- Grand Tower Channel Improvement Study, Upper Mississippi River. Photograph shows proposed dredge cut and dike system just below Grand Tower, Ill. Scales: horizontal, 1:600; vertical, 1:150. (*Courtesy of U.S. Waterways Experiment Station.*)

original channel is usually more nearly uniform, it is probable that the results should be used with caution. They should be regarded as qualitative rather than as quantitative.

Comparisons of model tests with actual river operation, made by the U.S. Waterways Experiment Station, have been reported by Vogel[1] and Thompson[2] and the results are encouraging. Even though model tests conducted in such a fashion can be relied on only for qualitative information, they can give positive information as to areas where scour or deposits may be expected to occur.

Figures 21 to 23 illustrate a number of typical movable-bed models.

c. Fixed-bed Models. Models of river channels are frequently built with fixed beds, *i.e.*, beds in which no erosion can occur. They are useful chiefly in investigating the effect of permanent changes in the channel caused by dredging, by obstructions

[1] VOGEL, H. D., Hydraulic Laboratory Results and Their Verification in Nature, *Trans. A. S. C. E.* **101**, 597–613, 1936.

[2] THOMPSON, P. W., The Use and Trustworthiness of Small-scale Hydraulic Models, *Civil Eng.*, **8**, 255–257, 1938.

such as bridge piers, or by levees. The investigation may be concerned with changes in navigation conditions or with backwater during floods. Models of this type have been used to investigate navigation conditions and backwater caused by coffer-damming operations during the construction of dams. The principles of design are identical with those for movable-bed models, except that it is not necessary to consider tractive force. In cases where an obstruction is so great that sudden changes in the water surface occur, it is better to use an undistorted model, if at all possible. If a distorted model is necessary, the degree of distortion should be kept as small as possible.

Figure 24 illustrates a portion of a large river model. The distortion is very great, about twenty times, but the results appear to be satisfactory.

d. Distorted vs. Undistorted Models. In general, an undistorted model is preferable to a distorted model where limitations of space and funds do not preclude its use.

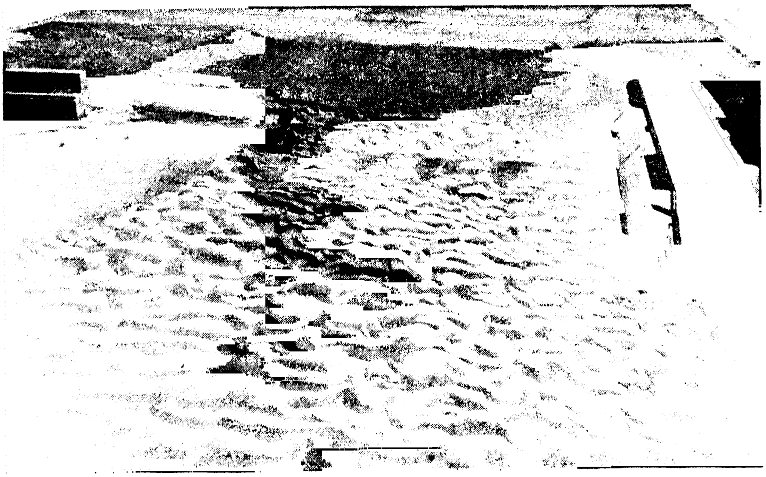


FIG. 23.—1:100 scale model of Pickwick Landing Dam, T.V.A., showing erosion during second stage of construction. (*Courtesy of Tennessee Valley Authority.*)

It reproduces flow phenomena accurately where gravitational forces predominate. River banks, earth cuts, and dikes have their natural slopes, making it possible to mold them in sand. Its disadvantages are: surface slopes are extremely flat; it is difficult to measure depths in small-scale models; it is difficult to produce bed load movement; and it is apt to be very expensive.

A distorted model has certain advantages: sufficient tractive force for bed load movement may be obtained, water surface slopes are steeper and much easier to determine, the width and length of the model may be decreased, the cost being thus lowered considerably. It has certain disadvantages which cannot be overlooked: the magnitude and distribution of velocities are incorrectly reproduced, the degree of error depending upon the amount of distortion; river bends, earth cuts, and dikes may become so steep that they cannot be molded satisfactorily in sand or other movable material; it is not adapted to satisfying gravitational and frictional forces simultaneously; and strictly speaking is not applicable to any case of nonuniform flow.

O'Brien[1] has pointed out some of the defects of distorted models and shown how their use leads to improper conclusions in some instances. As a practical matter,

[1] O'BRIEN, M. P., Analyzing Hydraulic Models for Effect of Distortion, *Eng. News-Record*, **109**, 313–315, 1932.

distorted models are necessary and considerable attention must be given to devising ways and means of improving their performance and interpreting results obtained from them.

4. Miscellaneous Models. Under this heading may be grouped those models which are not readily classified otherwise.

a. Harbor and Tidal Models. Tidal models have been studied by the U.S. Waterways Experiment Station and by the Tidal Model Laboratory at the University of California. Some of the methods and equipment used by the U.S. Waterways Experiment Station have been described by Tiffany.[1]

FIG. 24.—Mississippi River flood-control model. Mississippi River, Helena, Ark., to Donaldsonville, La. Portion of model showing operation of the White River emergency reservoir. Scales: horizontal, 1:2000; vertical, 1:100. (*Courtesy of U.S. Waterways Experiment Station.*)

b. Ship Models. The testing of ship models is a specialized field. Most of the ship testing in the United States has been done by the Navy's Bureau of Ships at the David Taylor Model Basin. A few universities maintain small towing tanks. The field is of interest to hydraulic engineers because it was during a series of tests on ship models that Froude discovered the relationship that now bears his name. The Taylor Model Basin also tests ship propellers for thrust and cavitation characteristics.

c. Turbine Models. Hydraulic turbines are designed largely on the basis of model tests conducted by the manufacturers. The principal manufacturers maintain their own laboratories and very little turbine testing is done elsewhere. As a result, the field is specialized, with most of the experimental data in the hands of the manufacturers.

5. Electric Analogies. The pattern of flow in some hydraulic structures, especially at the entrance to spillways and sluices, may be described approximately by the equations for potential flow. The variation from the flow pattern described by the hydrodynamical equations is largely near the boundaries in such cases and is caused

[1] TIFFANY, J. B., Small-scale Simulation of Tidal Phenomena, *Civil Eng.*, **8**, 537–539, 1938.

by fluid friction. The equations for the potential flow of a frictionless fluid are identical with those for the flow of electricity through a conducting medium. Accordingly, it is possible to simulate the hydraulic flow pattern for those cases where the flow can be represented in two dimensions. An electrical conductor having a shape similar to a section parallel to the flow is arranged so that known electrical voltages can be applied at those sections where the hydraulic pressures are known. The distribution of voltage, and the corresponding pressure distribution in the hydraulic system, can then be conveniently measured with a voltmeter. In potential flow, the flow lines are always perpendicular to the pressure lines and are thus easily determined. The spacing of the flow lines also indicates the velocity distribution. This method is particularly useful in determining local velocity distribution in irregular sections when investigating the possibility of cavitation. An example of this application is described by Rouse and Hassan.[1]

4. MATERIALS AND METHODS OF CONSTRUCTION

1. Materials Available. *a. Steel.* Small steel shapes are excellent for use as supporting frames for large models. The surfaces of spillway models may be constructed of sheet metal in order to obtain smooth surfaces and thus properly simulate frictional forces. Such surfaces require extremely careful workmanship at spillway crests, for example. Sheet metal presents some difficulty in making satisfactory joints. Soldered joints must be made very firmly to prevent their opening up. Once constructed, however, models of steel will hold their shape permanently. This fact is a great advantage where extensive tests are required to determine discharge coefficients. In such work, the shape of the model is of extreme importance. Figure 26 shows the framework of a steel spillway model.

b. Concrete. Models may be built satisfactorily of concrete where the design is known to be final. However, if the tests indicate that revision is desirable, concrete does not afford the opportunity for easy changes that is given by wood or sheet metal. Elaborate wooden forms must sometimes be built before concrete can be poured. In such cases, it is sometimes cheaper and frequently more satisfactory to make the model of wood in the first place. Concrete is especially satisfactory for such models as sluice entrances or other models where warped surfaces occur. It is particularly useful in fixed-bed river models. A river bed molded or paved in concrete will hold its shape indefinitely. It is advantageous also in that it may be made much smoother than a sand surface. This is frequently desirable in river models. Figure 27 shows an application of concrete.

c. Wood. In many respects, wood is the most satisfactory material to use for model construction. It is readily worked into any desired shape, is inexpensive, lends itself admirably to changes in design, and is easily handled. Steel or concrete models are frequently so heavy that they must be moved by means of a crane. The principal disadvantage of wood models is that they absorb water and may swell or warp out of shape. Similarity is thus destroyed and errors introduced. Swelling can be reduced, but not eliminated, by boiling the wood in paraffin or by soaking it thoroughly with linseed oil and then covering with several coats of paint. As far as is known, there is no satisfactory treatment of wood that will prevent absorption of water.

d. Molding Plaster. The various molding plasters may frequently be used as a substitute for concrete where permanence is not desired. They set very quickly, whereas concrete requires at least one day. Their chief disadvantage is that they wear away rapidly under the action of moving water. They are useful also in the

[1] ROUSE, H., and HASSAN, M. M., Cavitation-Free Inlets and Contractions, *Mech. Eng.*, **71**, No. 3, 1949, p. 213.

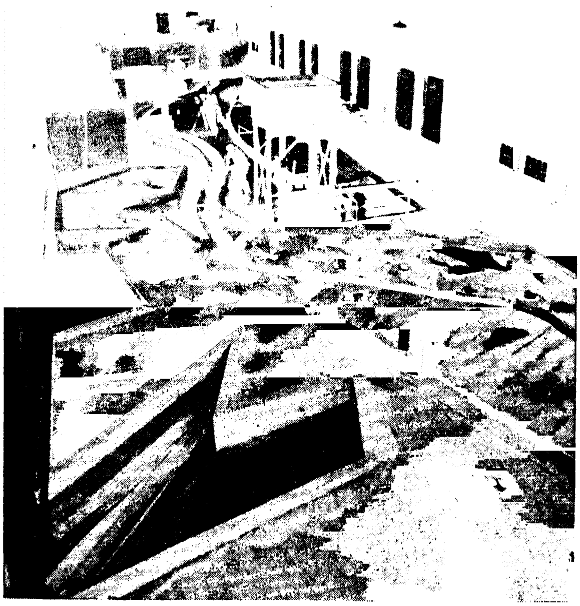


FIG. 25.—Model of Cape Cod Canal made for Corps of Engineers, U.S. Army, to study currents, water surface profiles, etc., of enlarged and improved canal. Scales: horizontal, 1:600; vertical, 1:60. (*Study made under the direction of Dr. K. C. Reynolds, Dept. of Civil and Sanitary Engineering, M. I. T.*)

FIG. 26.—Framework of a steel spillway model. The ogee crest of sheet metal is rolled over the templates. (*Courtesy of Tennessee Valley Authority.*)

manufacture of cores for concrete models. The plaster core may be soaked and chipped out with comparative ease after the larger concrete model has been completed (see Fig 28).

FIG. 27. Concrete mold used for making 24 concrete piers for a 1:100 model of Pickwick Landing Dam. (*Courtesy of Tennessee Valley Authority.*)

e. Plastics. There are several plastics on the market which are very useful in some types of model construction. Two of these, which are known by the trade names of Lucite and Plexiglas, are transparent They may be had in sheets, rods, and tubes of various dimensions. These materials are plastic when heated and the sheets can be formed by pressing between molds They may be machined and cemented. They have been used extensively for models of complicated conduit intersections, turbine scroll cases, draft tubes, and the like (see Fig 29). Many plastics are also available in powder or pellet form and can be molded into any desired form by heated presses.

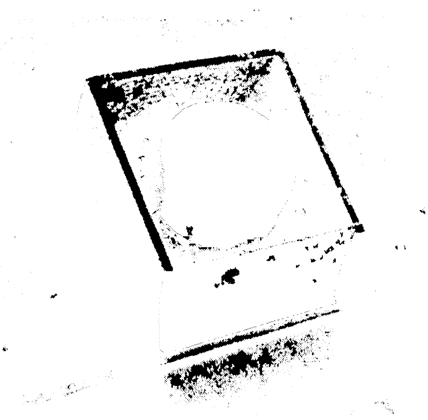


FIG. 28. Plaster core in mold ready for concrete casting of a model penstock entrance for Hiwassee Dam. (*Courtesy of Tennessee Valley Authority.*)

f. Glass. Glass is no longer used as much as it once was. Pyrex tubing has been used for models of circular conduits where it was desired to observe the flow conditions, and it is still occasionally used for this purpose. The principal use of glass at the present time is for the construction of transparent panels in the sides or bottom of testing flumes. The use of transparent plastic for this purpose is not satisfactory because of its low modulus of elasticity. Where high strength is required, special strength glass may be obtained.

g. Sand. Models built of sand are common. Most river models are molded of sand, which may be paved with concrete to prevent erosion. In the case of a movable-bed model, erosion is, of course, desirable and necessary. Sand or gravel is also used

at the foot of spillway models to determine the erosive tendencies of spillway aprons. When used for this purpose, the material should be uniform rather than well graded. A well-graded material is not satisfactory since the fine fractions of the material move much more readily than the coarser portions. The fines are soon carried away, and only the large fragments of rock are left. When this happens, it is practically impossible to duplicate tests since the material in which the erosion occurs has changed its characteristics. Sand has been partly superseded for the construction of movable-bed models by lighter materials such as lignite, haydite, sawdust, and other light materials.

2. Building Models of Structures. The actual building of models of hydraulic structures after the scale has been determined and the material chosen is principally

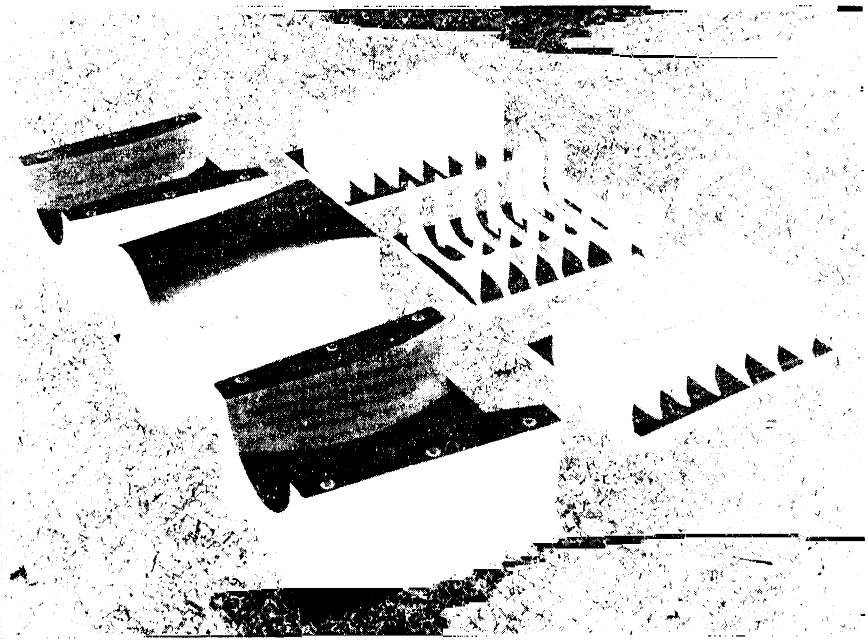


FIG. 29.—Molds for the Pyralin transition section of the discharge conduit, Sardis Dam model. Scale, 1:25. (*Courtesy of U.S. Waterways Experiment Station.*)

a matter of obtaining sufficiently skilled workmen and seeing that the model is built to the required specifications. It is important that models of structures be built accurately at those points where rapid changes in direction of flow occur, such as weir crests and gate slots, since changes in direction govern pressure distribution and discharge. This fact is especially important when the discharge characteristics of a structure are investigated.

Piezometer openings are a frequent source of trouble if not carefully made. The opening should present no surfaces that would change the flow in any respect, since changes in direction of flow cause changes in pressure. The openings should be flush with the surface and have no projecting burrs, either beyond the opening or inside it. The opening should be slightly beveled or rounded rather than square-cornered, since square-cornered recesses cause contraction effects at their downstream side. Disturbances may also be minimized by making the opening as small as will serve the purpose.

Models are usually set in a flume or channel having sides high enough to hold the headwater at proper elevation. Occasionally a special chamber is constructed to supply the appropriate conditions for an unusual structure.

It is necessary to provide a supply of water and a means of regulating and measuring it. It is usually most convenient to measure the flow when it enters the model. The triangular weir in a stilling chamber is frequently employed, although a diaphragm orifice or a Venturi meter may be used equally well. A device is necessary at the lower end of the model to provide the necessary back pressure or tail-water depth. This may be a valve or adjustable tailgate.

A number of actual models have been illustrated earlier in the chapter.

3. Methods of Building River Models. *a. General.* General requirements for the construction of river models are ample space, sufficient water supply, and protec-

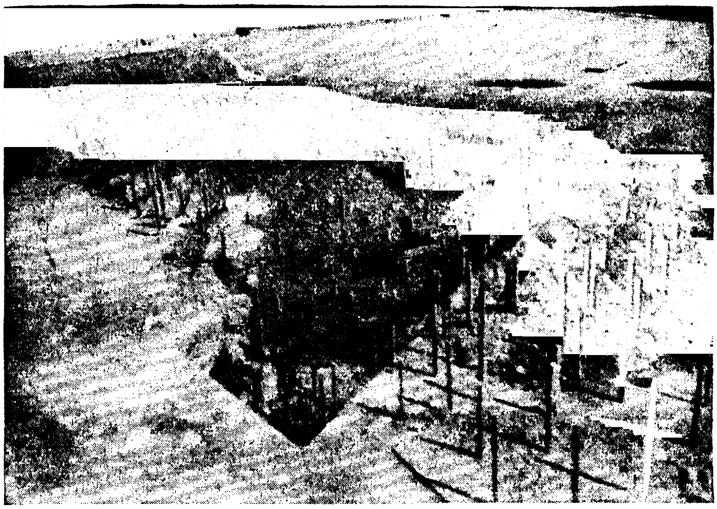


FIG. 30.—Construction of topography at Rock Island Dam Site. Dowels were set with their tops at ground level. Sand was filled in around them, and covered with concrete which was molded to their tops. (*Courtesy of Prof. C. M. Allen, Worcester Polytechnic Institute.*)

tion against inclement weather. On the assumption that these requirements have been met, the general procedure is similar for all cases. The approximate outline of the model is laid out, and a waterproof basin is fashioned. This may be accomplished by excavating a tight clay soil out of doors or by building a concrete or brick wall on a concrete floor inside. The desired topography may be reproduced by several methods. Pegs may be set with their tops at the elevation of the ground surface (Fig. 30) and the topography molded up to them. Sheet-metal templates cut to represent cross sections may be set and the bed of the river molded to their tops (Fig. 31).

River models require the same general provisions for water supply and tail-water regulation as do structure models. The normal rating curve of a stream may be simulated with these two control devices. The discharge is regulated and measured by means of the control weir at the inlet, and the gage height is controlled by means of an adjustable tailgate.

b. Fixed-bed Models. Fixed-bed models can be constructed by the methods previously described. The topography should be molded in sand or gravel to within

FIG. 31.—1:60 scale models of Union Village and Knightville spillways, showing method of construction. (*Courtesy of Prof. C. M. Allen, Worcester Polytechnic Institute.*)

FIG. 32.—Molding a movable-bed model. Chain of Rocks Channel Improvement Study, Upper Mississippi River. Scales: horizontal, 1:600; vertical, 1:125. (*Courtesy of U.S. Waterways Experiment Station.*)

½ or 1 in. of the final surface and the remainder placed in concrete. Occasionally when only temporary service is required, it is possible to prevent erosion for a short time by molding the topography in sand. The surface may then be dusted lightly with dry cement. The cement settles lightly on the sand and binds the top layers into a thin skin. The protective surface when set is $\frac{1}{32}$ to $\frac{1}{16}$ in. thick and, of course, will not support the weight of an observer. In order to prevent this covering from being damaged by breaking up or floating away when the model is filled with water, it is necessary to perforate it with small holes rather closely spaced, say 6 in. to 1 ft apart. This method of fixing a sand bed is extremely rapid.

c. Movable-bed Models. The construction of movable-bed models usually requires the preparation of two levels of topography. The lower level should be more or less permanent and once constructed should need no further attention. It may be built

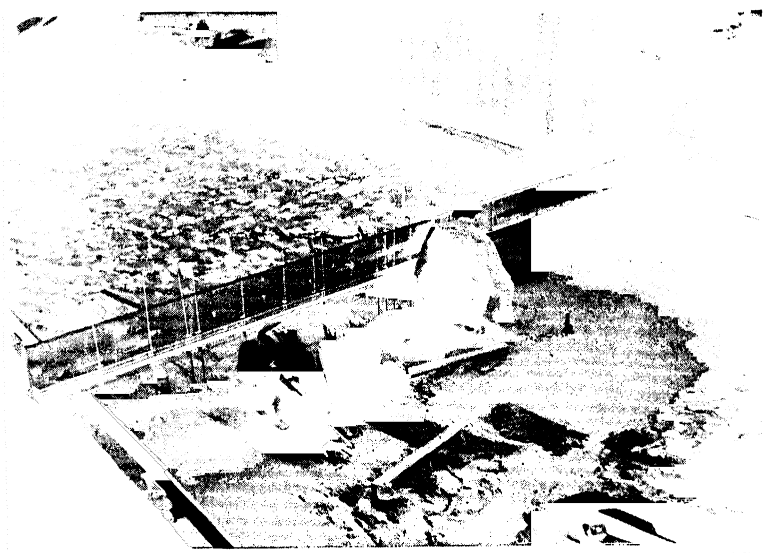


FIG. 33.—Molding the bed of a 1:150-scale model of Watts Bar Dam, T.V.A. (*Courtesy of Tennessee Valley Authority.*)

as a fixed-bed model. The upper level is expected to be eroded during the progress of each test, hence will need to be replaced. The material composing this layer should be small enough and light enough to move readily with the tractive forces available. Molding of the bed is guided by templates supported on rails at either side of the bed of the model. Figure 32 illustrates the process.

A modification of this method consists of the use of an adjustable template which is composed of a stiff beam carrying a number of rods adjustable to any elevation. The model bed is molded to the bottoms of these rods (see Fig. 33). For models of this type, it is necessary either to keep the original templates on hand and ready for immediate use in remolding the bed, or else to utilize the adjustable template described. When the latter course is employed, it is desirable to have two or more of the adjustable templates available so that replacing of the bed for another test may be expedited.

5. EQUIPMENT

Certain items of equipment are indispensable to a hydraulic laboratory. These indispensable items are an adequate water supply, plenty of space, suitable measuring instruments, and last, but not least, a thoroughly qualified staff.

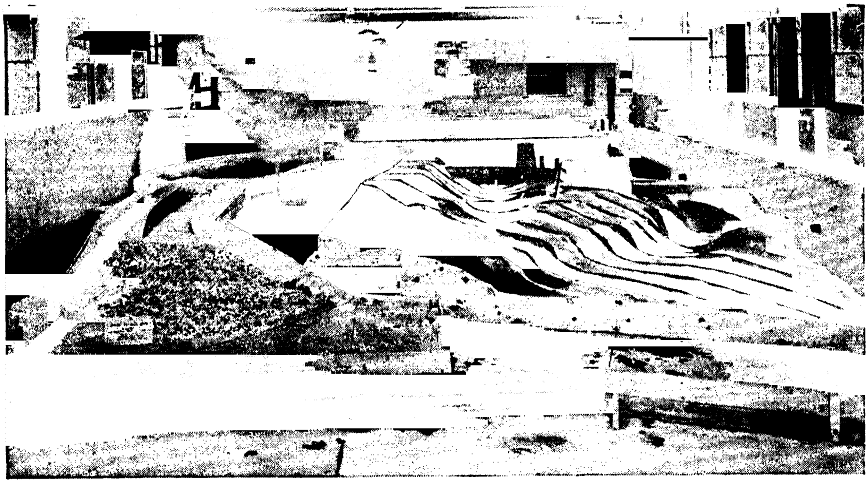


FIG. 31.—1:60 scale models of Union Village and Knightville spillways, showing method of construction. (*Courtesy of Prof. C. M. Allen, Worcester Polytechnic Institute.*)

FIG. 32.—Molding a movable-bed model. Chain of Rocks Channel Improvement Study, Upper Mississippi River. Scales: horizontal, 1:600; vertical, 1:125. (*Courtesy of U.S. Waterways Experiment Station.*)

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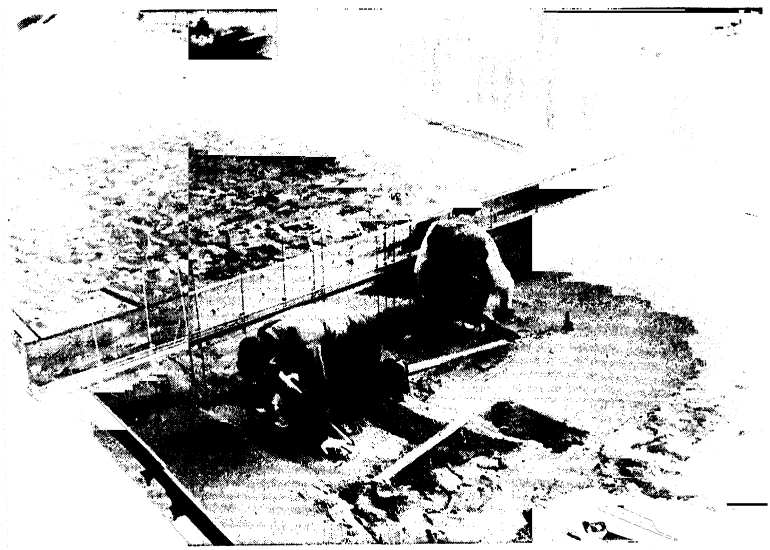

Fig. 33.—Molding the bed of a 1:150-scale model of Watts Bar Dam, T.V.A. (*Courtesy of Tennessee Valley Authority.*)

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1. Water Supply. The amount of water to be supplied will necessarily depend on the type of work that is planned for the laboratory and somewhat on the source of supply available. Large-scale spillway models demand a much greater water supply, for example, than river models. An ordinary river model of average size will usually operate with less than 3 or 4 cfs, but a model of three spillway gates 40 ft by 40 ft at a scale of 1:25 will require in the neighborhood of 40 cfs. A laboratory is fortunate if it can draw its supply by gravity from a reservoir, a river, or an irrigation canal, under such circumstances that sufficient head is available. Most installations, however, depend on pumped supplies. A circulated system of water should include a supply reservoir large enough to supply the pumps for 5 or 10 min continuous pumping at the maximum rate. This should be sufficient capacity for most purposes.

Since water delivered direct from a pump is apt to vary considerably in pressure and discharge because of fluctuations in voltage and the level of the supply sump, it is desirable to supply most models by gravity. This can be accomplished by pumping the water to an elevated supply tank equipped with a long overflow weir which will serve to iron out minor fluctuations in discharge. The weir length should be approximately 20 ft for each cfs pump capacity. This length will supply an automatic control of sufficient accuracy. The supply tank should be provided with separate outlets for all the models that are to be in operation simultaneously. Failure to recognize this requirement has resulted in considerable difficulty in many laboratories. Two models operating from one supply line are not independent of one another because of the friction loss in that portion of the supply line which is common to both models.

If the total water supply is large, it is more economical to supply it with a number of small pumps, since the total capacity of the system will only be required a relatively small portion of the time. In many instances, pumps have been so arranged that they may be connected in parallel to deliver the maximum discharge to the supply tank or in series to supply lesser amounts of water at an unusually high pressure for special purposes. This feature should not be overlooked in the initial design of a laboratory.

2. Space. There should be sufficient space for the construction of any models required. The space available should be level and under cover, except where the climate is unusually mild. Wind, rain, and excessive heat and cold interfere materially with experimental work. A large area unencumbered by fixed equipment, which can be reached by pipe lines from elevated water-supply tanks, is in general the best suited for hydraulic experimentation. It may or may not be covered, but if economically feasible, a protecting roof is extremely desirable. Provision must be made for carrying waste water back to a supply sump, if a pumped circulating supply is used.

Descriptions of American and European laboratories have been published in several places[1,2,3] and will be found useful in planning a new one. "Hydraulic Laboratory Practice"[1] although old contains many floor plans.

3. Flumes. It is convenient to test models in permanent flumes equipped with a weir or other measuring device at the inlet and a regulating tailgate at the outlet. Flumes used for tests of structure models are usually about square in cross section, although the width is frequently less than the depth. The dimensions may vary from 1 by 1 ft to as much as 10 by 10 ft. A common size is 3 ft by 3 ft. A flume should be long enough to provide space for baffles to smooth out the flow, an unobstructed approach to the model equivalent to at least six or eight times the depth of flow (more length is better), and sufficient length below the model to observe erosion conditions

[1] Hydraulic Laboratory Practice, A.S.M.E., 1929.

[2] Bureau of Standards, Hydraulic Laboratories in the United States, Hydraulic Laboratory Bulletin, Series B, first revision, October, 1935.

[3] HOOPER, L. J., Representative Hydraulic Laboratories in the United States and Canada, *Jour. Boston Soc. Civil Eng.*, **25**, No. 1, sec. 2, 1938.

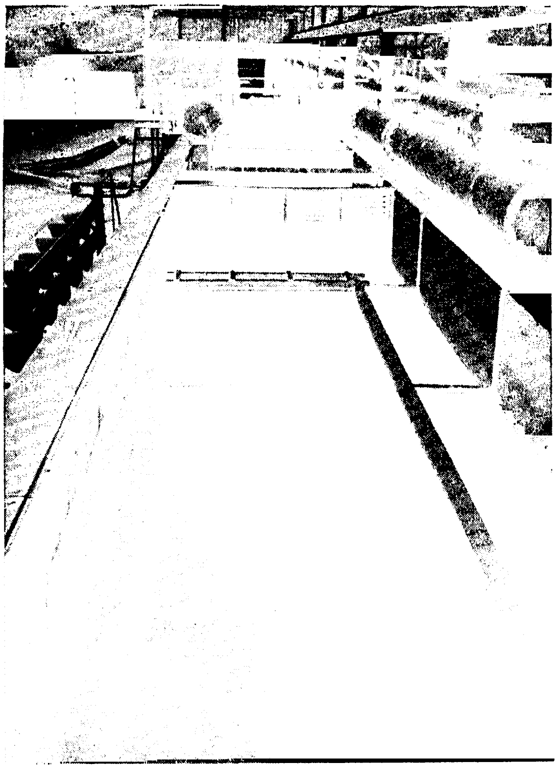


FIG. 34.—Upper end of 8-ft by 8-ft flume in laboratory of T.V.A. Note type of baffles used. (*Courtesy of Tennessee Valley Authority.*)

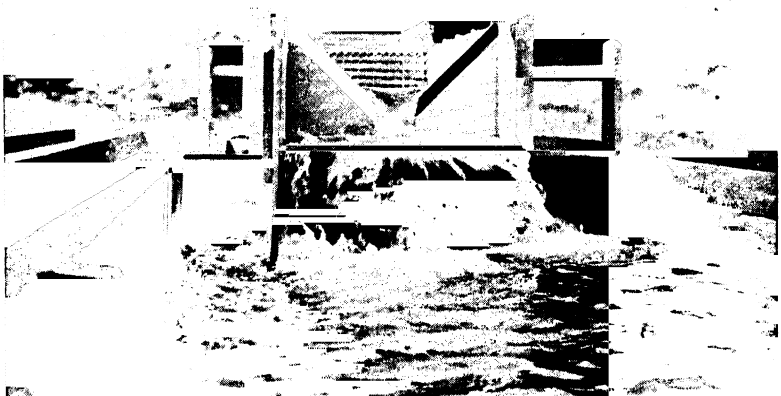


FIG. 35.—Triangular weir supplying water to the forebay of a river model. Note baffles at left for smoothing out the flow. (*Courtesy of Tennessee Valley Authority.*)

and to provide proper simulation of the getaway conditions. It may be desirable to include a sand trap to stop the loss of sand or gravel from the flume while erosion tests are being carried on. A flume may be made of steel, concrete, wood, or sheet metal supported by wooden or steel frames. Steel is probably the most satisfactory for all conditions since it has the most rigidity. It does not dry out or warp as wood does, it retains its shape better than sheet metal, and it is more flexible to use than concrete. Glass panels are frequently provided for observation purposes. A few flumes have been so built that they could be tilted throughout their entire length. This type requires expensive construction and is used only for special purposes, such as transportation of bed load or a study of the effects of distortion by tilting an entire model.

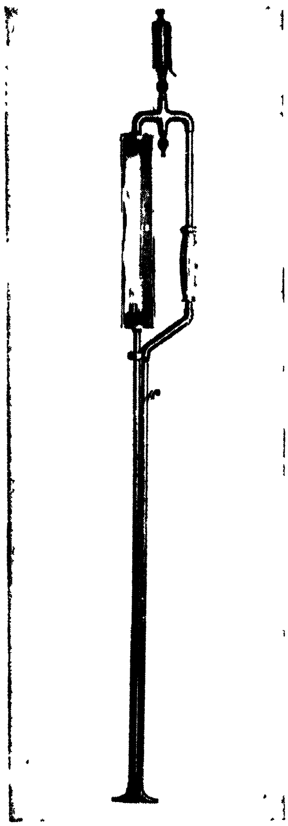


FIG. 36.—Bentzel velocity tube. Measures velocity from 0.3 to 5 fps.

River models may also be constructed in flumes which are generally much wider than their depth. A typical flume for river models may be 1 ft deep, 20 ft wide, and 80 to 100 ft long. It may be built simply by constructing a wall of brick or concrete blocks on a smooth level floor and molding the model inside the basin thus formed. If frequent changes in the model outlines are needed, precast concrete blocks may be used with bolts cast in to support rails for templates.

4. Measuring Instruments. The results obtained from hydraulic models are no better than the accuracy of the instruments used for measuring. Discharges, velocities, lengths, elevations, and time must all be measured, and suitable equipment must be available.

a. Discharge. Rates of discharge may be measured by weirs, Venturi meters, or diaphragm orifices. The Venturi meter and diaphragm orifice do not cover a great range of discharge. A meter that is suitable for measuring large quantities shows a very small head differential for small quantities. If this type of meter is used, it will probably be necessary to use a by-pass meter for small rates of flow. The orifice has an additional disadvantage in that it loses a considerable amount of head, which is undesirable when water must be pumped or drawn from an elevated tank. Weir boxes with triangular weirs at the ends have been found suitable by a number of laboratories.

In addition to the portable measuring devices, such as weirs and orifices, each laboratory should be equipped with at least one standard volumetric or gravimetric measuring standard. If a laboratory is being built new, it is easy to make provision for a volumetric measuring basin in the original design. Small rates of discharge may be measured with standard platform scales and easily constructed sheet-metal tanks. Portable measuring equipment should be calibrated against the primary standard at regular intervals.

b. Velocities. A number of instruments have been developed for measuring velocities in hydraulic models. The Pitot tube may be used where velocities are high enough to give a suitable head reading. For measurement of velocities in open-channel models, which are usually of the order of 1 fps, miniature current meters

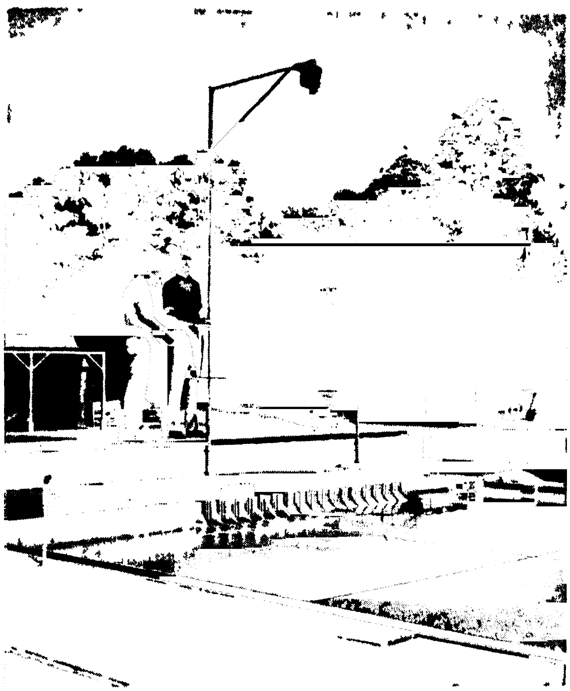


FIG. 37 Photographic method of measuring surface velocities (Courtesy of Tennessee Valley Authority.)

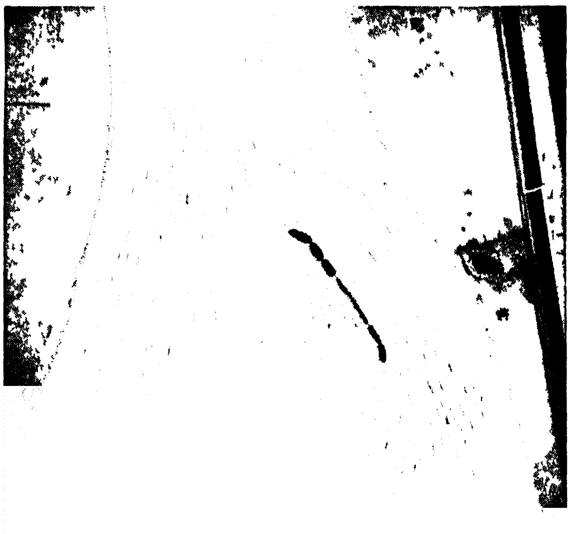


FIG. 38.—Typical picture showing results of photographic method of measuring surface velocities. (*Courtesy of Tennessee Valley Authority.*)

having propellers approximately 1 in. in diameter or the Bentzel velocity tube are very satisfactory. Figure 36 illustrates the Bentzel velocity tube.

Surface velocities may be measured photographically. This has been accomplished by sprinkling confetti on the water surface and photographing it from vertically above with a time exposure varying from ½ to 2 sec. The confetti shows up as streaks on the pictures. The streaks are proportional to the surface velocity. This method requires an accurate determination of camera shutter speed. It is illustrated by Figs. 37 and 38.

c. Elevations. Water-surface slopes must be measured very accurately at times. An engineer's level in good adjustment will usually be found satisfactory in connection with the standard hook gage reading to 0.001 ft. In special cases, it may be

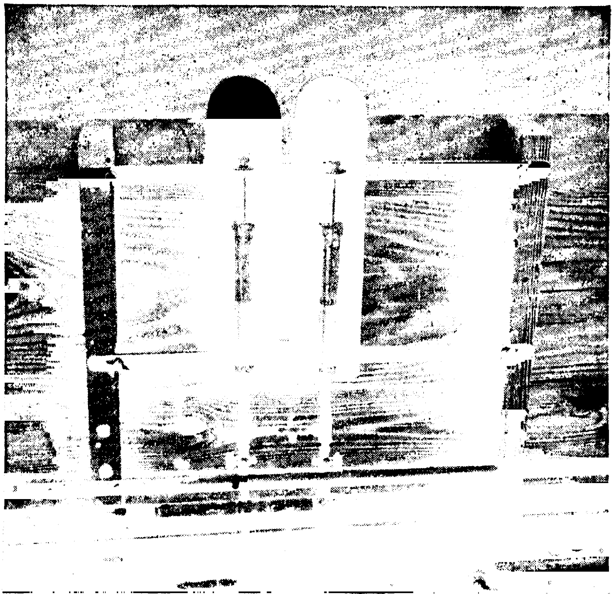


Fig. 39.—Microgages reading to 0.0001 ft used for accurate determination of water-surface slopes. (*Courtesy of Tennessee Valley Authority.*)

desirable to read elevations even closer. Gages have been built reading to 0.0001 ft, but there is no commercial level available for determining zeros with that accuracy. Such gages are generally used to determine relative elevations and are referred one to the other by reading the elevation of a common pool of water (see Fig. 39).

d. Time. The measurement of time offers no particular problems. Any of the standard measuring devices for measuring time are satisfactory in model work. Well-adjusted pocket watches, stop watches, and electric clocks may be used. Occasionally a chronograph is desirable for special work.

5. Photographic Equipment. It has been found that photographic records of laboratory tests are extremely valuable. The equipment of a laboratory should include at least one hand camera of the ground-glass focusing type with interchangeable holders for cut film or film pack, double extension bellows, supplementary lenses for long and short focal lengths, and a substantial tripod. The picture size should be large enough so that contact prints may be made without the necessity for enlargements. In addition, frequent use will be found for a 5- by 7-in. or 8- by 10-in. view camera and for a 16-mm motion-picture camera. It is also convenient, although

not absolutely necessary, to have a darkroom equipped for the emergency development of negatives.

6. Personnel. An expert staff is the most vital factor in the success of hydraulic-model experimentation. The engineer in charge of a hydraulic laboratory must have a thorough knowledge of model laws, their applicability, and their limitations. He should have a good understanding of all the sciences basic to engineering. He should also be sufficiently acquainted with construction practice to recognize the practical difficulties involved. The same criteria apply in lesser degree to the subordinate members of the staff.

A small shop containing tools for working in both wood and metal should be a part of the laboratory equipment and should be staffed by the best machanics and carpenters available. It cannot be overemphasized that the success of a model-testing program depends almost entirely on the construction of the models, the execution of the tests, and the interpretation of the results. These require skilled workmen, adequate equipment, and an expert, highly trained staff of engineers.

6. GENERAL

1. Results to Be Expected. There has been a tendency toward regarding models as the solution of all hydraulic problems. This is not true, although much information can be obtained from models that cannot be determined in any other way. In structures, or hydraulic phenomena in which friction plays a relatively unimportant part, the hydraulic model gives excellent results if not built to too small a scale. Magnitudes and directions of currents, dynamic forces, and discharges may be determined with considerable accuracy. Although extensive data on the performance of the prototype in comparison with model tests are not yet available, such data as have been presented indicate that the agreement of models of this type with the results of the prototype is excellent.[1]

Models of river channels may be expected to indicate the probable direction and magnitudes of velocities, the presence of undesirable eddies, tendencies toward scour and deposit, and at least an approximation of backwater effects. These results can be obtained from models with complex outlines and boundary conditions which are entirely impossible to analyze by any other means.

2. Operation of Models. Little needs to be said about the details of operation of models. Each one brings its own problems. In general, however, observations should be made in sufficient numbers to ensure accuracy. Gages should be read several times for each determination of water-surface elevation because of the fluctuations in water level which always occur due to turbulence and the possibility of storage of water between two points of observation. Surges in long channels also cause fluctuation of water level.

It is important to make duplicate tests occasionally in order to be sure that the test procedure has not changed. An apparent variation in results may sometimes be traced to an unconscious change in testing methods. In general, proper operation of a model will follow a complete and proper understanding of the problems to be investigated, the results desired, and the limitations of the method used.

Barnes and Jobes[2] have written an excellent description of the methods used at Case School of Applied Science in testing models of the Muskingum Conservancy District dams.

3. Limitations of Models. The limitations of models should be understood as well as their possibilities. Since the laws governing hydraulic phenomena conflict

[1] Conformity Between Model and Prototype, A Symposium. *Trans. A. S. C. E.*, **109**, 1–193, 1944 (includes a brief bibliography).

[2] BARNES, G. E., and JOBES, J. G., Construction and Testing of Hydraulic Models, Muskingum Watershed Project, *Trans. A. S. C. E.*, **103**, 227–244, 1938.

in their requirements, the model in general can be designed to solve only one sort of problem at a time. A model designed to act in accordance with Froude's law may duplicate the phenomena governed by that law very accurately, but the effects of friction, if they exist, may, and frequently are, not represented clearly. The result is apt to be a blind reliance on the results of the model tests without a proper interpretation of the modifying influences of friction. Conversely, a model designed to simulate friction forces may not be correct with respect to gravity forces. The operation of surface tension is not represented in either of these cases. At times it may be necessary to build separate models in order to study different phases of the same problem.

Phenomena involving waves are extremely difficult to study by means of models, particularly when they occur in rivers or harbors, because of the small scales necessary and the impossibility of distorting waves. Gravity waves in the prototype, when scaled down, may become capillary waves in the model. These two types of waves follow different laws, and transference of capillary-wave phenomena to the prototype leads only to absurdities.

Transference of the results on river models, both movable and fixed bed, to nature should be carried out with extreme caution, with due regard being given to the conditions of the tests. Indications of scour and deposit are usually qualitative only. The distribution of velocities, particularly in distorted models, should be utilized with caution, and even water surfaces in river models should be accepted only after careful consideration.

4. Costs. Although model testing is a standard method of procedure in the design of important structures, the costs of model tests are not widely known. The only published costs are those of Barnes and Jobes[1] for the model tests on the dams of the Muskingum Conservancy District. Here, model tests were made on structures whose construction cost was approximately $23,000,000. The cost of the model tests including overhead was approximately $42,000, about 0.2 per cent of the total cost. The authors point out that the figures must be used with caution because model studies are likely to be unrelated in character. The costs given are stated to be: "for a concentrated extensive program of studies generally similar and related, under a carefully planned and rigorous schedule. Furthermore, for these studies, work ceased the moment major decisions on design could be made, and the models were built in an economical manner for this immediate purpose only."

It is probable that tests on a single project or on a number of projects differing widely in character would be more expensive. There seems no doubt, however, that economies effected by model testing are more than likely to pay for the entire testing program.

[1] BARNES, G. E., and JOBES, J. G., Construction and Testing of Hydraulic Models, Muskingum Watershed Project, *Trans. A. S. C. E.*, **103**, 227–244, and following discussion, 245–255, 1938.

SECTION 25

HYDROLOGY

By Phillip Z. Kirpich

A brief discussion of hydrology is presented in this section. The scope of this book, together with space limitations, does not permit a complete treatment of the subject, for which reference is made to several splendid texts published during recent years on the broader application of the subject. The treatment of flood flows presented here is confined largely to the aspects of flood control which are more or less common to most of the foregoing sections of this book.

PRECIPITATION

1. Measurement. Nonrecording precipitation gages of good design provide a means of magnifying precipitation depth so that it is more easily observed. The standard U.S. Weather Bureau gage,[1,2,*] for example, with a receiver area of 50.3 sq. in. (8 in. diameter) and a measuring tube area of 5.03 sq in. (2.53 in. diameter), provides a magnification of 10. Automatic recording gages, which produce a chart record, are manufactured by several instrument companies. Results obtained from recording-gage charts should be checked by making periodic volumetric measurements in order to guard against faulty operation of the gage mechanism.

To minimize error, gages should be located to avoid poor exposure. The ideal location is a large, flat, open area free from large trees or structures which might intercept precipitation. On the other hand, to reduce wind effects, low barriers such as bushes or fences are desirable at a distance from the gage not less than twice their height.

Observers equipped with nonrecording gages usually record only daily amounts of precipitation and depths of snowfall on the ground. It is often advantageous to instruct them to record, in addition, the times of beginning and ending of heavy precipitation. Used in conjunction with the record of a near-by recording gage, this information is of great use in delineating storm characteristics and areal distribution of precipitation. There are at present in the United States 8,000 nonrecording gages and 3,000 recording gages, or about one gage per 275 sq miles.[3]

2. Sources of Data. In the United States, the principal source of precipitation data is the U.S. Weather Bureau. Many other agencies, both public and private, maintain gages. However, in a given locality it is always best to obtain data first from the appropriate regional office of the Weather Bureau, as this agency often publishes data obtained from other agencies. An excellent series of 79 river basin maps, locating both precipitation- and stream-gaging stations, is available.[4]

3. Adjustment of Records. A precipitation record is often obtained from a poorly exposed gage or from one whose location has been changed during the period of record. If the exposure was satisfactory for part of the record (at least 5 years or more), an adjustment of the remainder of the record can be made. The adjustment should be made by comparing the ratio between the recorded values of the annual or seasonal precipitation with the corresponding average value for a group of base stations in the vicinity. Compute the ratios for each year or season, and examine them for indications of any sharp changes or trends, which, if present, indicate modifications in the

* Superior numbers refer to items in the Bibliography at the end of this section.

regime of the station. In lieu of computing the ratios, the *double-mass-curve* technique[5] may be used. Either of these two methods is based on the fact that seasonal precipitation at stations in the same general locality are usually consistent with one another. However, this is not true for short-period precipitation, for which this type of adjustment is not recommended.

4. Estimates for Missing Records. Ratios established as described in Art. 3 can be used to estimate the missing portion of a record. As an example, available records for Stations A and B in the same general locality are as follows:

Period	Station A	Station B
1930	41.0 in.	
1931	40.2 in.	
1932	36.0 in.	
1933–1949 (avg)	37.5 in.	39.0 in.

The precipitation at Station B for the years 1930, 1931, and 1932 are estimated by proportion to be 42.6, 41.9, and 37.4 in., respectively.

5. Geographical Distribution. The following basic factors determine the amount of mean annual precipitation at a station on the earth's surface[6]: (1) latitude, (2) position and size of the continental land mass in which the station is located, (3) distance of the station from the coast or other source of moisture, (4) temperature of ocean and coastwise currents with respect to adjacent land masses, (5) extent and altitude of adjacent mountain ranges, *i.e.*, *orographic effects*, (6) altitude of the station.

Considering latitude alone, the generalized world pattern is composed of a series of belts corresponding to the circulation of the atmosphere. At the equator, there is a belt of relatively low pressure known as the doldrums, where intense solar radiation heats the air and causes it to expand and rise. Warm moisture-laden winds converge on the region and produce high precipitation from frequent thunderstorms. At about 30° north and south latitude, there are high-pressure belts, called the horse latitudes, where warm, dry air descends and precipitation is low. From about latitudes 35° to 65° interaction of the moisture-laden prevailing westerlies with cold, dry polar air generates storms of the frontal type (see Art. 9) and produces abundant precipitation. Convection-type thunderstorms also occur in this zone in summer, but produce less total precipitation than the frontal storms even though the short-duration intensities at individual points are generally much higher. From latitude 65° to the poles, dry polar air predominates increasingly and causes a decrease in precipitation.

The wide variations in mean annual precipitation in the United States (Fig. 1) are an example of the large departures from the generalized world precipitation pattern caused by factors (2) through (6). The latter factors produce even greater variations in Asia. High pressure builds up in winter over the cold continental land mass and produces a dry wind (winter monsoon) blowing outward to the Indian Ocean. The summer monsoon is wet and blows in the reverse direction. Thus the normal planetary circulation of the atmosphere is greatly modified by the large size of the Asiatic continent. The combined effects of the wet summer monsoon and the high altitude of the Himalayas in northern India, which it crosses, explain why this region is one of very high precipitation. (Mean annual precipitation at Cherrapunji, India, is 428 in.)

6. Seasonal Distribution. The belts of atmospheric circulation described in the preceding article move north in summer and south in winter (opposite seasons in the Southern Hemisphere), causing marked changes in the depth and type of precipitation for the various seasons of the year. In the United States, the variation in depth

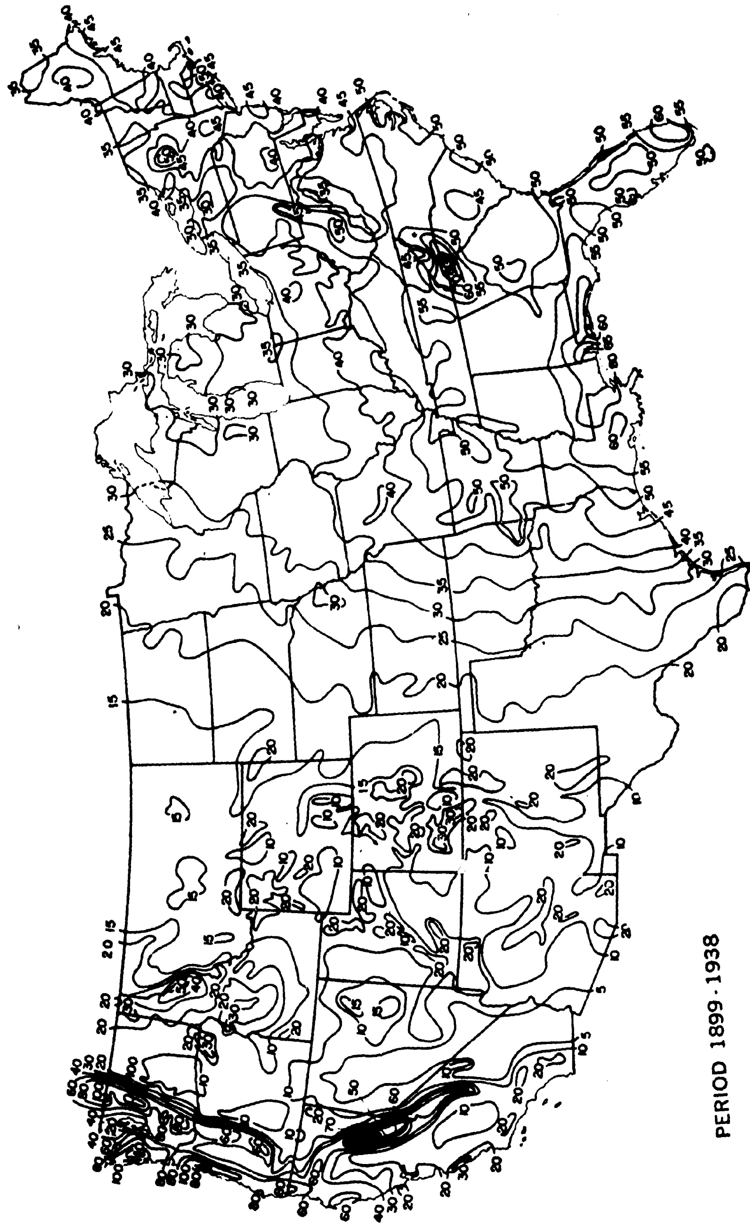


FIG. 1.—Average annual precipitation in the United States.

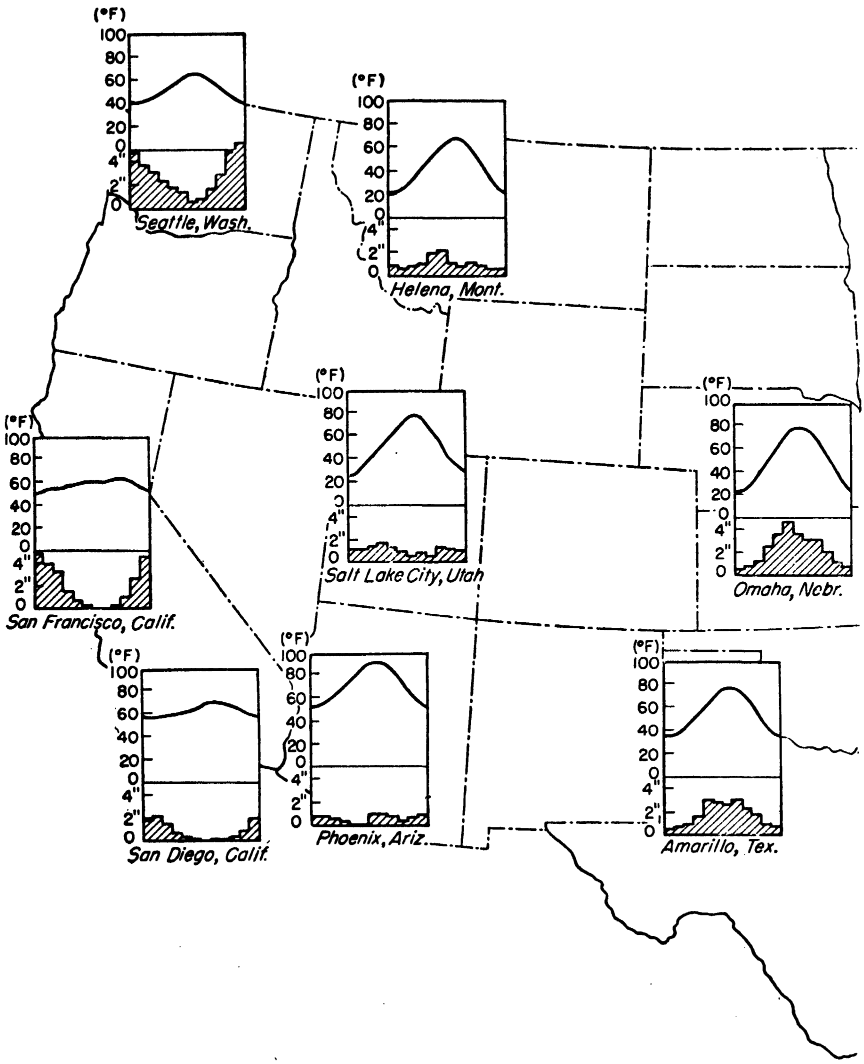
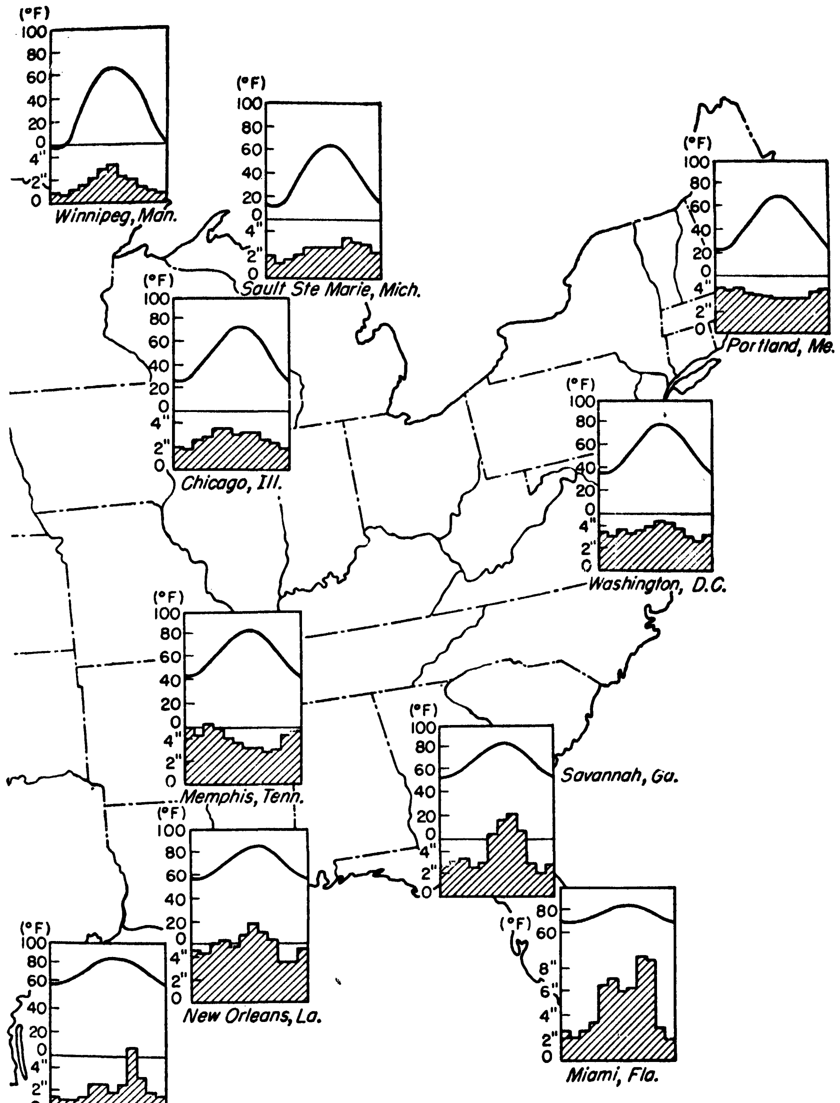


Fig. 2.—Monthly mean temperature and precipi-

between summer and winter is greatest on the West Coast (Fig. 2) because of two principal factors: (1) the northward movement in summer of the dry high-pressure belt of the horse latitude and (2) the presence of cool coastal water. The latter lowers the temperature below the dew point and hence reduces the moisture content of air carried landward by the prevailing westerlies. Subsequently, when this air strikes the warm land mass, dehumidification takes place, and almost all opportunity for precipitation is lost.



tation for selected stations in the United States.

The East Coast, on the other hand, shows a fairly uniform seasonal depth of precipitation. The origin of precipitation is distinctly different, however, in summer and winter. Summer precipitation is mostly of the convectional thunderstorm type and increases toward the south owing to increased summer convectional activity in that direction. Winter precipitation is almost entirely of the cyclonic or frontal type caused by the interaction of polar and tropical air masses. A further description of storm types is given in Art. 9.

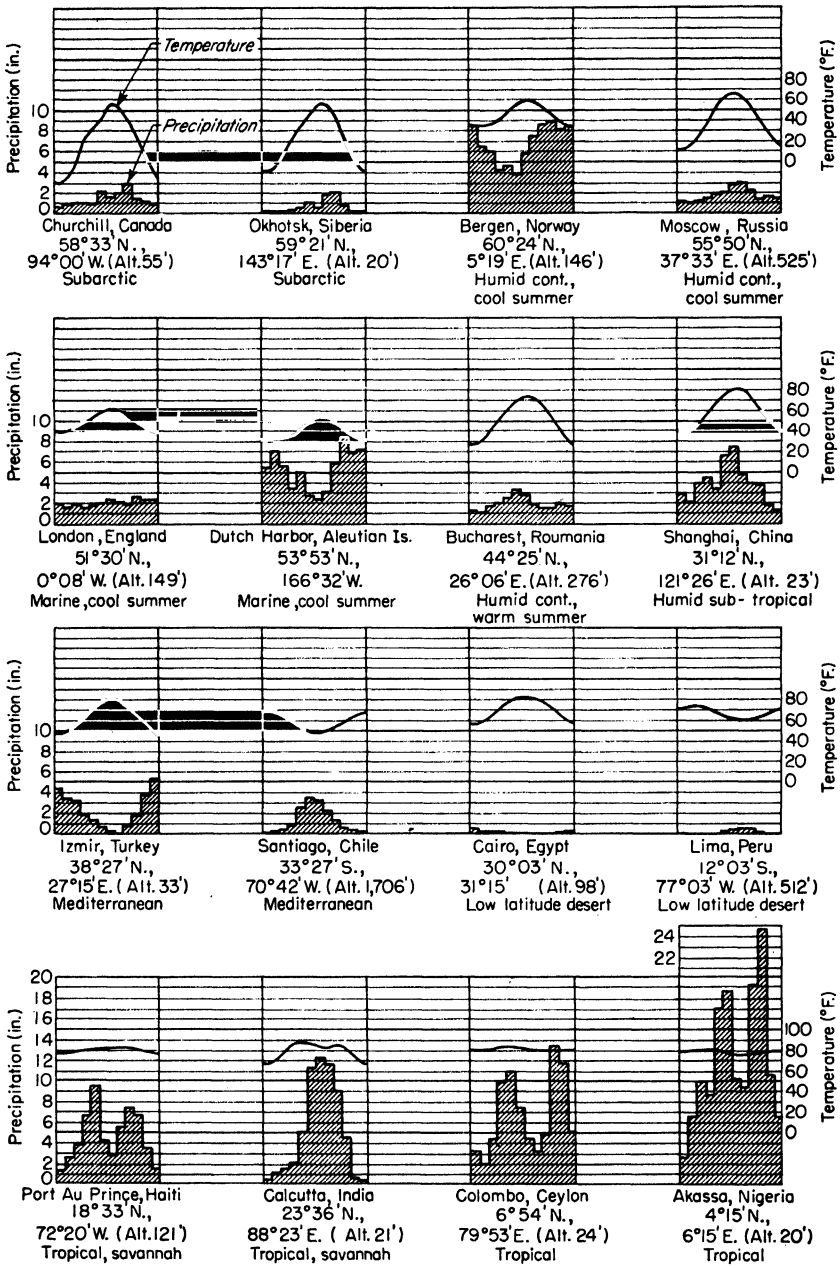


FIG. 3.—Monthly mean temperature and precipitation for selected world stations outside the United States.

In the interior of the country particularly west of the 95th meridian, still other seasonal patterns exist. In the Great Plains region, as at Omaha, Neb., summer thunderstorms produce high precipitation as compared with winter, when the region is covered by cold, dry polar air. In the Plateau Region, as at Salt Lake City, Utah, mountain ranges seal off incoming moist air during all seasons.

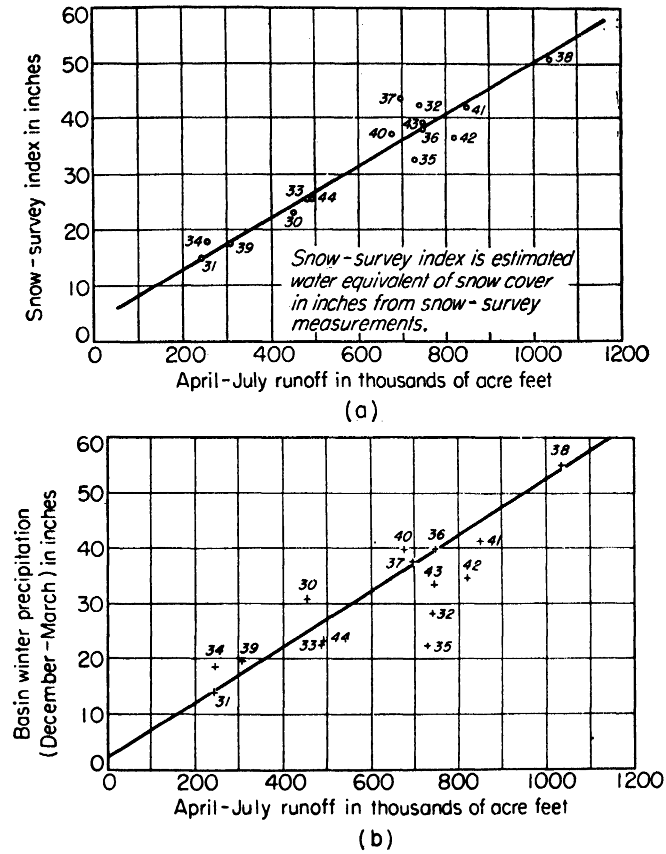


FIG. 4.—Comparison of runoff correlations for the Tuolumne River at Hetch Hetchy, Calif.[7] (a) Snow-survey data, (b) precipitation data.

Figure 3 gives monthly temperatures and precipitation for selected stations throughout the world, representative of climatic conditions varying from subarctic to tropical. Climatic types are after Trewartha.[6]

7. Snow. Investigations to date are insufficient for the determination of adequate relationships among the many variables involved in snow problems. However, satisfactory empirical relations can often be derived if there are sufficient snow-cover measurements and precipitation, temperature, and runoff records.

Figure 4 shows correlations between (1) a snow-survey index for April 1 and the ensuing April–July runoff, and (2) the basin winter precipitation and the ensuing April–July runoff. It is noted that the first correlation is much the better of the two. The snow-survey index is the estimated water equivalent on the basin in inches determined as follows:

1. Average water equivalents were measured along 11 snow courses, each at a constant elevation from 5,700 to 10,300 ft in the Sierra Nevada (see Ref. 8 for snow-surveying methods).
2. The basin was divided into three zones according to altitude.
3. The average water depth for each zone was computed by averaging the courses in that zone.
4. The average basin depth, or snow-survey index, is the weighted average of the zones, the weights being taken as proportional to the area of the zone.

Another type of correlation, shown in Fig. 5, gives the "degree-day factor" for use in short-period predictions. The degree-day factor multiplied by the weighted degree

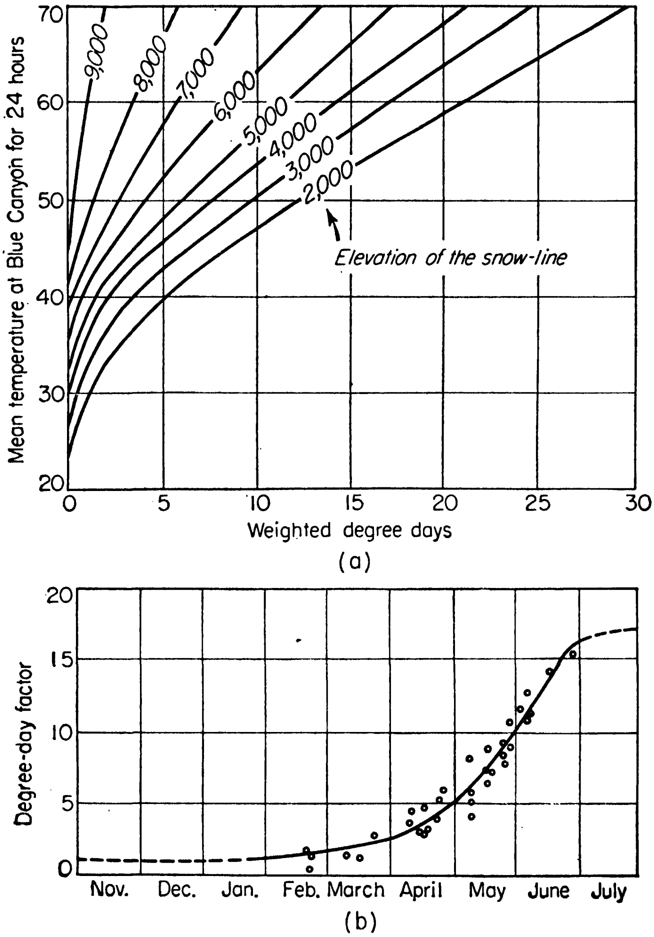


FIG. 5.—Snow-melt correlations utilizing degree-day factors.[45]

days gives snow melt in inches. Degree days are measured from 32F as a base; a day having an average temperature of 44F would be equivalent to 12 degree days. In Fig. 5(*a*), the mean temperature at an index station (Blue Canyon) and the elevation of the snow line (the lower limit of snow cover) are used as parameters to get "weighted" degree days. The latter, multiplied by the degree-day factor, which is a function of the date, gives snow melt in inches.

The setting up of design storms of storm rainfall is described in Art. 11. Rainfalls from winter storms are often critical for large areas, particularly if augmented by snow

melt. Because in this case we are interested in maximum rather than in average conditions, the degree-day method described in the preceding paragraph would give too low a value of the snow melt, and it is necessary to study the physics of melting snow more closely.

Light[9] and Wilson[10] have done this, Light presenting the formula

$$D = V[0.00736(T - 32)10^{-0.0000156Z} + 0.0231(e - 6.11)]$$

in which D is the snow melt in 6 hr in inches, V is the wind velocity 50 ft above the snow surface in miles per hour, T is the air temperature 10 ft above the snow surface in degrees Fahrenheit, e is the vapor pressure in millibars, and Z is the station elevation in feet above mean sea level. If Z is less than a few thousand feet, its effect may be neglected, and the formula reduces to

$$D = V(0.0074T + 0.023e - 0.37)$$

Light's formula, which is empirical in part, evaluates snow melt caused by turbulent heat exchange between the air (including water vapor in the air) and the snow.

Snow melt by rainfall can be computed from

$$D_r = \frac{P(T_w - 32)}{144}$$

derived from the fact that the heat of fusion of ice is 144 Btu/lb, in which D_r is snow melt in inches, P is rainfall in inches, and T_w is the wet-bulb temperature, assumed equal to the rain temperature. D_r is usually small compared with D; the wind, high humidity, and temperature accompanying warm rains cause more snow melt than the rain.

Studies of storms with large depths of snow melt in the Ohio River basin in western Pennsylvania showed D to be only 60 to 72 per cent of that indicated by the formula.[11] Unfortunately there are insufficient determinations at present to evaluate the percentages for general application. Studies for a particular basin, if justified, should include detailed meteorologic and hydrologic analyses of past storms and the resulting runoff to establish values of the variables in the formula for D during successive time periods. The runoff hydrographs should be analyzed to determine variation in infiltration (see Art. 24) with time, runoff due to rainfall, and runoff due to snow melt. If K is the percentage of snow melt D, given by the theoretical formula, the variation of its value with time can be determined and applied later in setting up design storms (see Art. 11) for maximum conditions. A value of K of zero during the early part of the storm might be logical, as ripening of the snow cover takes place before the maximum rate of snow melt is reached. By "ripening" is meant the process prior to the occurrence of melting, in which the temperature of the snow is brought up to the melting point and the capacity of the snow to hold liquid water is exhausted.

8. Droughts. The safe (or "firm") capacity of projects depending on stream flow must, as discussed in Art. 21, be based on records sufficiently long to include low-flow or drought periods. The word "drought" as used here is merely a descriptive term, denoting a period of lower than average flow. If the stream-flow record is too short, it may be extended by developing correlations with precipitation records as described in Arts. 19 and 20.

Thornthwaite,[12] in a discussion of the definitions of drought from an agronomical point of view, points out that it cannot be defined merely as a shortage of rainfall, because both the demand of vegetation for water (which varies according to the time of the year) and the moisture available in the soil must be taken into account. He has developed curves for "potential evapo-transpiration," or the amount of water

that would be evaporated and transpired by plants, if it were available. Figure 6 is an example showing that at Columbus, Ohio, in an average year there is a surplus of rain from December to May; from May to July the excess of potential evapo-transpiration over precipitation is withdrawn from soil moisture; from July to October there is moisture deficiency, *i.e.*, drought; and from October to December precipitation again exceeds potential evapo-transpiration, causing soil moisture accretion. It is seen that the magnitude of drought depends not only on the quantity of precipitation, but also on its relative timing with respect to potential evapo-transpiration. At Independence, Iowa, during an average year, owing to better relative timing, there is no moisture deficiency, even though the total annual rainfall is less than at Columbus.

While the moisture deficiency at Columbus does not mean crop failure during an average year, it does indicate that crop yields would be increased if the deficiency were eliminated. The possible countermeasures listed by Thornthwaite are supplemental

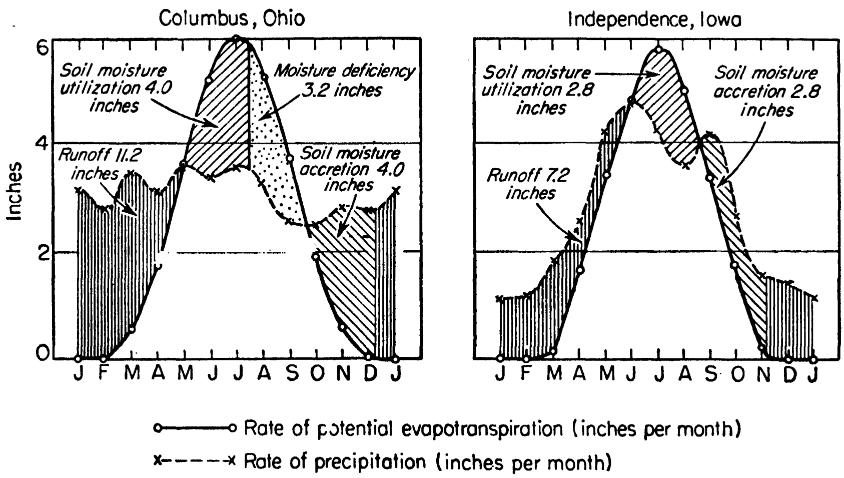


FIG. 6.—Potential evapo-transpiration and concurrent precipitation.[12]

irrigation and improved farming practices, which include adjustment of the crop calendar so that harvest will precede drought, and crop rotation to improve the soil structure and increase its water-storage capacity.

Acquaintance with the agronomist's point of view is of increasing importance to the hydrologist for two principal reasons: (1) If future scientific research and economic studies demonstrate the value of supplemental irrigation, even in "humid" regions like eastern United States and western Europe, it will be necessary to develop sources of supply to meet the new demand for water. (2) If farming practices are improved, actual evapo-transpiration prevailing at present will be increased so as to equal potential evapo-transpiration. Although crop yields will benefit, the net runoff to streams will be reduced (see Art. 15).

STORM RAINFALL

9. Storm Types. Certain features of the various types of storm should be considered in the design of flood-control and drainage structures, and spillways for dams. These features include season of probable occurrence, areal extent, frequency, and possibility of rapid succession of two or more storms.

Cyclones. The term cyclone denotes a storm area of low atmospheric pressure, roughly circular in shape, in which the winds blow spirally inward counterclockwise in

the Northern Hemisphere, clockwise in the Southern. There are two main types of cyclones each associated with a distinct storm type: the tropical cyclone (also called hurricane or typhoon) and the extratropical cyclone or frontal-type storm.

Tropical cyclones[13a] are comparatively small, violent storms originating in the doldrums belt about 15° of latitude from the equator. Hurricane winds, which often reach velocities greater than 100 mph, blow spirally inward about the center or eye of the storm. The resulting rapid convergence of warm moist air causes heavy precipitation. A total of 96.5 in. of rain fell at Silver Hill, Jamaica, B.W.I., in four days, during the passage of a hurricane. The diameter of a tropical cyclone varies from 100 to 600 miles. For data on frequencies and paths of hurricanes, see Ref. 14.

The *frontal-type storm* is associated with the extratropical cyclone. In fact, as stated by Bjerknes,[13b] the extratropical cyclone is formed, initially, by an unstable wave or eddy in the polar front, which forms the boundary of separation between northern polar air and southern tropical air. Extratropical cyclones vary greatly in size up to 1,000 miles in diameter and move generally eastward across the United States at a speed of 300 to 700 miles per day. As stated in Art. 6, almost all precipitation in winter on the East Coast of the United States is caused by frontal storms.

Thunderstorms are local atmospheric disturbances of short duration characterized by violent vertical air currents, gusty surface winds, torrential rain, lightning, thunder, and sometimes hail. The necessary conditions for thunderstorm formation are three: (1) the presence of a body of warm, moist air, (2) atmospheric instability in the zone overlying this body of air, and (3) an initial lifting agent. By "atmospheric instability" is meant a condition wherein the rate of decrease in temperature with height (lapse rate) exceeds the rate at which the warm, moist air mass cools by adiabatic expansion. The intensity of the storm depends on the magnitude of the three factors. The initial lifting agent is most commonly local thermal convection. Quiet, humid surface air over an intensely heated land area becomes warmer than surrounding air and causes an unstable condition in which the warm air rises and is replaced at the ground by cooler descending air. The latter also becomes heated and rises. If this unstable convective condition continues, towering cumulus clouds form and thundershowers occur. Local convectional storms usually occur following the hottest part of the day and produce a large percentage of the summer rainfall in the middle-latitude continents and almost all of the annual rainfall in the tropics.

A second cause of initial lifting is a well-developed cold front. A series of cold-front thunderstorms may develop as a continuous line (called a squall line) along the front. Unlike the convectional type, cold-front thunderstorms can occur at any time of the day and any season of the year, although they are rare during the winter months. If conditions are favorable, such a storm can produce large amounts of rainfall over a relatively wide area such as the storm of July 7–8, 1935, which caused record-breaking floods in the upper Susquehanna River basin in south central New York.

Another cause of thunderstorms is orographic lifting in mountainous areas, a factor tending to increase the frequency and intensity of thunderstorms in such areas. Intensity-duration relations for thunderstorms are discussed in Art. 10.

10. Point Rainfall. By point rainfall is meant rainfall at a single station as distinguished from rainfall over an area. For small areas of less than a few square miles, the areal rainfall and the point rainfall are nearly the same. For larger areas, it is necessary to average the rainfalls at more than one station (see Art. 11), owing to the fact that rainfall rarely occurs uniformly over the area in a given storm, particularly one of the thunderstorm type, which is the type producing the governing values for the design intensities of point rainfall.

Two types of problems can be outlined: (1) analysis of past storm precipitation data to give point or station rainfall vs. time; (2) determination of point rainfall vs.

time for design conditions. In the first type of problem it is often necessary to determine the distribution of precipitation at a nonrecording gage where measurements are made only once or twice a day. The time distribution can be shown graphically by either a hyetograph or a mass curve, as shown in Fig. 7. The hyetograph shows the average intensity during a selected time period (½ hr, in this case), while the mass curve shows the cumulative depth vs. time, the slope of the curve at any point giving the intensity. In many storms the percentage distribution of precipitation is the same at widely distributed points; hence the mass curve for a nonrecording gage can be determined by plotting the observed points and then sketching in the curve by comparing

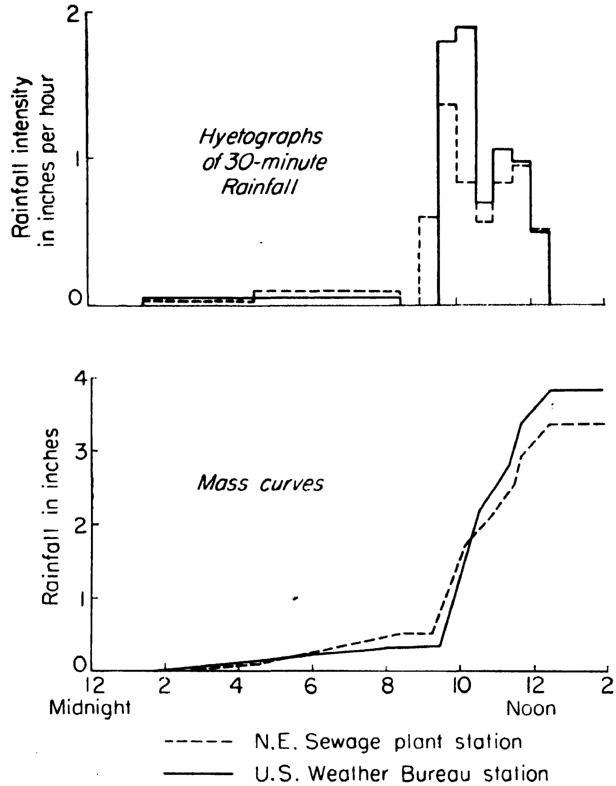


FIG. 7.—Hyetographs and mass curves for the storm of Aug. 3, 1950, at Philadelphia, Pa.

it with the known curve for a near-by recording gage. With complex storms, such as a cold-front thunderstorm, meteorological studies may be justified to give a closer approximation.[15]

The second type of problem concerns the setting up of a hypothetical storm for design conditions. This is often done so as to combine, for a given frequency of occurrence, the most critical features of several past storms into a single design storm, thereby simplifying design computations.

Two main steps are followed in setting up such a design storm: (1) the establishment of the intensity-duration relation for the design frequency selected; (2) the determination of the rainfall pattern or distribution with respect to time.

The first step would be done as follows: The maximum 5-min rainfall intensities during the storms of record are arranged in order of magnitude. If the record is 50

years long, the highest rainfall intensity has a frequency of 50 years, the second 25 years, the third 16.7 years, and so on. A smooth curve is plotted from which the intensity for any frequency can be determined. A similar computation is made for 10-min, 15-min, and other durations. Cross plots are then made giving curves of intensity vs. duration, with frequency as a parameter. Talbot, in 1891, found such curves to be of the form

$$i = \frac{a}{t + b}$$

where i is the intensity in inches per hour, t is the duration in minutes, and a and b are constants. Other mathematical expressions for the intensity-duration relation have been devised, but they have largely been superseded by Yarnell's studies in which he

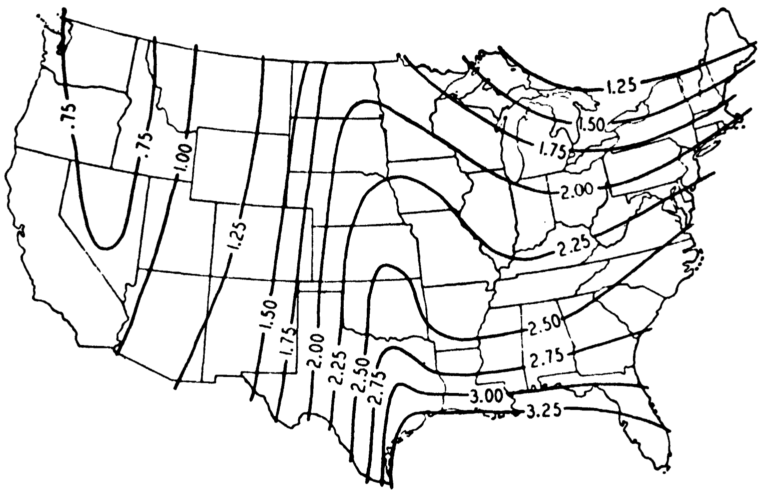


FIG. 8.—One-hour rainfall, in inches, to be expected once in 10 years in the United States.[16]

compiled point-rainfall depths and frequencies at 211 U.S. Weather Bureau stations equipped with automatic recorders based on records available through 1933.[16] He also prepared a series of 56 outline maps of the United States showing point-rainfall depths for various frequencies and durations. Analyses of individual station records

TABLE 1.—FREQUENCY OF 1-HR RAINFALLS

Frequency in years	Rainfall in per cent of 10-year rainfall	
	A	B
2	66	75
5	84	89
10	100	100
25	123	117
50	139	129
100	161	142

Use column A for places where the 1-hr, 10-year rainfall is less than 2.75 in., and column B for places where it is greater than 2.75 in.

brought up to date indicate that Yarnell's relations are valid with only minor changes, particularly for the eastern half of the United States. Figure 8 shows the 1-hr rainfall depths determined by Yarnell to have an average frequency of one occurrence in 10

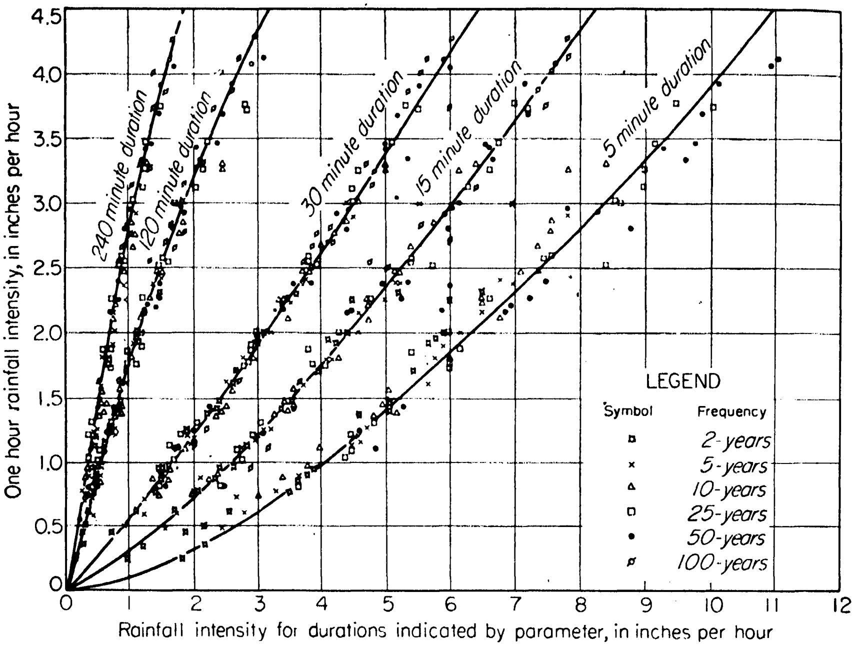


FIG. 9.—Relation of 1-hr rainfall intensities to intensities for durations of 5, 15, 30, 120, and 240 min.[17]

years. A study of the outline maps indicates that the 1-hr depths for other frequencies have the average values shown in Table 1 for any geographical location. The maximum deviation from the average for a particular location is generally no more than 5 per cent, which is sufficiently accurate for design purposes in most cases. With the 1-hr rainfall intensity established, the intensities for other durations can be determined from Fig. 9, which was also obtained from a study of Yarnell's maps.

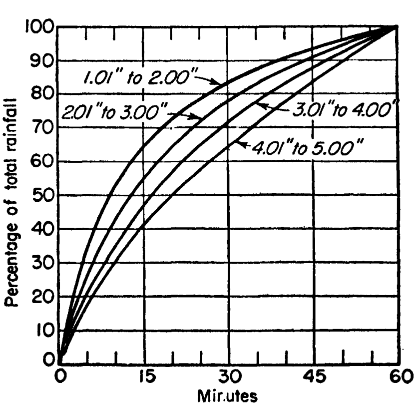


FIG. 10.—Typical mass curves of 1-hr thunderstorm rainfall.[18]

The second step, involving determination of the rainfall pattern, can be done as shown in Table 2. The assumption in this case, namely, that the 5-year, 15-min, 30-min, and other-duration rainfalls would all occur in the same storm, is a severe one and, therefore, conservative when used as a design criterion. The frequency of this storm would actually be less than once in 5 years. The sequence of depths, shown in the last column of Table 2, has been chosen so as to place the greatest intensities in the early part of the storm

in accordance with a recent Weather Bureau study[18] of heavy thunderstorms. Figure 10, taken from this study shows that, as the depth for the total storm duration increases, the rates for partial durations become more uniform.

11. Areal Rainfall. As stated in Art. 10, for areas greater than a few square miles, the point rainfall and the areal rainfall are usually different. If basin rainfall is defined as the average precipitation over a drainage basin during a given time period, there are two types of problems, as is the case of point rainfall: (1) analysis of past storms to give basin rainfall as a function of time and (2) determination of basin rainfall vs. time for design conditions.

TABLE 2.—COMPUTATION OF HYPOTHETICAL 5-YEAR DESIGN STORM FOR VICINITY OF LOUISVILLE, KY

1-hr, 10-year rainfall = 2.20 in. (Fig. 8)

1-hr, 5-year rainfall = 0.85 × 2.20 = 1.87 in. (Table 1)

Duration min	Intensity,[1] in./hr	Depth, in.	Depth increment, in.	Design-storm depths, in.
0	....	0		
			1.05	0.20
15	4.20	1.05		
			0.43	1.05
30	2.95	1.48		
			0.20	0.43
45	2.25	1.68		
			0.19	0.19
60	1.87	1.87		
			0.19	0.19
75	1.65	2.06		
			0.04	0.09
90	1.40	2.10		
			0.09	0.04
105	1.25	2.19		
			0.01	0.01
120	1.10	2.20		

[1] From Fig. 9.

The first problem is best analyzed with the use of the mass curve for point rainfall described in Art. 10. Various methods are used for weighting the point rainfalls to get the basin rainfall including simple arithmetic averages, the Thiessen method, and the isohyetal method. Simple averages are of satisfactory accuracy only when the rainfall varies slightly or when stations are equally spaced. In the Thiessen method, the weight of each station is proportional to its area of influence, which is best determined by plotting perpendicular bisectors to the lines joining adjacent stations, and measuring the basin area falling within the resulting polygon. Thiessen polygon lines are shown in the example in Fig. 23. In the isohyetal method, lines of equal rainfall are drawn by interpolating the station values (see Fig. 23). Straight-line interpolations are generally used unless known topographic influences indicate otherwise. From the isohyetal map, the average basin rainfall is computed by measuring the areas within successive isohyets. As a repetition of this process for many storms is laborious, the Thiessen method is often preferred even though it may not be so accurate. In some cases a successful compromise may be effected by modifying the Thiessen weights so as to give results in accordance with the isohyetal method as applied to only a sampling of the total number of storms to be analyzed.

The second problem, that of setting up a design storm giving basin rainfall vs.

time, is approached by detailed studies of past storms of record. These rainfall values are then used in design-flood computations for important structures, as explained in Art. 25. The underlying principle in these studies is that of *storm transposition*, in which it is assumed that past storms in a region can recur in a transposed position, which is critical with respect to the basin being studied. Unless the basin is narrow and elongated or has some other unusual shape, a further assumption is often made, namely, that the storm shape and orientation will conform sufficiently close to the shape of the basin so that reductions in the precipitation values need not be made. In mountainous regions, where orographic effects are pronounced, modifications can be introduced, as described in Art. 12.

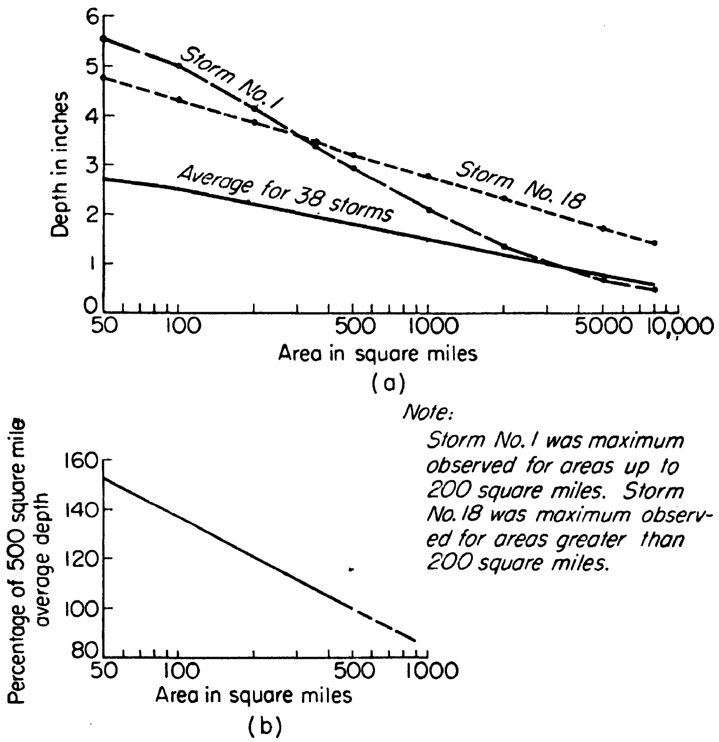


Fig. 11.—Area-depth relations, thunderstorm rainfall, Muskingum data (6-hr storms).

For areas up to about 500 sq miles in size, thunderstorm-type rainfall produces the governing values of mean basin rainfall. Unfortunately, there is a scarcity of data on the areal distribution of thunderstorm rainfall owing to the fact that a high density of rain gages—of one to from 10 to 20 per square miles—is needed to record adequately the sharp variations in thunderstorm rainfall, whereas the average density in the United States is only about one gage per 275 sq miles as stated in Art. 1. An exception to the general rule is the Muskingum River basin in Ohio, where a government-sponsored hydrologic research project has resulted in the installation of 500 gages in a basin area of 8,000 sq miles or one gage per 16 sq miles. Studies made of 38 relatively intense storms occurring during June, July, August, and September of the years 1937 through 1941 are described in Ref. 18. Figure 11(*a*) shows 6-hr-duration area-depth curves for areas of 50 to 8,000 sq miles for two of the most intense observed storms and for an average of the 38 storms. Figure 11(*b*) shows a curve giving the percentage increase

in average depth for areas 50 to 500 sq miles in size, in terms of the 500-sq mile depth. The latter curve is based on the average curve in Fig. 11(*a*). In the studies it was found that the deviations of individual storms from the curve in Fig. 11(*b*) increased as the size of the area decreased; however, as the average deviation was not large, the curve is believed to be of value.

Duration-depth relations for point rainfall, previously discussed, showed that in thunderstorms the higher intensities occur in the early part of the storm. However, when areal thunderstorm rainfall is considered, the effect of storm movement, causing nonsynchronization of the mass curves of rainfall at various points in the area, results in a more uniform rainfall pattern. A typical mass curve of thunderstorm rainfall for an area of 375 sq miles is shown in Fig. 12.

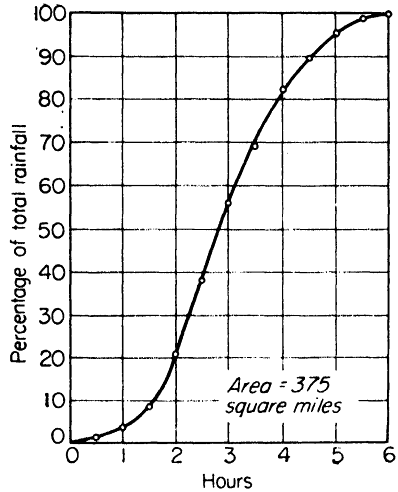


FIG. 12.—Typical mass curve of thunderstorm rainfall over a basin.

Figure 13, showing the maximum rainfalls observed in the United States for various durations and for areas of less than 1,000 sq miles, is a plot of data contained in Ref. 18, based on extensive cooperative studies by the Hydrometeorological Section of the U.S. Weather Bureau and the U.S. Army Engineers.

For areas greater than about 500 sq miles, the governing values of mean basin rainfall are produced by frontal cyclonic storms rather than by thunderstorms. The task

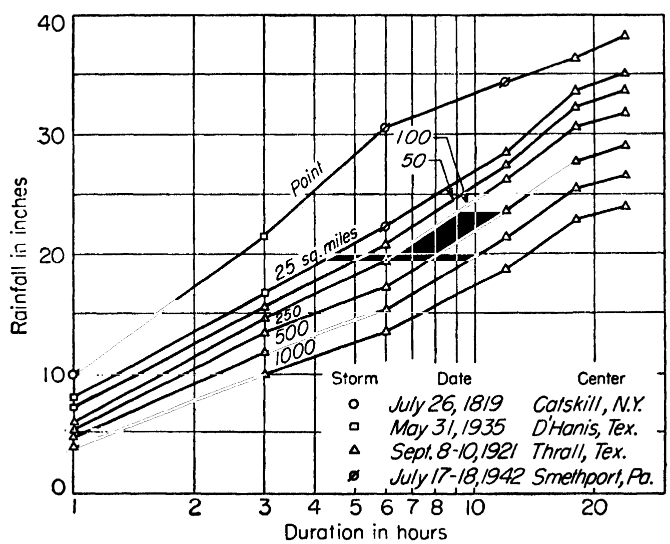


FIG. 13.—Maximum observed United States rainfalls for areas less than 1,000 sq miles.

of analyzing in a comprehensive manner the precipitation data for such large-area storms requires too great an expenditure of labor to be justified in the investigations for a single project. Fortunately, however, the extensive projects for flood control and

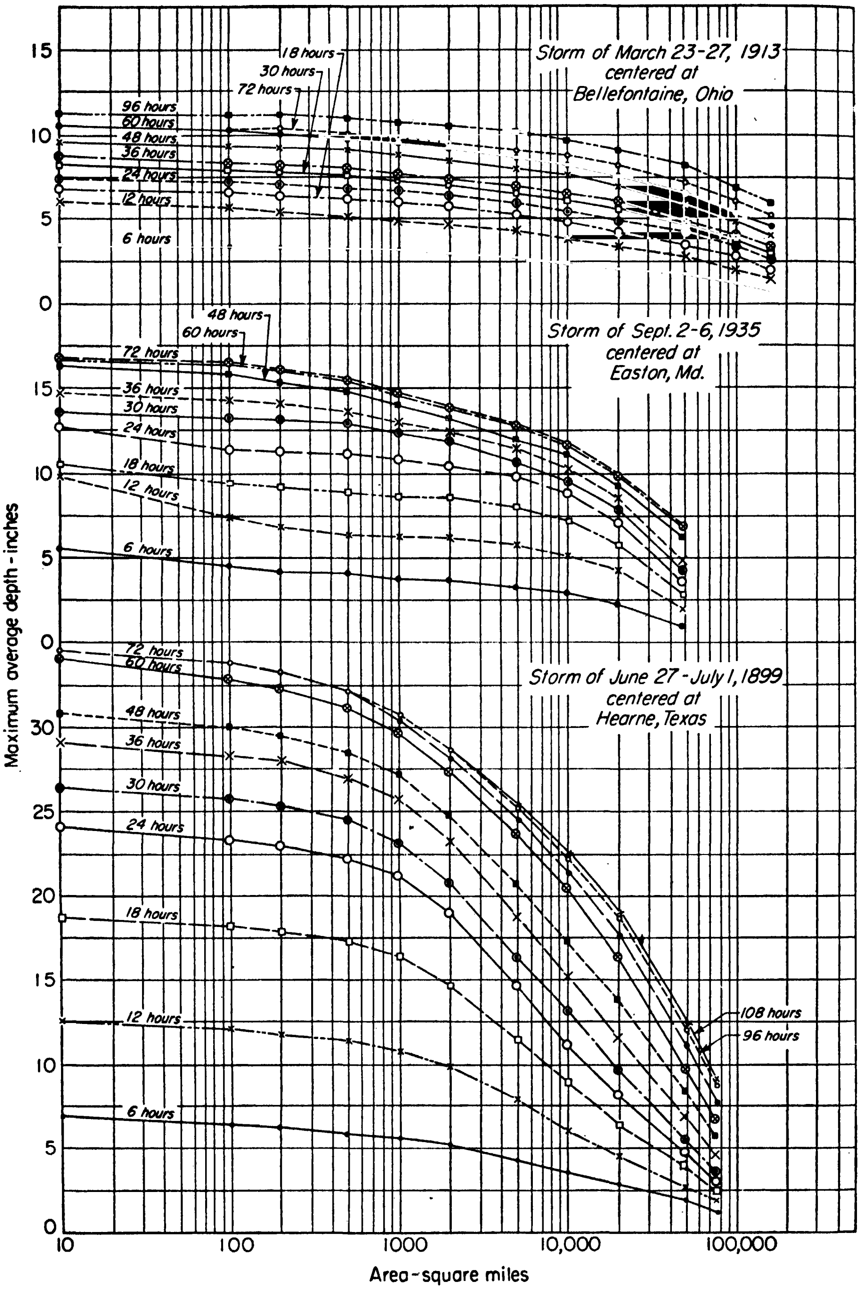


FIG. 14.—Depth-area-duration curves for three large-area United States storms.

river basin development of agencies of the Federal government have greatly stimulated studies of large-area storms. These studies, started in 1937 by the Corps of Engineers in cooperation with the Hydrometeorological Section of the Weather Bureau, have included detailed examinations of original precipitation records for both recording and nonrecording gages, and meteorological studies of the storm history to aid in the plotting of mass curves. Following the plotting of the mass curves, isohyetal maps and depth-area duration curves, similar to those shown in Fig. 14, were drawn. To date, the results for 394 major large-area storms have been published in Ref. 19, which is continually being revised and supplemented as additional studies are completed. Data for 150 of these storms are also presented by Bernard in Ref. 20 which includes a description by Hathaway of the methods of analysis used.

12. Orographic Effects. The orographic effect, or lifting of air passing over a mountain barrier, was listed in Art. 5 as one of the factors governing the depth of annual precipitation. When associated with thunderstorms or cyclonic activity, the orographic effect may greatly influence the areal distribution of storm rainfall. Therefore, in mountainous regions it is necessary to modify the simple procedure of storm transportation described in the preceding article.

A method[32a] that has been used with some success is to express storm rainfall at various stations in the basin as a percentage of the mean annual precipitation. If this is done, it is often found that the recorded rainfall depths tend to be equal percentages of the mean annual rainfall in spite of the orographic effects which cause large differences in the depths of both storm rainfall and mean annual precipitation. Isopercential lines (lines of equal percentage) can be drawn and, in the storm transposition, the pattern of the isopercential lines, rather than of the isohyetal lines, can be transposed. Linsley[21] lists the following minimum meteorological conditions before consideration of the foregoing theory can be applied: (1) the inflow directions of storms crossing the area do not vary excessively; (2) the air-mass characteristics are reasonably the same from storm to storm; and (3) the area under consideration is small enough so that variation in latitude of storm tracks does not affect the distribution within the zone.

RUNOFF

13. Stream Gaging. Two steps are ordinarily followed in measuring the flow of natural streams: (1) a record of stage or water level at a given cross section is obtained; (2) a stage-discharge relation or "rating curve" is developed. The stage record is obtained by direct observation of a properly placed scale called a staff gage or of a wire-weight gage, or is recorded automatically on a graph by means of a continuous water-level recorder. Recorders have the obvious advantage of not requiring continuous attendance and are particularly useful where stream levels are subject to sharp variations. Instrument manufacturers have developed recorders which operate unattended for up to 60 days. Detailed information on the placing and housing of recorders can be obtained from the instrument manufacturers or Refs. 22 and 23.

The establishment of the stage-discharge relation is a complex subject on which only an outline can be given. In almost all cases, rating curves of acceptable accuracy are obtainable only by current-meter measurements whereby, for a particular stage, velocities in the cross section are determined at a sufficient number of points to compute the discharge within the allowable margin of error. In lieu of current-meter measurements, velocities are sometimes computed by various hydraulic formulas including those for uniform open channels, weirs, and contracted openings (see Appendixes A and B), or surface velocities observed by floats are multiplied by coefficients to give the average cross-sectional velocity. With the exception of geometrically perfect weirs and specifically designed contracted openings, such as the Parshall flume, these

methods are not recommended except for rough determination of the discharge. Obviously, in some instances, such as the determination of unusual flood discharges of short duration, current-meter measurements are not feasible, requiring the use of the approximate methods as the only alternative.

The stage-discharge relation is a permanent one only in case the controlling stream cross section is stable and is located away from backwater effects due to a downstream reservoir or tributary. If the channel slope is flat, the translation of a flood wave is also likely to affect the relation. In the presence of these factors, repeated gagings are necessary to evaluate their effects. The problem of the "shifting control" is often solved by constructing a low sill. Improved accuracy at small flows can be obtained by constructing it in the shape of a flat V.

14. Sources of Data. The U.S. Geological Survey maintains approximately 5,000 gaging stations in the United States and publishes the results annually in a series of *Water Supply Papers*. The average daily, monthly, and annual flows are published as well as the instantaneous maximum and minimum flows for the year. Instantaneous flows at other times are obtainable by special request. Other Federal agencies, some state and local government agencies, and private utilities compile stream-flow records and can provide valuable information.

15. Water Loss or Evapo-transpiration. The term "evapo-transpiration" denotes water returned to the atmosphere as (1) evaporation from land surfaces, (2) evaporation from "interception," which refers to precipitation intercepted by vegetation and not reaching the ground, and (3) transpiration, whereby water, drawn from the soil by the roots of plants as part of their life process, is then released through pores into the atmosphere. The term "consumptive use" is often used interchangeably with evapo-transpiration, and its determination is of prime importance in the design of irrigation projects (see Sec. 17). Let P denote the volume of annual precipitation on a drainage basin, E the evapo-transpiration, and R the runoff. Then

$$P = E + R \qquad \text{and} \qquad R = P - E$$

From the latter equation it is seen that, from the point of view of runoff, evapo-transpiration is a loss. Hence, it is often referred to as "water loss." For drainage basins having large water areas in lakes or swamps, water loss equals the sum of evapo-transpiration plus the evaporation from the water areas. Subterranean flow or "deep seepage" sometimes occurs to or from a basin in sufficient magnitude to require adjustment of the runoff factor (see Art. 17).

By subtracting mean annual runoff from mean annual precipitation, Williams[24] has compiled mean annual water losses for many drainage basins in humid parts of the United States, with results as shown in Fig. 15. He also found (Fig. 16) an approximate correlation between mean annual water loss (evapo-transpiration) and mean annual temperature. A noteworthy feature of Fig. 15 is the fact that the water-loss lines are approximately parallel to the temperature lines east of longitude 95°. West of this longitude, the water-loss lines turn sharply and become perpendicular to the temperature lines. This is explained by the rapidly decreasing precipitation westward which fails to satisfy the evapo-transpiration demands that would otherwise occur at the prevailing temperature.

Quantitative data on consumptive use of water by various crops and by forest and range lands under various systems of management are necessary not only for determination of water-supply requirements for irrigation systems (see Sec. 17) but also to determine what the effect on basin annual and seasonal runoff will be of large-scale schemes for irrigation or water-shed management. References 25 and 26 are examples of broad investigations involving determinations of consumptive use and evaluation of large-scale land-use changes on basin runoff quantity and distribution.

16. Direct and Base Runoff. Although the total runoff over a long period of time, such as a year, may be known as, for example, by deducting evapo-transpiration from basin precipitation, fluctuations over shorter periods of time require a closer look at the mechanics of the runoff process. Referring to Fig. 17, it is seen that, from the total

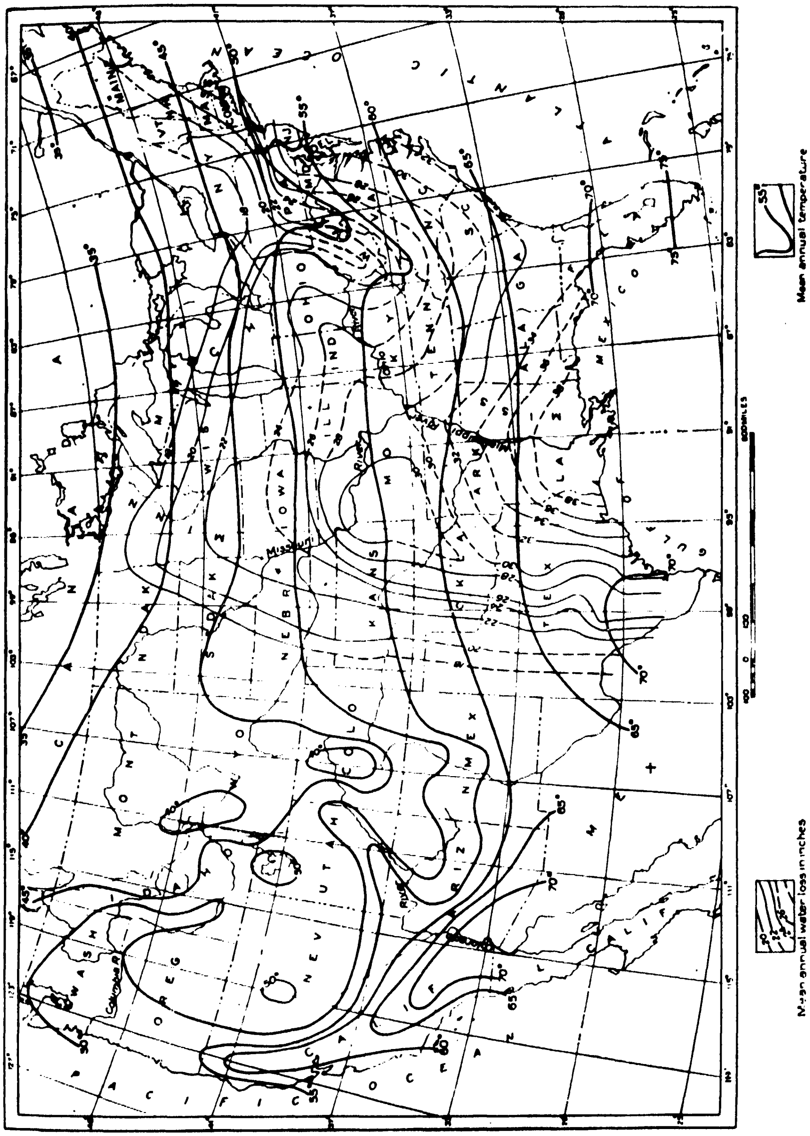


FIG. 15.—Generalized lines of mean annual water loss and temperature in the United States.[24]

rainfall measured by a gage on the ground, the amount necessary to wet vegetation must be deducted. This amount, measured in inches over the drainage basin, is called *interception* and is returned to the atmosphere by evaporation. Rainfall reaching the ground moves through the soil surface, a process which is called *infiltration*. Infiltra-

tion occurs both prior to and during the occurrence of surface runoff. The infiltration capacity or rate at which the soil is able to absorb rainfall is a variable depending on many factors (see Art. 24). Water which has infiltrated the surface passes first through the *belt of soil water.* In this belt, water is withdrawn by the transpiration of

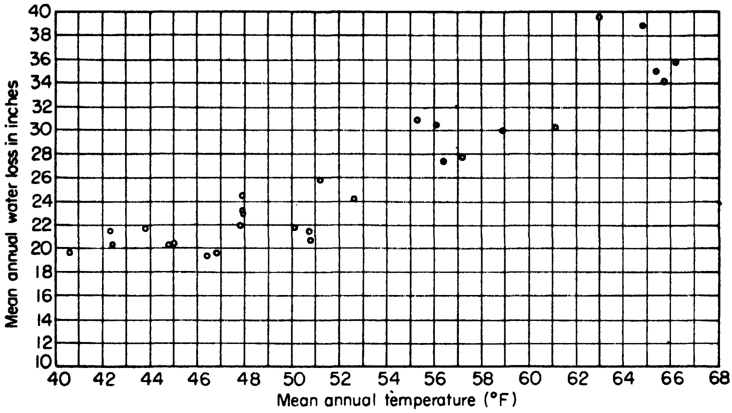


FIG. 16.—Mean annual evapo-transpiration vs. mean annual temperature for selected basins with the mean annual precipitation exceeding 20 in.[24]

plants and by evaporation from the soil, which in arid climates may reach depths as great as 20 ft.[27] Proceeding downward under the action of gravity, water leaving the belt of soil water passes through an intermediate belt (see Art. 17) and reaches the *water table.* The water table is a surface marking the upper limit of a zone or under-

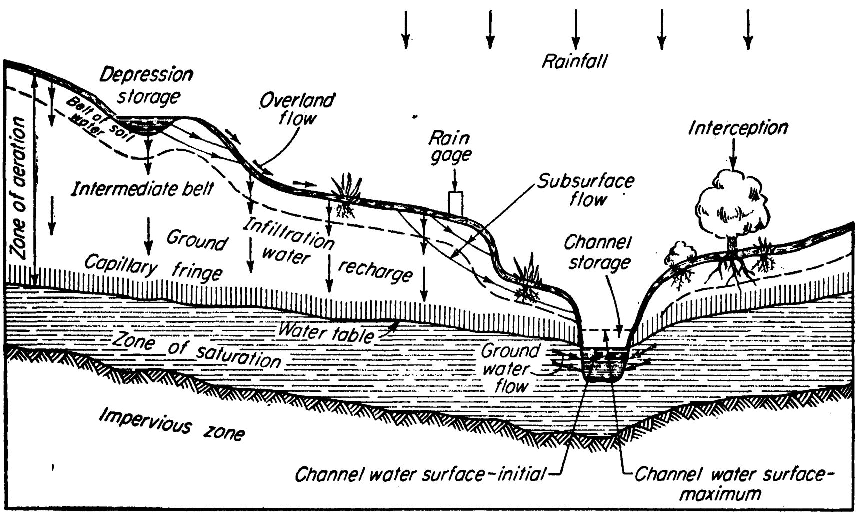


FIG. 17.—Generalized cross section defining runoff terms.

ground reservoir in which the soil is completely saturated. If water is added from above, the volume in the underground reservoir increases, causing a rise in the water table. This addition of water from above is called *ground-water recharge.* The relatively slow movement of water from the ground-water reservoir or zone of satura-

tion to a stream channel is called ground water or *base flow*. When the rate of rainfall exceeds the infiltration capacity, water accumulates on the ground surfaces as a thin film or sheet, and *overland flow* begins. A volume of water required to fill small surface depressions, termed *depression storage*, is abstracted following which the depressions overflow, and overland flow enters one of the myriads of channels and subchannels found throughout the drainage basin. The volume of water in transit in the overland-flow sheet is called *surface detention*. The sum of interception and depression storage is called *initial abstraction*. The term *rainfall excess* denotes the total volume (expresses in inches over the basin) of overland flow, or from the preceding definitions, it is precipitation less initial abstraction less infiltration. Local geological and soil conditions in some basins permit another path of movement to the stream channel, namely, through upper soil layers (see Fig. 17), when the term *subsurface flow* or *interflow* is applied. The term *direct runoff* denotes the sum of the overland and subsurface flow. Increasing rates of runoff (direct plus base) reaching a stream channel cause a rise in water surface and an increase in *channel storage*, which is the volume under the water-surface profile. This volume becomes large during floods, particularly when streams have wide flood plains. Figure 18, showing the discharge hydrograph of a stream and the graph of rainfall at a near-by recording gage, serves to illustrate further the above definitions.

17. Ground-water Runoff. Although water is present in the ground within all the belts between the impervious zone and the surface (Fig. 17) the term "ground water," as used in hydraulic engineering, refers only to water recoverable by springs and wells. The development of wells as a source of water supply is treated in Sec. 19.

The space between the ground surface and the water table is called the *zone of aeration* and includes the *capillary fringe*, where water is held in the soil pores by capillarity, the *intermediate belt*, where suspended water (called "vadose" water) is held by molecular attraction, and the belt of soil water. Plants extend their roots into the intermediate belt to various depths, while trees usually extend their roots into the zone of saturation.

The term *aquifer* refers to a water-bearing geologic formation, *i.e.*, one that is saturated with water. If confined between impervious strata, aquifers may contain water under pressure, in which case they are called *artesian*.

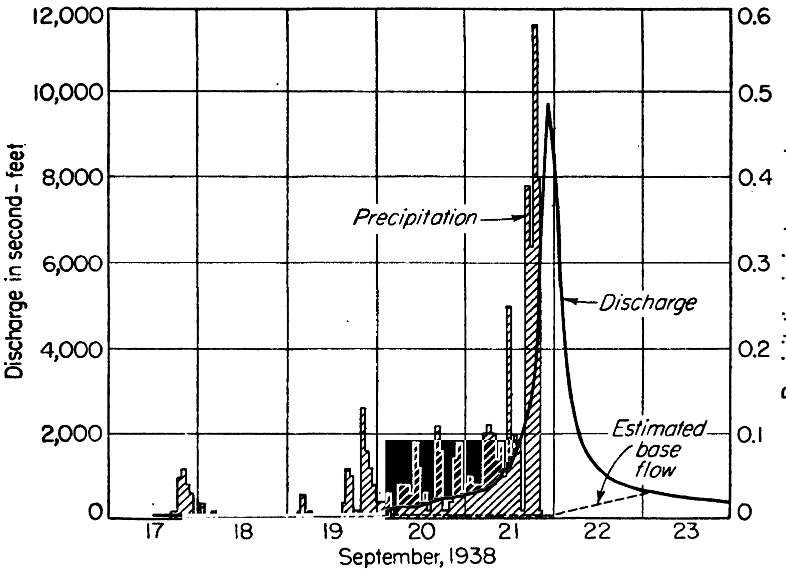
After the occurrence of direct runoff accompanying a stream rise, stream flow during the descending limb or *recession* side of a flow hydrograph occurs as outflow from channel storage and outflow from ground-water storage (ground-water or base flow). The former outflow occurs relatively rapidly following which flow is entirely from ground-water storage. From studies of many hydrographs, it has been found[29] that ground-water depletion curves for a given drainage basin are always nearly the same; hence the term *normal ground-water depletion curve* is used. It has been found further that this curve, or at least segments of it, follows a simple inverse exponential function[30] of elapsed time of the form

$$Q_t = Q_0 K^{-t}$$

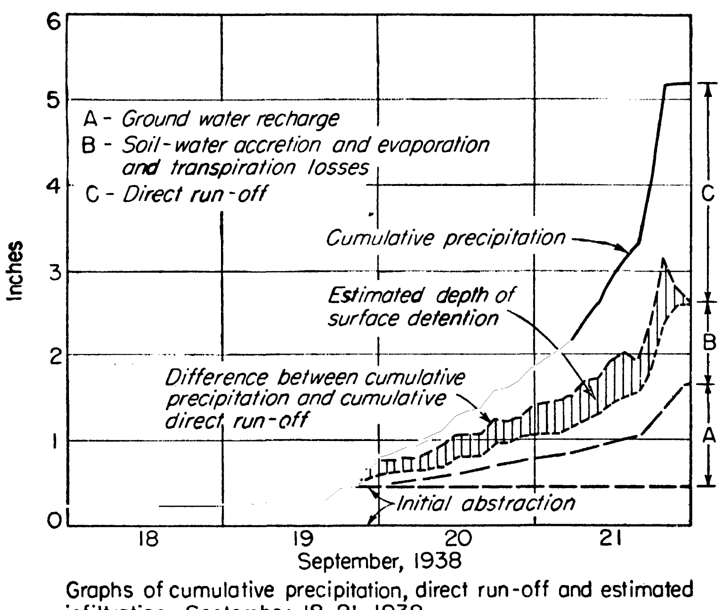
where Q_0 is the discharge at any instant, Q_t is the discharge t days later, and K is the "daily depletion factor." As Q_t is the derivative of storage with respect to time, integration of this equation gives

$$S_0 = \frac{Q_0}{\log_e K}$$

where e is the base of natural logarithms and S_0 is the ground-water storage available for runoff at the time of Q_0. From this, it is seen that the discharge at any time is proportional to the water remaining in storage. The value of K can be determined by plotting observed recession curves on semilogarithmic paper, taking care to select



Hydrograph of discharge and graph of hourly precipitation at Northfield, Vt., September 17-23, 1938.



Graphs of cumulative precipitation, direct run-off and estimated infiltration, September 18-21, 1938

FIG. 18.—Analyses of rainfall and runoff in the basin of Dog River at Northfield Falls, Vt.[28]

periods of little or no direct runoff. In Fig. 19, the recession constant K is the average slope of the stream-flow hydrographs, plotted on semilogarithmic paper, for 3 years during which typically low summer flows were preceded by periods of relatively high direct runoff.

18. Annual Runoff. Figure 20 is a map of the United States showing lines of equal average annual runoff. The indicated mean annual runoff for a given region is a generalized one, local variations due to geology and soil cover, especially to topography, causing departures from the average for the region. A detailed study of variations in mean annual runoff in the Connecticut River basin in New England[32b] showed differences of 100 per cent between valleys and near-by mountain peaks, with values for the basin as a whole ranging from 18 to 40 in. The principal reason for the

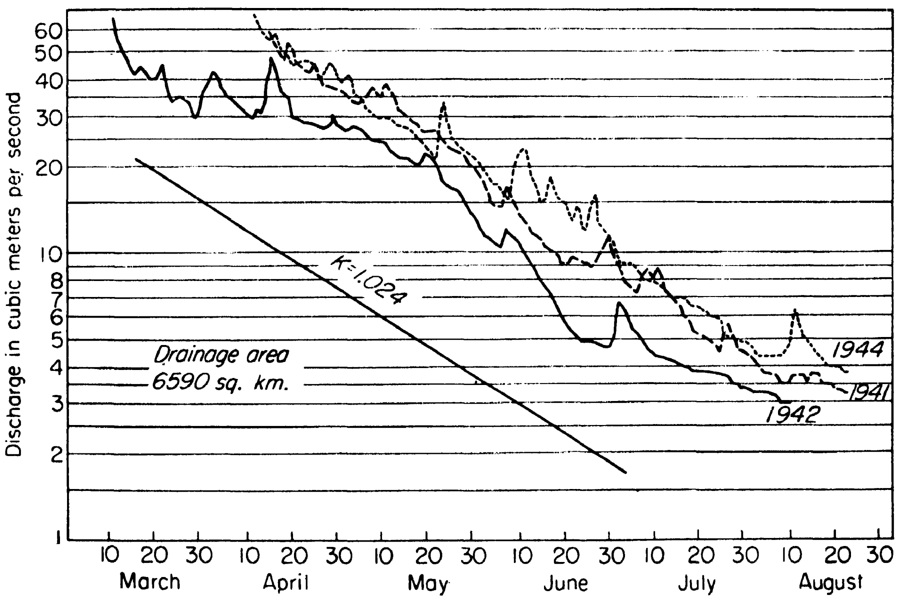


FIG. 19.—Semilogarithmic plotting of stream flow and derivation of the recession constant K. Gediz River at Kizkoprusu, Turkey.

variations, in this case, is the difference in precipitation caused by the mountainous terrain. It is apparent, therefore, that the values indicated in Fig. 20 give the average runoff over large areas and that there may be substantial errors if the values are applied to small ungaged areas, especially in mountainous regions.

It is of interest to compare mean annual precipitation and runoff over the United States as shown in Figs. 1 and 20. In central Nebraska, the mean annual runoff of about 1 in. is only about 5 per cent of the precipitation of 20 in., whereas in Pennsylvania the runoff is 20 in., or 50 per cent of the precipitation of 40 in. Note that in both cases the difference between precipitation and runoff or evapo-transpiration, is approximately the same—20 in.

19. Seasonal and Long-term Variations. Figure 21 shows the seasonal variations in the United States and a stream in the Caribbean area in terms of average monthly percentages of the annual runoff. The United States data are from Ref. 33a. In the autumn and winter, runoff is generally low in most of the United States as soil moisture, depleted during the summer, is replenished and as much precipitation occurs in the form of snow. Spring and early summer are periods of high flow, caused by melt-

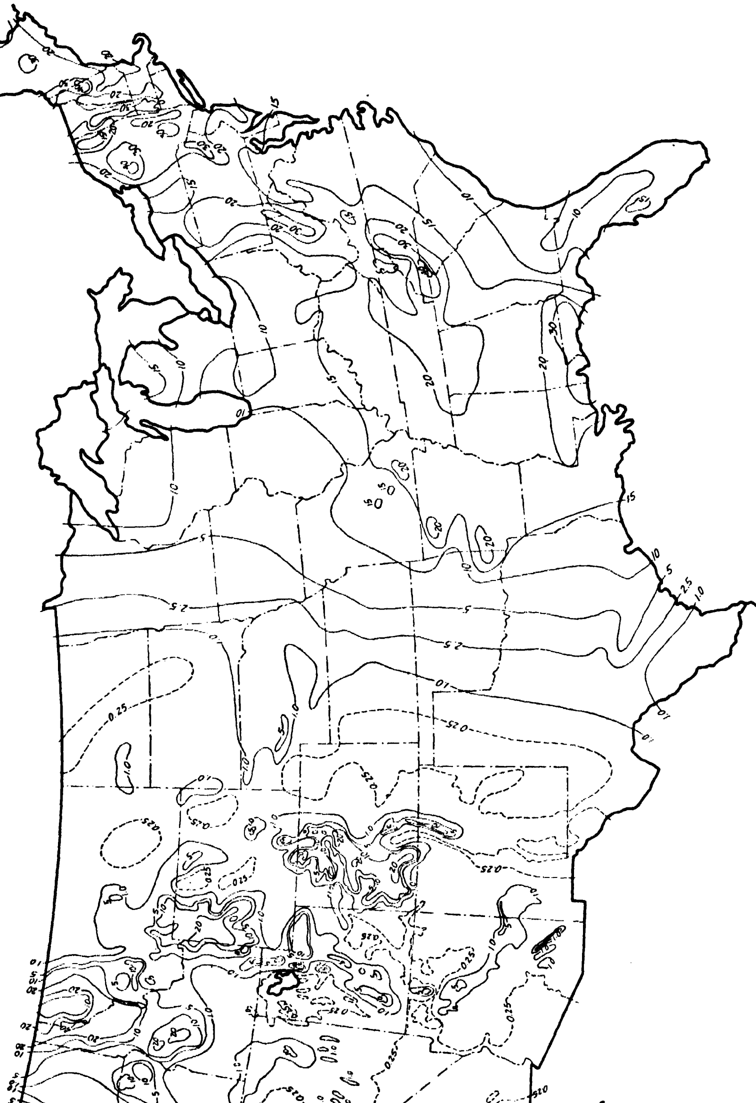


Fig. 20.—Average annual runoff in the United States.[31]

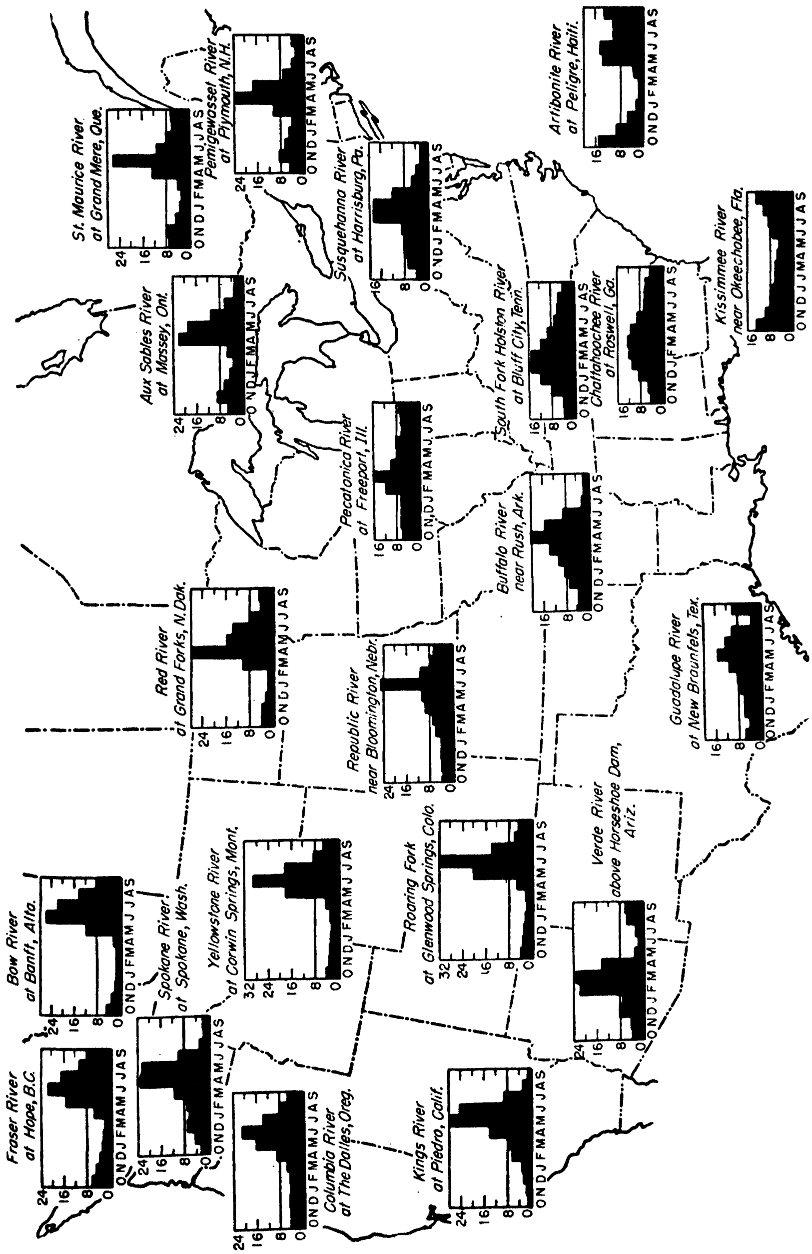


FIG. 21.—Average percentage distribution of annual runoff by months.

ing snow in northern and high-altitude areas and the high moisture content of the soil as a result of accumulations during the winter.

Long-term variations in stream flow are best studied by computing 10-year moving averages and plotting them against the terminal year. A series of 16 such plotted graphs, for various streams in the United States having records for 40 or more years, is described on page 68 of Ref. 33. The principal conclusion to be drawn from these studies is that there is no definite trend in the quantity of runoff, either upward or downward, for the United States as a whole. The graphs for some of the streams have seemingly regular cyclic variations; however, graphs for other streams, showing similar periodic variations for part of the record, do not repeat the periodicities with sufficient regularity to enable forecasting of future stream flows.

20. Estimates for Missing Records. Estimates of mean monthly or annual stream flow, when records are incomplete or even entirely lacking, may sometimes be made; however, the problem is one requiring great caution and judgment, and thorough familiarity with the hydrologic characteristics of the region and drainage basin being studied.

If a partial record is available, it may be completed by making correlations with near-by records for concurrent periods. Such correlations are best expressed graphically to enable evaluations of the reliability of the correlation, based on the relative uniformity or scatter of plotted points. Correlations with precipitation and temperature may produce useful results when estimating short gaps in a runoff record. Long-term precipitation records may be used to estimate annual runoff at a stream-gaging station for which only short-term records are available. In this case, trial correlations may be made involving precipitation for the current year and one or two previous years. The number of previous years to be considered will depend on the relative quantity of ground-water recharge carried over from year to year. Ordinarily, at least 10 years of runoff records are required to determine adequately the relative weights of the precipitation factors. Graphical and least-squares techniques suitable for determining the weights are described in Refs. 34 and 44*a*.

Estimates of monthly and annual runoff at a place entirely lacking in stream-gaging measurements should be approached with great caution, particularly in arid and semi-arid regions where the quantity of flow during drought periods is greatly affected by climatic and physiographic features. A possible solution is to obtain spot discharge measurements several times a year and then estimate the runoff by comparison with a long-term station in the same general locality; however, for important projects a gaging station should be installed as an essential first step in the project investigations. In humid regions such as Eastern United States where fairly uniform conditions prevail, it may be permissible to compute runoff as being proportional to the drainage area. A sufficient number of gages should be available in the region to establish whether or not seasonal runoffs are in fact proportional to the drainage area; estimates for drought periods should be made with caution as described above for arid regions.

21. Utilization Studies. The various techniques for determining reservoir-storage requirements to meet water-utilization demands are described in Sec. 1, Reservoirs. These techniques, which include the flow-duration curve, the mass curve, and the residual mass curve, are based on stream-gaging records with corrections applied for evaporation from the reservoir surface (see Art. 29). Inasmuch as future operation of the reservoir and the expectation of safe yields are based on past records, it is apparent that these records must be of sufficient length to include samples of extremely dry and wet periods that occur at infrequent intervals. Where stream-flow records are short, it will be necessary to extend them, as described in Art. 20. Ordinarily only runoff quantities for periods longer than a month can be obtained by this method, but such data would be ample in studying a reservoir of relatively large storage capacity.

On the other hand, with small reservoirs taking run-of-the-river flow, it would be necessary to know the expected instantaneous (or possible average daily) minimum. Studies of ground-water depletion curves (see Art. 17), correlated with minimum flows during the period of observed runoff records, may be useful in this connection.

FLOODS

22. The Flood Hydrograph. The runoff hydrograph for a stream, covering a time interval during which a period of direct, *i.e.*, storm, runoff occurred, was shown and discussed in Art. 16 and will now be examined in greater detail.

An essential first step in flood hydrograph analysis is the separation of storm runoff and base runoff, following which it is sometimes desirable to separate the storm runoff into surface and subsurface runoff. In Fig. 22, lines *AC* and *DE* are normal ground-water depletion curves (see Art. 17). Ground-water recharge occurs, in the time interval *AB*, the points *A* and *B* defining, respectively, the beginning and the end of the flood hydrograph. While the form of the ground-water hydrograph between *A* and *B* is largely indeterminate, it need not ordinarily be established exactly, as the ground-water flow is usually a small part of the total flow. Furthermore, if a consistent procedure is followed in separating the ground-water flow, both in analyzing and synthesizing flood hydrographs, results will be accurate enough for practical purposes. In Fig. 22 the ground-water hydrograph has been drawn arbitrarily as a reverse curve with the point of inflection about midway between *A* and *B*.

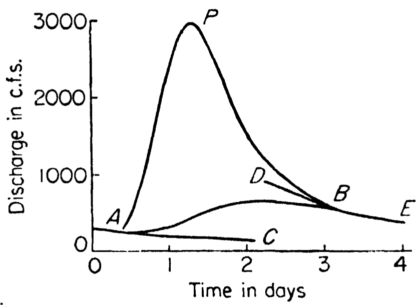
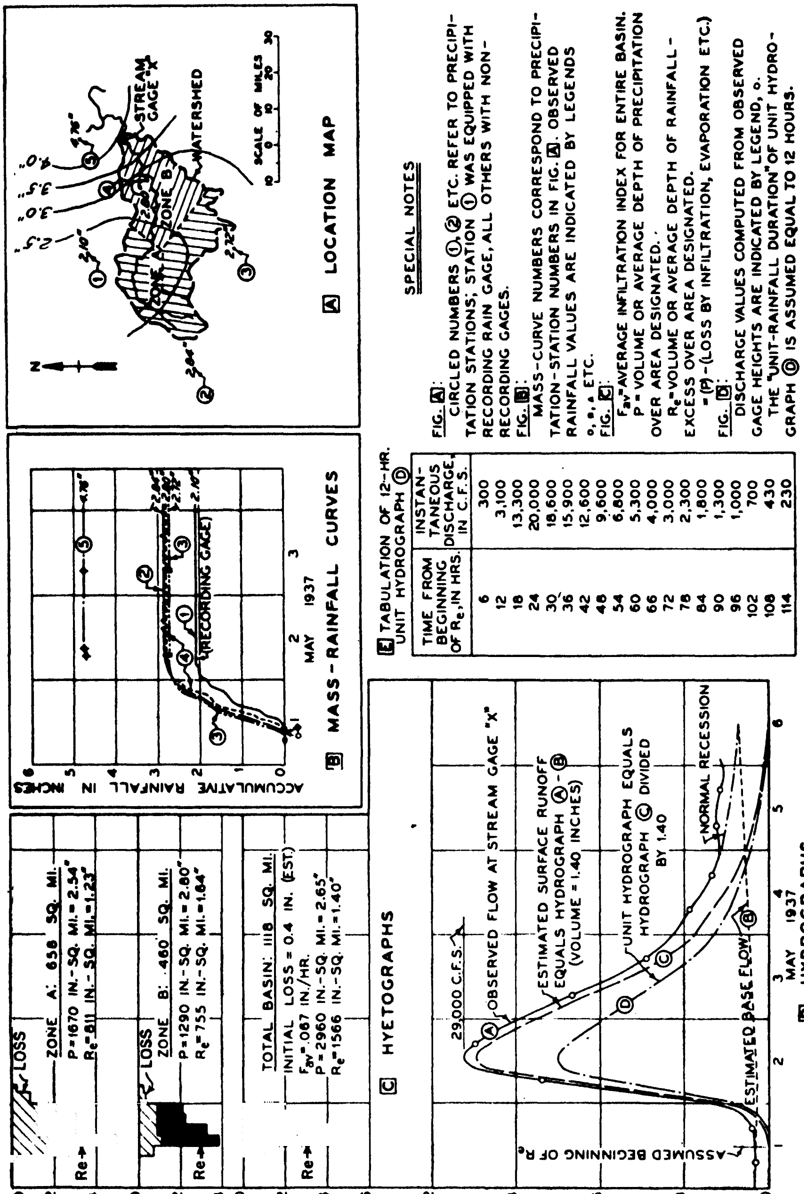


FIG. 22. Separation of direct and base runoff.

The hydrograph of direct runoff is then *APB*. Barnes[30] has developed an analytical method for separation of the direct-runoff hydrograph into surface and subsurface flow. However, while desirable for research, this refinement is not usually made in design studies.

The factors determining the magnitude and time of the peak, and the shape of the direct-runoff hydrograph can be placed in two groups: meteorological factors and drainage-basin characteristics. The meteorological factors include intensity of rainfall and its variation with time and place, depth and condition of snow on the ground, and temperature. The drainage-basin characteristics include a subgroup of topographic factors comprising size, shape (including length and width), drainage density (length of all stream channels in the basin divided by the area of the basin), land slope, channel discharge capacity and slope, and a second subgroup of geological and agronomical factors comprising surface and subsurface soil characteristics and vegetal cover. While almost all these factors can be expressed in quantitative terms,[35,36] a satisfactory explicit function for the direct-runoff hydrograph involving these factors has never been derived. However, a long step forward in flood-hydrograph analysis occurred with the introduction in 1932 by Sherman[37] of the unit hydrograph.

23. The Unit Hydrograph. Suppose that rainfall excess on a given drainage basin has a volume of 1 in. of depth, that it is uniformly distributed throughout the basin, and that it occurs at a constant rate with respect to time, beginning at time $= 0$ and ending at time $= t_R$. The resulting observed hydrograph of direct runoff would also have a volume of 1 in. (according to the definition of rainfall excess in Art. 16) and would represent the integrated effect of all the drainage-basin characteristics described above. Let this hydrograph be called a *unit hydrograph* for duration t_R. If the rain-



TIME FROM BEGINNING OF R_e, IN HRS.	INSTANTANEOUS DISCHARGE, IN C.F.S.
6	300
12	3,100
18	13,300
24	20,000
30	18,600
36	15,900
42	12,600
48	9,600
54	6,800
60	5,300
66	4,000
72	3,000
78	2,300
84	1,800
90	1,300
96	1,000
102	700
108	430
114	230

FIG. 23.—Computation of a unit hydrograph from an isolated storm.[38a]

fall-excess volume is n in., uniformly distributed as before, ordinates of the resulting discharge hydrograph can be obtained simply by multiplying ordinates of the unit hydrograph by n. If rainfall excess is reasonably uniform throughout the basin, here is a powerful tool for synthesizing flood hydrographs.

The preceding theory has some limitations. As discharge rates increase, overland

and channel velocities increase and tend to produce a more sharply peaked hydrograph. On the other hand, valley storage and overland detention also become greater, these factors having a dampening effect on the hydrograph. In a given drainage basin, if the velocity effects are greater than the storage effects, unit hydrographs based on minor observed flood rises are apt to have low peak values and should not be used. If, with rising flood stages, storage effects become pronounced, the final flood hydrograph should be corrected as described in Art. 27. Experience with observed flood hydrographs permits the generalization that unit hydrographs should not be used for drainage areas greater than about 2,000 sq miles, when valley storage effects, as well

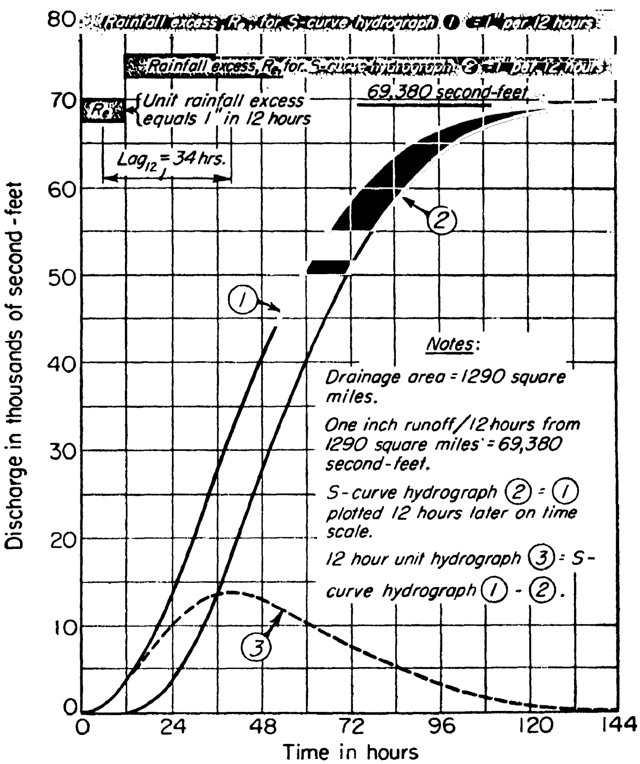


Fig. 24.—S-curve hydrographs.

as variations in rainfall excess in the drainage basin, tend to become too great to be reflected in the unit hydrograph alone.

Hydrologists are coming to recognize a lower limit, as well, in applicability of the unit-hydrograph principle.[33b] For very small areas of less than about 1 sq mile, the relative importance of overland flow detention is very great and other methods of flood-hydrograph determination are recommended (see Art. 28).

The techniques of unit-hydrograph derivation are summarized briefly in the following steps with references made to the example in Fig. 23:

1. Study the rainfall and runoff records and select for analysis storms which are isolated, intense, and occurring uniformly over the basin. A storm having all these characteristics will obviously be rare, particularly in a short record, and it will be necessary to select storms approaching that ideal as closely as possible.

2. Plot mass-rainfall curves, an isohyetal map for the storm, and hyetographs showing time distribution of rainfall, loss, and rainfall excess. Determination of the loss and its distribution is partly a matter of judgment. The total volume of the loss equals the basin rainfall minus the volume of direct runoff

(see step 3). The rate of loss is greatest in the early part of the storm, but it may be rather uniform, particularly with wet soil conditions from antecedent rainfall. A fuller discussion of loss rates is contained in Art. 24.

3. Plot the observed discharge hydrograph and separate and subtract the base flow (see Art. 22), giving the hydrograph of direct runoff with a volume of 1.40 in. Dividing the ordinates by 1.40 gives the unit hydrograph for rainfall-excess duration t_R of 12 hr, this time interval being the duration of rainfall excess indicated by the hyetographs.

4. To obtain a unit hydrograph for other values of t_R, proceed as shown in Fig. 24, and Table 3.

5. Repeat the process with additional storms and develop unit hydrographs for the same value of t_R so that they can be compared and averaged. If a storm has characteristics too complex for unit-hydrograph development, it may be necessary to estimate a trial unit hydrograph and reconstitute the hydrograph as described in the following paragraph.

TABLE 3.—COMPUTATION OF UNIT HYDROGRAPH OF ONE DURATION FROM UNIT HYDROGRAPH OF ANOTHER DURATION[38d]

Time, hr	Computation of S-curve hydrograph from known 12-hr unit hydrograph			Computation of 6-hr unit hydrograph from 12-hr S-curve hydrograph		
	12-hr unit hydrograph 3 in cfs	12-hr S-curve hydrograph 2 in cfs	12-hr S-curve hydrograph 1 in cfs	12-hr S-curve hydrograph 1 shifted 6 hr	Runoff from 0.5 in. R_e in 6 hr (col. — 4col. 5)	6-hr unit hydrograph, 2 (col. 6) in cfs
(1)	(2)	(3)	(4)	(5)	(6)	(7)
6	900		900		900	1,800
12	3,400		3,400	900	2,500	5,000
18	6,900	+ 900	7,800	3,400	4,400	8,800
24	10,100	3,400	13,500	7,800	5,700	11,400
30	12,300	+ 7,800	20,100	13,500	6,600	13,200
36	13,600	13,500	27,100	20,100	7,000	14,000
42	13,900	20,100	34,000	27,100	6,900	13,800
48	13,200	27,100	40,300	34,000	6,300	12,600
54	11,800	34,000	45,800	40,300	5,500	11,000
60	10,300	40,300	50,600	45,800	4,800	9,600
66	8,950	45,800	54,750	50,600	4,150	8,300
72	7,650	50,600	58,250	54,750	3,500	7,000
78	6,400	54,750	61,150	58,250	2,900	5,800
84	5,250	58,250	63,500	61,150	2,350	4,700
90	4,200	61,150	65,350	63,500	1,850	3,700
96	3,200	63,500	66,700	65,350	1,350	2,700
102	2,280	65,350	67,630	66,700	930	1,860
108	1,580	66,700	68,280	67,630	650	1,300
114	1,100	67,630	68,730	68,280	450	900
120	750	68,280	69,030	68,730	300	600
126	500	68,730	69,230	69,030	200	400
132	300	69,030	69,330	69,230	100	200
138	150	69,230	69,380	69,330	50	100
144	50	69,330	69,380	69,380	0	0

All discharges are instantaneous values at the end of the hour designated in column (1). Drainage area equals 1,290 sq miles.

Given: 12-hr unit-hydrograph values in columns (1) and (2).

Procedure: 1. Compute S-curve hydrographs 1 and 2, which is the sum of a series of 12-hr unit hydrographs spaced 12 hr apart. Computation procedure is indicated by the arrows in the table.

2. Shift S-curve hydrograph 1 6 hr as in column (5) and subtract from column (4) giving, in column (6), the runoff from a rainfall excess of ½ in. in 6 hr.

3. Multiply values in column (6) by 2 to get the rainfall-excess volume of 1 in.

TABLE 4.—COMPUTATION OF HYPOTHETICAL HYDROGRAPH[1]

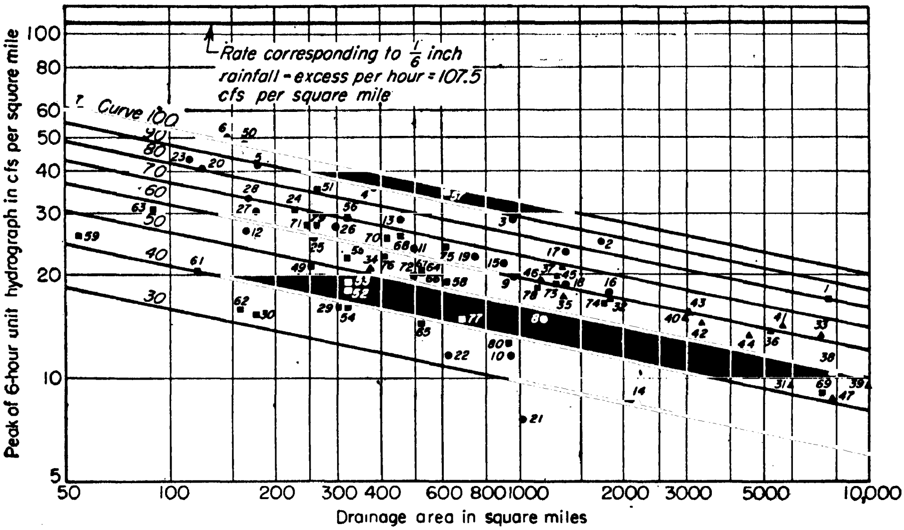
Time, hr	12-hr unit hydrograph, cfs	Rainfall excess, in. per 12 hr	Surface runoff from rainfall-excess units, cfs: Rainfall-excess, in. 0.7	3.8	10.9	1.8	Subtotal	Base flow, cfs	Total discharge, cfs
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
6	800	0							
12	2,900								
18	5,900	0.7	560	......	....	......	560	2,200	2,760
24	8,600	....	2,030	......	....	......	2,030	2,200	4,230
30	10,500	3.8	4,130	3,040	....	......	7,170	2,200	9,370
36	11,600	....	6,020	10,200	....	......	16,220	2,200	18,400
42	11,800	10.9	7,350	22,400	8,720	......	38,470	2,200	40,700
48	11,300	....	8,120	32,700	31,600	......	72,420	2,200	74,600
54	10,100	1.8	8,260	39,900	64,300	1,440	113,900	2,200	116,000
60	8,800	....	7,910	44,100	93,700	5,220	150,930	2,200	153,000
66	7,600	....	7,070	44,800	114,400	10,600	176,870	2,200	179,000
72	6,500	....	6,160	42,900	126,400	15,500	190,960	2,200	193,000
78	5,500	....	5,320	38,400	128,600	18,900	191,220	2,200	193,000
84	4,500	....	4,550	33,400	123,200	20,900	182,050	2,200	184,000
90	3,500	....	3,850	28,900	110,100	21,200	164,050	2,200	166,000
96	2,700	....	3,150	24,700	95,900	20,300	144,050	2,200	146,000
102	1,960	....	2,450	20,900	82,800	18,200	124,350	2,200	127,000
108	1,330	....	1,890	17,100	70,800	15,800	105,590	2,200	108,000
114	940	....	1,370	13,300	60,000	13,700	88,370	2,200	90,600
120	630	....	930	10,300	49,000	11,700	71,930	2,200	74,100
126	430	....	660	7,450	38,200	9,900	56,210	2,200	58,400
132	250	....	440	5,050	29,400	8,100	42,990	2,200	45,200
138	130	....	300	3,570	21,400	6,300	31,570	2,200	33,800
144	50	....	180	2,390	14,500	4,860	21,930	2,200	24,100
150	......	....	90	1,630	10,200	3,530	15,450	2,200	17,700
156	......	....	40	950	6,870	2,390	10,250	2,200	12,500
162	......	....	.....	490	4,690	1,690	6,870	2,200	9,070
168	......	....	.....	190	2,730	1,130	4,050	2,200	6,250
174	......	....	.....	......	1,420	770	2,190	2,200	4,390
180	......	....	.....	......	550	450	1,000	2,200	3,200
186	......	....	.....	......	....	230	230	2,200	2,430
192	......	....	.....	......	....	90	90	2,200	2,290

[1] Drainage area = 1,100 sq miles. All discharges are instantaneous values at the end of the hour given in column (1).

Application. Two examples, each illustrating the computation of a flood hydrograph by use of the unit hydrograph, are given in Table 4 and Fig. 26. Within the limitations cited above, the unit hydrograph has become an invaluable aid in the determination of flood hydrographs required in the design and operation of spillways, flood-control and multiple-purpose reservoirs, drainage pumping stations, and other water-control structures. Coupled with the principles of storm transposition (see Art.

25), the unit-hydrograph method permits much more reliable estimates of design-flood discharges than was formerly possible.

Synthetic Unit Hydrographs. Since the introduction in 1932 of the unit hydrograph,[37] its use has become widespread in the United States, particularly among agencies of the Federal government. Certain characteristics of observed unit hydrographs and of the drainage basins producing them have been compiled in the form of standard data sheets by participating agencies of the Subcommittee on Hydrologic Data of the Federal Inter-agency River Basin Committee. The purpose of the compilation, which is still continuing, is to permit exchange of basic information on unit hydrographs among interested agencies in order to obtain means of evaluating and



SOURCE OF DATA

▲ "Report of cooperative hydrologic investigations", by Penna. Dept. of Forests and Waters, U.S Weather Bureau and U.S. Geological survey.

● Transactions of American Geophysical Union, part II, 1940, pp. 649-659.

■ Hydrological reports prepared by various district offices of U.S. Engineer Department.

FIG. 25.—Six-hour unit hydrograph peaks vs. drainage area.[38d] Numbers beside points identify basins listed in Table 5.

comparing observed unit hydrographs and of deriving synthetic unit hydrographs for drainage basins having little or no stream-flow measurements. Drainage basin characteristics compiled include the drainage area; the length L along the longest watercourse from the gaging station to the head of the stream (not to the divide); the length L_{ca} along the stream from the gaging station to a point opposite the center of gravity of the drainage basin; the drainage density or total length of all streams (including intermittent streams) divided by the drainage area; the average stream slope or total fall of the longest watercourse divided by L; the average slope of tributary streams (a tributary stream being defined as one whose drainage area is less than 10 per cent of the total drainage area); the average slope of the main streams (a main stream being defined as one whose drainage area is more than 10 per cent of the drainage area); and the maximum, minimum, and mean elevations of the drainage basin. Unit-hydrograph characteristics compiled include the duration of rainfall excess (also called unit duration); peak of the unit hydrograph Q_p in cubic feet per second and q_p in cubic feet per second per square mile; time t_p from the center of rainfall excess to the peak; time

t_v from the center of rainfall excess until 50 per cent of the runoff volume has occurred; Snyder coefficient $C_t = (t_p/LL_{ca})^{0.3}$; and Snyder coefficient $640C_p = q_p t_p$.

It has been found that, if adjacent streams have similar characteristics, the values of C_t and $640C_p$ are about the same; in the absence of stream-flow records, therefore, these coefficients can assist in the derivation of a synthetic unit hydrograph. The curves in Fig. 25 were taken from Ref. 38*d* (also given in Ref. 46*a*), which describes curve 100 as being an enveloping curve of the plotted points and of some additional

TABLE 5.—IDENTIFICATION OF BASINS IN FIG. 25

Point No.	Stream	Location	Point No.	Stream	Location
1	Washita River, Okla.	Near Mouth	37	Youghiogheny River, Pa.-W.Va.	Connellsville, Pa.
2	Youghiogheny River, Pa.	Suthersville, Pa.			
3	Black River, Ark.	Leeper, Ark.	38	Susquehanna River, Pa.	Towanda, Pa.
4	Casselman River, Pa.	Markelton, Pa.	39	Susquehanna River, Pa.	Wilkes-Barre, Pa.
5	So. Fk. Ten Mile Creek, Pa.	Jefferson, Pa.	40	W. Br. Susquehanna River, Pa.	Renova, Pa.
6	Turtle Creek, Pa.	E. Pittsburgh, Pa.	41	W. Br. Susquehanna River, Pa.	Williamsport, Pa.
7	Canacadea Creek, N.Y.	Almond dam site, N.Y.			
8	Tygart River, W.Va.	Tygart dam site, W.Va.	42	Juniata River, Pa.	Newport, Pa.
9	Cheat River, W.Va.	Rowlesburg, W.Va.	43	Delaware River	Port Jervis, N.Y.
10	Allegheny River, Pa.	Kinzua, Pa.	44	Delaware River	Belvidere, N.J.
			45	Lehigh River, Pa.	Bethlehem, Pa.
11	L. Beaver Creek, Pa.	E. Liverpool, Pa.			
12	Sugar Creek, Pa.	Sugar Creek, Pa.	46	Schuylkill River, Pa.	Pottstown, Pa.
13	Redbank Creek, Pa.	Redbank Cr. dam site, Pa.	47	Susquehanna River, Pa.	Towanda, Pa.
			48	Canisteo River, N.Y.	Arkport Dam, N.Y.
14	Allegheny River, Pa.	Above Kinzua, Pa.	49	Otselic River, N.Y.	Whitney Pt., N.Y.
15	Clarion River, Pa.	Clarion, Pa.	50	Westfield River, Mass.	Knightville dam site, Mass.
16	Kiskiminitas River, Pa.	Vandergrift, Pa.			
17	Cheat River, W.Va.	Beaver Hole, W.Va.	51	Loyalhanna Creek, Pa.	New Alexandria, Pa.
18	Conemaugh River, Pa.	Bow, Pa.	52–56	Mahoning Creek, Pa.	Dayton, Pa.
19	West Fork River, W.Va.	Enterprise, W.Va.	57–58	Coldwater River, Miss.	Coldwater, Miss.
20	Laurel Hill Creek, Pa.	Ursina, Pa.	59	Saddle River, N.J.	Lodi, N.J.
			60	Whippany, N.J.	Morristown, N.J.
21	French Creek, Pa.	Utica, Pa.			
22	French Creek, Pa.	Saegerstown, Pa.	61	Ramapo River, N.J.	Mahwah, N.J.
23	Buffalo Creek, W.Va.	Barrackville, W.Va.	62	Ramapo River, N.J.	Pompton Lakes, N.J.
24	Dunkard Creek, Pa.	Bobtown, Pa.	63	Wanaque River, N.J.	Wanaque, N.J.
25	Chartiers Creek, Pa.	Carnegie, Pa.	64–68	Redbank Creek, Pa.	Pa.
			69	Washita River, Okla.	Durwood, Okla.
26	Oil Creek, Pa.	Rouseville, Pa.	70	Strawberry River, N.Y.	Poughkeepsie, N.Y.
27	Raccoon Creek, Pa.	Moffatts Mills, Pa.			
28	Yellow Creek, Pa.	Hammondsville, Pa.	71	Petit Jean River, Ark.	Boonville, Ark.
29	Brokenstraw Creek, Pa.	Youngsville, Pa.	72	Petit Jean River, Ark.	Blue Mt. dam site, Ark.
30	Millers River, Mass.	Birch Hill, Mass.	73	N. Br. White River, Mo.	Tecumseh, Mo.
			74	N. Br. White River, Mo.	Norfolk dam site, Ark.
31	Allegheny River, Pa.	Franklin, Pa.	75	Eleven Pt. River, Mo.	Bardley, Mo.
32	Allegheny River, Pa.	Vandergrift, Pa.			
33	Monongahela River, Pa.-W.Va.	Dam No. 2, Pa.	76	Fourche la Fave River, Ark.	Gravelly, Ark.
34	West Fork River, W.Va.	Clarksburg, W.Va.	77	Fourche la Fave River, Ark.	Nimrod dam site, Ark.
35	Tygart River, W.Va.	Fetterman, W.Va.			
			78	Little Red River, Ark.	Greer's Ferry, Ark.
			79	Row-Willamette River, Ore.	Dorena (Star), Ore.
36	Monongahela River, Pa.-W.Va.	Charleroi, Pa.	80	Illinois River, Okla.	Tahlaquah, Okla.

points (not shown) for drainage areas larger than 10,000 sq miles. The curves parallel to curve 100 correspond to percentages of the latter, as indicated. These curves and Table 5 can be used to assist in estimating the synthetic unit-hydrograph peak.

Reference 39 contains about 60 unit hydrographs for drainage basins throughout the state of Illinois.

24. Infiltration Theory. Infiltration was defined in Art. 16. It is necessary to estimate the total volume of infiltration and its distribution in determining rainfall excess, as described in Art. 23. Infiltration has been measured directly on small plots of land and related to soil type, vegetal cover, and antecedent soil-moisture conditions.[33c,40] These measurements, having been made under laboratory conditions,

cannot be applied directly to natural drainage basins of varying soil, cover, and topography; however, they have been very useful in studying means by which agronomic methods can increase infiltration capacity (the maximum rate at which the soil can absorb rainfall) thereby reducing surface runoff and soil erosion.

Other basic differences between small plots and natural drainage basins include (1) the presence of channelization in the latter which increases the speed of water particles compared with the overland flow velocities of small plots, and (2) quick subsurface flow which may be present in natural areas. Despite these differences, which tend to make indicated loss rates much smaller than those obtained from experimental plots, the infiltration approach has been furthered persistently by many investigators[33c] who have introduced the term *infiltration index* to denote the computed loss rate for natural basins. Although it might appear that the two differences noted above would tend to vitiate the infiltration approach, there may be value in its use in a practical case if it is found, for example, that the records for several flood periods for a drainage basin can be resynthesized by using a constant unit hydrograph and consistent values of the infiltration index. The example given in Art. 25 under Past Floods illustrates the use of the infiltration index. The results of a large number of determinations of the infiltration index for drainage basins in various parts of the United States are given in Refs. 38*e* and 46*b*. Reference 41 contains infiltration values derived from precipitation and runoff measurements on flat, turfed, airfield areas varying in size from 7.5 to 69.1 acres. Reference 39 contains similarly derived data for natural basins in Illinois.

25. Flood Discharge Estimates. Methods are summarized in this article for making discharge estimates for (1) past floods and (2) design floods.

Past Floods. Reliable stream-gaging observations, if available, form the best basis of determining the discharge of a past flood; however, if such observations are faulty or incomplete, other methods can provide a check on the discharge determinations. The general procedure is to assemble all known facts regarding the flood, including the areal and temporal distribution of precipitation, temperature, soil conditions, river stages, and flood discharges in adjacent river basins or in the same basin at upstream and downstream points. With all these facts, along with a knowledge of pertinent characteristics of the various drainage basins involved, it is possible, in many cases, to piece together the history of a past flood occurrence including a hydrograph giving the desired peak discharge.

Figure 26 is an example of such a determination. The mass rainfall curve in (*a*) was derived by plotting an isohyetal map of rainfall for the basin, computing the total basin rainfall (7.1 in.), and determining the time distribution by comparison with mass curves (not shown) at near-by stations. The mass infiltration curve, the slope of which equals the infiltration index, was developed in the course of comprehensive studies of the storm of July 1942, and the concurrent runoff, covering a large area and many drainage basins in northwestern Pennsylvania and southwestern New York. In (*b*) are shown hyetographs for rainfall, infiltration, and rainfall excess. The rainfall rate equals the slope of the mass rainfall curve in (*a*). During the time interval 11 P.M. to midnight, July 17, the rainfall rate exceeded the infiltration rate (average); hence, the volume of infiltration is as given in curve 1. From midnight to 3 A.M. July 18, the rainfall was less than the indicated infiltration rate in curve 1. The volume of infiltration during this period is, therefore, the same as the rainfall. This is shown in (*a*) by shifting curve 1, as indicated by the arrow, giving curve 2. From 3 to 5 A.M. the rainfall rate again exceeded the infiltration rate, as indicated by a segment of curve 1 shifted to curve 2. A similar procedure is followed during the remainder of the storm period. The shifting of the infiltration curve follows a suggestion made by C. F. Izzard.* The 1-hr synthetic unit hydrograph has the Snyder coefficients indi-

* *Proc. A. S. C. E.*, May, 1948, p. 767.

cated in the figure, as determined from observed coefficients for similar drainage basins in the same region. The 2- and 3-hr graphs were derived from the 1-hr graph utilizing the *S*-curve procedure (not shown) described in Art. 23. Finally, the flood hydrograph in (*d*) was computed from the unit hydrographs in (*c*) and the rainfall excess volumes in (*b*).

Design Floods. Design-flood hydrographs are needed in the design of reservoirs, spillways, flood channels, and other water-control structures. Methods applicable to drainage systems, including small natural areas of a few square miles or less, and to urban drainage are described in Art. 28. Economic factors are of prime importance in the selection of a design flood. For example, it may be found in the case of a local flood-protection project that physical limitations make it uneconomical to set the

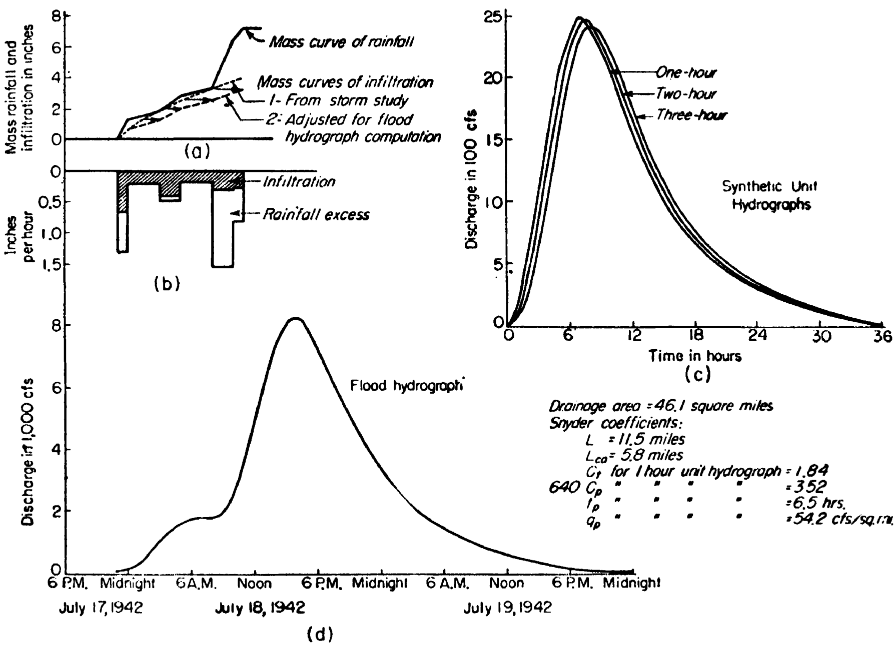


FIG. 26.—Computation of a hydrograph for a past flood. Dodge Creek at Portville, N. Y.

design capacity any higher than the maximum flood of record. In this instance, the design flood equals the maximum past flood; it should of course be established concurrently that the project is economically justified even with the limited protection. As a second example, the severity of the design flood for a reservoir spillway should be related to the economic loss resulting from the possible failure of the impounding dam or from downstream inundation.

The procedure for developing a design flood is the same as that described in the previous example relating to a past flood, except that the magnitude of the three main components—precipitation, infiltration index, and unit hydrograph—are changed to correspond to more critical conditions. Just how critical these conditions must be is related to the economic problem as previously discussed. Taking the infiltration index first, the most severe condition might be to use the minimum values observed in the region for similar physiographic conditions, or a value of zero would be used in the case of melting snow. The peak of a unit hydrograph derived from observations of a minor flood may have to be increased as much as 50 per cent to reflect conditions

during a major flood.[38] The determination of the design precipitation or design storm may require considerable meteorological research, as summarized in Art. 11. Snow-melt problems in connection with design floods are discussed in Art. 7.

Prior to the development of current techniques based on the unit hydrograph and flood routing analysis, a large number of empirical formulas were used to estimate flood discharges for design purposes. The formulas, many of which are given and evaluated in Ref. 42, make use of various combinations of factors such as drainage area, average width of the basin, average basin slope, and frequency. These formulas are of extremely local application but may be useful in extrapolating the available data. A second method, and one that still has value, is to plot for a given region all known maximum discharges in csm (cubic feet per second per square mile) against drainage area in square miles. If plotted on logarithmic paper, a straight line will usually define the envelope of maximum points. As the only factor here considered is drainage area, it is obvious that such a plot provides only another basis of comparing the design-flood discharge with the past flood history of the region being studied. Studies of maximum flood discharges throughout the United States[43] have shown the slope of the envelope curve to be -0.5 in most cases, giving a formula of the type

$$Q = C\sqrt{A}$$

where Q is the discharge in cubic feet per second, A is the drainage area in square miles, and the coefficient C is known as the *Myers rating*. The Myers ratings of the envelope curves for various regions in the United States, as determined from published records of the U.S. Geological Survey, are as follows:[44]

North Atlantic slope	4,800
South Atlantic and Eastern Gulf of Mexico drainage	6,300
Ohio River basin	5,800
Missouri River basin	3,300
Lower Mississippi River basin	6,400
Colorado River basin	2,500
Pacific slope basin in California	5,100
Pacific slope basin in Washington and Upper Columbia River basin	4,600
Pacific slope basins in Oregon and Lower Columbia River basin	5,800

26. Frequency Analyses. Flood-frequency analyses are generally made for one of two purposes: (1) as a guide to judgment in determining the capacity of a structure, such as a highway bridge opening or cofferdam where it is considered permissible to take a calculated risk, and (2) as a means of estimating the probable flood damage prevented by a system of flood-protection works over a period of years, usually equal to the estimated economic life of the works. In the first case, the magnitude of the flood discharge which will be equaled or exceeded in a certain period of years is desired. In the second case, it may be necessary to consider, in addition to the peak-flood discharge, factors such as duration of flooding, time between flood peaks, and month of occurrence (agricultural damage, for example, might be significant only during the growing season). In both cases, the reliability of the frequency estimates depends on the length of the observed record rather than on the specific method of mathematical analysis used to obtain the frequency relation. Thus, if a 50-year record is available, the peak discharge of a 10-year flood based on this record might be stated with assurance; however, the peaks of the 50- or 100-year floods could be estimated only very approximately.

The procedure followed in deriving a frequency relation for peak discharges consists of three steps: (1) compilation of flood peaks in order of magnitude, (2) computation of recurrence intervals, and (3) plotting. In compiling the flood peaks, either the "annual-flood" method or the "partial-duration series" method is used. In the former, only the highest peak in each water year is listed. In the latter, all floods

above a base are compiled. This eliminates an objection to the annual-flood method, namely, that the second highest peak during a given year may be greater than the highest peak during another year. However, in selecting peaks for the partial-duration series, caution should be exercised so as not to include consecutive peaks caused by storms which are not independent meteorological events. If flood damage is of primary concern, the time interval between peaks affects the amount of the damage and hence is a factor in deciding on which peaks to include in the compilation. "Historic floods" are severe floods which occurred prior to the beginning of the continuous stream-gaging record but whose peaks are established on the basis of satisfactory evidence. They should be included in the compilation, but their recurrence interval is computed differently as described subsequently.

The recurrence interval of each flood peak is computed by one of the following formulas:

$$T_r = \frac{N}{M} \tag{1}$$

$$T_r = \frac{N}{M - 0.5} \tag{2}$$

$$T_r = \frac{N + 1}{M} \tag{3}$$

where T_r is the recurrence interval in years, N is the length of continuous record in years, and M is the order of magnitude of the flood peak. Reference 44c discusses these formulas and shows that the shape of the final curve, particularly in the frequent-flood range, is little affected by the choice of the formula. The writer prefers formula (1) because of its simplicity and because refinements in making frequency estimates do not appear justified, considering the degree of accuracy of the basic data and the fact that, for the less frequent floods, the length of record is generally too short to make any positive predictions with regard to frequency.

When historic floods are included, the recurrence intervals for all floods equal to or greater than these floods are computed in the same way except that the value of N is made equal to the time in years between the earliest historic flood and the end of the continuous record. The final plot of peak discharge vs. recurrence interval can be made on probability or semilogarithmic paper. It is convenient in flood-damage studies to use percentage chance of occurrence rather than the recurrence interval T_r in years. The percentage chance of occurrence, $P = 1/T_r$.

27. Flood Waves in Rivers. The computation of flood-wave modification by reservoir storage was discussed in Sec. 1. One of the basic premises in this type of computation is that the inflow at the head of the reservoir is felt immediately throughout the reservoir. The flood-wave velocity is, therefore, infinite. Furthermore, the outflow rate is simply a function of the stage. The computation of flood-wave modification by storage in river channels and valleys—a process called "flood routing"—is much more complicated. The differential equations and specialized mathematical procedures have been well developed by various writers[47,48,49]; however, their application is too laborious for practical use and requires the employment of approximate methods of solution. On the other hand, it is probable that further progress in the development of electronic computing machines will enable improvements and refinements in present methods.

The method now in most common use in the United States, known as the Muskingum method[50] developed by G. T. McCarthy, assumes that the storage in a reach of the river at any time is a constant factor times the outflow plus another constant factor times the inflow, or

$$S = AO + BI$$

where S is storage, O is outflow rate, I is inflow rate, and A and B are constants. The more usual form of expression is

$$S = K[xI + (1 - x)O]$$

in which K and x are constants; $100x$ and $100(1 - x)$ are the percentage weights of I and O, respectively.

Values of K and x can be determined in two ways: (1) Compute water-surface profiles for various values of O, using I as a parameter (assume that the discharge in the reach varies uniformly with distance from O at the lower end to I at the upper end), then compute the storage under the profiles, and then use curve-fitting techniques to define the relation among S, O, and I, thus giving K and x. (2) From known hydrographs, based on stream-gaging measurements, of I at the upper end and O at the lower end of the reach, compute mass inflow, mass outflow, and storage which is the difference of the latter two quantities; for an assumed value of x, plot storage vs. weighted flow $xI + (1 - x)O$. It will be found that such a plot will have the form of a loop. Successive trials of x values will narrow the loop until it approaches a straight line, the slope of which is K. To get a sufficiently narrow loop, it may be necessary in some cases to vary x and K within various discharge ranges. The second method is more accurate and less laborious but requires reliable stream-gaging measurements.

After the K and x values have been determined, flood routing computations are made by application of the fundamental condition that, in any time interval Δt, inflow minus outflow equals change in storage Δs. Using subscripts 1 and 2 to denote beginning and end of Δt,

$$\Delta S = S_2 - S_1 = \frac{\Delta t}{2}(I_1 + I_2) - \frac{\Delta t}{2}(O_1 + O_2)$$

By combining this expression with the previous one for storage S, there results

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

where $C_0 = \dfrac{-Kx + \Delta t/2}{K(1 - x) + \Delta t/2}$

$$C_1 = \frac{Kx + \Delta t/2}{K(1 - x) + \Delta t/2}$$

$$C_2 = \frac{K(1 - x) - \Delta t/2}{K(1 - x) + \Delta t/2}$$

The value of $\Delta t/2$ should be chosen so that it is more than Kx, otherwise C_0 will be negative. An electronic routing device based on the preceding equation, recently developed by the U.S. Weather Bureau,[51] permits rapid determination of outflow hydrographs and of the effects of adjusting the K and x values.

28. Small-area Floods. As pointed out in Art. 23, the unit-hydrograph principle has been successfully applied to natural drainage basins varying in size from about 1 to 2,000 sq miles. In the following cases it has doubtful applicability: (1) natural areas smaller than 1 sq mile, (2) agricultural areas drained artificially by ditches and subsurface pipes, (3) urban built-up areas drained artificially by storm sewers. One reason for the lack of applicability of the unit hydrograph in these three cases is the lack of measurements of stream flow and concurrent precipitation on a scope comparable to that available for larger natural drainage basins; on the other hand, it is expected that the U.S. Soil Conservation Service will publish many useful data on small natural areas in the near future. A second reason opposing use of the unit hydrograph is the fact that the drainage density (see Art. 22) of areas smaller than a few square miles

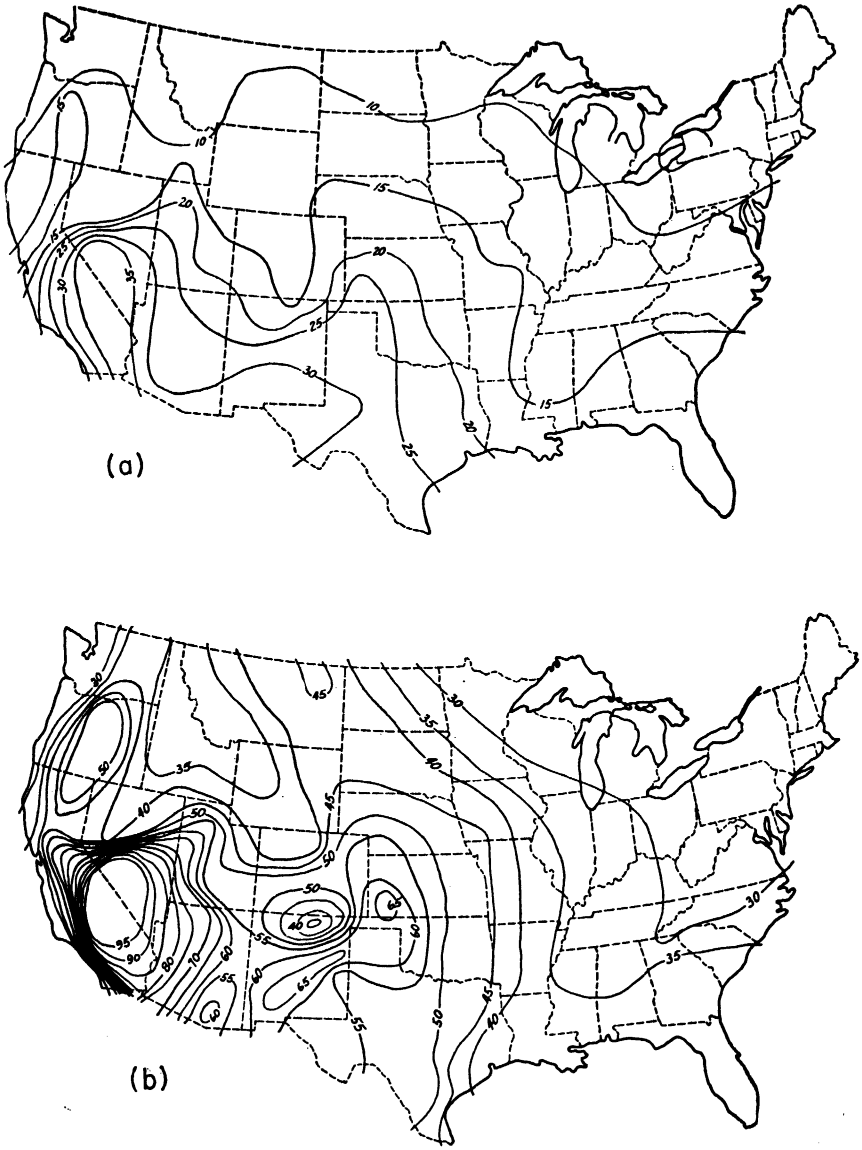


FIG. 27.—Mean evaporation in inches from U.S. Weather Bureau Class A land pans.[55] (*a*) November to April, (*b*) May to October.

is much less than for larger areas; as a result, the volume of surface detention (see Art. 16) is relatively greater. This tendency is even more pronounced in the case of very flat areas, such as airfields.

The principal methods now in use for small areas are three: (1) the so-called "rational" method, (2) the overland-flow hydrograph method, and (3) the use of empirical factors called "drainage moduli" in agricultural drainage work, based only

on size of drainage area. The use of method (1) is described in Sec. 22, Sewerage, while further data on the coefficients and a critique of the method are given in Ref. 52. Method (2) is described in Refs. 33*f* and 53.

29. Evaporation. In this article only evaporation from free-water surfaces is discussed. The factors affecting the rate of evaporation are[33d] air and water temperature, relative humidity, wind velocity, barometric pressure, and salinity of the water. Several formulas involving these factors have been developed of which the following, due to Meyer,[54] is an example:

$$E = C(V - v)\left(1 + \frac{W}{10}\right)$$

where E is the evaporation in inches for a given unit of time, V is the saturation vapor pressure in inches of mercury at the water temperature, v is the actual vapor pressure of the air about 25 ft above the ground in inches of mercury, W is the wind velocity in miles per hour about 25 ft above the ground, and C is a coefficient proportional to the unit of time used and varying with the depth of the body of water. For 24-hr periods, values of C varying from 0.36, for ordinary reservoirs about 25 ft deep, to 0.50 for shallow bodies of water have been determined.[54]

Measurements of evaporation in the United States have most commonly been made with the standard U.S. Weather Bureau Class A pan consisting of an unpainted galvanized iron pan 4 ft in diameter, 10 in. deep, and set on a wood grillage 6 in. above ground to permit circulation of air under the pan. Horton[55] has summarized the measurements made at 108 Weather Bureau evaporation stations and 42 other stations with results as shown in Fig. 27.

To estimate reservoir evaporation from pan measurements coefficients are applied; investigations in California indicated values of 0.65 to 0.70 from February to May, 0.75 to 0.80 from June to August, and 0.85 to 0.95 for the remainder of the year.[33e]

On large projects where evaporation is an important consideration, extensive investigations may be justified including supplemental pan measurements and meteorological measurements at a proposed reservoir site. Such supplemental measurements, even though of short duration, may be compared with similar long-term observations at regular Weather Bureau stations, thus enabling a more reliable estimate of evaporation at the reservoir site.

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APPENDIX A

HYDRAULIC FORMULAS

By W. L. Voorduin

I. OPEN CHANNELS

1. Theorem of Bernoulli, applied to open channels (see Fig. 1).

$$\frac{V_A^2}{2g} + h_e = \frac{V_B^2}{2g} + h_l$$

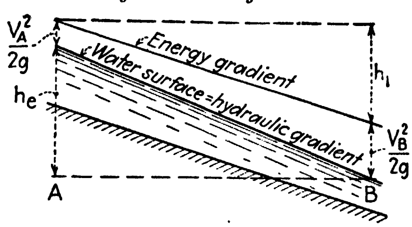


Fig. 1.

where V_A and V_B = mean velocity, fps, at A and B.
g = acceleration due to gravity = 32.17 ft/sec.²
$V_A^2/2g$ and $V_B^2/2g$ = velocity head, ft, at A and B.
h_l = head loss, ft, due to friction and turbulence, between A and B.
h_e = difference in elevation of water surface at A and B.

2. Friction losses are usually determined by the following formulas:

a. Chézy formula:

$$V = C\sqrt{RS}$$

where V = mean velocity, fps,
R = hydraulic radius, ft, = cross-sectional area of water prism divided by wetted perimeter.
S = hydraulic gradient.
C = coefficient.

b. Ganguillet and Kutter formula for the coefficient C in the Chézy formula:

$$C = \frac{41.6 + \dfrac{0.00281}{S} + \dfrac{1.811}{n}}{1 + \dfrac{\left(41.6 + \dfrac{0.00281}{S}\right) n}{\sqrt{R}}}$$

where S = hydraulic gradient.
n = coefficient of roughness.
R = hydraulic radius, ft.

Figure 2 is a graph for the solution of the Kutter formula. Table 1 shows values for the coefficient of roughness n for different channel linings.

Table 1.—Coefficients of Roughness for Different Channel Linings in Clean, Straight Channels

Type of lining	Condition	n (Kutter and Manning)	γ (Bazin)
Glazed coating or enamel	In perfect order	0.010	
Timber	Planed boards, carefully laid	0.010	0.06
	Planed boards, inferior workmanship or aged	0.012	
	Unplaned boards, carefully laid	0.012	0.16
	Unplaned boards, inferior workmanship or aged	0.014	
Metal	Smooth	0.010	0.06
	Riveted	0.015	0.30
	Slightly tuberculated	0.020	
Masonry	Neat cement plaster	0.010	0.06
	Sand and cement plaster	0.012	
	Concrete, steel troweled	0.012	
	Wood troweled	0.013	
	Brick, in good condition	0.013	0.16
	Rough	0.015	0.30
	Masonry in bad condition	0.020	
Stonework	Smooth, dressed ashlar	0.013	0.16
	Rubble set in cement	0.017	0.46
	Fine, well-packed gravel	0.020	
Earth	Regular surface in good condition	0.020	0.85
	In ordinary condition	0.0225	1.30
	With stones and weeds	0.025	1.75
	In poor condition	0.035	
	Partially obstructed with debris or weeds	0.050	

c. Bazin formula for the coefficient C in the Chézy formula:

$$C = \frac{87}{0.552 + \frac{\gamma}{\sqrt{R}}}$$

where R = hydraulic radius, ft.
γ = coefficient of roughness.

Figure 3 is a graph for the solution of the Bazin formula. Values of γ for different channel linings are given in Table 1.

d. Manning formula:

$$1.486\, R^{2/3}S^{1/2} \qquad \text{or} \qquad V = \frac{1.486}{n} R^{1/6}\sqrt{RS},$$

where V = mean velocity, fps.
R = hydraulic radius, ft.
S = hydraulic gradient.
n = coefficient of roughness.

Figure 4 is a nomograph for the solution of the Manning formula. Values for the coefficient of roughness n for different channel linings correspond to values for n in the Kutter formula and are given in Table 1.

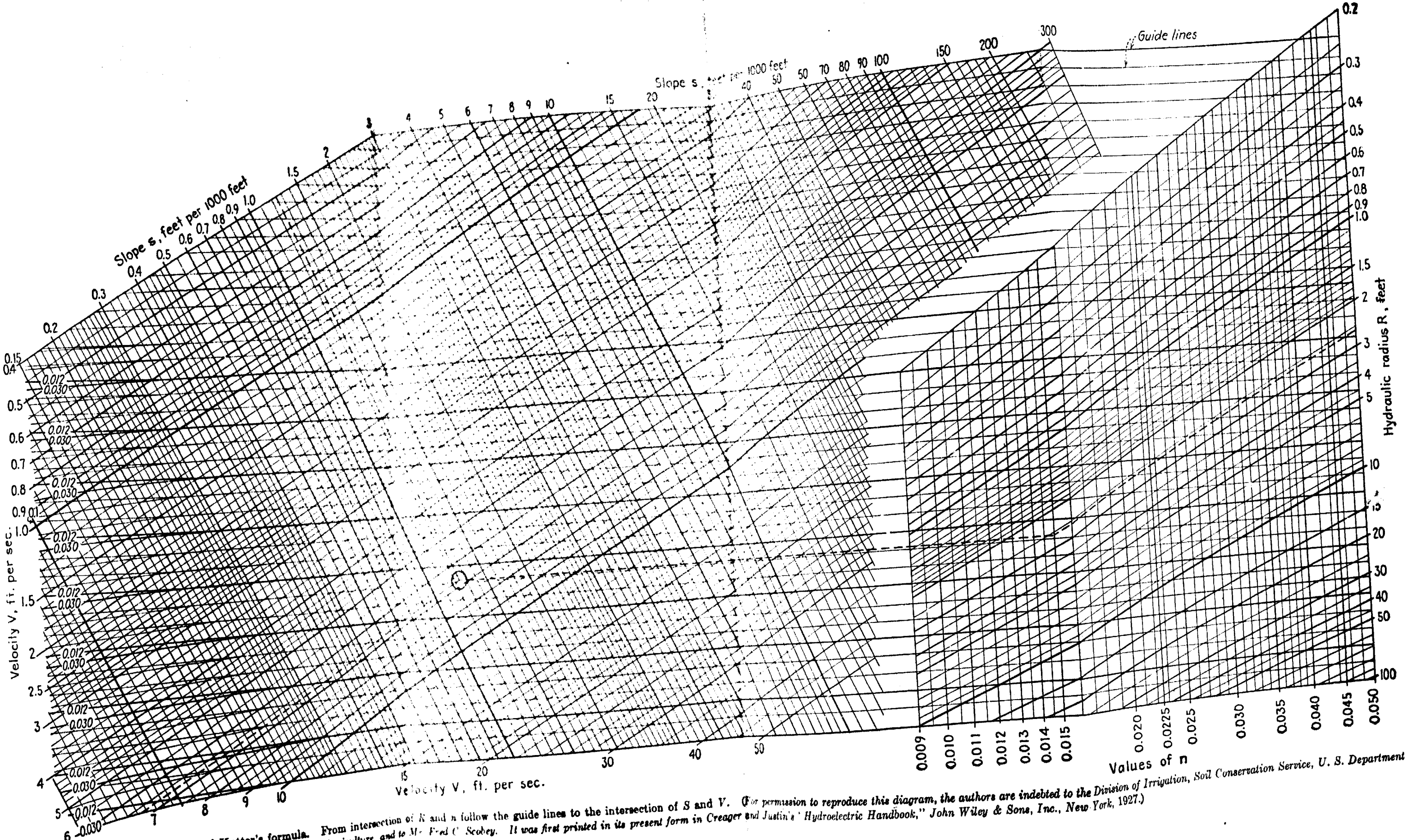


FIG. 2.—Diagram for the solution of Kutter's formula. From intersection of R and n follow the guide lines to the intersection of S and V. (For permission to reproduce this diagram, the authors are indebted to the *Division of Irrigation, Soil Conservation Service, U. S. Department of Agriculture*, and to Mr. Fred C. Scobey. *It was first printed in its present form in Creager and Justin's "Hydroelectric Handbook," John Wiley & Sons, Inc., New York, 1927.*)

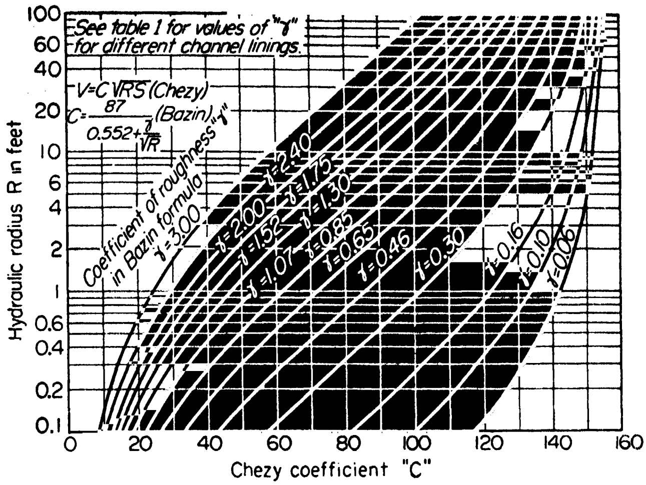


FIG. 3.—Graphical solution of Bazin formula.

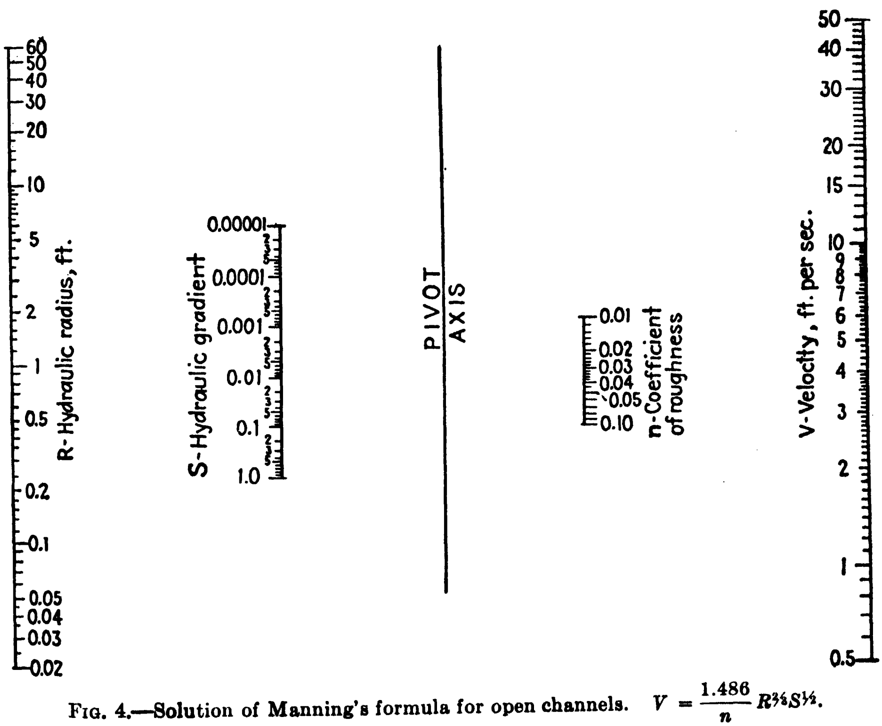


FIG. 4.—Solution of Manning's formula for open channels. $V = \frac{1.486}{n} R^{2/3} S^{1/2}$.

Figure 5 is a comparison of the coefficients of roughness in the Kutter, Bazin, and Manning formulas for equal coefficient C in the Chézy formula.

For clean channels with gentle curvature and long tangents, add 0.001; for sharp curvature, add 0.002 to preceding values of n and corresponding amounts to values of γ.

For water carrying small amount of silt, add 0.001, for silt-laden streams, add 0.002 to values of n and corresponding amounts to values of γ.

For very weedy reaches and irregular channels, the values of the coefficient of roughness may be considerably higher than shown in the preceding table.

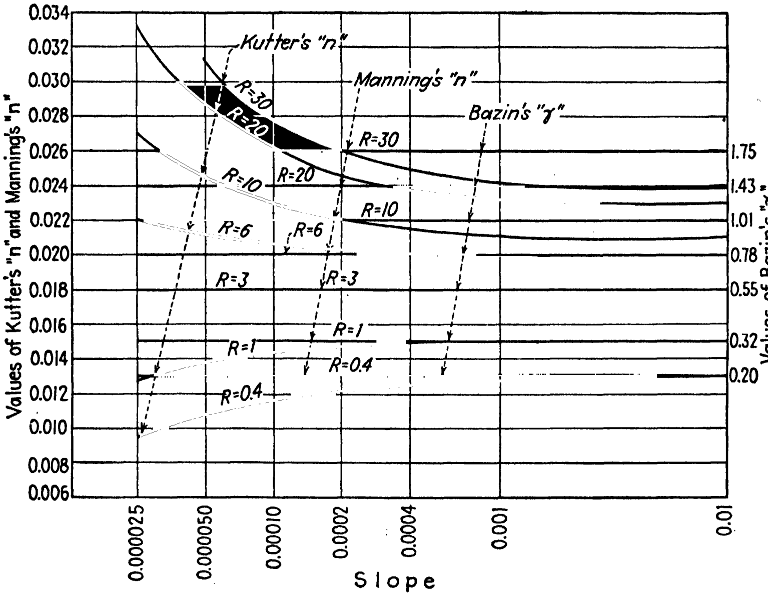


Fig. 5.—Comparison of roughness coefficients for Ganguillet and Kutter, Manning, and Bazin formulas for equal value of $C = 100$ in Chézy formula.

3. Minor Losses Due to Turbulence.

a. Sudden *contraction* or inlet losses (see Fig. 6).

Head loss due to turbulence between A and B:

$$C_i\left(\frac{V_B^2}{2g} - \frac{V_A^2}{2g}\right)$$

where C_i = 0.5 for sharp-cornered entrance.
= 0.25 for round-cornered entrance.
= 0.05 for bellmouthed entrance.
= 0.1 safe for design.

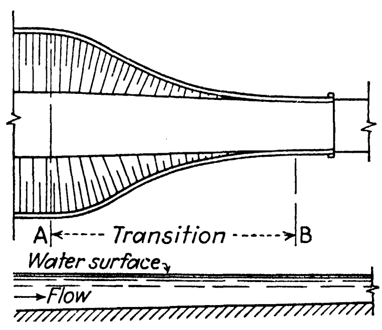


Fig. 6.—Typical inlet.

Total decrease in water-surface elevations at A and B = head loss due to friction + head loss due to turbulence + difference in velocity head.

b. Sudden *enlargement* or outlet losses (see Fig. 7).

Head loss due to turbulence between A and B:

$$C_0\left(\frac{V_A^2}{2g}-\frac{V_B^2}{2g}\right)$$

where C_0 = 1.0 for sudden enlargement.
= 0.1 for bellmouth outlet.
= 0.2 safe for design.

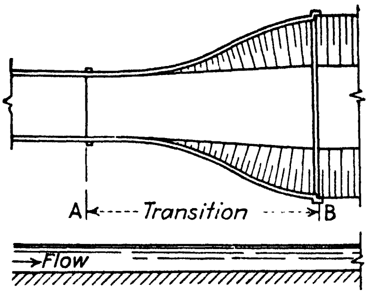


FIG. 7.—Typical outlet.

Total increase in water-surface elevations at A and B = difference in velocity head —head loss due to friction—head loss due to turbulence.

c. Losses due to bends and curves in the alignment of the channel are usually taken into account by an increase in the coefficient of roughness, depending in amount on the degree of curvature (see Table 1).

d. Backwater due to bridge piers. Figure 8 shows the result of model tests on bridge piers as channel obstructions by David L. Yarnell (*U. S. Dept. Agr. Bull.* 442).

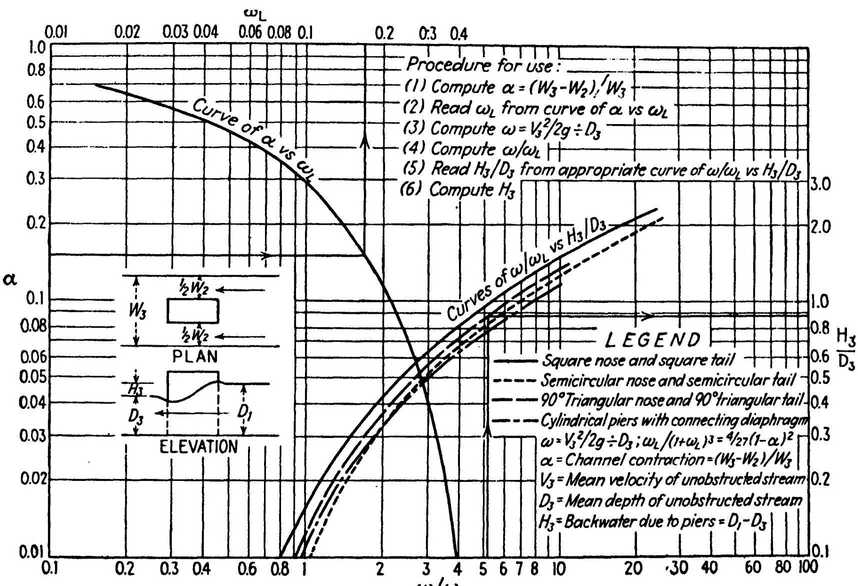


FIG. 8.—Backwater at bridge piers. Results of Yarnell's model tests. (*U. S. Dept. Agr., Tech. Bull.* 442.)

4. Critical Depth. The critical depth for a given flow of water in a channel is that depth for which depth + velocity head is a minimum. Conversely, for a given total head (= depth + velocity head), the discharge is a maximum at critical depth (see Fig. 9).

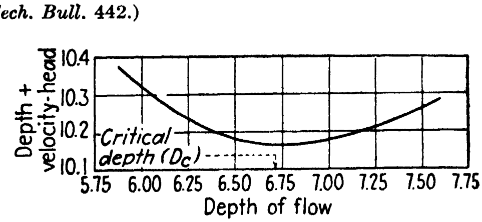


FIG. 9.—Critical depth.

For trapezoidal cross sections, with width at water surface $= B$ and bottom width $= b$:

$$D_c = \frac{4B}{5B + b} \times (\text{total head})$$

For rectangular cross sections ($B = b$):

$$\begin{aligned} D_c &= \tfrac{2}{3} \times (\text{total head}) = \tfrac{2}{3}H_t \\ \text{Velocity head} &= \tfrac{1}{3}H_t \\ \text{Maximum discharge (at critical depth)} &= \text{area} \times \text{velocity} \\ &= b \times \tfrac{2}{3}H_t \times \sqrt{2gH_t/3} \\ &= 3.087bH_t^{3/2} \end{aligned}$$

For V-shaped cross sections ($b = 0$):

$$\begin{aligned} D_c &= \tfrac{4}{5} \times (\text{total head}) = \tfrac{4}{5}H_t \\ \text{Velocity head} &= \tfrac{1}{5}H_t \\ \text{Maximum discharge} &= \tfrac{1}{2} \times B \times \tfrac{4}{5}H_t \sqrt{2gH_t/5} \\ &= 1.435BH_t^{3/2} \end{aligned}$$

5. Hydraulic Jump. For a discussion of the hydraulic jump, reference is made to Sec. 7, Spillways and Stream-bed Protection Works.

II. PIPES

1. Theorem of Bernoulli, applied to pipes (see Fig. 10).

$$\frac{V_A^2}{2g} + h_A + h_e = \frac{V_B^2}{2g} + h_B + h_l$$

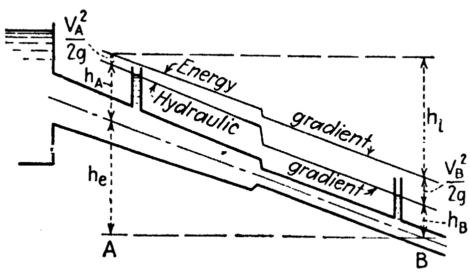


FIG. 10.

where V_A and V_B = mean velocity, fps, at A and B.
g = acceleration due to gravity.
$\frac{V_A^2}{2g}$ and $\frac{V_B^2}{2g}$ = velocity head, ft, at A and B.
h_A and h_B = pressure head, ft, at A and B.
h_l = head loss, ft, due to friction and turbulence, between A and B.
h_e = difference in elevation of center line of pipe at A and B.

2. Friction losses are usually determined by the following formulas:

a. Chézy formula $V = C\sqrt{RS}$, modified by *Darcy* to apply to pipes:

$$h_l = f\frac{l}{d}\frac{V^2}{2g}$$

where h_l = friction loss, ft.
l = length of pipe, ft.
d = inside diameter of pipe, ft.
V = mean velocity, fps.
g = acceleration due to gravity.
f = coefficient.

Fanning's values of f for smooth straight pipes are shown in Fig. 11.[1]

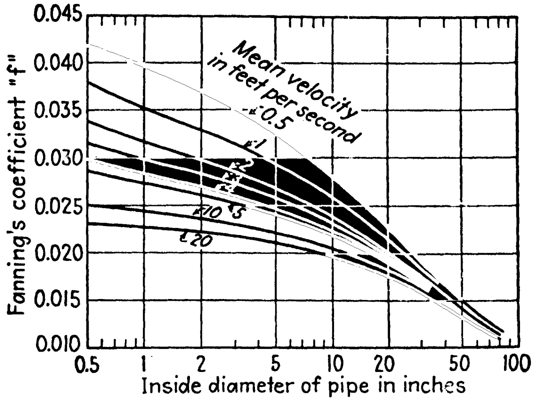


FIG. 11.

b. Hazen and Williams formula:[2]

$$V = 1.318CR^{0.63}S^{0.54}$$

where V = mean velocity, fps.
S = hydraulic gradient.
R = hydraulic radius, ft.
= ¼ inside diameter, ft.
C = friction coefficient.

Average values of C are given in Table 2.

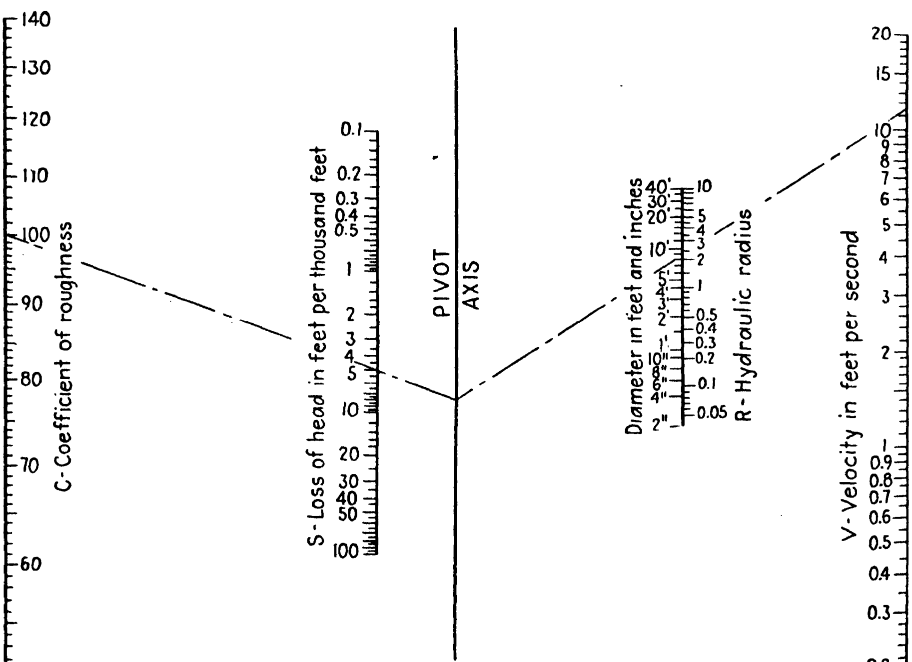
Figure 12 is a graph for the solution of the Hazen and Williams formula.

[1] Values of f as given in KING, H. W., "Handbook of Hydraulics," 2d ed., p. 205, McGraw-Hill Book Company, Inc., New York, 1939.

[2] See, WILLIAMS, GARDNER S., and ALLEN HAZEN, "Hydraulic Tables," John Wiley & Sons, Inc., New York, 1905.

Table 2.—Values of *C* in Hazen and Williams Formula

Type of pipe	Condition		"C"
Cast iron	New	All sizes	130
	5 years old	12″ and over	120
		8″	119
		4″	118
	10 years old	24″ and over	113
		12″	111
		4″	107
	20 years old	24″ and over	100
		12″	96
		4″	89
	30 years old	30″ and over	90
		16″	87
		4″	75
	40 years old	30″ and over	83
		16″	80
		4″	64
		40″ and over	77
		24″	74
		4″	55
Welded steel	Values of *C* the same as for cast-iron pipe, 5 years older		
Riveted steel	Values of *C* the same as for cast-iron pipe, 10 years older		
Wood-stave	Average value, regardless of age		120
Concrete or concrete-lined	Large sizes, good workmanship, steel forms		140
	Large sizes, good workmanship, wooden forms		120
	Centrifugally spun		135
Vitrified	In good condition		110



$V = 1.318\, C R^{0.63} S^{0.54}$

Fig. 12.—Solution of Hazen and Williams formula.

c. Scobey's Formulas.

Wood-stave pipes:[1]

$$V = 1.62D^{0.65}H^{0.555}$$

or

$$Q = 1.272D^{2.65}H^{0.555}$$

where V = velocity, fps.
D = inside diameter of pipe, ft.
Q = quantity of water flowing, cfs.
H = loss of head, ft per 1,000 ft.

Figure 13 is a nomograph for the solution of Scobey's formula for wood-stave pipe. With this formula no coefficient of roughness is required.

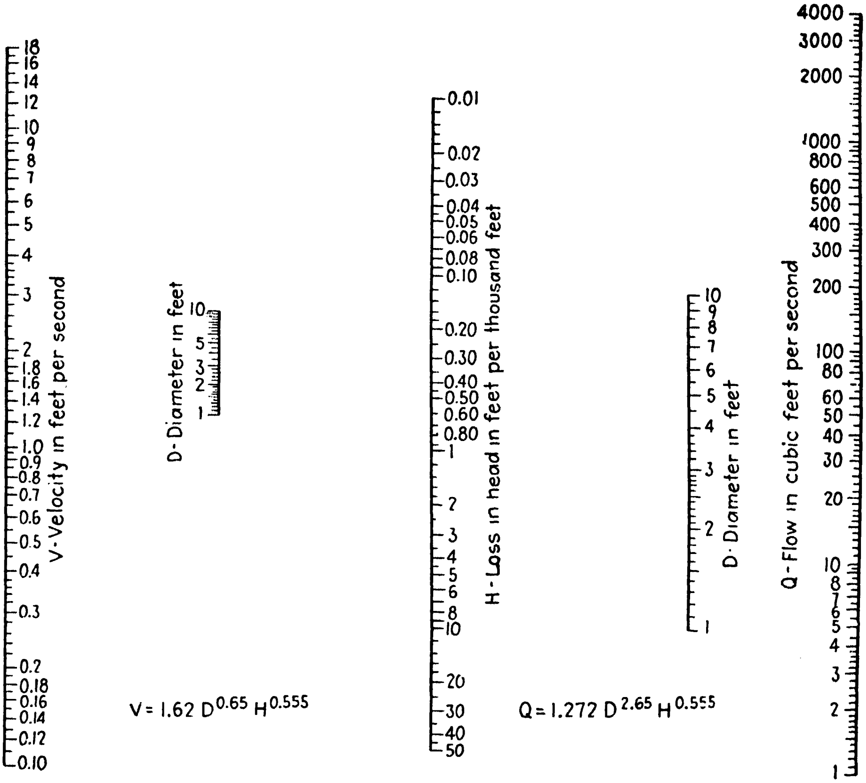


FIG. 13.—Solution of Scobey's formula for wood-stave pipe.

[1] See, Flow of Water in Wood-stave Pipes, *U.S. Dept. Agr., Bull.* 376, 1916, revised 1926.

Concrete pipes:[1]

$$V = C_s d^{0.625} H^{0.5}$$

or

$$Q = 0.00546 C_s d^{2.625} H^{0.5}$$

or

$$V = 4.73 C_s D^{0.625} H^{0.5}$$

or

$$Q = 3.72 C_s D^{2.65} H^{0.5}$$

where V = velocity, fps.
D = inside diameter of pipe, ft.
Q = quantity of water flowing, cfs.
d = inside diameter of pipe, in.
H = loss of head, ft per 1,000 ft.
C_s = coefficient.

TABLE 3.—VALUES OF C_s IN SCOBEY'S FORMULA FOR CONCRETE PIPES

Class	Condition	C_s
1	Old California cement pipes; generous supply of mortar in joints, mortar squeeze not removed Also: class 2 pipes conveying sewage	0.267
2	Dry-mix precast in short units, washed inside with cement mortar, moderate care; monolithic pipe over rough wood forms; cement-gun finish, not troweled	0.310
3	Wet-mix precast in short units; dry-mix precast in long units; monolithic pipe on steel forms; small cement-lined iron pipe; concrete pipe made under pressure and mechanically troweled with neat cement. Also: class 4 pipes conveying sewage or detritus-laden water	0.345
4	Glazed-interior pipes; large cement-lined iron pipes; monolithic pipes with joint scars or surface irregularities removed; highest quality precast pipe, made against oiled steel form, with joints as smooth as remainder, untouched with brush or "wash" process	0.370

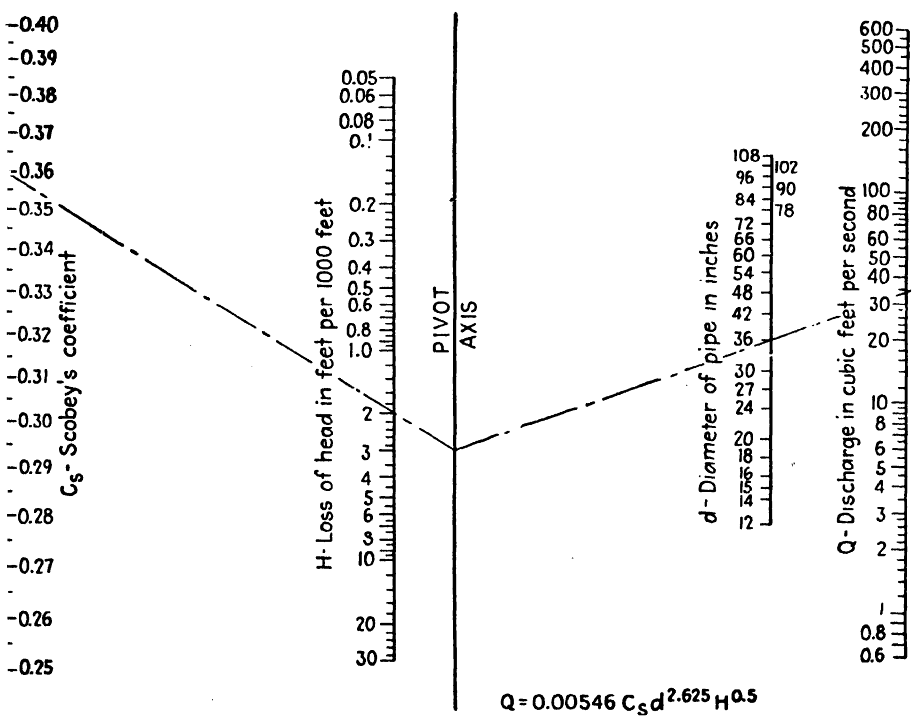


FIG. 14.—Scobey's formula for concrete pipes.

[1] See, Flow of Water in Concrete Pipes, *U.S. Dept. Agr. Bull.* 852, 1902.

Steel pipes:[1]

$$V = \frac{D^{0.58}H^{0.526}}{K_s^{0.526}}$$

$$Q = \frac{0.785D^{2.58}H^{0.526}}{K_s^{0.526}}$$

where V = velocity, fps.
D = inside diameter of pipe, ft.
Q = quantity of water flowing, cfs.
H = loss of head, ft per 1,000 ft.
K_s = coefficient.

Variation of K_s with time: $K_s = K_s'e^{0.015t}$, for the more acid waters of eastern United States.

$$K_s = K_s'e^{0.01t}, \text{ for relatively inactive waters}$$

where K_s = value for new pipe as given above.
t = age of pipe, years.
e = Naperian base = 2.7183.

TABLE 4.—VALUES OF K_s IN SCOBEY'S FORMULAS FOR STEEL PIPES

Class	Condition	Type	K_s
1	Full-riveted pipe, having both longitudinal and girth seams held by one or more lines of rivets with projecting heads from capacity standpoint; pipe with countersunk rivetheads on interior belongs in class 3	*a.* New sheet metal up to $\frac{3}{16}''$ thick *b.* New plate metal $\frac{3}{16}''$ to $\frac{7}{16}''$ thick, with either taper or cylinder joints *c.* New plate metal $\frac{1}{2}''$ up, with either taper or cylinder joints, and for plate $\frac{1}{4}''$ to $\frac{7}{16}''$ thick, when butt-jointed *d.* New butt-strap pipe of plate, $\frac{1}{2}''$ up	0.38 0.44 0.48 0.52
2	Girth-riveted pipe, having no retarding rivetheads in the continuous-seamed longitudinal joints, but having the same girth seams as full-riveted pipe	New sheet- and plate-metal pipe, such as lock-bar and hammer-weld pipe with lap or flange-riveted field (girth) joints; electric weld, hammer-weld and drawn pipe with riveted bump joints; and all other types with surface continuous except for girth belt of rivetheads between field units	0.34
3	Continuous interior pipe, having interior surface unmarred by plate offsets or by projecting rivetheads in either longitudinal or girth seams. Not necessarily described as smooth	New sheet- and plate-metal pipe such as pipe with full-welded crimped slip joint, lock bar with welded flange or leaded sleeve connections, bell and spigot, bolted coupling pipes all belong to this class	0.32

[1] See, The Flow of Water in Riveted Steel and Analogous Pipes, *U.S. Dept. Agr., Bull.* 150, 1930.

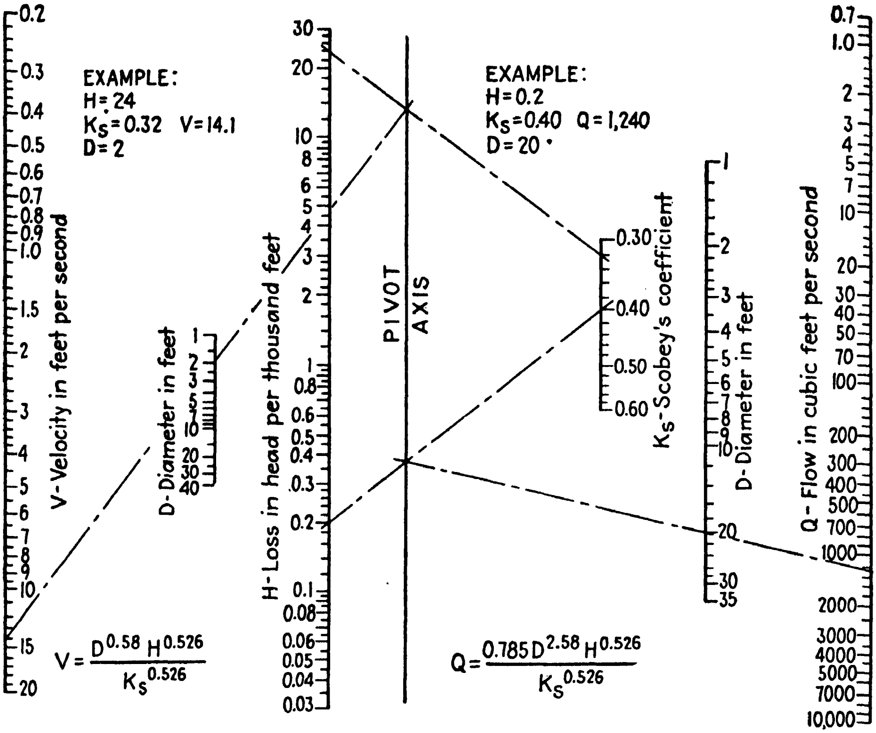


Fig. 15.—Solution of Scobey's formulas for steel pipe.

Figure 15 is a nomograph for the solution of Scobey's formulas for steel pipes.

d. Manning formula, as applicable to pipes:

$$V = \frac{0.59}{n} D^{2/3}S^{1/2}$$

where V = velocity, fps.
D = inside diameter of pipe, ft.
S = hydraulic gradient.
n = coefficient of roughness (see Table 5).

Figure 16 is a nomograph for the solution of **Manning's formula for the flow** of water in pipes.

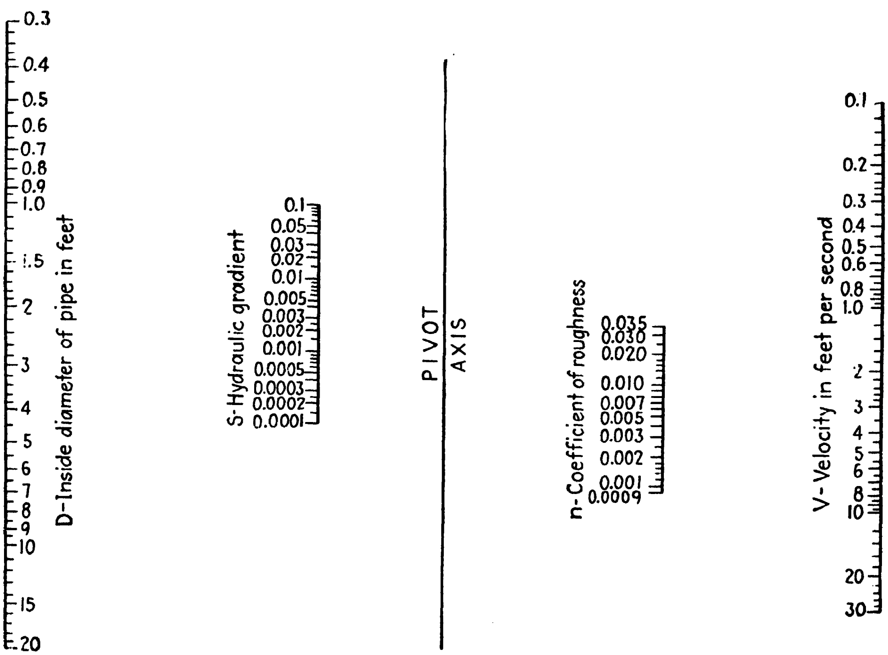
e. Ganguillet and Kutter formula is the same as given under **Art. I-2*b*.**

Table 5.—Values[1] of Coefficient of Roughness n in Manning and Ganguillet and Kutter Formulas

Type of pipe	Condition	n			
		Best	Good	Fair	Bad
Cast iron	Clean, uncoated	0.012	0.013	0.014	0.015
	Clean, coated	0.011	0.012	0.013	
	Dirty or tuberculated	.....	.....	0.015	0.035
Wrought iron	Commercial, black	0.012	0.013	0.014	0.015
	Commercial, galvanized	0.013	0.014	0.015	0.017
Lock bar or welded	Smooth and clean	0.010	0.011	0.013	
Brass or glass	Smooth	0.009	0.010	0.011	0.013
Riveted steel or spiral steel	Clean	0.013	0.015	0.017	
Vitrified sewer pipe	....	0.011	0.013	0.015	0.017
Common clay drainage tile	....	0.011	0.012	0.014	0.017
Concrete	Rough joints	.....	0.016	0.017	
	Dry mix, rough forms	.....	0.015	0.016	
	Wet mix, steel forms	.....	0.012	0.014	
	Very smooth	.....	0.011	0.012	
Wood stave	....	0.010	0.011	0.012	0.013

[1] Values are based on tables by Horton and values recommended by King. See H. W. King, "Handbook of Hydraulics," McGraw-Hill Book Company, Inc., New York, 1939.

Values given under good or fair may be used for designing.



$$V = \frac{0.59}{n} D^{2/3} S^{1/2}$$

Fig. 16.—Manning's formula for pipes.

8. Minor Losses Due to Turbulence.[1]

a. Entrance losses: $$h_e = C_e\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

where h_e = entrance loss due to turbulence, ft.
V_2 = velocity in pipe, fps.
V_1 = velocity of approach, fps.
g = acceleration due to gravity.
C_e = entrance loss coefficient.

When $V_1 = 0$, $$h_e = C_e\frac{V_2^2}{2g}$$

For inward projecting entrance: $C_e = 0.75$. For sharp-cornered entrance: $C_e = 0.50$. For rounded entrance: $C_e = 0.25$. For bellmouthed entrance: $C_e = 0.05$ or less. Value for practical design: $C_e = 0.10$.

b. Outlet losses: $$h_0 = C_0\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

where h_0 = outlet loss due to turbulence, ft.
V_1 = velocity in pipe above outlet, fps.
V_2 = velocity below outlet, fps.
g = acceleration due to gravity.
C_0 = outlet loss coefficient.

For discharge into still water: $V_2 = 0$; $$h_0 = C_0\frac{V_1^2}{2g}$$

For sudden enlargement: $C_0 = 1.0$. For bellmouthed outlet: $C_0 = 0.10$. Value for practical design: $C_0 = 0.20$.

c. Contraction losses: $$h_c = C_c\frac{V^2}{2g}$$

where h_c = loss of head, ft, due to sudden contraction.
C_c = coefficient.
V = velocity, fps, in smaller pipe.

For ratio of diameters = 1.5: $C_c = 0.30$. For ratio of diameters = 2.0: $C_c = 0.35$. For ratio of diameters = 2.5: $C_c = 0.40$. For ratio of diameters = 5.0: $C_c = 0.50$.

d. Enlargement losses: $$h_e = C_e\frac{V^2}{2g}$$

where h_e = loss of head, ft, due to sudden enlargement.
C_e = coefficient.
V = velocity, fps, in smaller pipe.

For ratio of diameters = 1.5: $C_e = 0.35$. For ratio of diameters = 2.0: $C_e = 0.60$. For ratio of diameters = 2.5: $C_e = 0.75$. For ratio of diameters = 5.0: $C_e = 1.00$.

For gradual enlargement, multiply preceding values of C_e by $\sin\theta$ where θ is angle between sides of the taper.

e. Bend losses: $$h_b = C_b\frac{V^2}{2g}$$

where h_b = excess loss of head in 90-deg bend over straight pipe.
C_b = coefficient of bend loss for 90-deg bend in pipe.

For ratio of radius of bend to diameter of pipe = 5 and over: $C_b = 0.60$. For ratio of radius of bend to diameter of pipe = 3 and over: $C_b = 0.65$. For ratio of radius of bend to diameter of pipe = 2 and over: $C_b = 0.75$. For ratio of radius of bend to diameter of pipe = 1 and over: $C_b = 1.0$.

Fuller gives: For 45-deg bends: ¾ loss of 90-deg bends of same radius. For 22½-deg bends: ½ loss of 90-deg bends of same radius.

[1] See also p. 410.

Example: The dotted line shows that the resistance of a 6-inch Standard Elbow is equivalent to approximately 16 feet of 6-inch Standard Pipe

Note: For sudden enlargements or sudden contractions, use the smaller diameter, d, on the pipe size scale.

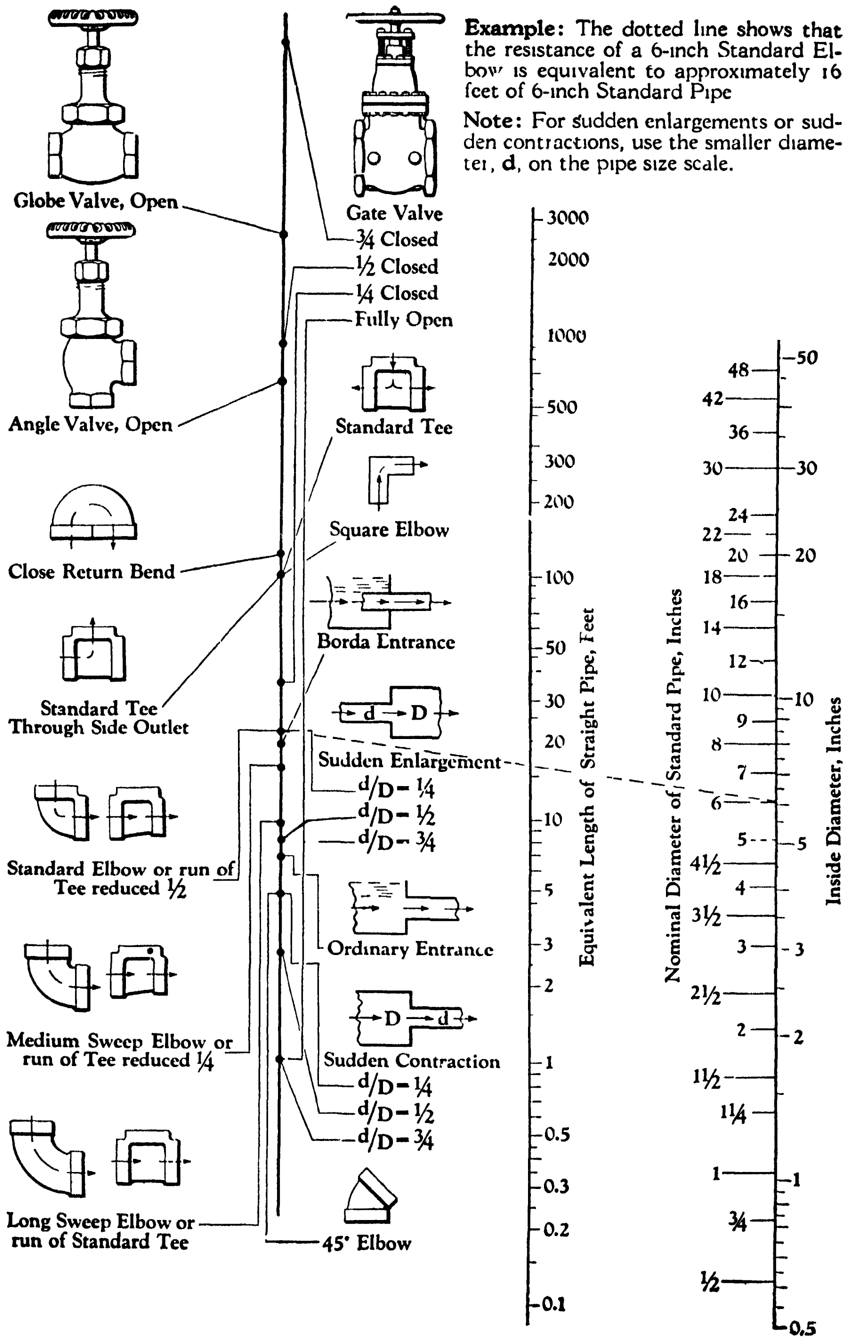


FIG. 17.—Resistance of valves and fittings to flow of fluid. (*Printed by permission of the Valve World, The Crane Company, Chicago*)

f. **Losses in fittings:**

$$h_f = C_f \frac{V^2}{2g}$$

where h_f = loss of head, ft.

V = velocity in pipe, fps.

C_f = coefficient of loss in fittings.

For globe valve, wide open: $C_f = 10.0$.

For angle valve, wide open: $C_f = 5.0$.

For gate valve, wide open: $C_f = 0.2$.

Losses in fittings in terms of equivalent length of straight pipe are shown on Fig. 17.

4. Venturi Meter Formulas.

$$Q = CA_1 \frac{1}{\sqrt{r^2 - 1}} \sqrt{2gh} \quad \text{or} \quad Q = CA_2 \sqrt{\frac{r^2}{r^2 - 1}} \sqrt{2gh}$$

$$M = \text{meter constant} = \frac{A_1}{\sqrt{r^2 - 1}} = A_2 \sqrt{\frac{r^2}{r^2 - 1}}$$

$$Q = CM \sqrt{2gh}$$

where Q = rate of discharge, cfs.

r = ratio of area at entrance to area at throat $= A_1/A_2$ or for circular areas $D_1^2 D_2^2$ if D_1 and D_2 are diameters at entrance and at throat. (Usually r is greater than 4, smaller than 16.)

g = acceleration due to gravity.

h = difference in level (ft) at which a liquid stands in two vertical tubes, which are connected to the side holes at entrance and throat, respectively, and are either open to the atmosphere, or under equal pressure or connected through a closed space *in vacuo*.

C = discharge coefficient, which varies between 0.97 and 1.00 for well-made Venturi tubes of suitable size for conditions of operation.

Figure 19 is a graph for values of the discharge coefficient C.

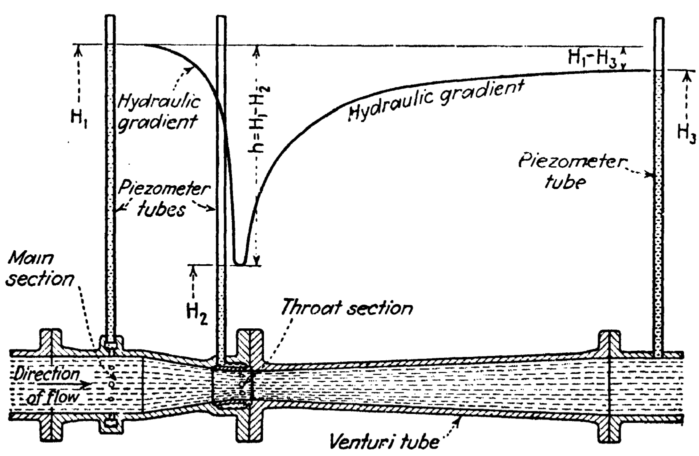


FIG. 18.—Section of Venturi tube.

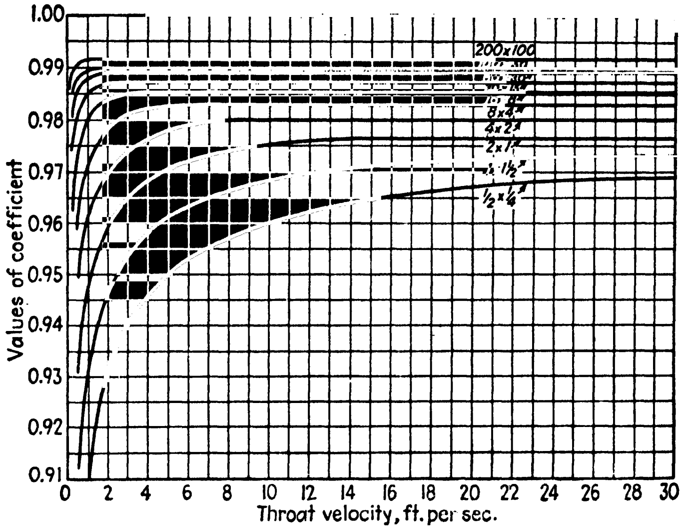


Fig. 19.—Values of discharge coefficient C.

III. UNDERGROUND FLOW OF WATER

1. Darcy Formula.

$$V = K\frac{p}{l} \quad \text{or} \quad Q = KAS.$$

where V = velocity of moving water, fps.
p = difference in pressure. ft.
Q = quantity of water flowing, cfs.
l = length of column of soil.
S = slope of ground-water level (p/l)
A = permeable area.
K = a constant depending largely on porosity and which is determined experimentally.

2. Hazen Formula.[1]

$$V = cd^2\frac{h}{l}(0.70 + 0.03t)$$

where V = velocity of water, meters daily, in a solid column of the same area as that of sand.
c = a constant factor (generally 1,000).
d = the effective size of particles, mm.
l = the thickness of sand through which the water passes.
t = the temperature of the water, deg C.
h = the loss of head in passing through the sand.

The effective size is that size, such that 10 per cent by weight of the particles are smaller than it.

The diameter of a sand grain is defined as that of a sphere of equal volume.

3. Slichter's Formula.

$$Q = 11.3\frac{pd^2A}{lk}[1 + 0.0187(t - 32°)]$$

[1] See Report, Massachusetts State Board of Health, 1892

where Q = discharge, cfs.
l = length of column of soil, ft.
p = difference in pressure or loss of head, ft.
A = area of cross section of the soil column, sq ft.
d = effective size of the soil particles composing the soil.
k = constant which depends on the porosity of the soil.
t = temperature of flowing water, deg F.

If K = transmission constant $= 11.3 \frac{d^2}{k} [1 + 0.0187(t - 32°)]$,

then

$$Q = K \frac{p}{l} A$$

Table 6.—Values of Slichter's Transmission Constant K for Various Effective Sizes and Porosity for Water at 60F

Effective size, mm	Porosity, %						Kind of soil
	30	32	34	36	38	40	
0.01	0.000033	0.000040	0.000050	0.000060	0.000072	0.000085	Silt
0.02	0.000131	0.000162	0.000198	0.000239	0.000286	0.000339	
0.03	0.000296	0.000364	0.0004460	0.000538	0.000645	0.000763	
0.04	0.000527	0.000648	0.0007940	0.000958	0.001145	0.001355	
0.05	0.000822	0.001012	0.001240	0.001495	0.001790	0.002120	Very fine sand
0.06	0.001182	0.001458	0.001784	0.002150	0.002580	0.003050	
0.07	0.001610	0.001983	0.002430	0.002930	0.003510	0.004155	
0.08	0.002105	0.002590	0.003175	0.003825	0.004585	0.005425	
0.09	0.002660	0.003280	0.004018	0.004845	0.005800	0.006860	
0.10	0.003282	0.004050	0.004960	0.005980	0.007170	0.008480	Fine sand
0.12	0.004725	0.005830	0.007130	0.008620	0.01032	0.01220	
0.14	0.006430	0.007940	0.009720	0.01172	0.01404	0.01662	
0.15	0.007390	0.009120	0.01115	0.01345	0.01611	0.01910	
0.16	0.008410	0.01036	0.01268	0.01531	0.01835	0.02170	
0.18	0.01064	0.01311	0.01605	0.01940	0.02320	0.02745	
0.20	0.01315	0.01620	0.01983	0.02390	0.02865	0.03390	
0.25	0.02050	0.02530	0.03100	0.03740	0.04480	0.05300	Medium sand
0.30	0.02960	0.03640	0.04460	0.05380	0.06450	0.07630	
0.35	0.04025	0.04960	0.06075	0.07330	0.08790	0.1039	
0.40	0.05270	0.06480	0.07940	0.09575	0.1145	0.1355	
0.45	0.06650	0.08200	0.1005	0.1211	0.1450	0.1718	
0.50	0.08220	0.1012	0.1240	0.1495	0.1780	0.2120	Coarse sand
0.55	0.09940	0.1225	0.1500	0.1810	0.2165	0.2565	
0.60	0.1182	0.1458	0.1784	0.2150	0.2580	0.3050	
0.65	0.1390	0.1710	0.2095	0.2530	0.3030	0.3580	
0.70	0.1610	0.1983	0.2430	0.2930	0.3510	0.4155	
0.75	0.1850	0.2278	0.2785	0.3365	0.4030	0.4770	
0.80	0.2105	0.2590	0.3175	0.3825	0.4585	0.5425	
0.85	0.2375	0.2925	0.3580	0.4325	0.5175	0.6125	
0.90	0.2660	0.3280	0.4018	0.4845	0.5800	0.6860	
0.95	0.2965	0.3650	0.4470	0.5400	0.6460	0.7650	
1.00	0.3282	0.4050	0.4960	0.5980	0.7170	0.8480	Fine gravel
2.00	1.315	1.620	1.983	2.390	2.865	3.390	
3.00	2.960	3.640	4.460	5.380	6.450	7.630	
4.00	5.270	6.480	7.940	9.575	11.45	13.55	
5.00	8.220	10.12	12.40	14.95	17.90	21.20	

Table 6 gives values of the transmission constant K for various effective sizes of soil particles and water of 60F.

Table 7 gives variations of the transmission constant K, as given in Table 6, with the temperature. The values in these tables are taken from those given by Prof. Slichter.[1]

Table 7.—Variations of Transmission Constant K for 60F with the Temperature

Temperature, deg F	Percentage of K_{60} as given in Table 6	Temperature, deg F	Percentage of K_{60} as given in Table 6
32	64		
35	67	70	115
40	73	75	123
45	80	80	130
50	86	85	139
55	93	90	147
60	100	95	155
65	108	100	164

4. Piping Velocities.

Table 8.—Theoretical Velocities Required to Move Particles of Various Sizes by Jet Action[1]

Diameter of particle, mm	Velocity, fpm	Diameter of particle, mm	Velocity, fpm
5.0	43.4	0.1	6.0
3.0	33.7	0.08	5.5
1.0	19.4	0.05	4.3
0.8	17.4	0.03	3.4
0.5	13.8	0.01	1.94
0.3	9.6		

[1] As given in *Trans. A.S.C.E.*, **87**, 50, 1924.

IV. SHARP-CRESTED WEIRS

1. Sharp-crested Weirs with Free Discharge (see Fig. 20).

a. Francis weir formula:

$$Q = Cl[(h + h_v)^{3/2} - h_v^{3/2}]$$

where Q = discharge, cfs.

l = effective length of crest, ft (= total crest length corrected for end contractions).

h = measured head on crest, ft, upstream from weir beyond beginning of surface curve.

h_v = velocity of approach.

C = coefficient (Francis's value C = 3.33).

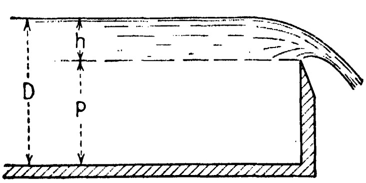


Fig. 20.

For complete end contractions:

$$l = l' - 0.1nh$$

where l' = total measured length on crest.

n = number of end contractions.

[1] See, Slichter, Charles S., The Rate of Movement of Underground Waters, *U.S. Geol. Survey*, Water Supply and Irrigation Paper 140.

For partial end contraction (sharp-nosed pier):

$$l = l' - 0.04nh$$

where n = number of partial end contractions.

b. Bazin weir formula:

$$Q = \frac{2}{3}\sqrt{2g}\, lh^{3/2}\left(0.6075 + \frac{0.01476}{h}\right)\left(1 + 0.55\frac{h^2}{D^2}\right)$$

where Q = discharge, cfs.

l = effective length of weir, ft.

h = measured head on crest, ft, upstream from weir. Beyond beginning of surface curve.

D = depth of water upstream from weir.

g = acceleration due to gravity.

c. Swiss Society of Engineers and Architects weir formula:

$$Q = \frac{2}{3}\sqrt{2g}\, lh^{3/2}\left(0.615 + \frac{0.615}{305h + 1.6}\right)\left(1 + 0.5\frac{h^2}{D^2}\right)$$

Nomenclature same as for Bazin formula.

TABLE 9.—VALUES OF $\left(3.228 + 0.435\frac{h_e}{P}\right)$ IN THE REHBOCK FORMULA FOR WEIR DISCHARGE

Head h, ft	Height of crest (P), ft (above bottom of channel of approach)															
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.1	3.273	3.269	3.266	3.263	3.260	3.258	3.256	3.255	3.253	3.252	3.251	3.248	3.247	3.245	3.244	3.243
0.2	3.317	3.309	3.302	3.296	3.291	3.287	3.283	3.280	3.277	3.275	3.272	3.268	3.265	3.262	3.260	3.258
0.3	3.360	3.348	3.338	3.330	3.322	3.316	3.311	3.306	3.301	3.298	3.294	3.288	3.283	3.279	3.275	3.272
0.4	3.404	3.388	3.374	3.363	3.353	3.345	3.338	3.331	3.326	3.320	3.316	3.308	3.301	3.296	3.291	3.287
0.5	3.447	3.427	3.411	3.397	3.384	3.374	3.365	3.357	3.350	3.343	3.338	3.328	3.319	3.312	3.306	3.301
0.6	3.491	3.467	3.447	3.430	3.416	3.403	3.392	3.382	3.274	3.366	3.359	3.347	3.337	3.329	3.322	3.316
0.7	3.534	3.506	3.483	3.463	3.447	3.432	3.419	3.408	3.398	3.389	3.381	3.367	3.356	3.346	3.337	3.330
0.8	3.578	3.546	3.519	3.497	3.478	3.461	3.446	3.434	3.422	3.412	3.403	3.387	3.374	3.362	3.353	3.345
0.9	3.621	3.585	3.556	3.530	3.509	3.490	3.474	3.459	3.446	3.435	3.425	3.407	3.392	3.379	3.368	3.359
1.0	3.665	3.625	3.592	3.564	3.540	3.519	3.501	3.485	3.471	3.458	3.446	3.426	3.410	3.396	3.384	3.374

Head h, ft	Height of crest (P), ft (above bottom of channel of approach)											
	4	5	6	7	8	9	10	12	14	16	18	20
1.0	3.337	3.315	3.301	3.290	3.283	3.277	3.272	3.264	3.259	3.255	3.252	3.250
1.1	3.348	3.324	3.308	3.297	3.288	3.281	3.276	3.268	3.262	3.258	3.255	3.252
1.2	3.359	3.333	3.315	3.303	3.293	3.286	3.280	3.272	3.265	3.261	3.257	3.254
1.3	3.370	3.341	3.323	3.309	3.299	3.291	3.285	3.275	3.269	3.264	3.260	3.256
1.4	3.381	3.350	3.330	3.315	3.304	3.296	3.289	3.279	3.272	3.266	3.262	3.259
1.5	3.392	3.359	3.337	3.321	3.310	3.301	3.293	3.283	3.275	3.269	3.264	3.261
1.6	3.402	3.368	3.344	3.328	3.315	3.306	3.298	3.286	3.278	3.272	3.267	3.263
1.7	3.413	3.376	3.352	3.334	3.321	3.310	3.302	3.290	3.281	3.274	3.269	3.265
1.8	3.424	3.385	3.359	3.340	3.326	3.315	3.307	3.293	3.284	3.277	3.272	3.267
1.9	3.435	3.394	3.366	3.346	3.332	3.320	3.311	3.297	3.287	3.280	3.274	3.269
2.0	3.446	3.402	3.373	3.353	3.337	3.325	3.315	3.301	3.290	3.283	3.276	3.272

d. **Rehbock formula:**[1]

$$Q = \left(3.228 + 0.435\frac{h_e}{P}\right) l h_e^{3/2}$$

$$h_e = h + 0.0036$$

Other nomenclature same as for Bazin formula.

e. Comparison of coefficients in preceding formulas:

All formulas may be written: $Q = Klh^{3/2}$

where for Francis formula: $K = 3.33\left[\left(1 + \frac{h_v}{h}\right)^{3/2} - \left(\frac{h_v}{h}\right)^{3/2}\right]$

For Bazin formula: $K = \left(3.248 + \frac{0.079}{h}\right)\left(1 + 0.55\frac{h^2}{D^2}\right)$

For Swiss Society of Engineers and Architects formula:

$$K = \left(3.288 + \frac{1}{92.8h + 0.49}\right)\left(1 + 0.5\frac{h^2}{D^2}\right)$$

For Rehbock formula: $K = \left(3.228 + 0.435\frac{h_e}{P}\right)$

Figure 21 shows values of K computed from the preceding formulas. Values of K, as given in the Test Code for Hydraulic Power Plants and Their Equipment of the American Society of Mechanical Engineers, are also given on Fig. 21. In all cases the coefficient K includes the correction for velocity of approach.

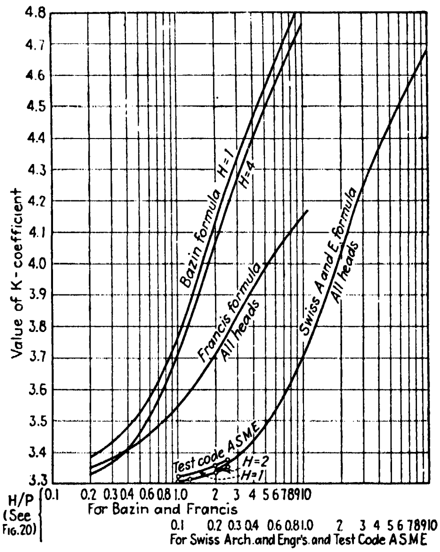


FIG. 21.—Coefficients for sharp-crested weir formulas. $Q = KLH^{3/2}$.

[1] American Society of Mechanical Engineers, Test Code for Hydraulic Prime Movers, 1938.

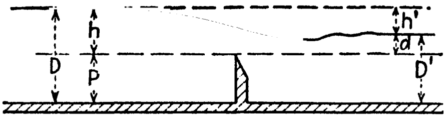


FIG. 22.—Submerged sharp-crested weir.

2. Submerged Sharp-crested Weirs.

a. Fteley and Stearns submerged-weir formula:

$$Q = Cl\sqrt{h'}\left(h + \frac{d}{2}\right)$$

where Q = discharge, cfs.
l = effective length of weir, ft.
h = measured head on crest, ft, upstream from weir beyond beginning of surface curve.
h' = difference in elevation of water surfaces ($= h - d$).
d = depth of submergence.
c = coefficient.

TABLE 10.—VALUES OF FTELEY AND STEARNS SUBMERGED-WEIR COEFFICIENT C[1]

d/h	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.....	3.330	3.331	3.335	3.343	3.360	3.368	3.371	3.372	3.370
0.1	3.365	3.359	3.352	3.343	3.335	3.327	3.318	3.310	3.302	3.294
0.2	3.286	3.278	3.271	3.264	3.256	3.249	3.241	3.234	3.227	3.220
0.3	3.214	3.207	3.201	3.194	3.188	3.182	3.176	3.170	3.165	3.159
0.4	3.155	3.150	3.145	3.140	3.135	3.131	3.127	3.123	3.119	3.116
0.5	3.113	3.110	3.107	3.104	3.102	3.100	3.098	3.096	3.095	3.093
0.6	3.092	3.091	3.090	3.090	3.089	3.089	3.089	3.090	3.090	3.091
0.7	3.092	3.093	3.095	3.097	3.099	3.102	3.105	3.109	3.113	3.117
0.8	3.122	3.127	3.131	3.137	3.143	3.150	3.156	3.164	3.172	3.181
0.9	3.190	3.200	3.209	3.221	3.233	3.247	3.262	3.280	3.300	3.325

[1] See, KING, H. W., "Handbook of Hydraulics," 2d ed., p. 98, McGraw-Hill Book Company, Inc., New York, 1939.

b. Francis submerged-weir formula:

$$Q = 3.33l\sqrt{h'}\,(h + 0.381d)$$

Nomenclature same as for Fteley and Stearns submerged-weir formula.

c. Herschel submerged-weir formula:[1]

$$Q = 3.33l(nh)^{3/2}$$

Nomenclature same as for Fteley and Stearns submerged-weir formula.

TABLE 11.—VALUES OF HERSCHEL'S SUBMERGED-WEIR COEFFICIENT n*

d/h	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.000	1.004	1.006	1.006	1.007	1.007	1.007	1.006	1.006	1.005
0.1	1.005	1.003	1.002	1.000	0.998	0.996	0.994	0.992	0.989	0.987
0.2	0.985	0.982	0.980	0.977	0.975	0.972	0.970	0.967	0.964	0.961
0.3	0.959	0.956	0.953	0.950	0.947	0.944	0.941	0.938	0.935	0.932
0.4	0.929	0.926	0.922	0.919	0.915	0.912	0.908	0.904	0.900	0.896
0.5	0.892	0.888	0.884	0.880	0.875	0.871	0.866	0.861	0.856	0.851
0.6	0.846	0.841	0.836	0.830	0.824	0.818	0.813	0.806	0.800	0.794
0.7	0.787	0.780	0.773	0.766	0.758	0.750	0.742	0.732	0.723	0.714
0.8	0.703	0.692	0.681	0.669	0.656	0.644	0.631	0.618	0.604	0.590
0.9	0.574	0.557	0.539	0.520	0.498	0.471	0.441	0.402	0.352	0.275

* See KING, H. W., "Handbook of Hydraulics," 2d ed., p. 99, McGraw-Hill Book Company, Inc., New York, 1939.

[1] *Trans. A.S.C.E.*, **14**, 1885.

V. SPILLWAY DAMS

1. General Formula for Ogee Spillways with Free Discharge.

$$Q = ClH^{3/2}$$

where Q = discharge, cfs.
l = effective length of crest, ft.
H = head on crest, ft.

If velocity of approach is appreciable,

$$Q = Cl\left(H + \frac{V^2}{2g}\right)^{3/2}$$

where C = coefficient of discharge = 3.9 to 4.0 for head for which the spillway crest is designed. For heads less than the design head the value of C decreases approximately as shown in Fig. 24.

2. Submerged Ogee Spillways. Figures 24 to 26 may be used to estimate the effect of submergence on the coefficient of discharge. Figures 24 and 25 show the

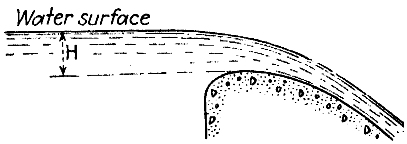


FIG. 23.—Ogee dam discharge.

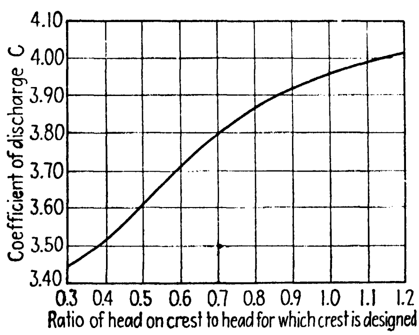


FIG. 24.—Coefficients of free discharge for ogee weirs.

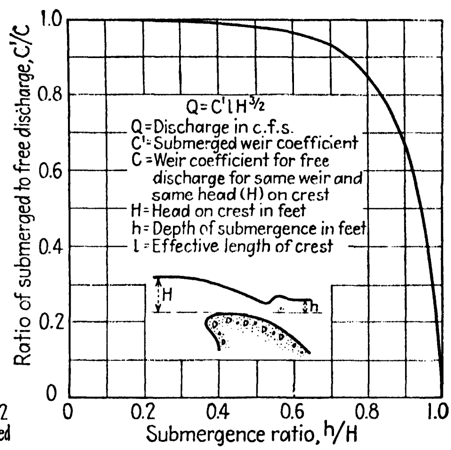


FIG. 25.—Submerged discharge curve.

head-discharge relationship in simplified form. These curves are satisfactory for preliminary studies for cases in which the depth of stream bed below the fixed crest is relatively large.

Figure 26 shows the results of more comprehensive investigations by the Bureau of Reclamation. The experimental relationships between upstream and downstream channel depths, head on the crest, and degree of submergence are clearly shown.

3. Broad-crested Weirs. Theoretical equation for discharge:

$$Q_{max} = 3.087l(h + h_v)^{3/2}$$

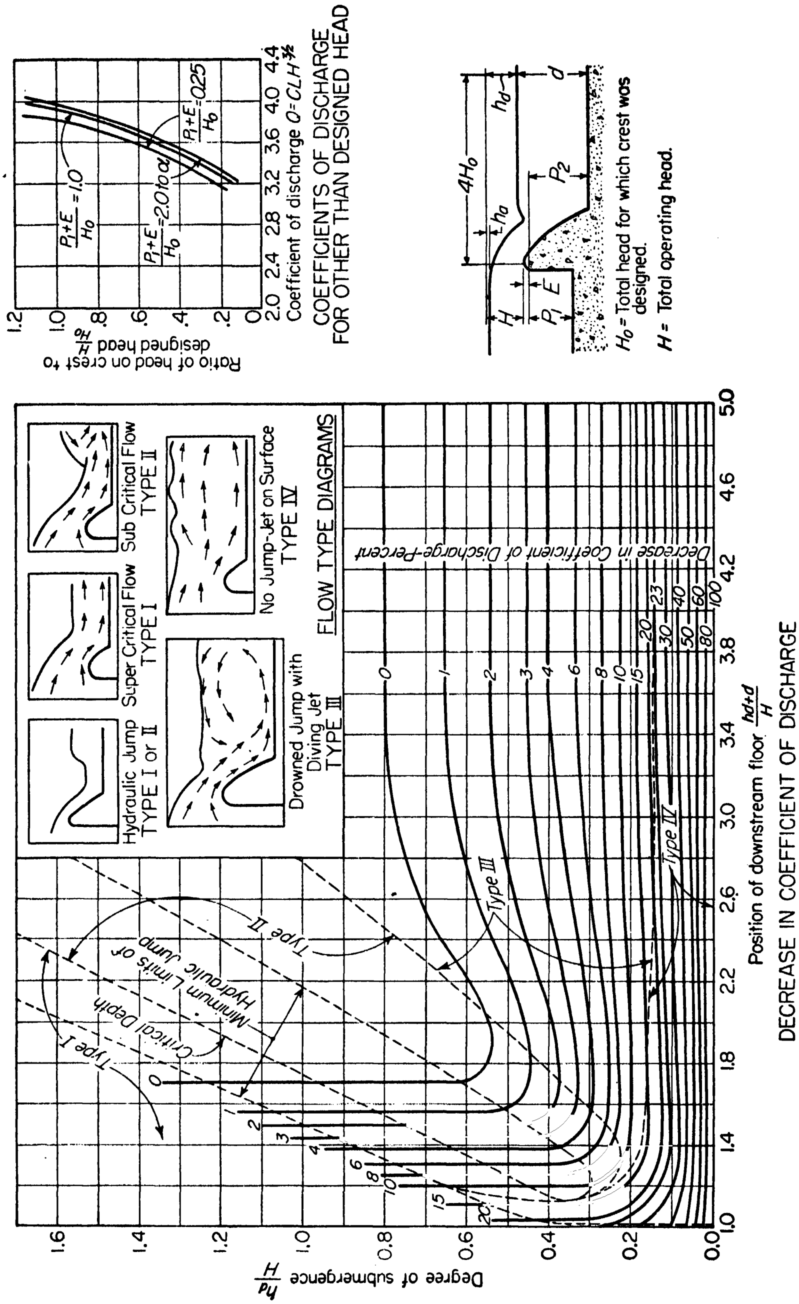


Fig. 26.—Submerged weir coefficients. (*Bureau of Reclamation.*)

where Q_{max} = maximum theoretical discharge, assuming no turbulence or friction losses.

l = unobstructed weir length.

$h + h_v$ = head on crest and velocity head of approach.

To allow for losses Q should be reduced by 10 to 15 per cent from theoretical value.

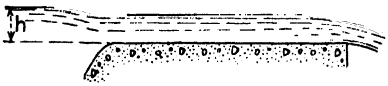


FIG. 27.—Broad-crested weir.

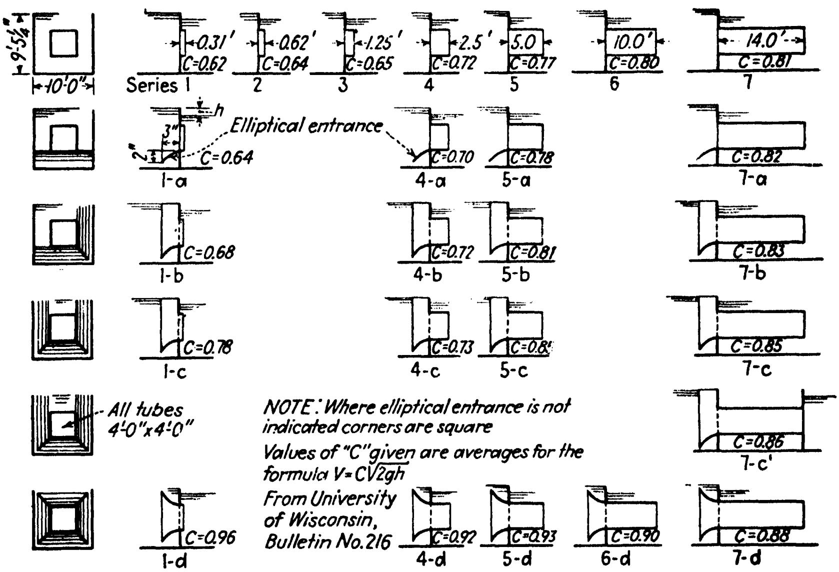


FIG. 28.—Coefficient of discharge. Flow through submerged tubes.

VI. DISCHARGE THROUGH ORIFICES AND TUBES

1. General Formula.

$$Q = CA\sqrt{2gh}$$

where Q = discharge, cfs.

C = Coefficient of discharge.

A = area of orifice, sq ft.

h = head on center line of orifice with free discharge or difference in upstream and downstream water levels for a submerged orifice.

Figure 28 shows average values for the coefficient C for various types of orifices. This coefficient is not greatly affected by submergence.

APPENDIX B

GRAPHICAL AIDS TO HYDRAULIC COMPUTATIONS

By Kenneth E. Sorensen

For many of the hydraulic computations described in preceding sections of this handbook, the practicing engineer will find graphical solutions of considerable value. Results will, in most cases, have an accuracy well within the requirements of the problem. The advantages of graphical solutions lie in the appreciable saving in time, continuous visual check against gross errors of individual numerical values, simple mechanical operations, and in many problems, the elimination of trial-and-error solutions.

Friction Slope in Tunnels and Channels. The most elementary graphical aid is the nomogram for solution of explicit algebraic equations, such as those given throughout the handbook. Two such nomograms that the author has also found useful in hydraulic design are those of Figs. 1 and 2, which solve Manning's formula for friction slope in tunnels and channels.

Surge Tanks. The nomogram of Fig. 3 can be used for the solution of the implicit algebraic equation for surge rise in simple and restricted orifice surge tanks due to instantaneous shutdown of all turbines. The nomogram is used as follows:

1. From the intersection of Kh_c and curve A, draw a straight line through the value of C_1/C_2 to intersect line B. (Note: For simple tanks $C_2 = 0$ and the line will be vertical.)
2. From this intersection draw a straight line through point C to intersect curve A.
3. Read KH_s on scale at left and divide value by K for solution of H_s.

It should be noted that, for the example given, a lesser initial Q would give a higher surge, *i.e.*, a line through the value C_1/C_2 and tangent to curve A would increase the value of KH_s. The point of tangency with curve A represents the value of h_c (and therefore, Q) which will result in the maximum surge.

Water Hammer. Trial-and-error computations required by arithmetic integration of water-hammer equations may be avoided by the nomographic method of **Fig. 4.** For the elementary case of a single, unbranched pipe or penstock,

a = velocity of pressure wave in penstock, fps.
L = total length of penstock, ft.
V = velocity of flow in penstock, fps.
H = instantaneous pressure at orifice, ft.
h = instantaneous pressure rise at orifice above initial pressure, ft.
B = orifice discharge coefficient relating pressure at orifice and velocity in penstock ($V = B\sqrt{H}$).

Then the classical difference equations for time interval $\Delta T = 2L/a$ sec are:

$$\Delta h_n = -\frac{a}{g}\Delta V_n$$
$$\Delta h_n = h_n + h_{n-1}$$
$$V_n = B_n\sqrt{H_n}$$

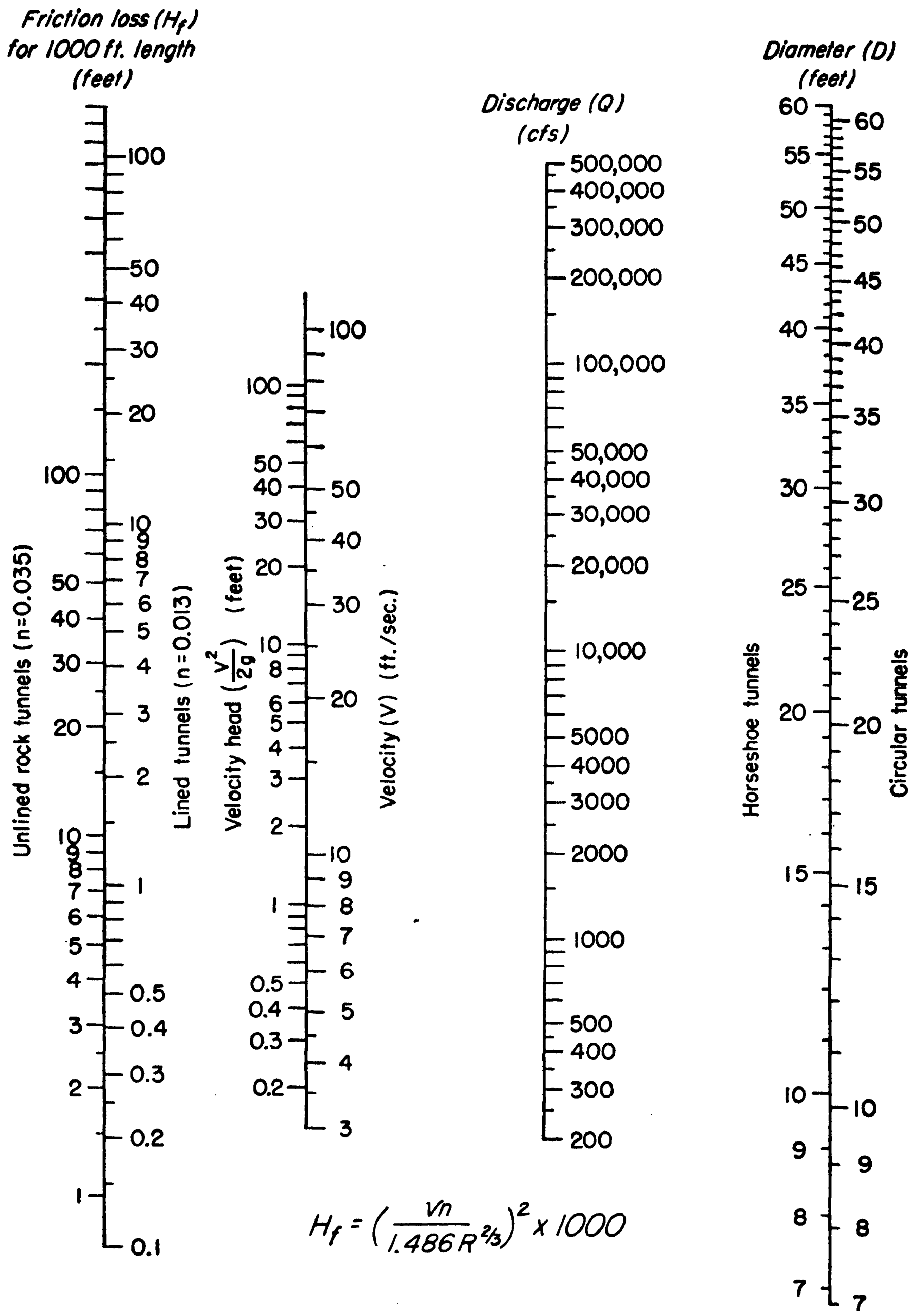


FIG. 1.—Friction loss in tunnels, flowing full.

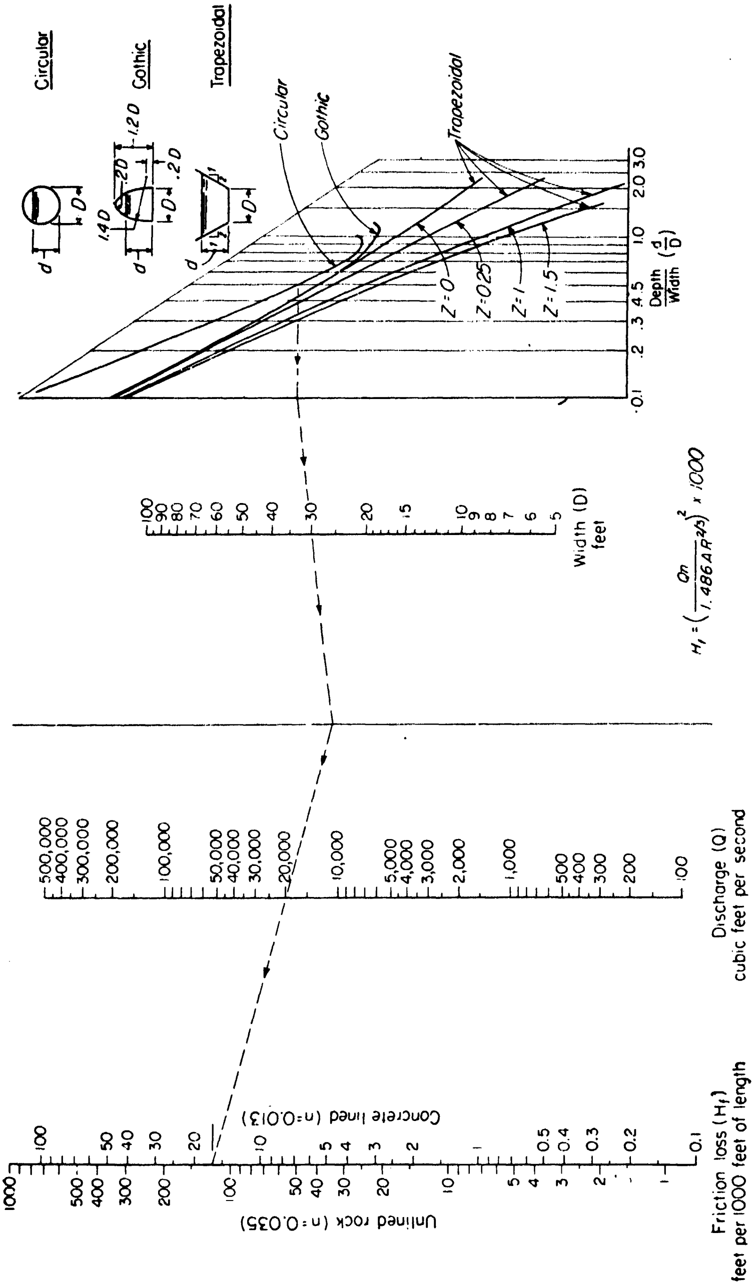


Fig. 2.—Friction loss, tunnels and channels.

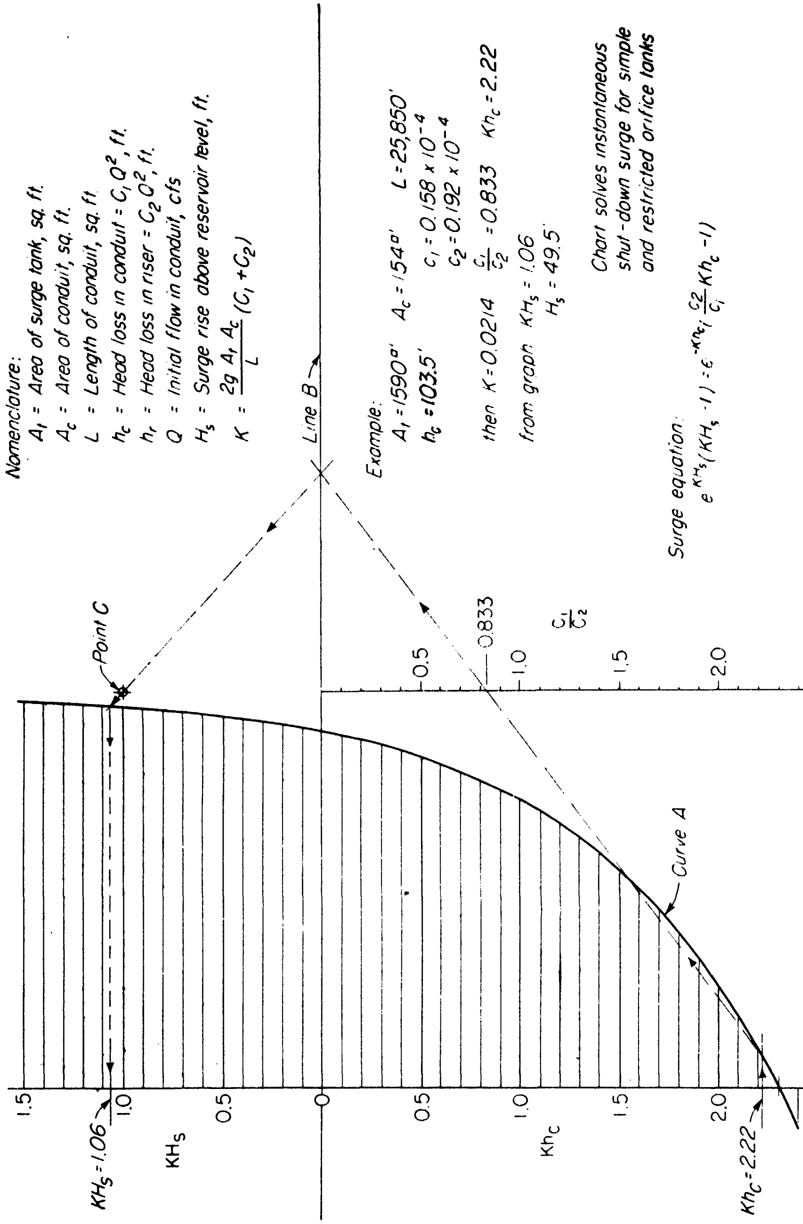


FIG. 3.—Surge tanks.

If H_0 is the pressure at the orifice at the initial steady state prior to opening or closing the orifice, then Fig. 4 can be used to determine the values of h/H_0 for successive intervals of $\Delta T = 2L/a$ sec, as follows:

1. Tabulate values of $\frac{a}{g} \frac{B}{\sqrt{H_o}}$ for successive periods $\Delta T = \frac{2L}{a}$.

2. From the coordinate $\frac{h}{H_o} = 0$ and initial value of $\frac{a}{g} \frac{B}{\sqrt{H_o}}$, draw a straight line to point A. Where this line intersects the value of $\frac{a}{g} \frac{B}{\sqrt{H_o}}$ at the end of the first interval, ΔT, read $\frac{h}{H_o}$.

3. Through the intersection obtained in step 2 and point B, draw a straight line to intersect line C.

4. From this intersection draw a straight line to point A. Where this line intersects the value of $\frac{a}{g} \frac{B}{\sqrt{H_o}}$ at the end of the second interval, read $\frac{h}{H_o}$.

5. Continue as in steps 3 and 4 until the critical values of water hammer have been determined.

If the value of $\frac{a}{g} \frac{B}{\sqrt{H_0}}$ should not suit the scale shown, all values of $\frac{a}{g} \frac{B}{\sqrt{H_o}}$ may be multiplied by any convenient common factor greater or less than unity, provided, however, that points A and B also be shifted vertically (A downward and B upward) by the same factor, using scale D.

Several common and important problems in the field of hydraulics require the solution of a differential equation of the first order in the form

$$\frac{df_1(y)}{dx} + f_2(y) = f_3(x) \tag{1}$$

in which x is the independent variable and y is the dependent variable. A graphical solution of this equation is possible if the functions $f_1(y)$, $f_2(y)$, and $f_3(x)$ and the values of x_1 and y_1 at some one point are known. Equation (1) may be approximated as

$$\Delta f_1(y) = f_1(y_2) - f_1(y_1) = [f_3(x) - f_2(y)]\,\Delta x \tag{2}$$

Furthermore, if the assumption is made that

$$f_2(y) = \tfrac{1}{2}[f_2(y_2) + f_2(y_1)] \tag{3}$$

and that $f_3(\bar{x})$ is the mean value of $f_3(x)$ between points (x_1,y_1) and (x_2,y_2), then

$$f_1(y_2) - f_1(y_1) = [f_3(\bar{x}) - f_2(y_1)]\frac{\Delta x}{2} + [f_3(\bar{x}) - f_2(y_2)]\frac{\Delta x}{2} \tag{4}$$

Figure 5 demonstrates a graphical method for solving Eq. (4). Curve 1 is plotted with $f_1(y)$ as ordinates and $f_2(y)$ as abscissas. Line 2 is drawn vertically at a distance from the origin equal to $f_3(\bar{x})$. If, from point y_1 on curve 1, line 4 is drawn at slope $\Delta x/2$ to intersect line 2, and if from that intersection line 5 is drawn at slope $-\Delta x/2$ to intersect curve 1 at y_2, then

$$f_1(y_2) - f_1(y_1) = [f_3(\bar{x}) - f_2(y_1)]\frac{\Delta x}{2} + [f_3(\bar{x}) - f_2(y_2)]\frac{\Delta x}{2} \tag{5}$$

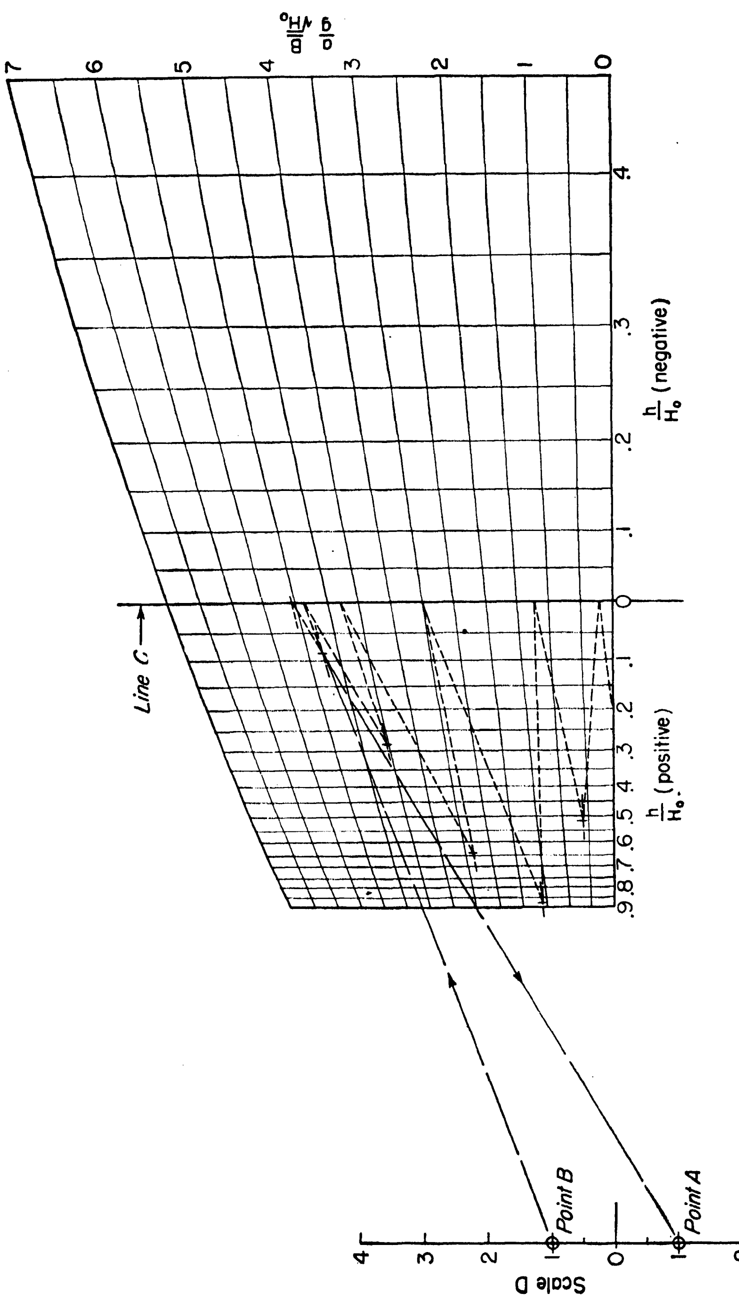


FIG. 4.—Water hammer.

which is identical to Eq. (4). To permit a plot of $y = F(x)$, curve 3 is drawn with y as ordinates and $f_2(y)$ as abscissas. By projecting from curve 1 through curve 3, each new value of y_2 obtained may be plotted opposite the corresponding value of x. Once y_2 has been found, it serves as y_1 for the next interval Δx, and the graphical process is repeated.

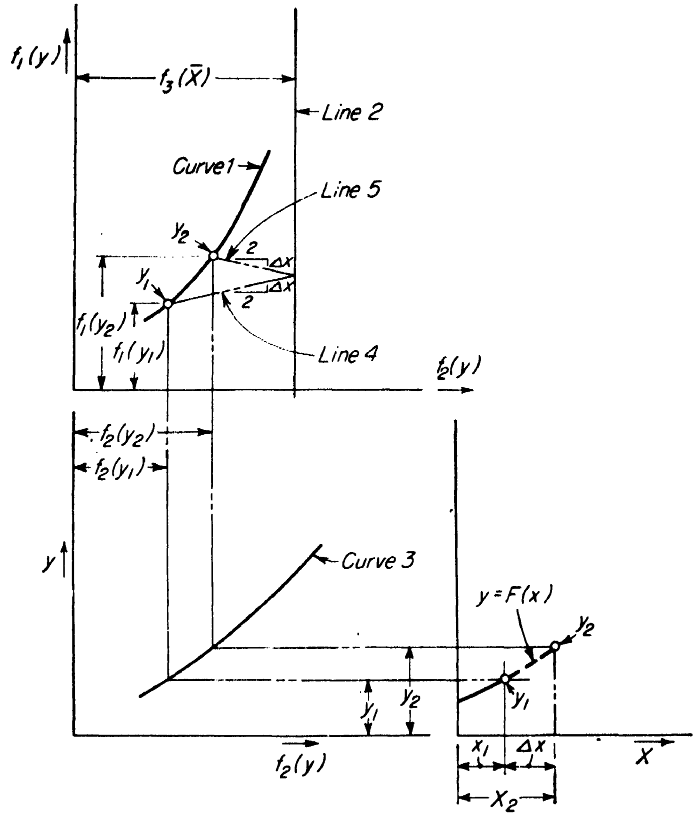


Fig. 5.

Reservoir Flood Routing. The equation for reservoir flood routing is

$$\frac{dV}{dT} = I - Q \tag{6}$$

in which reservoir volume $V = f_1(h)$.
Inflow $I = f_3(T)$
Outflow $Q = f_2(h)$.

so that

$$\frac{df_1(h)}{dT} + f_2(h) = f_3(T) \tag{7}$$

which is of the form of Eq. (1).

Figure 6 shows the graphical solution of Eqs. (6) and (7). Curve 1 is plotted with coordinates Q and V. Curve 3 is plotted with coordinates h and V (reservoir volume curve), Both h and Q can be plotted as functions of time as the graphical solution progresses. As $f_3(T) = I$, it is convenient to plot the inflow hydrograph to the same scale as Q. Then line 2 can be drawn by projecting horizontally from the average value of I for each period ΔT.

If wedge storage is to be considered and is known, a family of curves may be drawn for curve 1, each labeled with the appropriate value of I. Then, as shown in Fig. 7, the curve corresponding to I_1 is used for the initial point (h_1) and the curve corresponding to I_2 is used for the final point (h_2). However, curve 1 corresponding to zero wedge storage must be used for projecting to curve 3 in obtaining the plot of h against time.

Water-power Studies. Closely allied to reservoir flood routing is the water-power study involving determination of reservoir drawdown required to maintain constant

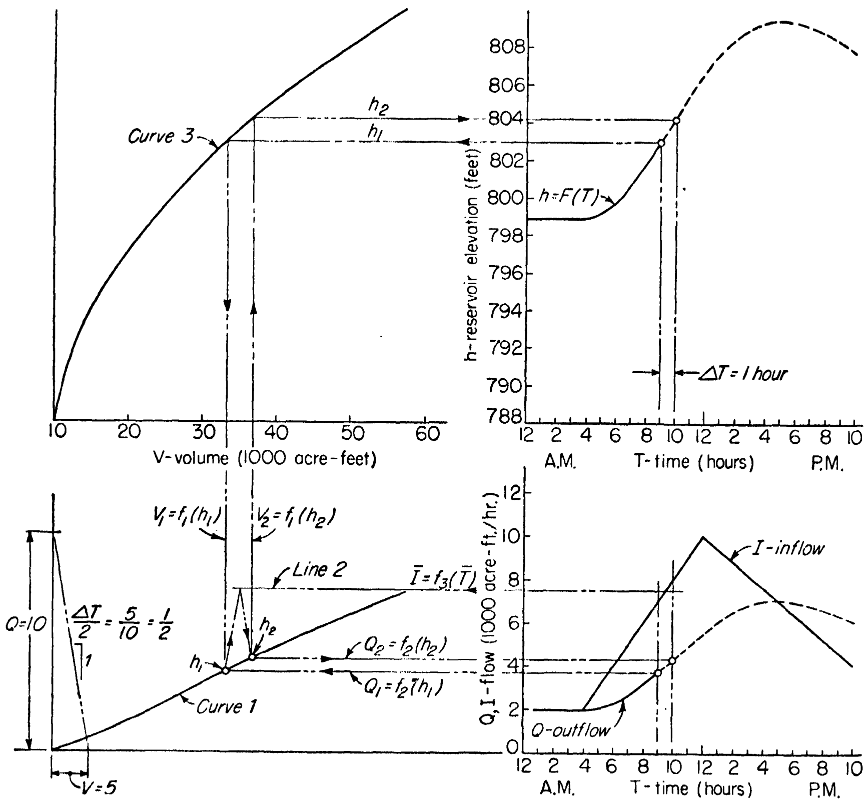


FIG. 6.

power output of plant under varying conditions of reservoir inflow. The equation for power is

$$\text{Hp} = Qf_2'(h) \tag{8a}$$

or

$$Q = f_2(h,\text{Hp}) \tag{8b}$$

in which Q is the discharge through the turbines and $f_2(h,\text{Hp})$ is a function of reservoir elevation that includes effect of tail-water rise and hydraulic and mechanical efficiency.

If $f_2(h,\text{Hp})$ is substituted for $f_2(h)$ in Eq. (7), then the method of Fig. 6 will permit a plot of reservoir elevation against time for any specific power output.

Nonuniform Flow in Channels. Bernoulli's theorem for flow in channels (Fig. 8a) may be expressed as

$$\frac{dE}{dL} + S = 0 \tag{9}$$

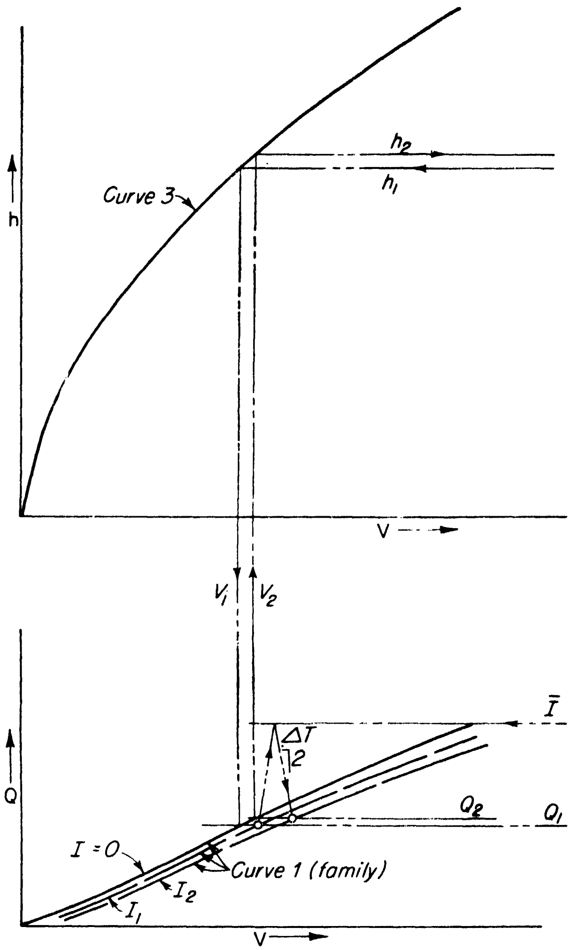


FIG. 7.

in which

$$\text{total energy } E = D + \frac{V^2}{2g} - S_0 L$$

$$D + \frac{V^2}{2g} = f_1(D) \quad (Q \text{ being constant}).$$

$$\text{bottom slope } S_0 = f_3(L).$$

$$\text{friction slope } S = f_2(D).$$

Equation (9) then takes the form

$$\frac{df_1(D)}{dL} + f_2(D) = f_3(L) \tag{10}$$

In the case of certain symmetrical artificial channels such as shown in Fig. 8*b*, it is desirable to alter Eq. (10) somewhat. In accordance with generally accepted hydraulic practice,

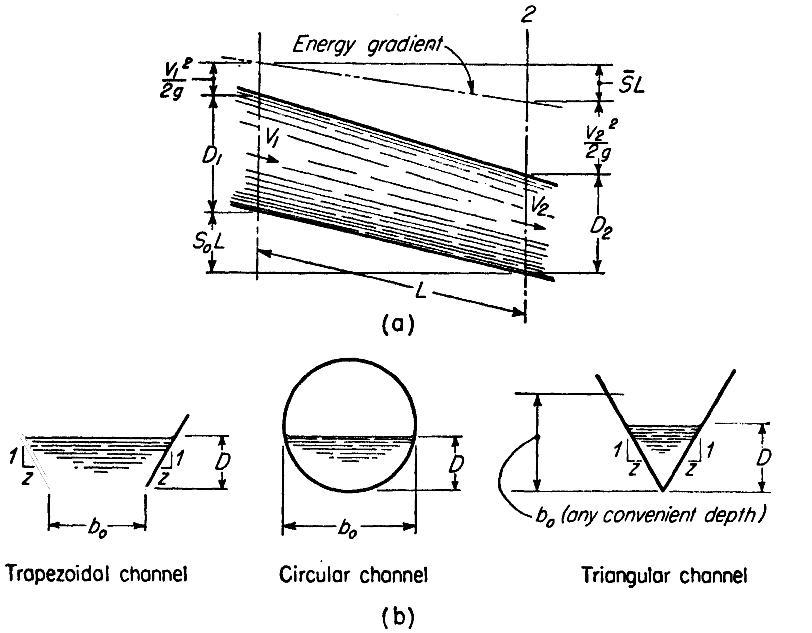


FIG. 8.

$$f_1(D) = D + \frac{V^2}{2g} = D + \frac{Q^2}{2gA^2} \tag{11a}$$

$$f_2(D) = S = \frac{Q^2n^2}{2.2082A^2R^{4/3}} \tag{11b}$$

$$\text{Area } A = ab_0^2 \tag{11c}$$

$$\text{Hydraulic radius } R = rb_0 \tag{11d}$$

$$a = F_1\left(\frac{D}{b_0}\right) \tag{11e}$$

$$r = F_2\left(\frac{D}{b_0}\right) \tag{11f}$$

Then,

$$f_1(D) = b_0\frac{D}{b_0} + \frac{Q^2}{2ga^2b_0^4} \tag{12a}$$

$$f_2(D) = \frac{Q^2n^2}{2.2082a^2r^{4/3}b_0^{16/3}} \tag{12b}$$

Further, if

$$\frac{D}{b_0} = \rho \tag{13a}$$

$$\phi(\rho) = \frac{1}{2ga^2} \tag{13b}$$

$$\psi(\rho) = \frac{1}{2.2082a^2r^{4/3}} \tag{13c}$$

$$M = \frac{b_0^5}{Q^2} \tag{13d}$$

$$N = \frac{S_0b_0^4}{Q^2} \tag{13e}$$

$$P = \frac{n^2}{b_0^{1/3}} \tag{13f}$$

then
$$f_1(D) = \frac{b_0}{M}[\rho M + \phi(\rho)] \tag{14a}$$
$$f_2(D) = \frac{b_0}{M} \cdot P\psi(\rho) \tag{14b}$$
$$f_3(L) = S_0 = \frac{b_0}{M} \cdot N \tag{14c}$$

and Eqs. (9) and (10) can be written
$$\frac{d[\rho M + \phi(\rho)]}{P\,dL} + \psi(\rho) = \frac{N}{P} \tag{15}$$

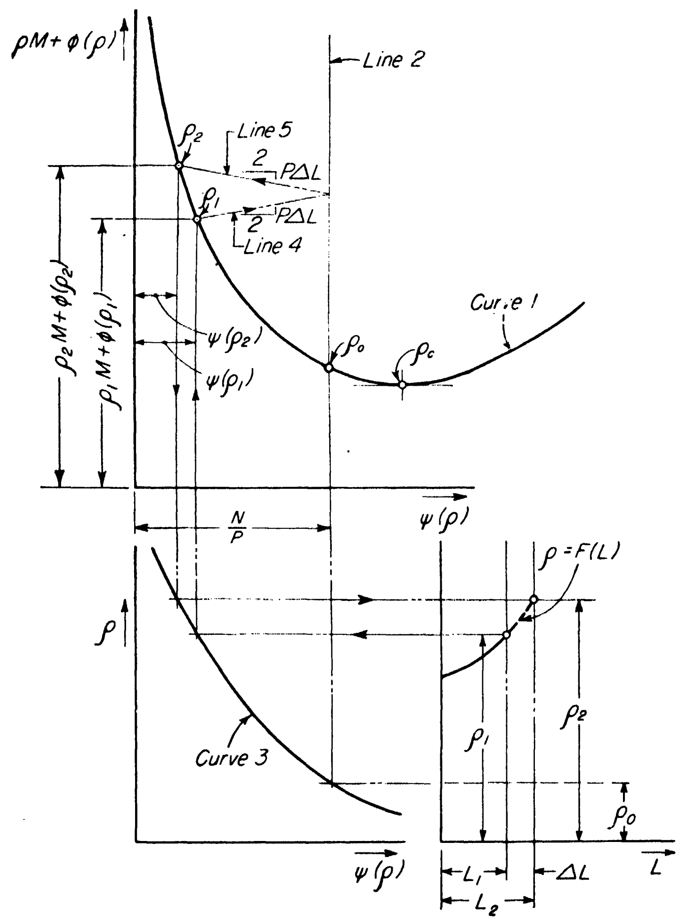


FIG. 9.

Figure 9 shows a graphical solution of Eq. (15). Curve 1 is plotted with ordinates of $\rho M + \phi(\rho)$ and abscissas of $\psi(\rho)$. Line 2 is drawn at a distance N/P from the origin. Curve 3 is drawn with ordinates of ρ (or D) and with abscissas of $\psi(\rho)$. Lines 4 and 5 are drawn at slopes $P\ \Delta L/2$ in the process of the graphical solution.

Certain characteristics of Eq. (15) and Fig. 9 are worthy of mention:

1. The functions $\phi(\rho)$ and $\psi(\rho)$ are functions of channel shape only.
2. Curve 1 is dependent on channel shape, Q, and b_0 only.
3. Distance N/P is dependent on the values of Q, b_0, S_0, and n only.
4. Slope $P\ \Delta L/2$ is dependent on b_0, n, and ΔL only.
5. The low point of curve 1 occurs at the value of $\rho_c = D_c/b_0$ in which D_c is the critical depth of the channel for flow Q.
6. The intersection of line 2 with curve 1 coincides with the value of $\rho_0 = D_0/b_0$ in which D_0 is the depth of channel at which uniform flow occurs.

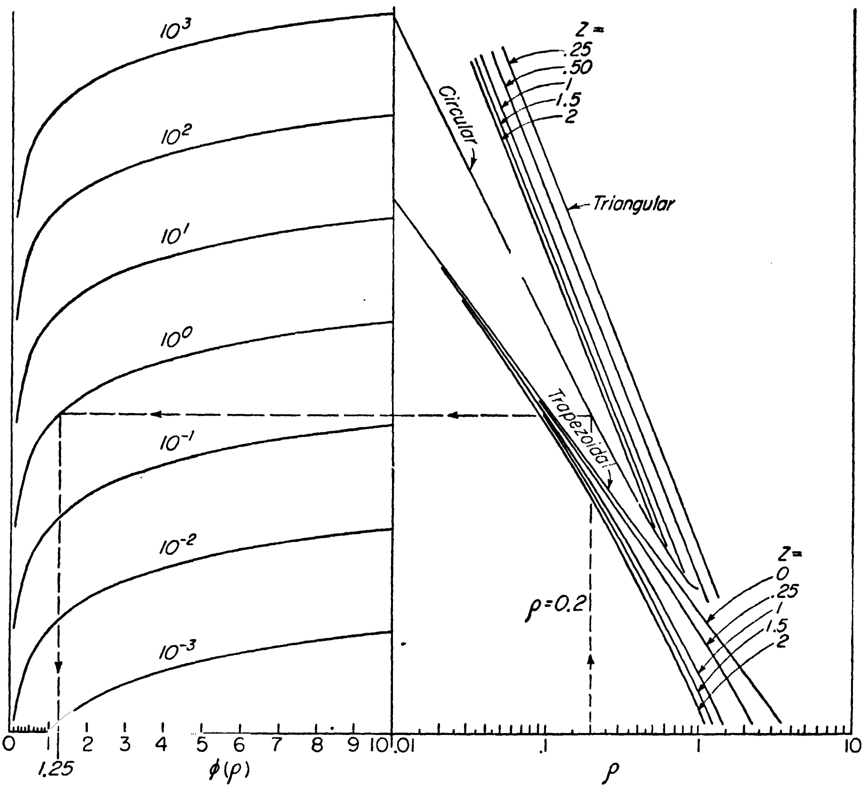


FIG. 10.

It should be noted that the solution described above is the reverse of the usual methods in that depths are determined at given increments of length, rather than the lengths being determined between given increments of depth. No trial and error is involved, not even for cases where S_0 and n may change at points along the channel. If, at some point in the water surface plot, it is desired to change the increment of length (ΔL) being used, only the slopes of lines 4 and 5 are affected. The smaller ΔL is chosen, the more accurate the solution becomes, particularly in regions where the curvature of curve 1 is sharp.

Values of $\phi(\rho)$ for the channels shown in Fig. 8*b* can be obtained from Fig. 10.

The values of $\psi(\rho)$ for circular and triangular channels can be obtained from Fig. 11. Values of $\psi(\rho)$ for rectangular and trapezoidal channels can be obtained from Table 114 of "Handbook of Hydraulics" by H. W. King, as $\psi(\rho)$ of Eq. (15) is identical with $(1/K')^2$ as used by Prof. King.

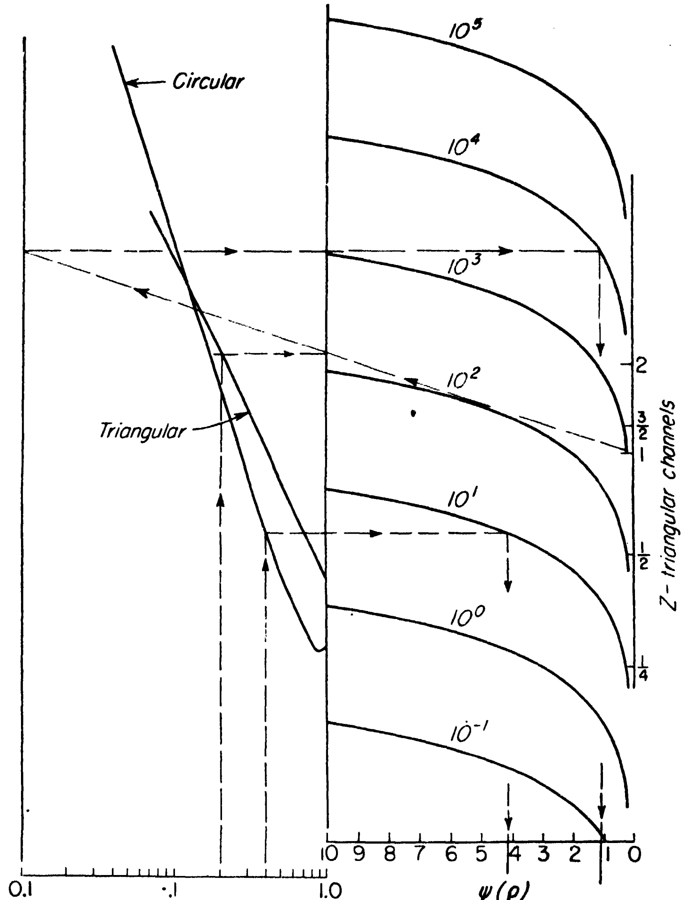


FIG. 11.

Figure 12 shows a sample problem for the solution of a backwater curve in a rectangular channel.

In the case of natural channels or artificial channels of more complicated section, it is more convenient to express the functions in terms of D rather than ρ. If $b_0 = 1$, then Eq. (15) becomes

$$\frac{d[DM' + \Phi(D)]}{d\dfrac{L}{P'}} + \Psi(D) = \frac{N'}{P'} \tag{16}$$

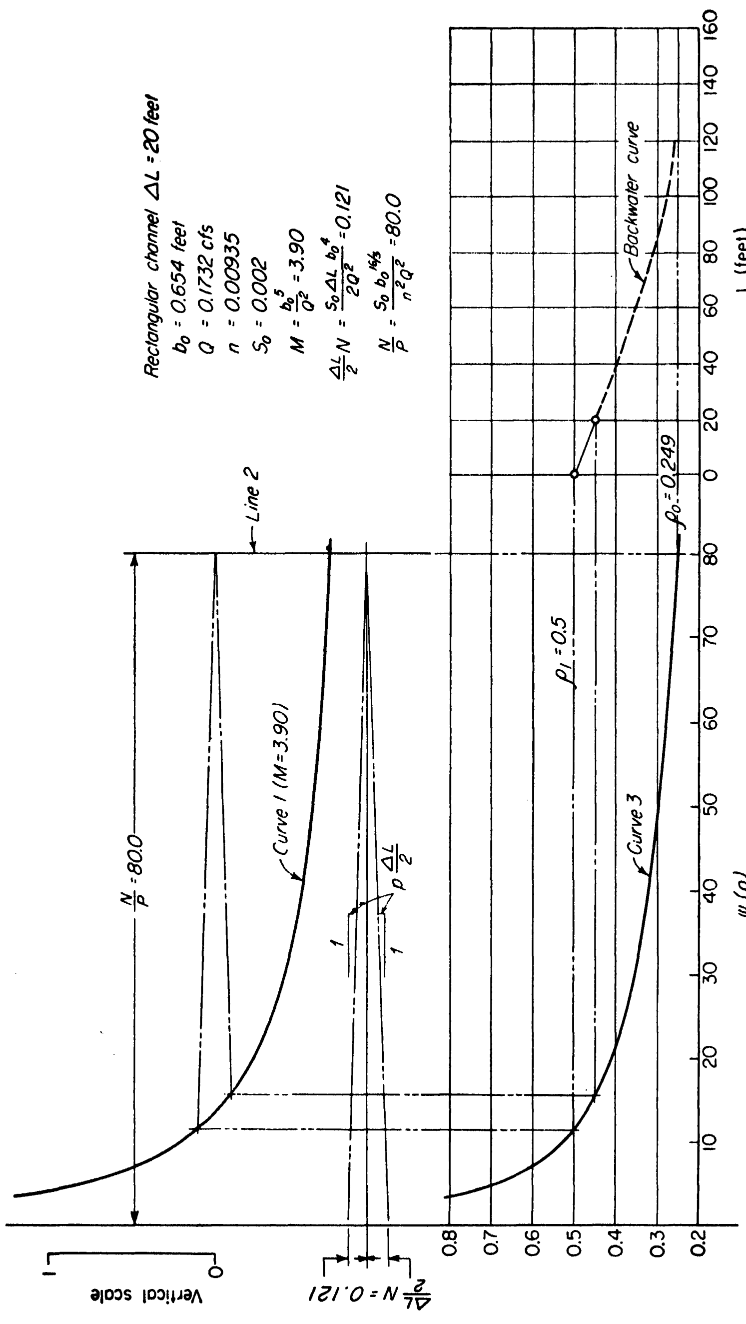

Rectangular channel ΔL = 20 feet
b_0 = 0.654 feet
Q = 0.1732 cfs
n = 0.00935
S_0 = 0.002
$M = \frac{b_0^5}{Q^2} = 3.90$
$\frac{\Delta L}{2} N = \frac{S_0 \Delta L\, b_0^4}{2Q^2} = 0.121$
$\frac{N}{P} = \frac{S_0\, b_0^{16/3}}{n^2 Q^2} = 80.0$
Line 2
$\frac{N}{P} = 80.0$
Curve 1 (M = 3.90)
$p \frac{\Delta L}{2}$
Curve 3
Backwater curve
$\rho_1 = 0.5$
$\rho_0 = 0.249$
L (feet)
ψ (ρ)
Vertical scale
$\frac{\Delta L}{2} N = 0.121$

FIG. 12.

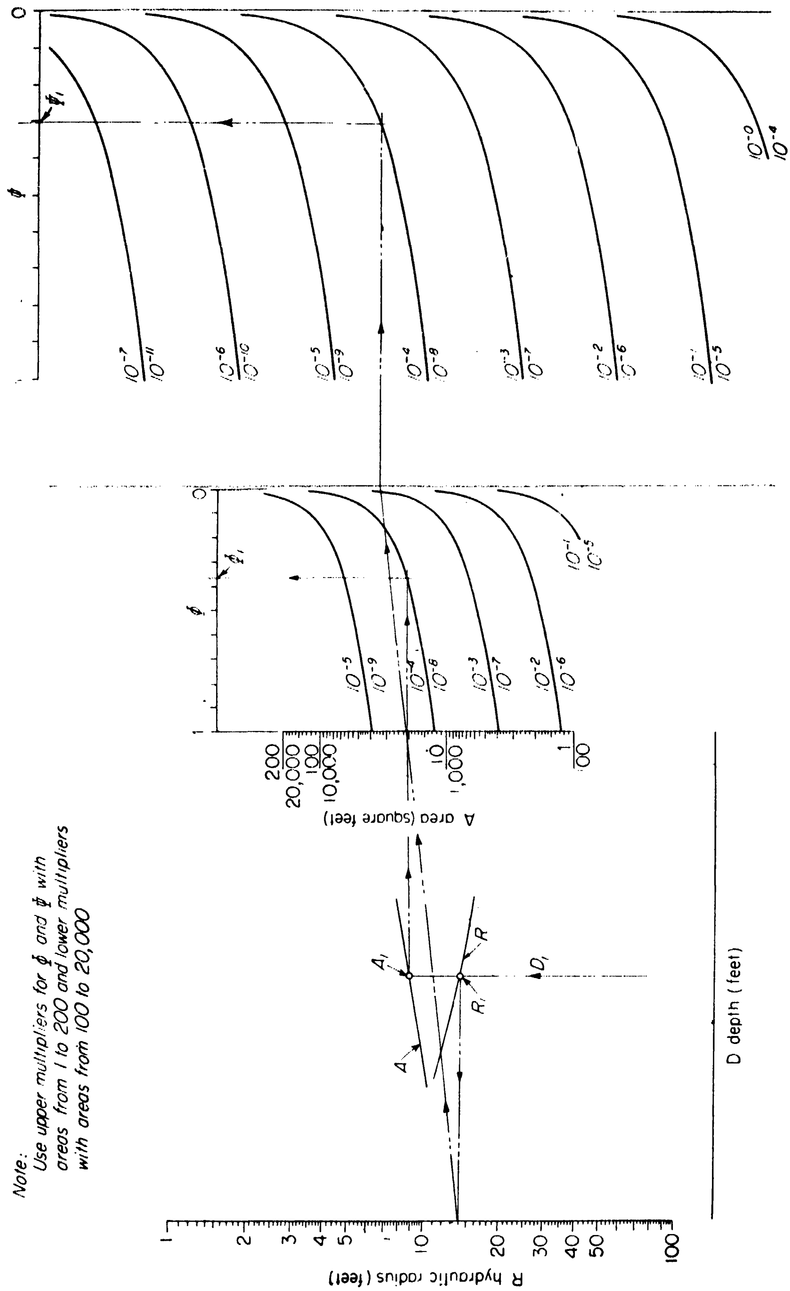

Note:
Use upper multipliers for ϕ and ψ with areas from 1 to 200 and lower multipliers with areas from 100 to 20,000
R hydraulic radius (feet)
A area (square feet)
D depth (feet)
ϕ
ψ_1
A_1
R_1
D_1
A
R
10^{-7} 10^{-11}
10^{-6} 10^{-10}
10^{-5} 10^{-9}
10^{-4} 10^{-8}
10^{-3} 10^{-7}
10^{-2} 10^{-6}
10^{-1} 10^{-5}
10^{-0} 10^{-4}

FIG. 13.

in which

$$\Phi(D) = \frac{1}{2gA^2} \tag{16b}$$

$$\Psi(D) = \frac{1}{2.2082A^2R^{4/3}} \tag{16c}$$

$$M' = \frac{1}{Q^2} \tag{16d}$$

$$N' = \frac{S_0}{Q^2} \tag{16e}$$

$$P' = n^2 \tag{16f}$$

The graphical solution of Eq. (16) is then the same as for Eq. (15). Values of $\Phi(D)$ and $\Psi(D)$ can be calculated directly or the chart of Fig. 13 may be used. Figure 14 shows the graphical solution of a backwater curve in a trapezoidal channel for which curves 1 and 3 were plotted from the functions obtained from Fig. 13.

Surge Tanks. For the surge-tank problem as depicted by Fig. 15, the following differential equations govern:

$$\frac{L}{gA_c}\frac{dQ}{dt} = -(y + H) \tag{17}$$

$$A_t\frac{dy}{dt} = Q - I \tag{18}$$

These equations may be solved graphically, if the following assumptions are made:

$$y_2 = y_1 + \frac{dy}{dt}\cdot dt$$

$$y = \tfrac{1}{2}(y_1 + y_2)$$

$$Q_2 = Q_1 + \frac{dQ}{dt}\cdot dt$$

$$Q = \tfrac{1}{2}(Q_1 + Q_2)$$

$I = \bar{I}$ = average flow to turbines during period dt. Equations (17) and (18) may then be put into the form

$$\left(\frac{L}{gA_c} + \frac{dt^2}{4A_t}\right)\frac{dQ}{dt} + H = -y_1 - (Q_1 - \bar{I})\frac{dt}{2A_t} \tag{19}$$

and further, if

$$\Delta T = dt$$

$$M = \frac{1}{\dfrac{L}{gA_c} + \dfrac{\Delta T^2}{4A_t}}$$

$$N = \frac{\Delta T}{2A_t}$$

then

$$\frac{dQ}{M\,dt} + H = -y_1 - N(Q_1 - \bar{I}) \tag{20}$$

and

$$y_2 = y_1 + N(Q_1 - \bar{I}) + N(Q_2 - \bar{I}) \tag{21}$$

Equation (20) is of the form of Eq. (1) and can be solved in a similar manner.

For the specific problem of instantaneous and complete reject of load by the turbine $\bar{I} = 0$ and Eqs. (20) and (21) become

$$\frac{dQ}{M\,dt} + H = -y_1 - NQ_1 \tag{22}$$

and

$$y_2 = y_1 + N(Q_1 + Q_2) \tag{23}$$

The graphical solution of these equations is shown on Fig. 16. Curve 1 is plotted with ordinates of Q and abscissas of H. Line 2 is drawn at slope of 1 to 1 through the

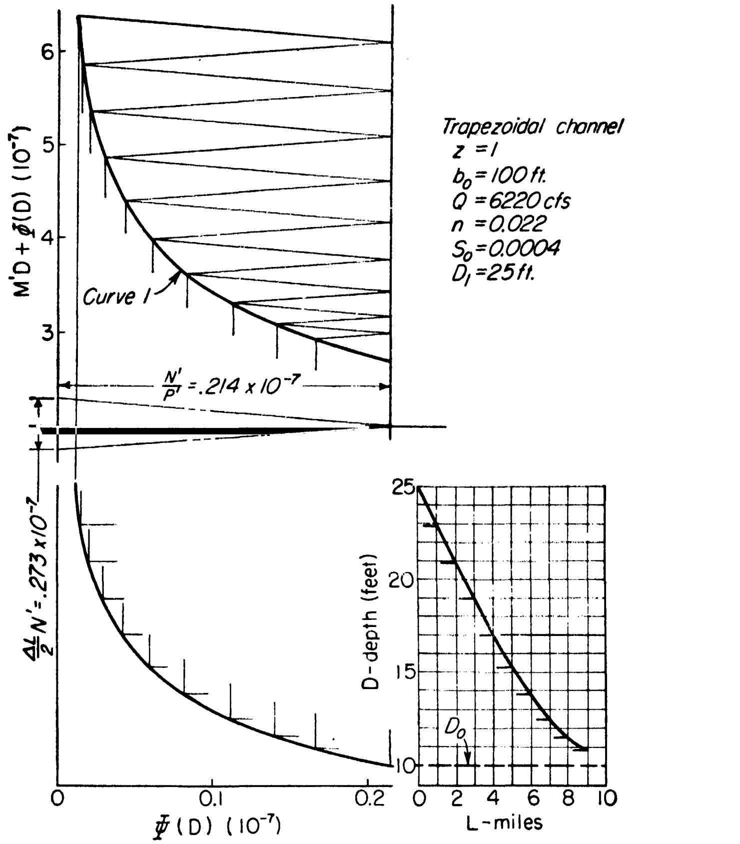

Trapezoidal channel
z = 1
b_0 = 100 ft.
Q = 6220 cfs
n = 0.022
S_0 = 0.0004
D_1 = 25 ft.
Curve 1
$\frac{N'}{P'}$ = .214 x 10^{-7}
$\frac{\Delta L}{2} N'$ = .273 x 10^{-7}
M'D + ϕ(D) (10^{-7})
Ψ(D) (10^{-7})
D-depth (feet)
L-miles
D_0

FIG. 14.

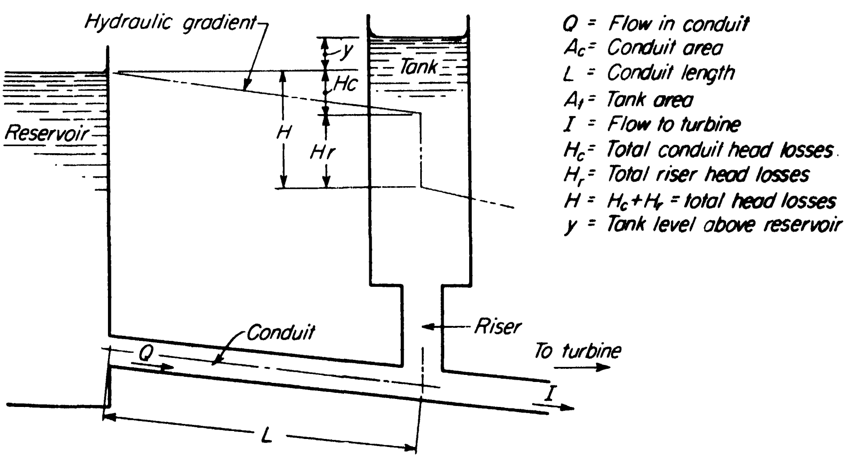

Hydraulic gradient
Reservoir
Tank
y
Hc
H
Hr
Conduit
Riser
To turbine
Q
I
L
Q = Flow in conduit
A_c = Conduit area
L = Conduit length
A_t = Tank area
I = Flow to turbine
H_c = Total conduit head losses
H_r = Total riser head losses
H = H_c + H_r = total head losses
y = Tank level above reservoir

FIG. 15.

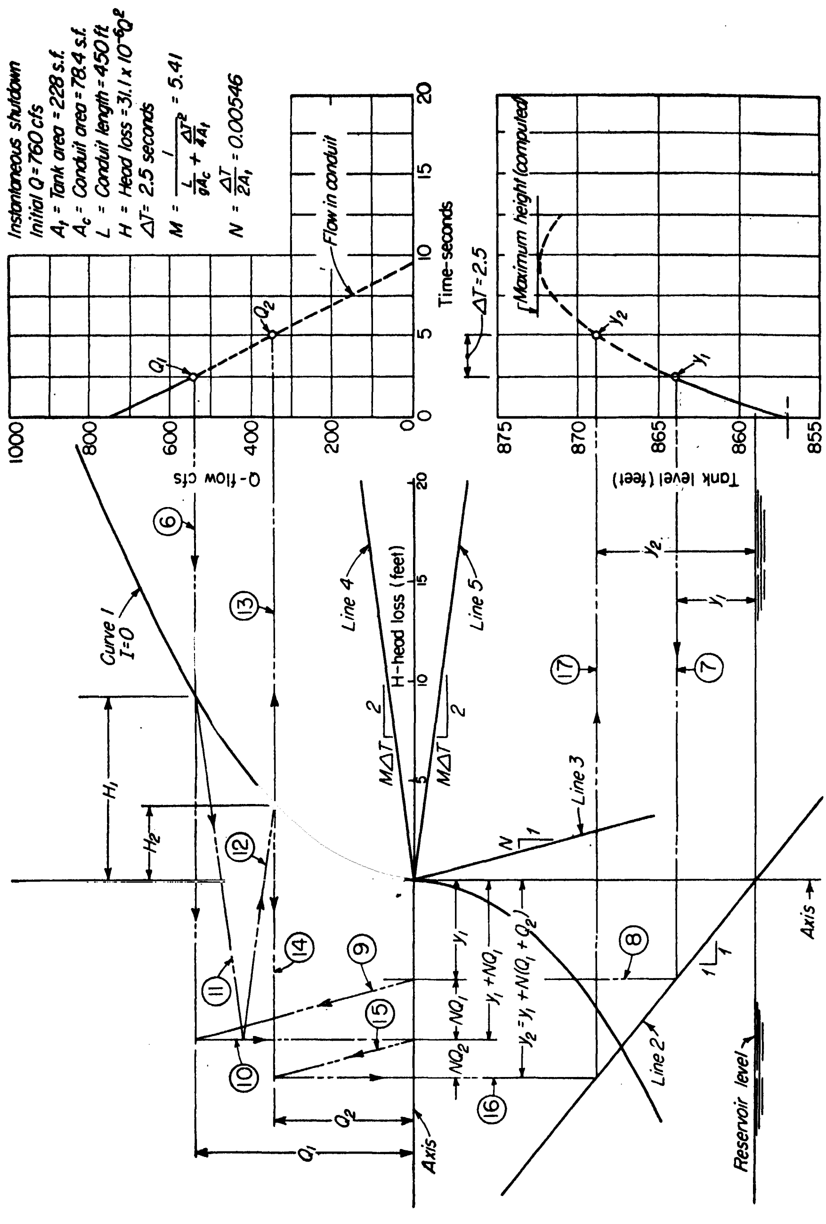

Instantaneous shutdown
Initial Q = 760 cfs
A_t = Tank area = 228 s.f.
A_c = Conduit area = 78.4 s.f.
L = Conduit length = 450 ft
H = Head loss = $31.1 \times 10^{-6} Q^2$
ΔT = 2.5 seconds
$M = \frac{1}{\frac{L}{gA_c} + \frac{\Delta T^2}{4A_t}} = 5.41$
$N = \frac{\Delta T}{2A_t} = 0.00546$
Flow in conduit
Time-seconds
Maximum height (computed)
ΔT=2.5
Q-flow cfs
Tank level (feet)
H-head loss (feet)
Curve I
I=0
Line 4
Line 5
Line 3
Line 2
$M\Delta T / 2$
$y_2 = y_1 + N(Q_1 + Q_2)$
$y_1 + NQ_1$
Reservoir level
Axis

FIG. 16.

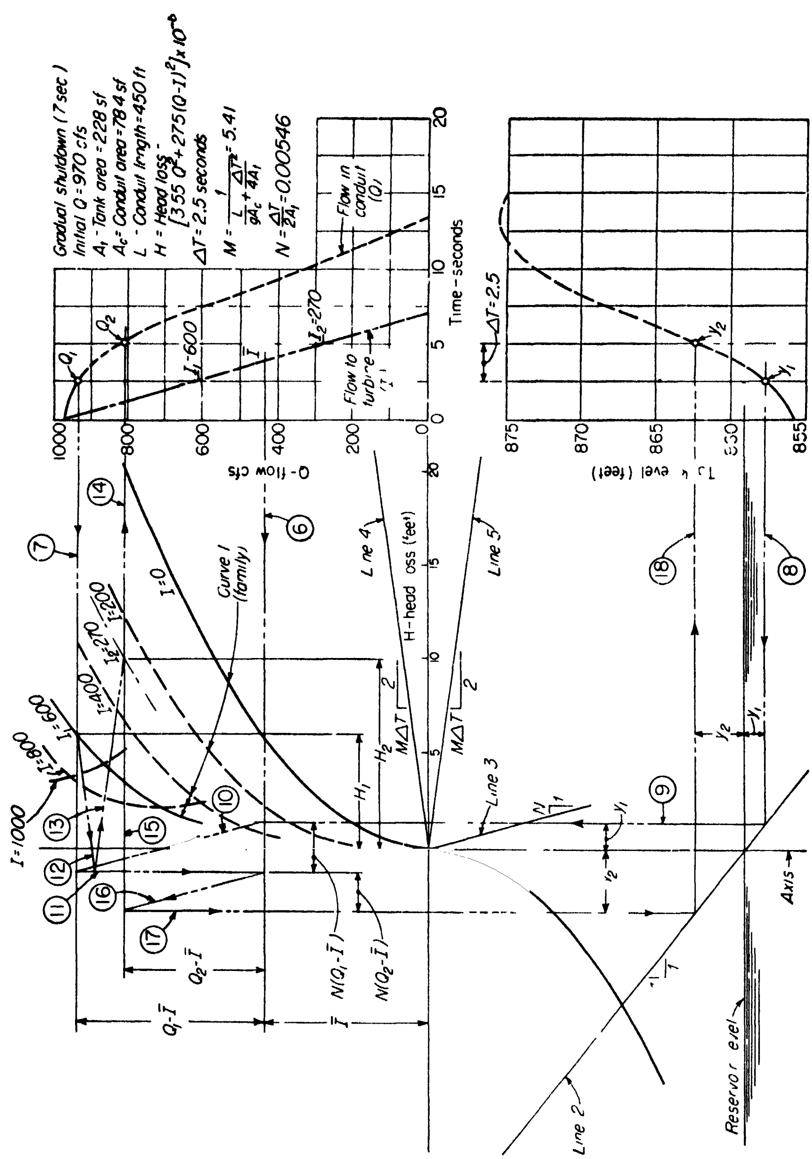

Gradual shutdown (7 sec)
Initial Q = 970 cfs
A_1 - Tank area = 228 sf
A_c = Conduit area = 78.4 sf
L - Conduit length = 450 ft
H = Head loss = $[3.55\,Q^2 + 2.75(Q-I)^2] \times 10^{-6}$
ΔT = 2.5 seconds
$M = \dfrac{1}{\frac{L}{gA_c} + \frac{\Delta T^2}{4A_1}} = 5.41$
$N = \dfrac{\Delta T}{2A_1} = 0.00546$
Flow in conduit (Q)
Flow to turbine (I)
Time - seconds
Q - flow cfs
H - head loss (feet)
Curve I (family)
Line 2
Line 3
Line 4
Line 5
Reservoir level
Axis
ΔT = 2.5

FIG. 17.

intersection of the vertical axis with reservoir level ($y = 0$). Line 3 is drawn at slope $1/N$. Lines 4 and 5 are drawn with slopes $+ M\ \Delta T/2$ and $- M\ \Delta T/2$. If y_1 and Q_1 are known, the value of Q_2 can be found as follows:

1. Project line 6 horizontally from Q_1.
2. Project line 7 horizontally from y_1 to Line 2 and line 8 vertically to the horizontal axis.
3. Draw line 9 parallel to line 3, and line 10 vertically downward from the intersection with line 6.
4. From the intersection of line 6 with curve 1, draw line 11 parallel to line 4 until reaching line 10.
5. Draw line 12 parallel to line 5 to intersect curve 1.
6. From the above intersection, draw line 13 horizontally to end of period ΔT, giving the value of Q_2.

To obtain the value of y_2, the following construction is made:

1. Draw line 14 horizontally from Q_2.
2. Draw line 15 parallel to line 3 to intersect line 14.
3. Draw line 16 vertically to line 2 and draw line 17 horizontally to end of period ΔT, giving y_2.

The above steps are then repeated for the next interval, ΔT, using new values of y_1 and Q_1.

In cases of gradual reject or acceptance of load by the turbines, Eqs. (20) and (21) must be satisfied. If I is not equal to zero, then H becomes a function of I as well as Q and the problem is similar to that of flood routing wherein wedge storage is considered. The solution is as shown on Fig. 17. Curve 1 becomes a family of curves for which I is a parameter, as H, in general, may be expressed as

$$H = H_c + H_r = C_1Q^2 + C_2(Q - I)^2 \tag{24}$$

Lines 2, 3, 4, and 5 are constructed as for instantaneous shutdown. The steps taken for the solution are as follows:

1. Draw line 6 horizontally from $\bar{I}$ for period ΔT.
2. Project line 7 from Q_1 and lines 8 and 9 from y_1.
3. Draw line 10 parallel to line 3 from the intersection of line 6 and 9.
4. Draw line 11 vertically through the intersection of lines 7 and 10.
5. From the intersection of line 7 with curve 1 corresponding to I_1 draw line 12 parallel to line 4.
6. From the intersection of lines 12 and 11 draw line 13 parallel to line 5 until intersection with curve 1 corresponding to I_2.
7. Draw line 14 horizontally to the right to give Q_2 at end of period ΔT.
8. Draw line 15 horizontally to the left from Q_2.
9. Draw the line 16 parallel to line 3.
10. From the intersection of lines 11 and 16, project line 17 downward to line 2 and line 18 horizontally to give y_2 at end of period ΔT.

The process is then repeated for the next period, ΔT, with new values of y_1 and Q_1.

Much of the foregoing material has been published in Graphical Solution of Hydraulic Problems, *Proc. A.S.C.E.*, Separate No. 116, February, 1952, by Kenneth E. Sorensen.

INDEX

Q

R